Cosmological First-order Phase Transition and Gravitational Waves

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Based on : 2206.01148, 2305.00074, and recent works

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Contents

Symmetry breaking

Our current Universe is known in a symmetry-broken phase which is triggered by several spontaneously symmetry breakings in the early universe, for example,

- GUT to SM, $SU(5) \to SU(3) \times SU(2) \times U(1)$
- ~~Inflation (in a sence)~~
- Electro-weak phase transition, $SU(2) \times U(1) \to U(1)_{\rm em}$
- QCD phase transition…

Leading to various topological defects in the early universe:
Phase transitions, domain walls, cosmic strings, monopoles, textures

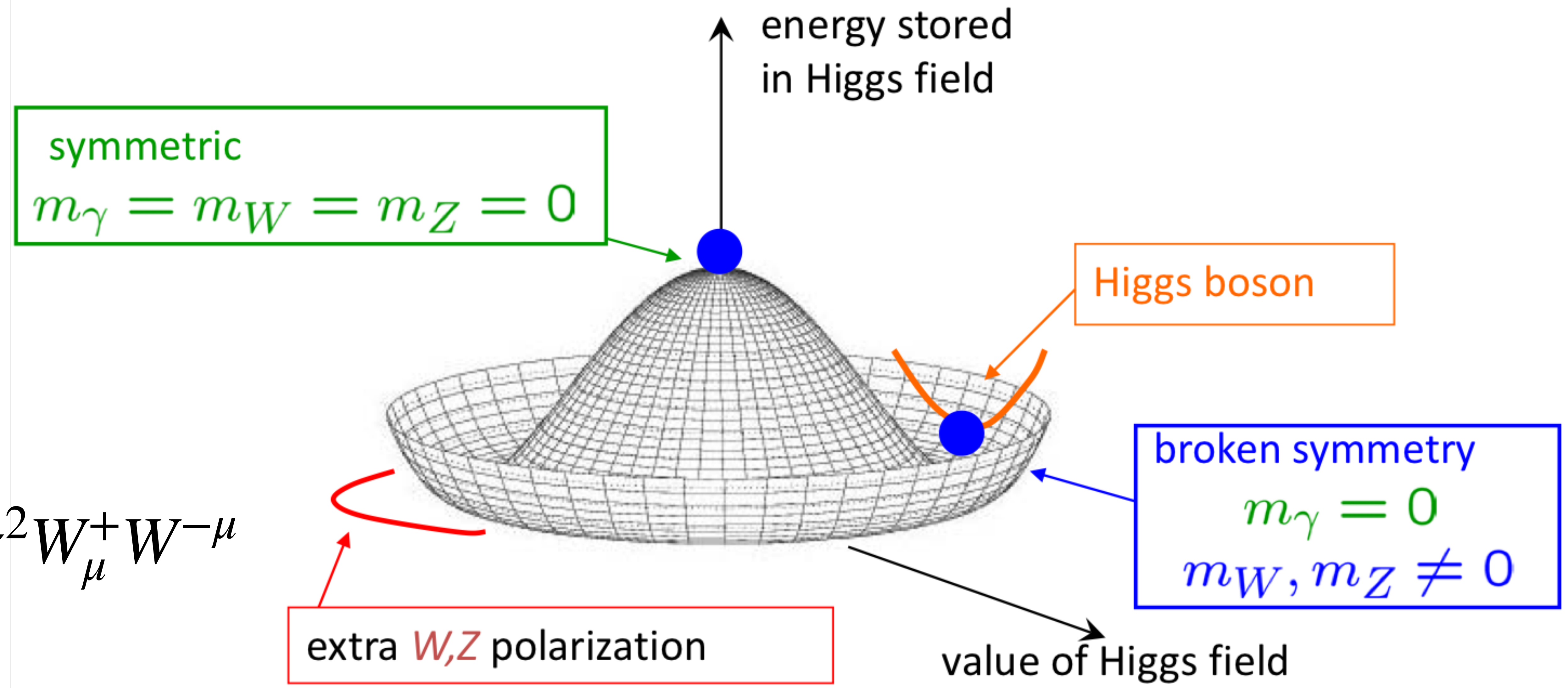
Higgs Mechanism

EW phase transition as an example $SU(2) \times U(1) \rightarrow U(1)_{\mathrm{em}}$

$$\mathscr{L}_\varphi = -\frac{1}{2}\left(\mathbf{D}^\mu \varphi\right)^\dagger \left(\mathbf{D}_\mu \varphi\right) - V\left(|\varphi|^2\right)$$

$$\Big\downarrow\ \chi = |\varphi|$$

$$\mathscr{L}_\varphi = -\frac{1}{2}\partial^\mu \chi \partial_\mu \chi - V(\chi) + \frac{g^2 + g'^2}{8}\chi^2 Z^\mu Z_\mu + \frac{g^2}{4}\chi^2 W^+_\mu W^{-\mu}$$



Gauge bosons gain masses except for photons

Phenomenology

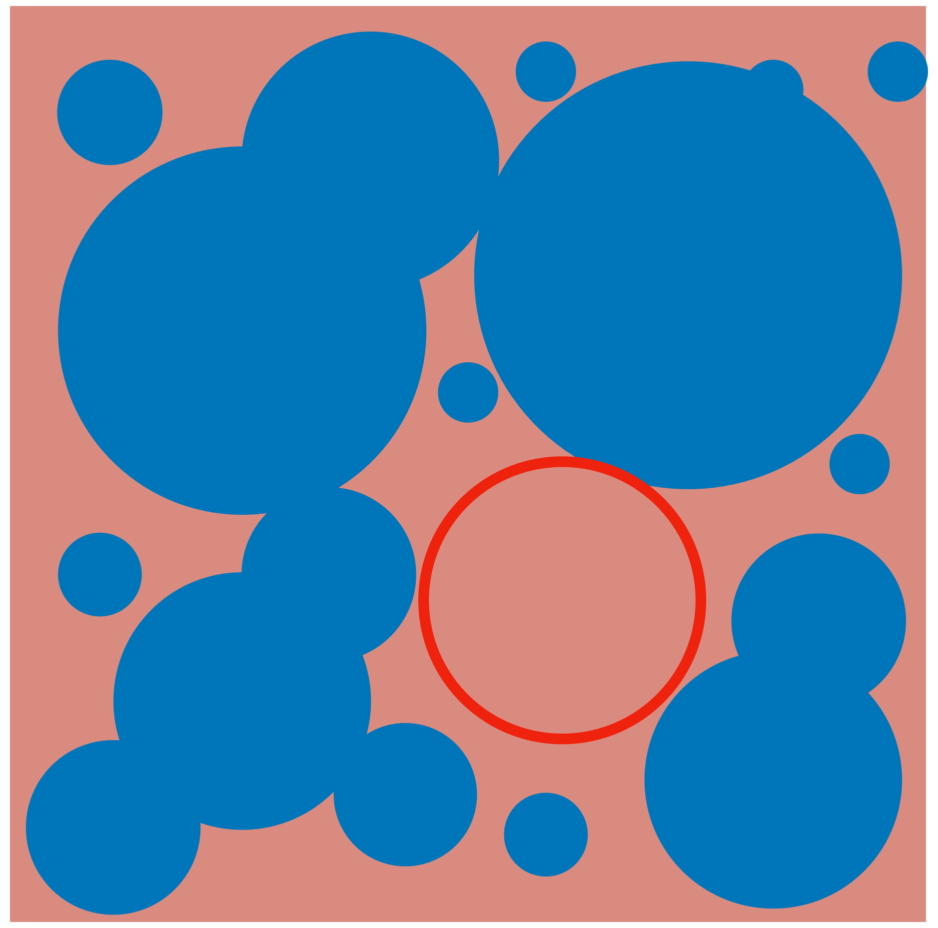
The study on first order phase transitions can tell us:

- New physics beyond Particle Physics Standard Model (since the SM only gives a cross over but not phase transition),
- electroweak baryogenesis,
- primordial black holes (curvature perturbations),
- primordial magnetic field,
- stochastic gravitational wave background…

Sakharov Three Conditions

1. Baryon number violation
2. CP violation
3. Thermal non-equilibrium

Curvature perturbations OR



Bubble collisions

Bubble remnants

…

to form PBHs

Phenomenology

Kuroda et al., arXiv:1511.00231

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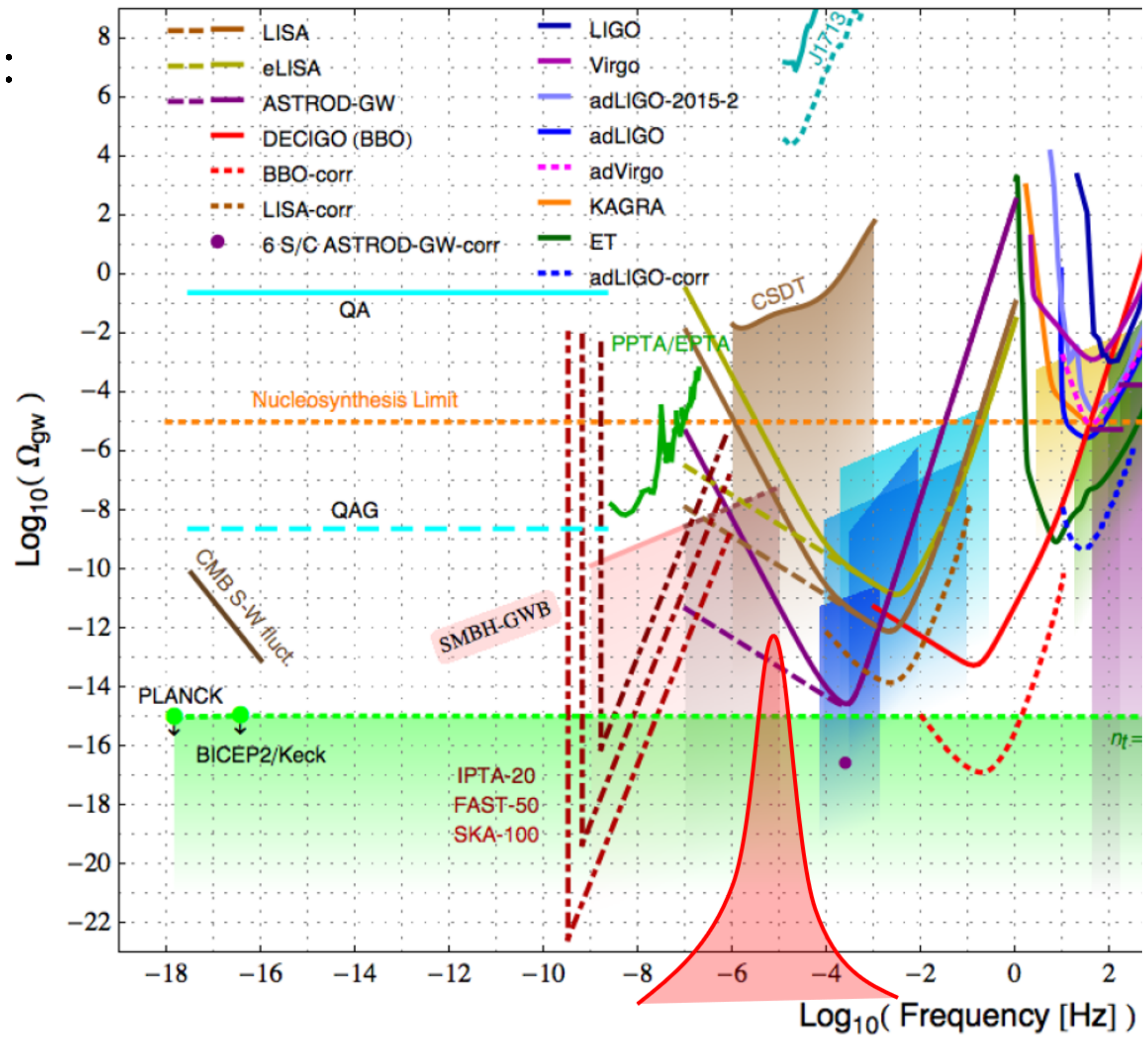
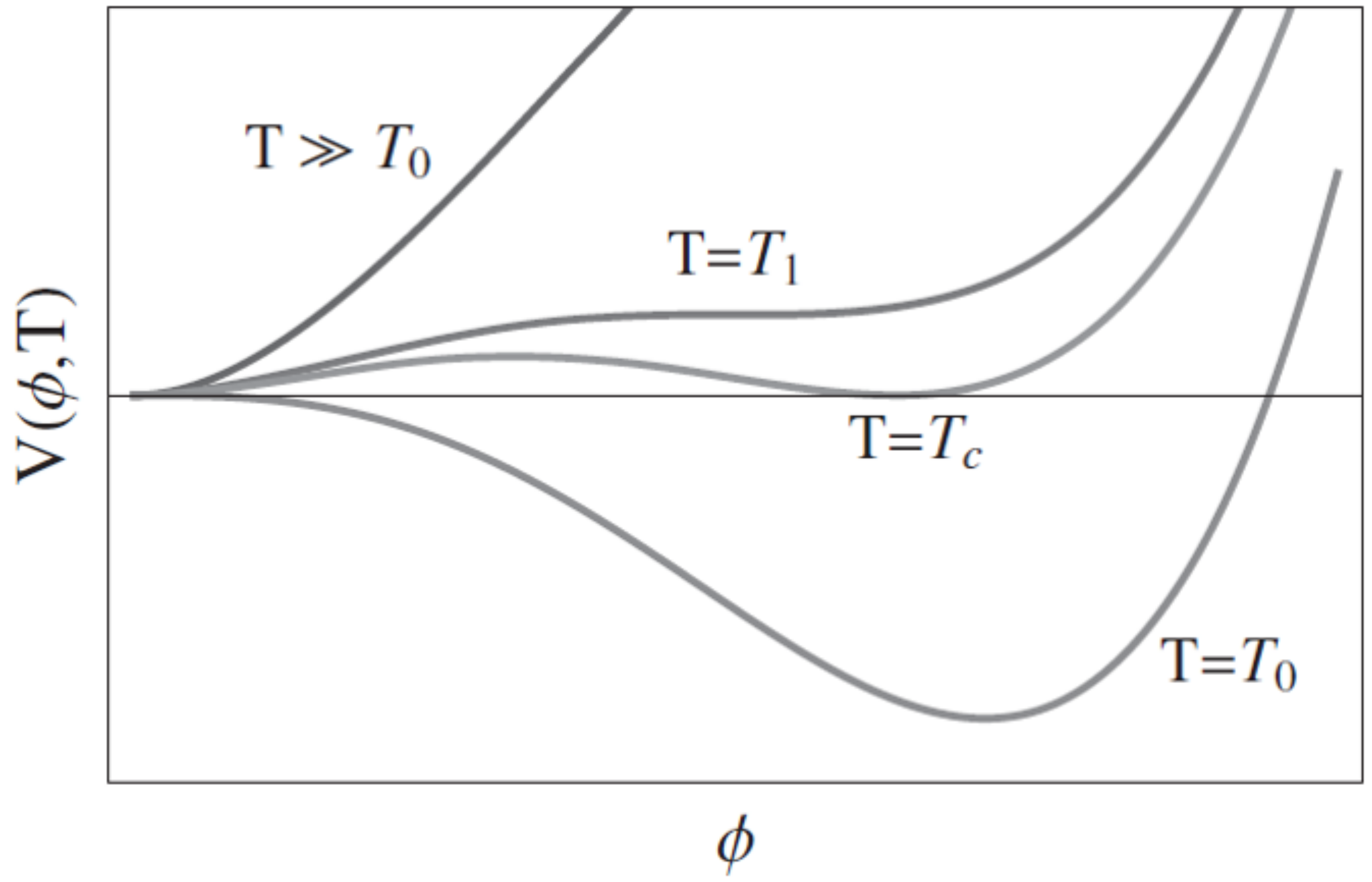
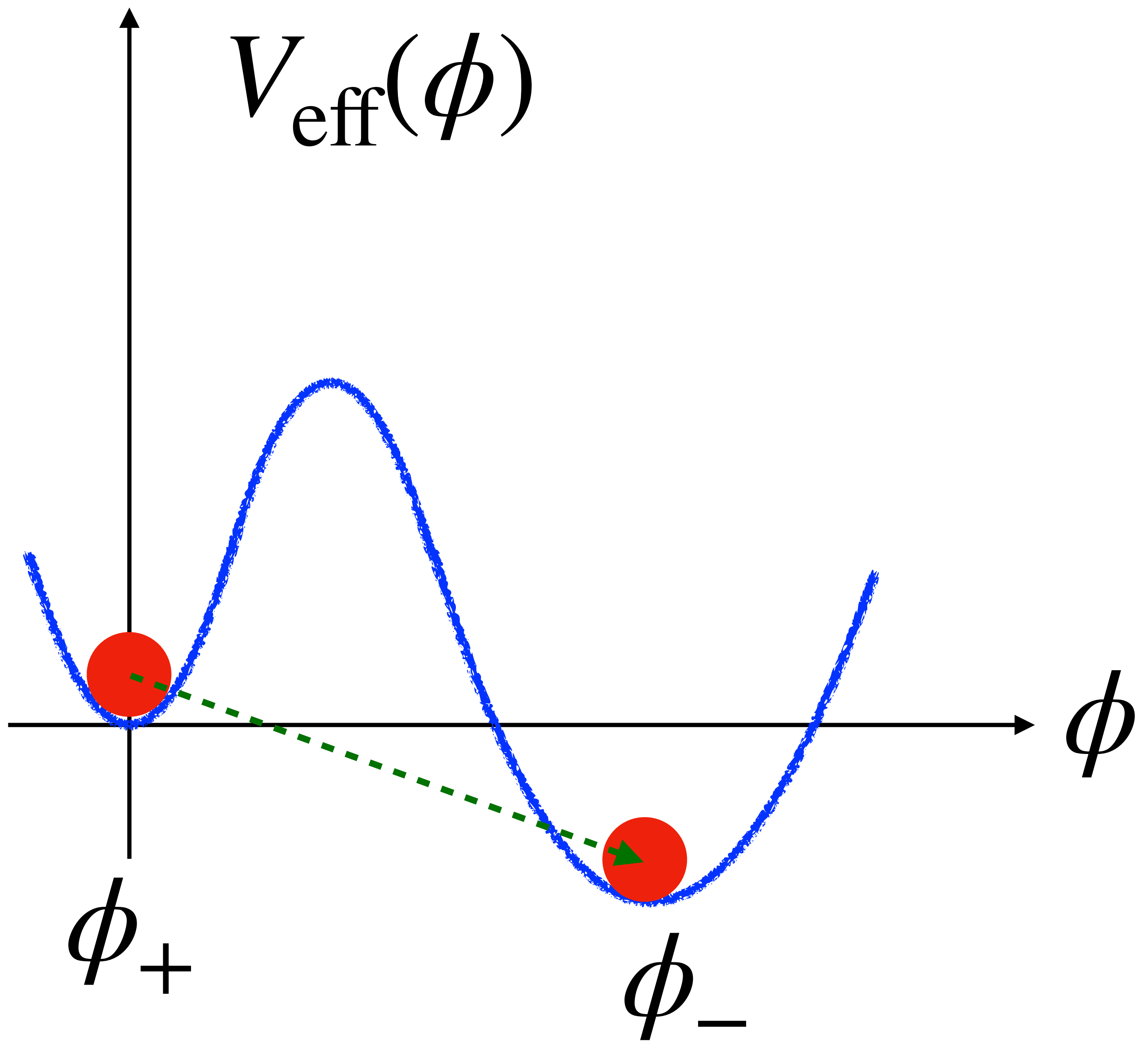


Quantum tunneling

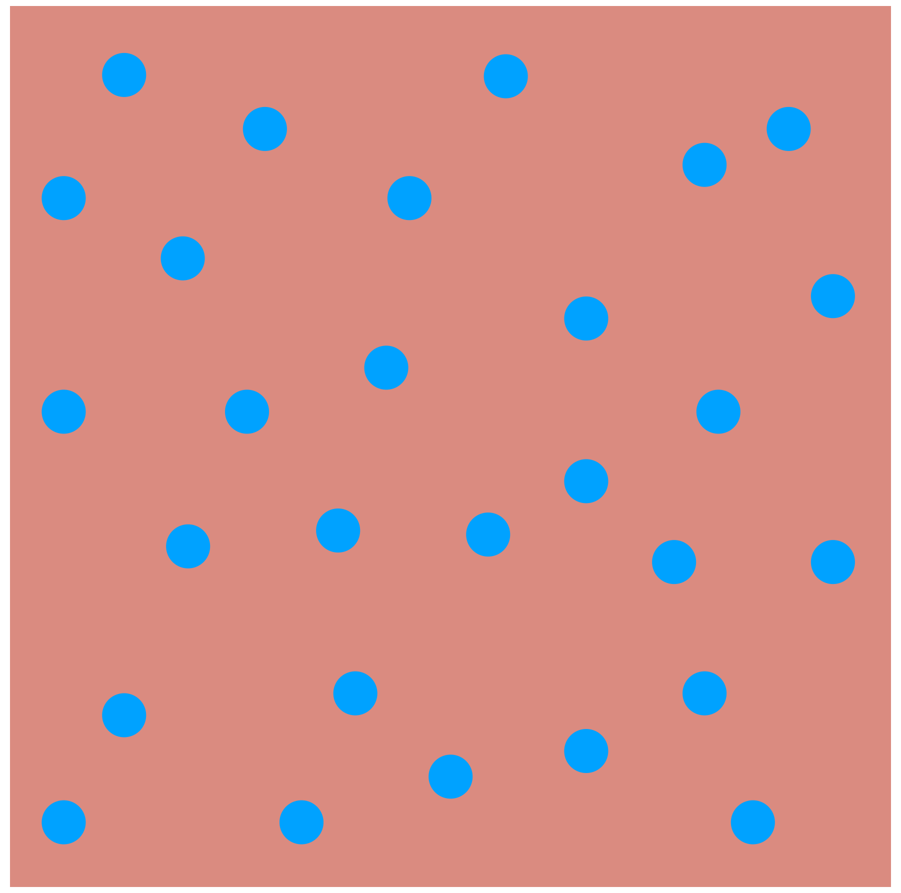
The symmetry breaking is usually described by (effective) scalar field ϕ, with a potential possessing both a true vacuum and a false vacuum.



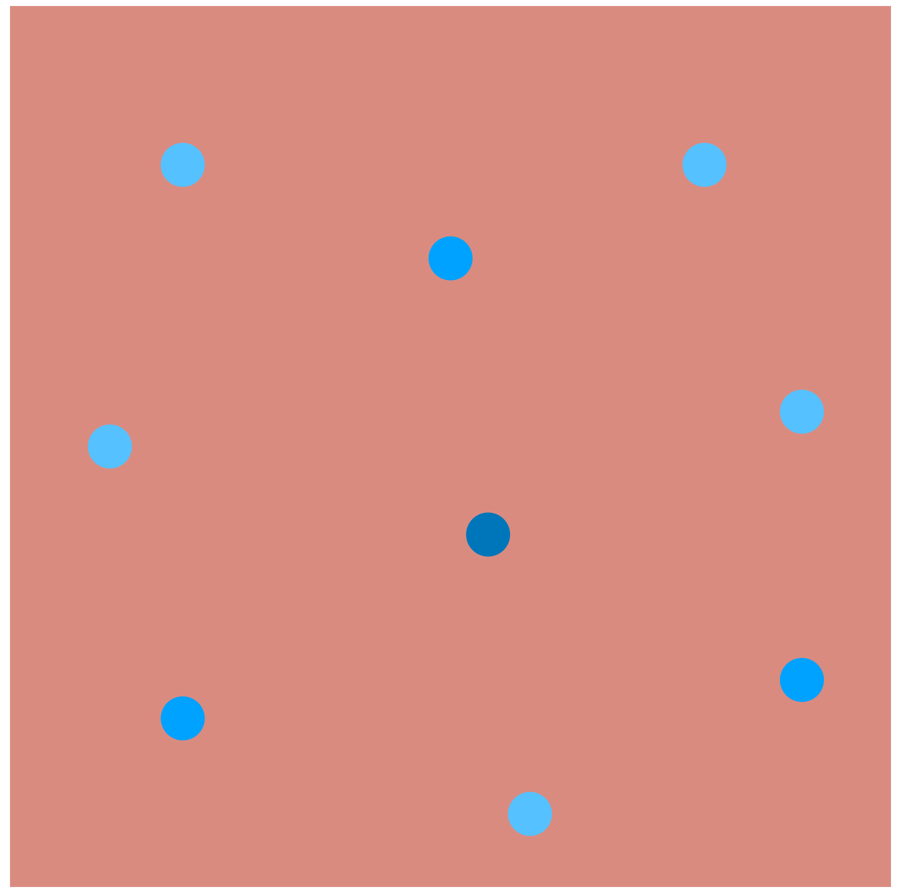
Gravitational Waves Volume 2: Astrophysics and Cosmology,
Michele Maggiore, Oxford University Press



Life of a bubble

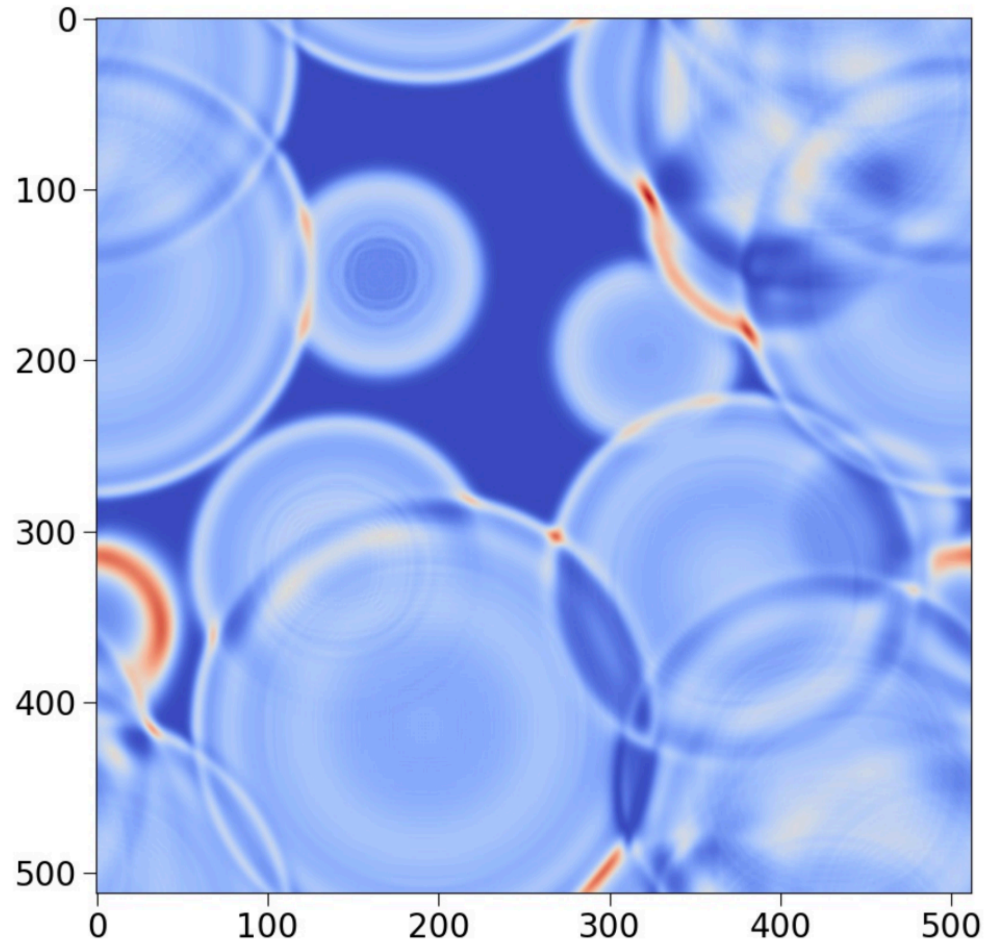


$$\Gamma(t) = \Gamma_0 e^{-\frac{1}{2}\beta_2^2 (t-t_m)^2}$$



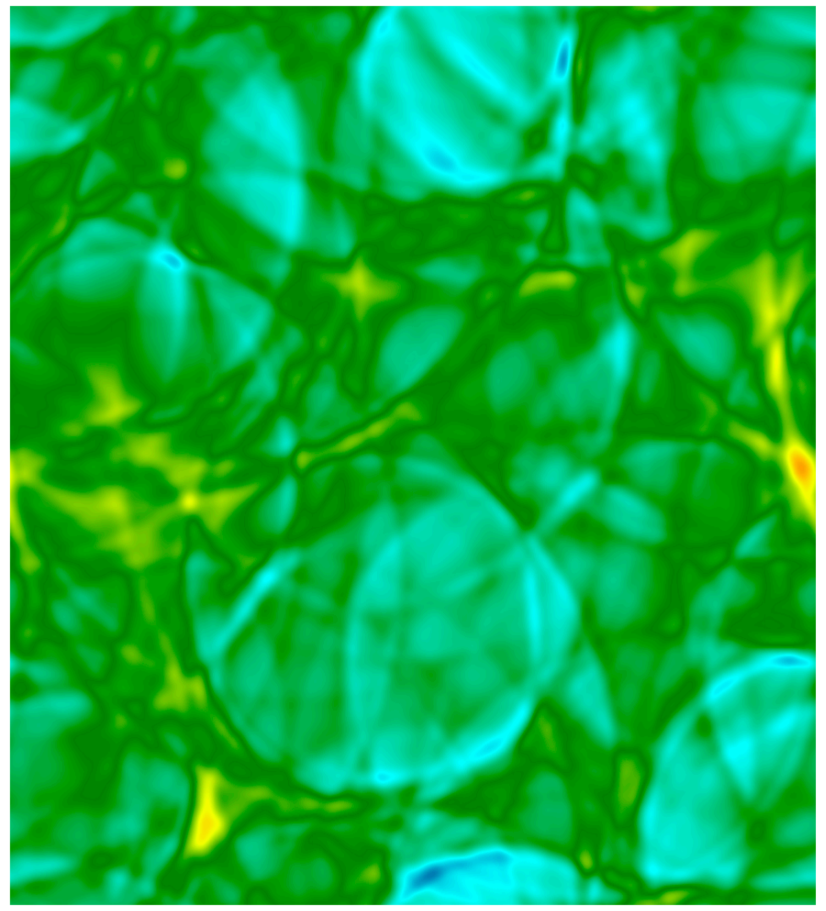
$$\Gamma(t) = \Gamma_0 e^{\beta_1 (t-t_f)}$$

Y. Di et al., arXiv:2012.15625



Wall collisions

Hinmarsh et al., arXiv:1304.2433



Sound waves

- Nucleation → • Expansion → • Collision

Contents

Equation of Motions

Assuming a fast first-order phase transition with radiation-like cosmic fluid and a scalar field with a flat background

$$\left.\begin{aligned}\nabla_\mu T^{\mu\nu}_\phi &\equiv [\nabla_\mu \nabla^\mu \phi - V_0'(\phi)]\nabla^\nu \phi = +f^\nu \\ \nabla_\mu T^{\mu\nu}_f &\equiv \sum_{i=\mathrm{B,F}} g_i \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{k^\mu k^\nu}{E_i(\mathbf{k})}\nabla_\mu f_i = -f^\nu\end{aligned}\right\} \left(k^\mu \partial_\mu + m_i F^\mu_i \frac{\partial}{\partial k^\mu}\right)\Theta(k^0)\delta(k^2+m_i^2) f_i(x,k) = C[f_i]$$

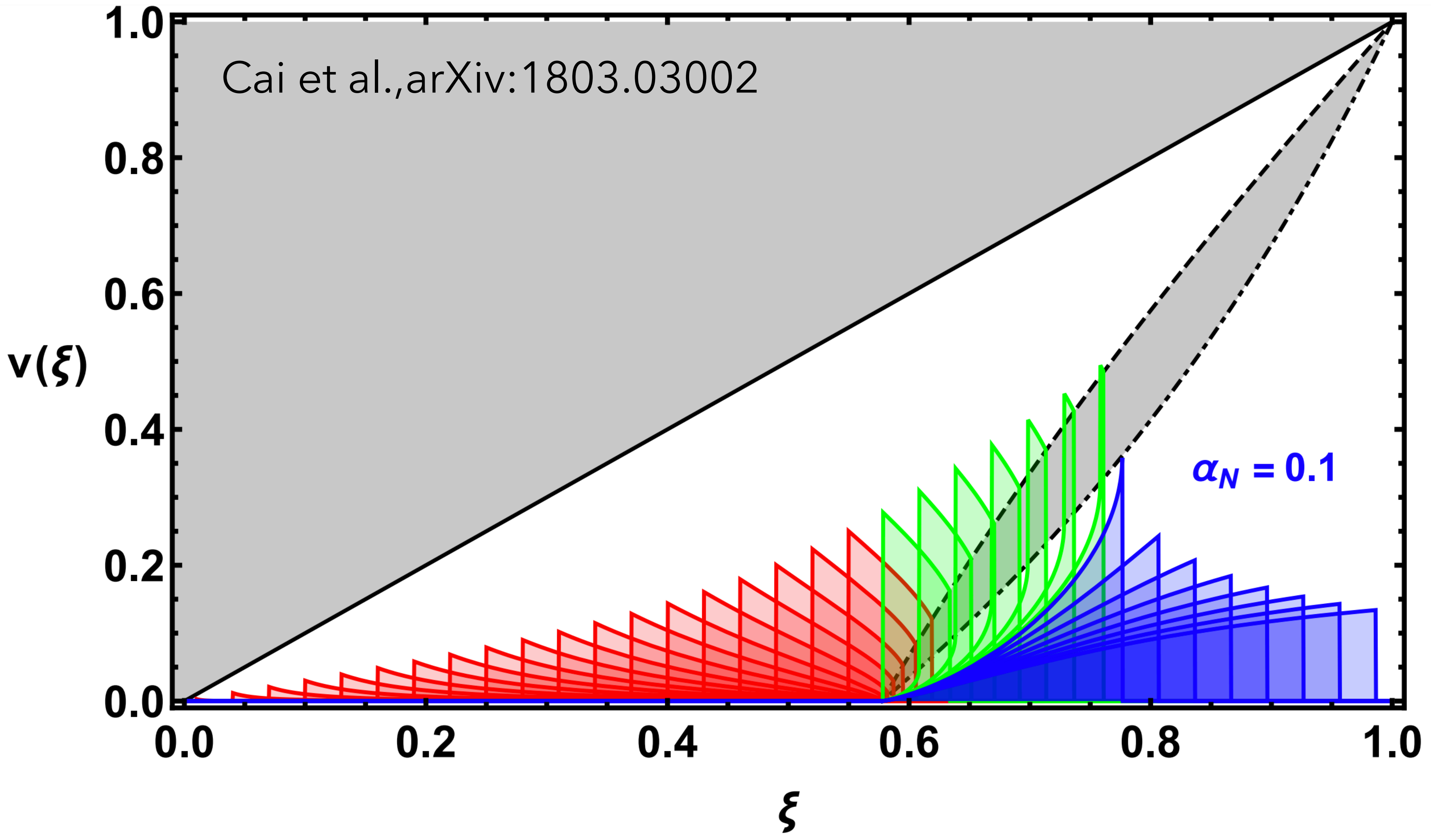
The plasma can be described by perfect fluid $T^{\mu\nu} = (\rho + p)u^\mu u^\nu + p\eta^{\mu\nu}$.

$$\begin{aligned} u_\nu \nabla_\mu T^{\mu\nu} &= 0 \\ \bar{u}_\nu \nabla_\mu T^{\mu\nu} &= 0 \end{aligned} \quad \Longrightarrow \quad \begin{aligned} \nabla_\mu (w u^\mu) - u^\mu \partial_\mu p &= 0 \\ \bar{u}^\nu u^\mu w \nabla_\mu (u_\nu) - \bar{u}^\mu \partial_\mu p &= 0 \end{aligned}$$

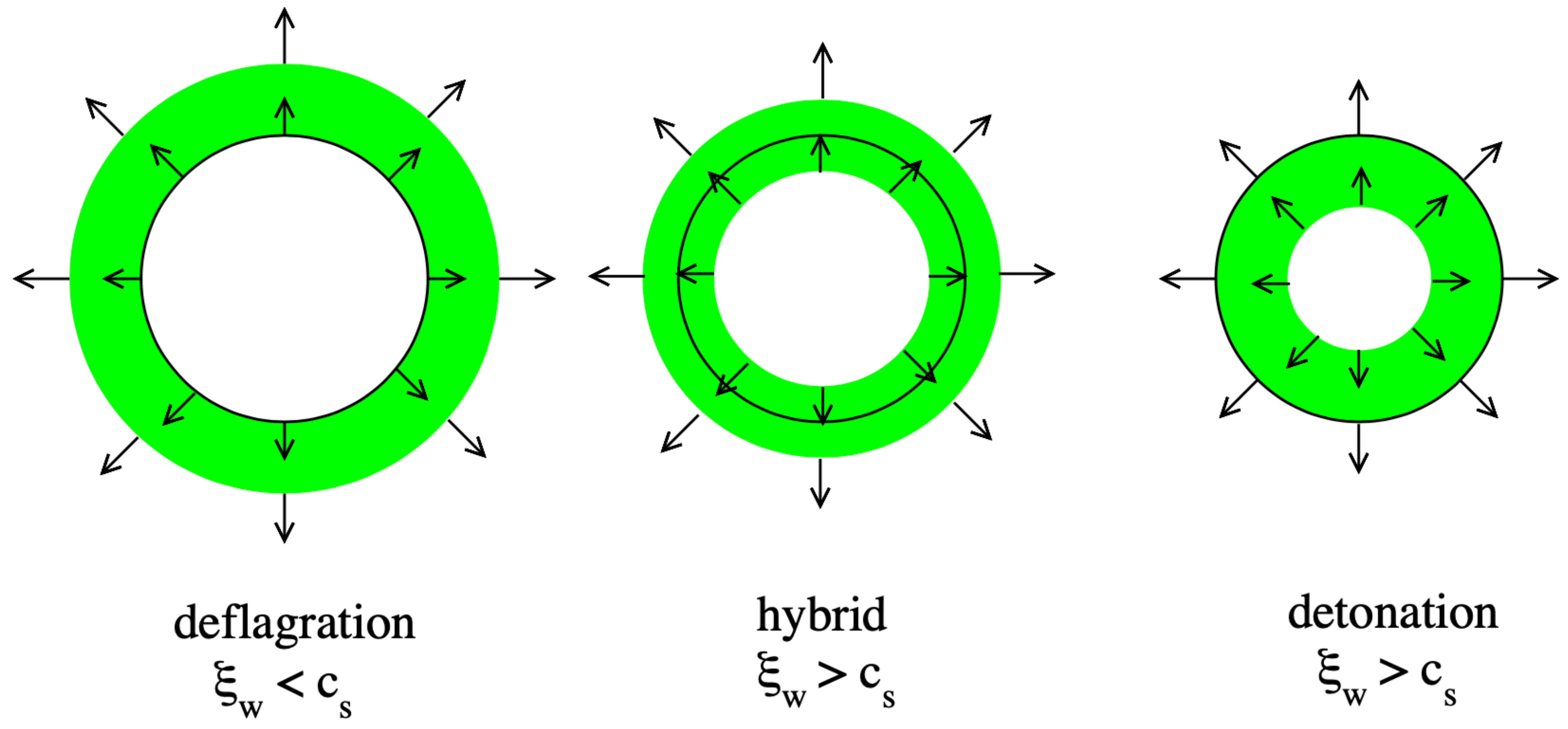
Equation of Motions

Projecting the conservation equation parallel and perpendicular to the bulk flow direction and rewriting the equations in self-similarity coordinate $\xi \equiv |\vec{x}|/t = r/t$ lead to

$$D\frac{v}{\xi} = \gamma^2(1 - \xi v)\left(\frac{\mu^2}{c_s^2} - 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi} \qquad \frac{\mathrm{d}w}{\mathrm{d}\xi} = w\gamma^2\mu\left(\frac{1}{c_s^2} + 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}$$



Cai et al.,arXiv:1803.03002



Espinosa et al.,arXiv:1004.4187

Efficiency Factor

Define efficiency factor as the ratio of bulk kinetic energy over the vacuum energy

$$\kappa_v = \int w(\xi)v^2\gamma^2\ 4\pi\xi^2 \mathrm{d}\xi \Big/ \left(\frac{4\pi}{3}\Delta V_0 \cdot \xi_w^3\right) = \frac{4}{\alpha_N \xi_w^3}\int_0^1 \frac{w(\xi)}{w_N} v^2\gamma^2\xi^2 \mathrm{d}\xi$$

where α_N is the phase transition strength factor in far front of the bubble wall

$$\alpha_N = \frac{\Delta V_0}{a_N T_N^4} = \frac{3\Delta V_0}{4 w_N}$$

with bag equation of state

$$p = \frac{1}{3}aT^4 - \Delta V_0, \quad \rho = aT^4 + \Delta V_0 .$$

Beyond bag EOS

The effective potential, $V_{\rm eff}(\phi, T) = V_0(\phi) + V_T(\phi, T)$, with zero-temperature contribution V_0 and finite-temperature contribution V_T. To the one-loop order, V_T reads

$$V_T^{1-\rm loop} = \sum_{i=\rm B,F} \pm g_i T \int \frac{{\rm d}^3\vec{k}}{(2\pi)^3} \ln\left[1 \mp e^{-\frac{\sqrt{\vec{k}^2+m_i^2}}{T}}\right] \equiv \frac{T^4}{2\pi^2}\sum_{i=\rm B,F} g_i J_i\left(\frac{m_i^2}{T^2}\right)$$

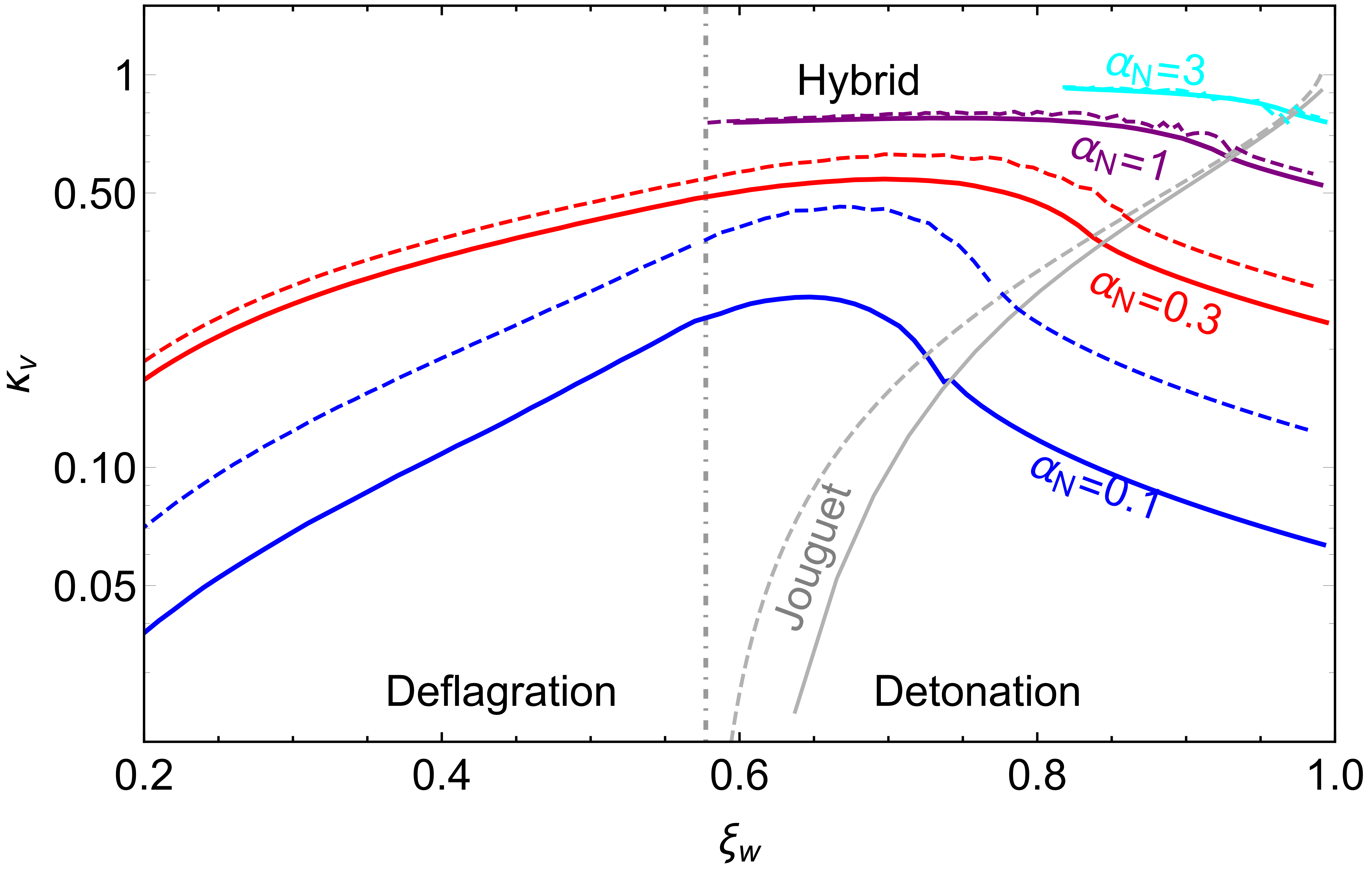
which leads to a modified equation of state

$$p = -\mathcal{F} = -V_0(\phi) + \frac{1}{3}aT^4 - bT^2 + cT$$

$$\rho = T\partial_T p - p = V_0(\phi) + aT^4 - bT^2$$

$$c_s^2 = \frac{\partial p}{\partial \rho} = \frac{1}{3}\left(1 - \frac{4bT - 3c}{12aT^3 - 6bT}\right) = \frac{1}{3} - \frac{b}{3aT^2} + \frac{c}{4aT^3} - \frac{b^2}{6a^2T^4} + \mathcal{O}(T^{-5})$$

Beyond bag EOS



Contents

GW radiation

GW sources during the phase transitions: $\nabla_\mu (T^{\mu\nu}_\phi + T^{\mu\nu}_f) = 0$

- Bubble collisions : Oscillations of the phase transition field ϕ with thin wall approximation

$$\nabla_\mu T^{\mu\nu}_\phi = [\nabla_\mu \nabla^\mu \phi - V_0'(\phi)]\nabla^\nu \phi = +f^\nu$$

R. Jinno and M. Takimoto, arXiv:1605.01403, 1707.03111

- Sound waves : Treating bulk fluids as freely propagating waves

$$\nabla_\mu T^{\mu\nu}_f = \sum_{i=\mathrm{B,F}} g_i \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \frac{k^\mu k^\nu}{E_i(\mathbf{k})} \nabla_\mu f_i = -f^\nu$$

C. Caprini, R. Durrer and G. Servant , arXiv:0711.2593

M. Hindmarsh, arXiv:1608.04735

M. Hindmarsh and M. Hijazi, arXiv:1909.10040

H.K. Guo, K. Sinha, D. Vagie, G. White, arXiv:2207.08537

- Turbulence, primordial magnetic fields …

Simulations: M. Hindmarsh, S. J. Huber, K. Rummukainen, and D.J. Weir, arXiv:1704.05871
Y.F. Di, J.L. Wang, R.Y. Zhou, L.G. Bian, R.G. Cai and J. Liu, arXiv:2012.15625

GW radiation

GW sources during the phase transitions:

R. Jinno et al., arXiv:1605.01403

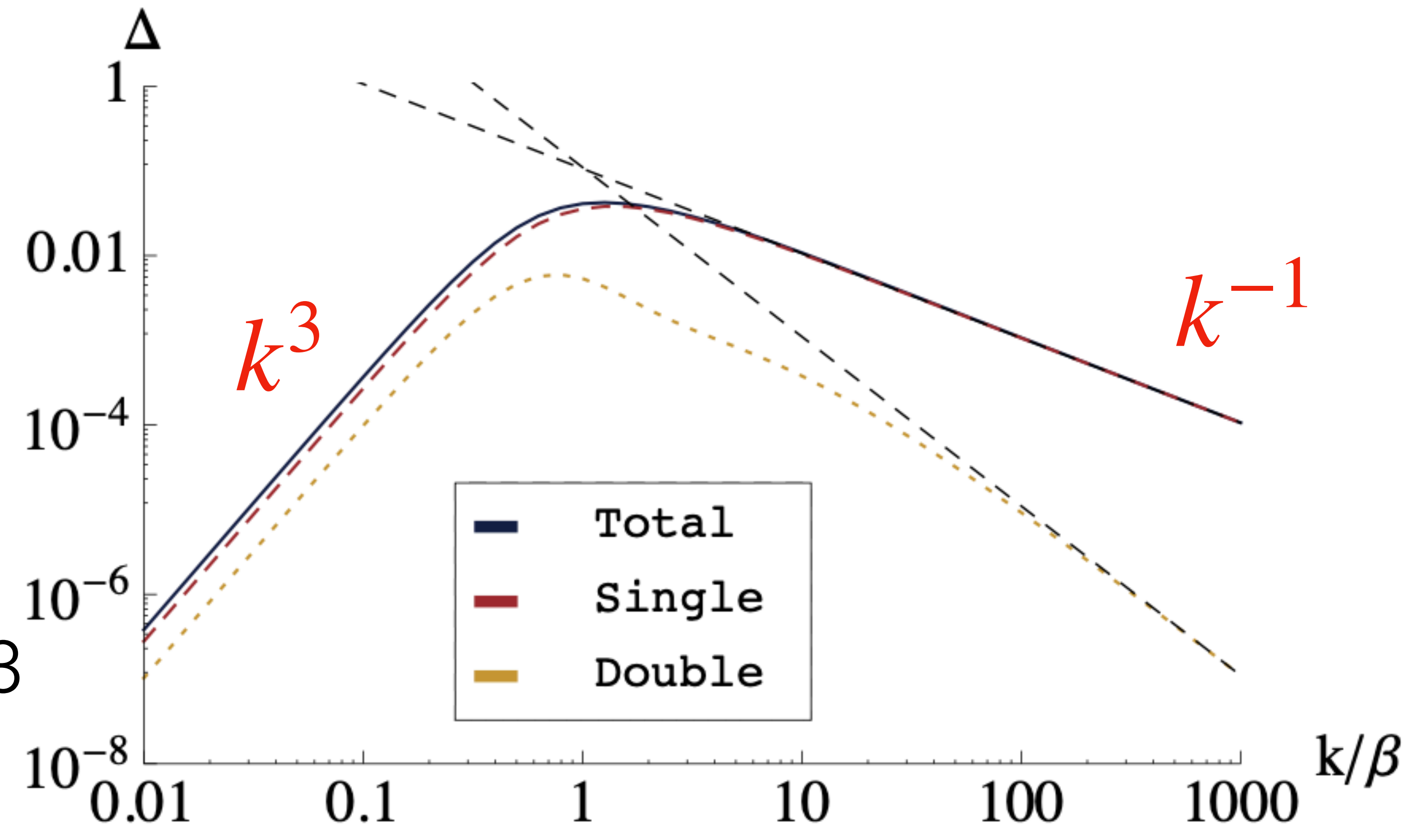
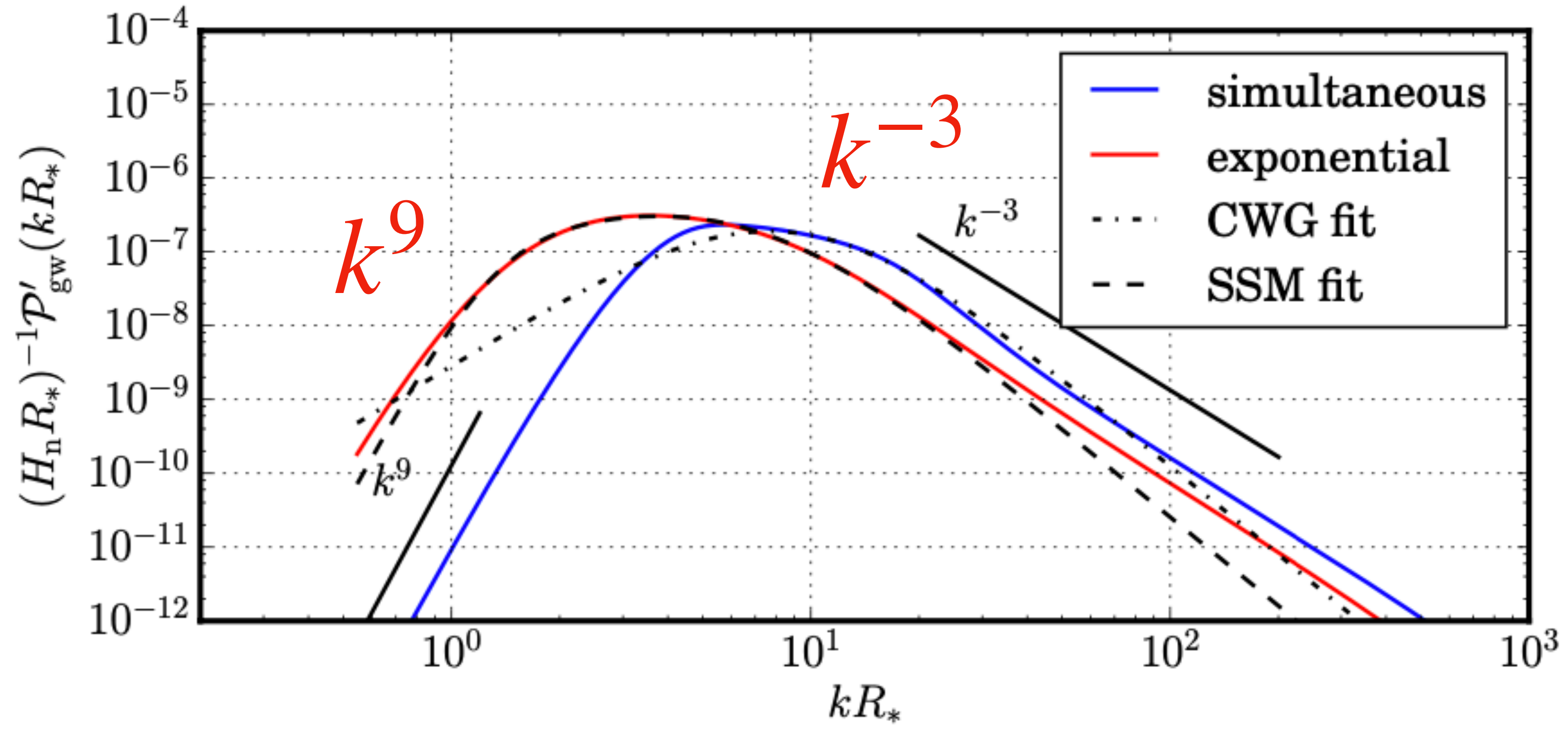


FIG. 6: Plot of the GW spectrum Δ (blue). Single- and double-bubble spectra $\Delta^{(s)}$ (red) and $\Delta^{(d)}$ (yellow) are also plotted. Black lines are auxiliary ones proportional to k^{-1} and k^{-2}, respectively.

- Bubble collisions :

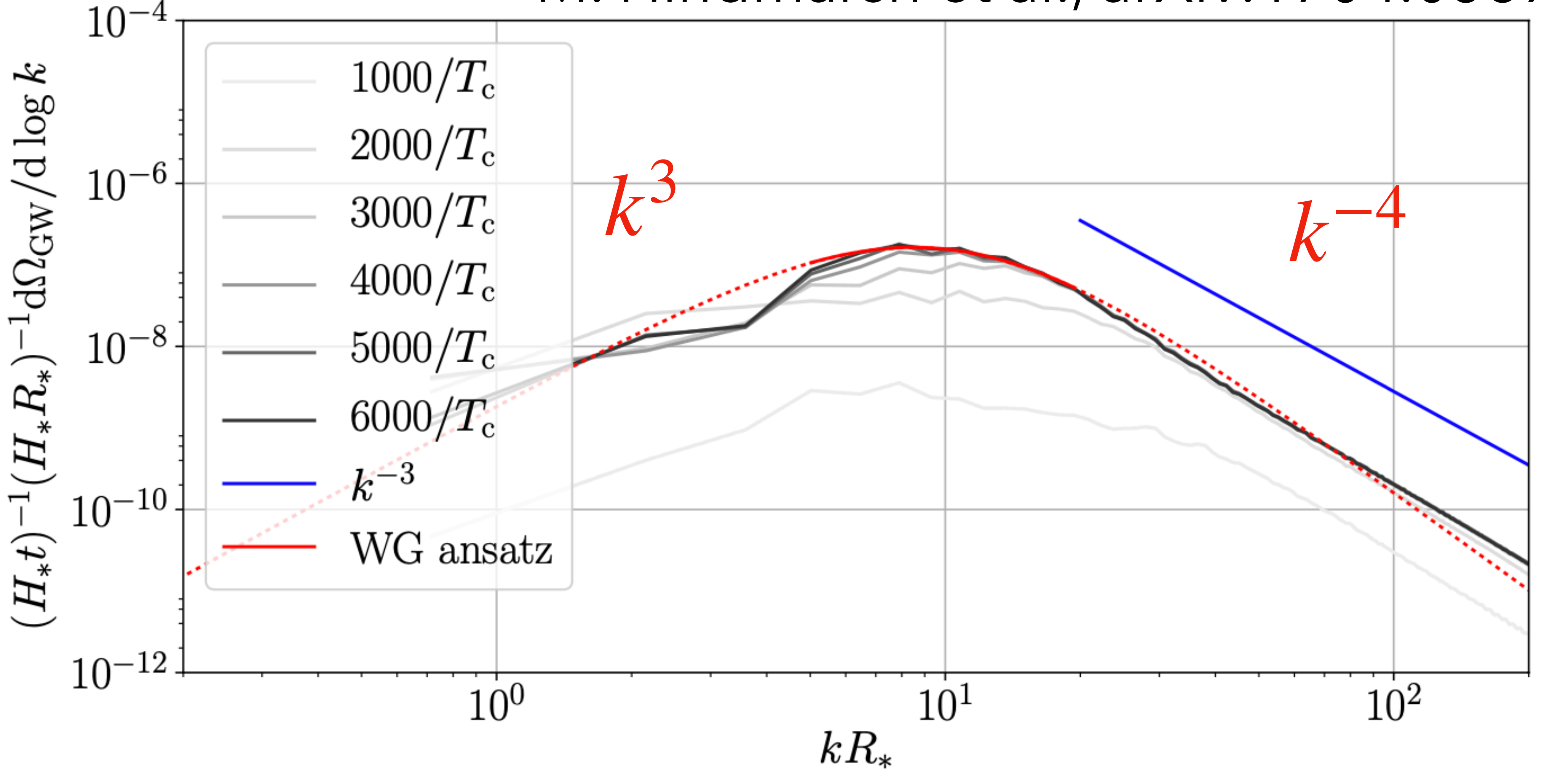
- Sound waves :



M. Hindmarsh and M. Hijazi, arXiv:1909.10040

(b) Intermediate, $v_{\mathrm{w}} = 0.92$

M. Hindmarsh et al., arXiv:1704.05871



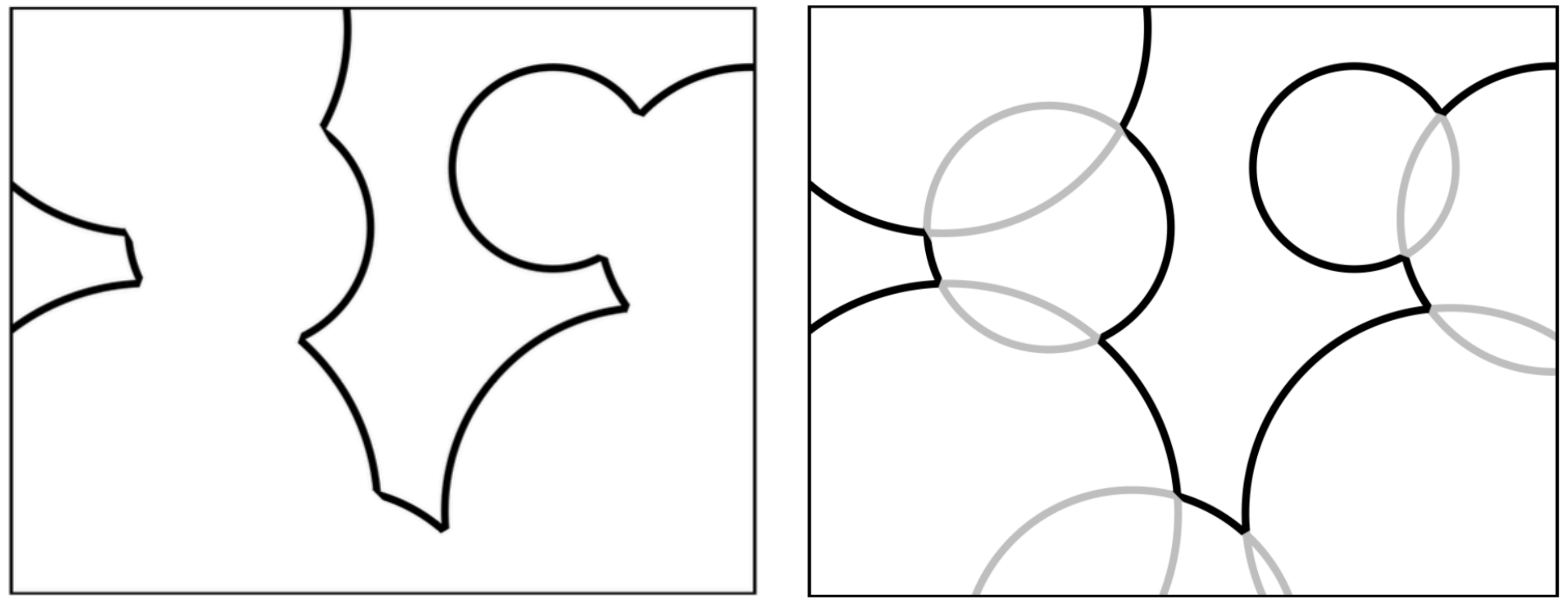
(b) Intermediate, $v_{\mathrm{w}} = 0.92$

Simulations:

Introduction

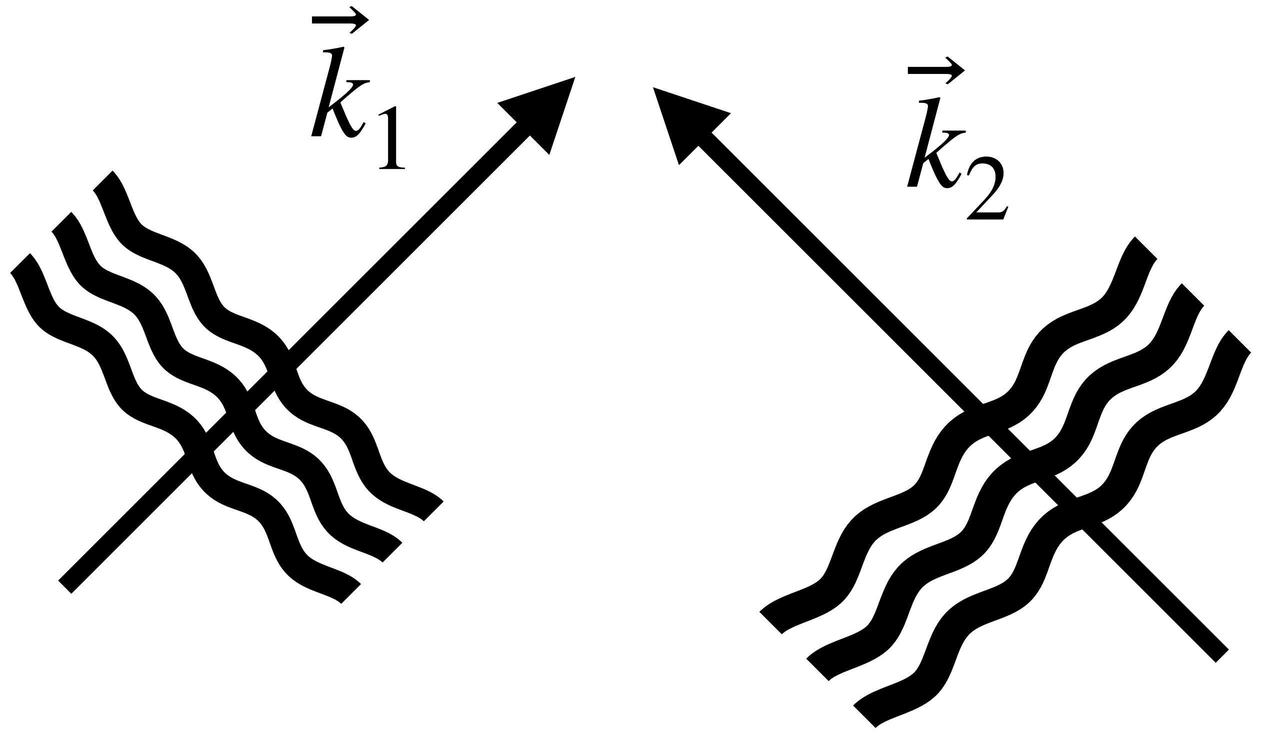
GW sources during the phase transitions:

- Bubble collisions :

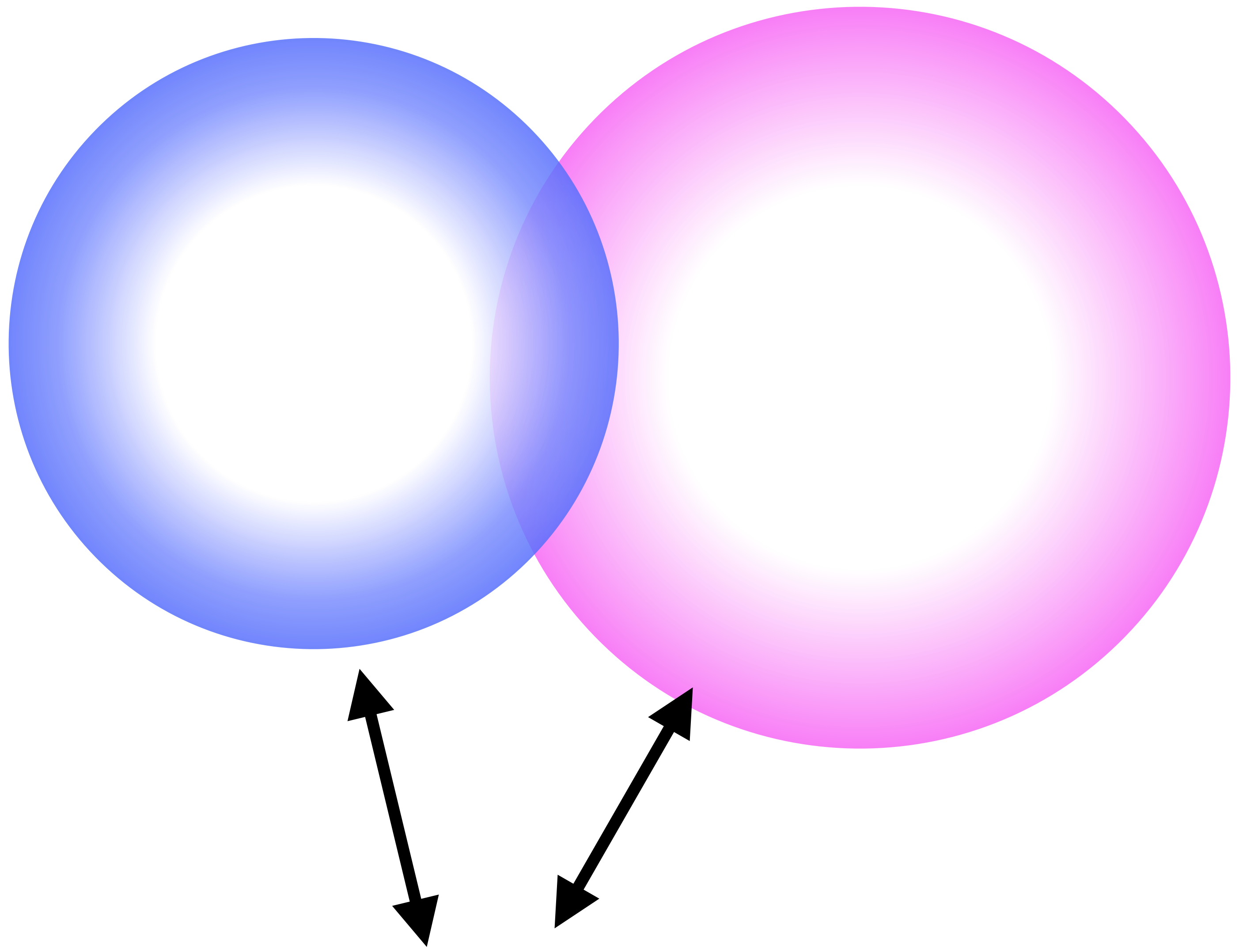


R. Jinno et al., arXiv:1605.01403   arXiv: 1707.03111

- Sound waves :



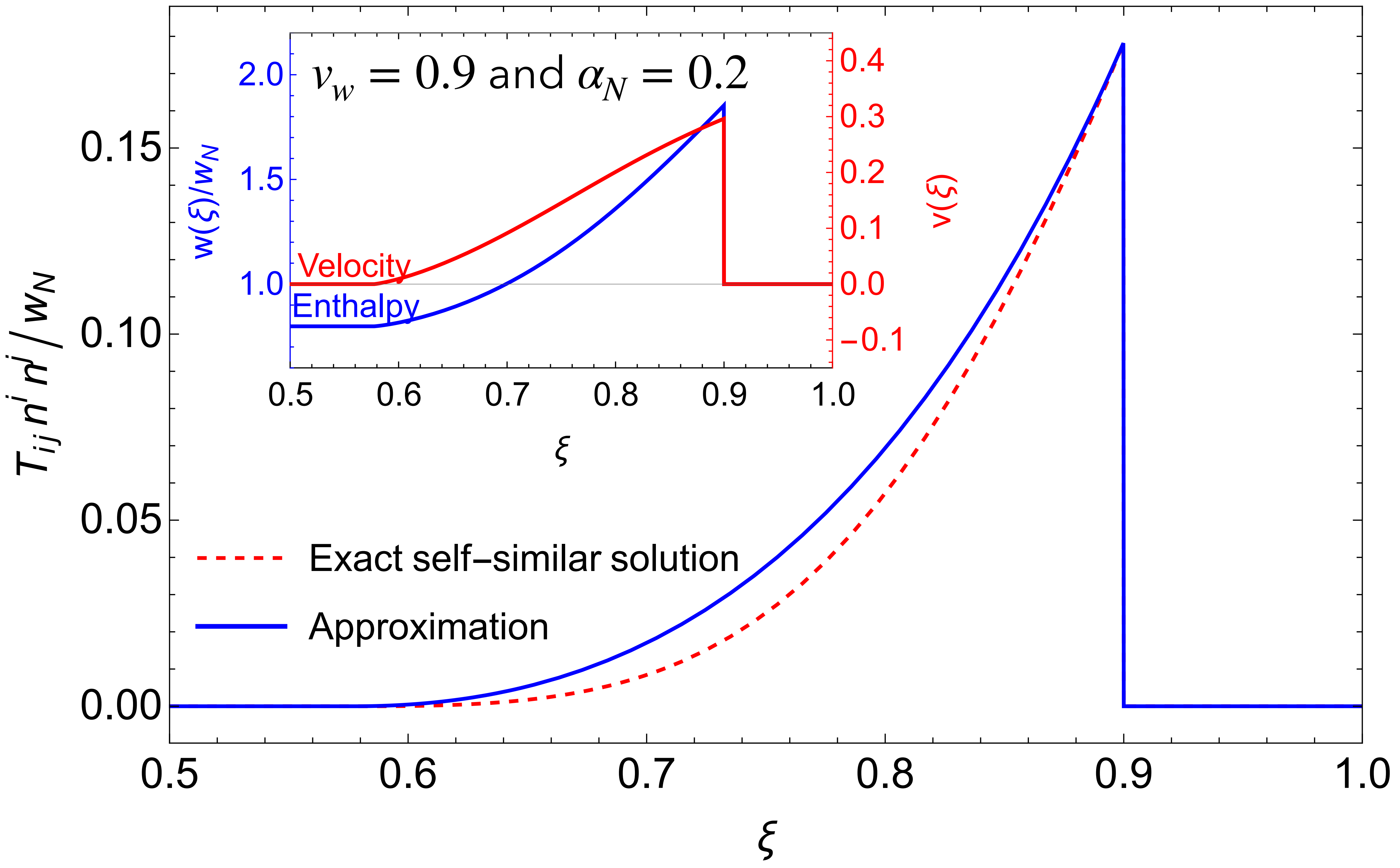
The process that our model cares



Existence of quadrupole for energy-momentum tensor

Sound shell model

For a steadily expanding bubble wall, the profiles of fluid velocity and thermodynamic quantities can be described as functions of a self-similar coordinate $\xi = r/t$ alone.



Perfect fluid approximation

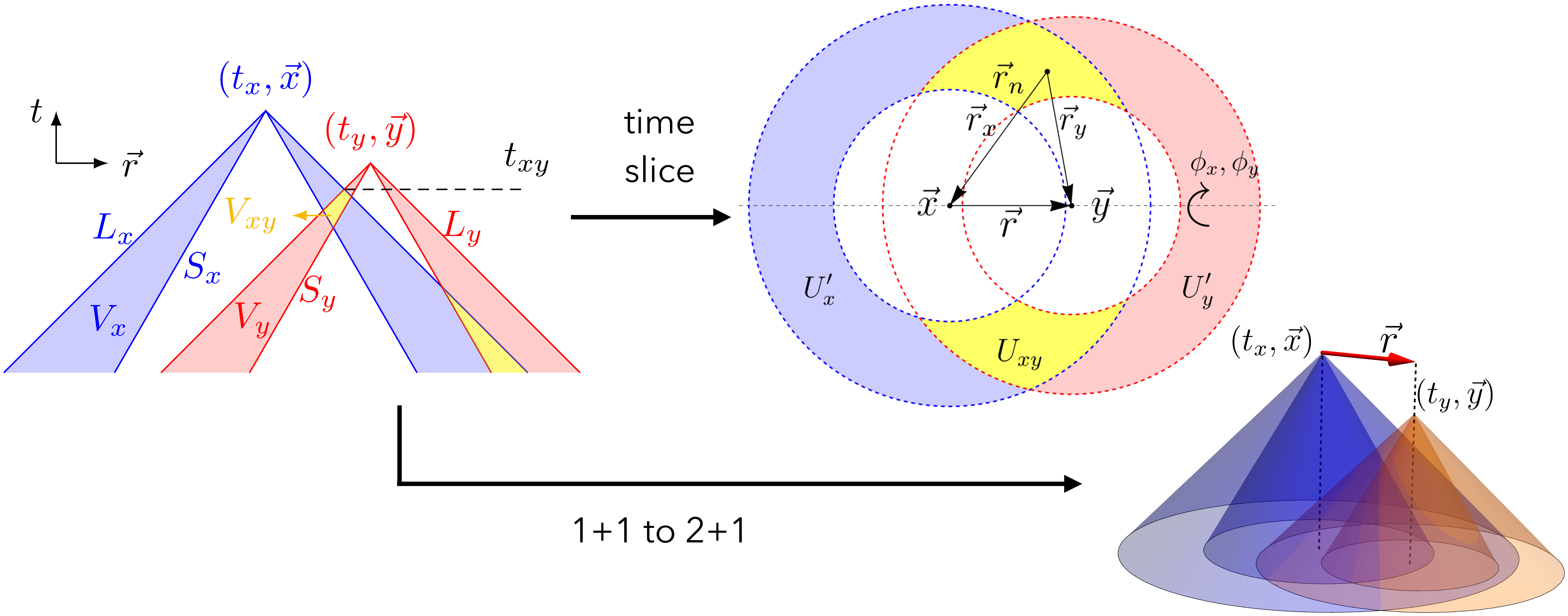
$$\hat{T}_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu}$$

Anisotropic, spacial part

$$T_{ij} = w\gamma^2 v_i v_j$$

Analytic derivation

Schematic images for where single-shell and double-shell might come from.

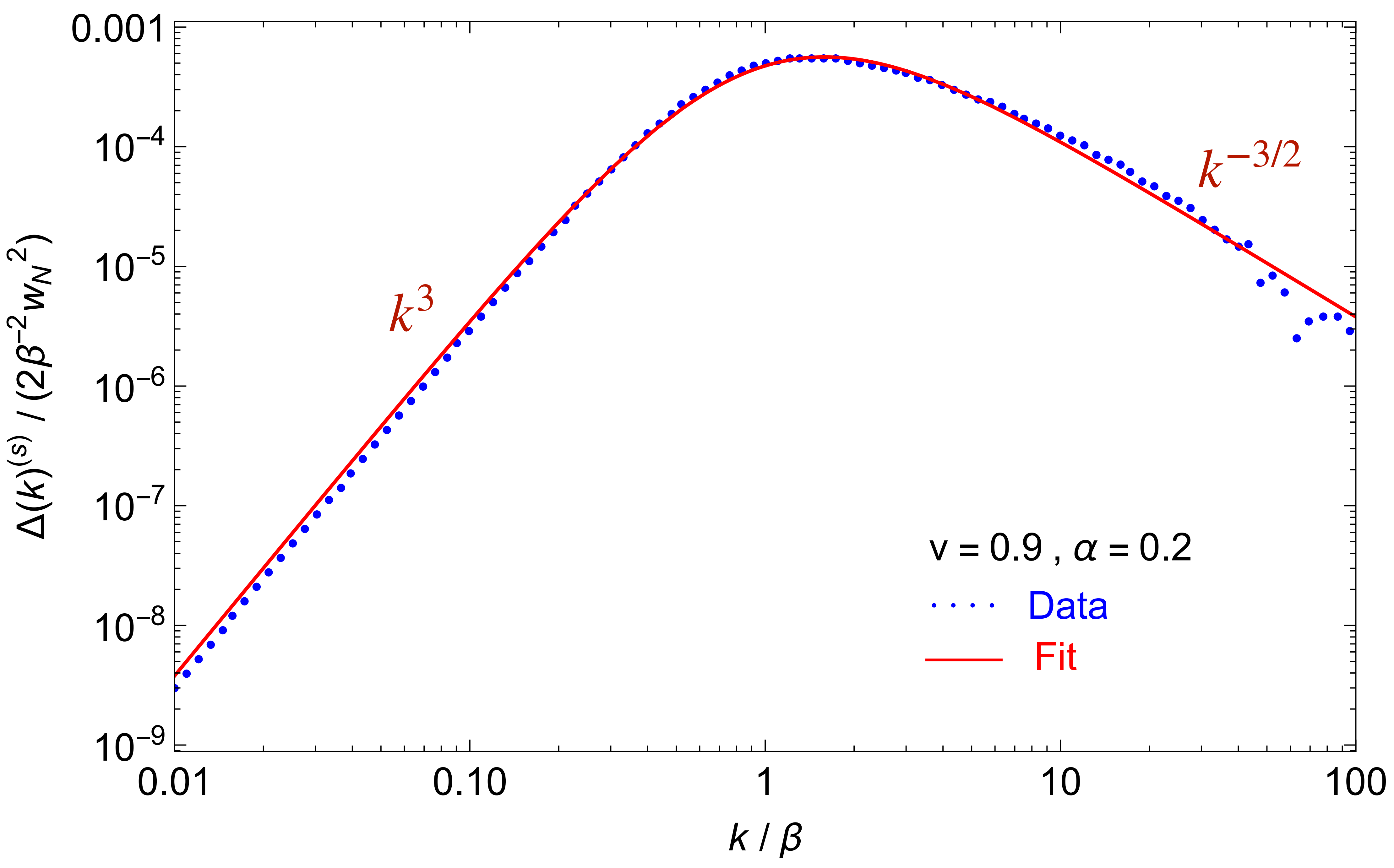


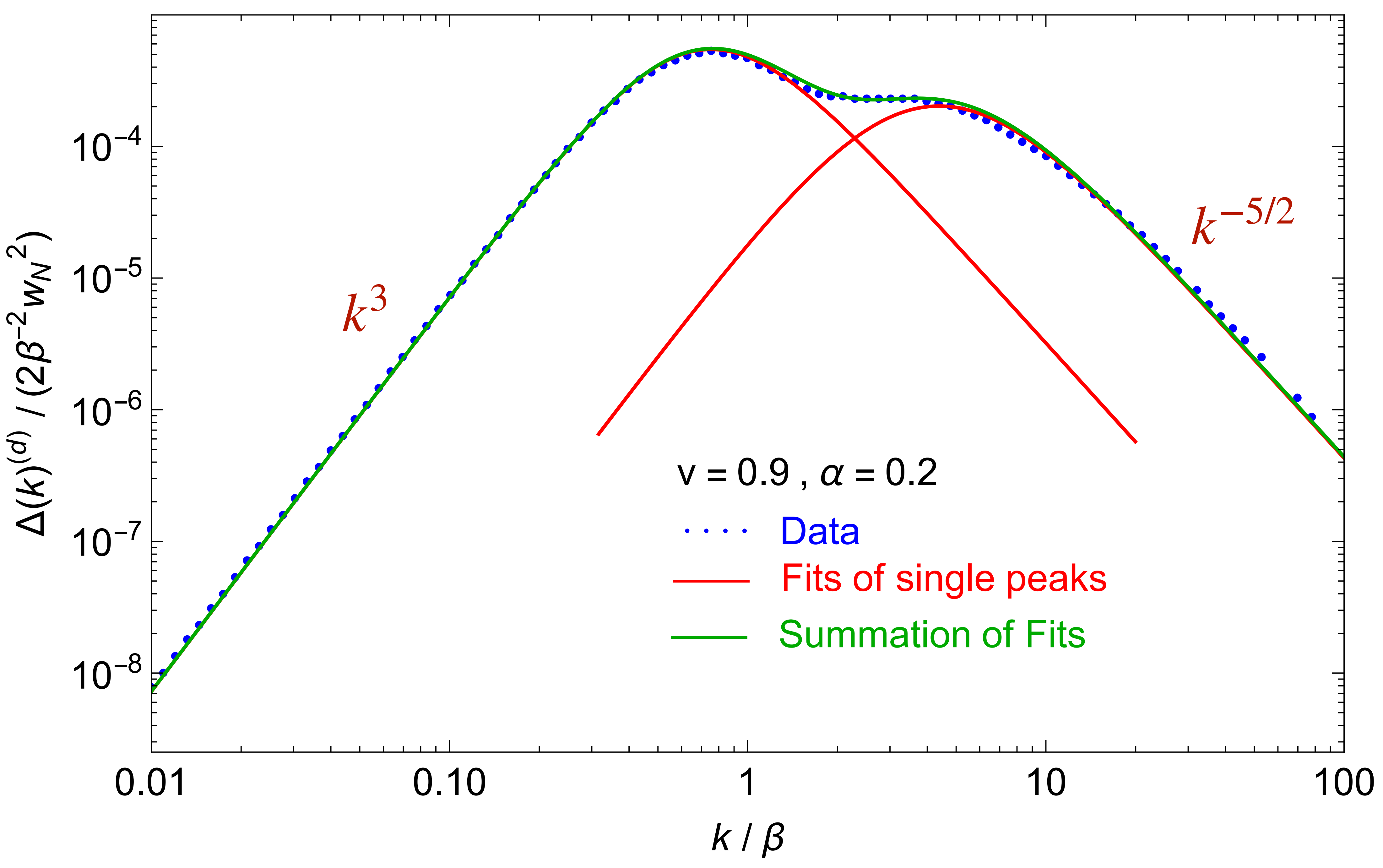
Numerical results

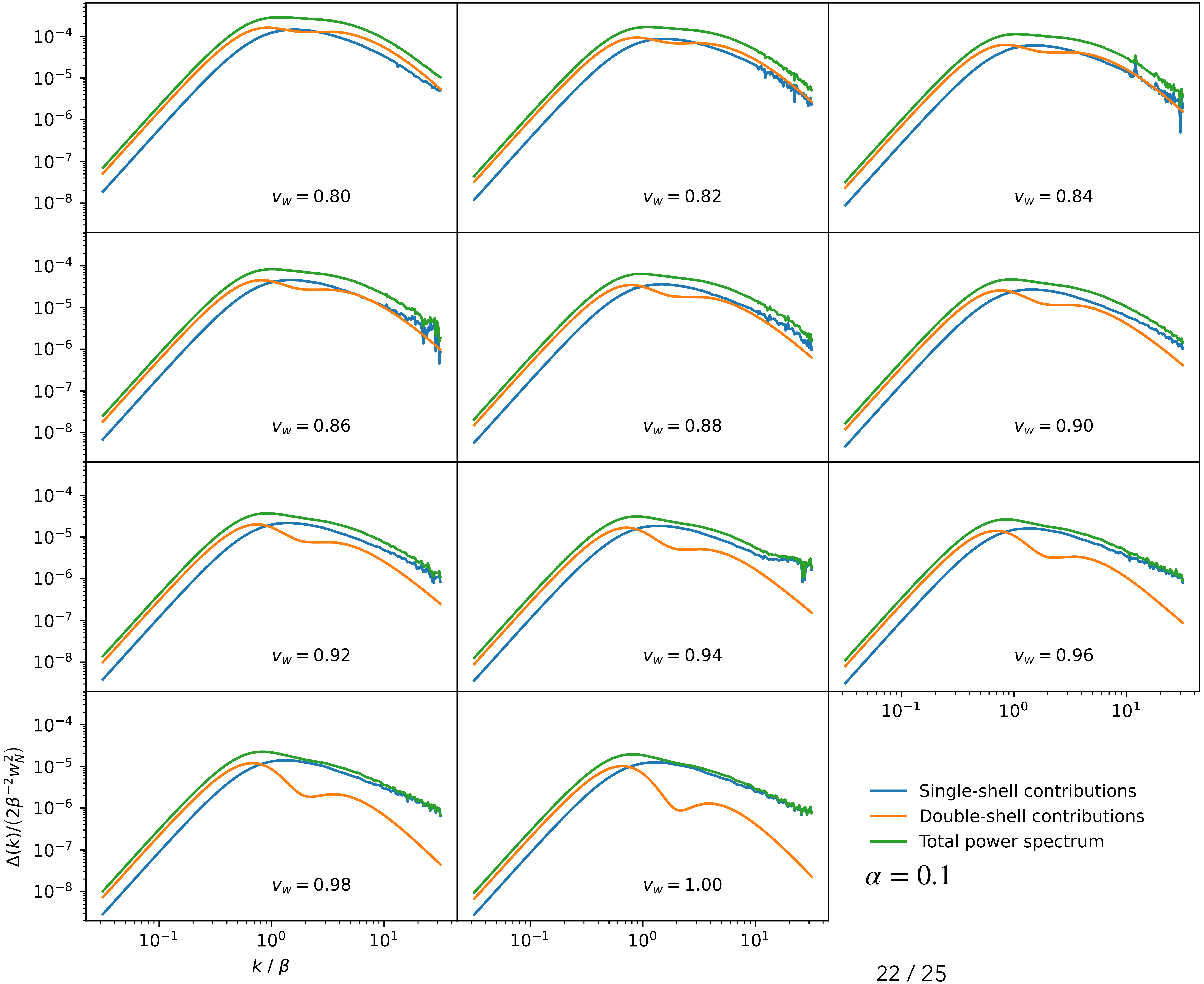
$$F_{n_1,n_2,\Delta}(k,k_*,F_*) = F_*\left(\frac{k}{k_*}\right)^{n_1}\left(\frac{1+(k/k_*)^{\Delta}}{2}\right)^{\frac{n_2-n_1}{\Delta}}$$

$$\Delta^{(s)} = F_{n_1,n_2,\Delta}(k,k_*,F_*)$$

$$\Delta^{(d)} = F_{n_1,n_2,\Delta_1}(k,k_{*1},F_{*2}) + F_{n_1,n_2,\Delta_2}(k,k_{*2},F_{*2})$$







- k^3 power law in low frequencies.
- Double-shell dominates in low frequencies, and single-shell gradually dominates in high frequencies as $v_w \to 1$.
- A broader dome as v_w decreases, which is found in numerical simulations.

M. Hindmarsh et al., arXiv:1704.05871

Contents

Discussions

Go beyond bag EOS

- One-loop correction with higher expansion around critical temperature.
- Less efficient GW radiation.

Sound shell model

- Numerical results and fitting formulas for power spectrum are provided.
- Low frequency k^3 ; High frequency $\longrightarrow$ Single-shell $k^{-2} \to k^{-1}$ as $v_w \to 1$
  Double-shell $k^{-5/2} \to k^{-3}$ as $v_w \to 1$
- A broader dome is discovered because of the double-shell domination at low v_w.

Discussions

Huber et al., arXiv:0709.2091

$$h_0^2\Omega_{\rm gw} = 1.84\times 10^{-6}\kappa^2\left(\frac{g_*}{100}\right)^{-1/3}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\alpha}{1+\alpha}\right)^2\left(\frac{v_w^3}{0.24+v_w^2}\right)$$

To be determined: κ, α, β, T_*, g_*, v_w. Further studies are needed!

Thank You!