Practical 1

Solution of First order Differential Equations

```
y = .
```

```
x = .
```

```
z = .
```

Plot and solve first order Differential Equation

Ques Solve $\frac{dy}{dx}$= y

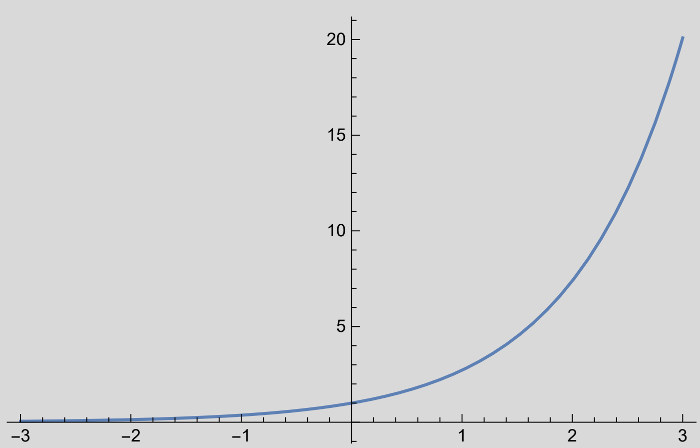
```
q1 = DSolve[{y'[x] == y[x]}, y[x], x]
```

$\{\{y[x] \to e^x C[1]\}\}$

```
q2 = y[x] /. sol1 /. {C[1] → 1}
```

$\{e^x\}$

```
Plot[{q2}, {x, -3, 3}]
```



Straight Integration

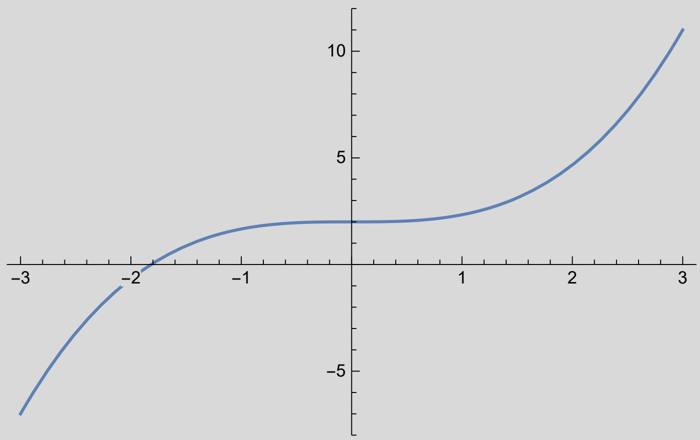
```
q3 = DSolve[y'[x] == x^2, y[x], x]
```

$\{\{y[x] \to \frac{x^3}{3} + C[1]\}\}$

```
q4 = y[x] /. q3 /. {C[1] → 2}
```

$$\left\{2+\frac{x^3}{3}\right\}$$

```
Plot[q4, {x, -3, 3}]
```



Another method

```
q5 = Table[y[x] /. q3 /. {C[1] → k}, {k, 2, 5}]
```

$$\left\{\left\{2+\frac{x^3}{3}\right\},\left\{3+\frac{x^3}{3}\right\},\left\{4+\frac{x^3}{3}\right\},\left\{5+\frac{x^3}{3}\right\}\right\}$$

```
Plot[q5, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink}, GridLines → Automatic,
  Frame → True, AxesOrigin → {0, 0}, AxesLabel → Automatic, ImageSize → Medium]
```

Plot: Options expected (instead of ImageSize → Medium) beyond position 2 in Plot[q5, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink}, ≪1≫, ≪1≫, AxesOrigin → {0, 0}, AxesLabel → Automatic, ImageSize → Medium]. An option must be a rule or a list of rules.

```
Plot[q5, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink}, GridLines → Automatic,
  Frame → True, AxesOrigin → {0, 0}, AxesLabel → Automatic, ImageSize → Medium]
```

Separable equations

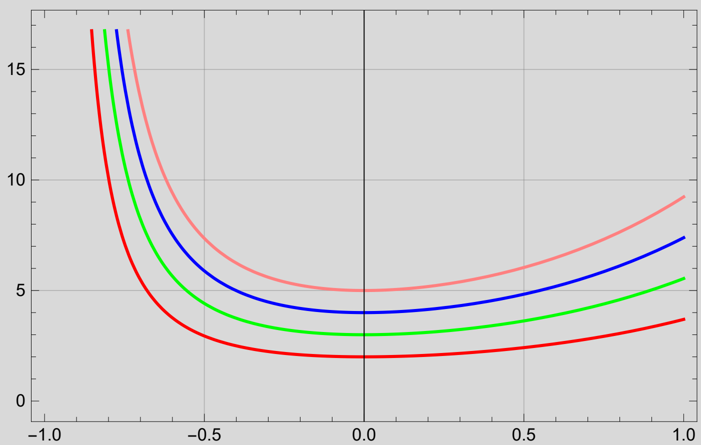
```
q1 = DSolve[y'[x] == 2 * x * y[x] / (x + 1), y[x], x]
```

$$\{\{y[x] \to e^{2\,(x-\text{Log}[1+x])}\,C[1]\}\}$$

```
q2 = Table[y[x] /. A /. {C[1] → k}, {k, 2, 5}]
```

$$\{\{2\,e^{2\,(x-\text{Log}[1+x])}\},\{3\,e^{2\,(x-\text{Log}[1+x])}\},\{4\,e^{2\,(x-\text{Log}[1+x])}\},\{5\,e^{2\,(x-\text{Log}[1+x])}\}\}$$

```
Plot[{q2}, {x, -1, 1}, PlotStyle → {Red, Green, Blue, Pink},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}]
```



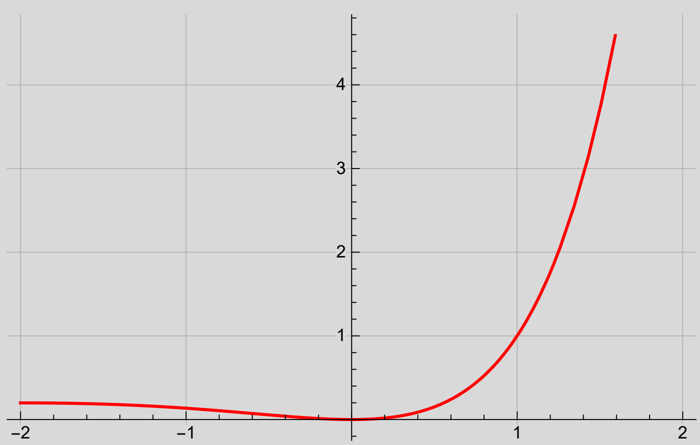
Initial Value Problem

This type of problem does not contain constant

```
q1 = DSolve[{y'[x] == y[x] + 2 / x * y[x], y[1] == 1}, y[x], x]
```

$\{\{y[x] \to e^{-1+x} x^2\}\}$

```
Plot[y[x] /. q1, {x, -2, 2}, PlotStyle → {Red}, GridLines → Automatic]
```



Homogenous Equations

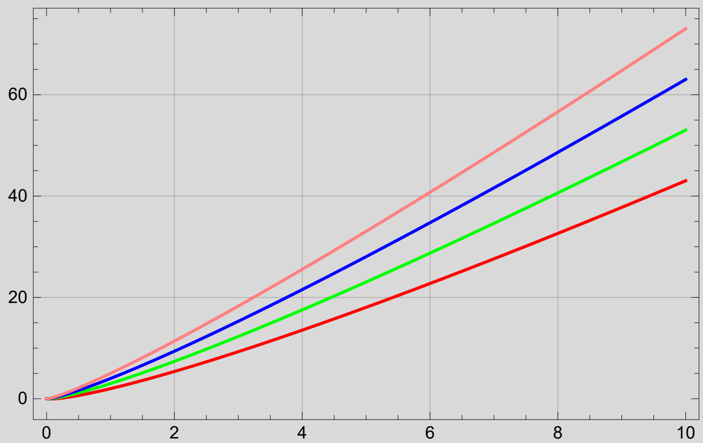
```
q1 = DSolve[y'[x] == (x + y[x]) / (x), y[x], x]
```

$\{\{y[x] \to x\, C[1] + x \operatorname{Log}[x]\}\}$

```
q2 = Table[y[x] /. q1 /. {C[1] → k}, {k, 2, 5}]
```

$\{\{2x + x \operatorname{Log}[x]\}, \{3x + x \operatorname{Log}[x]\}, \{4x + x \operatorname{Log}[x]\}, \{5x + x \operatorname{Log}[x]\}\}$

```
Plot[{q2}, {x, 0, 10}, PlotStyle → {Red, Green, Blue, Pink},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}]
```



Linear First Order Equations

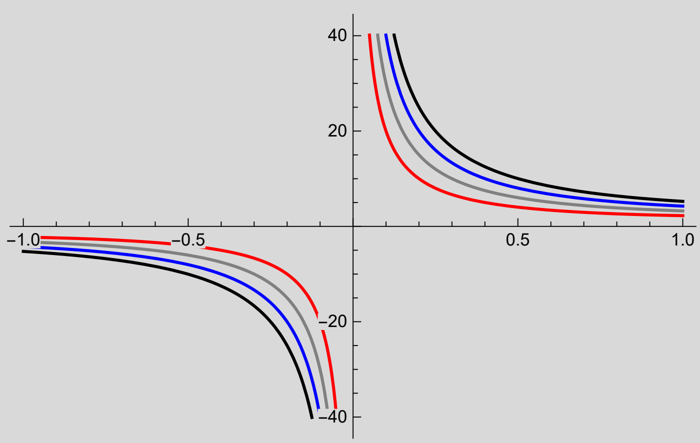
```
q1 = DSolve[y'[x] + y[x] / (x) == x^2, y[x], x]
```

$$\left\{\left\{y[x] \to \frac{x^3}{4} + \frac{C[1]}{x}\right\}\right\}$$

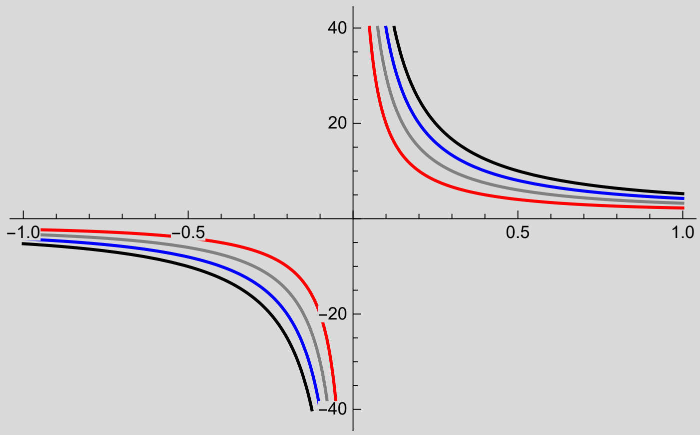
```
q2 = Table[y[x] /. sol5 /. {C[1] → k}, {k, 2, 5}]
```

$$\left\{\left\{\frac{2}{x} + \frac{x^3}{4}\right\}, \left\{\frac{3}{x} + \frac{x^3}{4}\right\}, \left\{\frac{4}{x} + \frac{x^3}{4}\right\}, \left\{\frac{5}{x} + \frac{x^3}{4}\right\}\right\}$$

```
Plot[{q2}, {x, -1, 1}, PlotStyle → {Red, Gray, Blue, Black}, PlotLegends → Automatic]
```



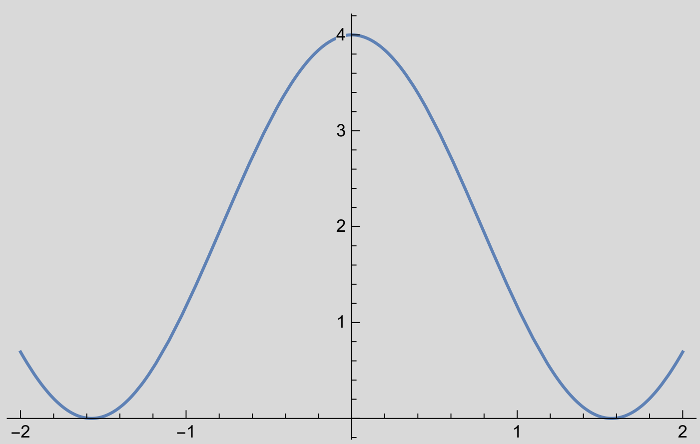
```
Plot[{q2}, {x, -1, 1}, PlotStyle → {Red, Gray, Blue, Black},
 PlotLegends → Placed[Automatic, Below]]
```




```
q3 = DSolve[{y'[x] * Tan[x] == 2 * y[x] - 8, y[Pi / 2] == 0}, y[x], x]
```

$\{\{y[x] \to 4\,\mathrm{Cos}[x]^2\}\}$

```
Plot[y[x] /. q3, {x, -2, 2}]
```



Bernoulli Equations

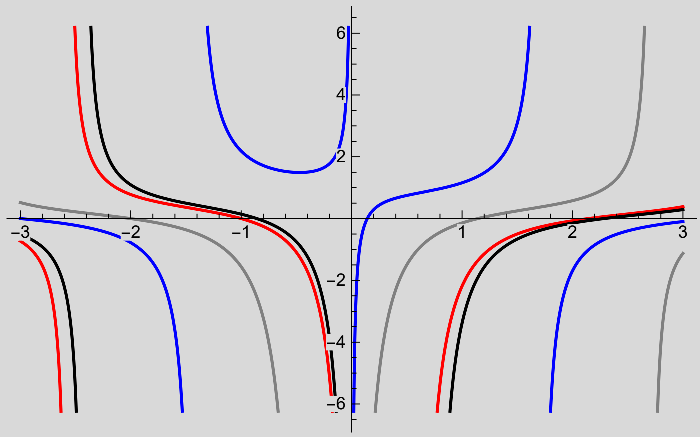
```
q1 = DSolve[x * y'[x] + y[x] == y[x]^2 x^2 + 1, y[x], x]
```

$\{\{y[x] \to \frac{\mathrm{Tan}[x + C[1]]}{x}\}\}$

```
q2 = Table[y[x] /. q1 /. {C[1] → k}, {k, 1, 4}]
```

$\{\{\frac{\mathrm{Tan}[1 + x]}{x}\}, \{\frac{\mathrm{Tan}[2 + x]}{x}\}, \{\frac{\mathrm{Tan}[3 + x]}{x}\}, \{\frac{\mathrm{Tan}[4 + x]}{x}\}\}$

```
Plot[{q2}, {x, -3, 3}, PlotStyle → {Red, Gray, Blue, Black},
 PlotLegends → Placed["Expressions", Right]]
```



Exact Equations

```
M1[x_, y_] := (xy^2 + x)
N1[x_, y_] := (yx^2)
Simplify[D[M1[x, y], y] - D[N1[x, y], x]]
```

0

```
eq1 = y'[x] == -M1[x, y[x]] / N1[x, y[x]]
```

$y'[x] == \frac{-x - x\, y[x]^2}{x^2\, y[x]}$

```
q1 = DSolve[eq1, y[x], x]
```

$\left\{\left\{y[x] \to -\frac{\sqrt{e^{2 C[1]} - x^2}}{x}\right\}, \left\{y[x] \to \frac{\sqrt{e^{2 C[1]} - x^2}}{x}\right\}\right\}$

```
q2 = DSolve[eq1, y[x], x]
```

$\left\{\left\{y[x] \to -\frac{\sqrt{e^{2 C[1]} - x^2}}{x}\right\}, \left\{y[x] \to \frac{\sqrt{e^{2 C[1]} - x^2}}{x}\right\}\right\}$

```
p[x_, y_] := -(5 x^2 - 2 y^2 + 11)
q[x_, y_] := (Sin[y] + 4 xy + 3)
Simplify[D[p[x, y], y] - D[q[x, y], x]]
```

4 y

Exercise

1. $(3 x + 2 y)\, dx + (2 x + y)\, dy = 0.$

```
M1[x_, y_] := (3 * x + 2 * y)
N1[x_, y_] := (2 * x + y)
Simplify[D[M1[x, y], y] - D[N1[x, y], x]]
```

```
0
```

```
eqn = y'[x] == -M1[x, y[x]] / N1[x, y[x]]
```

$$y'[x] == \frac{-3x - 2y[x]}{2x + y[x]}$$