Day 2 Probability tasks

Task 1
In a single toss of 2 fair (evenly-weighted) six-sided dice, find the probability that their sum will be at most 9.

Walkthrough
When tossing 2 dice the number of possible outcomes are 36. The first dice has 6 possible outcomes, {1, 2, 3, 4, 5, 6}, and the same applies for the second dice. 6 * 6 = 36.

Now we have to find the set of outcomes that have a sum that is at most 9.
Since we know that the largest sum of 2 dice is 12, it will be quicker to find the set of outcomes that have a sum greater than 9. Then we can use the complement to find the probability that a sum will be at most 9.
P(sum is at most 9) + P(sum is greater than 9) = 1

Starting with 6 on the first dice we can find the following sums that are greater than 9, {(6, 6), (6, 5), (6, 4), (5, 6), (5, 5), (4, 6)}

Using the compliment we have:
$$P(sum \leq 9) = 1 - P(sum > 9) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$$

Task 2
In a single toss of 2 fair (evenly-weighted) six-sided dice, find the probability that the values rolled by each die will be different and the two dice have a sum of 6.

Walkthrough
Again, the number of possible outcomes are 36. To get a sum

of 6, we have these possibilities {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}.

Since we have been asked to find the possibilities where the value rolled by each dice is different, we have to remove (3, 3). Then we have 4 outcomes.
If we let i denote the value on dice1 and j denote the value on dice2, then:

$$P(i \neq j \ \& \ sum = 6) = \frac{4}{36} = \frac{1}{9}$$

Task 3
There are 3 urns labeled X, Y and Z.

- Urn X contains 4 red balls and 3 black balls.
- Urn Y contains 5 red balls and 4 black balls.
- Urn Z contains 4 red balls and 4 black balls.

One ball is drawn from each of the 3 urns. What is the probability that, of the 3 balls drawn, 2 are red and 1 is black?

Walkthrough
To solve this task, it is effective to define the set of favourable outcomes. We have {RedRedBlack, RedBlackRed, BlackRedRed}. All of these options draw a ball from each urn and come up with 2 red balls and 1 black ball.

First we have to find the probabilities for drawing red and black balls from each urn:

$$P(Red \mid X) = \frac{4}{7} \qquad P(Black \mid X) = \frac{3}{7}$$
$$P(Red \mid Y) = \frac{5}{9} \qquad P(Black \mid Y) = \frac{4}{9}$$

$$P(Red \mid Z) = \frac{4}{8} \qquad P(Black \mid Z) = \frac{4}{8}$$

With these probabilities we can find the probability of drawing 2 red and 1 black ball, when drawing one ball from each urn:

$$P(RedRedBlack) = \frac{4}{7} * \frac{5}{9} * \frac{1}{2} = \frac{20}{126}$$

$$P(RedBlackRed) = \frac{4}{7} * \frac{4}{9} * \frac{1}{2} = \frac{16}{126}$$

$$P(BlackRedRed) = \frac{3}{7} * \frac{5}{9} * \frac{1}{2} = \frac{15}{126}$$

Summing up, we get:

$$P(2\ red\ and\ 1\ black\ ball) = \frac{51}{126} = \frac{17}{42}$$