Practical 8

Solve the following first order partial difference equations and hence plot their integral surfaces.

1. xu_x + yu_y = 2xy

```
pde1 = x * D[u[x, y], x] + y * D[u[x, y], y] == 2 * x * y;
sol1 = DSolve[pde1, u[x, y], {x, y}]
```

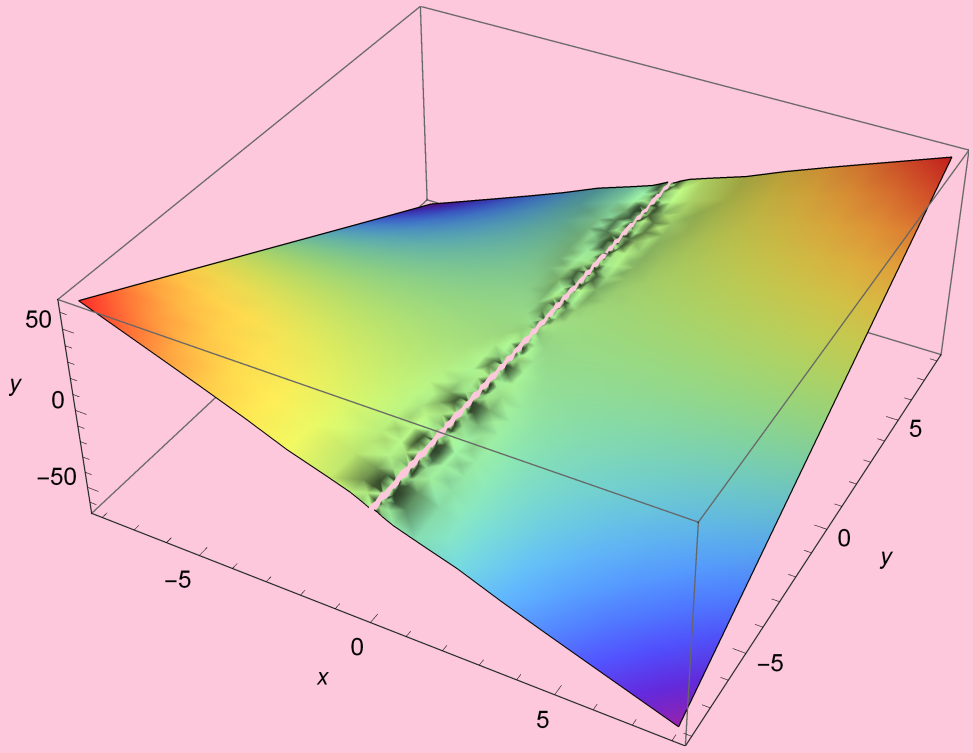
$$\left\{\left\{u[x, y] \to x\, y + C[1]\left[\frac{y}{x}\right]\right\}\right\}$$

a. C[a] = Sin a

```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → Sin[a]
```

$$x\, y + \text{Sin}\left[\frac{y}{x}\right]$$

```
Plot3D[parsol, {x, -8, 8},
 {y, -8, 8}, AxesLabel → {x, y, y},
 Mesh → None, ColorFunction → "Rainbow"]
```

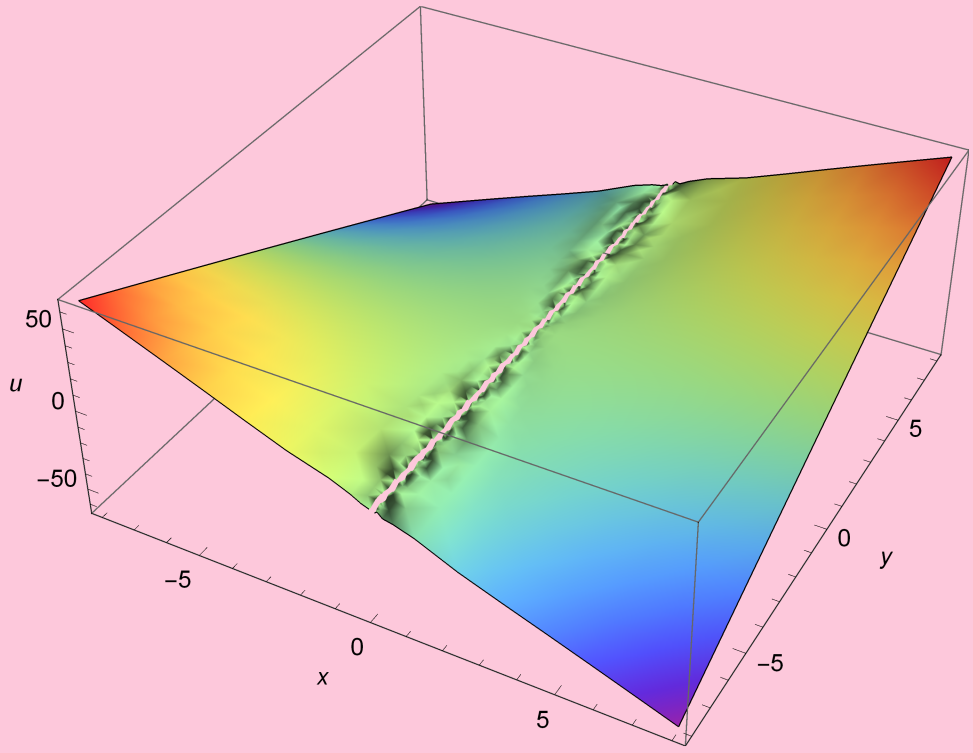


b. C[a] = Cos a

```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → Cos[a]
```

$$x\,y + \text{Cos}\left[\frac{y}{x}\right]$$

```
Plot3D[parsol, {x, -8, 8},
 {y, -8, 8}, AxesLabel → {x, y, u},
 Mesh → None, ColorFunction → "Rainbow"]
```

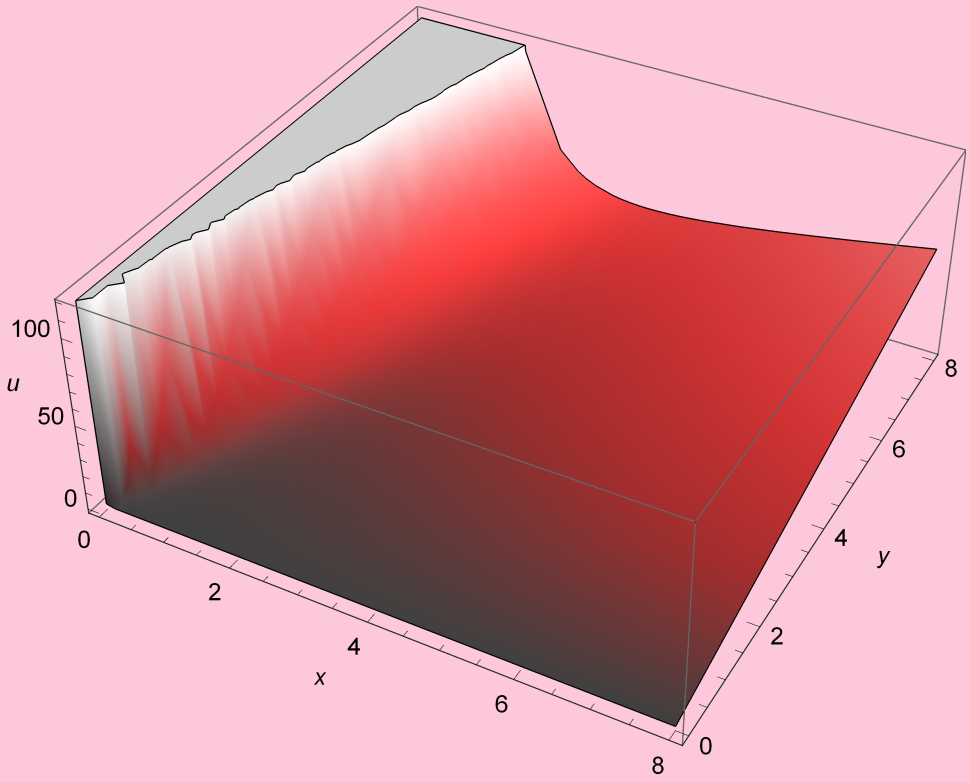


c. C[a] = a^3

```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → a^3
```

$$x\,y + \frac{y^3}{x^3}$$

```
Plot3D[parsol, {x, 0, 8},
 {y, 0, 8}, AxesLabel → {x, y, u},
 Mesh → None, ColorFunction → "CherryTones"]
```



Solve the following :
1. 3u_x + 2u_y = 0
2. u_x - u_y = 1

```
pde2 = 3*D[u[x, y], x] + 2*D[u[x, y], y] == 0;
sol1 = DSolve[pde2, u[x, y], {x, y}]
```

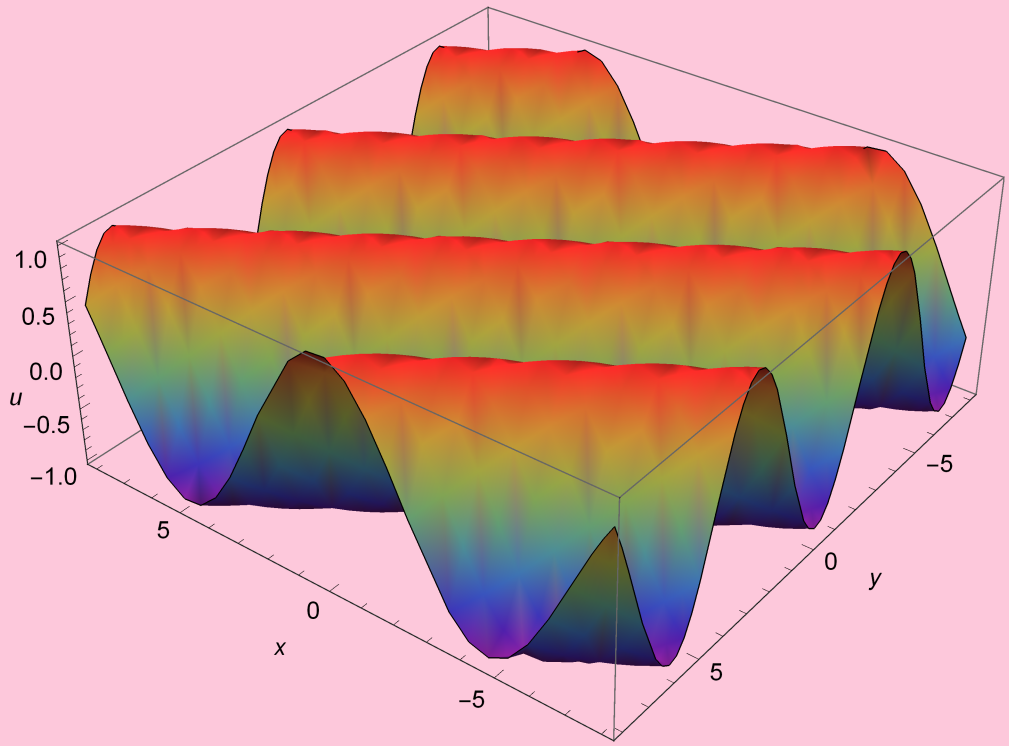
$$\left\{\left\{u[x, y] \to C[1]\left[\frac{1}{3}(-2x + 3y)\right]\right\}\right\}$$

a. C[a] = Sin a

```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → Sin[a]
```

$$\text{Sin}\left[\frac{1}{3}\,(-2\,x+3\,y)\right]$$

```
Plot3D[parsol, {x, -8, 8},
 {y, -8, 8}, AxesLabel → {x, y, u},
 Mesh → None, ColorFunction → "Rainbow"]
```

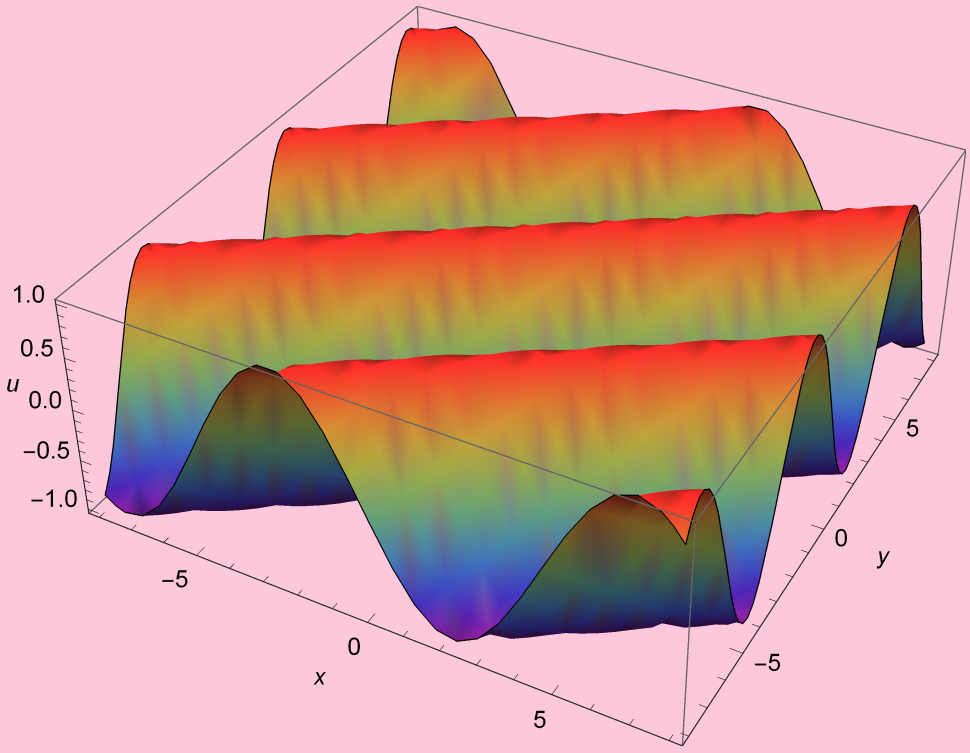


b. C[a] = Cos a

```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → Cos[a]
```

$$\text{Cos}\left[\frac{1}{3}\,(-2\,x+3\,y)\right]$$

```
Plot3D[parsol, {x, -8, 8},
 {y, -8, 8}, AxesLabel → {x, y, u},
 Mesh → None, ColorFunction → "Rainbow"]
```

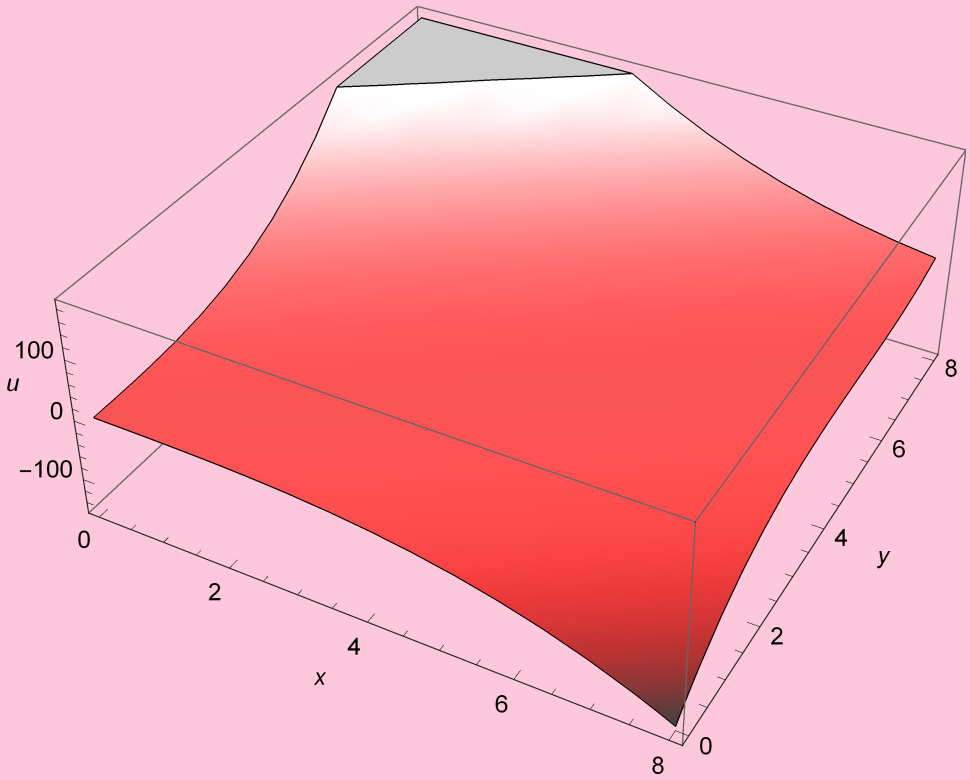


c. C[a] = a^3

```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → a^3
```

$$\frac{1}{27}(-2x+3y)^3$$

```
Plot3D[parsol, {x, 0, 8},
 {y, 0, 8}, AxesLabel → {x, y, u},
 Mesh → None, ColorFunction → "CherryTones"]
```



2. u_x - u_y = 1

```
pde3 = D[u[x, y], x] + D[u[x, y], y] == 1;
sol1 = DSolve[pde3, u[x, y], {x, y}]
```

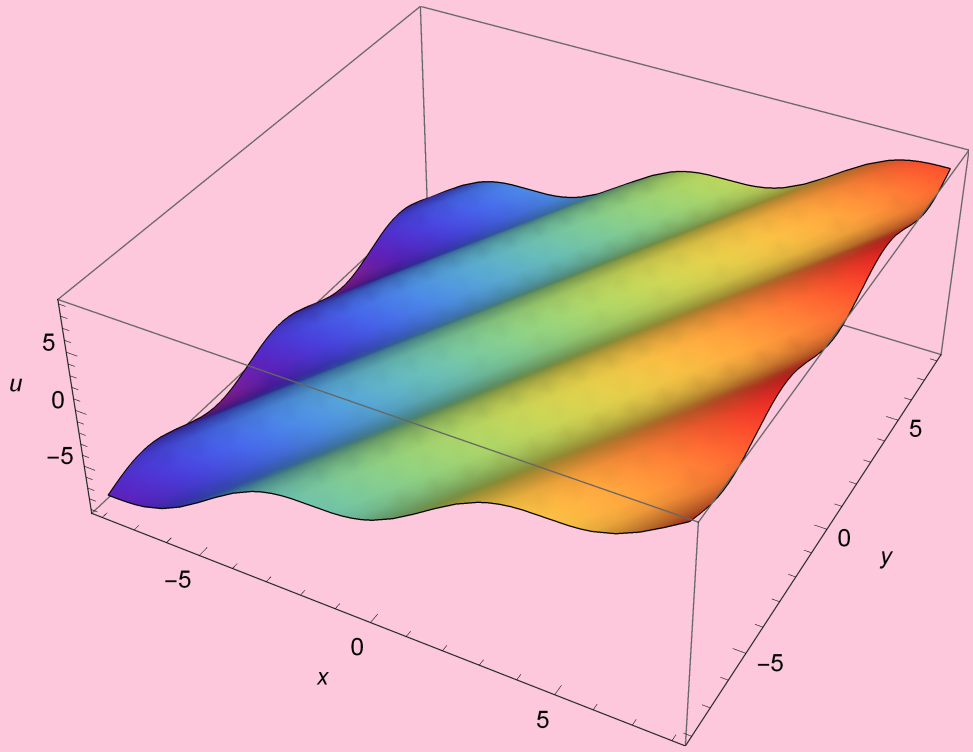
```
{{u[x, y] → x + C[1][−x + y]}}
```

a. C[a] = Sina

```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → Sin[a]
```

```
x - Sin[x - y]
```

```
Plot3D[parsol, {x, -8, 8},
 {y, -8, 8}, AxesLabel → {x, y, u},
 Mesh → None, ColorFunction → "Rainbow"]
```

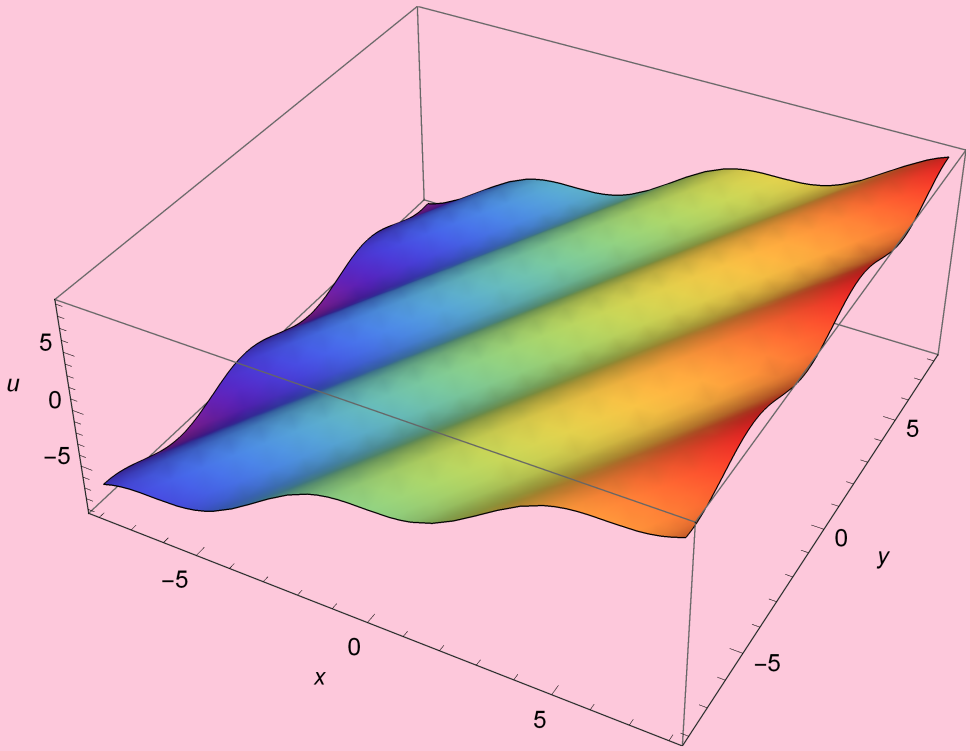


b. C[a] = Cos a

```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → Cos[a]
```

```
x + Cos[x - y]
```

```
Plot3D[parsol, {x, -8, 8},
 {y, -8, 8}, AxesLabel → {x, y, u},
 Mesh → None, ColorFunction → "Rainbow"]
```



c. C[a] = a^3

```
parsol = u[x, y] /. sol1[[1]] /. C[1][a_] → a^3
```

$$x + (-x + y)^3$$

```
Plot3D[parsol, {x, 0, 8},
 {y, 0, 8}, AxesLabel → {x, y, u},
 Mesh → None, ColorFunction → "CherryTones"]
```