Hydrodynamic sound shell model and the bubble wall velocity

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Based on : Phys.Rev.D 108 (2023) 2, L021502 (arxiv:2305.00074) and recent work

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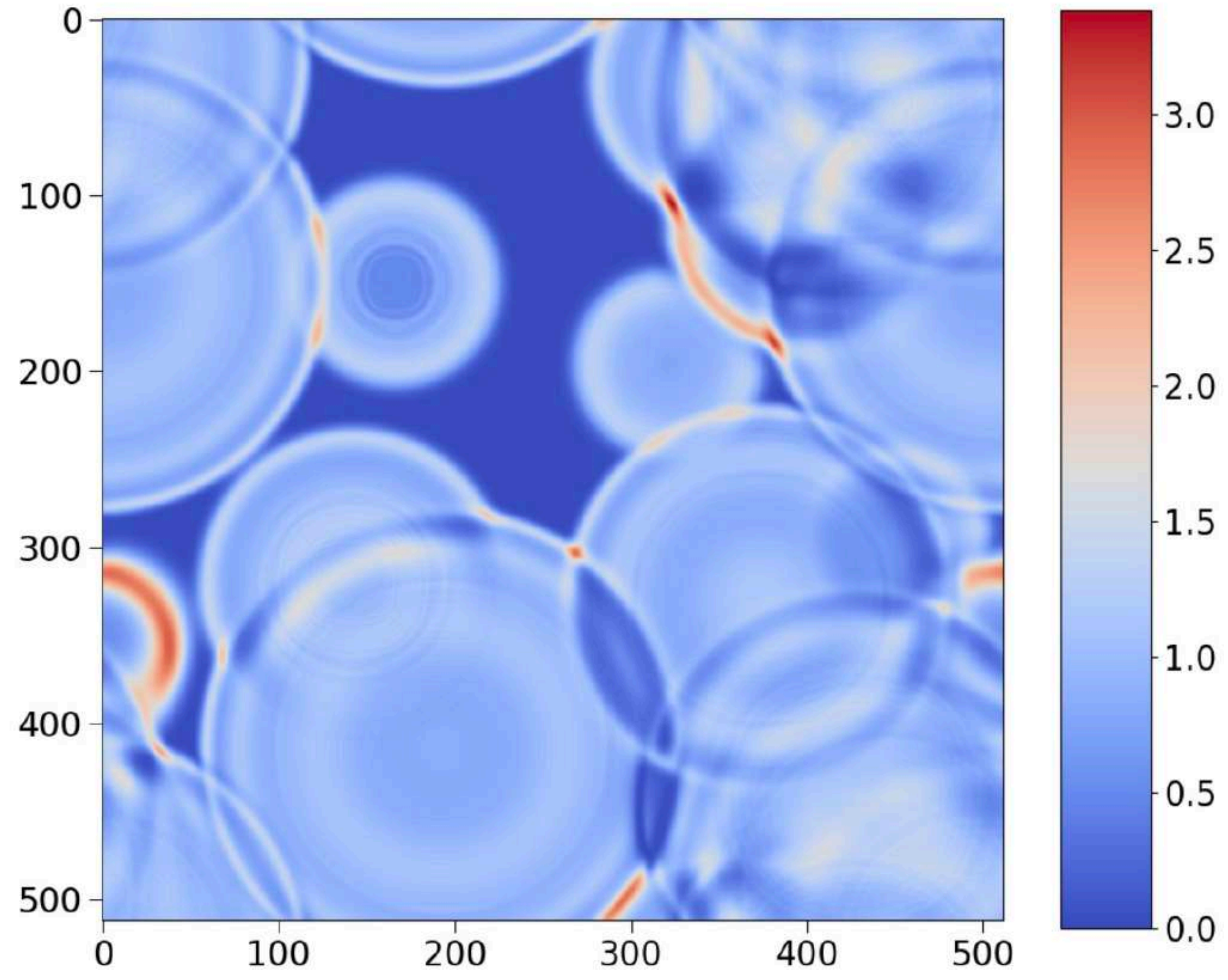
Introduction

Our universe suffers several phase transitions (PT) in the early time, such as electroweak phase transition and QCD phase transition. Gravitational waves (GWs) are one of the inspiring tools to probe the physics at that time.

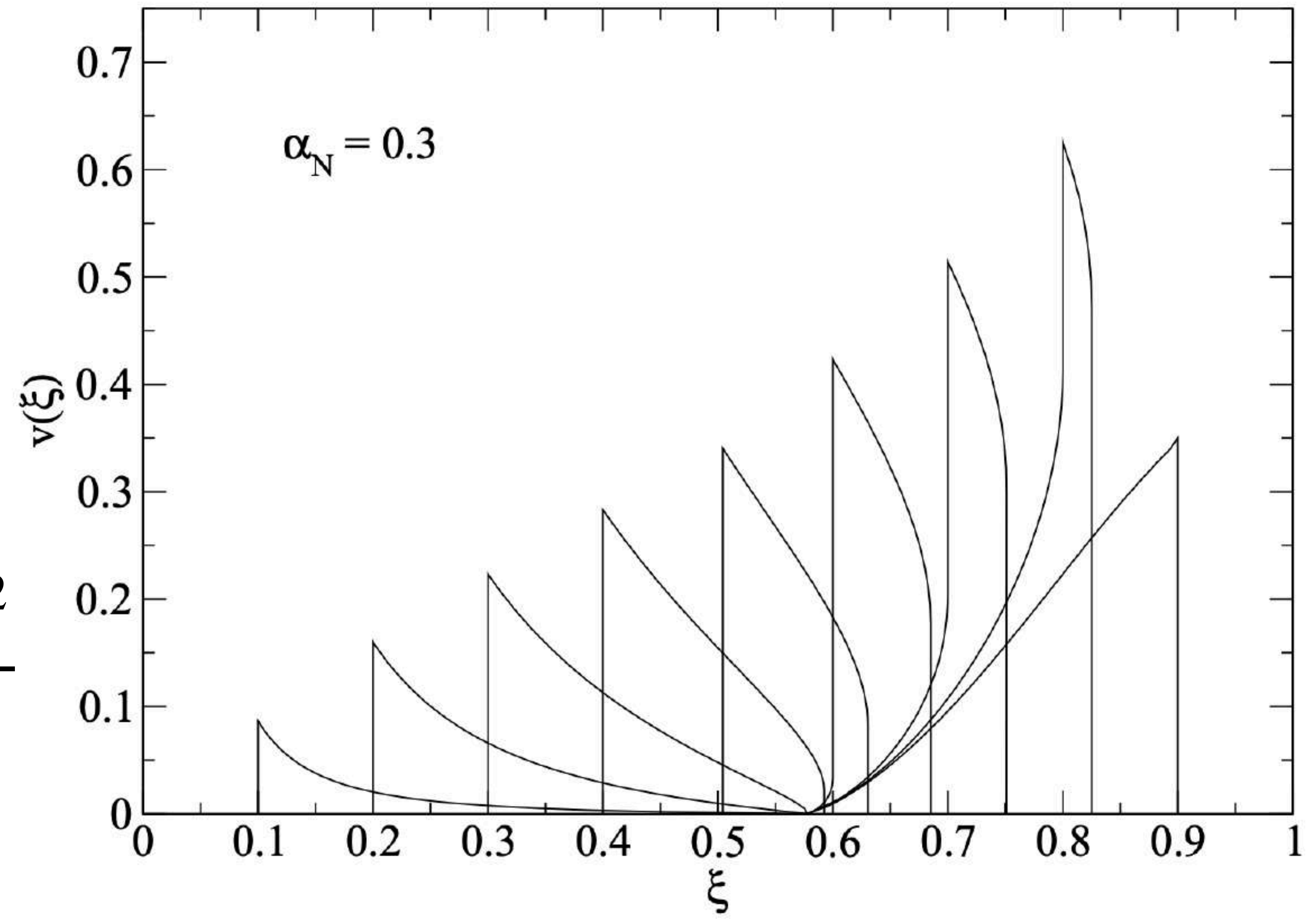


Y. Di et al.,arXiv:2012.15625

$$(\partial_t^2 - \nabla^2)\phi - \frac{\partial V}{\partial \phi} = 0$$

$$D\frac{v}{\xi} = \gamma^2(1 - \xi v)\left(\frac{\mu^2}{c_s^2} - 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}$$

$$\frac{\phi^2}{v^2}$$



J.R. Espinosa et al.,arXiv:1004.4187

Introduction

GW sources during the phase transitions: $\nabla_\mu (T^{\mu\nu}_\phi + T^{\mu\nu}_f) = 0$

- Bubble collisions : Oscillations of the phase transition field ϕ with thin wall approximation

$$\nabla_\mu T^{\mu\nu}_\phi = [\nabla_\mu \nabla^\mu \phi - V_0'(\phi)]\nabla^\nu \phi = +f^\nu$$

R. Jinno and M. Takimoto, arXiv:1605.01403, 1707.03111

- Sound waves : Treating bulk fluids as freely propagating waves

$$\nabla_\mu T^{\mu\nu}_f = \sum_{i=\mathrm{B,F}} g_i \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \frac{k^\mu k^\nu}{E_i(\mathbf{k})} \nabla_\mu f_i = -f^\nu$$

C. Caprini, R. Durrer and G. Servant , arXiv:0711.2593

M. Hindmarsh, arXiv:1608.04735

M. Hindmarsh and M. Hijazi, arXiv:1909.10040

H.K. Guo, K. Sinha, D. Vagie, G. White, arXiv:2207.08537

- Turbulence, primordial magnetic fields …

Simulations: M. Hindmarsh, S. J. Huber, K. Rummukainen, and D.J. Weir, arXiv:1704.05871
Y.F. Di, J.L. Wang, R.Y. Zhou, L.G. Bian, R.G. Cai and J. Liu, arXiv:2012.15625

Introduction

GW sources during the phase transitions:

R. Jinno et al., arXiv:1605.01403

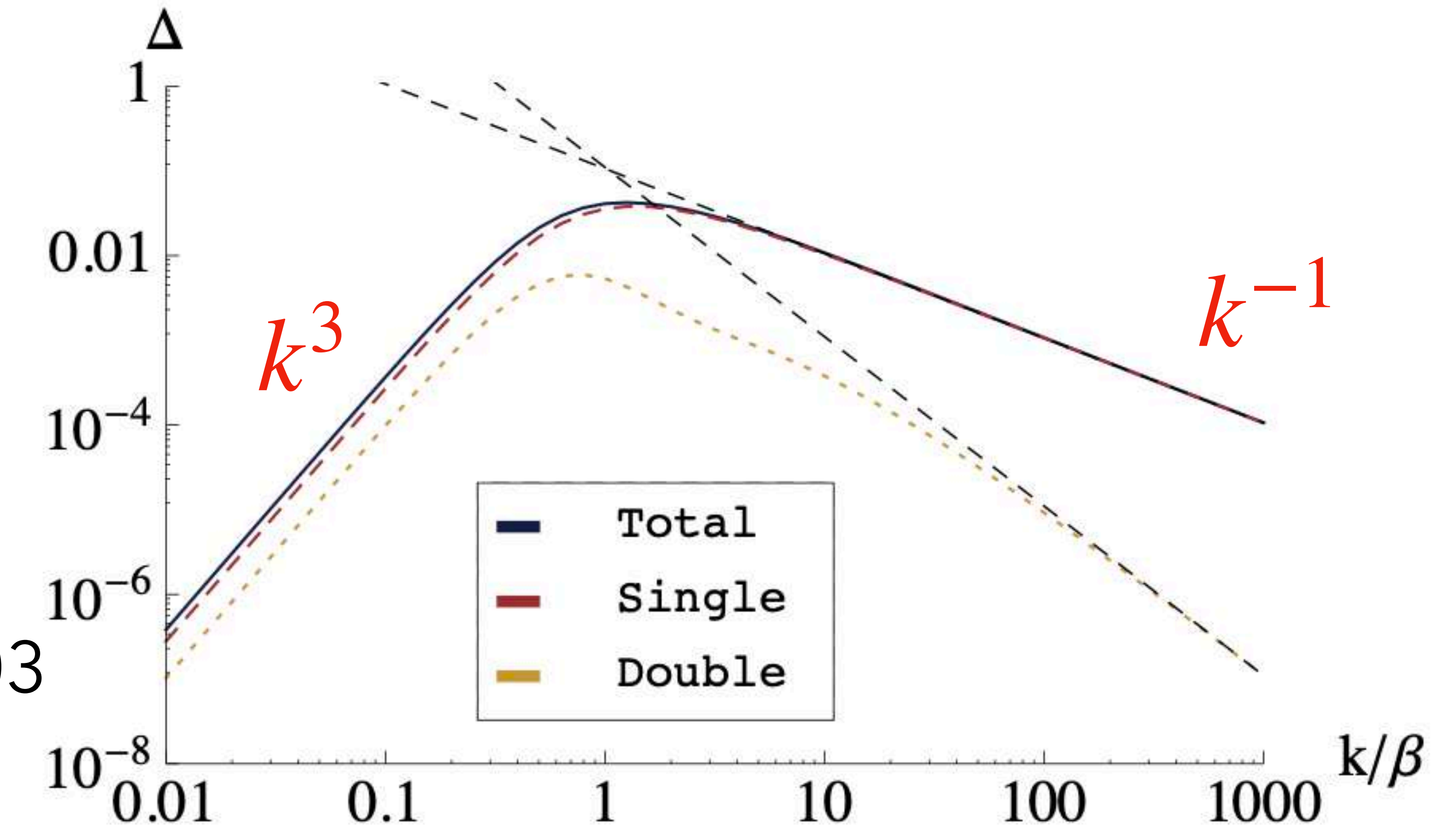
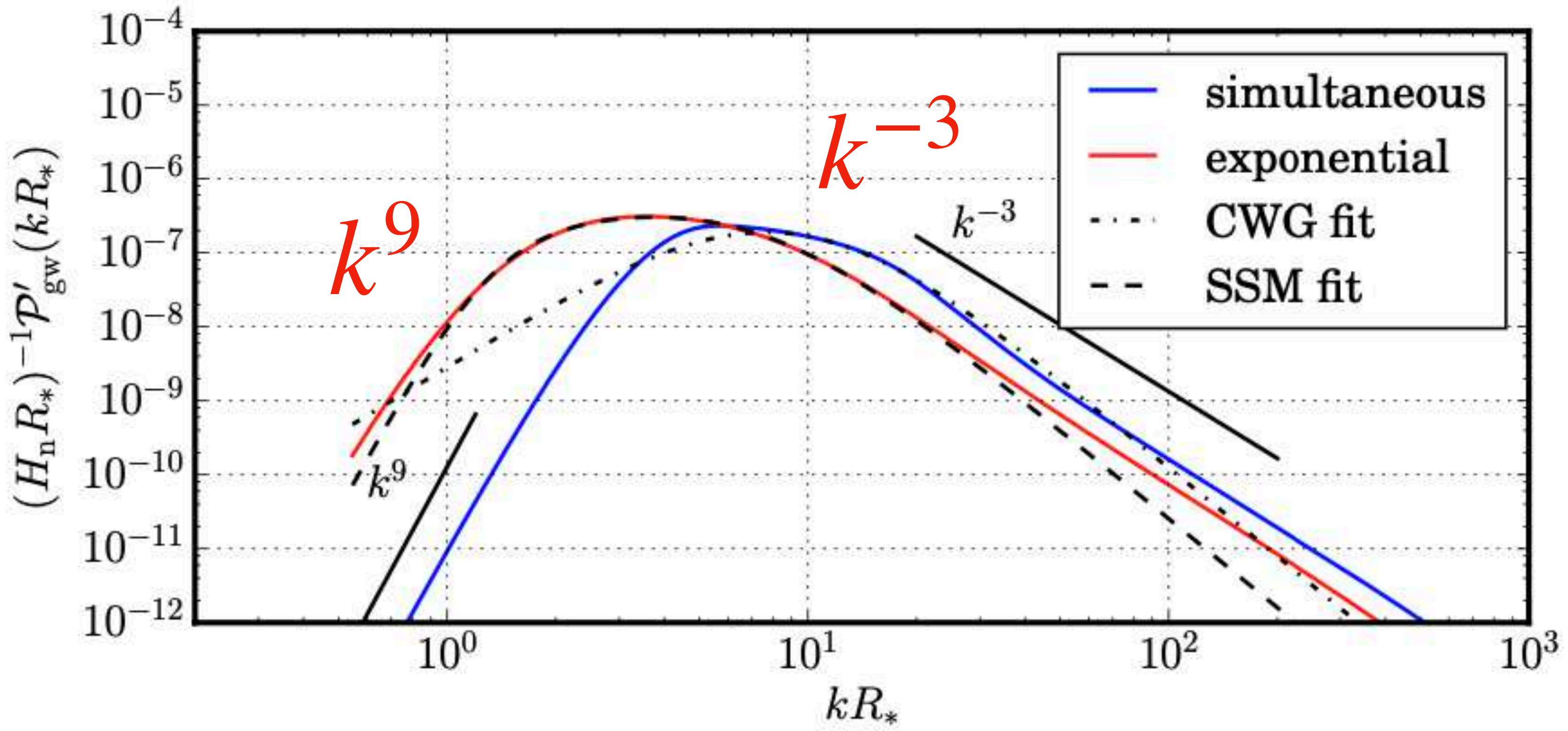


FIG. 6: Plot of the GW spectrum Δ (blue). Single- and double-bubble spectra $\Delta^{(s)}$ (red) and $\Delta^{(d)}$ (yellow) are also plotted. Black lines are auxiliary ones proportional to k^{-1} and k^{-2}, respectively.

- Bubble collisions :

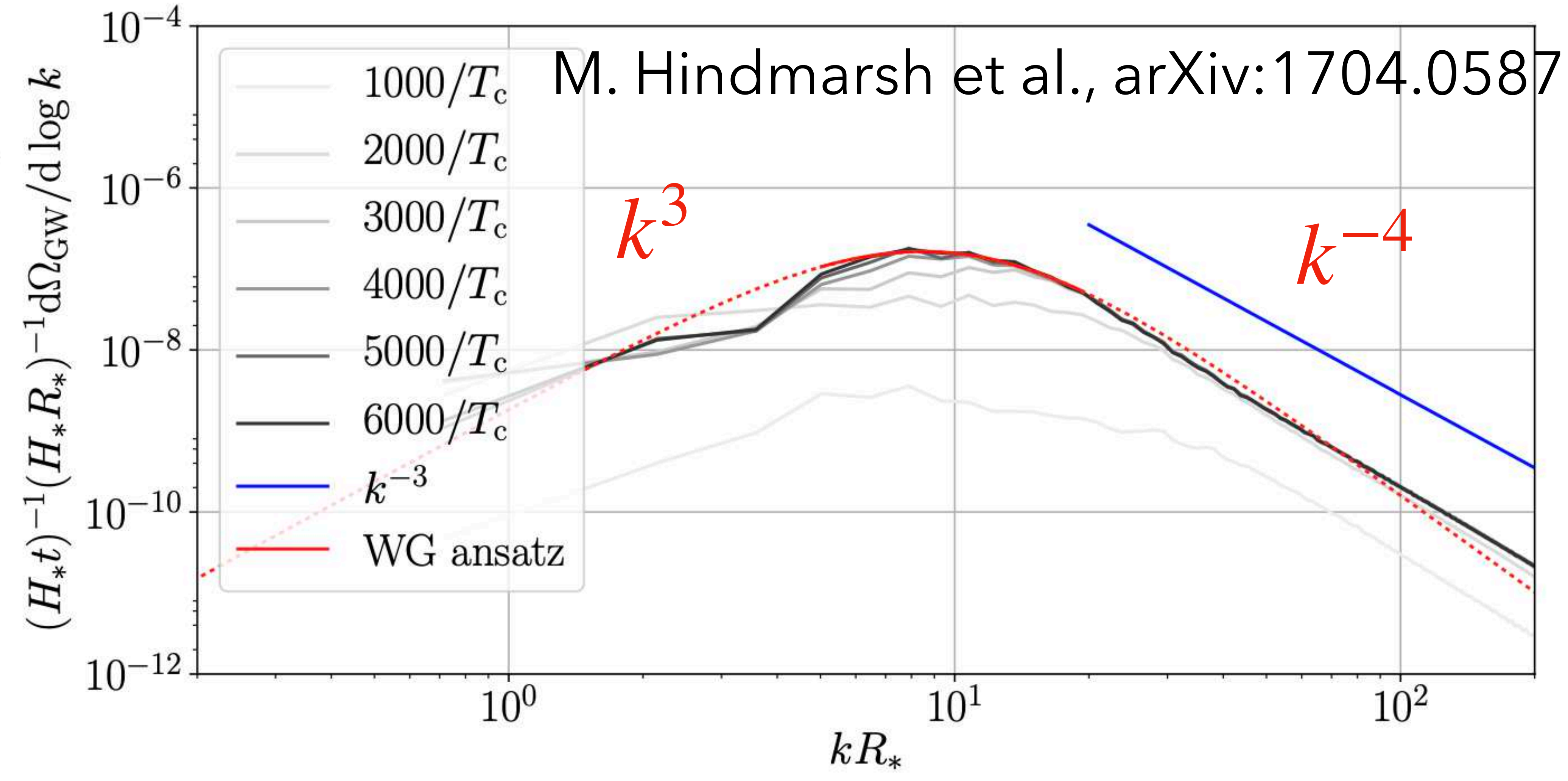
- Sound waves :



M. Hindmarsh and M. Hijazi, arXiv:1909.10040

(b) Intermediate, $v_w = 0.92$

M. Hindmarsh et al., arXiv:1704.05871



(b) Intermediate, $v_w = 0.92$

Simulations:

Introduction

GW sources during the phase transitions:

R. Jinno et al., arXiv:1605.01403

- Bubble collisions :

$$\frac{f_{\text{peak}}}{\beta} = \frac{0.35}{1 + 0.069v + 0.69v^4}$$

$$\Delta_{\text{peak}} = \frac{0.48v^3}{1 + 5.3v^2 + 5.0v^4}$$

- Sound waves :

M. Hindmarsh et al., arXiv:1704.05871

$$\mathcal{P}_{\text{gw}}(k) = 3\left(\Gamma\overline{U}_{\text{f}}^2\right)^2 (H\tau_{\text{v}})\,(HL_{\text{f}})\,\frac{(kL_{\text{f}})^3}{2\pi^2}\tilde{P}_{GW}(kL_{\text{f}})$$

M. Hindmarsh and M. Hijazi, arXiv:1909.10040

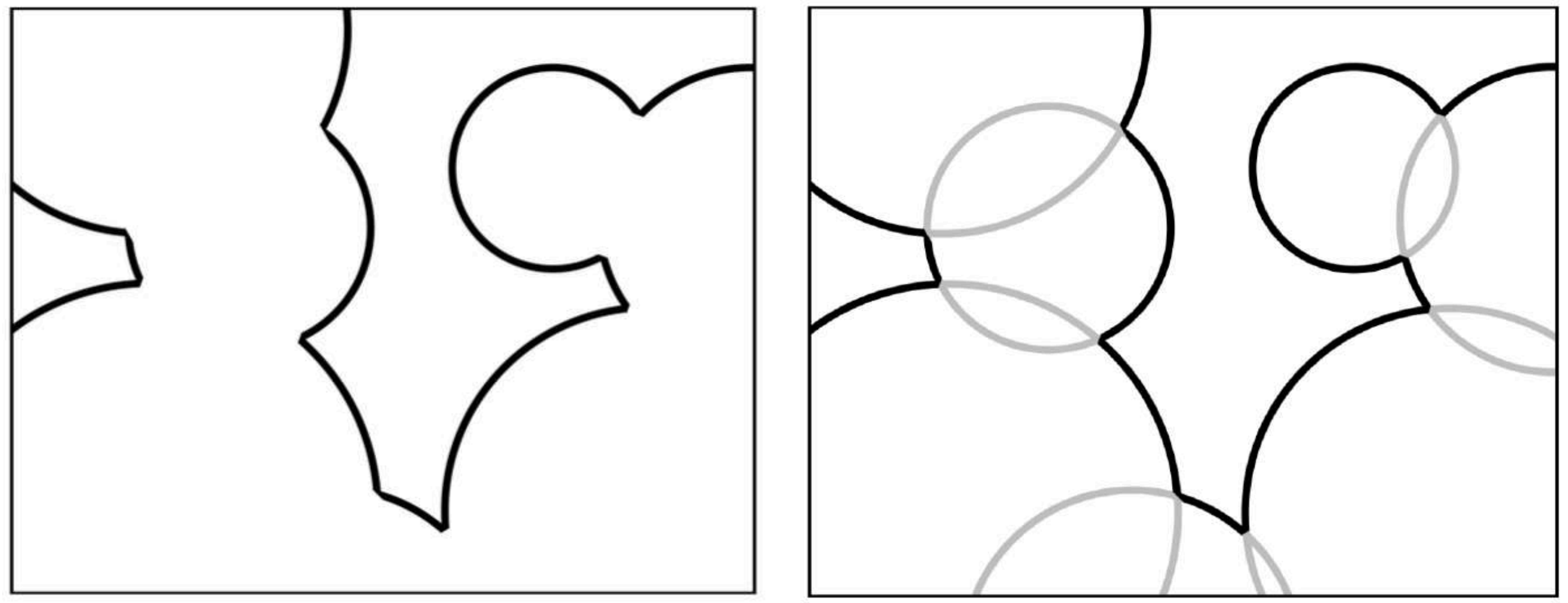
$$\frac{d\Omega_{\text{gw},0}}{d\ln(f)} = 0.68F_{\text{gw},0}\Gamma^2\overline{U}_{\text{f}}^4(H_{\text{n}}R_*)\tilde{\Omega}_{\text{gw}}C\left(\frac{f}{f_{\text{p},0}}\right)$$

No matter which kind of model, the bubble wall velocity is a very important parameter!

Introduction

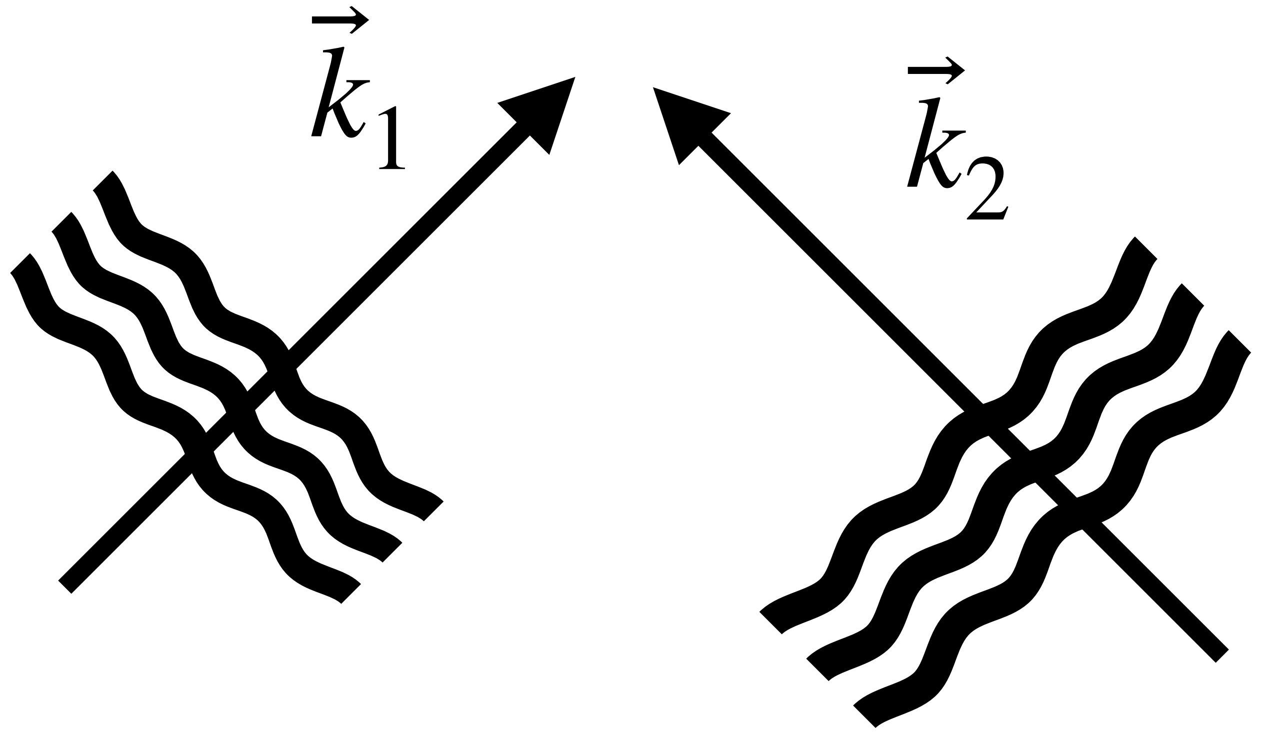
GW sources during the phase transitions:

- Bubble collisions :

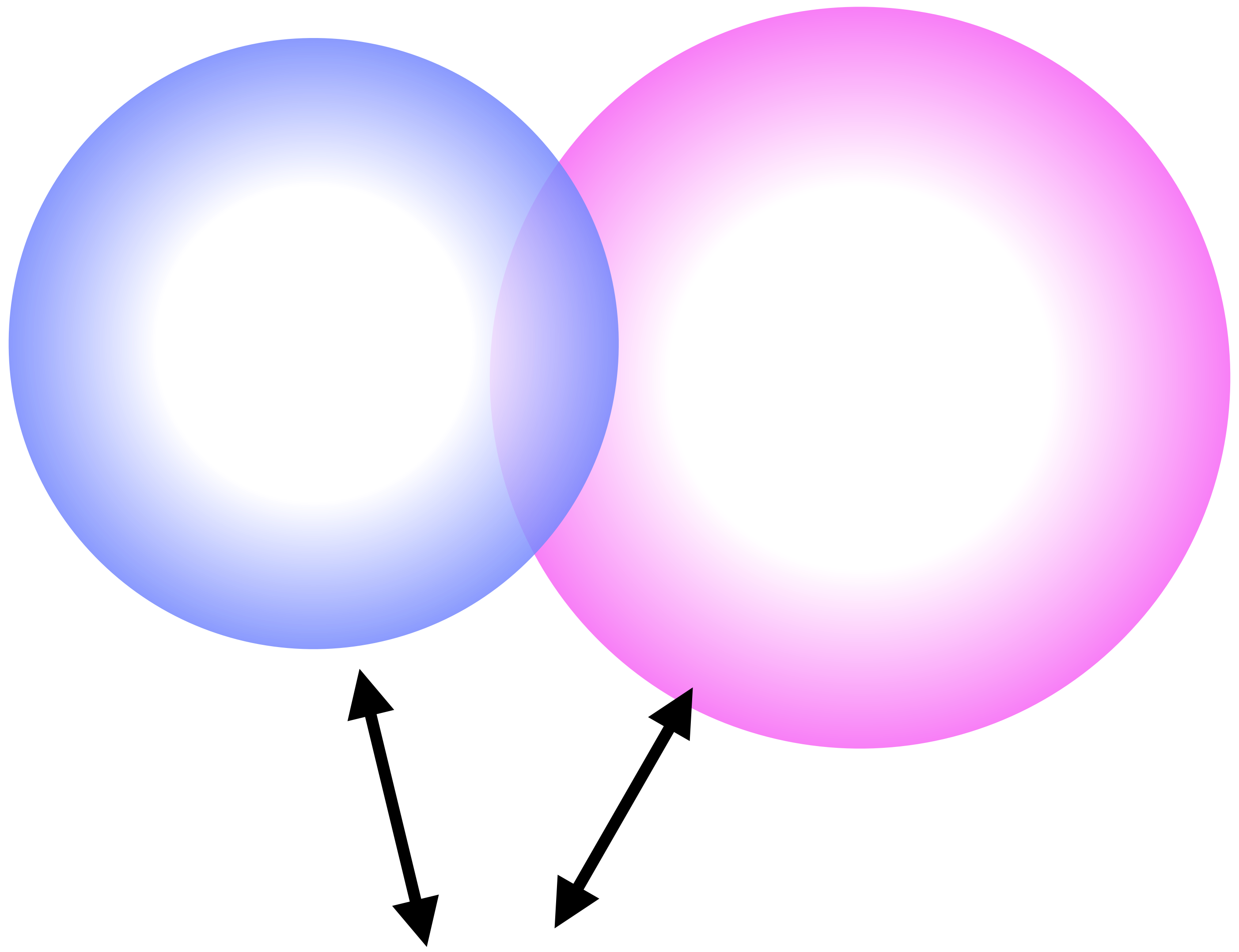


R. Jinno et al., arXiv:1605.01403 arXiv: 1707.03111

- Sound waves :



The process that our model cares



Existence of quadrupole for energy-momentum tensor

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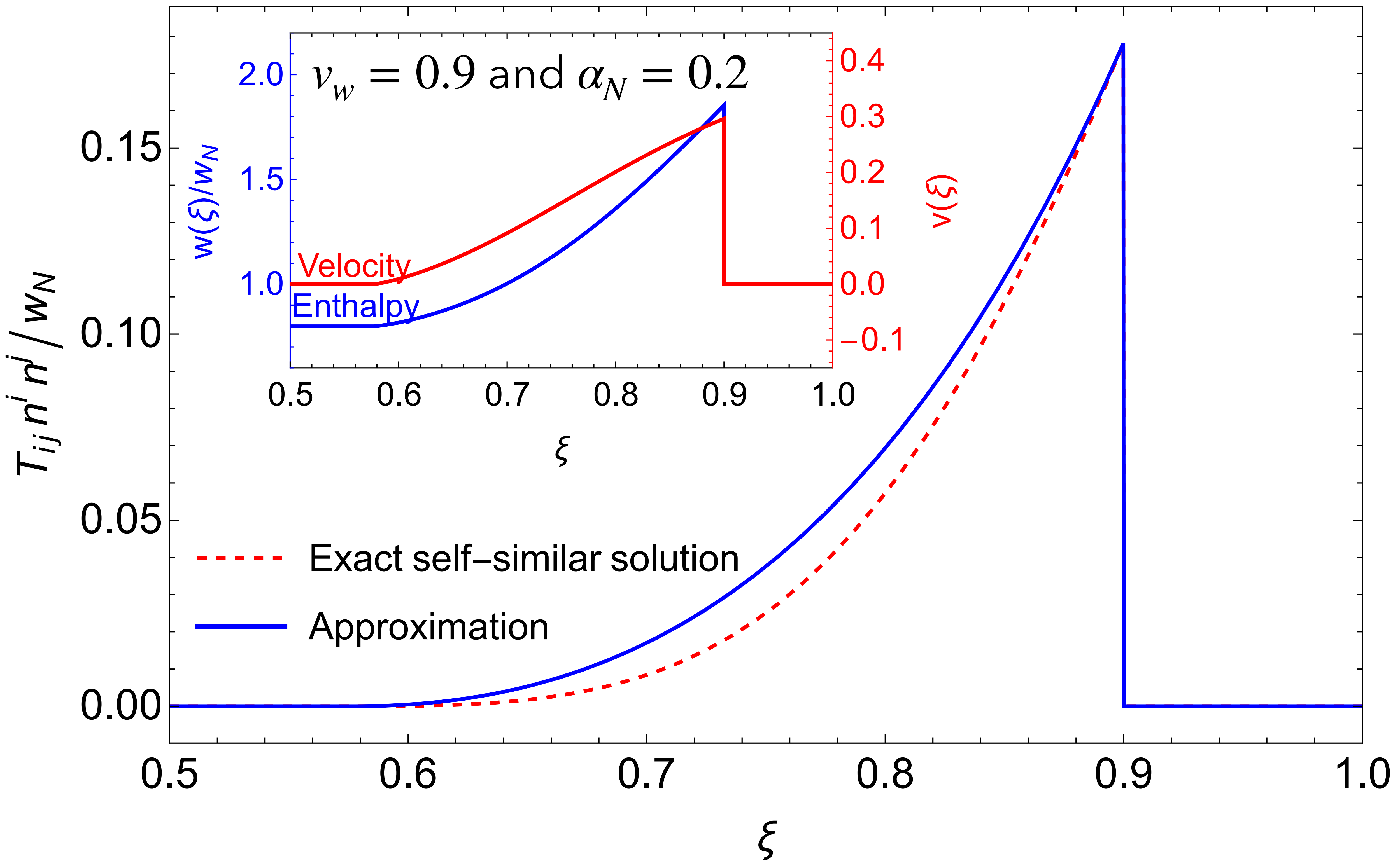
Sound shell model

For a steadily expanding bubble wall, the profiles of fluid velocity and thermodynamic quantities can be described as functions of a self-similar coordinate $\xi = r/t$ alone.



Perfect fluid approximation

$$\hat{T}_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu}$$

Anisotropic, spacial part

$$T_{ij} = w\gamma^2 v_i v_j$$

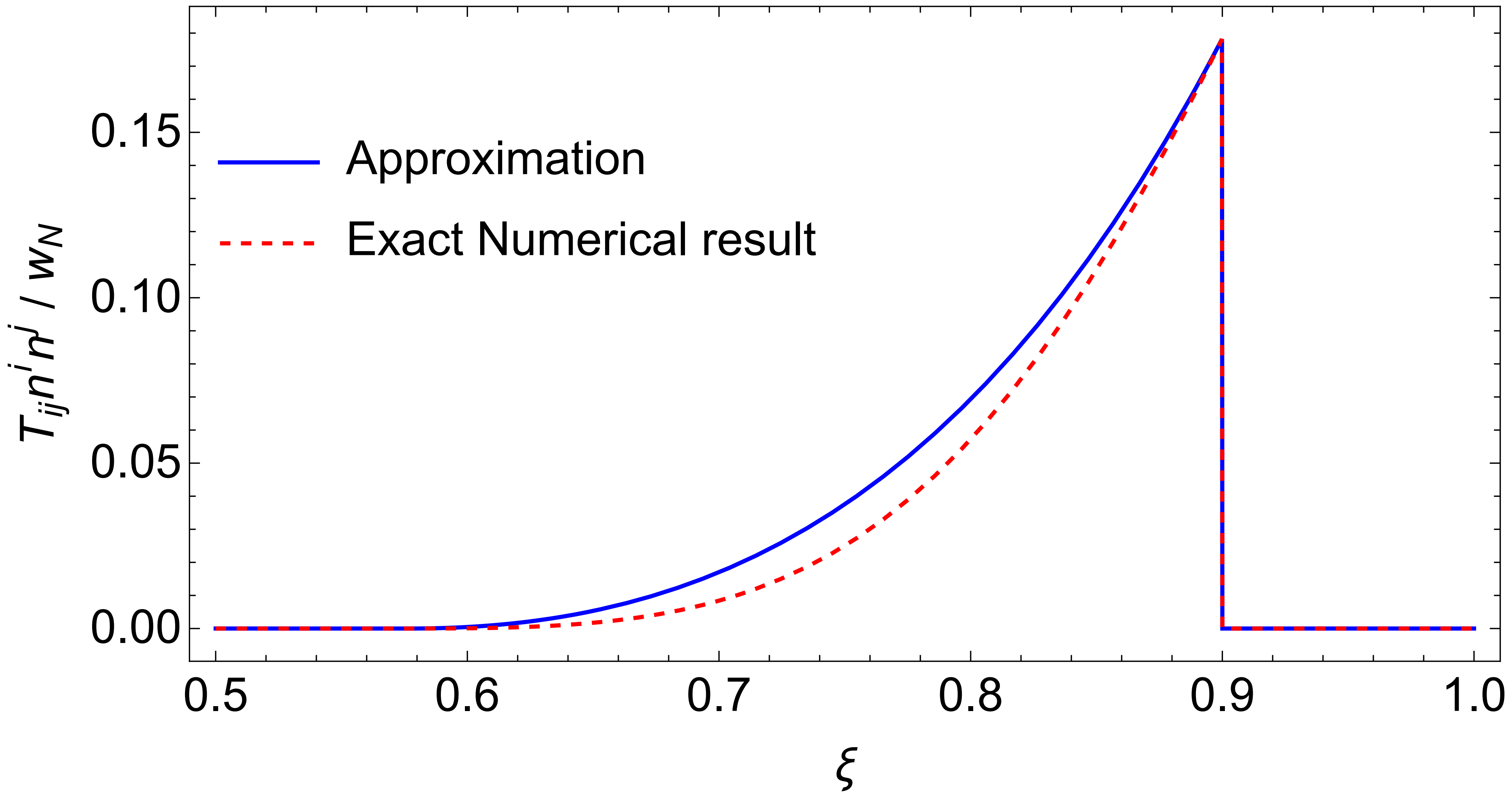
Sound shell model

Linear approximations

$$v_i(t,\vec{x}) = vn_i \simeq \begin{cases} \dfrac{v_m(r-R_1(t))}{R_2(t)-R_1(t)}n_i, & R_1(t) < r < R_2(t) \\ 0, & \text{otherwise} \end{cases}$$

$$\frac{w(t,r)}{w_N} \simeq \begin{cases} \dfrac{w_r(r-R_1(t))}{R_2(t)-R_1(t)}+1, & R_1(t) < r < R_2(t) \\ 1, & \text{otherwise} \end{cases}$$

where $R_1 = c_s(t-t_n)$ and $R_2 = v_w(t-t_n)$

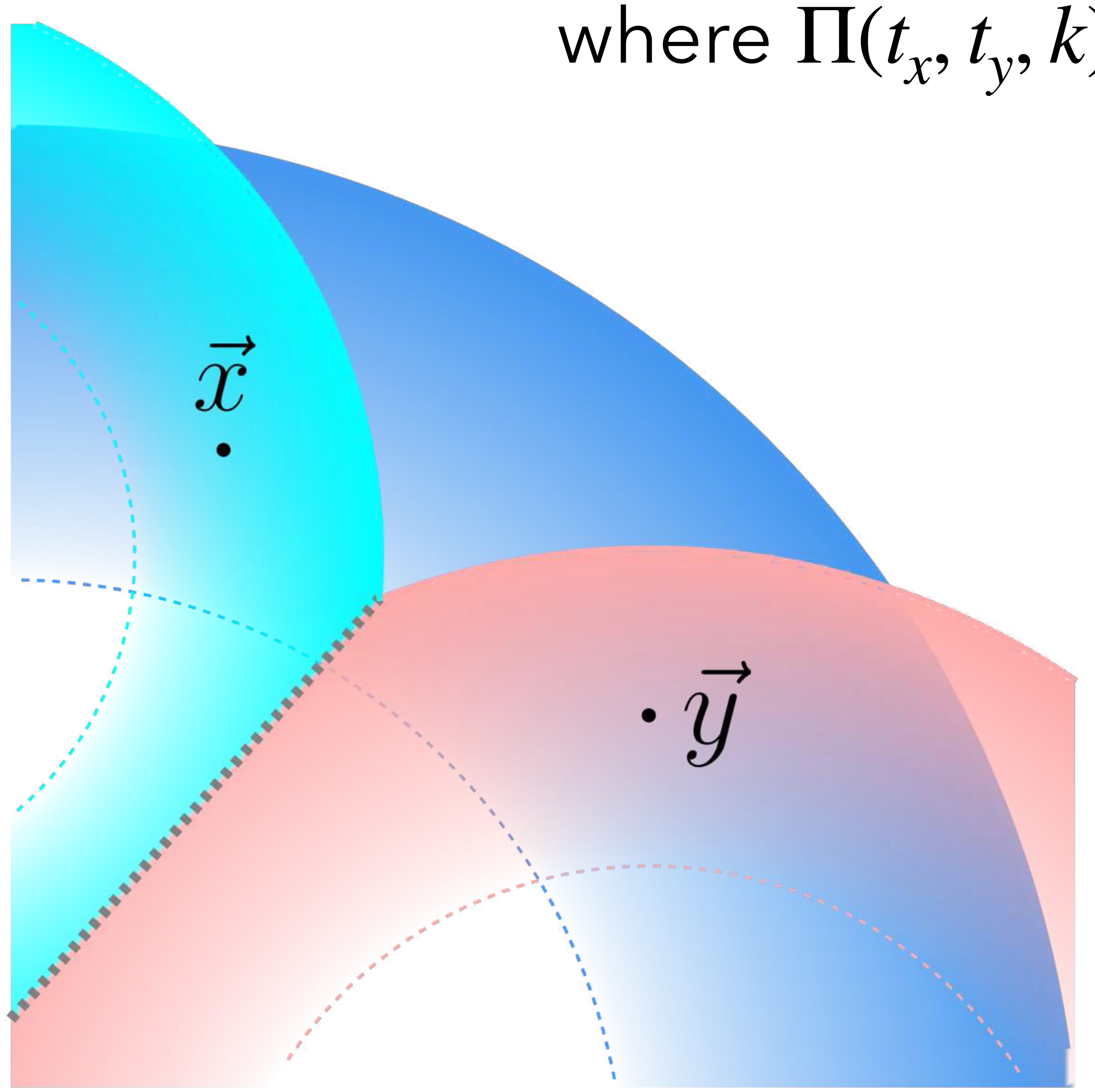


$$\begin{aligned} T_{ij} &= \frac{wv^2}{1-v^2}n_in_j \\ &= w_N\left(\frac{w_r}{v_w-c_s}\frac{r-c_s(t-t_n)}{t-t_n}+1\right) \\ &\quad\times \sum_{s=0}^{\infty}\left(\frac{v_m}{v_w-c_s}\frac{r-c_s(t-t_n)}{t-t_n}\right)^{2s+2} n_in_j\,. \end{aligned}$$

Analytic derivation

$$P_{\rm GW}(t,k) = \frac{1}{\rho_{\rm tot}}\frac{{\rm d}\rho_{\rm GW}}{{\rm d}\ln k} = \frac{2G}{\pi\rho_{\rm tot}}\frac{a_*^4}{a^4}k^3\int_{t_i}^{t_f}{\rm d}t_x\int_{t_i}^{t_f}{\rm d}t_y\cos(k(t_x-t_y))\Pi(t_x,t_y,k) = \frac{2G}{\pi\rho_{\rm tot}}\frac{a_*^4}{a^4}\Delta(k),$$

$$\text{where } \Pi(t_x,t_y,k) = \Lambda_{ij,kl}(\hat{k})\Lambda_{ij,mn}(\hat{k})\int {\rm d}^3r e^{i\vec{k}\cdot\vec{r}}\langle T_{kl}(t_x,\vec{x})T_{mn}(t_y,\vec{y})\rangle\,.$$

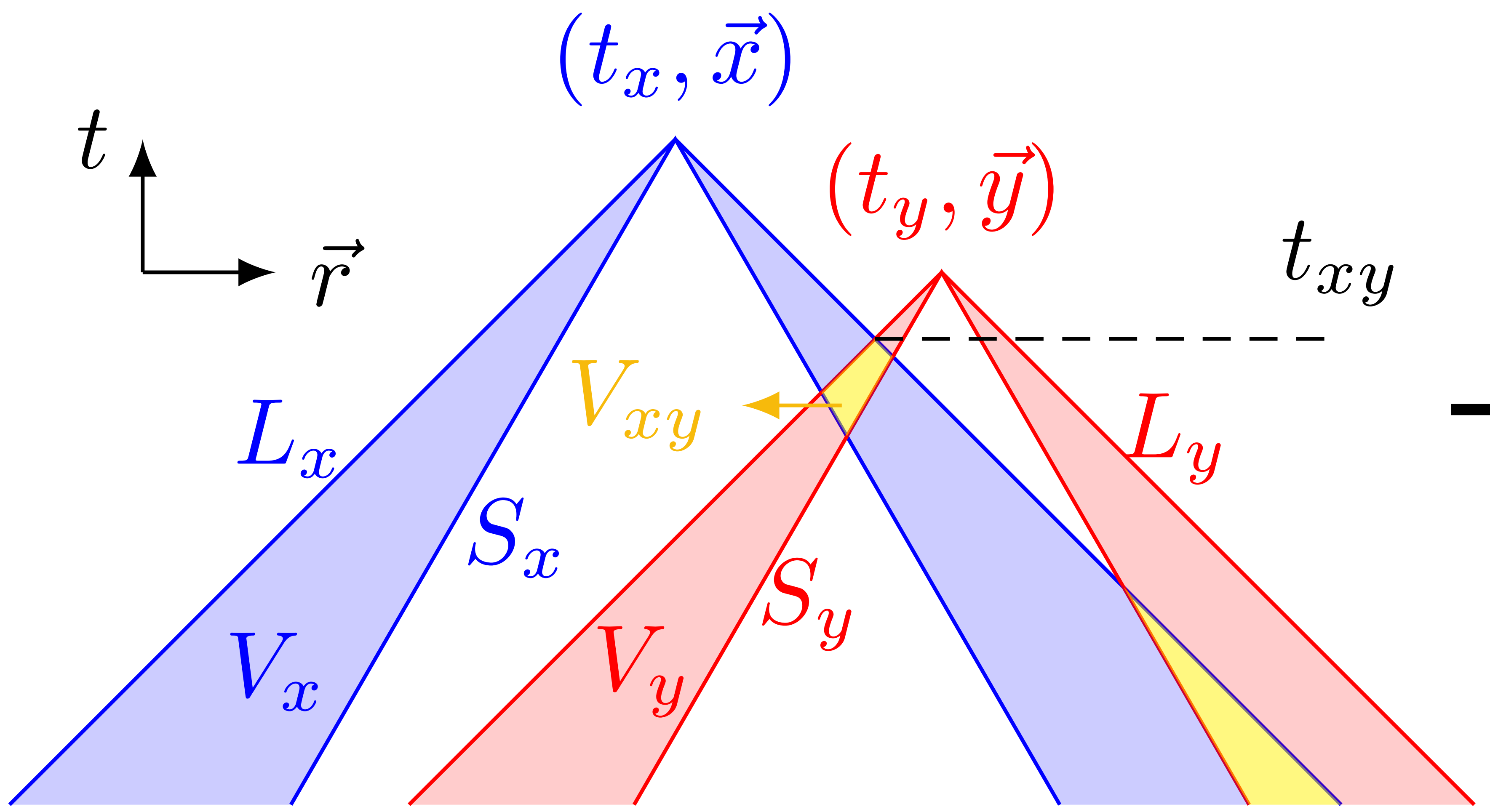


Contributions:

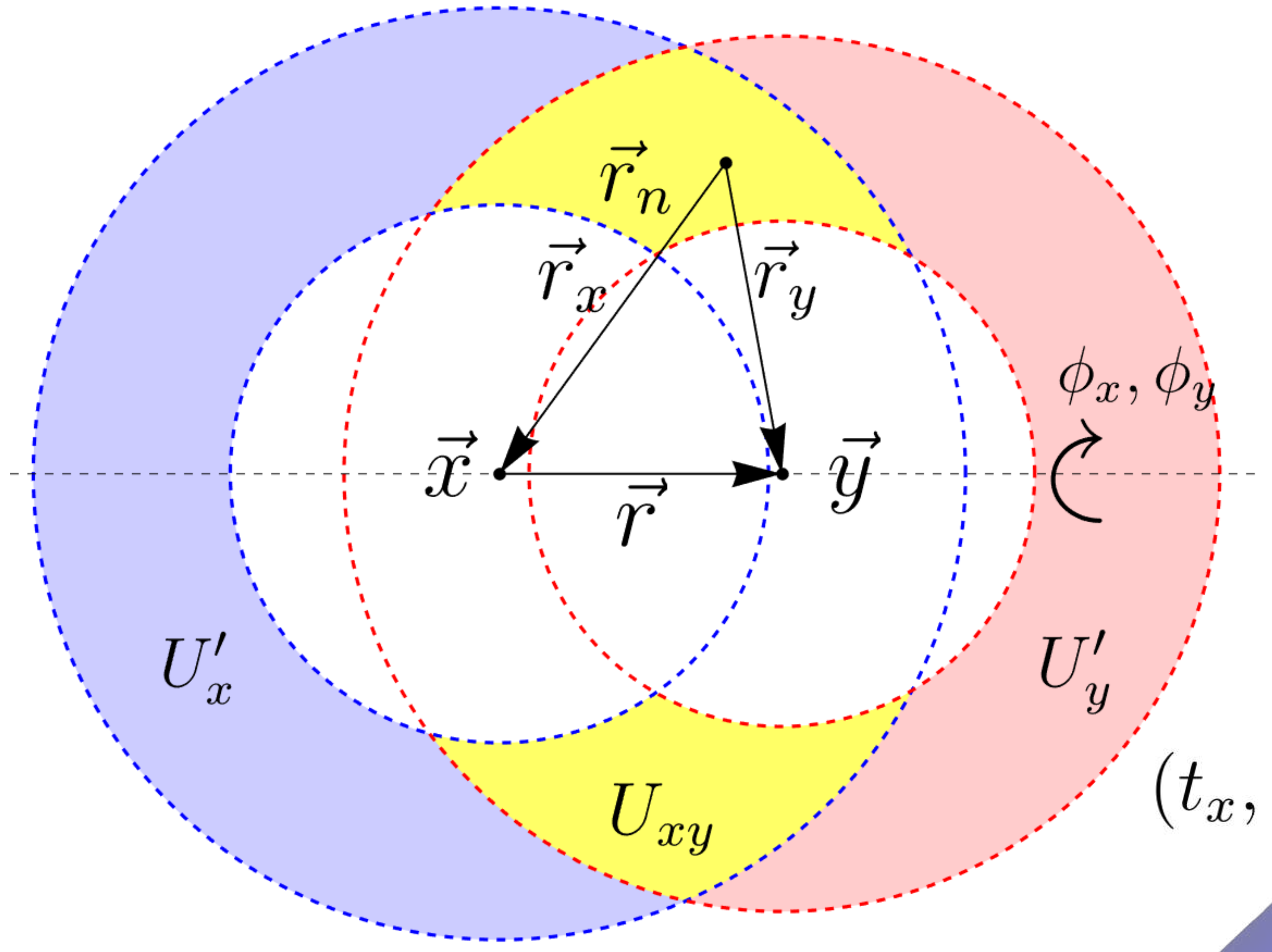
- $\vec{x}$ and $\vec{y}$ are in the same sound shell (single-shell)
- $\vec{x}$ and $\vec{y}$ are in two different sound shells (double-shell)

Analytic derivation

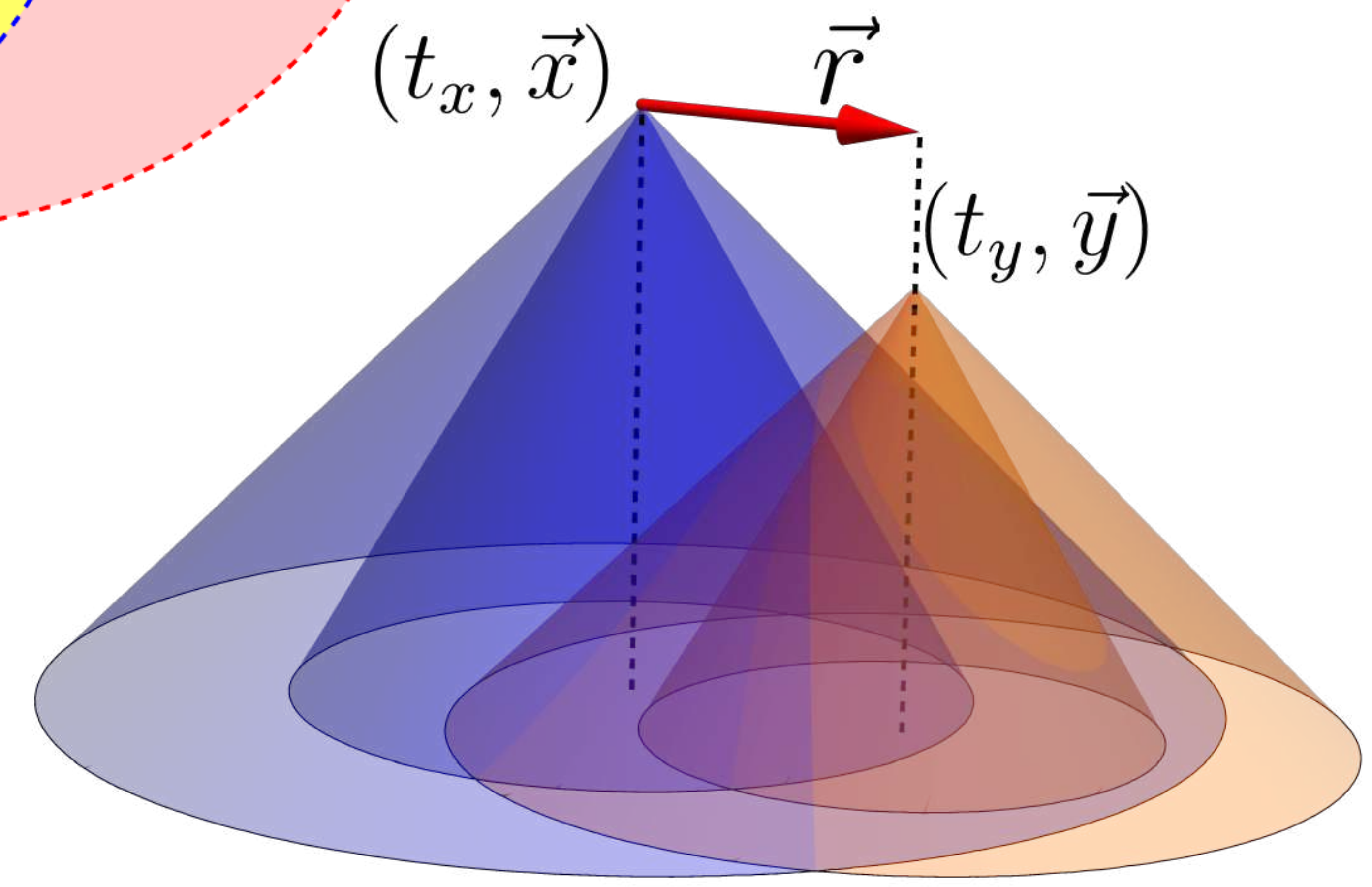
Schematic images for where single-shell and double-shell might come from.



time
slice



1+1 to 2+1



Analytic derivation

The total power spectrum: $\Delta(k) = \Delta(k)^{(s)} + \Delta(k)^{(d)}$

$$\Delta(k)^{(s)} = \frac{2}{v_w^3}\beta^{-2}\tilde{k}^3 \int_0^{+\infty} \mathrm{d}\tilde{t} \int_0^{+\infty} \mathrm{d}\tilde{r} \frac{\cos(\tilde{k}\tilde{t})}{\mathscr{I}(\tilde{t}, \tilde{r}/v_w)} \tilde{r}^6 \left(j_0(\tilde{k}\tilde{r}) F_a + \frac{j_1(\tilde{k}\tilde{r})}{\tilde{k}\tilde{r}} F_b + 2\frac{j_2(\tilde{k}\tilde{r})}{(\tilde{k}\tilde{r})^2} F_c \right)$$

$$\Delta(k)^{(d)} = \frac{1}{2\pi v_w^6}\beta^{-2}\tilde{k}^3 \int_0^{1/v_w} \mathrm{d}\tilde{t} \int_0^{+\infty} \mathrm{d}\tilde{r} \frac{\cos(\tilde{k}\tilde{t})}{\mathscr{I}(\tilde{t}, \tilde{r}/v_w)^2} \tilde{r}^{10} \left(\frac{j_2(\tilde{k}\tilde{r})}{(\tilde{k}\tilde{r})^2} G_x\left(\tilde{t}/\tilde{r}\right) G_y\left(\tilde{t}/\tilde{r}\right) \right)$$

where $\tilde{k} = k/\beta$, $\tilde{t} = t_d\beta$, $\tilde{r} = r\beta$,
and β is given in the bubble nucleation rate $\Gamma(t) = \Gamma(t_*)\exp[\beta(t - t_*)]$

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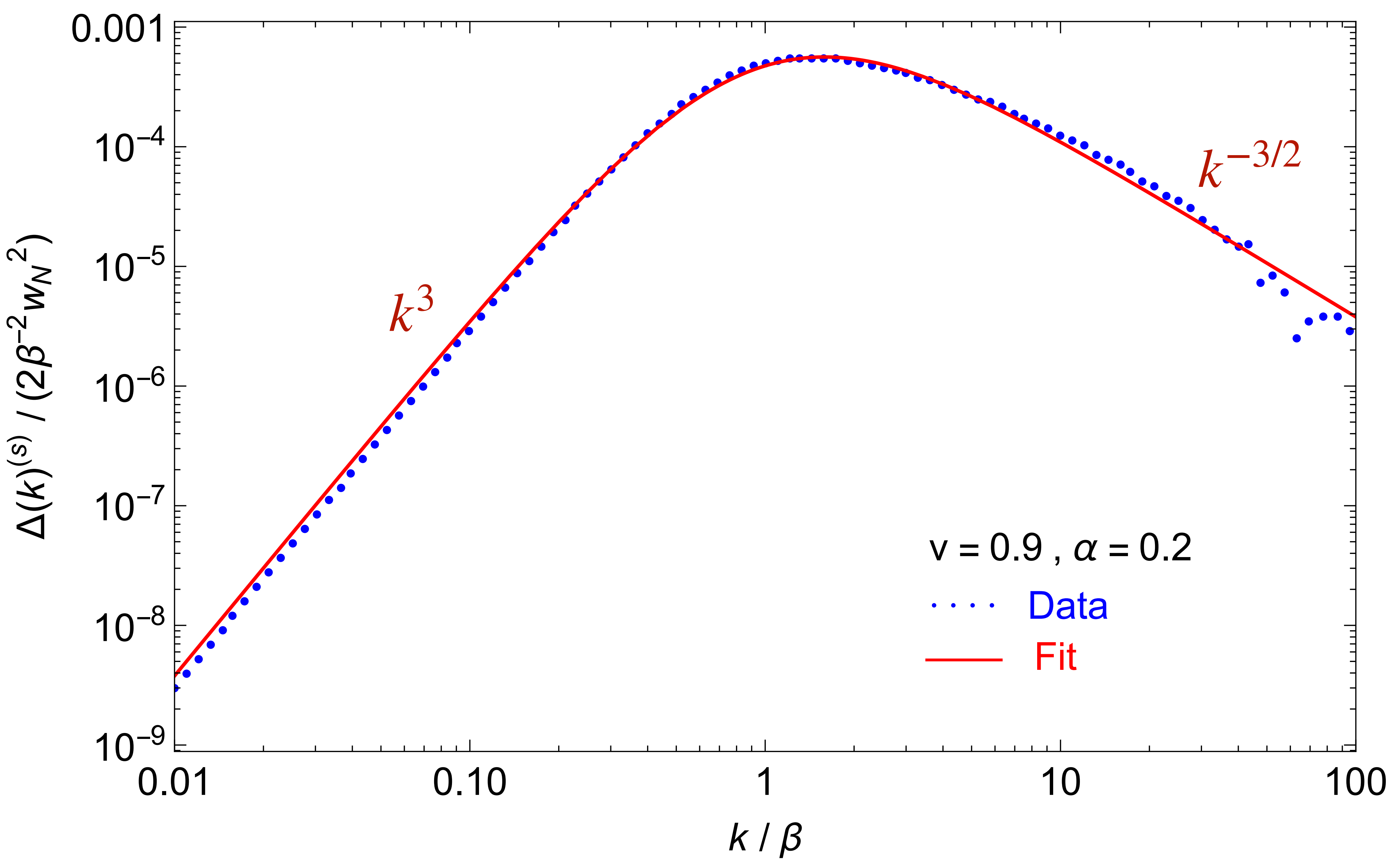
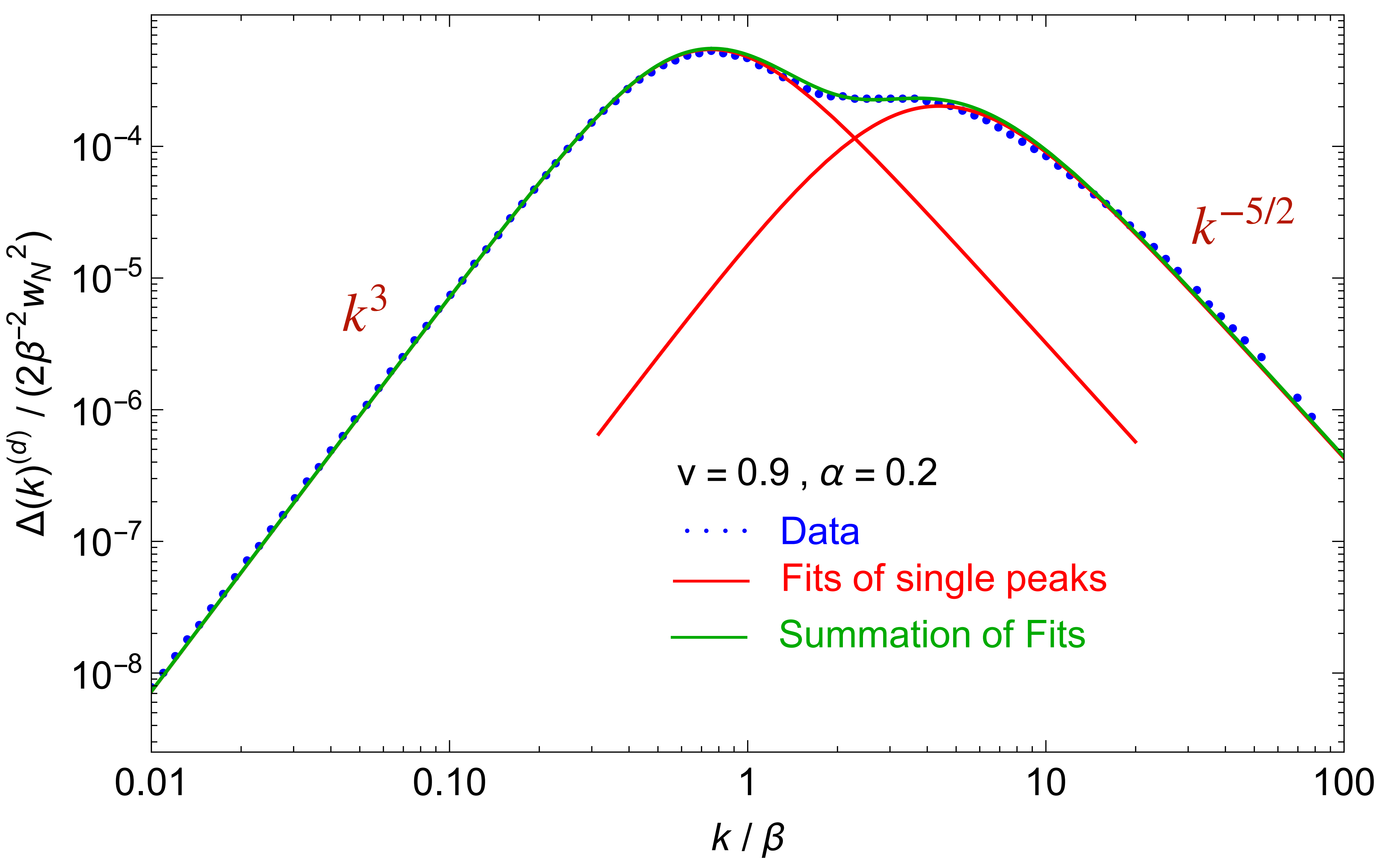
Numerical results

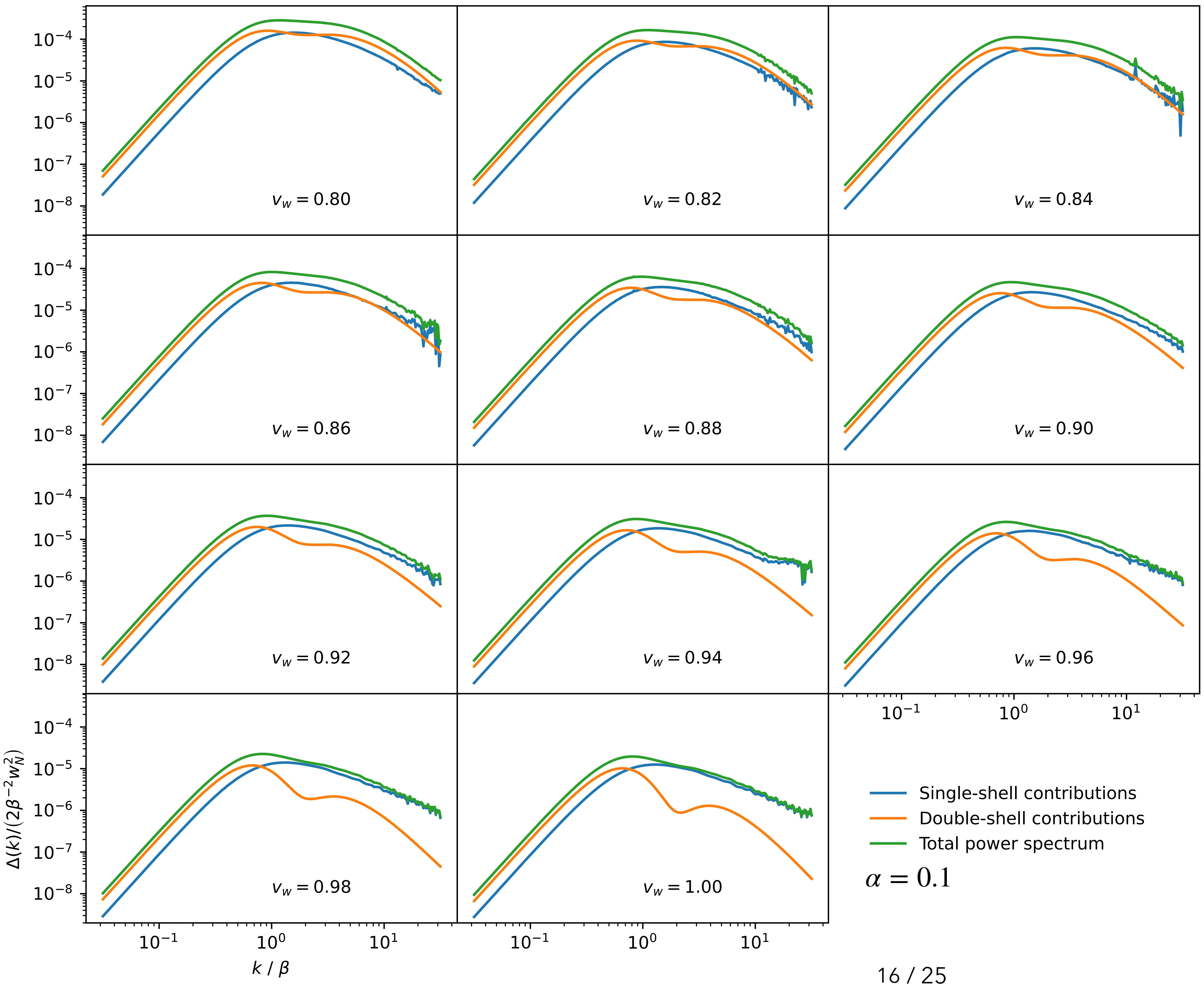
$$F_{n_1,n_2,\Delta}(k,k_*,F_*) = F_*\left(\frac{k}{k_*}\right)^{n_1}\left(\frac{1+(k/k_*)^{\Delta}}{2}\right)^{\frac{n_2-n_1}{\Delta}}$$

$$\Delta^{(s)} = F_{n_1,n_2,\Delta}(k,k_*,F_*)$$

$$\Delta^{(d)} = F_{n_1,n_2,\Delta_1}(k,k_{*1},F_{*2}) + F_{n_1,n_2,\Delta_2}(k,k_{*2},F_{*2})$$







- k^3 power law in low frequencies.
- Double-shell dominates in low frequencies, and single-shell gradually dominates in high frequencies as $v_w \to 1$.
- A broader dome as v_w decreases, which is found in numerical simulations.

M. Hindmarsh et al., arXiv:1704.05871

Peak frequency

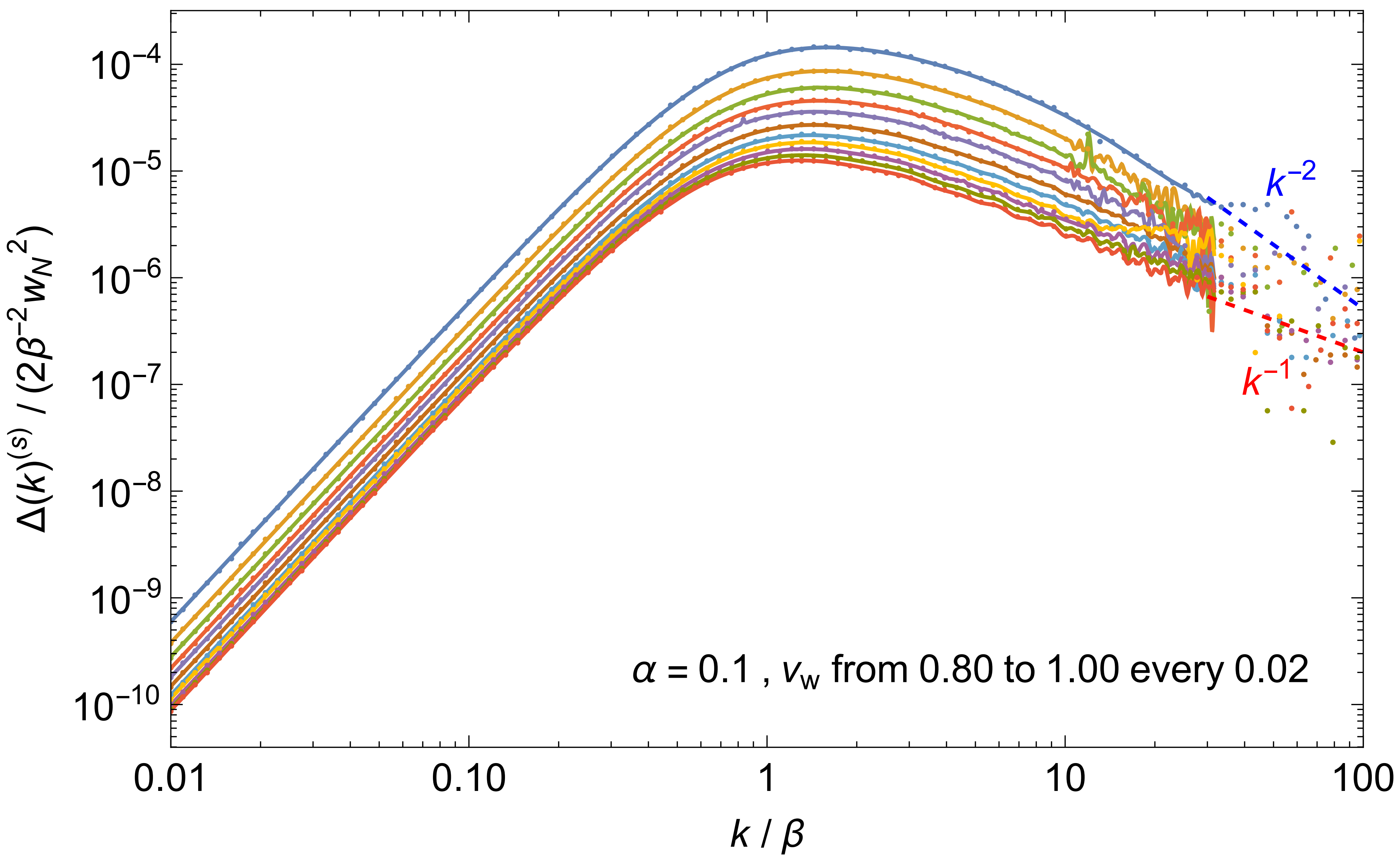
$$k^{(s)}_{\text{peak}} = 3.78 k_{w*}$$

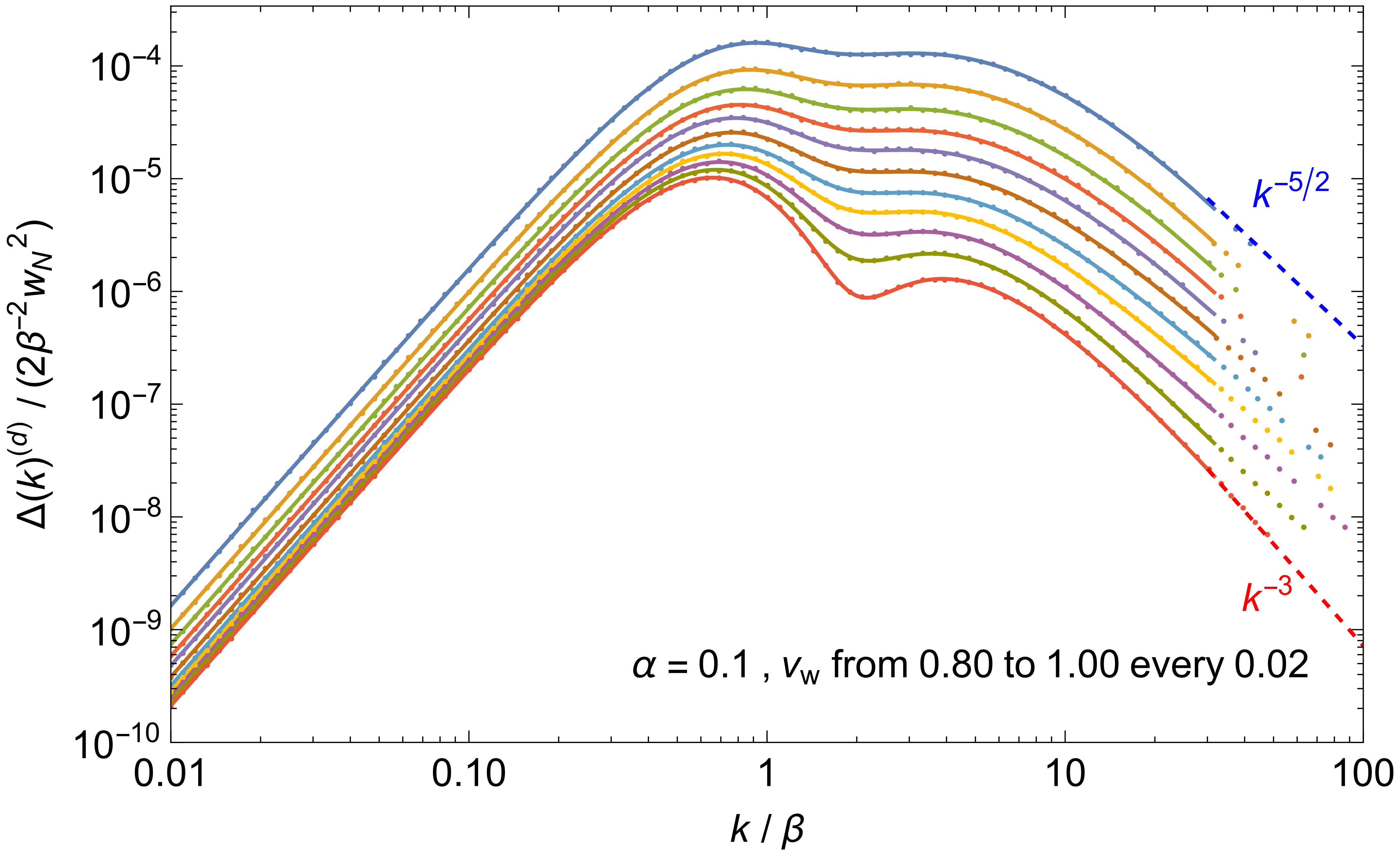
$$k^{(d)}_{\text{peak},1} = 1.54 k_{s*} \left(\frac{k_{s*}}{k_{w*}} \right)^{-0.74}$$

$$k^{(d)}_{\text{peak},2} = \frac{1 - (2 - \delta_1) v_w + (1 + \delta_1) v_w^2}{\delta_2 + (0.1 + \delta_3) v_w - (0.1 - \delta_3) v_w^2}$$

with $\delta_1 = 0.044$, $\delta_2 = 0.0051$, $\delta_1 = 0.0089$

where $k_{w*} = 1/R_* = (8\pi)^{1/3} v_w^{-1} \beta$ and
$k_{s*} = 1/L_{s*} = R_{w*}^{-1} v_w / (v_w - c_s)$





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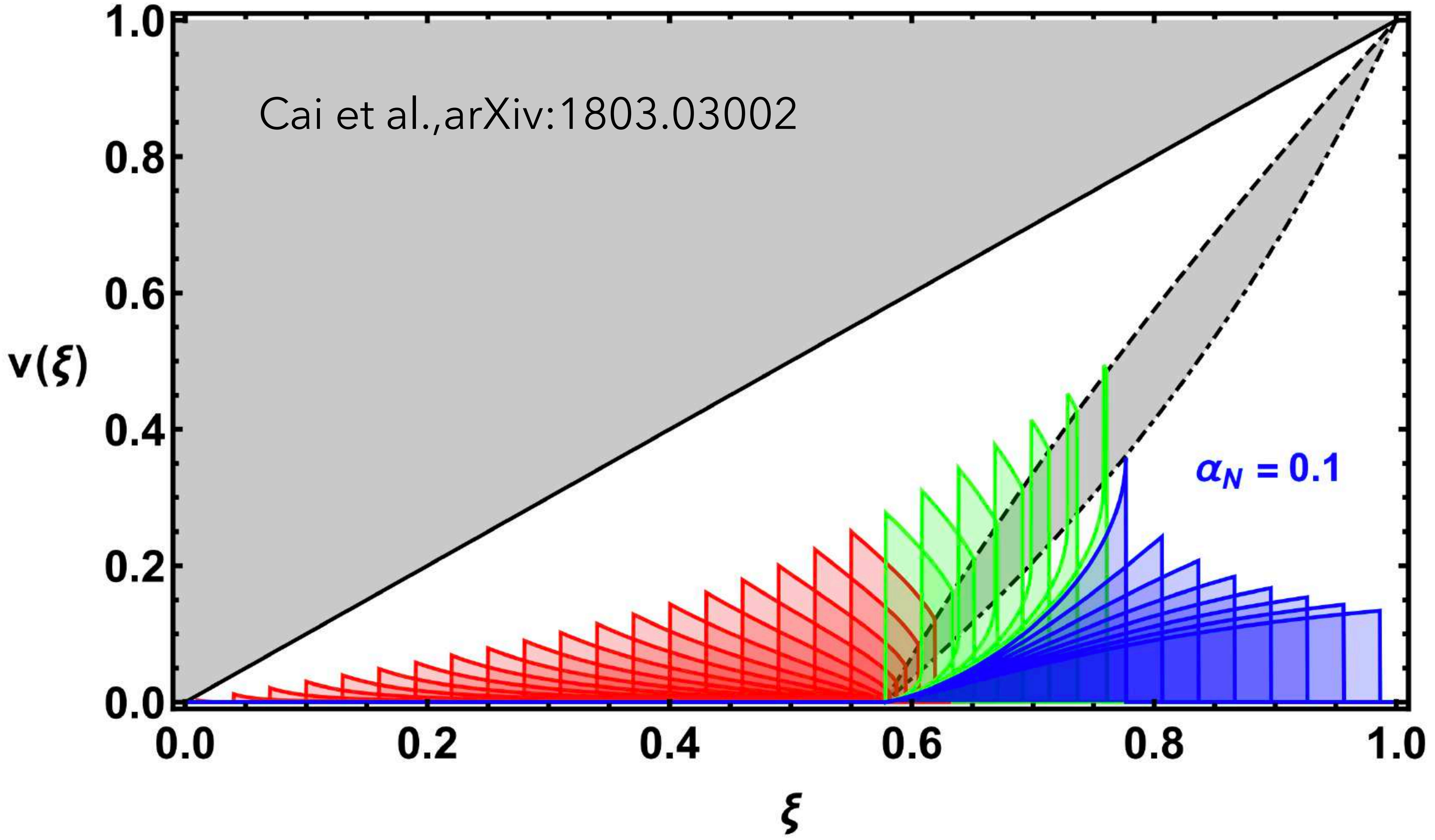
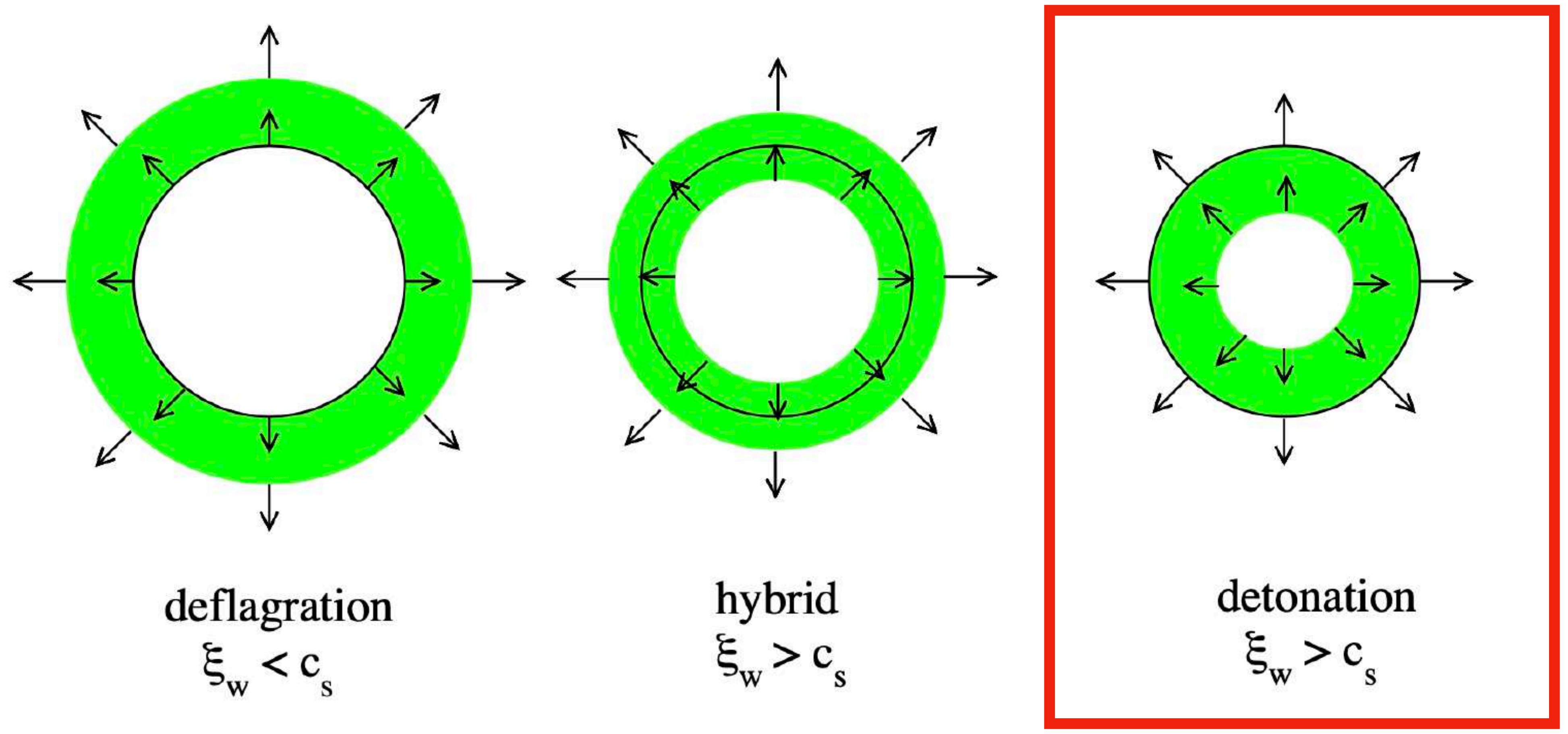
Equation of Motion

The plasma can be described by perfect fluid $T^{\mu\nu} = (\rho + p)u^\mu u^\nu + p\eta^{\mu\nu}$, whose equation of motion is given by $\nabla_\mu T^{\mu\nu} = 0$. Projecting the conservation equation parallel and perpendicular to the bulk flow direction and rewriting the equations in self-similarity coordinate $\xi \equiv |\vec{x}|/t = r/t$ lead to

$$D\frac{v}{\xi} = \gamma^2(1-\xi v)\left(\frac{\mu^2}{c_s^2} - 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi} \qquad \frac{\mathrm{d}w}{\mathrm{d}\xi} = w\gamma^2\mu\left(\frac{1}{c_s^2} + 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}$$



Our sound shell model could only describe this situation



Espinosa et al.,arXiv:1004.4187

Wall velocity

$$w_+\bar{\gamma}_+^2\bar{v}_+ = w_-\bar{\gamma}_-^2\bar{v}_-$$
$$w_+\bar{\gamma}_+^2\bar{v}_+^2 + p_+ = w_-\bar{\gamma}_-^2\bar{v}_-^2 + p_-$$

$$\Leftarrow \quad \nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad T\nabla_\mu(su^\mu) = 0 \; \text{?}$$

The EoM are derived from the conservation of stress tensor in the self-similar frame. The Taub junction condition consists of two conservation laws, the stress tensor and the particle number density conservation, the latter of which is given by

$$\nabla_\mu(nu^\mu) = 0$$

In the self similar frame, applying this to the junction interface $\xi = \xi_0$ results in

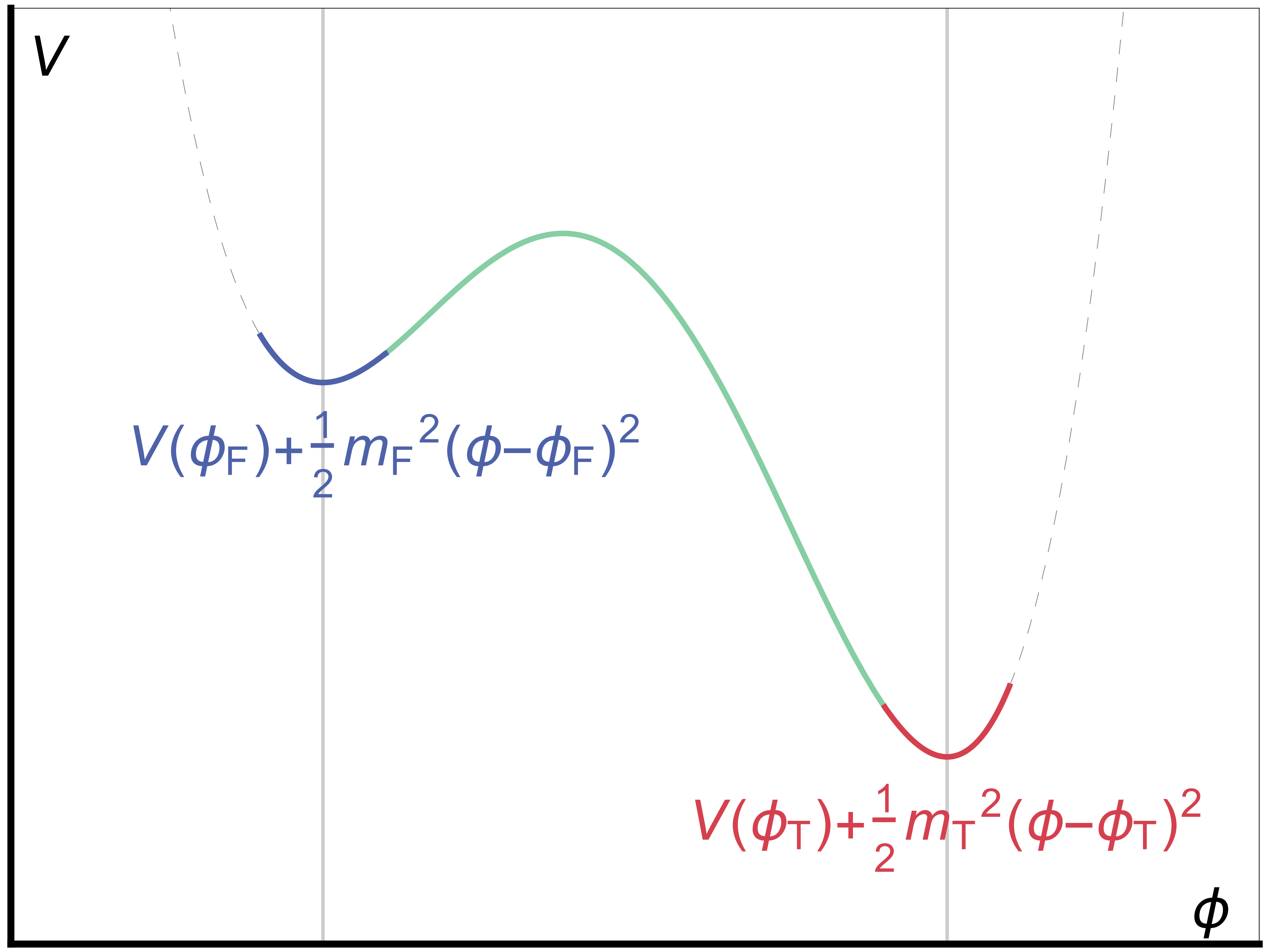
$$n_+\gamma_+(\xi_0 - v_+) = n_-\gamma_-(\xi_0 - v_-)$$

which could be matched with the hydrodynamical solutions of the system.

Wall velocity

Question: how to evaluate the particle number density?

$$\frac{n_+}{n_-} = \frac{n_{f,+} + n_{\phi,+}}{n_{f,-} + n_{\phi,-}} = \frac{b_+ T_+^3 + n_{\phi,+}}{b_- T_-^3 + n_{\phi,-}} = \frac{1 + \dfrac{n_{\phi,+}}{b_+ T_+^3}}{\dfrac{b_-}{b_+}\left(\dfrac{T_-}{T_+}\right)^3 + \dfrac{n_{\phi,-}}{b_+ T_+^3}}$$



$$n_{\phi,0} \simeq V(\phi_0)/m_0$$

$$n_B \simeq \frac{\zeta(3)}{\pi^2} T^3$$

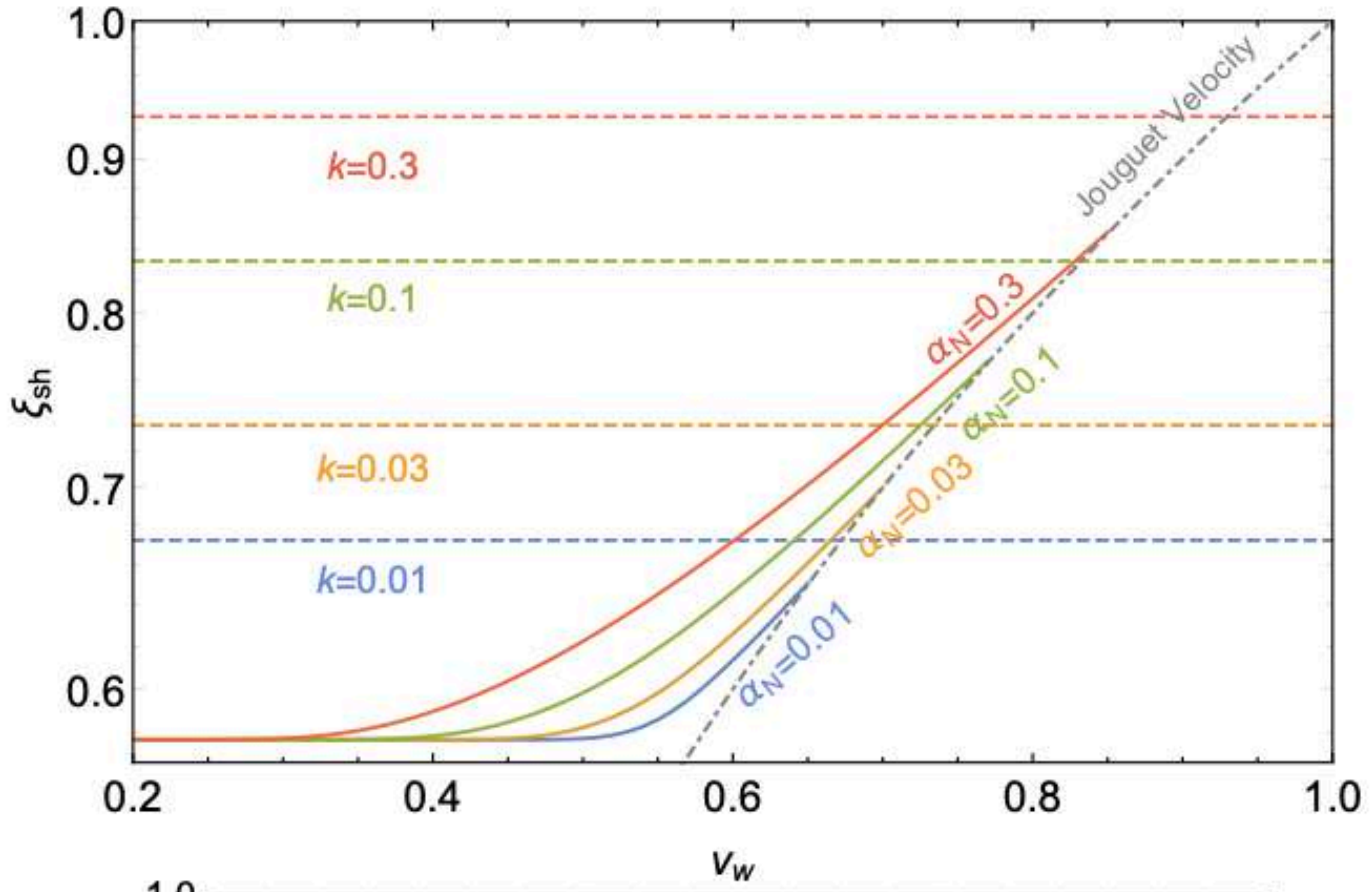
$$n_F \simeq \frac{3}{4}\frac{\zeta(3)}{\pi^2} T^3$$

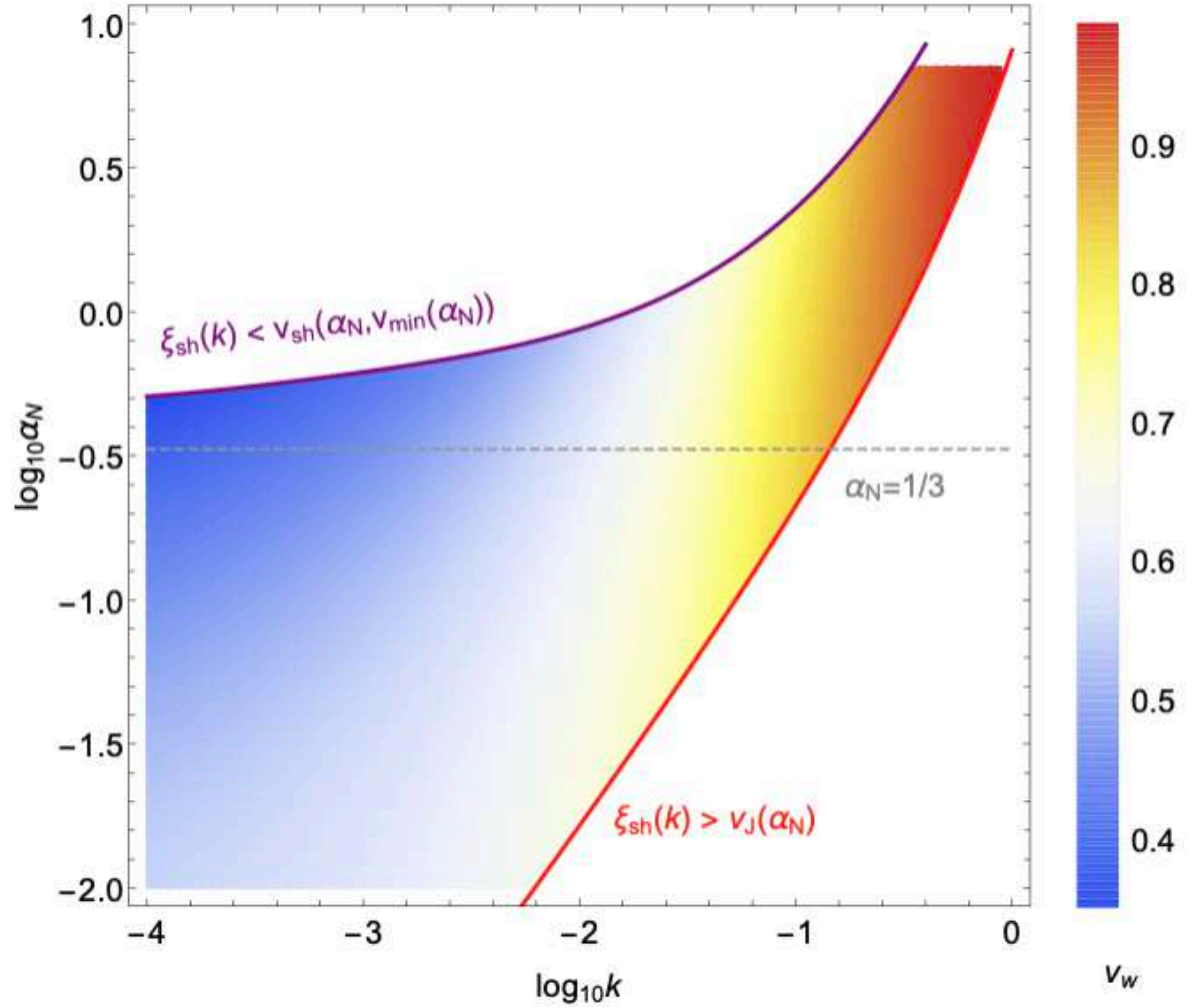
From finite temperature QFT

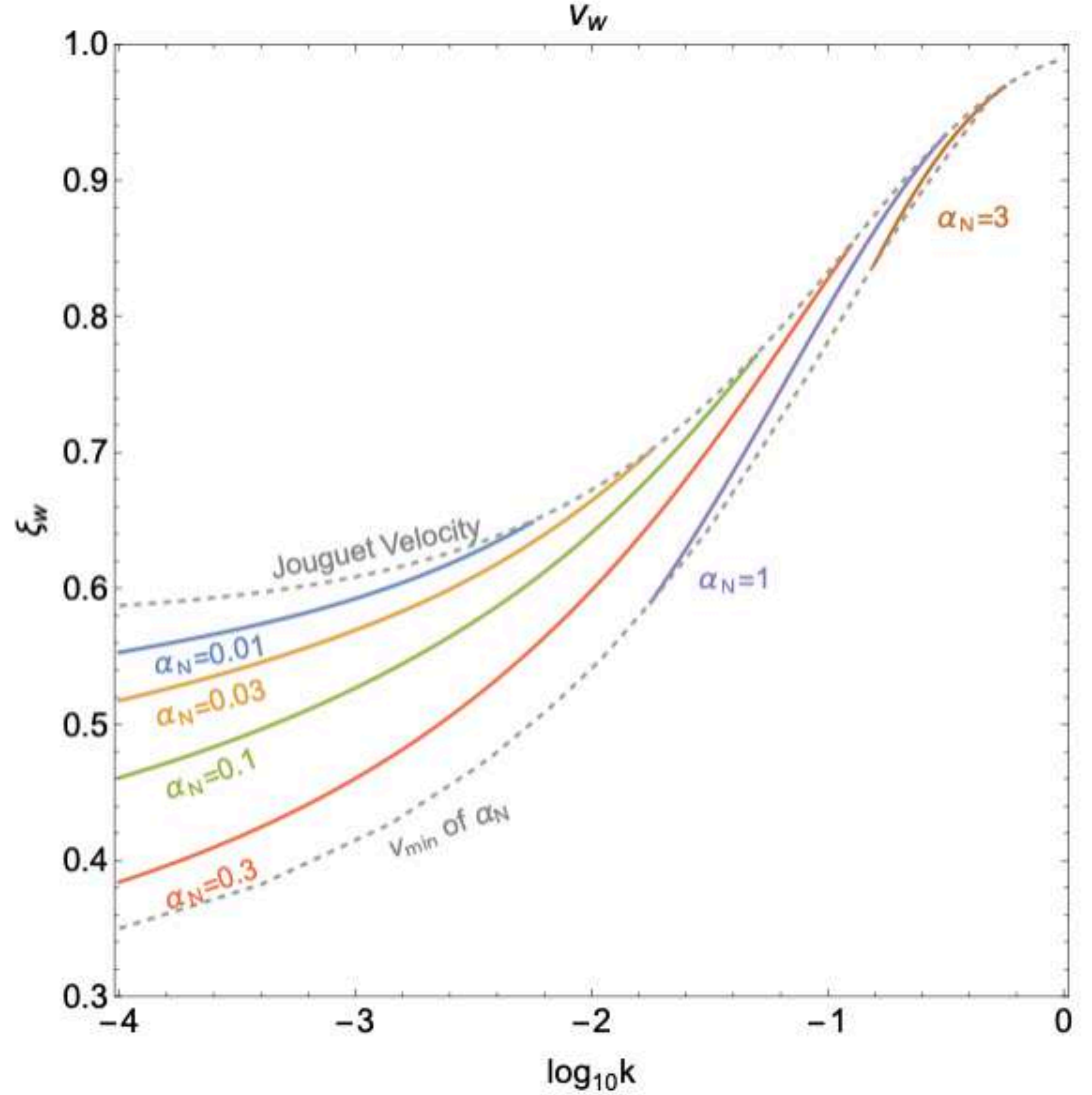
Deflagration and Hybrid

At the shock front $n_{\phi,+} \simeq n_{\phi,-}$ since the whole shock is at the false vacuum

$$\frac{n_+}{n_-}\frac{\xi_{sh}\sqrt{-1+10\xi_{sh}^2-9\xi_{sh}^4}}{1-\xi_{sh}^2} = 1 \quad \Rightarrow \quad \xi_w$$



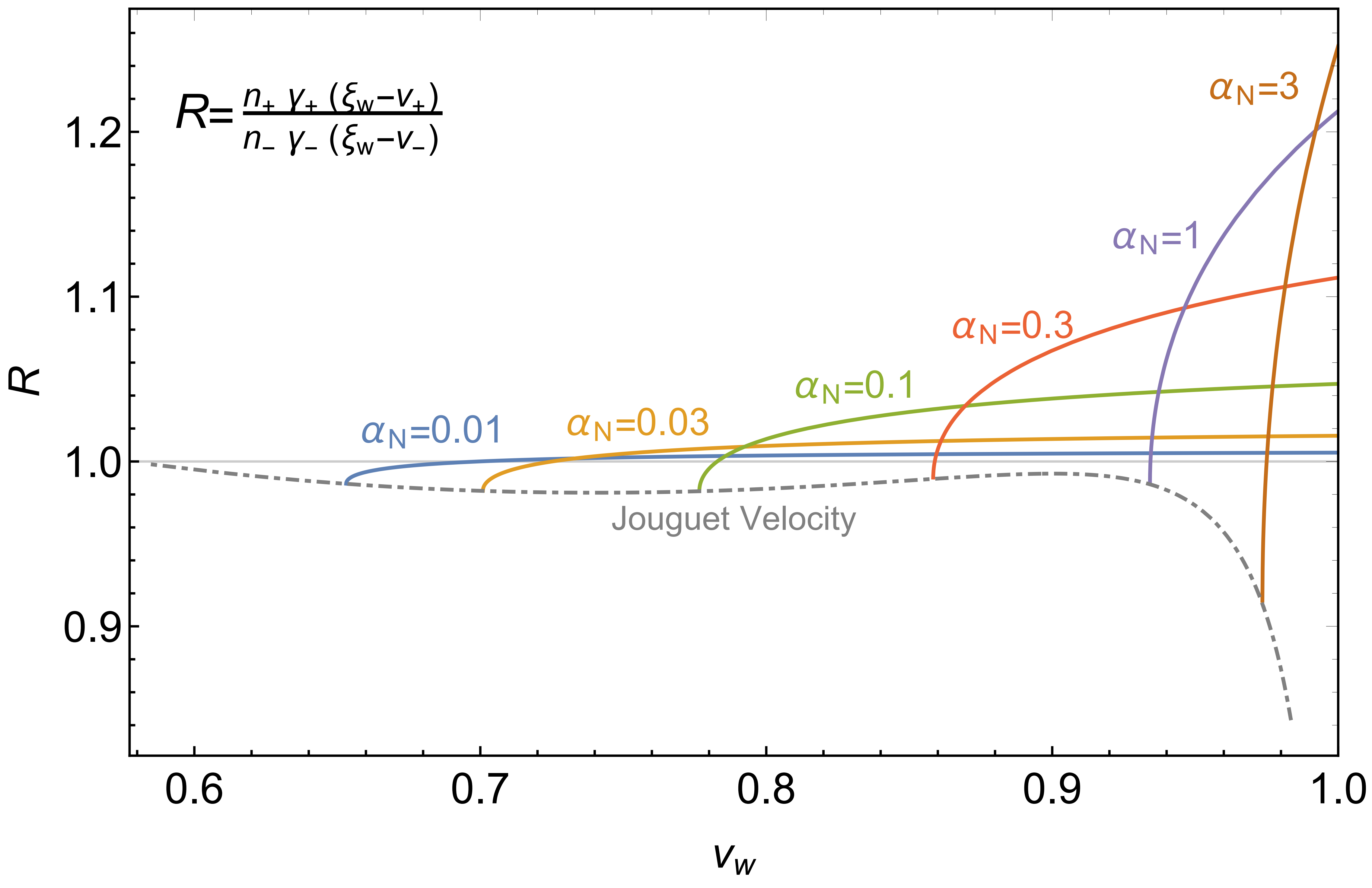


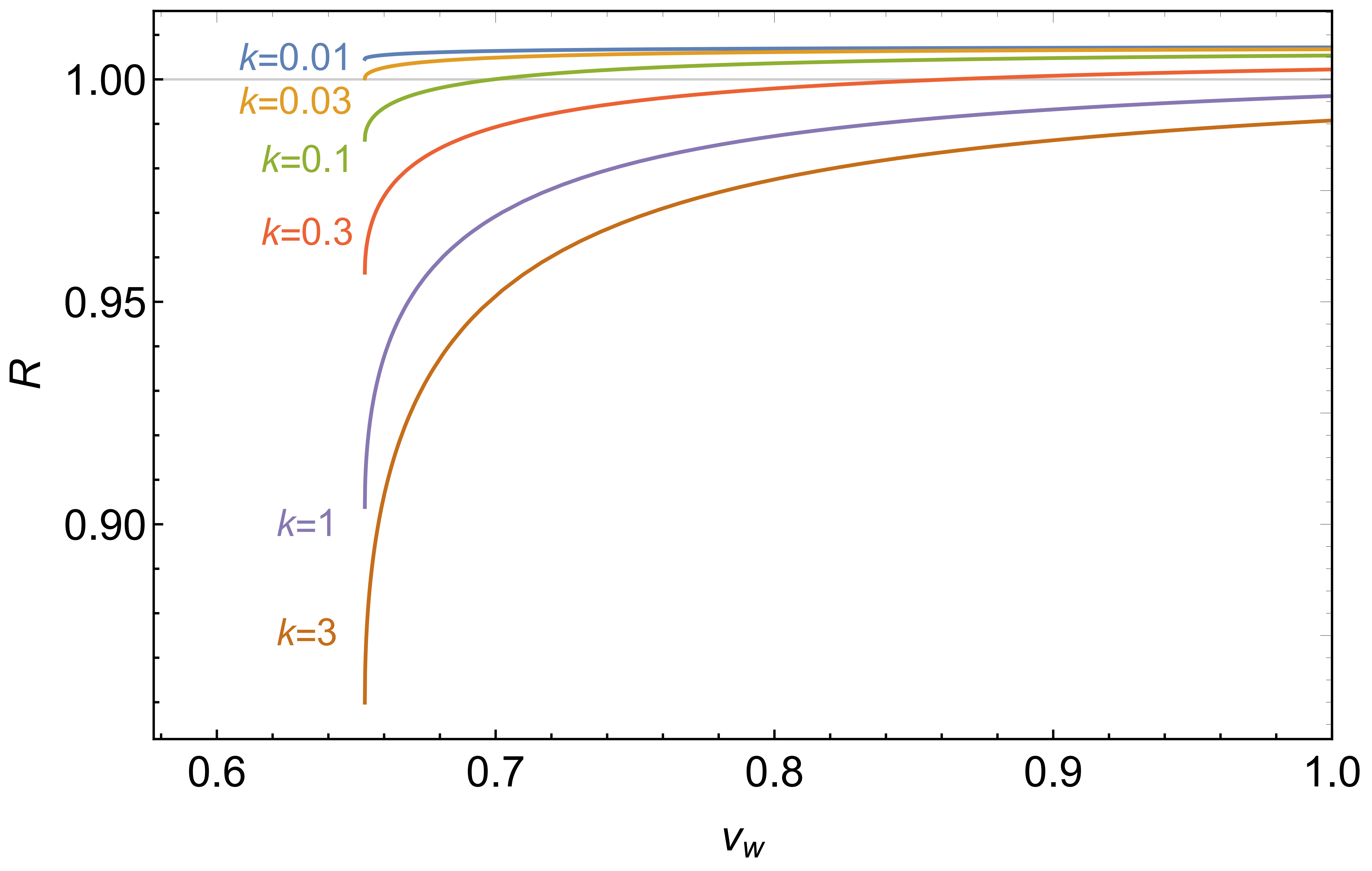


Detonation

$$\frac{n_+}{n_-} = \frac{b_+T_+^3 + n_{\phi,+}}{b_-T_-^3 + n_{\phi,-}} = \frac{1+\dfrac{n_{\phi,+}}{b_+T_+^3}}{\dfrac{b_-}{b_+}\left(\dfrac{T_-}{T_+}\right)^3 + \dfrac{n_{\phi,-}}{b_+T_+^3}} = \frac{1+k_+}{\dfrac{b_-}{b_+}\left(\dfrac{w_-}{w_+}\Big/\dfrac{a_-}{a_+}\right)^{3/4} + k_-}$$

$$R \equiv \frac{n_+}{n_-}\cdot\frac{v_w}{\gamma_-(v_w - v_-)} = \Gamma_w$$





Contents

Discussions

We analytically derived the GW power spectrum from the sound shell during the bubble collisions, as a summation of two contributions: single-shell and double-shell.

- Numerical results and fitting formulas for power spectrum are provided.
- Low frequency k^3 ; High frequency $\longrightarrow$ Single-shell $k^{-2} \to k^{-1}$ as $v_w \to 1$; Double-shell $k^{-5/2} \to k^{-3}$ as $v_w \to 1$
- A broader dome is discovered because of the double-shell domination at low v_w.

We also provided a method to estimate the bubble wall velocity with steady expansion from the conservation of particle number density flow, which works well for two kinds of expansion modes. For detonation, further studies on the corresponding field theories are needed to reach a better result.

Thank You!