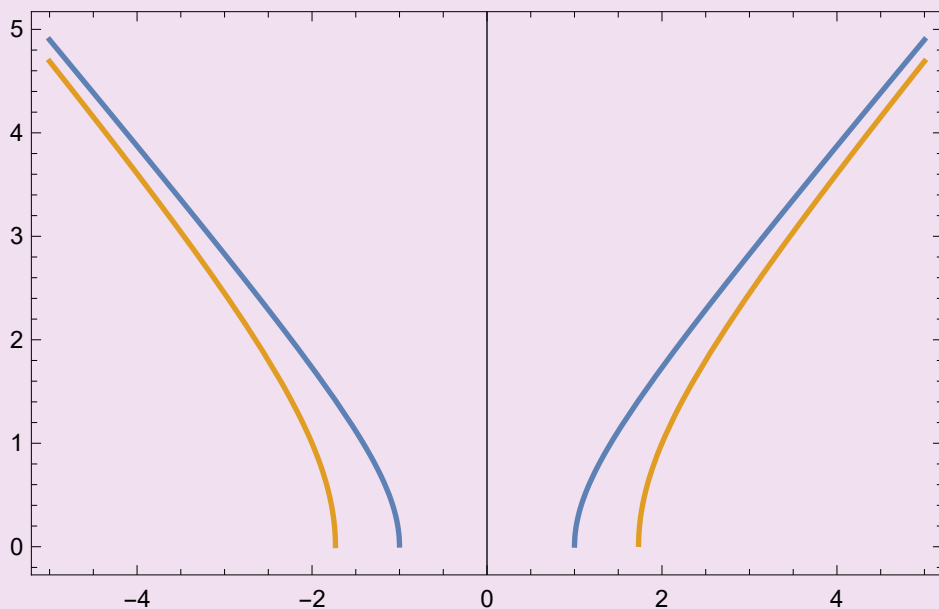# Practical 7

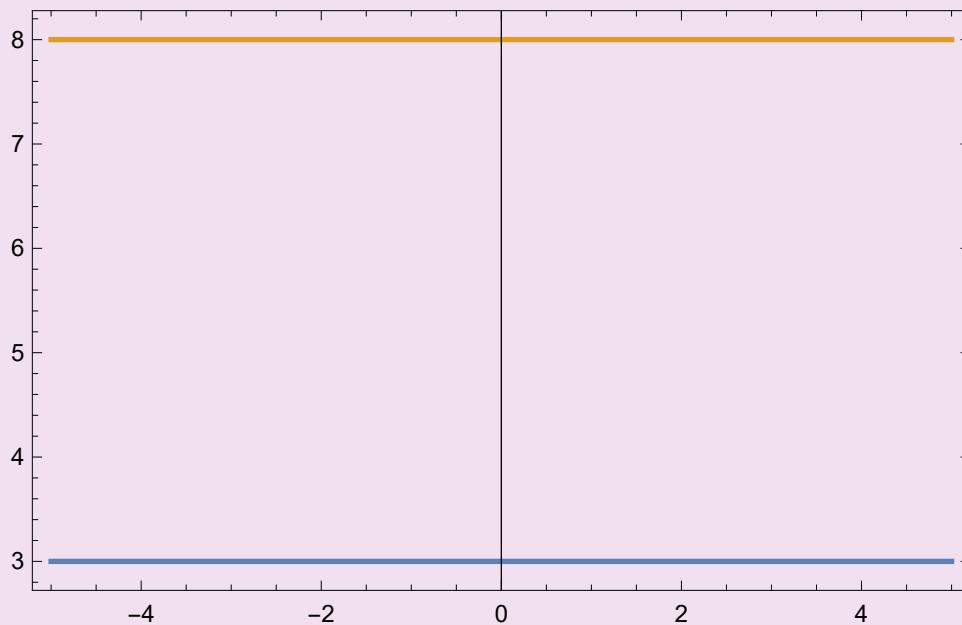Plotting the characteristic for first order PDE

1. $yu_x - xu_y = 0$

The characteristic system is given by $\frac{dx}{y} = \frac{dy}{x} = \frac{du}{0}$ and the characteristic equations are given by $x^2 + y^2 = c1$ and $u = c2$. Taking c1 = 1 and 3 and c2 = 3 and 8.

```
Plot[{Sqrt[x² - 1], Sqrt[x² - 3]},
  {x, -5, 5}, PlotStyle → Thick, Frame → True]
```
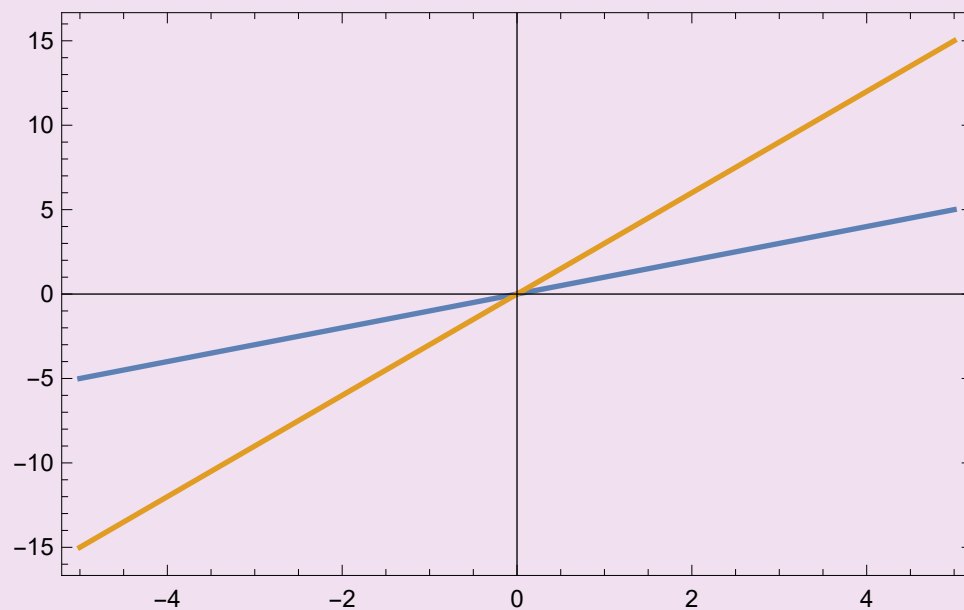
```
Plot[{3, 8}, {x, -5, 5}, PlotStyle → Thick, Frame → True]
```
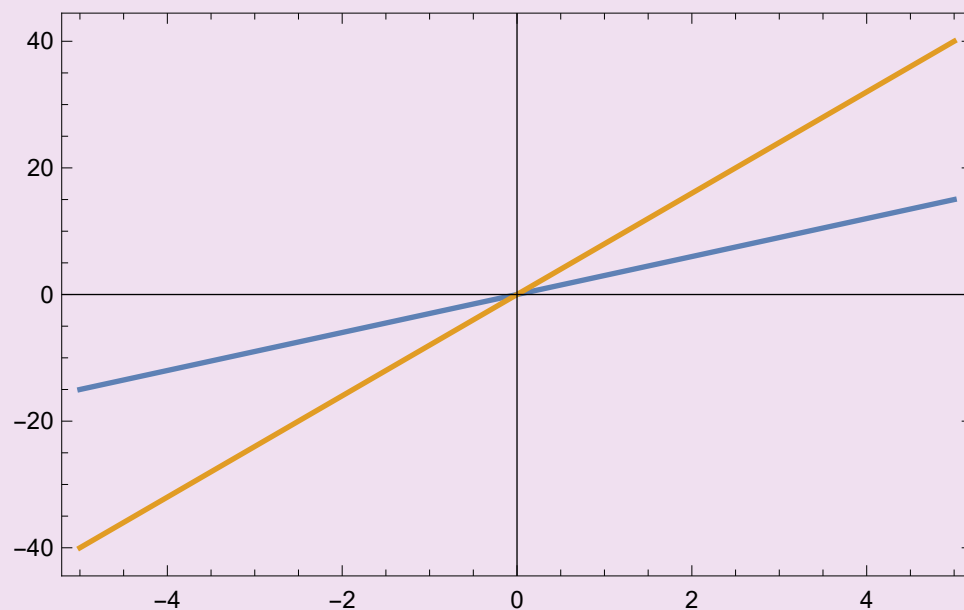

2. $xu_x + yu_y = u$

The characteristic system is given by $\frac{dx}{x} = \frac{dy}{y} = \frac{du}{u}$ and the characteristic equations are given by $\frac{y}{x} = c1$ and $\frac{u}{x} = c2$. Taking c1 = 1 and 3 and c2 = 3 and 8.

```
Plot[{x, 3 x}, {x, -5, 5}, PlotStyle → Thick, Frame → True]
```
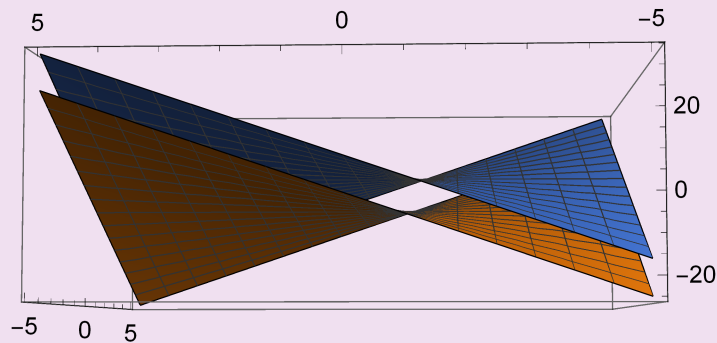


```
Plot[{3 x, 8 x}, {x, -5, 5},
 PlotStyle → Thick, Frame → True]
```
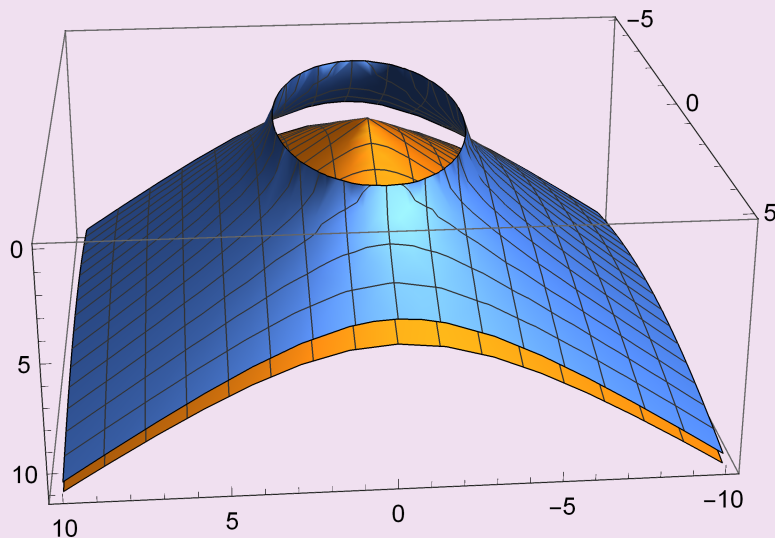

3. $(y + xu)u_x - (x + uy)u_y = x^2 - y^2$

The characteristic system is given by $\dfrac{dx}{y+ux} = \dfrac{dy}{-(x+uy)} = \dfrac{du}{x^2-y^2}$ and the characteristic equations are given by $xy + u = c_1$ and $x^2 + y^2 - u^2 = c_2$. Taking $c_1 = 0$ and 9 and $c_2 = 0$ and 10.

```
Plot3D[{-x*y, -x*y+9},
  {x, -5, 5}, {y, -5, 5}, PlotStyle → Thick]
```



```
Plot3D[{Sqrt[x² + y²], Sqrt[x² + y² - 10]},
  {x, -5, 5}, {y, -10, 10}, PlotStyle → Thick]
```
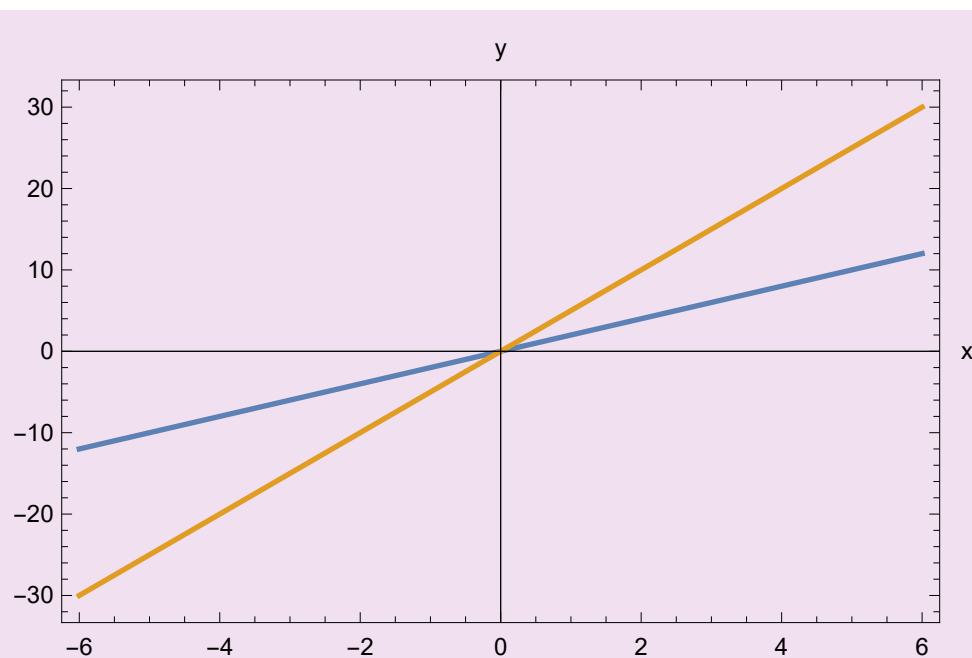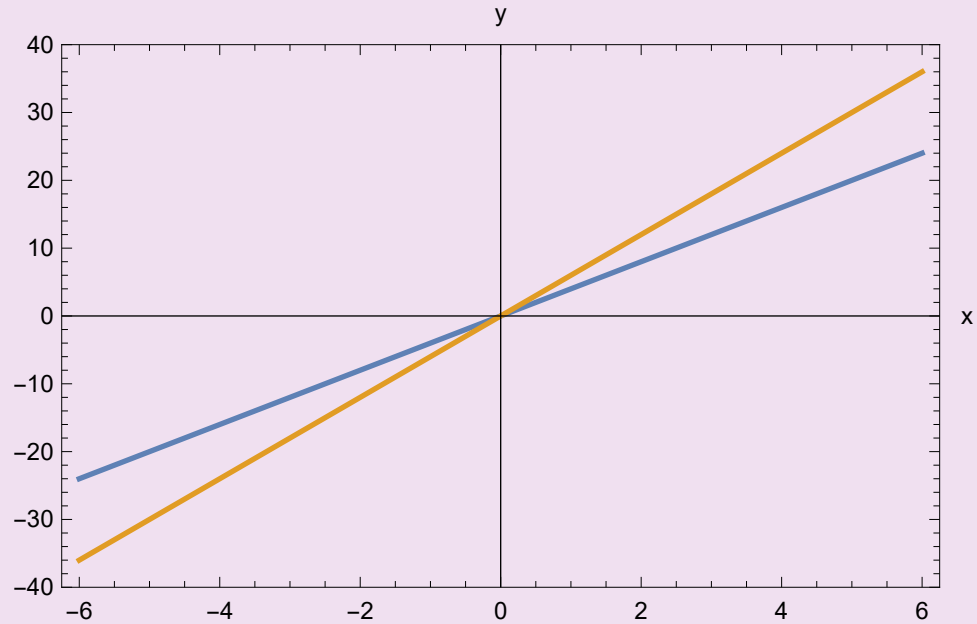

Questions

1. $xu_x - yu_y = u$

2. $y^2 u_x - xyu_y = x(u - 2y)$

3. $u(x+y) u_x - u(x - y)u_y = x^2 + y^2$

4. $u_x - u_y = 1$

1.

```
Plot[{2 x, 5 x}, {x, -6, 6}, PlotStyle → Thick,
  Frame → True, AxesLabel → {"x", "y"}]
```
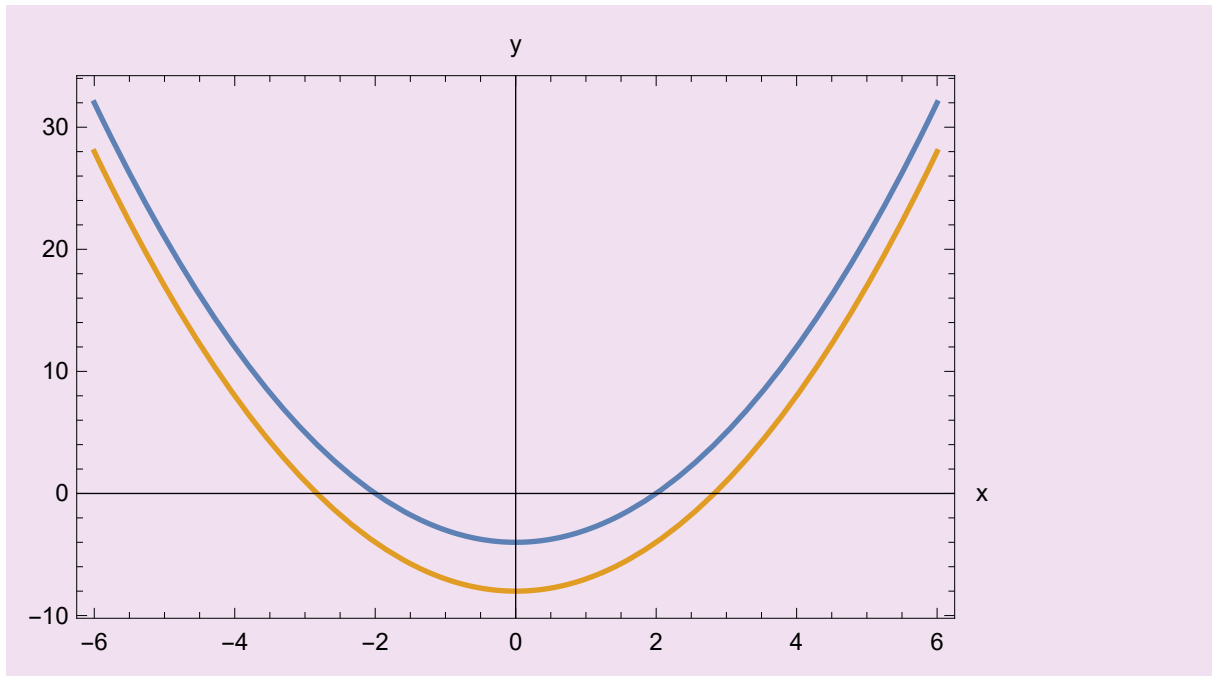
```
Plot[{4 x, 6 x}, {x, -6, 6}, PlotStyle → Thick,
  Frame → True, AxesLabel → {"x", "y"}]
```
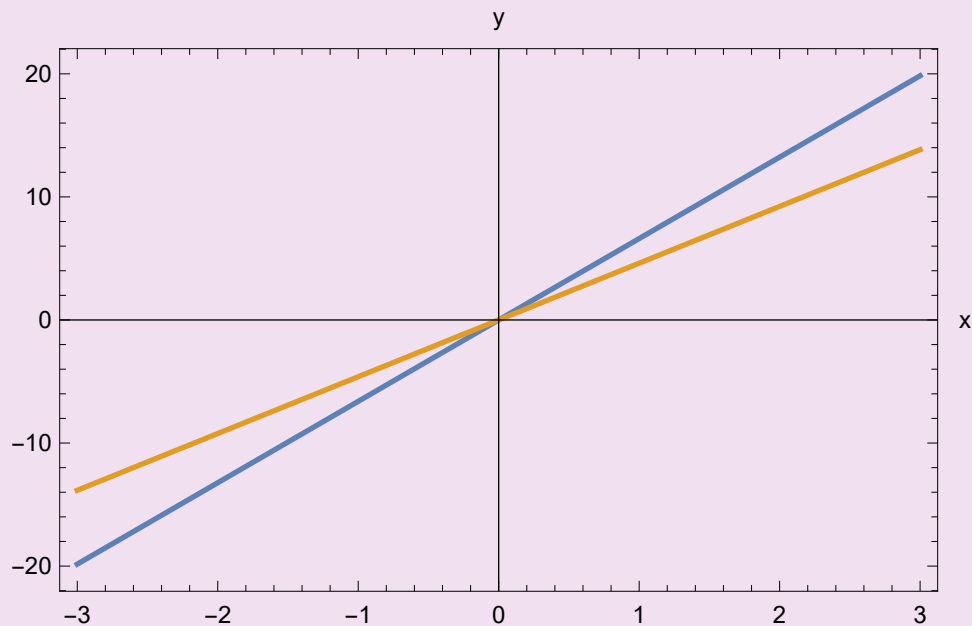

2.

```
Plot[{x² - 4, x² - 8}, {x, -6, 6}, PlotStyle → Thick,
 Frame → True, AxesLabel → {"x", "y"}]
```
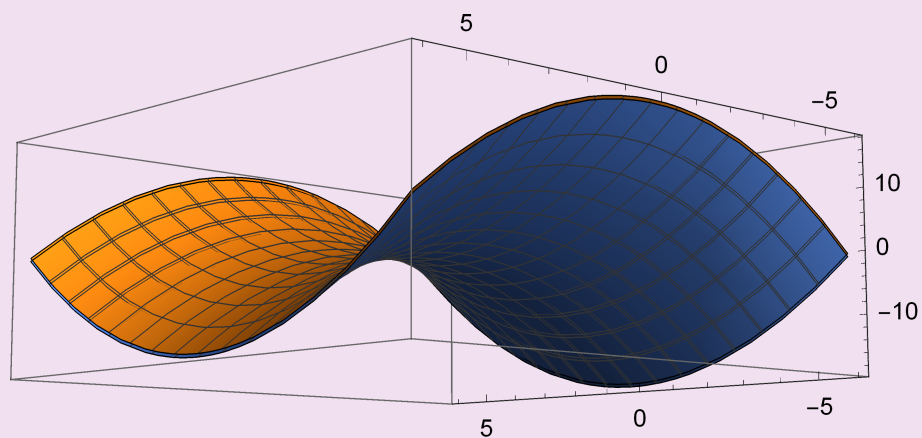
```
Plot[{y (8 - 2 * Log[2]), y (6 - 2 * Log[2])}, {y, -3, 3},
  PlotStyle → Thick, Frame → True, AxesLabel → {"x", "y"}]
```
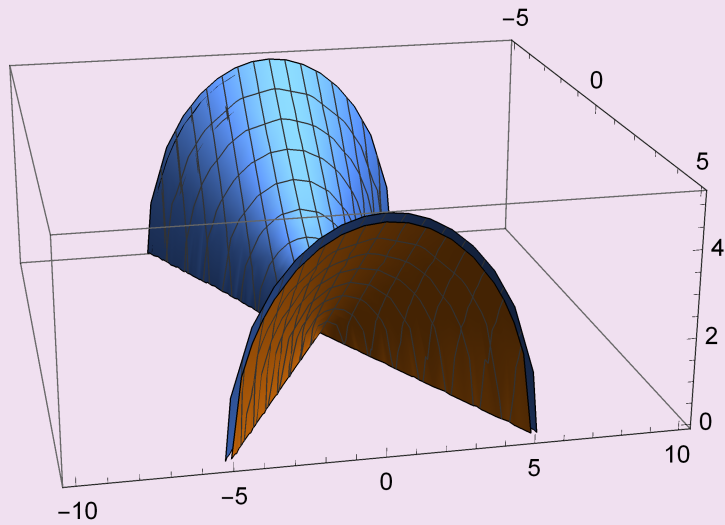

3.

$$\text{Plot3D}\left[\left\{\left(x^2 - y^2 - 1\right)/2, \left(x^2 - y^2 - 2\right)/2\right\},\right.$$
$$\left.\{x, -6, 6\}, \{y, -6, 6\}, \text{PlotStyle} → \text{Thick}\right]$$
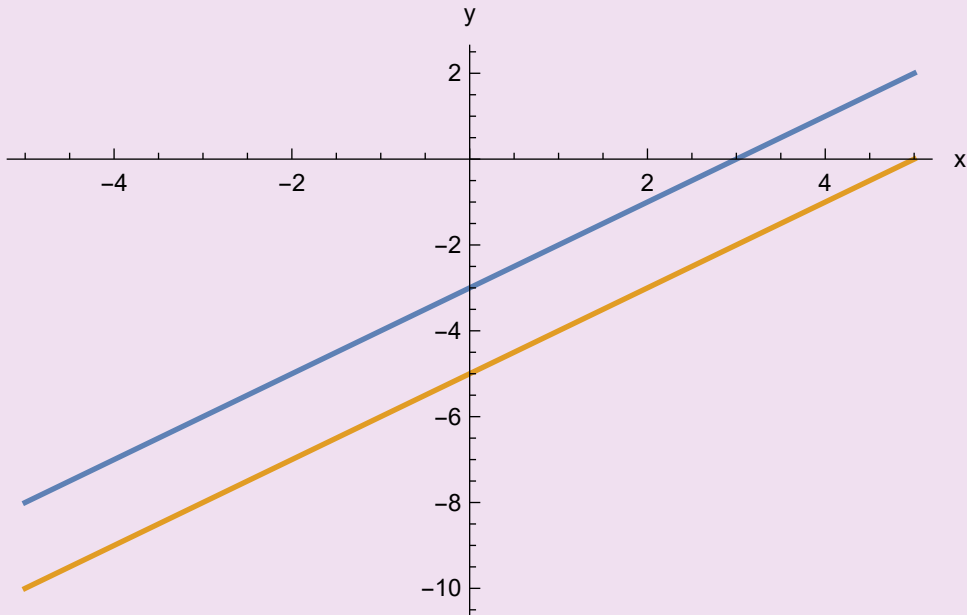
```
Plot3D[{Sqrt[x² - y²], Sqrt[x² - y² + 2]},
 {x, -5, 5}, {y, -10, 10}, PlotStyle → Thick]
```


4.

```
Plot[{x - 3, x - 5}, {x, -5, 5},
  PlotStyle → Thick, AxesLabel → {"x", "y"}]
```



```
Plot[{y - 0, y - 3}, {y, -5, 5},
  PlotStyle → Thick, AxesLabel → {"x", "y"}]
```