In Hawk paper, tables are given scaled by 2^{78}. Using a high precision package in Python, we downscale the entries to get Table 3. The displayed numbers in table 3 are cut off at the 20th decimal place, but in the Python program we still have access to all 78 decimal points.

The entries in the tables are as follows:

- $T_0[k] = Pr(|X| \geq 2 + 2k)$ when $X \sim \mathcal{D}_{2\mathbb{Z},\sigma}$
- $T_1[k] = Pr(|X| \geq 3 + 2k)$ when $X \sim \mathcal{D}_{2\mathbb{Z}+1,\sigma}$

Trivially, $Pr(|X| \geq 0) = 1$ when $X \sim \mathcal{D}_{2\mathbb{Z},\sigma}$ and $Pr(|X| \geq 1) = 1$ when $X \sim \mathcal{D}_{2\mathbb{Z}+1,\sigma}$. Using these entries, we can define the probability mass function of the distribution:

$$Pr(X = x) = \begin{cases} 1 - T_0[0] & \text{if } x = 0 \\ \frac{1}{2}(1 - T_1[0]) & \text{if } |x| = 1 \\ \frac{1}{2}(T_c[\dfrac{|x| - c}{2} - 1] - T_c[\dfrac{|x| - c}{2}]) & \text{if } |x| > c \end{cases}$$

The probability of a random variable $X \sim \mathcal{D}_{2\mathbb{Z}+c}$ being equal to some $x \in 2\mathbb{Z} + c$ is the probability that X is greater than or equal to x, minus the probability that X is greater than or equal to $x + 2$. These values can be retrieved from the tables, and we have that $Pr(|X| = |x|) = Pr(|X| \geq |x|) - Pr(|X| \geq |x + 2|)$, which can be looked up in the table as $Pr(|X| \geq |x|) = T_c[\dfrac{|x| - c}{2} - 1]$ and $Pr(|X| \geq |x + 2|) = T_c[\dfrac{|x| - c}{2}]$.

The scaling by $\frac{1}{2}$ is to account for negative/positive sign. We can define the probability mass function for "combined" $\mathcal{D}_{2\mathbb{Z},\sigma}$ and $\mathcal{D}_{2\mathbb{Z}+1,\sigma}$ as $\mathcal{D}_{\mathbb{Z},\sigma}$ as given by sampling algorithm. Since the parameter c which is 0 or 1 with probability $\frac{1}{2}$,

$$Pr(X = x) = \frac{1}{2} \begin{cases} 1 - T_0[0] & \text{if } x = 0 \\ \frac{1}{2}(1 - T_1[0]) & \text{if } |x| = 1 \\ \frac{1}{2}(T_c[\dfrac{|x| - c}{2} - 1] - T_c[\dfrac{|x| - c}{2}]) & \text{if } |x| > c \end{cases}$$

Denote now by $f(x) = Pr(X = x)$. We can compute $\sigma^2 = \mathbb{E}[X^2] - \mathbb{E}[X]^2$ as $\sum_x x^2 \cdot f(x) - 0$ and then $\sigma = \sqrt{\sigma^2}$. Note that we sum over all x that has a corresponding non-zero entry in the table. So for Hawk256 we sum from $x = -20$ to $x = 20$ since $Pr(X = -20) = Pr(X = 20) = \frac{1}{4}(T_0[\frac{20}{2} - 1] - T_0[\frac{20}{2}]) = \frac{1}{4}(T_0[9] - T_0[10]) = \frac{1}{4}(T_0[9] - 0)$, since entry 9 in T_0 is the last non-zero entry in the tables. Similarly, for Hawk 512 and Hawk 1024 we sum from $x = -26$ to $x = 26$ since there are 13 entries in the tables.

Below are the moments for X

Table 1: Moments of X

Degree	$\mathbb{E}[X^2]$	$\mathbb{E}[X^4]$
256	4.08040000000000212758	49.948992479999702471611
512	6.53313599999999999999	128.04559798348799999938
1024	6.74960399999999999999	136.671462470447999999396

Now, let $Z = \frac{X}{\sigma}$ such that $\mathbb{V}[Z] = \mathbb{E}[Z^2] - \mathbb{E}[Z]^2 = \mathbb{E}[\frac{X^2}{\sigma^2}] - 0 = \frac{\sigma^2}{\sigma^2} = 1$. Then $\mathbb{E}[Z^4] = \mathbb{E}[(\dfrac{X}{\sigma})^4] = \dfrac{\mathbb{E}[X^4]}{\sigma^4}$. Explicitly, the values for $\mathbb{E}[Z^4]$ are:

Table 2: Values for $\mathbb{E}[Z^4]$

Degree	$\mathbb{E}[Z^4]$	$\mathbb{E}[Z^4] - 3$
256	2.999999999999979001575261935069...	-2.099842473806493023232033185539 $\cdot 10^{-14}$
512	2.999999999999999999986268406268...	-1.373159373148697609358990321080 $\cdot 10^{-20}$
1024	2.999999999999999999987558603578...	-1.244139642155725628841431806681 $\cdot 10^{-20}$

Table 3: Distribution tables for Hawk

Degree	**Index**	T_0	T_1
256	0	0.60500764458817746227	0.30112171978068869027
	1	0.12111379328095156493	0.03890258577957197982
	2	0.00990366124775658544	0.00198867578314904092
	3	0.00031404023074329352	0.00003892746532120486
	4	$3.78338918128274943781 \cdot 10^{-6}$	$2.88107957977879477782 \cdot 10^{-7}$
	5	$1.71827120294345768175 \cdot 10^{-8}$	$8.02371725048700385830 4 \cdot 10^{-10}$
	6	$2.93316557610652772623 \cdot 10^{-11}$	$8.39327557089857137278 \cdot 10^{-13}$
	7	$1.87989625626860808692 \cdot 10^{-14}$	$3.29554767989818261073 \cdot 10^{-16}$
	8	$4.52170671790477090613 \cdot 10^{-18}$	$4.85555019568627245149 \cdot 10^{-20}$
	9	$4.06972861376089616036 \cdot 10^{-22}$	0
512	0	0.68783859123519186518	0.42167577949865736597 3
	1	0.22815616423571508097	0.10815323940371951438 8
	2	0.04466972048333950340	0.01600998546041348599 6
	3	0.00496451451795861930	0.00132898309827764356 2
	4	0.00030663435548965438	$6.09071651345428178398 9 \cdot 10^{-5}$
	5	$1.04060485623348598200 \cdot 10^{-5}$	$1.52825807349333708088 3 \cdot 10^{-6}$
	6	$1.92840254421950117664 \cdot 10^{-7}$	$2.08996680843278062652 5 \cdot 10^{-8}$
	7	$1.94496525936032772231 \cdot 10^{-9}$	$1.55393981568189441865 5 \cdot 10^{-10}$
	8	$1.06572930907894827110 \cdot 10^{-11}$	$6.27344637643491079338 7 \cdot 10^{-13}$
	9	$3.16941999635792541820 \cdot 10^{-14}$	$1.37418114581221312839 9 \cdot 10^{-15}$
	10	$5.11306475526383081512 \cdot 10^{-17}$	$1.63259644882856008556 1 \cdot 10^{-18}$
	11	$4.47306188044175245463 \cdot 10^{-20}$	$1.04886501671723909173 6 \cdot 10^{-21}$
	12	$1.98523347012726641969 \cdot 10^{-23}$	0
1024	0	0.69288508052237884893	0.42962686546922340680
	1	0.23617193265906574750	0.11428308553940400016
	2	0.04842133601191184882	0.01789248535056067734
	3	0.00574920368926730516	0.00160288957145916094
	4	0.00038713254406289563	$8.09018865846981160986 \cdot 10^{-5}$
	5	$1.46156213735215806631 \cdot 10^{-5}$	$2.28113759483951792305 \cdot 10^{-6}$
	6	$3.07433006382663749701 \cdot 10^{-7}$	$3.57649723218357077527 \cdot 10^{-8}$
	7	$3.59052157753520911942 \cdot 10^{-9}$	$3.11001989345370333228 \cdot 10^{-10}$
	8	$2.32386383797340793425 \cdot 10^{-11}$	$1.49779523761119320525 \cdot 10^{-12}$
	9	$8.32631949511072616142 \cdot 10^{-14}$	$3.99196058657956792236 \cdot 10^{-15}$
	10	$1.65056413522219193150 \cdot 10^{-16}$	$5.88541321937194339148 \cdot 10^{-18}$
	11	$1.80970574414351394708 \cdot 10^{-19}$	$4.79764755280756051425 \cdot 10^{-21}$
	12	$1.09187840856999653083 \cdot 10^{-22}$	0