Practical 2

Particular solutions of the non - homogenous equation
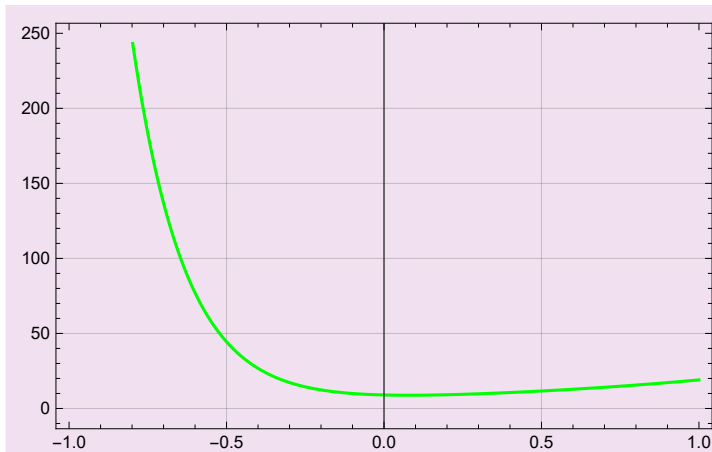
```
Sol = DSolve[y''[x] + 5 * y'[x] - 6 * y[x] == 0, y[x], x]
```

$\{\{y[x] \to e^{-6x} C[1] + e^x C[2]\}\}$

```
Sol1 = y[x] /. Sol[[1]] /. {C[1] → 2, C[2] → 7}
```

$2 e^{-6x} + 7 e^x$

```
Plot[{Sol1}, {x, -1, 1}, PlotStyle → {Green},
  Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic]
```



```
Sol = DSolve[y''[x] + y[x] == 0, y[x], x]
```

$\{\{y[x] \to C[1] \cos[x] + C[2] \sin[x]\}\}$

```
Sol1 = y[x] /. Sol /. {C[1] → 1, C[2] → 2}
Sol2 = y[x] /. Sol /. {C[1] → 1, C[2] → 3}
Sol3 = y[x] /. Sol /. {C[1] → 1, C[2] → 4}
Sol4 = y[x] /. Sol /. {C[1] → 1, C[2] → 5}
Plot[{Sol1, Sol2, Sol3, Sol4}, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink},
  Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic, ImageSize → 700,
  PlotLegends → LineLegend[{"Sol1", "Sol2", "Sol3", "Sol4"}, LegendFunction → "Frame"]]
```
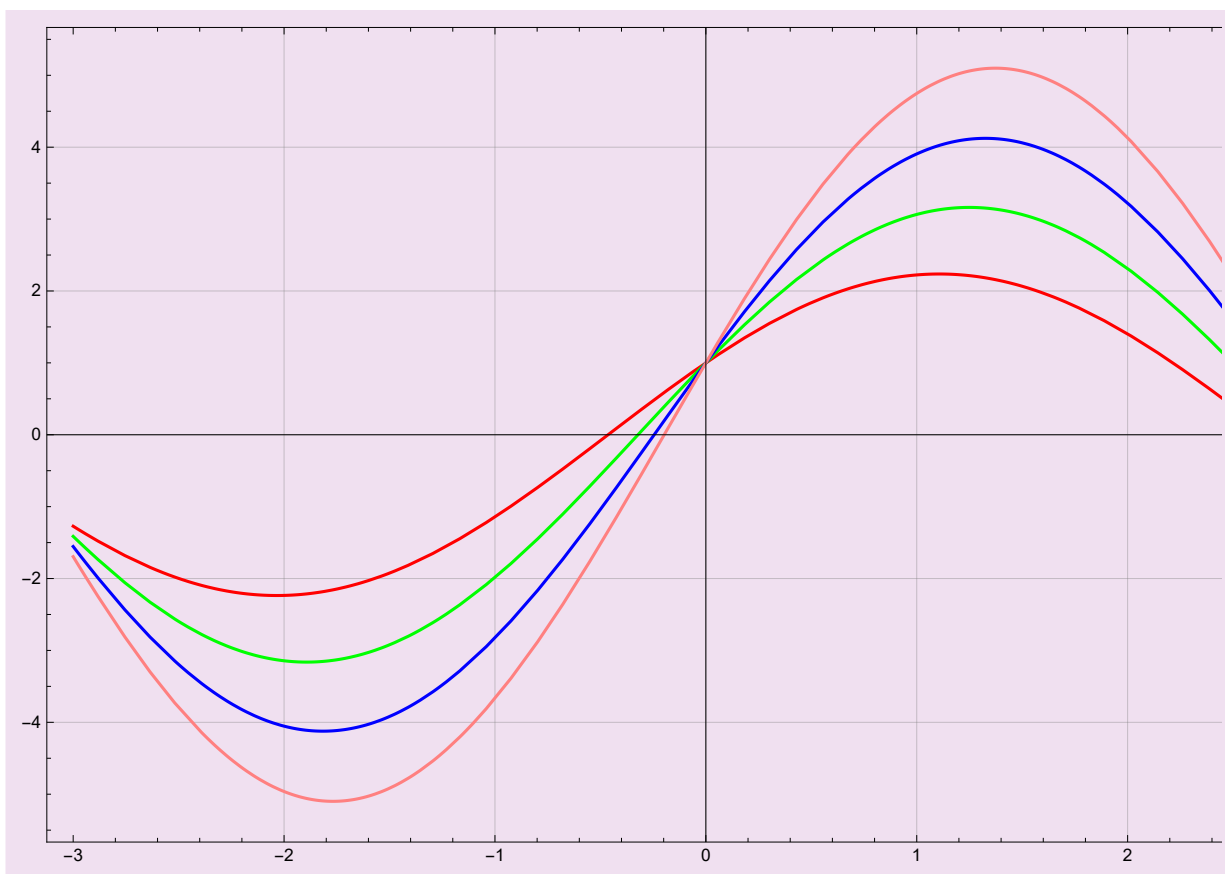
{Cos[x] + 2 Sin[x]}

{Cos[x] + 3 Sin[x]}
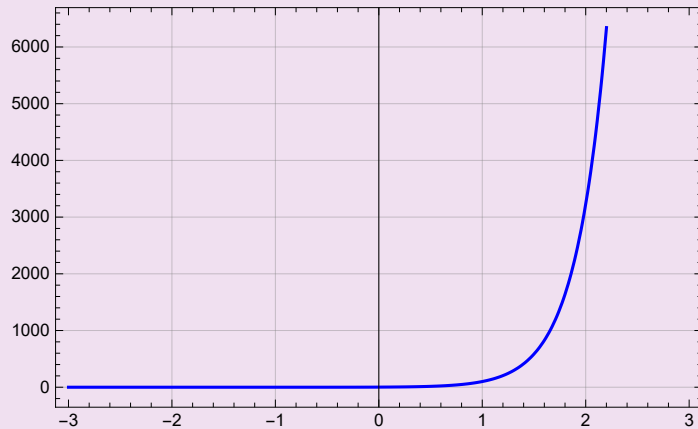
{Cos[x] + 4 Sin[x]}

{Cos[x] + 5 Sin[x]}


```
Sol2 = DSolve[y''[x] - 6 * y'[x] + 9 y[x] == 0, y[x], x]
```

$$\left\{\left\{y[x] \rightarrow \mathbb{e}^{3x} C[1] + \mathbb{e}^{3x} x C[2]\right\}\right\}$$

```
Sol3 = y[x] /. Sol2[[1]] /. {C[1] → 2, C[2] → 3}
Plot[{Sol3}, {x, -3, 3}, PlotStyle → {Blue},
 Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic]
```

$2 e^{3x} + 3 e^{3x} x$


```
Sol2 = DSolve[y''[x] - 6 * y'[x] + 9 y[x] == 0, y[x], x]
Sol3 = y[x] /. Sol2[[1]] /. {C[1] → 2, C[2] → 3}
Plot[{Sol3}, {x, -3, 3}, PlotStyle → {Blue},
 Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic]
```
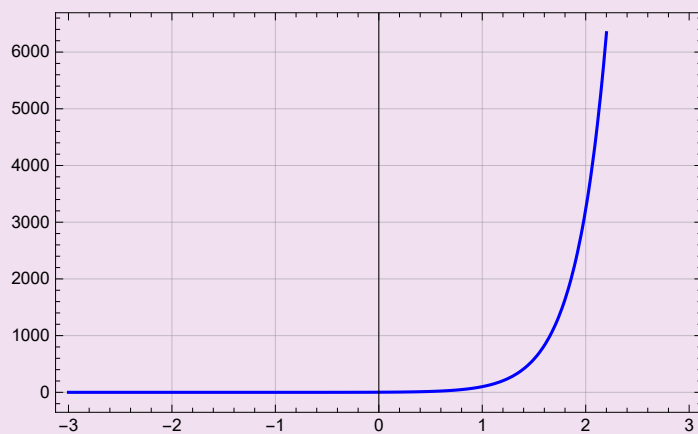
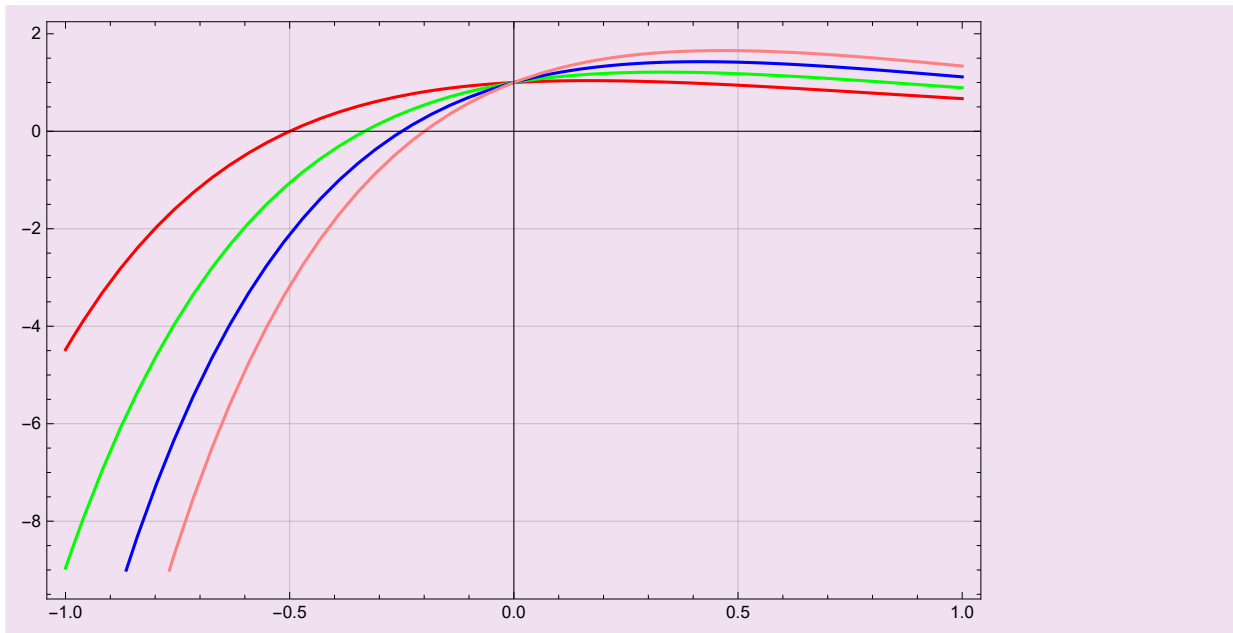$\left\{ \left\{ y[x] \to e^{3x} C[1] + e^{3x} x C[2] \right\} \right\}$

$2 e^{3x} + 3 e^{3x} x$


```
B = DSolve[4 y''[x] + 12 y'[x] + 9 y[x] == 0, y[x], x]
```

$\left\{ \left\{ y[x] \to e^{-3x/2} C[1] + e^{-3x/2} x C[2] \right\} \right\}$

```
B1 = Table[y[x] /. B /. {C[1] → 1, C[2] → K}, {K, 2, 5}]
```

$\left\{ \left\{ e^{-3x/2} + 2 e^{-3x/2} x \right\}, \left\{ e^{-3x/2} + 3 e^{-3x/2} x \right\}, \left\{ e^{-3x/2} + 4 e^{-3x/2} x \right\}, \left\{ e^{-3x/2} + 5 e^{-3x/2} x \right\} \right\}$

```
Plot[B1, {x, -1, 1}, PlotStyle → {Red, Green, Blue, Pink},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}, ImageSize → 500,
 PlotLegends → LineLegend[{"Sol1", "Sol2", "Sol3", "Sol4"}, LegendFunction → "Frame"]]
```



```
Sol4 = DSolve[y''[x] - y'[x] + y[x] == 0, y[x], x]
```

$$\left\{\left\{y[x] \to e^{x/2} C[1] \cos\left[\frac{\sqrt{3}\ x}{2}\right] + e^{x/2} C[2] \sin\left[\frac{\sqrt{3}\ x}{2}\right]\right\}\right\}$$

$$\left\{\left\{y[x] \to e^{x/2}\ \boxed{C[1]}\ \cos\left[\frac{\sqrt{3}\ x}{2}\right] + e^{x/2}\ \boxed{C[2]}\ \sin\left[\frac{\sqrt{3}\ x}{2}\right]\right\}\right\}$$

$$\left\{\left\{y[x] \to e^{x/2}\ \boxed{C[1]}\ \cos\left[\frac{\sqrt{3}\ x}{2}\right] + e^{x/2}\ \boxed{C[2]}\ \sin\left[\frac{\sqrt{3}\ x}{2}\right]\right\}\right\}$$

$$\left\{\left\{y[x] \to e^{x/2} C[1] \cos\left[\frac{\sqrt{3}\ x}{2}\right] + e^{x/2} C[2] \sin\left[\frac{\sqrt{3}\ x}{2}\right]\right\}\right\}$$

$$\left\{\left\{y[x] \to e^{x/2} C[1] \cos\left[\frac{\sqrt{3}\ x}{2}\right] + e^{x/2} C[2] \sin\left[\frac{\sqrt{3}\ x}{2}\right]\right\}\right\}$$
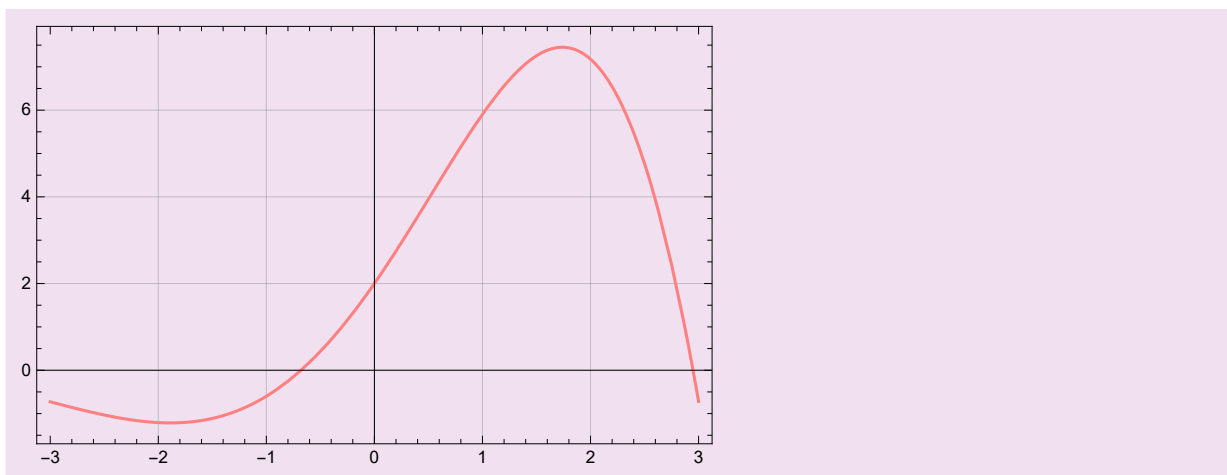
```
Sol5 = y[x] /. Sol4[[1]] /. {C[1] → 2, C[2] → 3}
```

$$2 e^{x/2} \cos\left[\frac{\sqrt{3}\ x}{2}\right] + 3 e^{x/2} \sin\left[\frac{\sqrt{3}\ x}{2}\right]$$

$$2 e^{x/2} \cos\left[\frac{\sqrt{3}\ x}{2}\right] + 3 e^{x/2} \sin\left[\frac{\sqrt{3}\ x}{2}\right]$$

$$2 e^{x/2} \cos\left[\frac{\sqrt{3}\ x}{2}\right] + 3 e^{x/2} \sin\left[\frac{\sqrt{3}\ x}{2}\right]$$

```
Plot[{Sol5}, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink},
 Frame → True, AxesOrigin → {0, 0}, GridLines → Automatic]
```
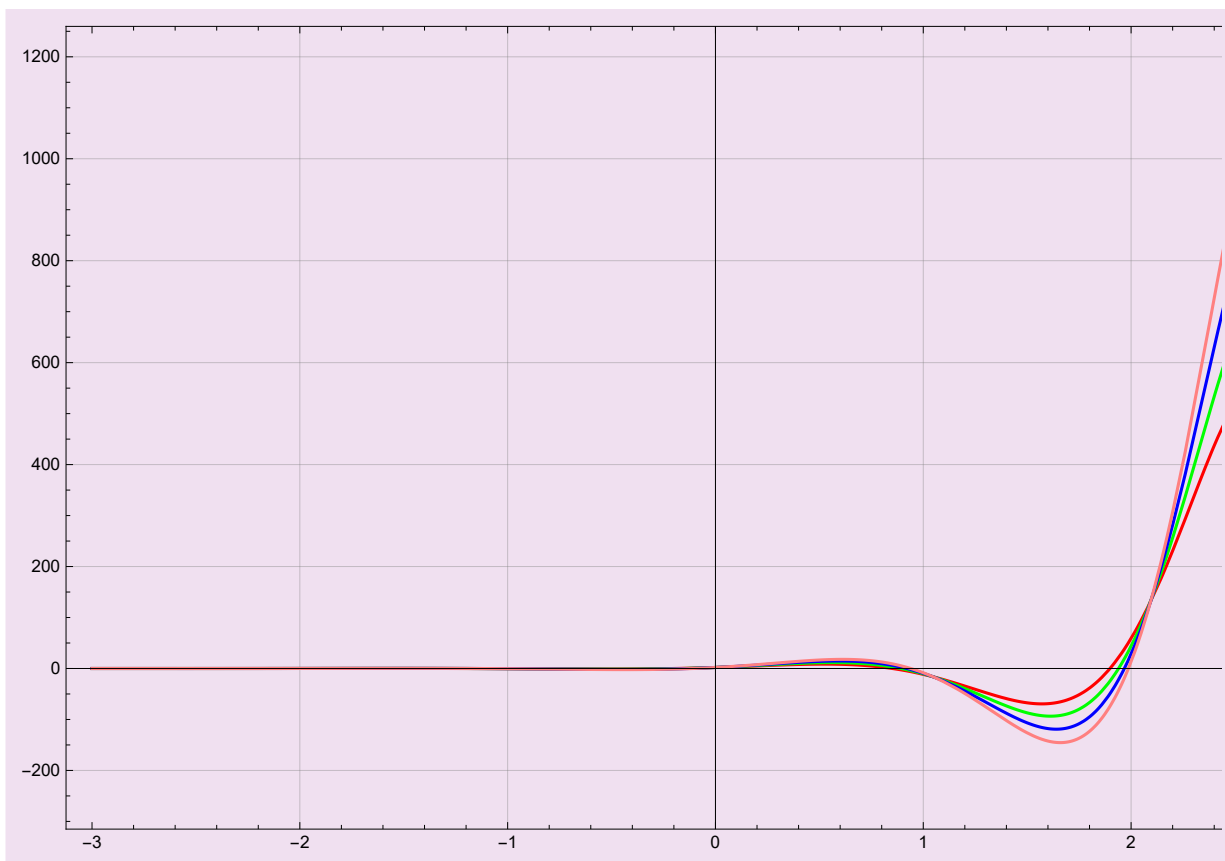


```
c = DSolve[y''[x] - 4 * y'[x] + 13 * y[x] == 0, y[x], x]
```

$$\left\{\left\{y[x] \to e^{2x} C[2] \cos[3x] + e^{2x} C[1] \sin[3x]\right\}\right\}$$

```
c1 = Table[y[x] /. c /. {C[1] → K, C[2] → 2}, {K, 3, 6}]
```

$$\left\{\left\{2 e^{2x} \cos[3x] + 3 e^{2x} \sin[3x]\right\}, \left\{2 e^{2x} \cos[3x] + 4 e^{2x} \sin[3x]\right\},\right.$$
$$\left.\left\{2 e^{2x} \cos[3x] + 5 e^{2x} \sin[3x]\right\}, \left\{2 e^{2x} \cos[3x] + 6 e^{2x} \sin[3x]\right\}\right\}$$

```
Plot[{c1}, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink}, GridLines → Automatic,
 Frame → True, AxesOrigin → {0, 0}, PlotRange → All, ImageSize → 700]
```
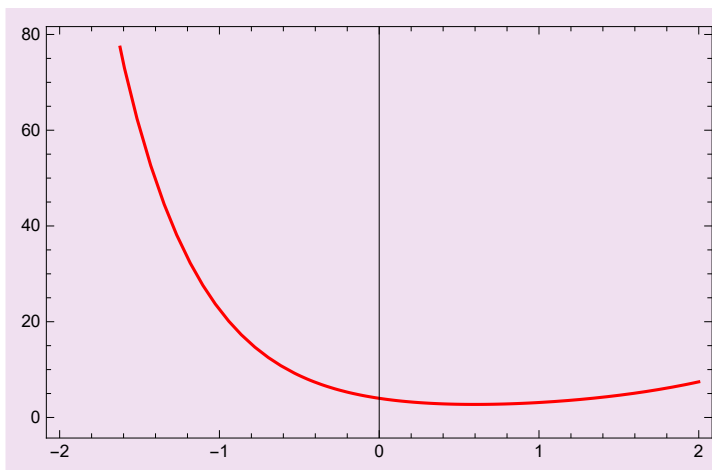


```
p = DSolve[{y''[x] + y'[x] - 2 * y[x] == 0, y[0] == 4, y'[0] == -5}, y[x], x]
```

$$\left\{\left\{y[x] \rightarrow e^{-2x}\left(3 + e^{3x}\right)\right\}\right\}$$

```
Plot[y[x] /. p, {x, -2, 2}, PlotStyle → {Red}, Frame → True]
```

```
r = DSolve[y''[x] - 2 * y'[x] - 3 * y[x] == 30 * Exp[2 * x], y[x], x]
```

$$\left\{ \left\{ y[x] \rightarrow -10\, e^{2x} + e^{-x}\, C[1] + e^{3x}\, C[2] \right\} \right\}$$

```
a = Table[y[x] /. r /. {C[1] → K, C[2] → 2}, {K, 3, 6}]
```

$$\left\{ \left\{ 3\, e^{-x} - 10\, e^{2x} + 2\, e^{3x} \right\}, \left\{ 4\, e^{-x} - 10\, e^{2x} + 2\, e^{3x} \right\}, \left\{ 5\, e^{-x} - 10\, e^{2x} + 2\, e^{3x} \right\}, \left\{ 6\, e^{-x} - 10\, e^{2x} + 2\, e^{3x} \right\} \right\}$$

```
Plot[{a}, {x, -3, 3}, PlotStyle → {Red, Green, Blue, Pink},
 ImageSize → 500, Frame → True, AxesOrigin → {0, 0}]
```
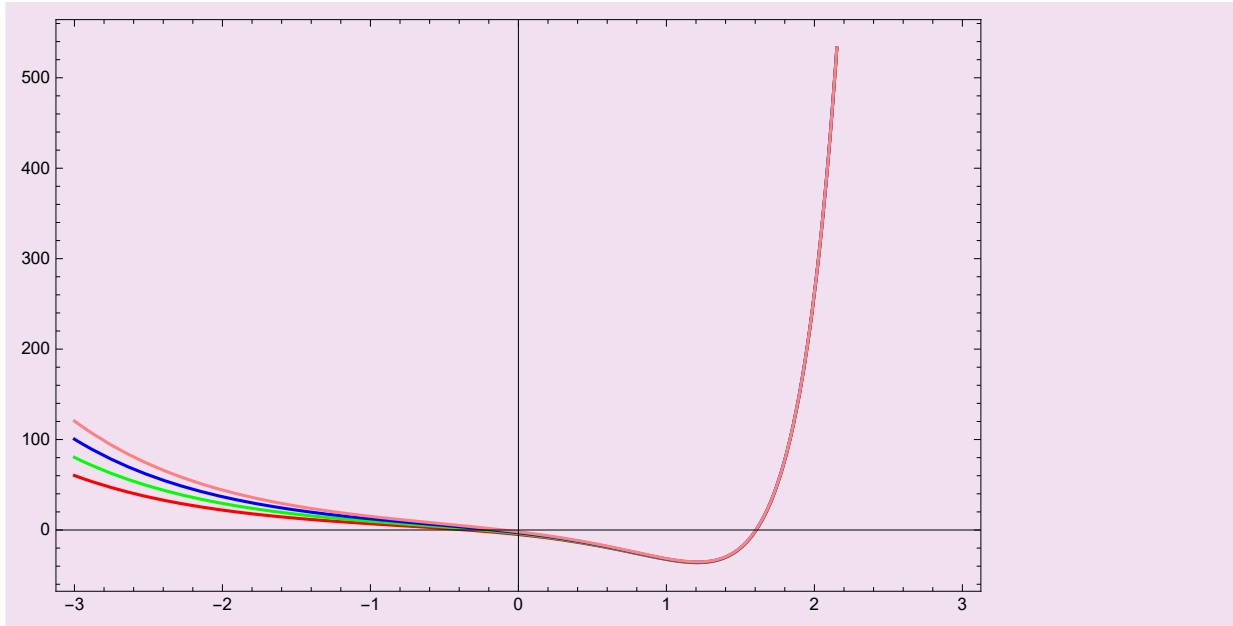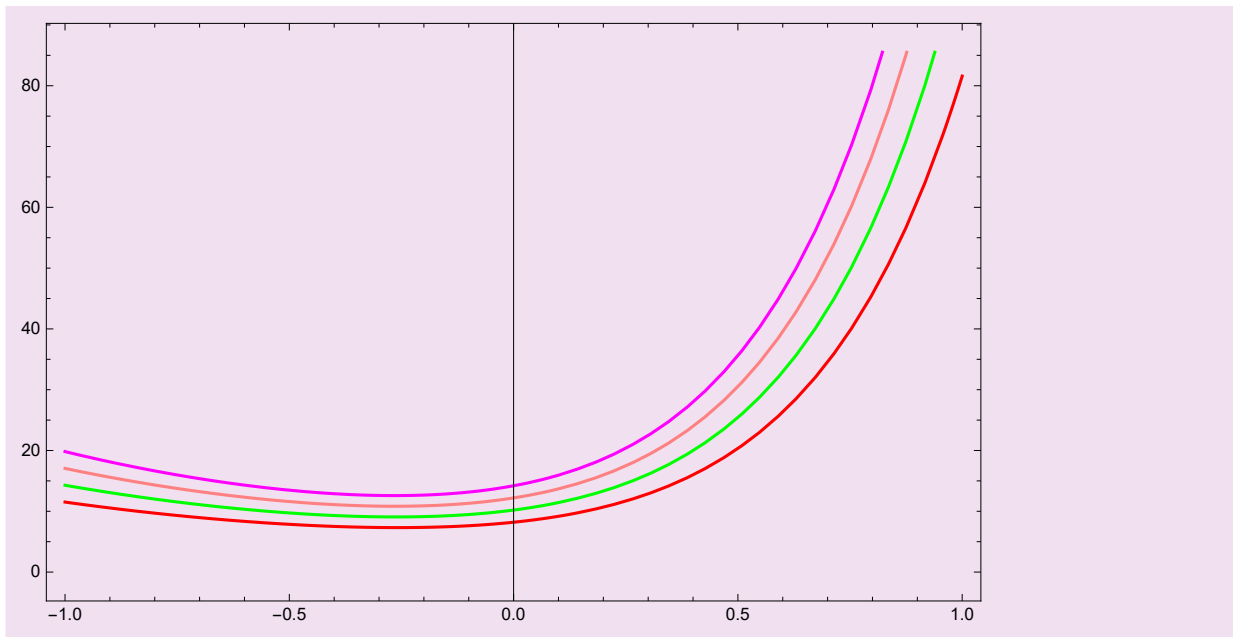


```
p = DSolve[y''[x] - 2 * y'[x] - 3 * y[x] == 2 * Sin[x], y[x], x]
```

$$\left\{ \left\{ y[x] \rightarrow e^{-x}\, C[1] + e^{3x}\, C[2] + \frac{1}{5} \left(Cos[x] - 2\, Sin[x] \right) \right\} \right\}$$

```
p1 = Table[y[x] /. p /. {C[1] → m, C[2] → m}, {m, 4, 7}]
```

$$\left\{ \left\{ 4\, e^{-x} + 4\, e^{3x} + \frac{1}{5} \left(Cos[x] - 2\, Sin[x] \right) \right\}, \left\{ 5\, e^{-x} + 5\, e^{3x} + \frac{1}{5} \left(Cos[x] - 2\, Sin[x] \right) \right\},$$
$$\left\{ 6\, e^{-x} + 6\, e^{3x} + \frac{1}{5} \left(Cos[x] - 2\, Sin[x] \right) \right\}, \left\{ 7\, e^{-x} + 7\, e^{3x} + \frac{1}{5} \left(Cos[x] - 2\, Sin[x] \right) \right\} \right\}$$
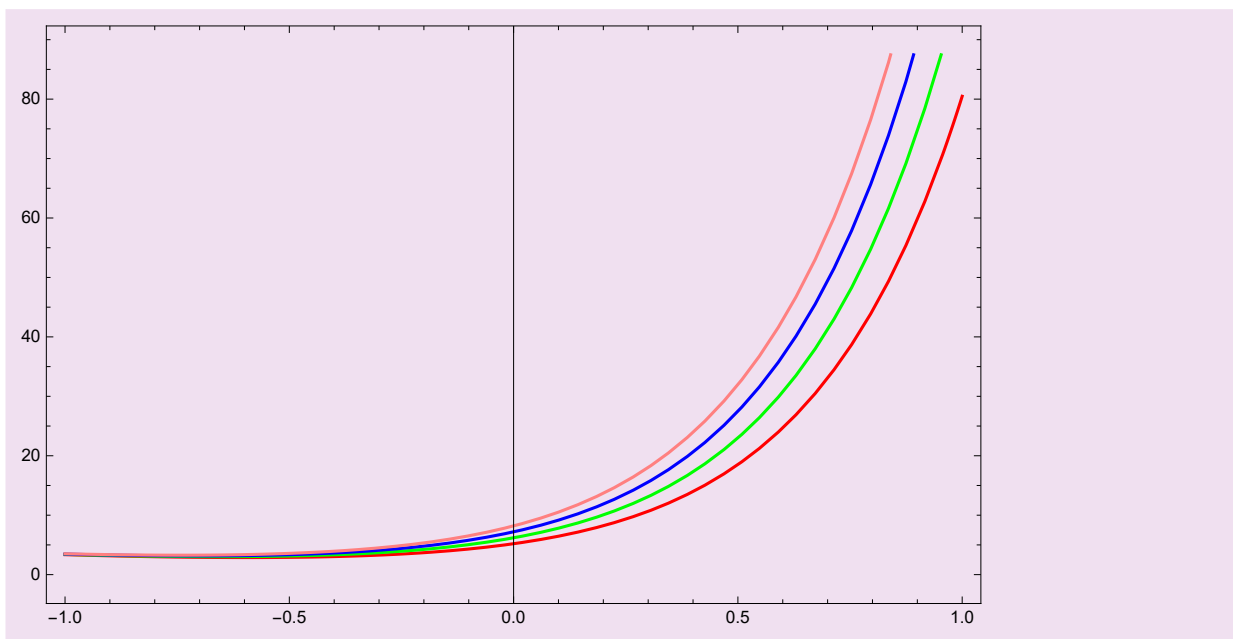
```
Plot[{p1}, {x, -1, 1}, PlotStyle → {Red, Green, Pink, Magenta},
 Frame → True, ImageSize → 500, AxesOrigin → {0, 0}]
```



```
p2 = Table[y[x] /. p /. {C[1] → 1, C[2] → m}, {m, 4, 7}]
```

$$\left\{\left\{\mathbb{e}^{-x} + 4\,\mathbb{e}^{3x} + \frac{1}{5}\,\left(\mathrm{Cos}[x] - 2\,\mathrm{Sin}[x]\right)\right\}, \left\{\mathbb{e}^{-x} + 5\,\mathbb{e}^{3x} + \frac{1}{5}\,\left(\mathrm{Cos}[x] - 2\,\mathrm{Sin}[x]\right)\right\},\right.$$
$$\left.\left\{\mathbb{e}^{-x} + 6\,\mathbb{e}^{3x} + \frac{1}{5}\,\left(\mathrm{Cos}[x] - 2\,\mathrm{Sin}[x]\right)\right\}, \left\{\mathbb{e}^{-x} + 7\,\mathbb{e}^{3x} + \frac{1}{5}\,\left(\mathrm{Cos}[x] - 2\,\mathrm{Sin}[x]\right)\right\}\right\}$$

```
Plot[{p2}, {x, -1, 1}, PlotStyle → {Red, Green, Blue, Pink},
 Frame → True, ImageSize → 500, AxesOrigin → {0, 0}]
```
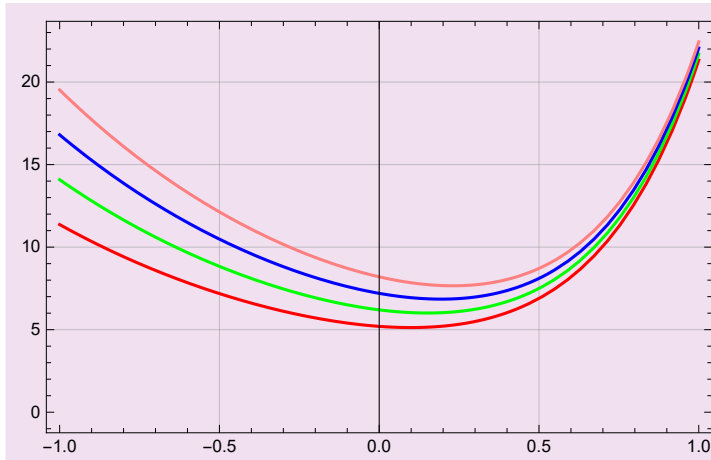
```
p3 = Table[y[x] /. p /. {C[1] → m, C[2] → 1}, {m, 4, 7}]
```

$$\left\{\left\{4\, e^{-x} + e^{3\,x} + \frac{1}{5}\, \left(\text{Cos}[x] - 2\, \text{Sin}[x]\right)\right\}, \left\{5\, e^{-x} + e^{3\,x} + \frac{1}{5}\, \left(\text{Cos}[x] - 2\, \text{Sin}[x]\right)\right\},\right.$$
$$\left.\left\{6\, e^{-x} + e^{3\,x} + \frac{1}{5}\, \left(\text{Cos}[x] - 2\, \text{Sin}[x]\right)\right\}, \left\{7\, e^{-x} + e^{3\,x} + \frac{1}{5}\, \left(\text{Cos}[x] - 2\, \text{Sin}[x]\right)\right\}\right\}$$

```
Plot[{p3}, {x, -1, 1}, PlotStyle → {Red, Green, Blue, Pink},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}]
```



```
q2 = DSolve[{y''[x] + y[x] == .001 * x^2, y[0] == 0, y'[0] == 1.5}, y[x], x]
```

$$\left\{\left\{y[x] \rightarrow -0.002 + 0.001\, x^2 + 0.002\, \text{Cos}[1.\, x] + 1.5\, \text{Sin}[1.\, x]\right\}\right\}$$

```
q3 = Table[y[x] /. q2[[1]]]
```

$$-0.002 + 0.001\, x^2 + 0.002\, \text{Cos}[1.\, x] + 1.5\, \text{Sin}[1.\, x]$$

```
{-0.002` + 0.001` x^2 + 0.002` Cos[1.` x] + 1.5` Sin[1.` x]}
Plot[q3, {x, -10, 10}, PlotStyle → {Red},
 GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}, PlotLegends → Automatic]
```

$$\left\{-0.002 + 0.001\, x^2 + 0.002\, \text{Cos}[1.\, x] + 1.5\, \text{Sin}[1.\, x]\right\}$$
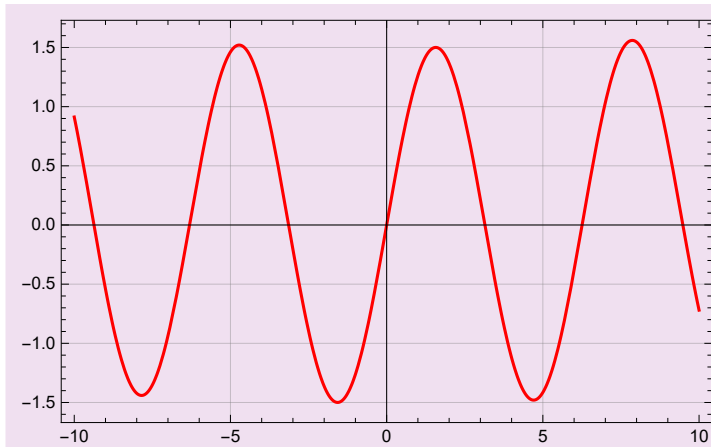
```
b = DSolve[x^2 * y''[x] - 2 * x * y'[x] - 4 * y[x] == 0, y[x], x]
```

$$\left\{\left\{y[x] \rightarrow \frac{C[1]}{x} + x^4 C[2]\right\}\right\}$$

```
c = Table[y[x] /. b /. {C[1] → K, C[2] → 2}, {K, 3, 6}]
```

$$\left\{\left\{\frac{3}{x} + 2 x^4\right\}, \left\{\frac{4}{x} + 2 x^4\right\}, \left\{\frac{5}{x} + 2 x^4\right\}, \left\{\frac{6}{x} + 2 x^4\right\}\right\}$$

```
Plot[{c}, {x, 0, 2}, PlotStyle → {Red, Green, Blue, Pink},
  GridLines → Automatic, Frame → True, AxesOrigin → {0, 0}, PlotLegends → Automatic]
```