Introduction to Cosmological First Order Phase Transition and the Energy Budget

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Spontaneously Symmetry Breaking

Our current Universe is known in a symmetry-broken phase which is triggered by several spontaneously symmetry breakings in the early universe, for example,

- GUT to SM, $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$,
- ~~Inflation (in a sense)~~,
- EW phase transition,
- QCD phase transition...

The symmetry breaking is usually described by (effective) scalar field ϕ, with a potential possessing both a true vacuum and a false vacuum.

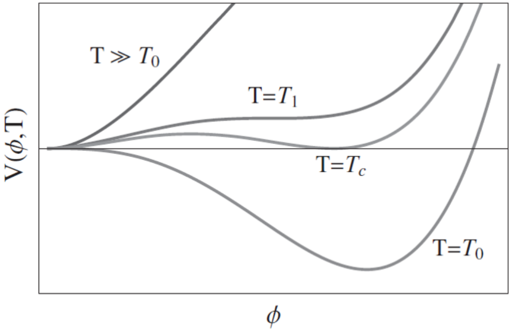


Figure 1: A typical effective potential of scalar field ϕ

Motivation

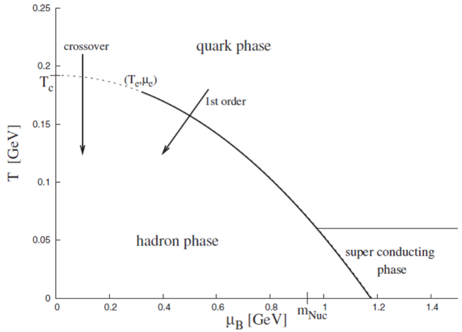


Figure 2: Phase diagram of a QCD system

The study on first order phase transitions can tell us:

- New physics beyond Particle Physics Standard Model (since the SM only gives a cross over but not phase transition),
- electroweak baryogenesis,
- primordial magnetic field,
- primordial black holes,
- stochastic gravitational wave background...

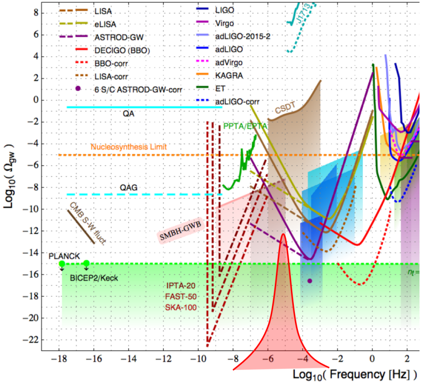


Figure 3: $\Omega_{\rm gw}$ v.s. frequency bands for several GW detectors [1]

$$h_0^2\Omega_{\rm gw} = 1.84\times 10^{-6}\kappa^2\left(\frac{H_*}{\beta}\right)^2\left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{v_b^3}{0.24+v_b^2}\right)\left(\frac{100}{g_*}\right)^{1/3} \tag{1}$$

$$f_{\rm EW} = 3.83\times 10^{-6}{\rm Hz}\left(\frac{\beta}{H_*}\right) \left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{106.75}\right)^{-1/6} g(v_b) \tag{2}$$

with κ, α, β, T_*, g_*, v_b determined by specific particle physics model.[2]

Consider a system with gravity and Higgs field,

$$S = \int_{\mathcal{M}} \mathrm{d}^4 x \sqrt{-g} \left(\frac{\mathcal{R}}{16\pi G} + \mathcal{L}_\phi \right) \tag{3}$$

By performing Wick rotation one could get a classical ϕ field profile like a Euclidean bubble with radius $R = \sqrt{\tau^2 + \mathbf{x}^2} = \sqrt{-t^2 + \mathbf{x}^2}$.

The profiles and expansion of bubble are given below, which lead to "thin-wall" approximation.

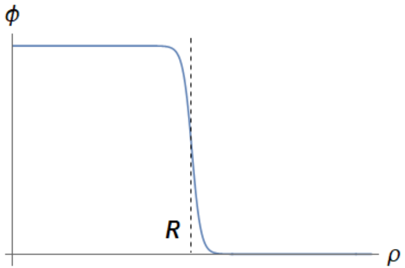


Figure 4: Profile of ϕ in radial direction.

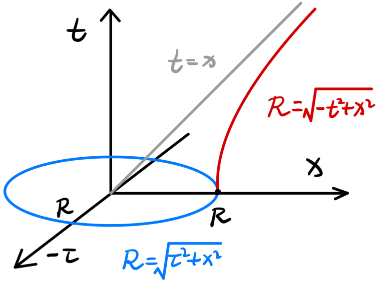
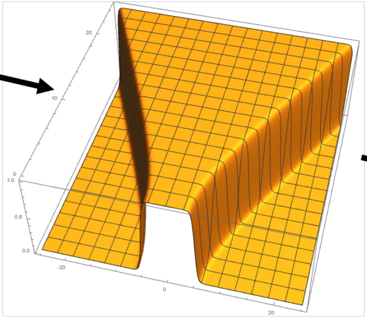


Figure 5: Bubble in Euclidean and Lorentz space-time.

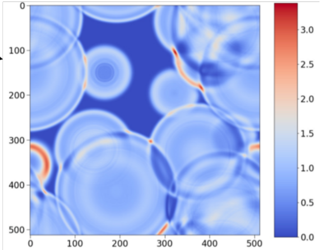
Fate of the bubbles

Bubble Nucleation Through Vacuum Fluctuations



Bubble Expansion

Bubble Collision



But Wait! We forget the other components of the cosmic fluid, which provides a friction force on the bubble wall. The bubbles can be divided into two types by the ability of the backreaction force of balancing the driving force,[3]

- Runaway bubble, which accelerates permanently and expands near the speed of light. The main contribution to SGWB comes from bubble collisions,
- Non-runaway bubble, which reaches a steady expansion. The main contribution to SGWB comes from sound waves,

We focus on the steady expansion of the bubbles. By solving the EoM, we could figure out three types of expansion modes, namely deflagration, hybrid and detonation, depending on how fast the bubble wall moves.

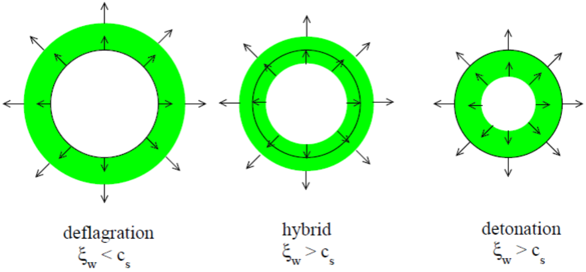


Figure 6: Pictorial representation of expanding bubbles of different types. The black circle is the phase interface (bubble wall). In green we show the region of non-zero fluid velocity[4]

Equation of Motion

The total energy-momentum tensor for the scalar-plasma system

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + p\eta^{\mu\nu} = w u^\mu u^\nu + p\eta^{\mu\nu}. \tag{4}$$

We project the energy-momentum conservation $\nabla_\mu T^{\mu\nu} = 0$ parallel and perpendicular to the bubble wall direction with $u^\mu = \gamma(v)(1, \vec{v})$ and $\tilde{u}^\mu = \gamma(v)(v, \vec{1})$ respectively [4], one can get

$$\begin{aligned} u^\mu \nabla_\mu \rho + w \nabla_\mu u^\mu =& 0, \qquad (5)\\ \tilde{u}^\mu \nabla_\mu p + w \tilde{u}_\nu u^\mu \nabla_\mu u^\nu =& 0. \qquad (6) \end{aligned}$$

Since there is no characteristic distance scale, the resulted featureless bubble expansion is self-similar and the solution of EoM can be expressed in a similarity coordinate $\xi \equiv |\vec{x}(t)|/t$. Rearrange the above projected equations in the similarity coordinate,

$$2\frac{v}{\xi} = \gamma^2(1-\xi v)\left(\frac{\mu(\xi,v)^2}{c_s^2}-1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}, \tag{7}$$

$$\frac{\mathrm{d}w}{\mathrm{d}\xi} = w\gamma^2\mu(\xi,v)\left(\frac{1}{c_s^2}+1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}, \tag{8}$$

where $c_s^2 = \mathrm{d}p(T)/\mathrm{d}\rho(T) = 1/3$ given by proper Equation-of-State and

$$\mu(\xi,v) = \frac{\xi - v}{1-\xi\cdot v} \tag{9}$$

Once the solution of EoM for the fluid velocity profile $v(\xi)$ is obtained, one can calculate the enthalpy profile from Eq. (8) as

$$w(\xi) = w(\xi_0) \exp\left[\int_{v(\xi_0)}^{v(\xi)} \left(\frac{1}{c_s^2} + 1\right) \gamma^2 \mu(\xi, v) \mathrm{d}v\right]. \tag{10}$$

And from $w = T\partial_T p$, the temperature profile can be obtained from EoM Eq. (6) as

$$T(\xi) = T(\xi_0) \exp\left[\int_{v(\xi_0)}^{v(\xi)} \gamma^2 \mu(\xi, v) \mathrm{d}v\right]. \tag{11}$$

With no boundary conditions, the solution family is shown below.

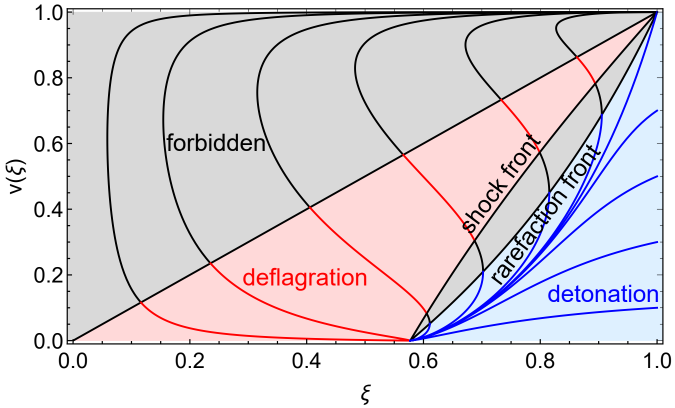


Figure 7: Solutions of EoM with $c_s^2 = 1/3$.

Junction Conditions

We still need the boundary conditions to connect symmetric phase and broken phase so that the EoM can be solved. Apply conservation law $\nabla_\mu T^{\mu\nu} = 0$ across the bubble wall one get $(T_+^{z\nu} - T_-^{z\nu})n_\nu = 0$, $(T_+^{t\nu} - T_-^{t\nu})n_\nu = 0$ with $n_\mu = (0,0,0,1)$, i.e.

$$w_+\bar{v}_+\bar{\gamma}_+^2 = w_-\bar{v}_-\bar{\gamma}_-^2, \tag{12}$$

$$w_+\bar{v}_+^2\bar{\gamma}_+^2 + p_+ = w_-\bar{v}_-^2\bar{\gamma}_-^2 + p_-, \tag{13}$$

The $+$ and $-$ subscript denote values in the front and back of the bubble wall respectively. The overbar denotes velocity measured in the bubble wall rest frame,

$$\bar{v}_\pm = \frac{\xi_w - v_\pm}{1 - \xi_w \cdot v_\pm} = \mu(\xi_w, v_\pm). \tag{14}$$

Some efforts to go beyond bag EoS are made [5, 6, 7, 8, 9], by introducing a specific EoS called ν-model,

$$p = c_s^2 a T^\nu - \epsilon, \quad \rho = a T^\nu + \epsilon, \tag{15}$$

where the square sound velocity $c_s^2 \equiv 1/(\nu - 1)$ as a constant in a given phase (symmetric or broken phase). With $\nu = 4$, it returns bag EoS,

$$p = \frac{1}{3} a T^4 - \epsilon, \quad \rho = a T^4 + \epsilon. \tag{16}$$

However, the sound velocity defined by $c_s^2 = \mathrm{d}p(T)/\mathrm{d}\rho(T)$ is not a constant even in the same phase but a function of T.

1. Symmetry Breaking
2. Bubbles
3. Hydrodynamics
4. **Modifications**
   - Thermodynamics
   - Numerical Results
5. Summary

We start with the effective potential [10, 11], $V_{\text{eff}}(\phi, T) = V_0(\phi) + V_T(\phi, T)$, with zero-temperature contribution V_0 and finite-temperature contribution V_T. To the one-loop order, V_T reads

$$V_T^{1-\text{loop}} = \sum_{i=\text{B,F}} \pm g_i T \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \ln\left[1 \mp e^{-\frac{\sqrt{\vec{k}^2+m_i^2}}{T}}\right] \equiv \frac{T^4}{2\pi^2} \sum_{i=\text{B,F}} g_i J_i\left(\frac{m_i^2}{T^2}\right), \quad (17)$$

where g_i is the number of degrees of freedom for particle species i. The integrals function reads

$$J_{\text{B/F}}(x) = \pm \int_0^\infty \mathrm{d}y \; y^2 \ln(1 \mp e^{-\sqrt{x+y^2}}). \quad (18)$$

The main contribution to V_T comes from the particle species with $m_i < T$, in which case the integrals can be expanded [12] as

$$J_{\rm B}\left(\frac{m_i^2}{T^2}\right) = -\frac{\pi^2}{90} + \frac{1}{24}\left(\frac{m_i}{T}\right)^2 - \frac{1}{12\pi}\left(\frac{m_i}{T}\right)^3 + \mathcal{O}\left(\frac{m_i^4}{T^4}\right), \tag{19}$$

$$J_{\rm F}\left(\frac{m_i^2}{T^2}\right) = -\frac{7}{8}\frac{\pi^2}{90} + \frac{1}{48}\left(\frac{m_i}{T}\right)^2 + \mathcal{O}\left(\frac{m_i^4}{T^4}\right), \tag{20}$$

for bosons and fermions respectively. To the leading order, this simply returns bag EoS,

$$V_T^{m_i\ll T}(\phi,T) = -\frac{1}{3}\frac{\pi^2}{30}\left(\sum_{i=\rm B} g_i + \frac{7}{8}\sum_{i=\rm F} g_i\right)T^4 \equiv -\frac{1}{3}\frac{\pi^2}{30}g_{\rm eff}T^4 \equiv -\frac{1}{3}\rho_{\rm rad} \equiv -p_{\rm rad}. \tag{21}$$

The thermal quantities can be evaluated by the effective potential as the identification of the free energy density, to the linear order in temperature

$$\mathcal{F} = V_{\rm eff}(\phi, T) = V_0(\phi) - \frac{1}{3}aT^4 + bT^2 - cT, \tag{22}$$

$$p = -\mathcal{F} = -V_0(\phi) + \frac{1}{3}aT^4 - bT^2 + cT, \tag{23}$$

$$\rho = T\partial_T p - p = V_0(\phi) + aT^4 - bT^2. \tag{24}$$

where the coefficients read

$$a = \sum_{i=\mathrm{B}} g_i + \frac{7}{8}\sum_{i=\mathrm{F}} g_i, \quad b = \frac{1}{24}\left(\sum_{i=\mathrm{B}} g_i m_i^2 + \frac{1}{2}\sum_{j=\mathrm{F}} g_j m_j^2\right), \quad c = \frac{1}{12\pi}\sum_{i=\mathrm{B}} g_i m_i^3. \tag{25}$$

Then the sound velocity gains a deviation from $1/3$,

$$c_s^2 = \frac{1}{3}\left(1 - \frac{4bT - 3c}{12aT^3 - 6bT}\right) = \frac{1}{3} - \frac{b}{3aT^2} + \frac{c}{4aT^3} - \frac{b^2}{6a^2T^4} + \mathcal{O}(T^{-5}). \tag{26}$$

In the phase where symmetry is not broken, almost all the particles are massless, thus $b_+ = c_+ = 0$ and the bag EoS is kept, i.e. $c_{s+}^2 = 1/3$, where the subscript $+$ denotes the quantities in the symmetric phase.

Actually one can expand Eq. (22) to higher order as

$$\mathcal{F} = V_{\text{eff}} = V_0(\phi) - \frac{1}{3}aT^4 + bT^2 - cT - d\ln\frac{T}{T_c} + \mathcal{O}(T^{-1}), \tag{27}$$

with

$$d = \frac{1}{32\pi^2}\left(-\sum_{i=\text{B}} g_i m_i^4 + \sum_{j=\text{F}} g_j m_j^4\right). \tag{28}$$

We have numerically shown that this term's contribution is too small to affect our result.

Now we need to solve the EoM (7),

$$2\frac{v}{\xi} = \gamma^2(1-\xi v)\left(\frac{\mu(\xi,v)^2}{c_s^2(T)} - 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}$$

with c_s^2 as a function of temperature which is determined by (11),

$$T(\xi) = T(\xi_0)\exp\left[\int_{v(\xi_0)}^{v(\xi)} \gamma^2\mu(\xi,v)\mathrm{d}v\right].$$

Solving Method

There are two methods to solve this equation.

1. Numerically direct solve this nonlinear integro-differential equation, if we know both the initial value of velocity and temperature.
2. Iterations. Noticing the deviation of c_s^2 from $1/3$ is small, one can solve the equation with $c_s^2 = 1/3$ to get 0-order profiles. Apply i-order profiles to compute new c_s^2 profiles and plug it into EoM to get $(i+1)$-order result. We only need to know the initial value of velocity.

Velocity Profiles: Hybrid As an Example

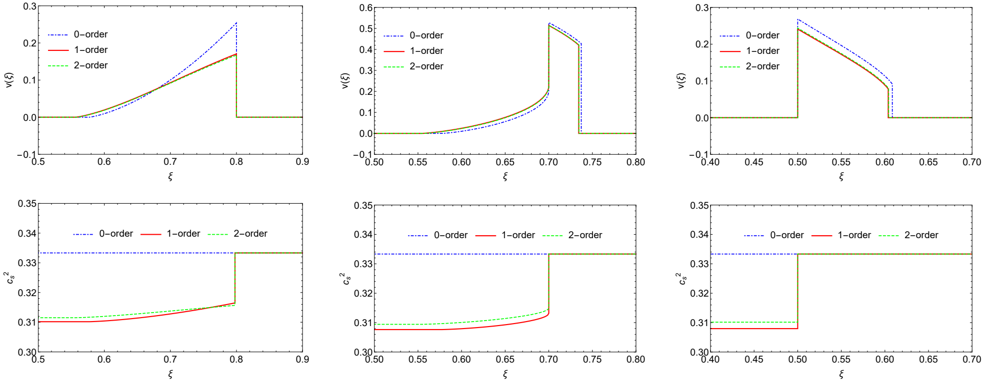


Figure 8: First 3 orders fluid and sound velocity profiles. We choose $\alpha_+ = 0.1$, $a_+/a_- = 1.2$, $b_-/(a_- T_N^2) = 2/25$, $c_-/(a_- T_N^3) = 2/125$.

Enthalpy and Temperature Profiles

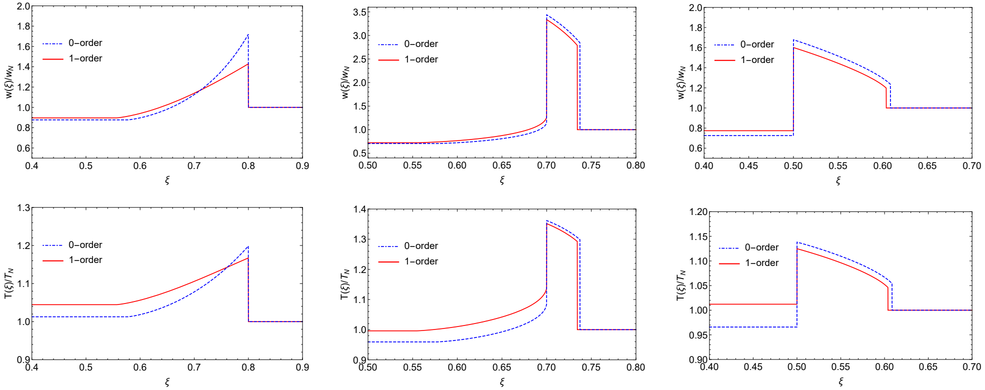


Figure 9: First 2 orders enthalpy and temperature profiles. We choose $\alpha_+ = 0.1$, $a_+/a_- = 1.2$, $b_-/(a_- T_N^2) = 2/25$, $c_-/(a_- T_N^3) = 2/125$.

Define efficiency factor as the ratio of bulk kinetic energy over the vacuum energy

$$\kappa_v = \int w(\xi) v^2 \gamma^2 \; 4\pi\xi^2 \mathrm{d}\xi \bigg/ \left(\frac{4\pi}{3} \Delta V_0 \cdot \xi_w^3 \right) = \frac{4}{\alpha_N \xi_w^3} \int_0^1 \frac{w(\xi)}{w_N} v^2 \gamma^2 \xi^2 \mathrm{d}\xi, \tag{29}$$

where α_N is the phase transition strength factor in far front of the bubble wall,

$$\alpha_N = \frac{\Delta V_0}{a_N T_N^4} = \frac{3\Delta V_0}{4 w_N}. \tag{30}$$

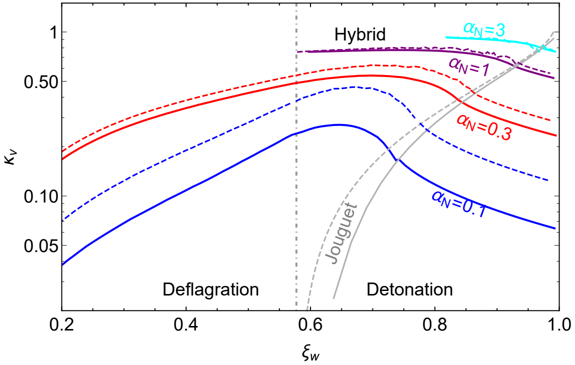


Figure 10: The numerical result of efficiency factors as a function of bubble wall velocity ξ_w, with parameters $a_+/a_- = 1.2$, $b_-/(a_- T_N^2) = 2/25$, $c_-/(a_- T_N^3) = 2/125$. The solid and dashed curves are efficiency factors under and beyond bag EoS respectively.

There are mainly three conclusions in our work:

1. A particle physics model independent method is provided to analyse FOPT beyond bag EoS from a hydrodynamics point of view.
2. A lower bound for phase transition strength factor α_N is found for steady expansion bubbles.
3. Fluid velocity and thermal quantities profiles are got from solving EoM. As a consequence efficiency factors are evaluated. The efficiency factor beyond bag EoS becomes smaller than that under bag EoS.

Thanks!

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