Hydrodynamic sound shell model

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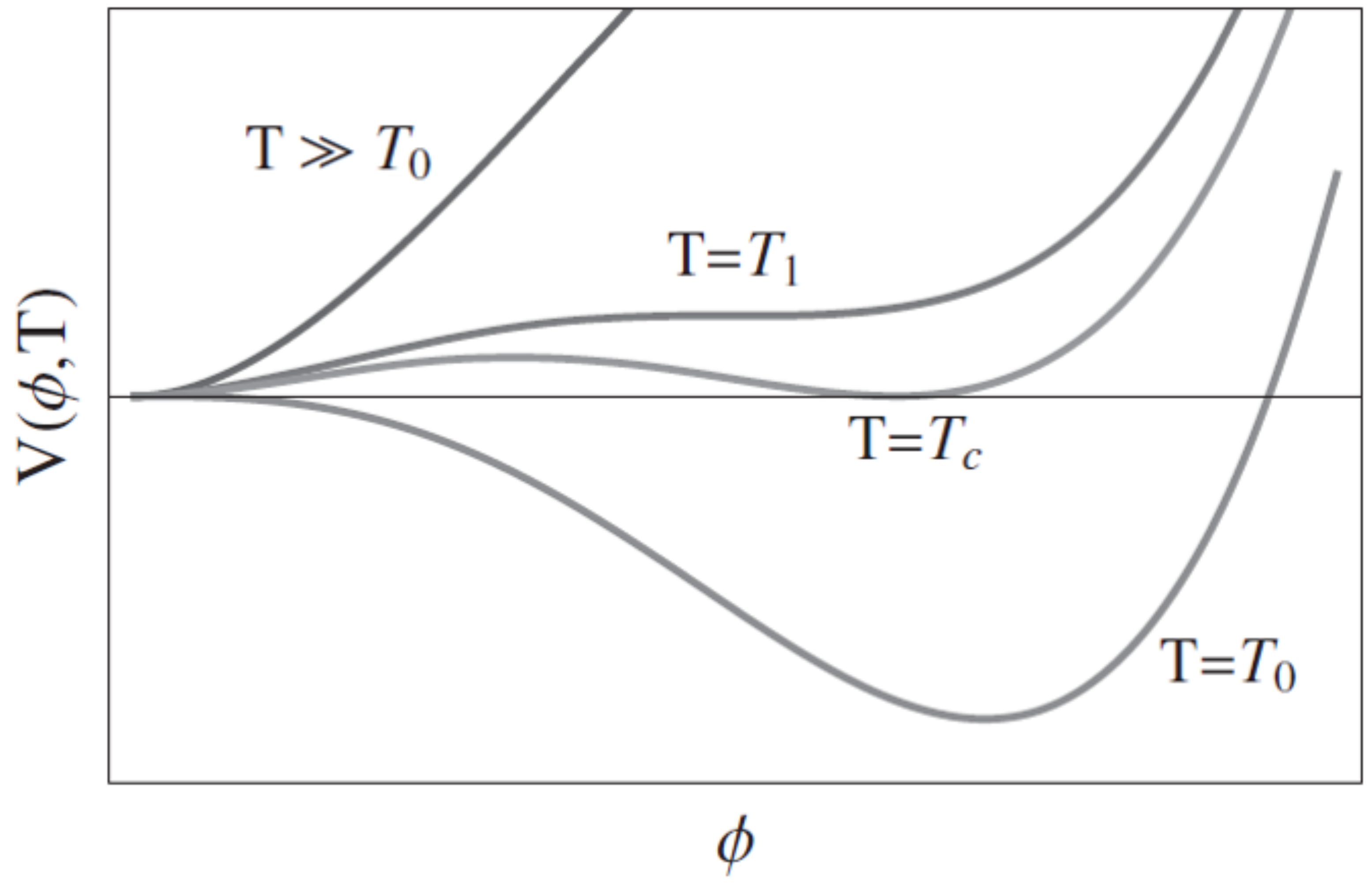
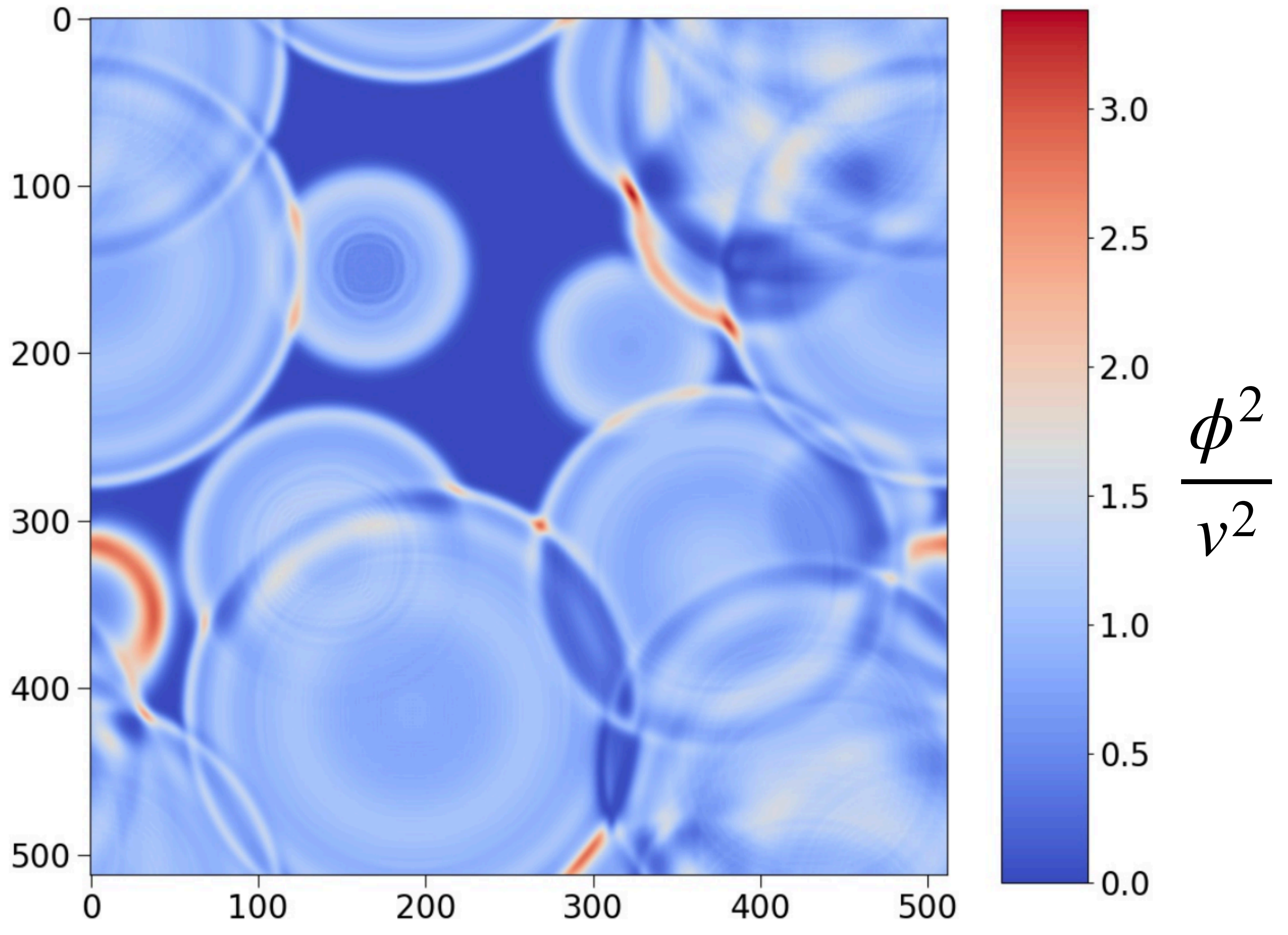
Introduction

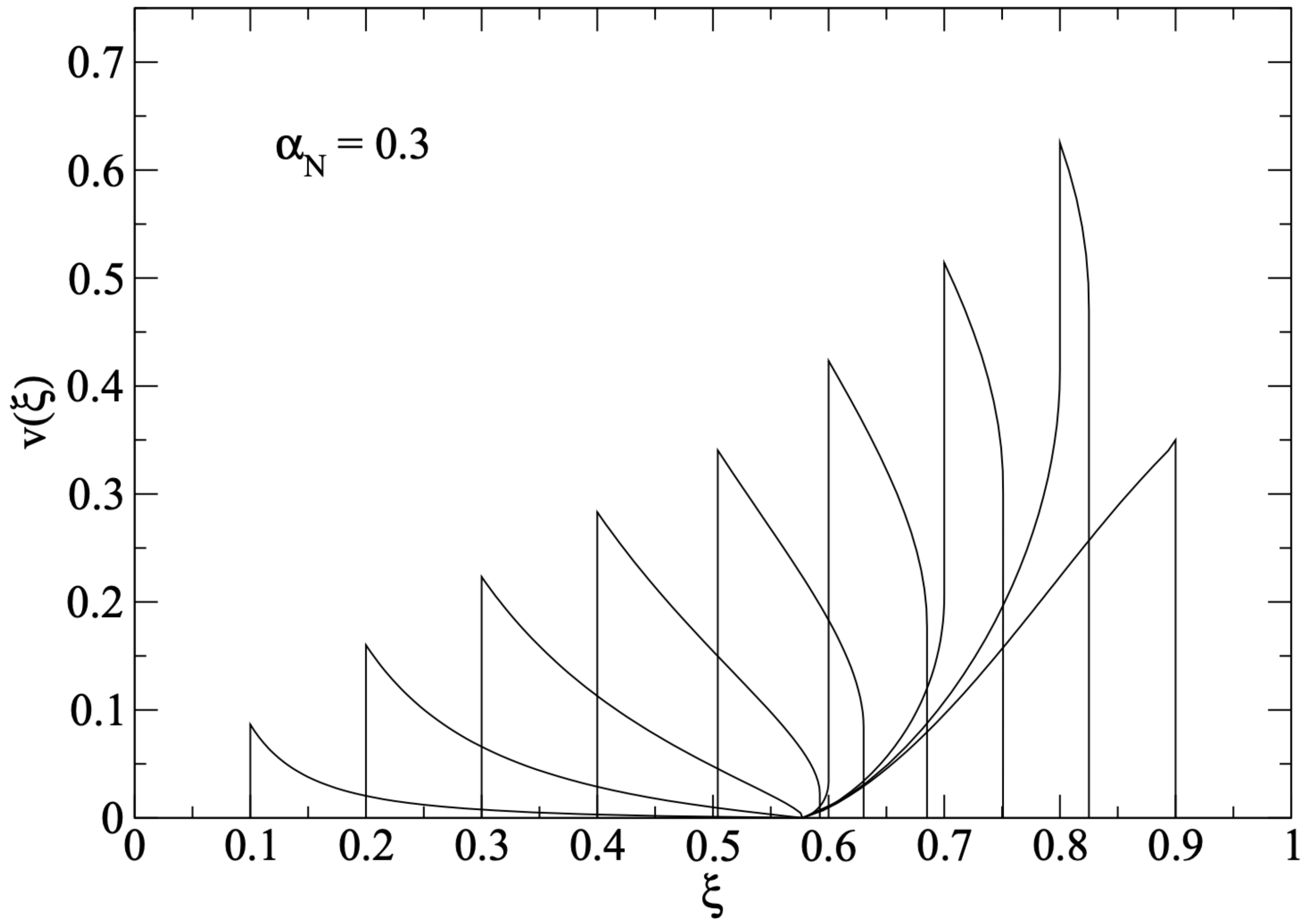
Our universe suffers several phase transitions (PT) in the early time, such as electroweak phase transition and QCD phase transition. Gravitational waves (GWs) are one of the inspiring tools to probe the physics at that time.



Credit: M. Maggiore, "Gravitational Waves Volume 2: Astrophysics and Cosmology"



Credit: J. Liu et al.,arXiv:2012.15625



Credit: J.R. Espinosa et al.,arXiv:1004.4187

Introduction

GW sources during the phase transitions: $\nabla_\mu (T_\phi^{\mu\nu} + T_f^{\mu\nu}) = 0$

- Bubble collisions : Oscillations of the phase transition field ϕ with thin wall approximation

$$\nabla_\mu T_\phi^{\mu\nu} = [\nabla_\mu \nabla^\mu \phi - V_0'(\phi)] \nabla^\nu \phi = + f^\nu$$

R. Jinno and M. Takimoto, arXiv:1605.01403, 1707.03111

- Sound waves : Treating bulk fluids as freely propagating waves

$$\nabla_\mu T_f^{\mu\nu} = \sum_{i=\mathrm{B,F}} g_i \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \frac{k^\mu k^\nu}{E_i(\mathbf{k})} \nabla_\mu f_i = - f^\nu$$

C. Caprini, R. Durrer and G. Servant , arXiv:0711.2593

M. Hindmarsh, arXiv:1608.04735

M. Hindmarsh and M. Hijazi, arXiv:1909.10040

H.K. Guo, K. Sinha, D. Vagie, G. White, arXiv:2207.08537

Simulations: M. Hindmarsh, S. J. Huber, K. Rummukainen, and D.J. Weir, arXiv:1704.05871
Y.F. Di, J.L. Wang, R.Y. Zhou, L.G. Bian, R.G. Cai and J. Liu, arXiv:2012.15625

Introduction

GW sources during the phase transitions:

- Bubble collisions :

- Sound waves :

Simulations:

R. Jinno et al., arXiv:1605.01403

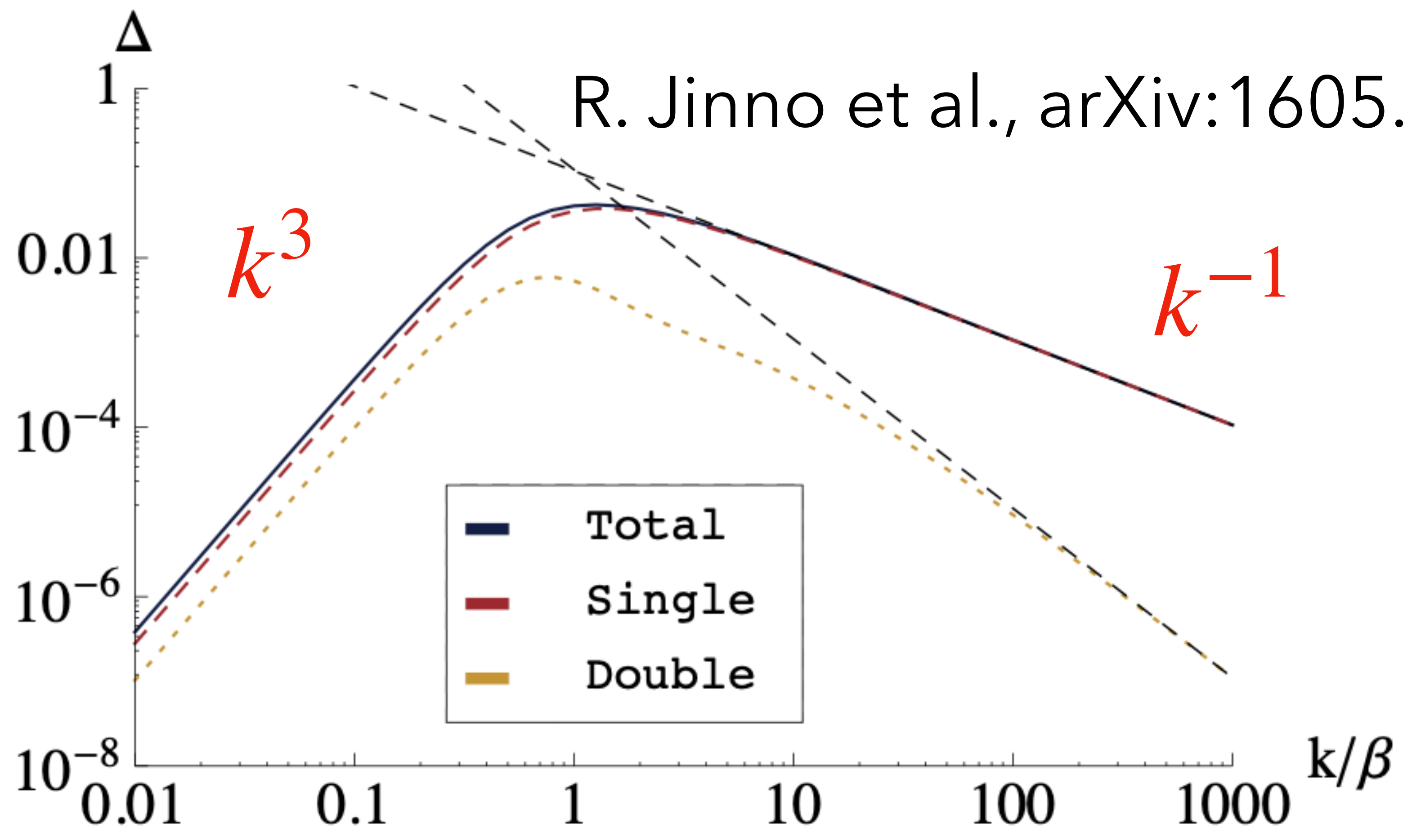
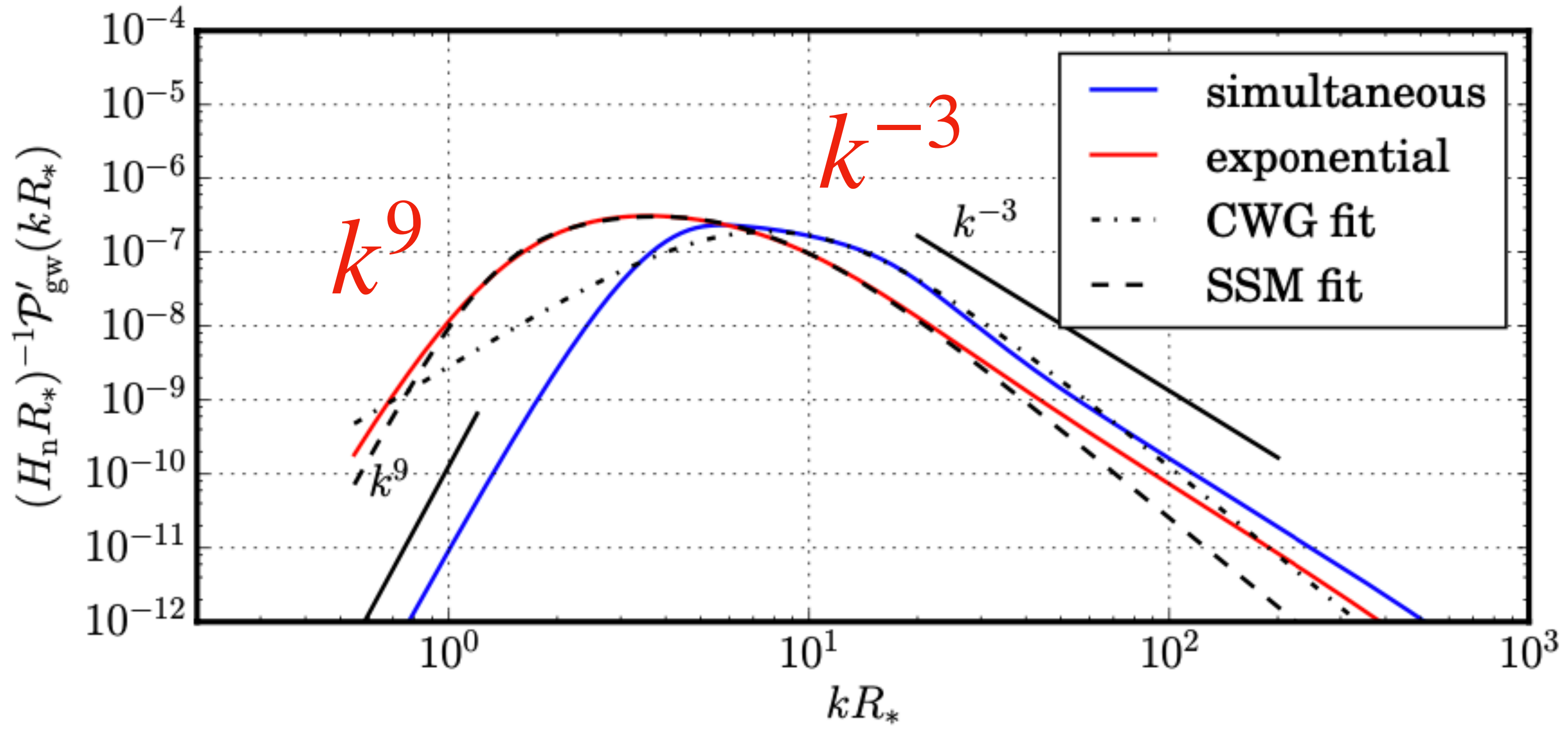


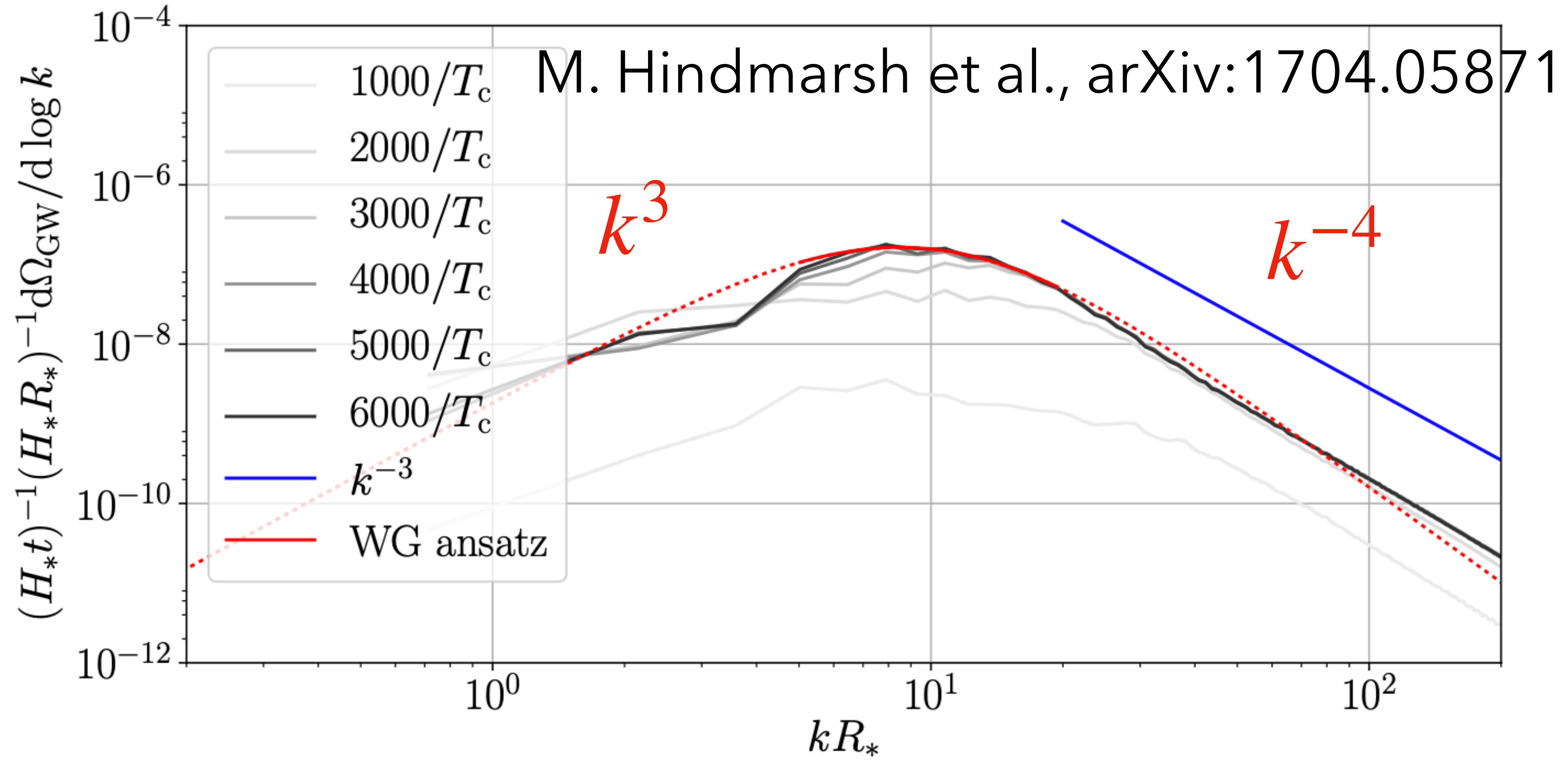
FIG. 6: Plot of the GW spectrum Δ (blue). Single- and double-bubble spectra $\Delta^{(s)}$ (red) and $\Delta^{(d)}$ (yellow) are also plotted. Black lines are auxiliary ones proportional to k^{-1} and k^{-2}, respectively.



M. Hindmarsh and M. Hijazi, arXiv:1909.10040

(b) Intermediate, $v_{\rm w} = 0.92$

M. Hindmarsh et al., arXiv:1704.05871



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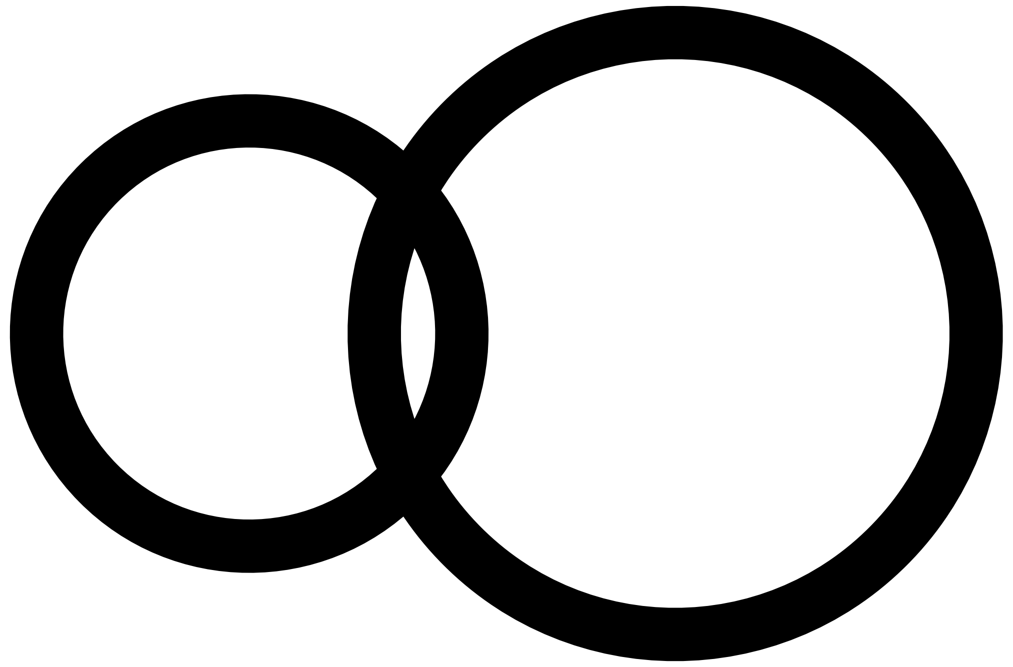
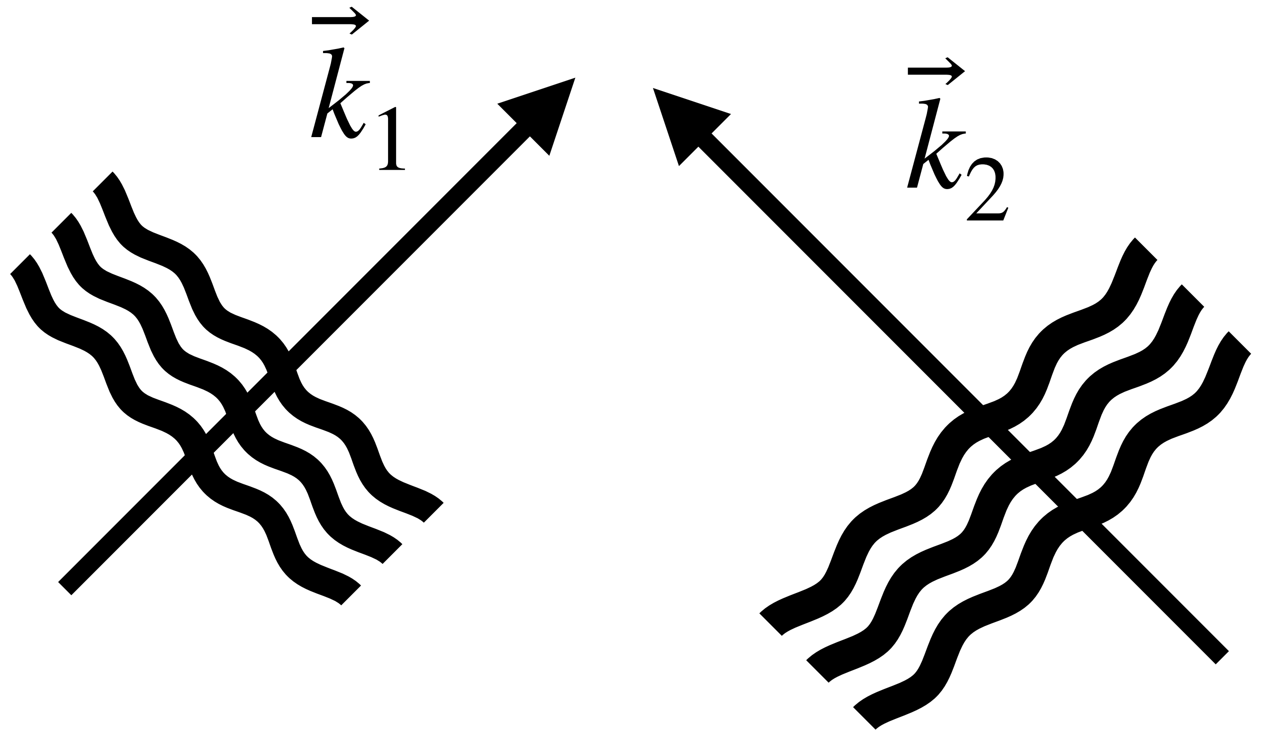
Introduction

GW sources during the phase transitions:

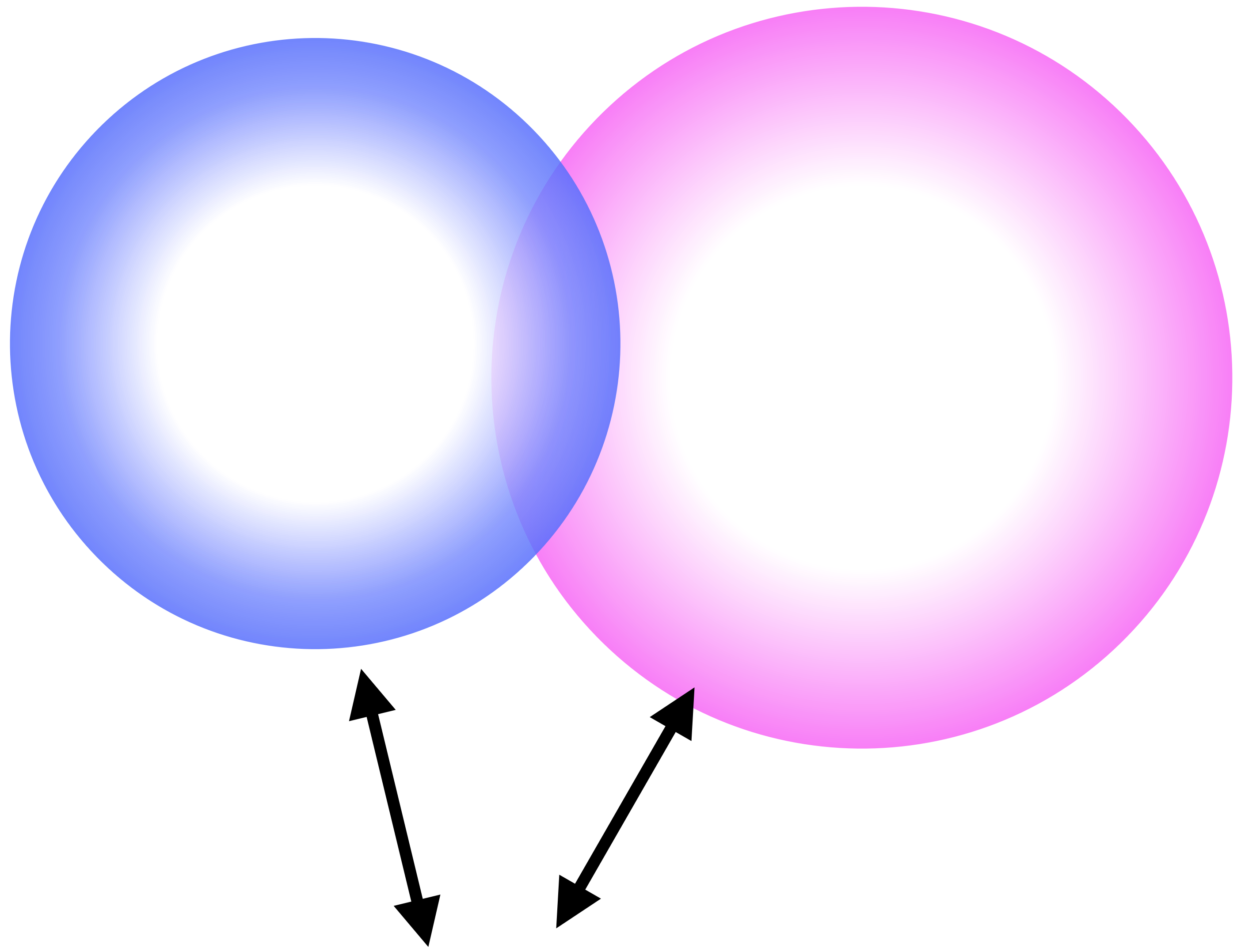
- Bubble collisions :



- Sound waves :



The process that our model cares



Existence of quadrupole for energy-momentum tensor

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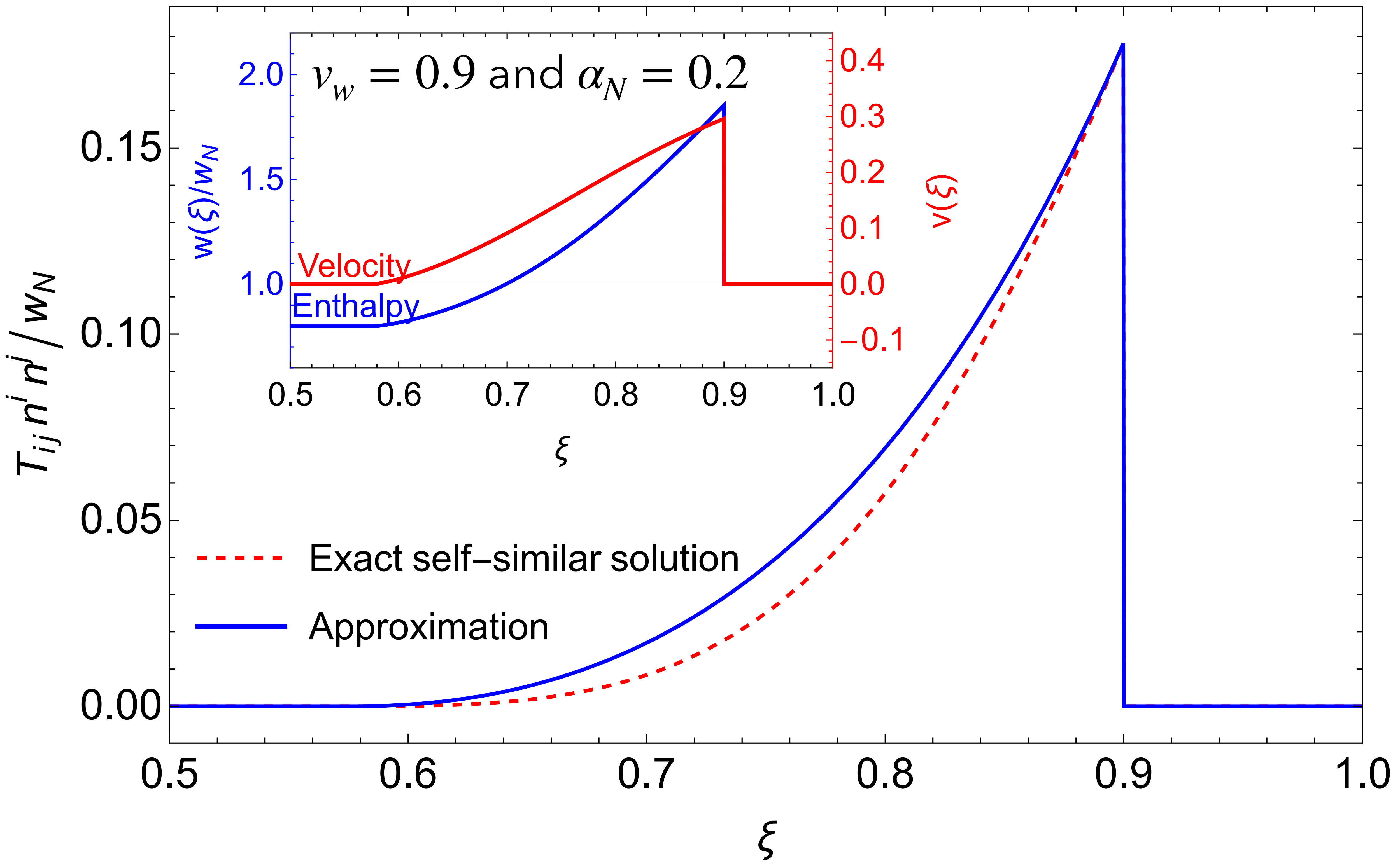
Sound shell model

For a steadily expanding bubble wall, the profiles of fluid velocity and thermodynamic quantities can be described as functions of a self-similar coordinate $\xi = r/t$ alone.



Perfect fluid approximation

$$\hat{T}_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu}$$

Anisotropic, spacial part

$$T_{ij} = w\gamma^2 v_i v_j$$

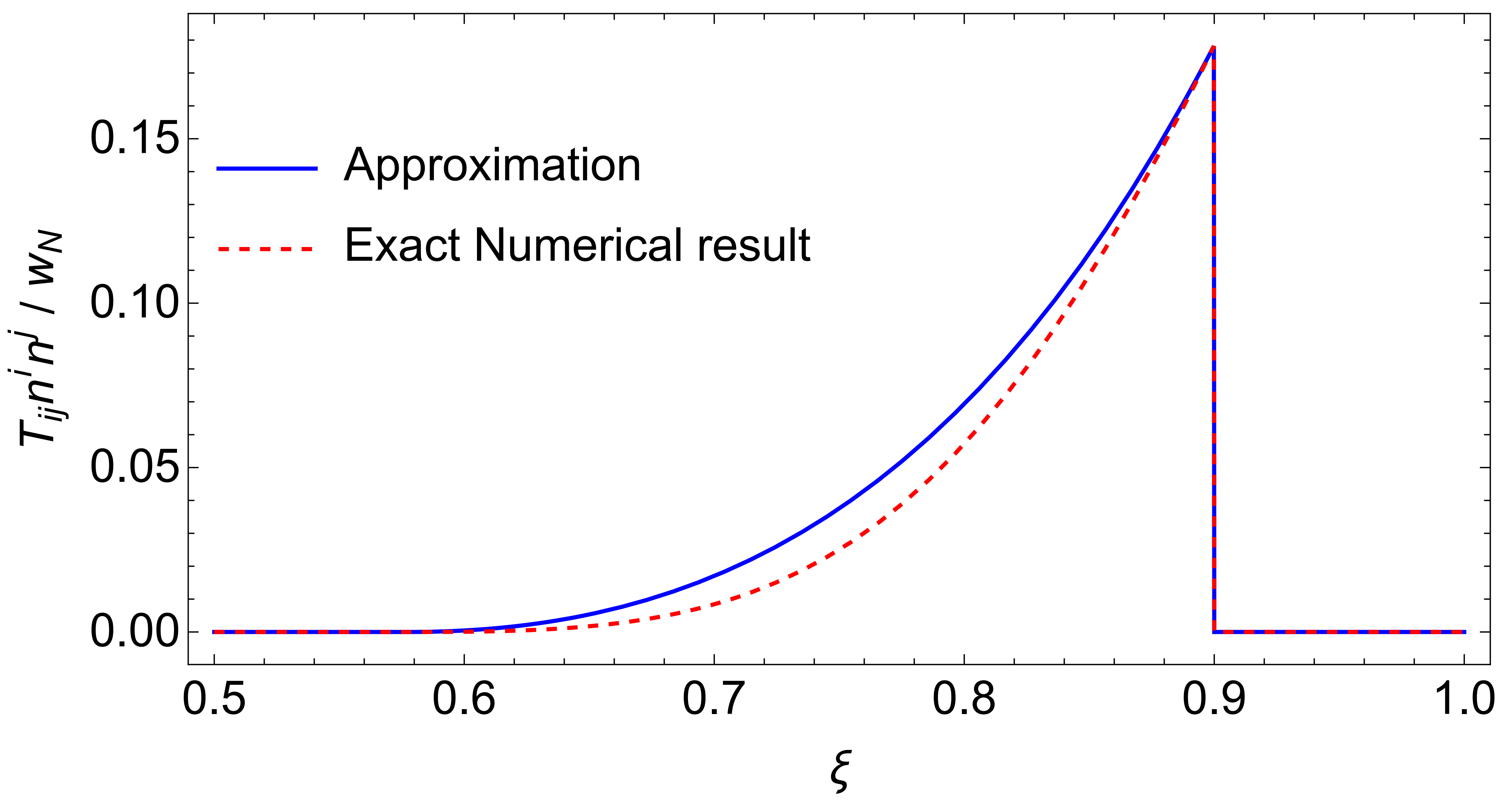
Sound shell model

Linear approximations

$$v_i(t,\vec{x}) = vn_i \simeq \begin{cases} \dfrac{v_m(r-R_1(t))}{R_2(t)-R_1(t)}n_i, & R_1(t) < r < R_2(t) \\ 0, & \text{otherwise} \end{cases}$$

$$\frac{w(t,r)}{w_N} \simeq \begin{cases} \dfrac{w_r(r-R_1(t))}{R_2(t)-R_1(t)} + 1, & R_1(t) < r < R_2(t) \\ 1, & \text{otherwise} \end{cases}$$

where $R_1 = c_s(t-t_n)$ and $R_2 = v_w(t-t_n)$



$$\begin{aligned} T_{ij} &= \frac{wv^2}{1-v^2}n_in_j \\ &= w_N\left(\frac{w_r}{v_w-c_s}\frac{r-c_s(t-t_n)}{t-t_n}+1\right) \\ &\quad\times \sum_{s=0}^{\infty}\left(\frac{v_m}{v_w-c_s}\frac{r-c_s(t-t_n)}{t-t_n}\right)^{2s+2} n_in_j\,. \end{aligned}$$

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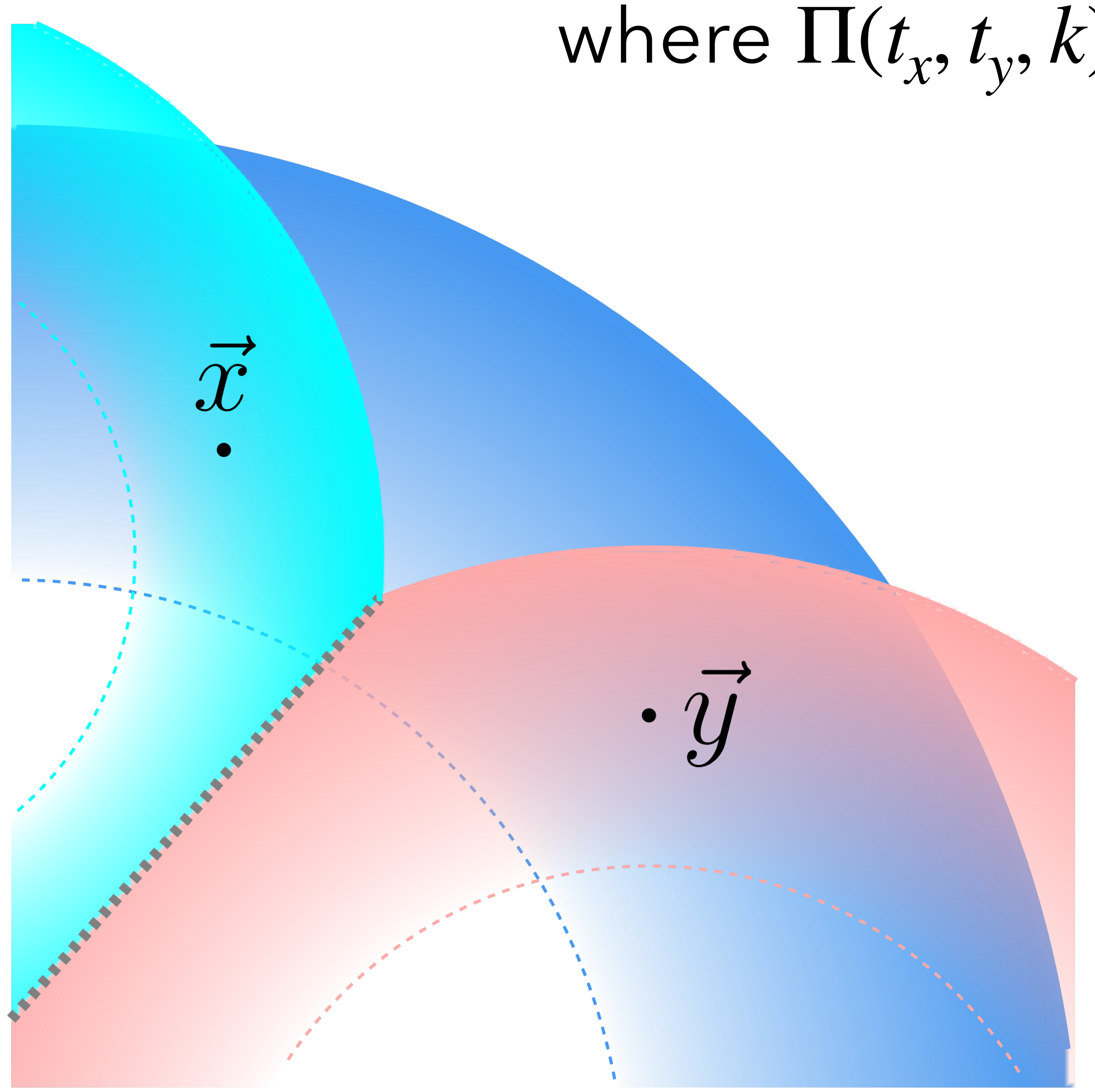
Analytic derivation

$$P_{\rm GW}(t,k) = \frac{1}{\rho_{\rm tot}}\frac{{\rm d}\rho_{\rm GW}}{{\rm d}\ln k} = \frac{2G}{\pi\rho_{\rm tot}}\frac{a_*^4}{a^4}k^3\int_{t_i}^{t_f}{\rm d}t_x\int_{t_i}^{t_f}{\rm d}t_y\cos(k(t_x-t_y))\Pi(t_x,t_y,k) = \frac{2G}{\pi\rho_{\rm tot}}\frac{a_*^4}{a^4}\Delta(k),$$

$$\text{where } \Pi(t_x,t_y,k) = \Lambda_{ij,kl}(\hat{k})\Lambda_{ij,mn}(\hat{k})\int {\rm d}^3r e^{i\vec{k}\cdot\vec{r}}\langle T_{kl}(t_x,\vec{x})T_{mn}(t_y,\vec{y})\rangle\,.$$

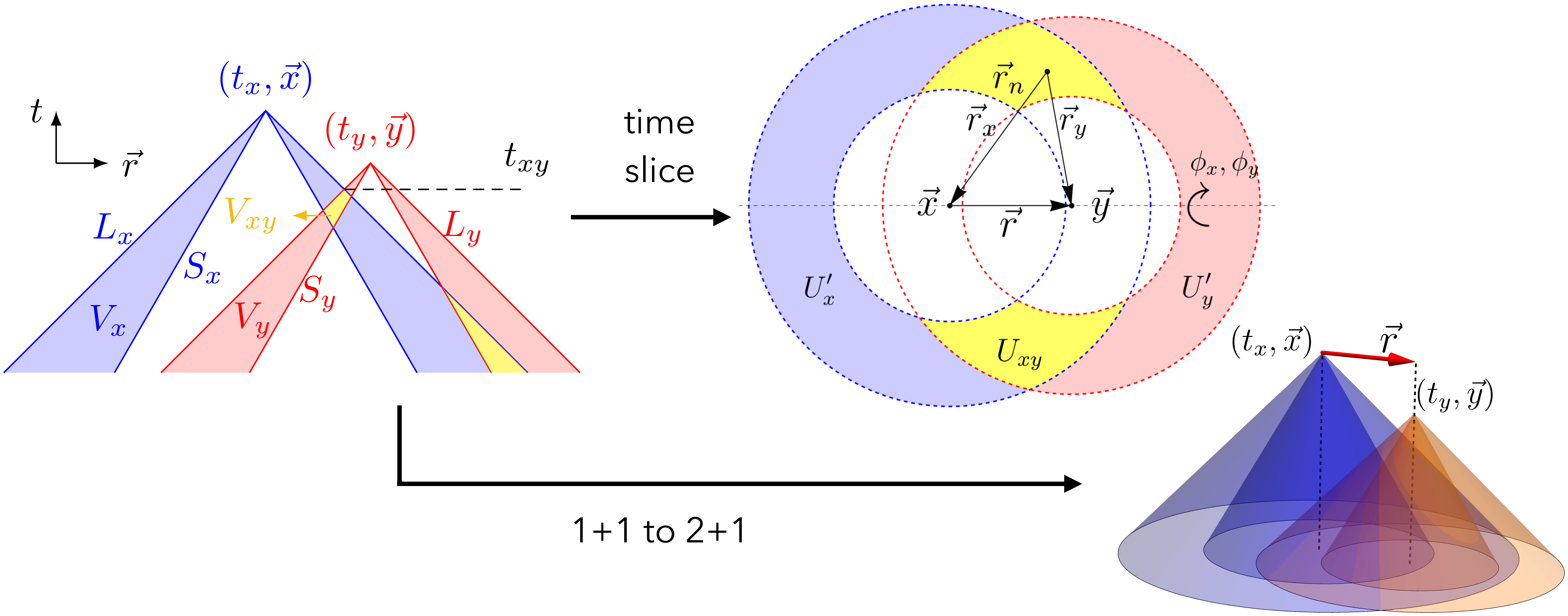


Contributions:

- $\vec{x}$ and $\vec{y}$ are in the same sound shell (single-shell)
- $\vec{x}$ and $\vec{y}$ are in two different sound shells (double-shell)

Analytic derivation

Schematic images for where single-shell and double-shell might come from.



Analytic derivation

The total power spectrum: $\Delta(k) = \Delta(k)^{(s)} + \Delta(k)^{(d)}$

$$\Delta(k)^{(s)} = \frac{2}{v_w^3}\beta^{-2}\tilde{k}^3 \int_0^{+\infty} \mathrm{d}\tilde{t} \int_0^{+\infty} \mathrm{d}\tilde{r} \frac{\cos(\tilde{k}\tilde{t})}{\mathcal{I}(\tilde{t}, \tilde{r}/v_w)} \tilde{r}^6 \left(j_0(\tilde{k}\tilde{r}) F_a + \frac{j_1(\tilde{k}\tilde{r})}{\tilde{k}\tilde{r}} F_b + 2\frac{j_2(\tilde{k}\tilde{r})}{(\tilde{k}\tilde{r})^2} F_c \right)$$

$$\Delta(k)^{(d)} = \frac{1}{2\pi v_w^6}\beta^{-2}\tilde{k}^3 \int_0^{1/v_w} \mathrm{d}\tilde{t} \int_0^{+\infty} \mathrm{d}\tilde{r} \frac{\cos(\tilde{k}\tilde{t})}{\mathcal{I}(\tilde{t}, \tilde{r}/v_w)^2} \tilde{r}^{10} \left(\frac{j_2(\tilde{k}\tilde{r})}{(\tilde{k}\tilde{r})^2} G_x\left(\tilde{t}/\tilde{r}\right) G_y\left(\tilde{t}/\tilde{r}\right) \right)$$

where $\tilde{k} = k/\beta$, $\tilde{t} = t_d\beta$, $\tilde{r} = r\beta$,
and β is given in the bubble nucleation rate $\Gamma(t) = \Gamma(t_*)\exp[\beta(t - t_*)]$

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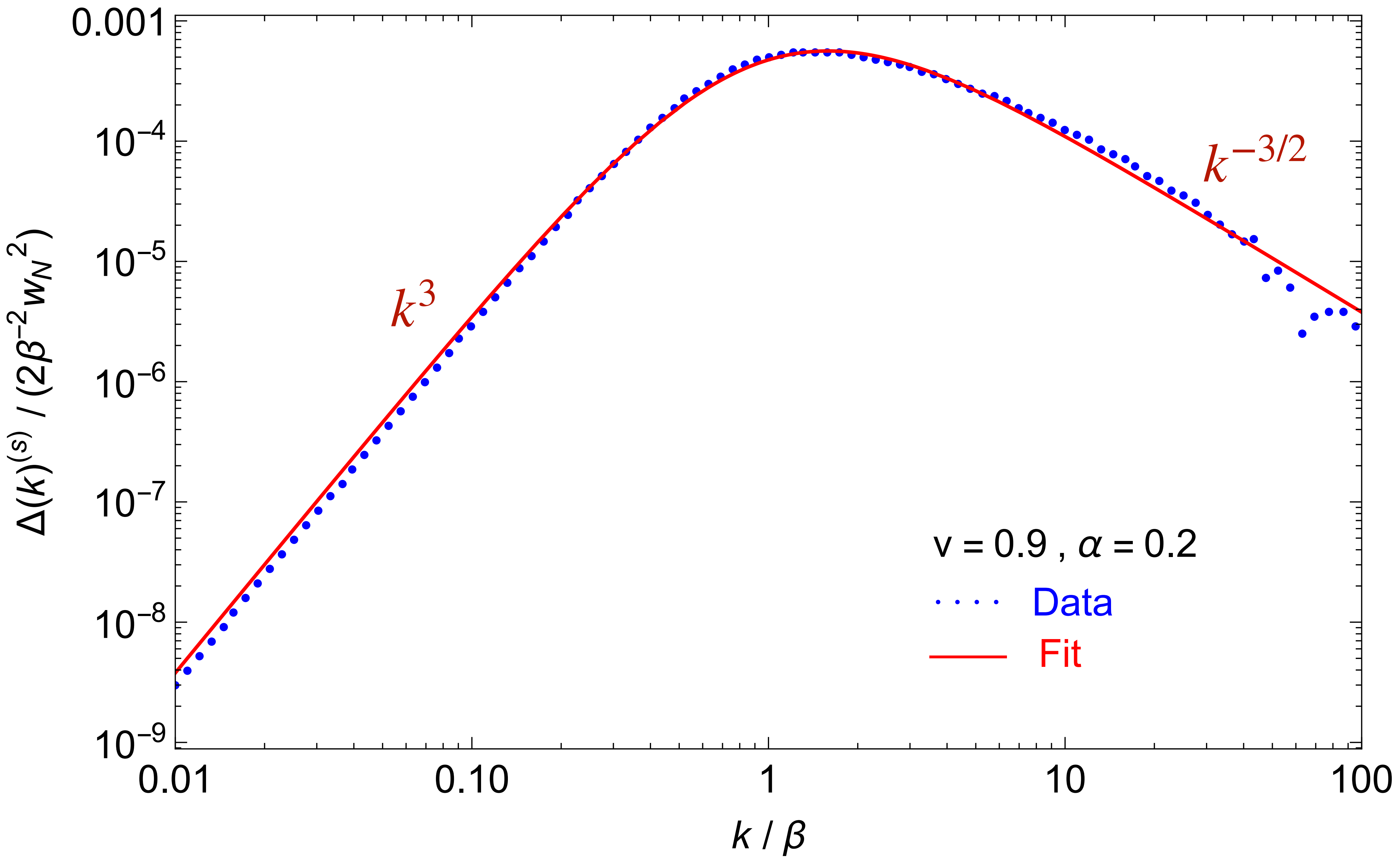
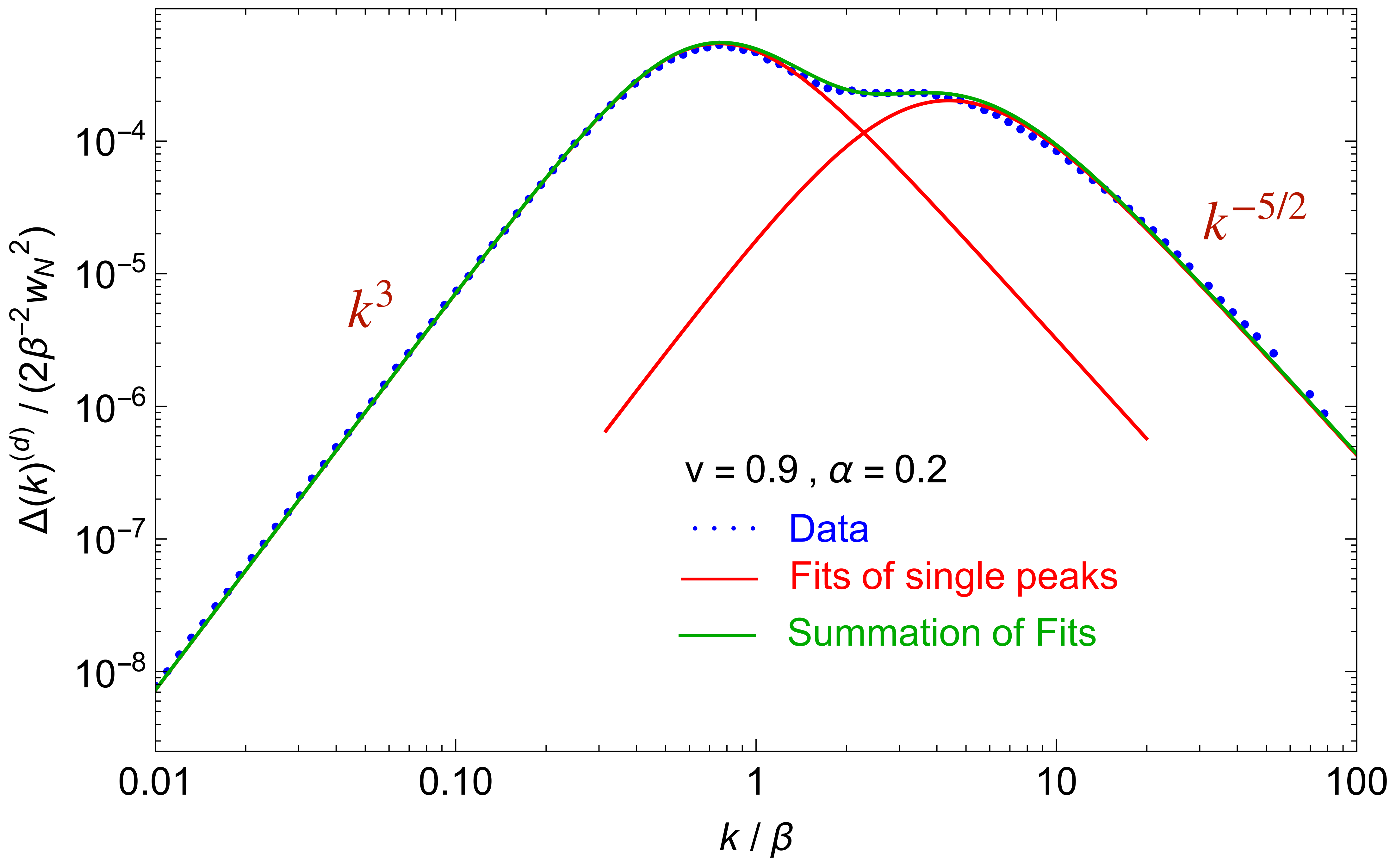
Numerical results

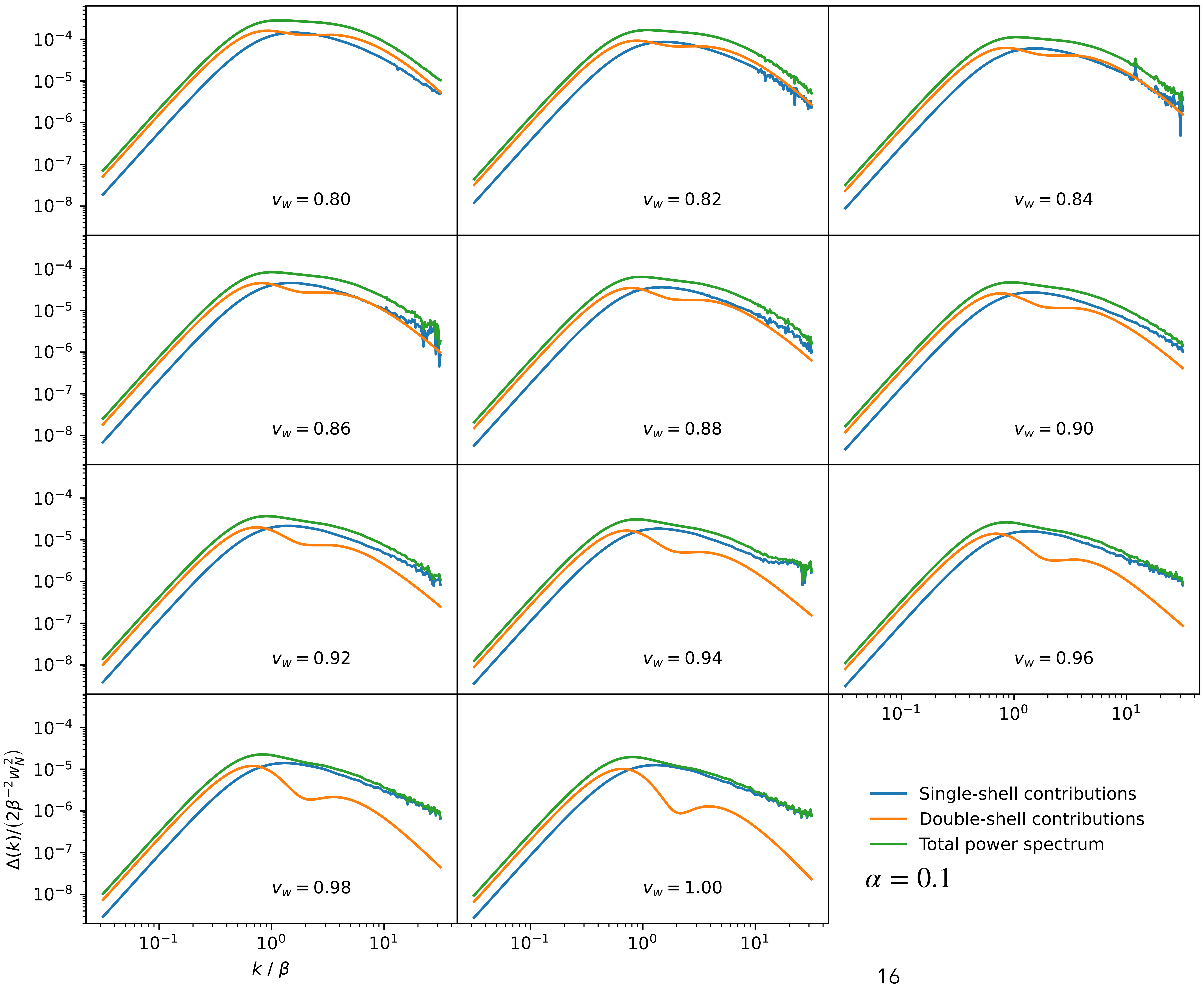
$$F_{n_1,n_2,\Delta}(k,k_*,F_*) = F_* \left(\frac{k}{k_*}\right)^{n_1} \left(\frac{1+(k/k_*)^{\Delta}}{2}\right)^{\frac{n_2-n_1}{\Delta}}$$

$$\Delta^{(s)} = F_{n_1,n_2,\Delta}(k,k_*,F_*)$$

$$\Delta^{(d)} = F_{n_1,n_2,\Delta_1}(k,k_{*1},F_{*2}) + F_{n_1,n_2,\Delta_2}(k,k_{*2},F_{*2})$$







- k^3 power law in low frequencies.
- Double-shell dominates in low frequencies, and single-shell gradually dominates in high frequencies as $v_w \to 1$.
- A broader dome as v_w decreases, which is found in numerical simulations.

M. Hindmarsh et al., arXiv:1704.05871

Peak frequency

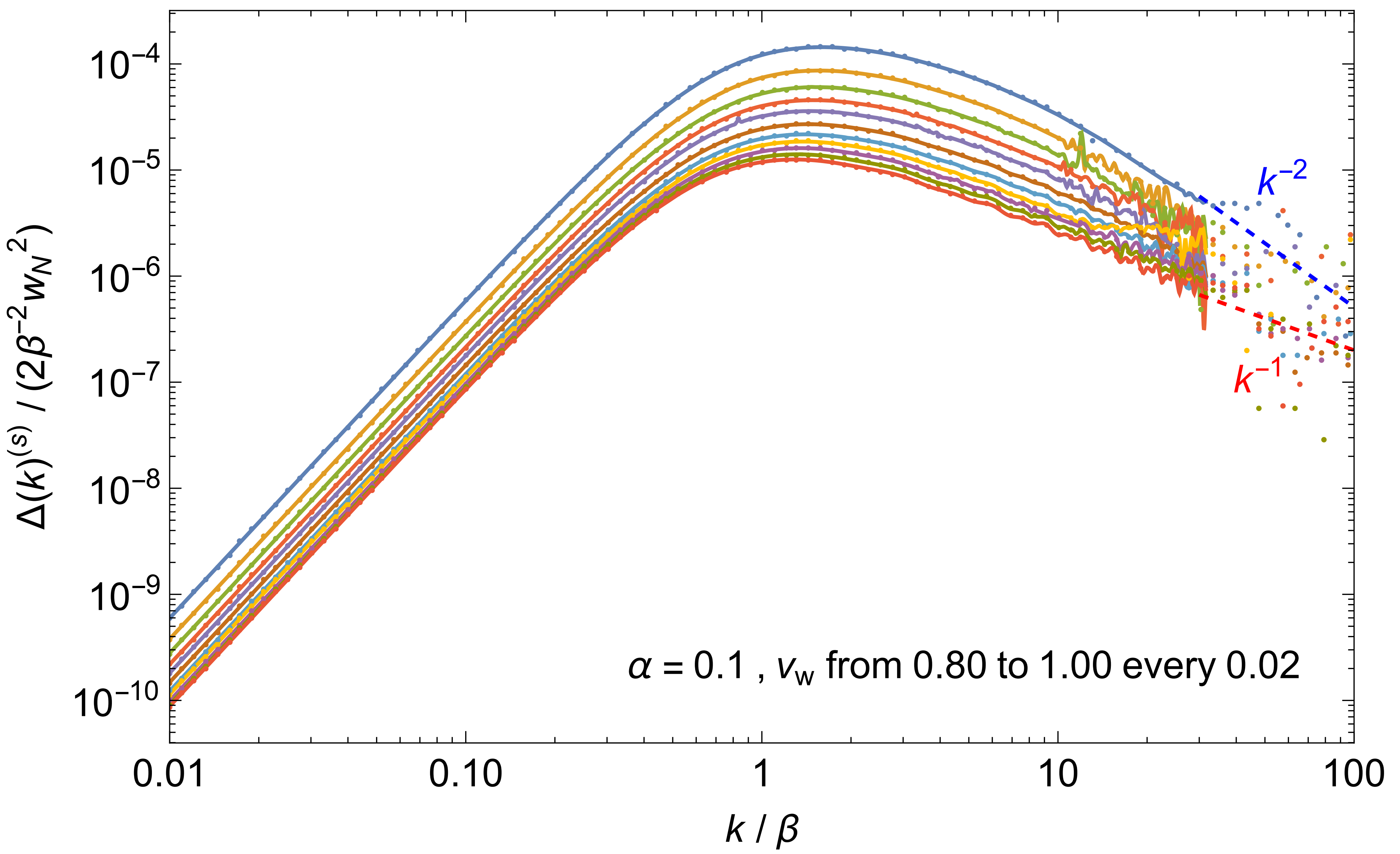
$$k^{(s)}_{\text{peak}} = 3.78 k_{w*}$$

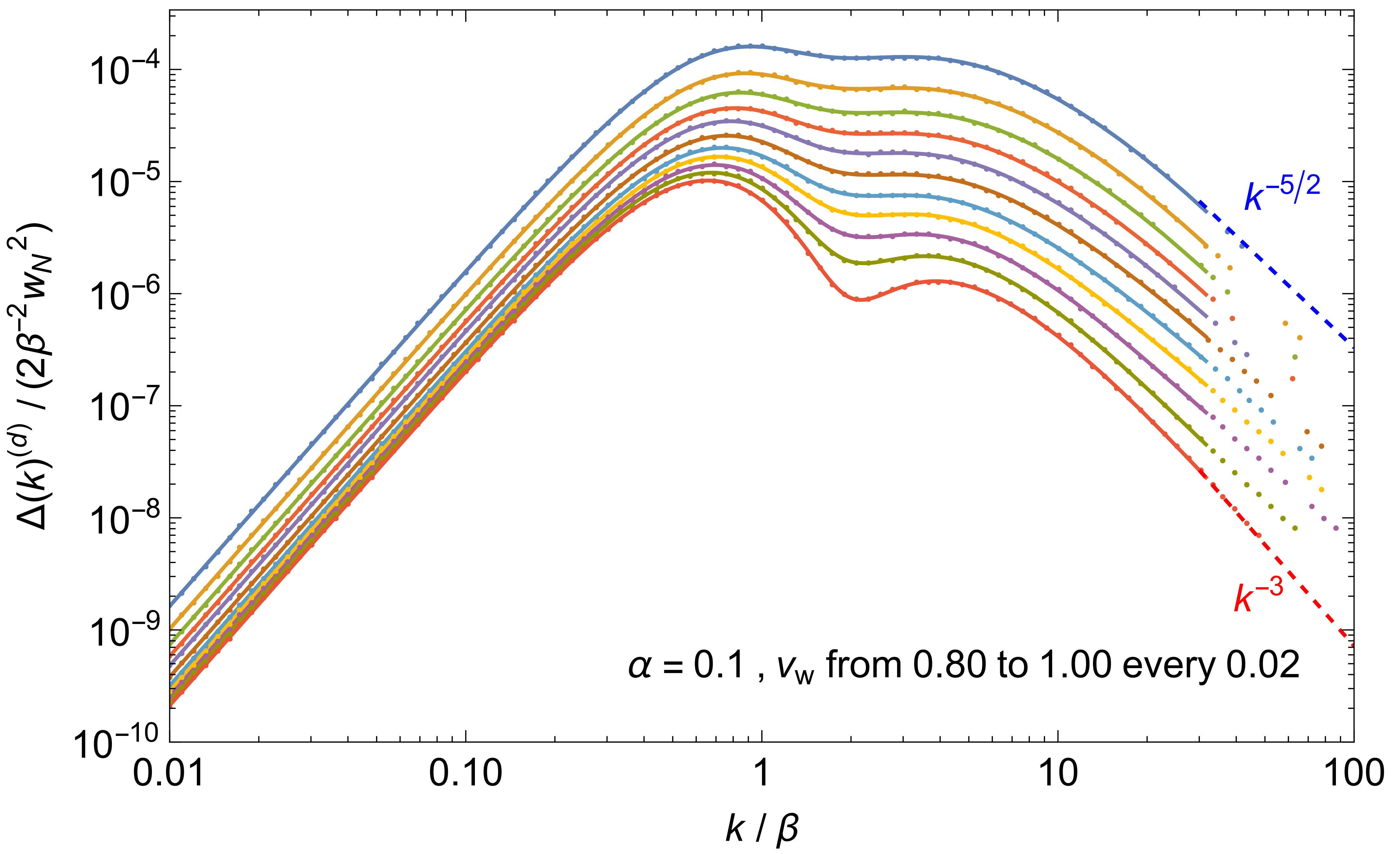
$$k^{(d)}_{\text{peak},1} = 1.54 k_{s*} \left(\frac{k_{s*}}{k_{w*}}\right)^{-0.74}$$

$$k^{(d)}_{\text{peak},2} = \frac{1 - (2-\delta_1)v_w + (1+\delta_1)v_w^2}{\delta_2 + (0.1+\delta_3)v_w - (0.1-\delta_3)v_w^2}$$

with $\delta_1 = 0.044,\ \delta_2 = 0.0051,\ \delta_1 = 0.0089$

where $k_{w*} = 1/R_* = (8\pi)^{1/3} v_w^{-1} \beta$ and $k_{s*} = 1/L_{s*} = R_{w*}^{-1} v_w/(v_w - c_s)$





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Discussions

We analytically derived the GW power spectrumfrom the sound shell during the bubble collisions, as a summation of two contributions: single-shell and double-shell.

- Numerical results and fitting formulas for power spectrum are provided.
- Low frequency k^3 ; High frequency $\longrightarrow$ Single-shell $k^{-2} \to k^{-1}$ as $v_w \to 1$ / Double-shell $k^{-5/2} \to k^{-3}$ as $v_w \to 1$
- A broader dome is discovered because of the double-shell domination at low v_w.

Some of the approximations, such as the simplified velocity and enthalpy profiles and the uncounted part of the collided parts of the sound shell, can be further studied to gain more accurate results. More general bubble nucleation models or the expansion of the universe can also be considered in future works. Similar method can be applied to calculate the GWs of thick wall bubble collision.

Thank You!

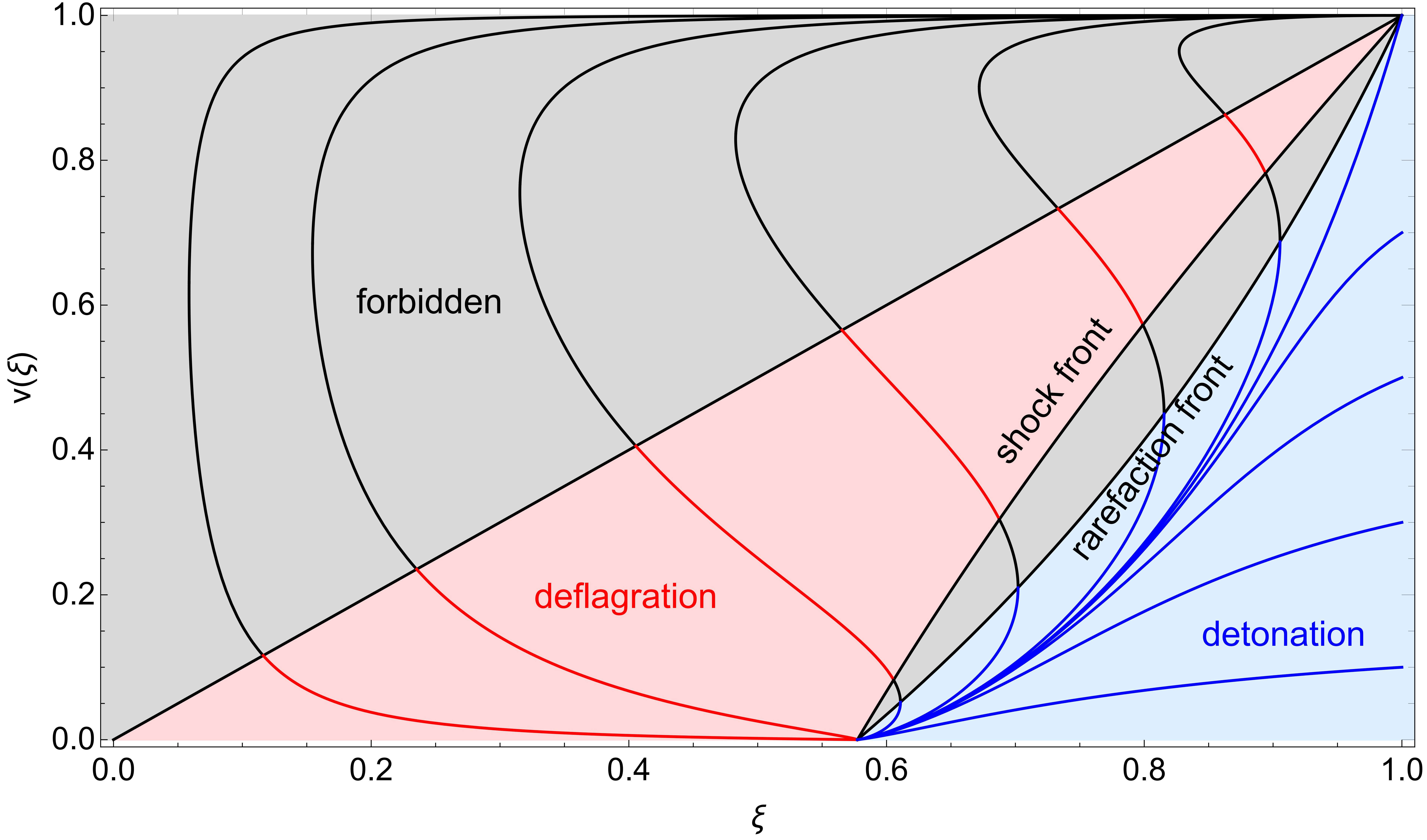
Appendix A

The plasma can be described by perfect fluid $T^{\mu\nu} = (\rho + p)u^\mu u^\nu + p\eta^{\mu\nu}$, whose equation of motion is given by $\nabla_\mu T^{\mu\nu} = 0$. Projecting the conservation equation parallel and perpendicular to the bulk flow direction and rewriting the equations in self-similarity coordinate $\xi \equiv |\vec{x}|/t = r/t$ lead to

$$2\frac{v}{\xi} = \gamma^2(1 - \xi v)\left(\frac{\mu^2}{c_s^2} - 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}$$

$$\frac{\mathrm{d}w}{\mathrm{d}\xi} = w\gamma^2\mu\left(\frac{1}{c_s^2} + 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}$$

Appendix A



forbidden
deflagration
shock front
rarefaction front
detonation
v(ξ)
ξ
0.0
0.2
0.4
0.6
0.8
1.0

Appendix A

Our model could only describe this situation

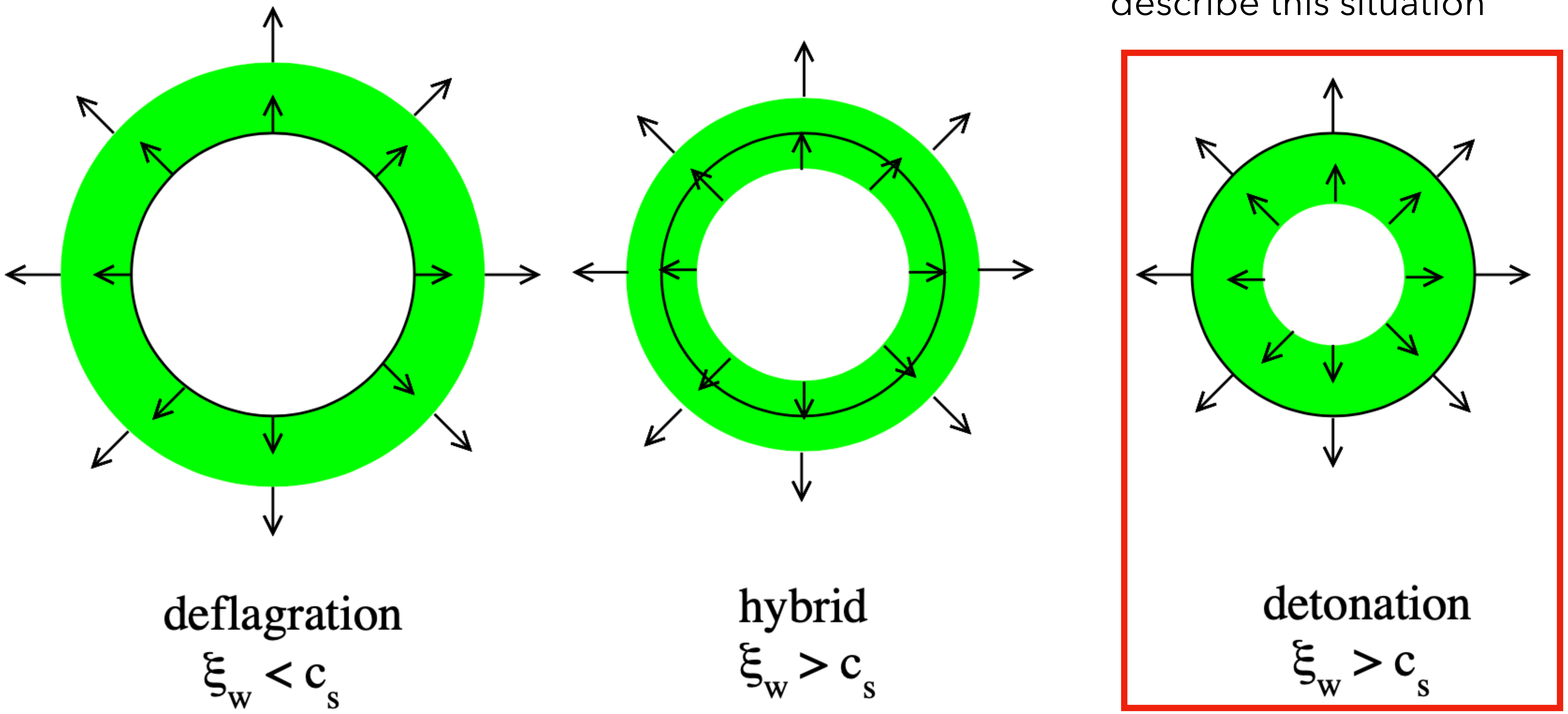


Figure 3: Pictorial representation of expanding bubbles of different types. The black circle is the phase interface (bubble wall). In green we show the region of non-zero fluid velocity.