Wave Toy for Method of Lines

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Abstract

An implementation of wavetoy in first order form suitable for use with the method of lines.

1 Purpose

WaveToy is the simple test thorn that comes with Cactus as standard. This is written so that it solves the wave equation

$$\partial_t^2 \phi = \partial_{x^i}^2 \phi^i \tag{1}$$

directly using the leapfrog scheme. This form of the equations isn't suitable for use with the method of lines.

The purpose of this thorn is to rewrite the equations in first order form

$$\begin{aligned} \partial_t \Phi &= \partial_{x^i} \Pi^i, & (2)\\ \partial_t \Pi^j &= \partial_{x^j} \Phi, & (3)\\ \partial_t \phi &= \Phi, & (4)\\ \partial_{x^j} \phi &= \Pi^j. & (5) \end{aligned}$$

The first three equations (which expand to six separate PDEs) will be evolved. The final equation is used to set the initial data and can be thought of as a constraint.

This will be implemented using simple second order differencing and the basic Runge-Kutta time integrators included within the method of lines. Unfortunately because of the large difference with the number of grid functions defined it has to be a new implementation (I believe)...

The results of this thorn can then be compared against the standard wavetoy results to compare accuracy, efficiency, etc...

2 How it works

The equations are evolved entirely using the method of lines thorn. So all we have to provide (for the evolution) is a method of calculating the right hand side of equation (2) and boundary conditions. The boundary conditions are standard from wavetoy itself. The right hand side is calculated using second order centred finite differences.

To be compatible with the method of lines thorn we must let it know that the GFs (ϕ, Π, Φ^j) exist and where they are stored. They should all have two time levels (although the addition of extra time levels may not cause problems it's just wasting space). The GFs corresponding to the right hand sides must also be registered with the MoL thorn. These should only have one time level.