Practical 4

Solution of Differential Equation by Variation of parameter

Second order differential equations

$\frac{d^2y}{dx^2} + p\ \frac{dy}{dx} + qy = f(x)$

Where p and q are constants f(x) is a non zero function of x

General solution of homogeneous equation $\frac{d^2y}{dx^2} + p\ \frac{dy}{dx} + qy = 0$

Particular solutions of the non - homogeneous equation $\frac{d^2y}{dx^2} + p\ \frac{dy}{dx} + qy = f(x)$

y''[x] + y[x] == 2Sin[x]

```
ClearAll
```

```
ClearAll
```

```
gsh = DSolve[y''[x] + y[x] == 0, y[x], x]
```

```
{{y[x] → C[1] Cos[x] + C[2] Sin[x]}}
```

```
gsh = y[x] /. gsh
```

```
{C[1] Cos[x] + C[2] Sin[x]}
```

```
y1 := Cos[x];
y2 := Sin[x];
f := 2*Sin[x];
w = y1*D[y2, x] - y2*D[y1, x]
w = Simplify[w]
```

```
Cos[x]^2 + Sin[x]^2
```

```
1
```

```
psn = -y1 * Integrate[y2 * (f/w), x] + y2 * Integrate[y1 * (f/w), x]
psn1 = Simplify[psn]
```

$$-\frac{1}{2}\cos[2x]\sin[x] - 2\cos[x]\left(\frac{x}{2} - \frac{1}{4}\sin[2x]\right)$$

$$\frac{1}{2}\left(-2x\cos[x] + \sin[x]\right)$$

```
gsh1 + psn1
```

$$gsh1 + \frac{1}{2}\left(-2x\cos[x] + \sin[x]\right)$$

```
ClearAll
```

```
ClearAll
```

Exa2 : y'' + 3*y' + 2*y = 30*e^(2 x)

```
yc2 = DSolve[y''[x] + 3*y'[x] + 2*y[x] == 0, y[x], x]
```

$$\{\{y[x] \to e^{-2x}C[1] + e^{-x}C[2]\}\}$$

```
y1 := Exp[-2*x]
y2 := Exp[-1*x]
f := 30*Exp[2*x]
w = y1*D[y2, x] - y2*D[y1, x]
w = Simplify[w]
```

$$e^{-3x}$$

$$e^{-3x}$$

```
yp2 = -y1 * Integrate[y2 * (f/w), x] + y2 * Integrate[y1 * (f/w), x]
yp2 = Simplify[yp2]
```

$$\frac{5e^{2x}}{2}$$

```
yc2 + yp2
```

$$\left\{\left\{\frac{5e^{2x}}{2} + \left(y[x] \to e^{-2x}C[1] + e^{-x}C[2]\right)\right\}\right\}$$

```
ClearAll
```

```
ClearAll
```

```
x =.
```

```
y =.
```

Exercise Questions

Q1. $\frac{d^2y}{dx^2} + y = \text{Cotx}$

```
S1 = DSolve[y''[x] + y[x] == Cot[x], y[x], x]
```

$$\left\{\left\{y[x] \to C[1]\,\text{Cos}[x] + C[2]\,\text{Sin}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]\text{Sin}[x] + \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]\text{Sin}[x]\right\}\right\}$$

Q2. $\frac{d^2y}{dx^2} - 2\,\frac{dy}{dx} + y = xe^x \ln x$

```
s2 = DSolve[y''[x] + 4*y'[x] + 5 y[x] == e^(-2 x)*Sec[x], y[x], x]
```

$$\left\{\left\{y[x] \to e^{-2x}\,C[2]\,\text{Cos}[x] + e^{-2x}\,C[1]\,\text{Sin}[x] + \left(e^{-2x}\,e^{-2x-2x\,(-1+\text{Log}[e])}\right.\right.$$
$$\left(i\,e^{2x}\,e^{2ix}\,\text{Cos}[x]\,\text{Hypergeometric2F1}\left[1,\,(1-i)+i\,\text{Log}[e],\,(2-i)+i\,\text{Log}[e],\,-e^{2ix}\right] +\right.$$
$$(1-i)\,e^{2x}\,\text{Cos}[x]\,\text{Hypergeometric2F1}\left[1,\,i\,(-1+\text{Log}[e]),\,(1-i)+i\,\text{Log}[e],\,-e^{2ix}\right] - i\,e^{2x}$$
$$e^{2ix}\,\text{Cos}[x]\,\text{Hypergeometric2F1}\left[1,\,(1-i)+i\,\text{Log}[e],\,(2-i)+i\,\text{Log}[e],\,-e^{2ix}\right]\text{Log}[e] +$$
$$i\,e^{2x}\,\text{Cos}[x]\,\text{Hypergeometric2F1}\left[1,\,i\,(-1+\text{Log}[e]),\,(1-i)+i\,\text{Log}[e],\,-e^{2ix}\right]\text{Log}[e] +$$
$$\left.(1+i)\,e^{2x+2x\,(-1+\text{Log}[e])}\,\text{Sin}[x] - e^{2x+2x\,(-1+\text{Log}[e])}\,\text{Log}[e]\,\text{Sin}[x]\right)\Big) \Big/$$
$$\left.\left.(2\,(-1+\text{Log}[e])\,((-1-i)+\text{Log}[e]))\right\}\right\}$$