Practical 5

Example: Solve the following system of equations:

$$\frac{dx}{dt} = -3x - y$$
$$\frac{dy}{dt} = x - 3y$$

```
eq1 = {x'[t] == -3*x[t] - y[t], y'[t] == -3 y[t] - x[t]}
```

```
{x′[t] == -3 x[t] - y[t], y′[t] == -x[t] - 3 y[t]}
```

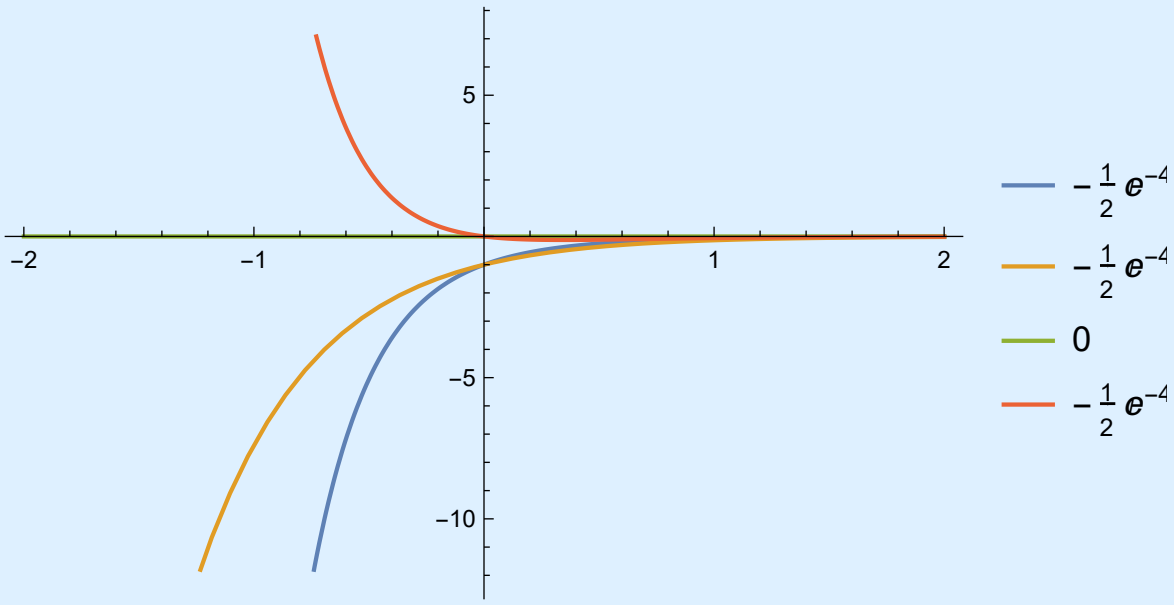
```
sol = DSolve[eq1, {y[t], x[t]}, t]
```

$$\left\{\left\{x[t] \to \frac{1}{2} e^{-4t}\left(1 + e^{2t}\right) C[1] - \frac{1}{2} e^{-4t}\left(-1 + e^{2t}\right) C[2],\right.\right.$$
$$\left.\left. y[t] \to -\frac{1}{2} e^{-4t}\left(-1 + e^{2t}\right) C[1] + \frac{1}{2} e^{-4t}\left(1 + e^{2t}\right) C[2]\right\}\right\}$$

```
tabx = Table[x[t] /. sol[[1, 1]] /. {C[1] → i, C[2] → j},
  {i, -1, 0}, {j, 0, 1}]
```

$$\left\{\left\{-\frac{1}{2} e^{-4t}\left(1 + e^{2t}\right), -\frac{1}{2} e^{-4t}\left(-1 + e^{2t}\right) - \frac{1}{2} e^{-4t}\left(1 + e^{2t}\right)\right\},\right.$$
$$\left.\left\{0, -\frac{1}{2} e^{-4t}\left(-1 + e^{2t}\right)\right\}\right\}$$

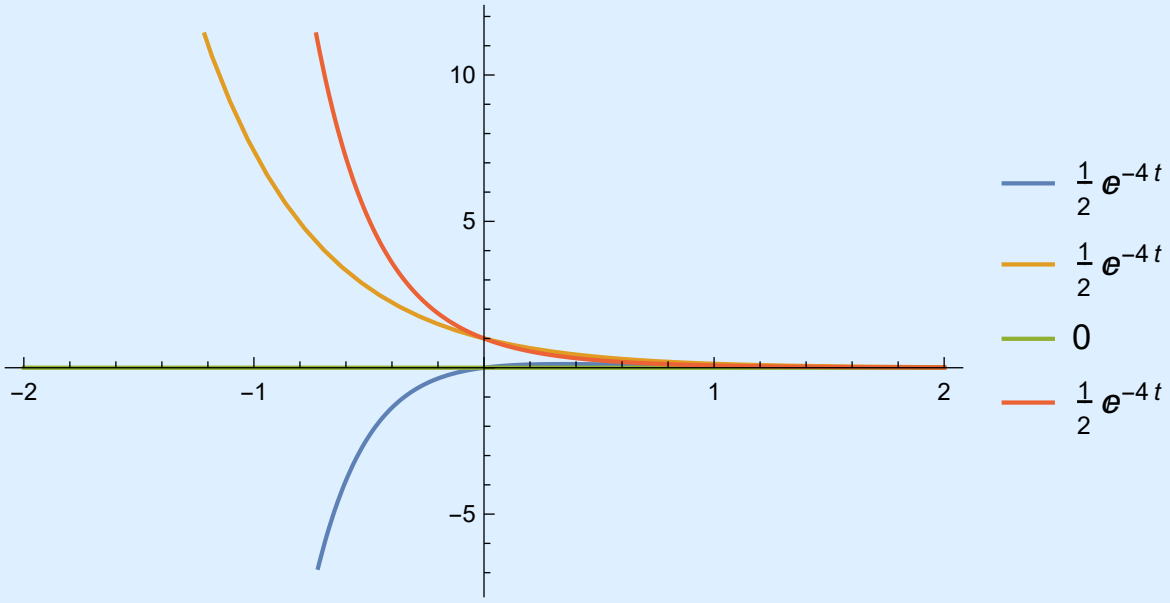
```
Plot[Evaluate[tabx], {t, -2, 2},
 PlotLegends → "Expressions"]
```



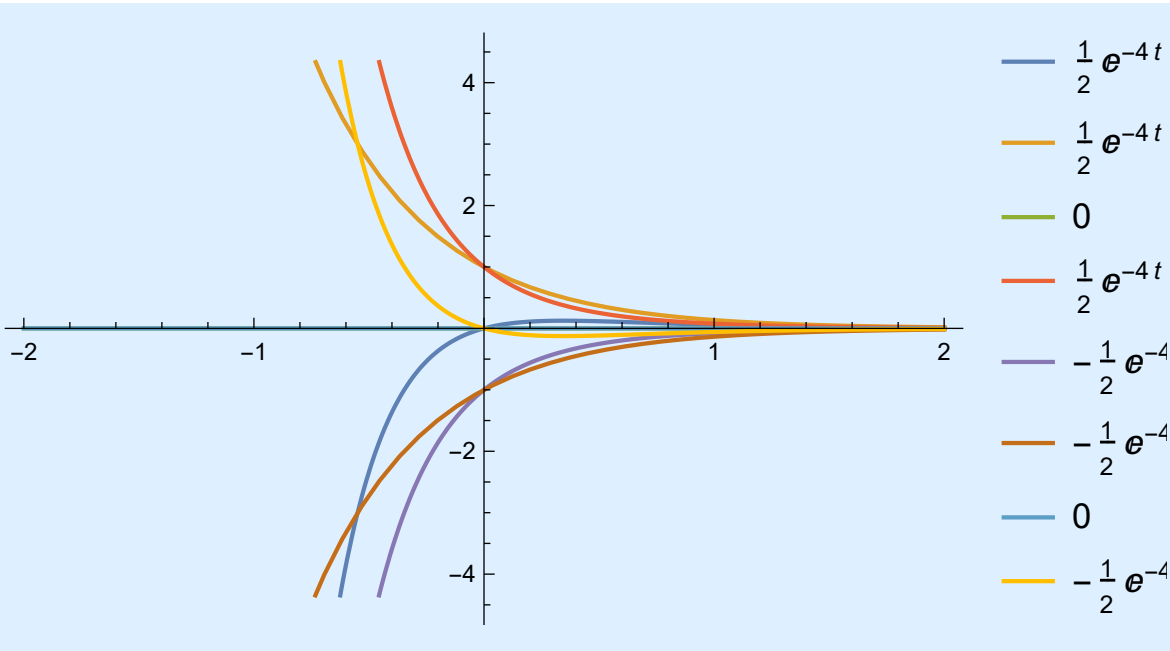

```
taby = Table[y[t] /. sol[[1, 2]] /. {C[1] → i, C[2] → j},
  {i, -1, 0}, {j, 0, 1}]
```

$$\left\{\left\{\frac{1}{2}\,e^{-4t}\left(-1+e^{2t}\right),\ \frac{1}{2}\,e^{-4t}\left(-1+e^{2t}\right)+\frac{1}{2}\,e^{-4t}\left(1+e^{2t}\right)\right\},\ \left\{0,\ \frac{1}{2}\,e^{-4t}\left(1+e^{2t}\right)\right\}\right\}$$

```
Plot[Evaluate[taby], {t, -2, 2},
 PlotLegends → "Expressions"]
```




```
Plot[Evaluate[{taby, tabx}],
 {t, -2, 2}, PlotLegends → "Expressions"]
```



Example: Solve the following systems of equations:

$\frac{dx}{dt} = y$
$\frac{dy}{dt} = 6x - y$
with initial conditions x(0) = 1, y(0) = -2

```
eq2 = {{x'[t] == y[t], y'[t] == -y[t] + 6 x[t]},
  x[0] == 1, y[0] == -2}
```

$$\{\{x'[t] == y[t], y'[t] == 6\,x[t] - y[t]\}, x[0] == 1, y[0] == -2\}$$

```
DSolve[eq2, {x[t], y[t]}, t]
```

$$\left\{\left\{x[t] \to \frac{1}{5} e^{-3t}\left(4 + e^{5t}\right), y[t] \to \frac{2}{5} e^{-3t}\left(-6 + e^{5t}\right)\right\}\right\}$$

```
{xsol[t_], ysol[t_]} = ExpandAll[
  {x[t], y[t]} /. Flatten[DSolve[eq2, {x[t], y[t]}, t]]]
```

$$\left\{\frac{4\,e^{-3t}}{5} + \frac{e^{2t}}{5}, -\frac{12}{5} e^{-3t} + \frac{2\,e^{2t}}{5}\right\}$$

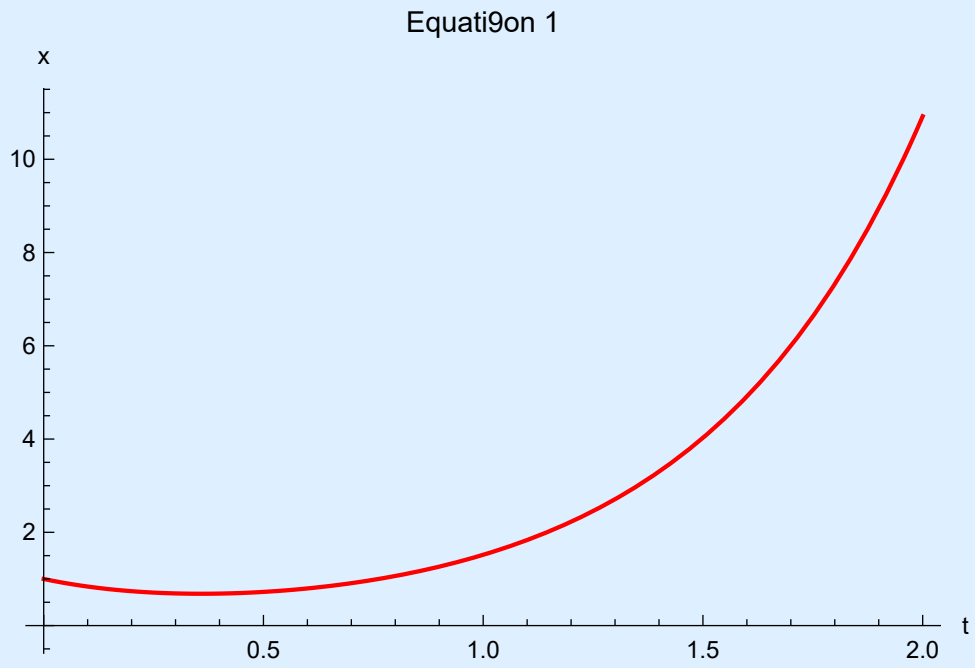
```
xsol[t]
```

$$\frac{4\,e^{-3t}}{5} + \frac{e^{2t}}{5}$$

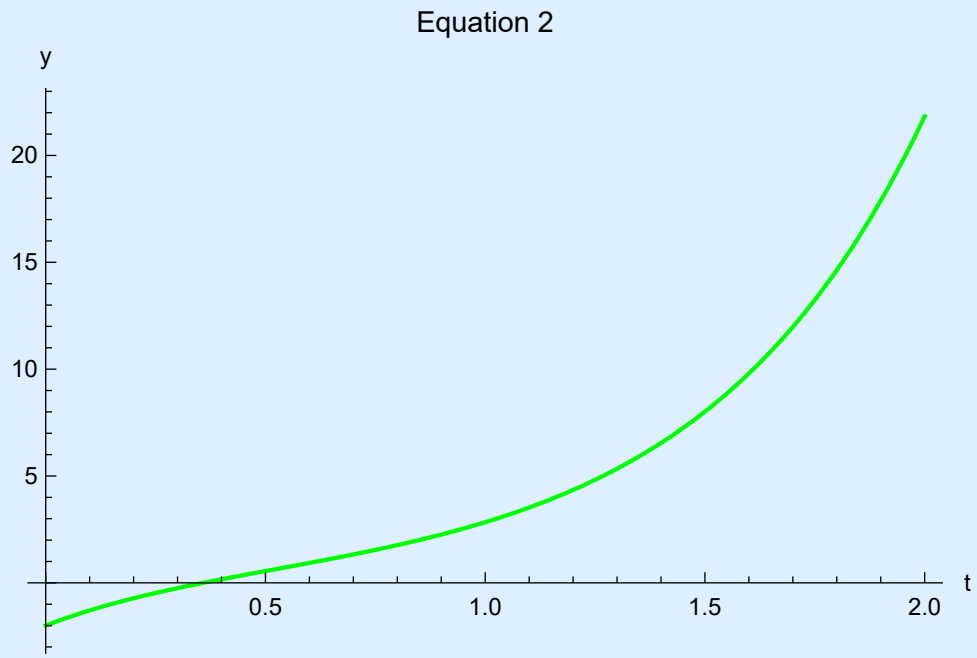
```
ysol[t]
```

$$-\frac{12}{5} e^{-3t} + \frac{2\,e^{2t}}{5}$$

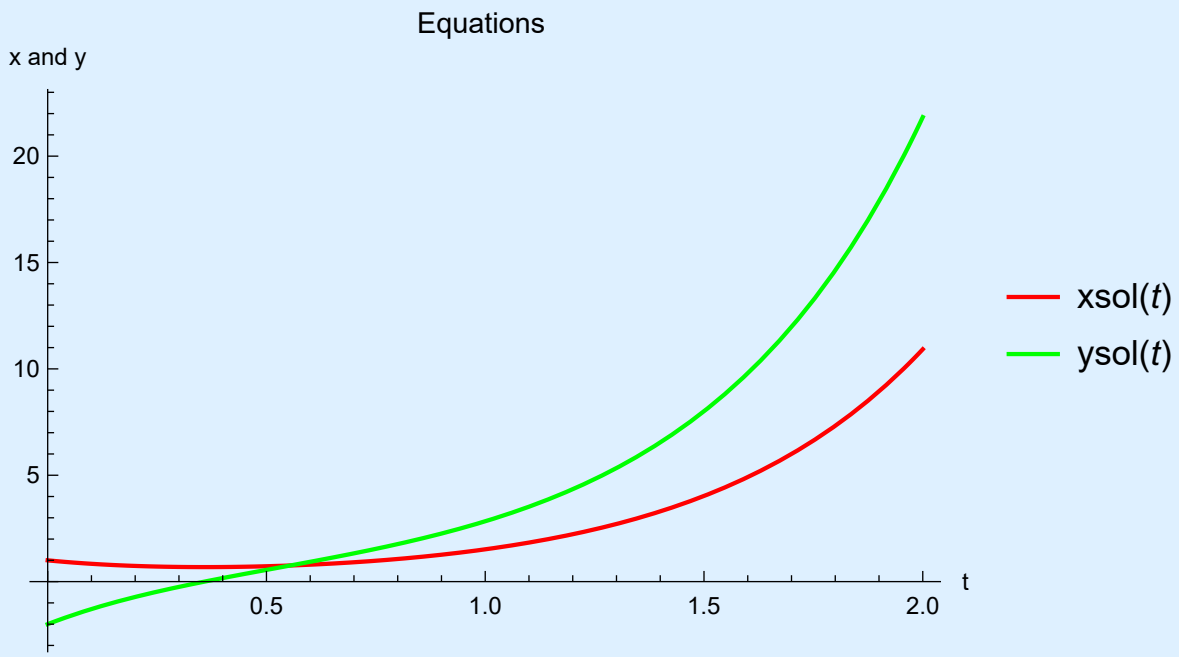
```
plot1 = Plot[xsol[t], {t, 0, 2}, AxesLabel → {"t", "x"},
  PlotLabel → "Equati9on 1", PlotStyle → {Red}]
```



```
plot2 = Plot[ysol[t], {t, 0, 2}, AxesLabel → {"t", "y"},
  PlotLabel → "Equation 2", PlotStyle → {Green}]
```



```
Plot[{xsol[t], ysol[t]}, {t, 0, 2},
 AxesLabel → {"t", "x and y"}, PlotLabel → "Equations ",
 PlotStyle → {Red, Green}, PlotLegends → "Expressions"]
```



$$\frac{dx}{dt} = 5x - 2y$$
$$\frac{dy}{dt} = 4x - y$$

```
eq = {x'[t] == 5*x[t] - 2*y[t], y'[t] == 4*x[t] - y[t]}
```

```
{x'[t] == 5 x[t] - 2 y[t], y'[t] == 4 x[t] - y[t]}
```

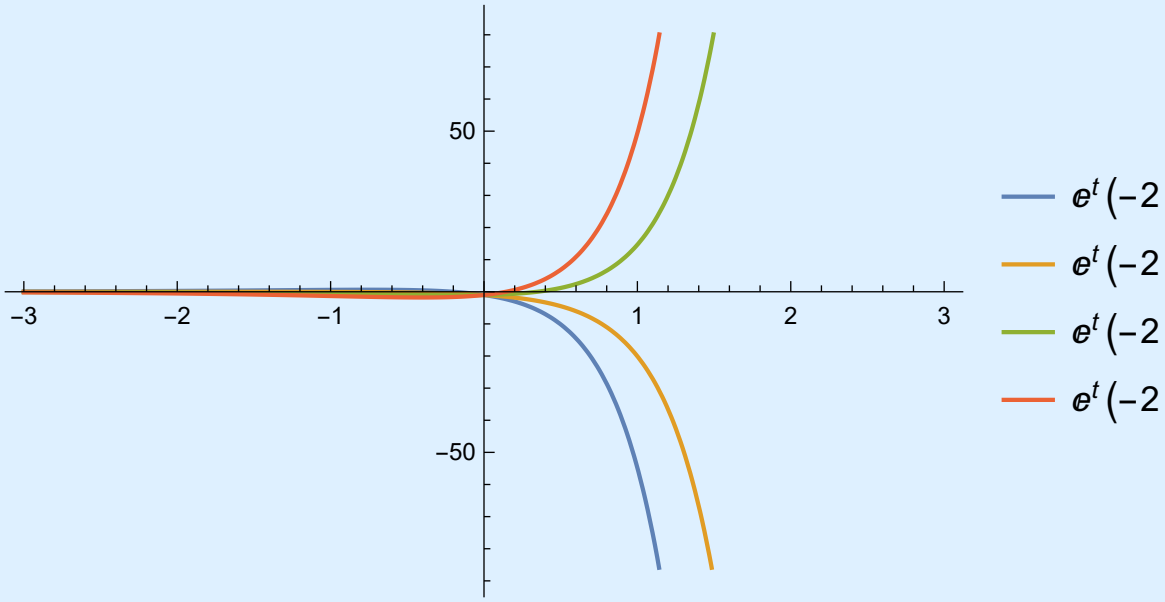
```
sol = DSolve[eq, {y[t], x[t]}, t]
```

```
{{x[t] → ℯ^t (-1 + 2 ℯ^(2 t)) C[1] - ℯ^t (-1 + ℯ^(2 t)) C[2],
  y[t] → 2 ℯ^t (-1 + ℯ^(2 t)) C[1] - ℯ^t (-2 + ℯ^(2 t)) C[2]}}
```

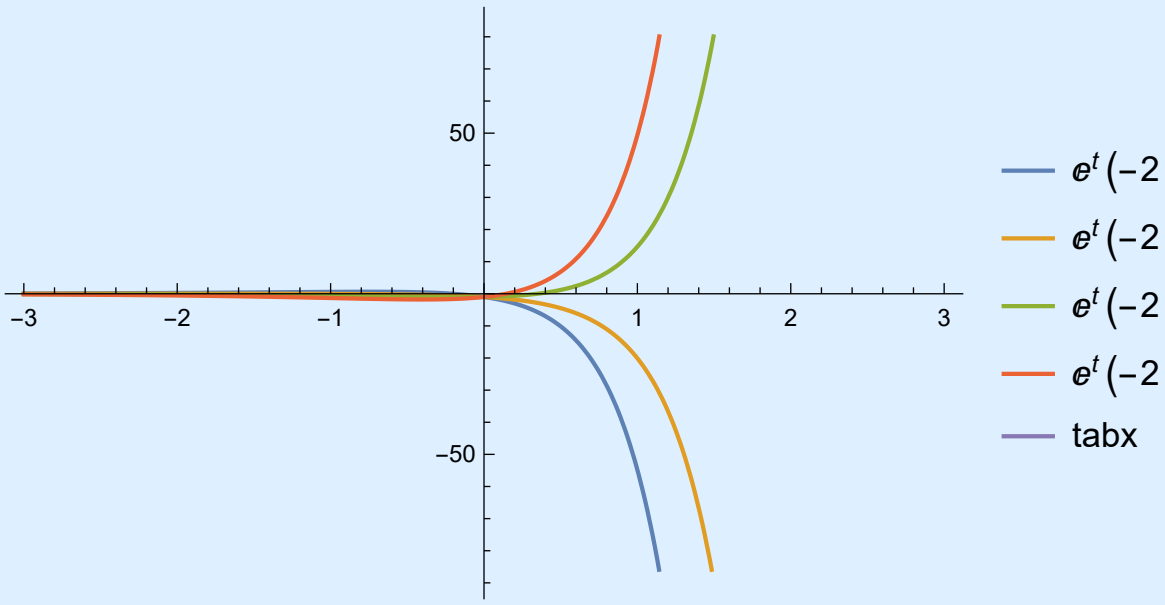
```
taby = Table[y[t] /. sol[[1, 2]] /. {C[1] → i, C[2] → j},
    {i, -2, 1}, {j, -1, -1}] // Flatten
```

$$\left\{e^t\left(-2+e^{2t}\right)-4\,e^t\left(-1+e^{2t}\right),\ e^t\left(-2+e^{2t}\right)-2\,e^t\left(-1+e^{2t}\right),\ e^t\left(-2+e^{2t}\right),\ e^t\left(-2+e^{2t}\right)+2\,e^t\left(-1+e^{2t}\right)\right\}$$

```
Plot[Evaluate[taby], {t, -3, 3}, PlotLegends → "Expressions"]
```



```
Plot[Evaluate[{taby, tabx}],
 {t, -3, 3}, PlotLegends → "Expressions"]
```




```
{x′[t] == 3 x[t] - 4 y[t], y′[t] == 2 x[t] - y[t]}
```

$$\frac{dx}{dt} = 3x - 4y$$
$$\frac{dy}{dt} = 2x - y$$

```
eq1 = {x'[t] == 3 * x[t] - 4 * y[t], y'[t] == -1 * y[t] + 2 * x[t]}
```

```
{x′[t] == 3 x[t] - 4 y[t], y′[t] == 2 x[t] - y[t]}
```

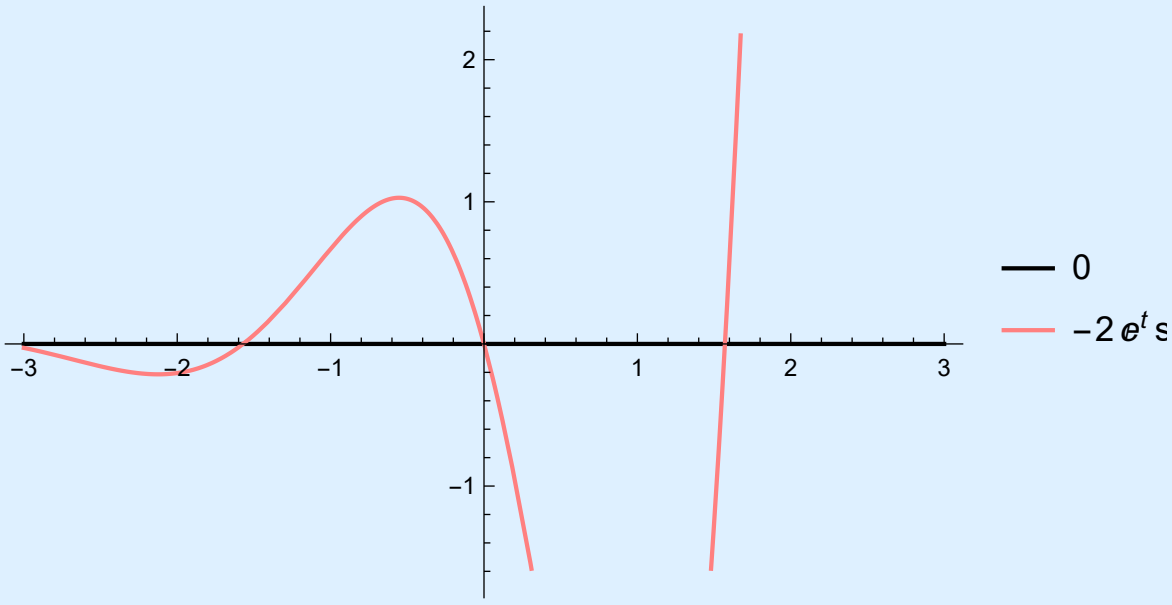
```
sol = DSolve[eq1, {y[t], x[t]}, t]
```

$$\left\{\left\{x[t] \to -2\, e^{t}\, C[2]\, \text{Sin}[2\, t] + e^{t}\, C[1]\, (\text{Cos}[2\, t] + \text{Sin}[2\, t]),\right.\right.$$
$$\left.\left. y[t] \to e^{t}\, C[2]\, (\text{Cos}[2\, t] - \text{Sin}[2\, t]) + e^{t}\, C[1]\, \text{Sin}[2\, t]\right\}\right\}$$

```
tabx = Table[x[t] /. sol[[1, 1]] /. {C[1] → i, C[2] → j},
    {i, 0, 0}, {j, 0, 1}] // Flatten
```

$$\left\{0, -2\, e^{t}\, \text{Sin}[2\, t]\right\}$$

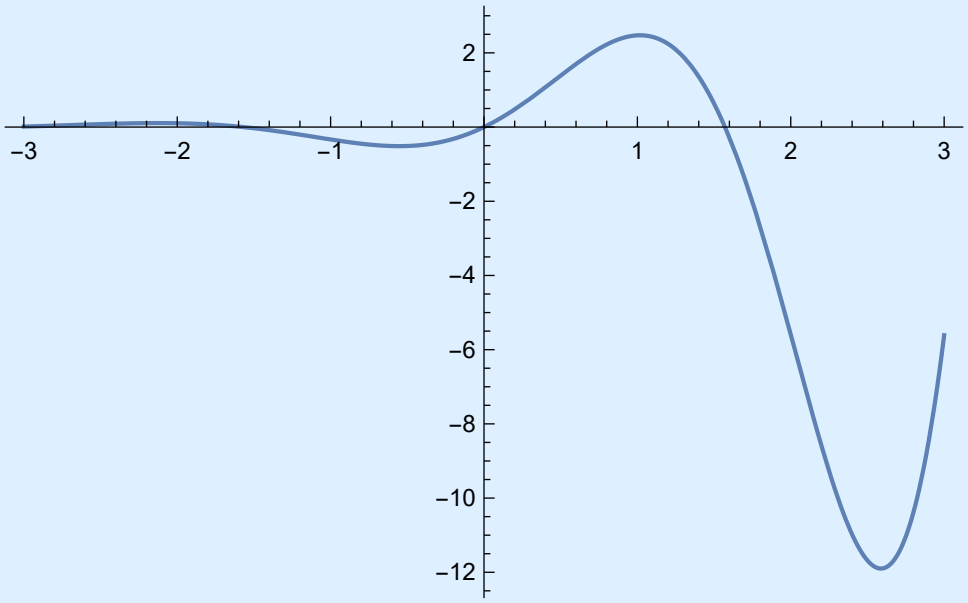
```
Plot[Evaluate[tabx], {t, -3, 3}, PlotLegends → "Expressions",
 PlotStyle → {Black, Pink, Brown}]
```



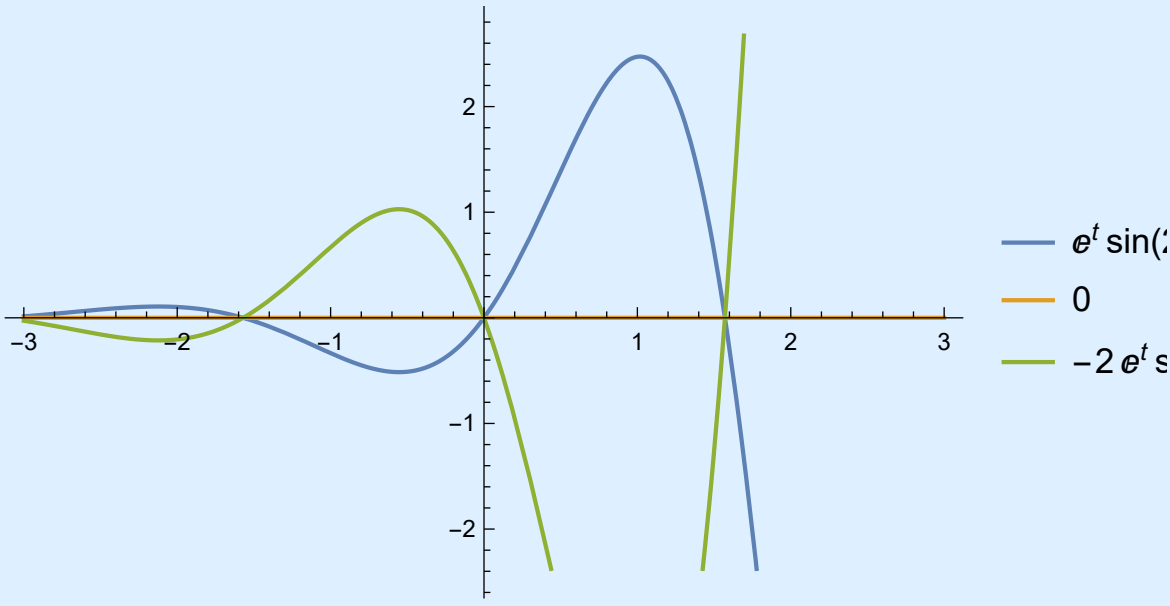

```
taby = Table[y[t] /. sol[[1, 2]] /. {C[1] → i, C[2] → j},
    {i, 1, 1}, {j, 0, 0}] // Flatten
```

$$\left\{e^{t}\, \text{Sin}[2\, t]\right\}$$

```
Plot[Evaluate[taby], {t, -3, 3}, PlotLegends → "Expressions"]
```




```
Plot[Evaluate[{taby, tabx}],
 {t, -3, 3}, PlotLegends → "Expressions"]
```



$\frac{dx}{dt} = -2x + 7y$ with x(0) = 9, y(0) = -1

$$\frac{dy}{dt} = 3x + 2y$$

```
eq2 = {{x'[t] == -2 * x[t] + 7 * y[t], y'[t] == 2 * y[t] + 3 * x[t]},
  x[0] == 9, y[0] == -1}
```

$$\{\{x'[t] == -2\,x[t] + 7\,y[t],\ y'[t] == 3\,x[t] + 2\,y[t]\},\ x[0] == 9,\ y[0] == -1\}$$

```
DSolve[eq2, {x[t], y[t]}, t]
```

$$\left\{\left\{x[t] \to e^{-5t}\left(7 + 2\,e^{10t}\right),\ y[t] \to e^{-5t}\left(-3 + 2\,e^{10t}\right)\right\}\right\}$$

```
{xsol[t_], ysol[t_]} = ExpandAll[
  {x[t], y[t]} /. Flatten[DSolve[eq2, {x[t], y[t]}, t]]]
```

$$\left\{7\,e^{-5t} + 2\,e^{5t},\ -3\,e^{-5t} + 2\,e^{5t}\right\}$$

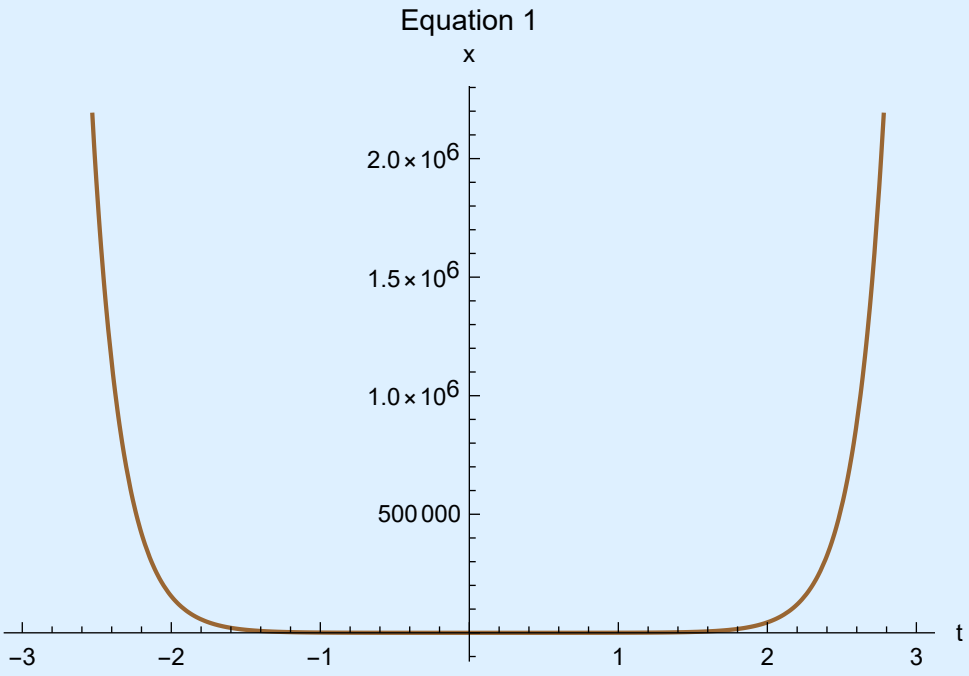
```
xsol[t]
```

$$7\,e^{-5t} + 2\,e^{5t}$$

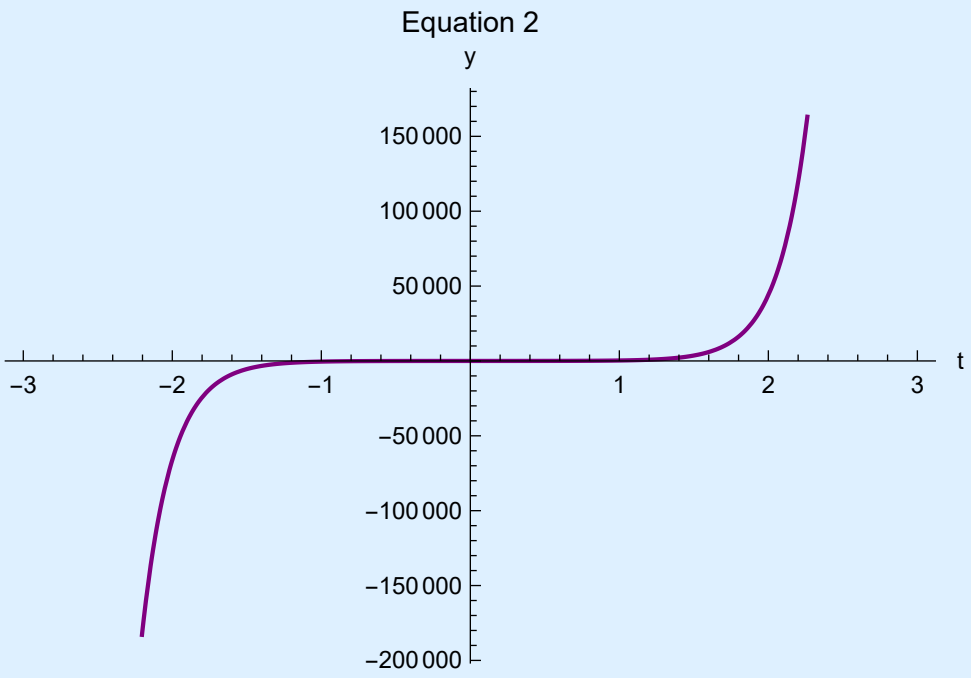
```
ysol[t]
```

$$-3\,e^{-5t} + 2\,e^{5t}$$

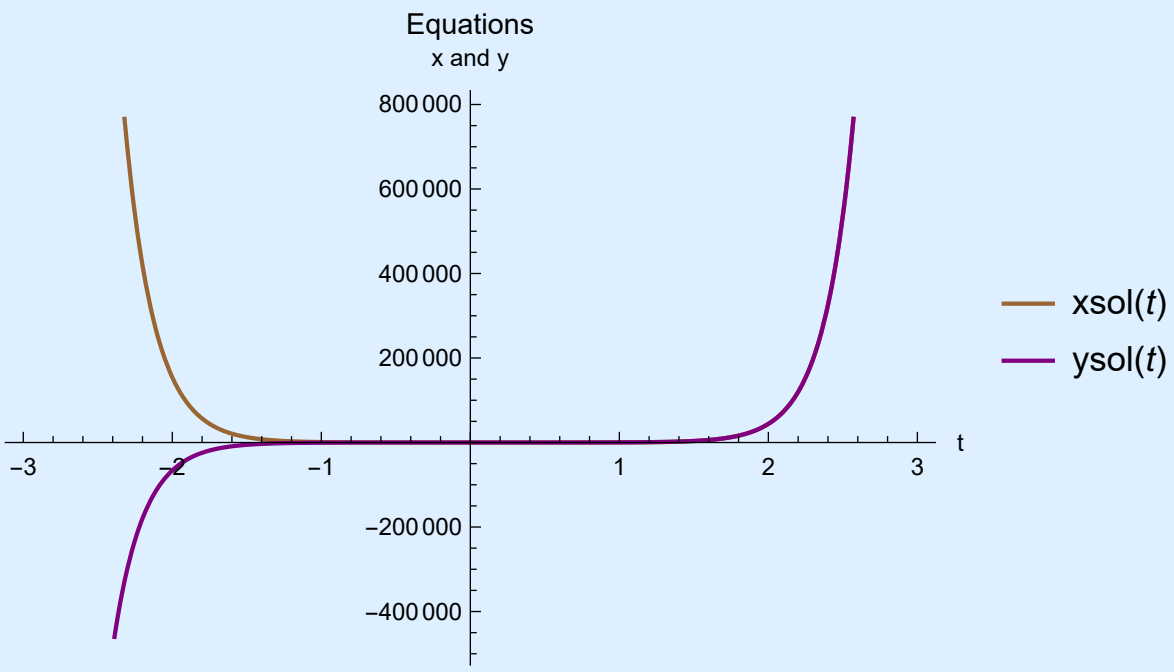
```
plot1 = Plot[xsol[t], {t, -3, 3}, AxesLabel → {"t", "x"},
  PlotLabel → "Equation 1", PlotStyle → {Brown}]
```



```
plot2 = Plot[ysol[t], {t, -3, 3}, AxesLabel → {"t", "y"},
  PlotLabel → "Equation 2", PlotStyle → {Purple}]
```



```
Plot[{xsol[t], ysol[t]}, {t, -3, 3},
 AxesLabel → {"t", "x and y"}, PlotLabel → "Equations",
 PlotStyle → {Brown, Purple}, PlotLegends → "Expressions"]
```



$$\frac{dx}{dt} = 7x - y \text{ with } x(0) = 1,\ y(0) = 3$$
$$\frac{dy}{dt} = 4x + 3y$$

```
eq2 = {{x'[t] == 7 * x[t] - 1 * y[t], y'[t] == 3 * y[t] + 4 * x[t]},
   x[0] == 1, y[0] == 3}
```

```
{{x'[t] == 7 x[t] - y[t], y'[t] == 4 x[t] + 3 y[t]},
 x[0] == 1, y[0] == 3}
```

```
DSolve[eq2, {x[t], y[t]}, t]
```

```
{{x[t] → -ℯ^(5 t) (-1 + t), y[t] → -ℯ^(5 t) (-3 + 2 t)}}
```

```
{xsol[t_], ysol[t_]} = ExpandAll[
  {x[t], y[t]} /. Flatten[DSolve[eq2, {x[t], y[t]}, t]]]
```

$$\left\{e^{5\,t} - e^{5\,t}\,t,\ 3\,e^{5\,t} - 2\,e^{5\,t}\,t\right\}$$

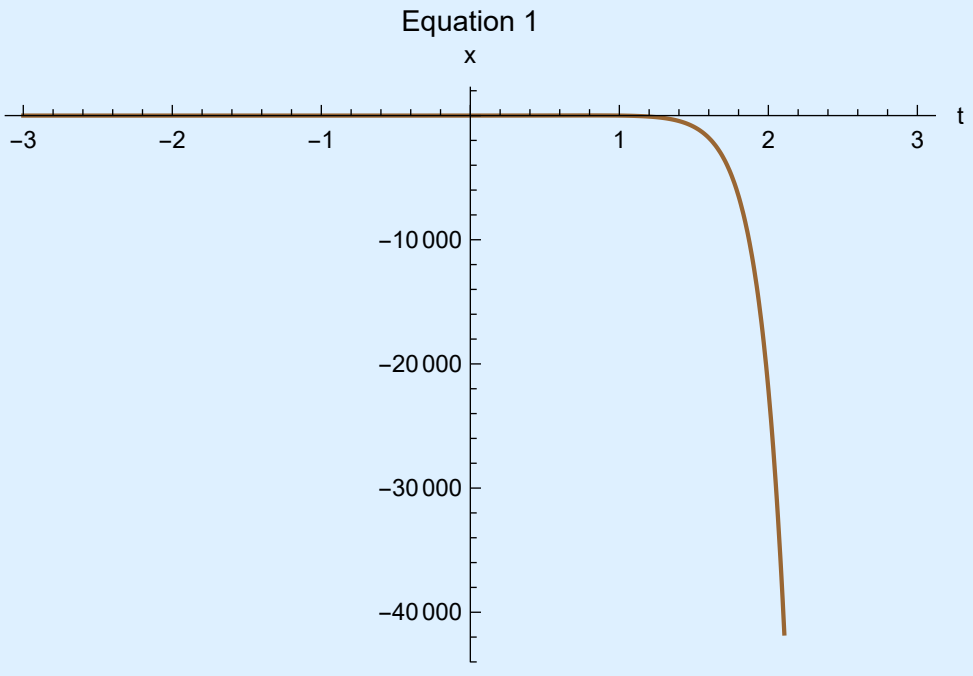
```
xsol[t]
```

$$e^{5\,t} - e^{5\,t}\,t$$

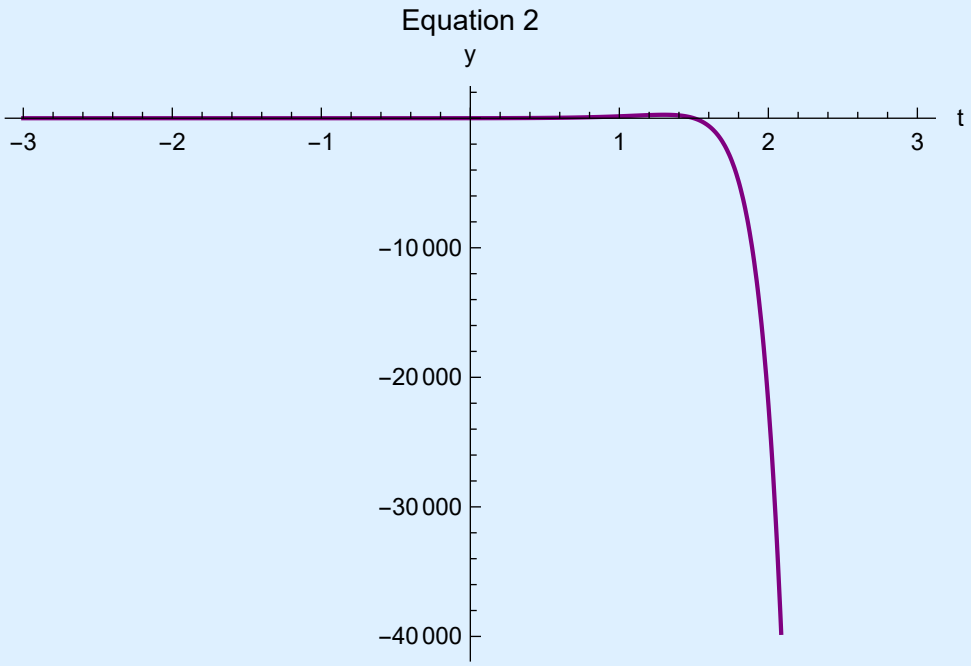
```
ysol[t]
```

$$3\,e^{5\,t} - 2\,e^{5\,t}\,t$$

```
plot1 = Plot[xsol[t], {t, -3, 3}, AxesLabel → {"t", "x"},
  PlotLabel → "Equation 1", PlotStyle → {Brown}]
```



```
plot2 = Plot[ysol[t], {t, -3, 3}, AxesLabel → {"t", "y"},
  PlotLabel → "Equation 2", PlotStyle → {Purple}]
```



```
Plot[{xsol[t], ysol[t]}, {t, -3, 3},
 AxesLabel → {"t", "x and y"}, PlotLabel → "Equations",
 PlotStyle → {Brown, Purple}, PlotLegends → "Expressions"]
```