Practical 3

Plotting of third order solution family of differential equations

Ques. 1 Solve third order differential equations $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 4y = 0$ and plot it's any three solutions.

```
sol = DSolve[y'''[x] - 5 y''[x] + 8 y'[x] - 4 y[x] == 0, y[x], x]
```

$$\left\{\left\{y[x] \to e^{x} C[1] + e^{2x} C[2] + e^{2x} x C[3]\right\}\right\}$$

```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 2/3}]
```

$$e^{x} + 2\,e^{2x} + \frac{2}{3} e^{2x} x$$

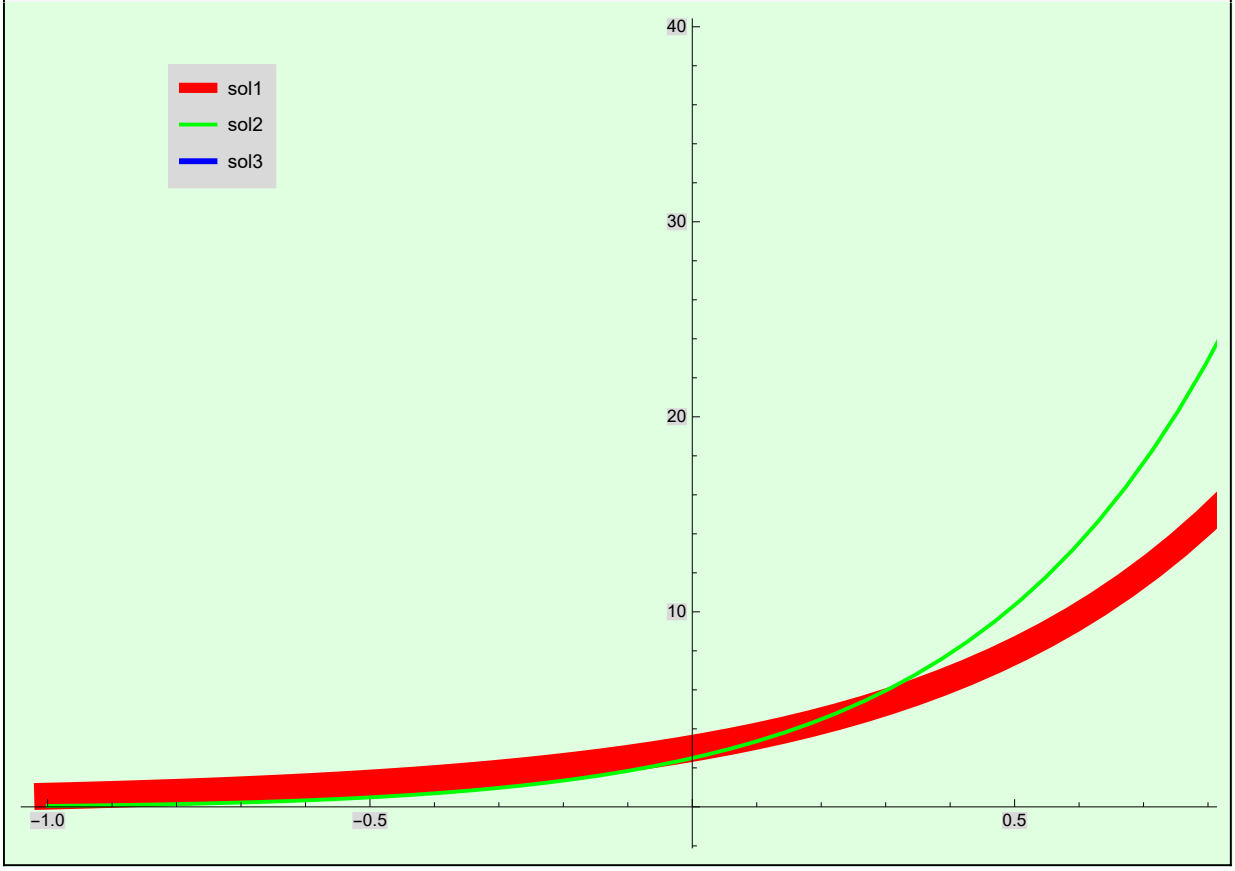
```
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 2, C[3] → 3}]
```

$$0.5\,e^{x} + 2\,e^{2x} + 3\,e^{2x} x$$

```
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C2[2] → -2, C[3] → 0.5}]
```

$$-e^{x} + 0.5\,e^{2x} x + e^{2x} C[2]$$

```
Plot[{sol1, sol2, sol3}, {x, -1, 1}, ImageSize → 700,
 PlotStyle → {{Red, Thickness[0.02]}, {Green, Thick}, {Blue, Thickness[0.01]}},
 PlotLegends → Placed[LineLegend["Expressions",
    LegendLayout → "Column", LegendFunction → "Frame"], {0.15, 0.87}]]
```




```
SOL = DSolve[y'''[x] + 3 y''[x] - 25 y'[x] + 21 y[x] == 0, y[x], x]
```

$$\left\{\left\{y[x] \to e^{-7x} C[1] + e^{x} C[2] + e^{3x} C[3]\right\}\right\}$$

```
sol1 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → 2, C[2] → 2, C[3] → 2/3}]
```

$$2\, e^{-7x} + 2\, e^{x} + \frac{2\, e^{3x}}{3}$$

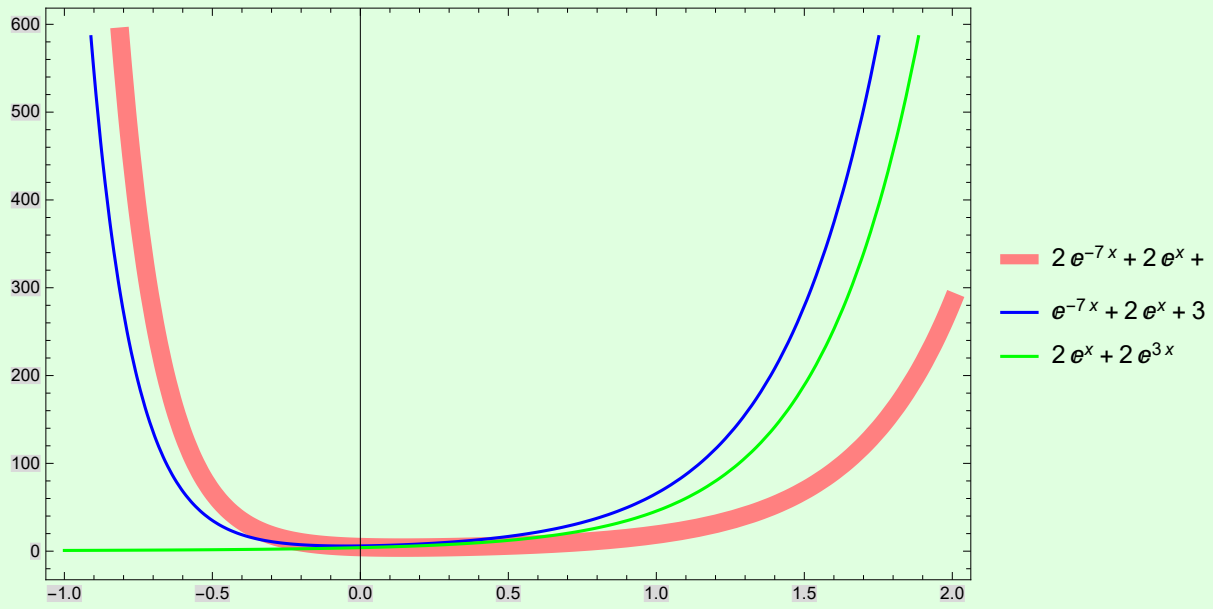
```
sol2 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
```

$$e^{-7x} + 2\, e^{x} + 3\, e^{3x}$$

```
sol3 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → 0, C[2] → 2, C[3] → 2}]
```

$$2\, e^{x} + 2\, e^{3x}$$

```
Plot[{sol1, sol2, sol3}, {x, -1, 2}, PlotStyle → {{Pink, Thickness[0.02]}, Blue, Green},
 Frame → True, ImageSize → 500, PlotLegends → {sol1, sol2, sol3}]
```




```
pol = DSolve[y'''[x] - 4 y''[x] - 25 y'[x] + 28 y[x] == 0, y[x], x]
```

$$\left\{\left\{y[x] \to e^{-4x} C[1] + e^{x} C[2] + e^{7x} C[3]\right\}\right\}$$

```
pol1 = Evaluate[y[x] /. pol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
```

$$e^{-4x} + 2\, e^{x} + 3\, e^{7x}$$

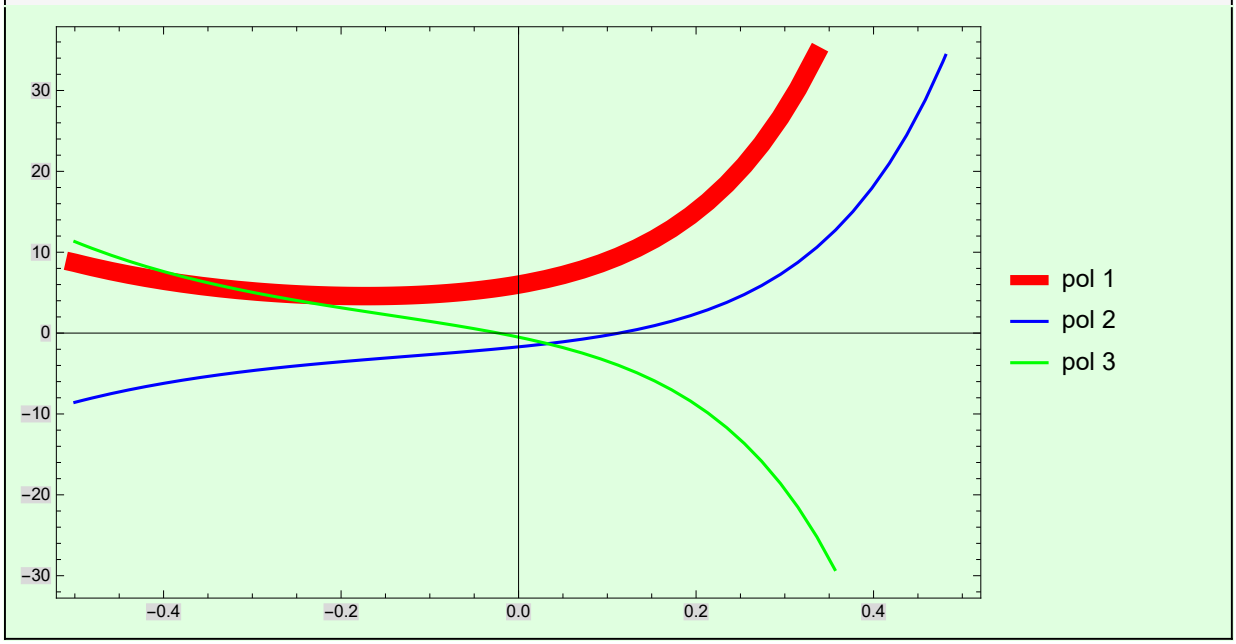
```
pol2 = Evaluate[y[x] /. pol[[1]] /. {C[1] → -1, C[2] → -2, C[3] → 1.3}]
```

$$-e^{-4x} - 2\, e^{x} + 1.3\, e^{7x}$$

```
pol3 = Evaluate[y[x] /. pol[[1]] /. {C[1] → 3/2, C[2] → 0.5, C[3] → -2.5}]
```

$$\frac{3\, e^{-4x}}{2} + 0.5\, e^{x} - 2.5\, e^{7x}$$

```
Plot[{pol1, pol2, pol3}, {x, -0.5, 0.5},
 PlotStyle → {{Red, Thickness[0.02]}, Blue, Green}, Frame → True,
 ImageSize → 500, PlotLegends → LineLegend[{"pol 1", "pol 2", "pol 3"}]]
```



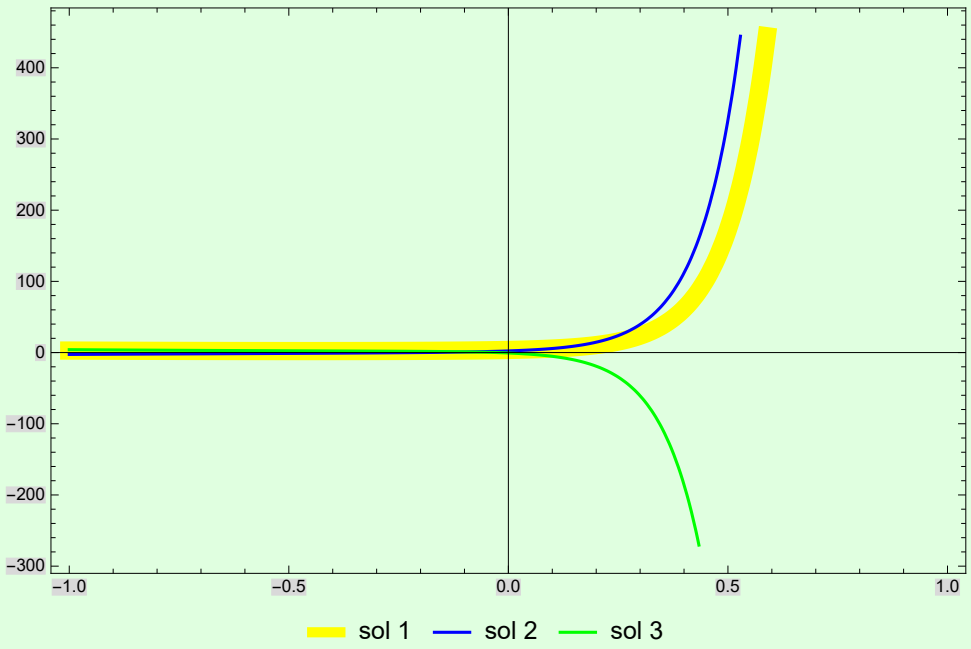
```
SOL = DSolve[y'''[x] - 13 y''[x] + 19 y'[x] + 33 y[x] == Cos[2 x], y[x], x]
sol1 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 2/3}]
sol2 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → -1, C[2] → 2, C[3] → 1.3}]
sol3 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → 3/2, C[2] → 0.2, C[3] → -2.3}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Yellow, Thickness[0.02]}, Blue, Green}, Frame → True,
 ImageSize → 500, PlotLegends → Placed[{"sol 1", "sol 2", "sol 3"}, Below]]
```

$$\left\{\left\{y[x] \to e^{-x} C[1] + e^{3x} C[2] + e^{11x} C[3] + \frac{17 \cos[2x] + 6 \sin[2x]}{1625}\right\}\right\}$$

$$e^{-x} + 2 e^{3x} + \frac{2 e^{11x}}{3} + \frac{17 \cos[2x] + 6 \sin[2x]}{1625}$$

$$-e^{-x} + 2 e^{3x} + 1.3 e^{11x} + \frac{17 \cos[2x] + 6 \sin[2x]}{1625}$$

$$\frac{3 e^{-x}}{2} + 0.2 e^{3x} - 2.3 e^{11x} + \frac{17 \cos[2x] + 6 \sin[2x]}{1625}$$



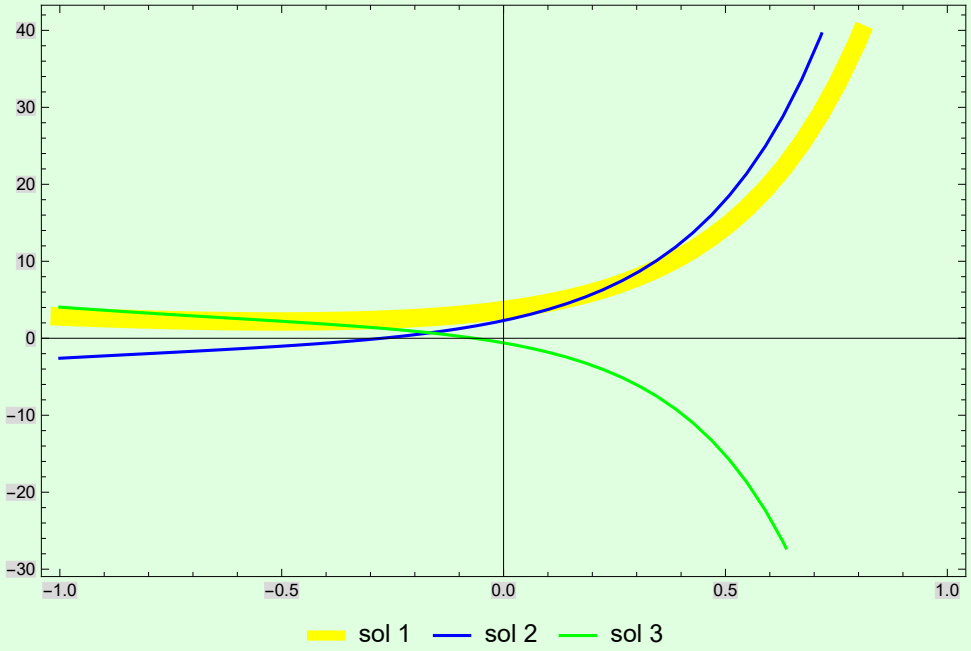
```
SOL = DSolve[y'''[x] - 6 y''[x] + 5 y'[x] + 12 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 2/3}]
sol2 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → -1, C[2] → 2, C[3] → 1.3}]
sol3 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → 3/2, C[2] → 0.2, C[3] → -2.3}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Yellow, Thickness[0.02]}, Blue, Green}, Frame → True,
 ImageSize → 500, PlotLegends → Placed[{"sol 1", "sol 2", "sol 3"}, Below]]
```

$$\{\{y[x] \to e^{-x} C[1] + e^{3x} C[2] + e^{4x} C[3]\}\}$$

$$e^{-x} + 2\, e^{3x} + \frac{2\, e^{4x}}{3}$$

$$-e^{-x} + 2\, e^{3x} + 1.3\, e^{4x}$$

$$\frac{3\, e^{-x}}{2} + 0.2\, e^{3x} - 2.3\, e^{4x}$$



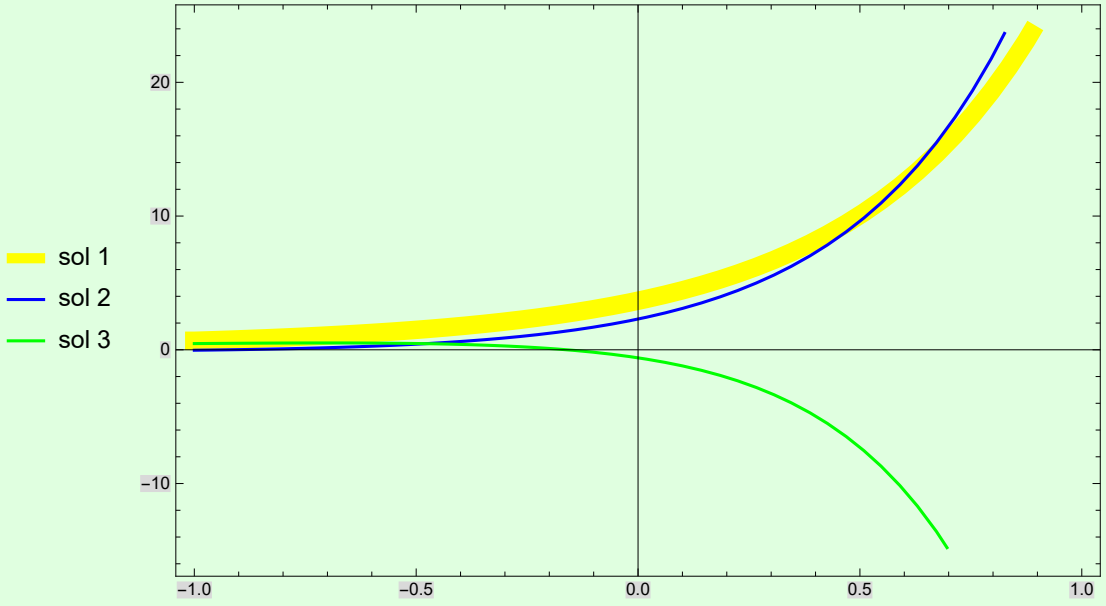
```
SOL = DSolve[y'''[x] - 6 y''[x] + 11 y'[x] - 6 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 2/3}]
sol2 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → -1, C[2] → 2, C[3] → 1.3}]
sol3 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → 3/2, C[2] → 0.2, C[3] → -2.3}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Yellow, Thickness[0.02]}, Blue, Green}, Frame → True,
 ImageSize → 500, PlotLegends → Placed[{"sol 1", "sol 2", "sol 3"}, Left]]
```

$$\{\{y[x] \to e^x C[1] + e^{2x} C[2] + e^{3x} C[3]\}\}$$

$$e^x + 2\, e^{2x} + \frac{2\, e^{3x}}{3}$$

$$-e^x + 2\, e^{2x} + 1.3\, e^{3x}$$

$$\frac{3\, e^x}{2} + 0.2\, e^{2x} - 2.3\, e^{3x}$$



```
SOL = DSolve[y'''[x] + y'[x] == Sec[x], y[x], x]
sol1 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 2/3}]
sol2 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → -1, C[2] → 2, C[3] → 1.3}]
sol3 = Evaluate[y[x] /. SOL[[1]] /. {C[1] → 3/2, C[2] → 0.2, C[3] → -2.3}]
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Yellow, Thickness[0.02]}, Blue, Green}, Frame → True,
 ImageSize → 500, PlotLegends → Placed[{"sol 1", "sol 2", "sol 3"}, Right]]
```

$$\left\{\left\{y[x] \to C[3] - x\,\text{Cos}[x] - C[2]\,\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + C[1]\,\text{Sin}[x] + \text{Log}[\text{Cos}[x]]\,\text{Sin}[x]\right\}\right\}$$

$$\frac{2}{3} - 2\,\text{Cos}[x] - x\,\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sin}[x] + \text{Log}[\text{Cos}[x]]\,\text{Sin}[x]$$

$$1.3 - 2\,\text{Cos}[x] - x\,\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Sin}[x] + \text{Log}[\text{Cos}[x]]\,\text{Sin}[x]$$

$$-2.3 - 0.2\,\text{Cos}[x] - x\,\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \frac{3\,\text{Sin}[x]}{2} + \text{Log}[\text{Cos}[x]]\,\text{Sin}[x]$$