Practical 6
Solution of Cauchy problem of First order PDE

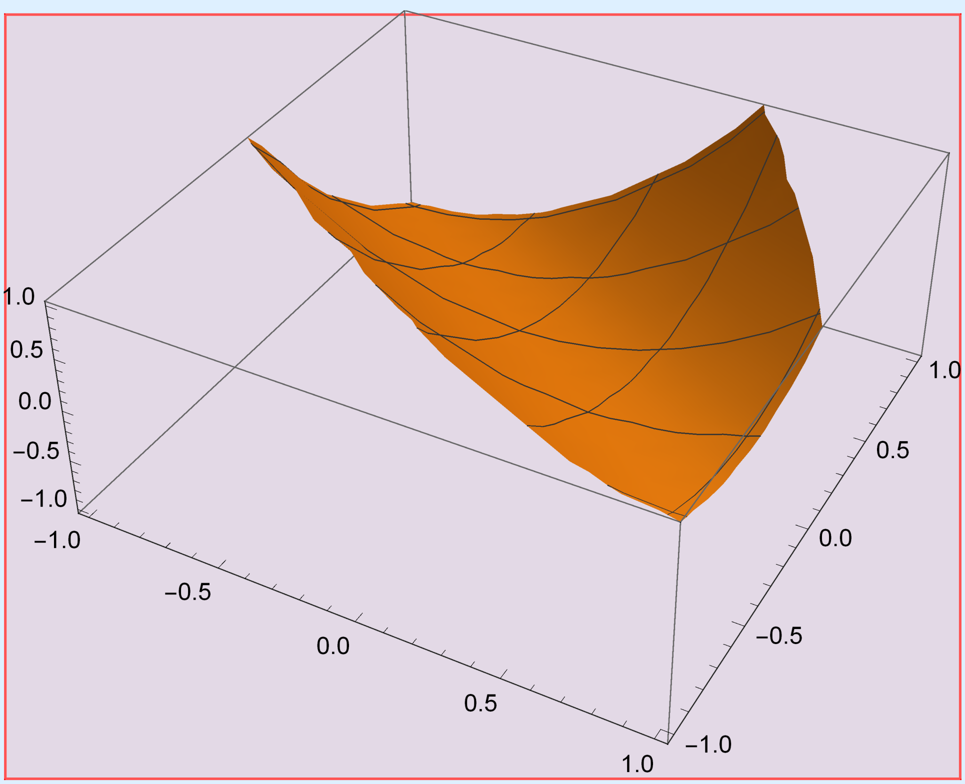
Q1. u_x - u_y = 1, u(x, 0) = x^2

In[1]:=
```
sol = DSolve[{D[u[x, y], x] - D[u[x, y], y] == 1,
   u[x, 0] == x^2}, u[x, y], {x, y}]
```

Out[1]= $\{\{u[x, y] \to x^2 - y + 2\,x\,y + y^2\}\}$

In[3]:=
```
Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -4, 4}]
```

Out[3]=



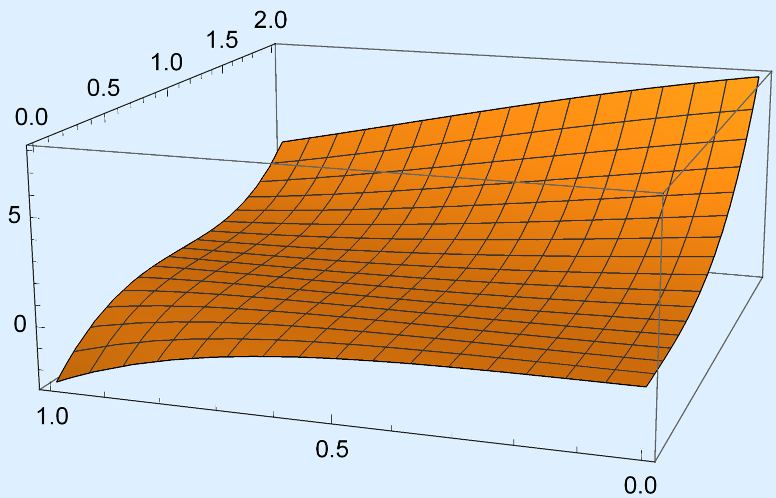
u_x + u_y = u, u(x, 0) = x^3

In[5]:=
```
sol2 = DSolve[{D[u[x, y], x] + D[u[x, y], y] == u[x, y],
   u[x, 0] == x^3}, u[x, y], {x, y}]
```

Out[5]= $\{\{u[x, y] \to -e^{y} (-x + y)^{3}\}\}$

In[6]:=
```
Plot3D[u[x, y] /. sol2, {x, 0, 2}, {y, 0, 1}]
```

Out[6]=



yu_x _xu_y = u, u(0, y) = y^3

In[7]:=
```
sol11 =
 DSolve[{y D[u[x, y], x] + x D[u[x, y], y] == u[x, y],
   u[0, y] == y^3}, u[x, y], {x, y}]
```

Out[7]= $\left\{\left\{u[x, y] \to -\frac{(-x^2 + y^2)^2}{x + \sqrt{y^2}}\right\}, \left\{u[x, y] \to -(-x^2 + y^2)\left(x + \sqrt{y^2}\right)\right\}\right\}$

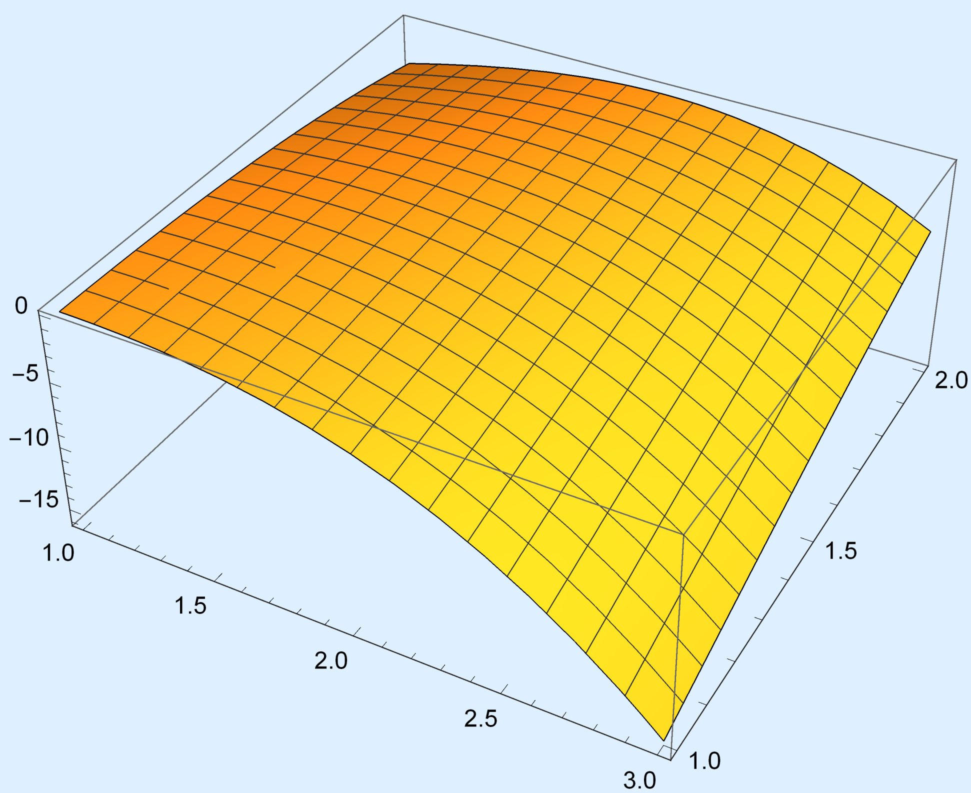
In[8]:=
```
sol11[[1, 1]]
```

Out[8]=
$$u[x, y] \to -\frac{\left(-x^2 + y^2\right)^2}{x + \sqrt{y^2}}$$

In[9]:=
```
sol11[[2, 1]]
```

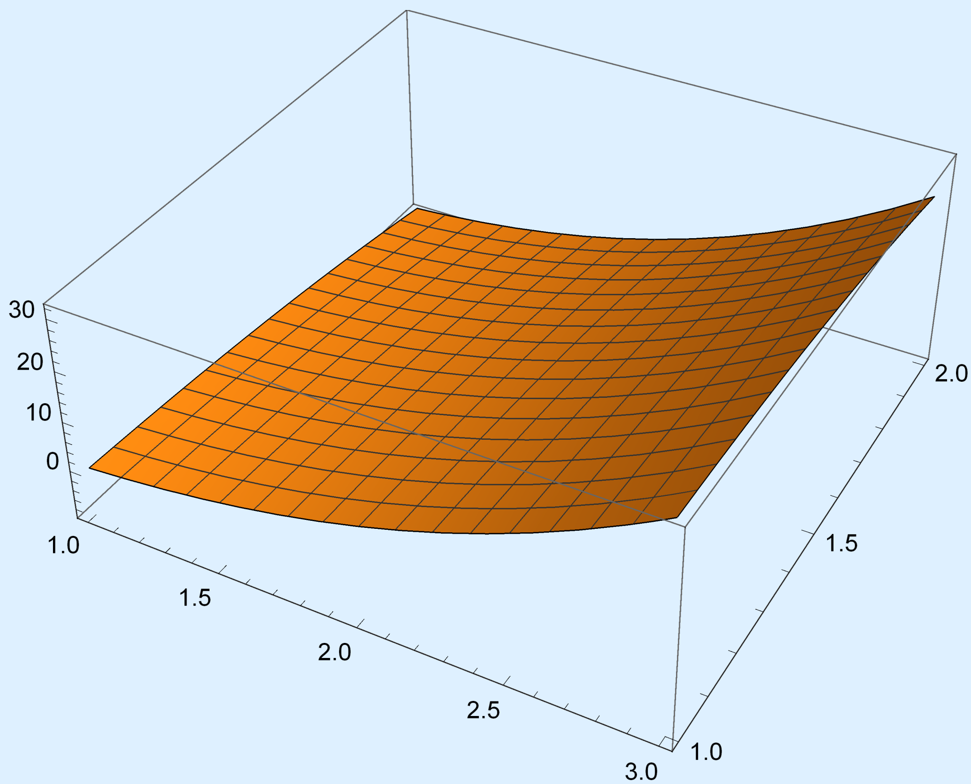
Out[9]=
$$u[x, y] \to -\left(-x^2 + y^2\right)\left(x + \sqrt{y^2}\right)$$

In[10]:=
```
Plot3D[u[x, y] /. sol11[[1, 1]], {x, 1, 3}, {y, 1, 2}]
```

Out[10]=


In[11]:=
```
Plot3D[u[x, y] /. sol11[[2, 1]], {x, 1, 3}, {y, 1, 2}]
```

Out[11]=



In[34]:=
```
ClearAll
```

Out[34]=
```
ClearAll
```

Questions:
Q1. u_x + xu_y = 0, u(0, y) = siny

In[12]:=
```
sol = DSolve[{D[u[x, y], x] - x*D[u[x, y], y] == 0,
   u[0, y] == Sin [y]}, u[x, y], {x, y}]
```

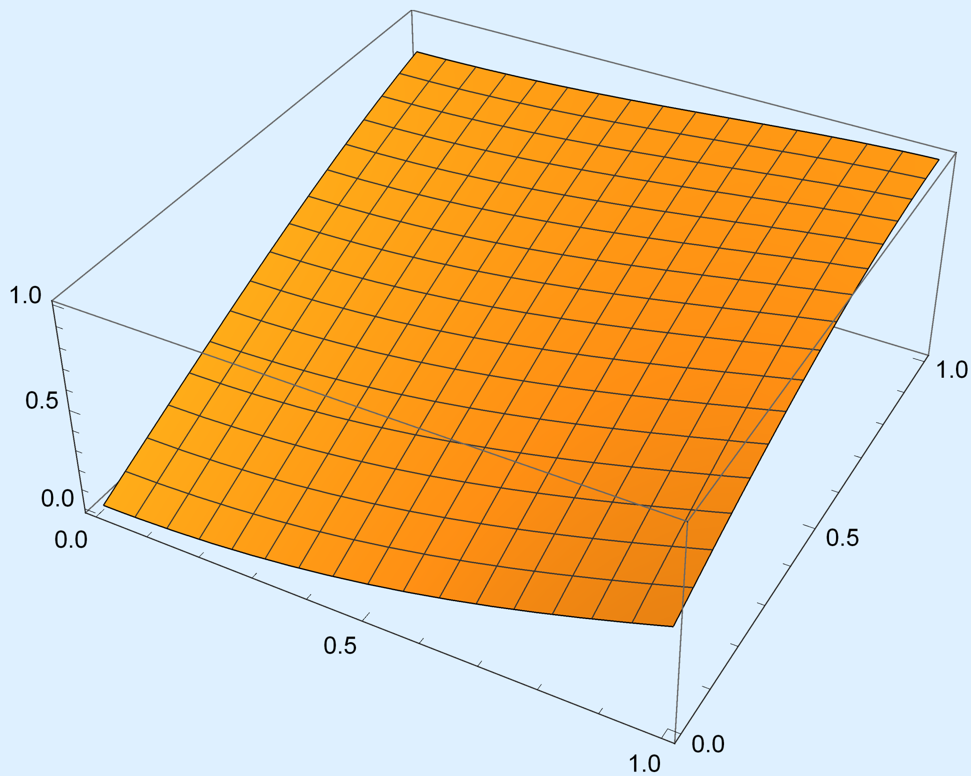
Out[12]=

$$\left\{\left\{u[x, y] \to \text{Sin}\left[\frac{1}{2}\left(x^2 + 2y\right)\right]\right\}\right\}$$

In[13]:=

```
Plot3D[u[x, y] /. sol, {x, 0, 1}, {y, 0, 1}]
```

Out[13]=



Q2. u(x+y)u_x + u(x-y)u_y = x^2 + y^2, u = 0, y = 2x

In[14]:=
```
sol = DSolve[{u[x, y] * (x + y) * D[u[x, y], x] +
      u[x, y] * (x - y) * D[u[x, y], y] == x^2 + y^2,
    u[x, 2 x] == 0}, u[x, y], {x, y}]
```

··· Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[14]=

$$\left\{\left\{u[x, y] \to -\sqrt{\frac{2}{7}}\sqrt{2 x^2 + 3 x y - 2 y^2}\right\}, \left\{u[x, y] \to \sqrt{\frac{2}{7}}\sqrt{2 x^2 + 3 x y - 2 y^2}\right\}, \left\{u[x, y] \to -\sqrt{\frac{2}{7}}\sqrt{2 x^2 + 3 x y - 2 y^2}\right\}, \left\{u[x, y] \to \sqrt{\frac{2}{7}}\sqrt{2 x^2 + 3 x y - 2 y^2}\right\}\right\}$$

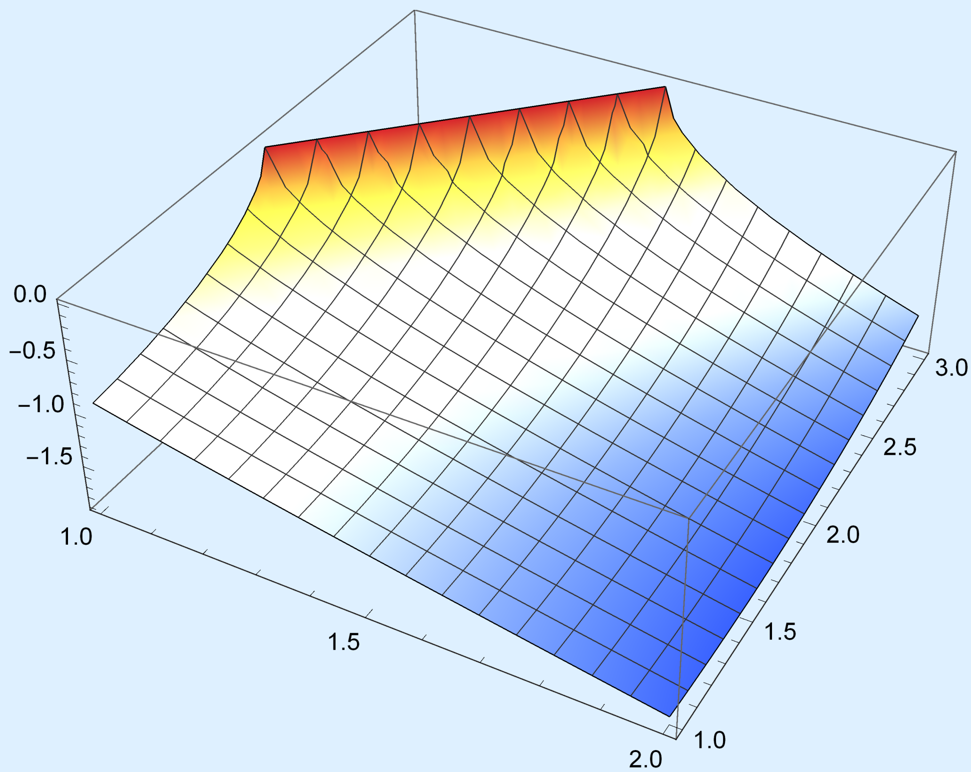
In[15]:=
```
sol[[1, 1]]
```

Out[15]=

$$u[x, y] \to -\sqrt{\frac{2}{7}}\sqrt{2 x^2 + 3 x y - 2 y^2}$$

In[16]:=
```
Plot3D[u[x, y] /. sol[[1, 1]], {x, 1, 2},
 {y, 1, 3}, ColorFunction → "TemperatureMap"]
```

Out[16]=



Q3. u_x + xu_y = (y - $\frac{1}{2}$ x^2)^2, u(0, y) = e^y

In[17]:=
```
sol =
 DSolve[{D[u[x, y], x] + x D[u[x, y], y] == (y - 1 / 2 x^2) ^2,
   u[0, y] == Exp[y]}, u[x, y], {x, y}]
```

Out[17]=

$$\left\{\left\{u[x, y] \to \frac{1}{4} e^{-\frac{x^2}{2}} \left(4 e^y + e^{\frac{x^2}{2}} x^5 - 4 e^{\frac{x^2}{2}} x^3 y + 4 e^{\frac{x^2}{2}} x y^2\right)\right\}\right\}$$

In[18]:=

```
Plot3D[u[x, y] /. sol, {x, 2, 4},
 {y, 1, 3}, ColorFunction → Hue]
```

Out[18]=