Interdisciplinary postgraduate program «Optics and Vision»

Foundations of Modern Optics

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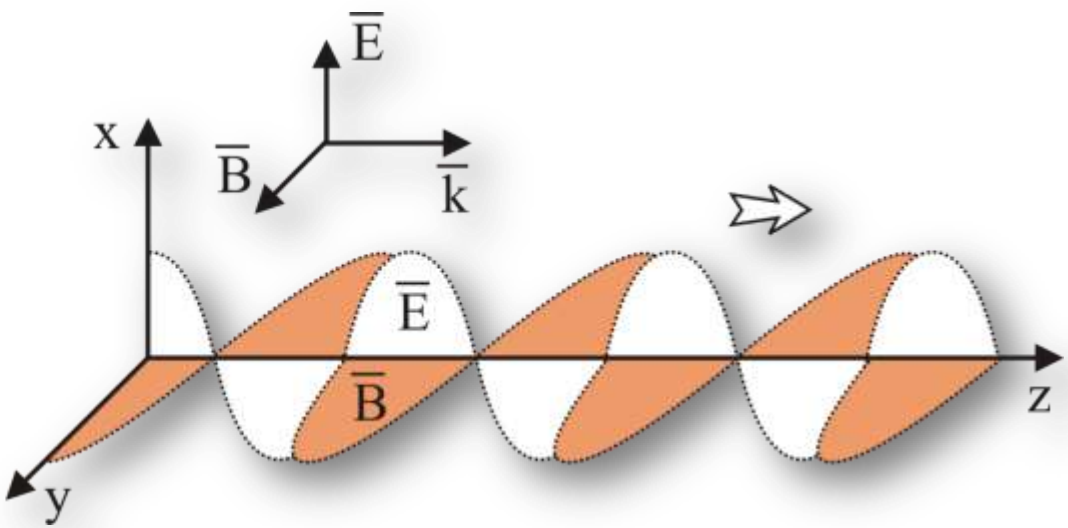
2 Basic principles

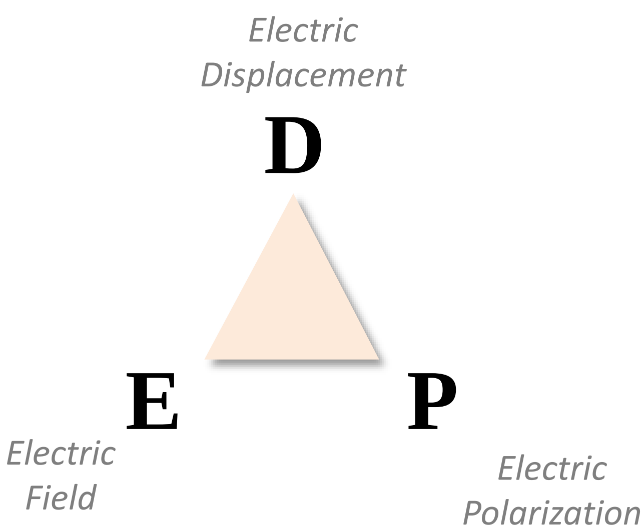
Electromagnetism

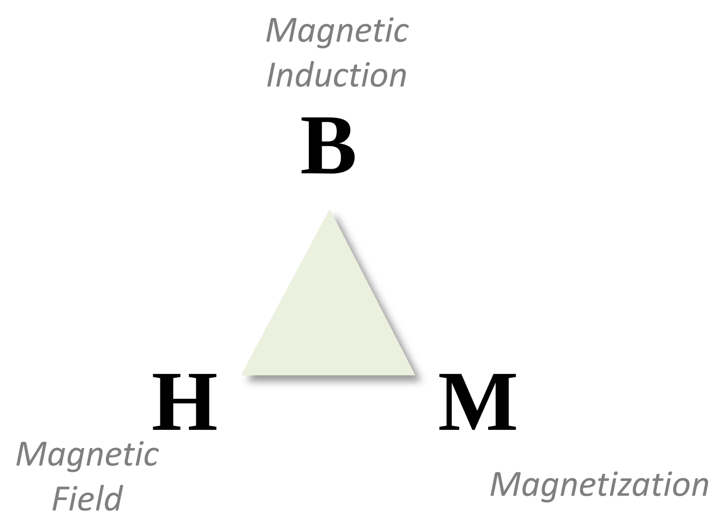


Maxwell Equations

$$\nabla \cdot \mathbf{D} = \rho, \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$$





Electric and magnetic properties are described using 3+3 vectors!

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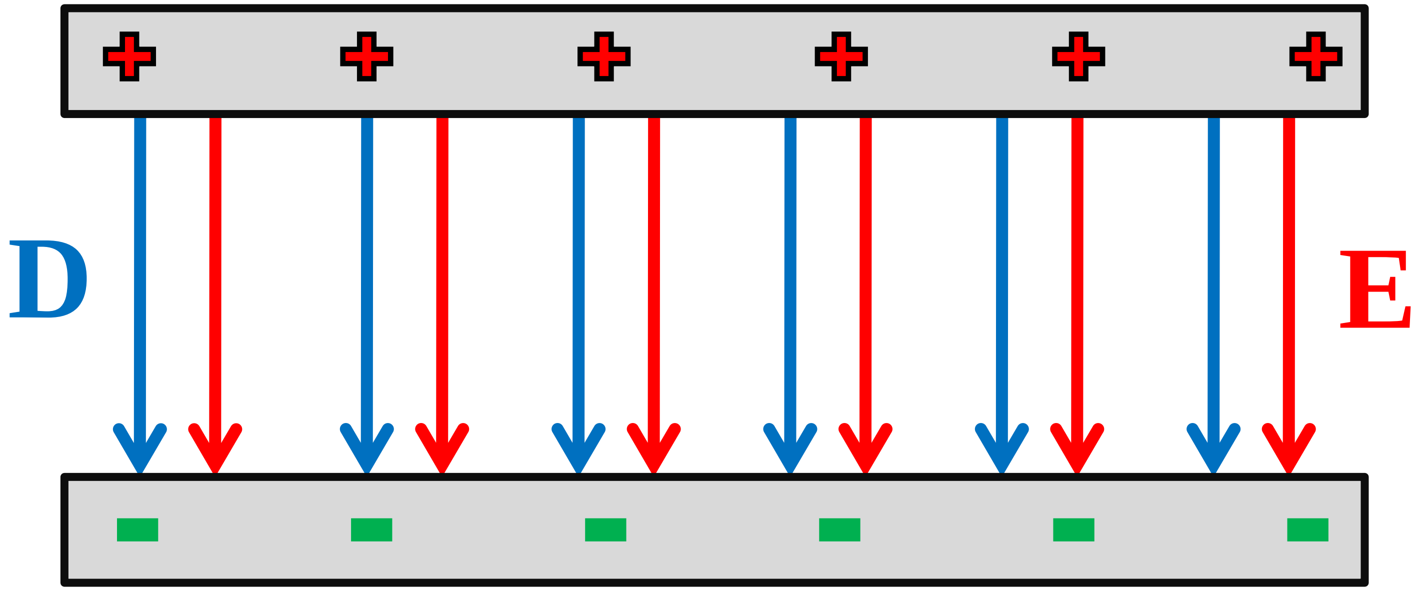
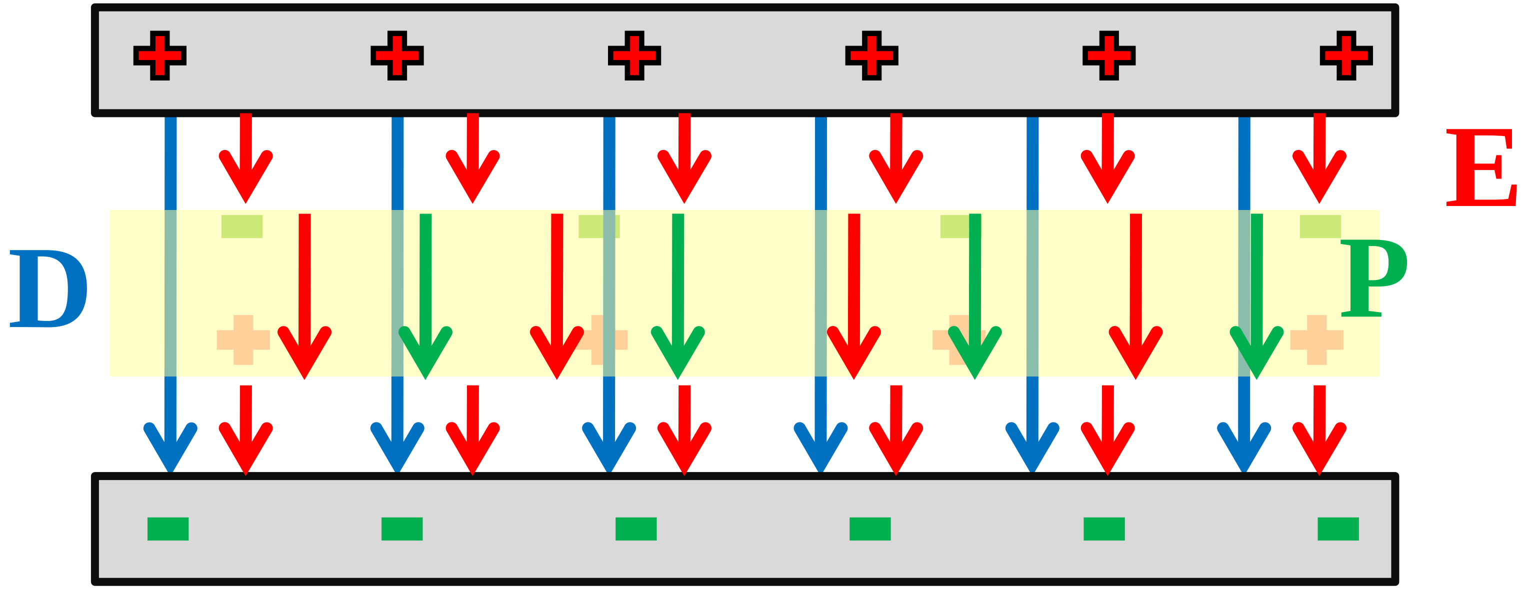
Why do we need so many vectors?



Interaction of field with matter



Material equations

Vacuum permittivity

$$\begin{rcases}\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P} \\ \mathbf{B} = \mu_o (\mathbf{H} + \mathbf{M})\end{rcases} \underset{\text{isotropic}}{\Rightarrow}$$

vacuum permeability

permittivity

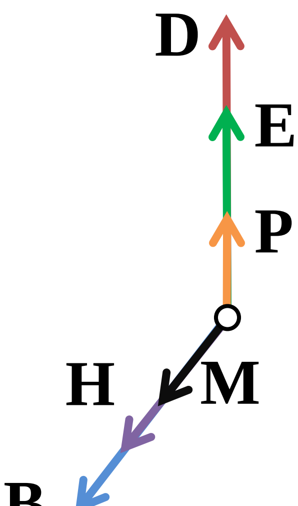
dielectric constand

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_r \varepsilon_o \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H} = \mu_r \mu_o \mathbf{H}$$

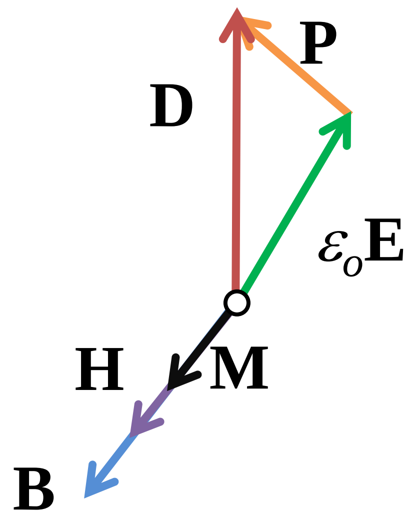
magnetic permeability

relative magnetic permeability



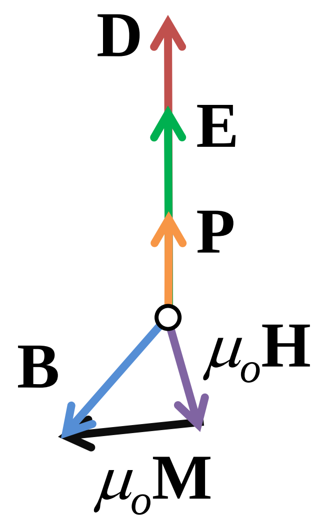
$\mathbf{D} /\!/ \mathbf{E}, \ \ \mathbf{B} /\!/ \mathbf{H}$

Electrically isotropic
Magnetically isotropic



$\mathbf{B} /\!/ \mathbf{H}$

Electrically anisotropic
Magnetically isotropic



$\mathbf{D} /\!/ \mathbf{E}$

Electrically isotropic
Magnetically anisotropic

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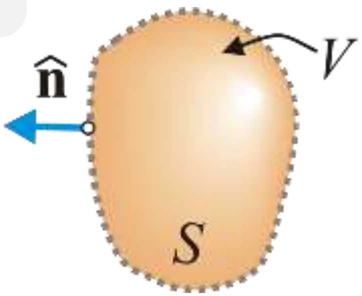
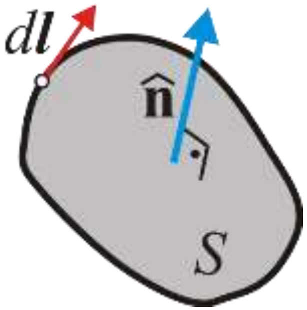
Maxwell equations (integral form)

$$\iiint_V \nabla \cdot \mathbf{D}\, dv = \iiint_V \rho\, dv \Rightarrow \boxed{\oiint_s \mathbf{D} \cdot \mathbf{n}\, ds = q}$$





$$\iiint_V \nabla \cdot \mathbf{B}\, dv = 0 \Rightarrow \boxed{\oiint_s \mathbf{B} \cdot \mathbf{n}\, ds = 0}$$

$$\iint_S (\nabla \times \mathbf{E}) \cdot \mathbf{n}\, ds = -\iint_S \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{n}\, ds \Rightarrow \boxed{\oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \iint_S \mathbf{B} \cdot \mathbf{n}\, ds}$$

$$\iint_S (\nabla \times \mathbf{H}) \cdot \mathbf{n}\, ds = \iint_S \mathbf{j} \cdot \mathbf{n}\, ds + \iint_S \frac{\partial \mathbf{D}}{\partial t} \cdot \mathbf{n}\, ds \Rightarrow \boxed{\oint_L \mathbf{H} \cdot d\mathbf{l} = I + \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot \mathbf{n}\, ds}$$

H/M wave equation

Derivation of the wave equation for an isotropic material

$$\left.\begin{array}{c}\nabla\times(\nabla\times\mathbf{E})=-\nabla\times\dfrac{\partial\mathbf{B}}{\partial t}=-\dfrac{\partial}{\partial t}(\nabla\times\mathbf{B})\\ \mathbf{B}=\mu\mathbf{H}\\ \nabla\times(\nabla\times\mathbf{E})=\nabla\cdot(\nabla\cdot\mathbf{E})-\nabla^2\mathbf{E}\end{array}\right\}\Rightarrow\left.\begin{array}{c}\nabla\cdot(\nabla\cdot\mathbf{E})-\nabla^2\mathbf{E}=-\mu\dfrac{\partial}{\partial t}(\nabla\times\mathbf{H})\\ \mathbf{D}=\varepsilon\mathbf{E}\\ \rho=0,\ \mathbf{j}=\mathbf{0}\Rightarrow\nabla\cdot\mathbf{E}=0\end{array}\right\}\Rightarrow$$

$$-\nabla^2\mathbf{E}=-\mu\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\mathbf{D}\right)\Rightarrow\boxed{\nabla^2\mathbf{E}-\varepsilon\mu\frac{\partial^2\mathbf{E}}{\partial t^2}=0}$$

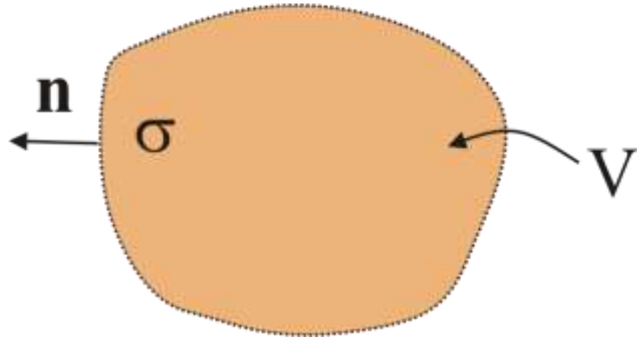
wave equation

$$\varepsilon\mu=\frac{1}{\upsilon^2}\Rightarrow\upsilon=\frac{1}{\sqrt{\varepsilon\mu}}=\frac{1}{\sqrt{\varepsilon_r\mu_r}}\frac{1}{\sqrt{\varepsilon_o\mu_o}}=\frac{c}{\sqrt{\varepsilon_r\mu_r}}$$

light velocity in vacuum

light velocity

Energy law of the E/M field. Poynting vector



$$\oiint_S (\mathbf{E}\times\mathbf{H})\cdot\mathbf{n}\,dS + \iiint_V \left(\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t} + \mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}\right)dv + \iiint_V \mathbf{j}\cdot\mathbf{E}\,dv = 0$$

Energy flow of E/M radiation

Electric and Magnetic energy density

Losses

$$\mathbf{S} = \mathbf{E}\times\mathbf{H}$$

Poynting vector

Harmonic plane waves

harmonic plane wave

$$\mathbf{A} = \mathbf{A}_o e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

$$\nabla \cdot \mathbf{A} = i\mathbf{k} \cdot \mathbf{A}, \quad \nabla \times \mathbf{A} = i\,\mathbf{k} \times \mathbf{A}, \quad \frac{\partial \mathbf{A}}{\partial t} = -i\omega\mathbf{A}, \quad \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\omega^2 \mathbf{A}$$

$$\mathbf{k}\cdot\mathbf{r} = (k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} + k_z\hat{\mathbf{z}}) \cdot (x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}) = k_x x + k_y y + k_z z$$

$$\nabla \cdot \mathbf{A} = \mathbf{A}_o e^{-i\omega t} \left(\frac{\partial}{\partial x}\hat{\mathbf{x}} + \frac{\partial}{\partial y}\hat{\mathbf{y}} + \frac{\partial}{\partial z}\hat{\mathbf{z}}\right) \cdot e^{i\mathbf{k}\cdot\mathbf{r}} =$$

$$= \mathbf{A}_o e^{-i\omega t} i\,(k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} + k_z\hat{\mathbf{z}})\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\Rightarrow \nabla \cdot \mathbf{A} = i\mathbf{k} \cdot \mathbf{A}$$

$$\nabla \times \mathbf{A} = \nabla \times [\mathbf{A}_o e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}] =$$

$$e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}\, \nabla \times \mathbf{A}_o - \mathbf{A}_o \times \nabla e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} =$$

$$= -e^{-i\omega t} \mathbf{A}_o \times \nabla e^{i\mathbf{k}\cdot\mathbf{r}} = i\,\mathbf{k} \times \mathbf{A}$$

Assuming that our E/M wave is plane and harmonic

$$\mathbf{D} = \mathbf{D}_o e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}, \quad \mathbf{E} = \mathbf{E}_o e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}, \quad \mathbf{B} = \mathbf{B}_o e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}, \quad \mathbf{H} = \mathbf{H}_o e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

Maxwell equations a simplified:

$$\left.\begin{array}{ll} \nabla\cdot\mathbf{D} = \rho, & \nabla\times\mathbf{E} = -\dfrac{\partial\mathbf{B}}{\partial t} \\ \nabla\cdot\mathbf{B} = 0, & \nabla\times\mathbf{H} = \mathbf{j} + \dfrac{\partial\mathbf{D}}{\partial t} \end{array}\right\} \Rightarrow \begin{array}{ll} i\mathbf{k}\cdot\mathbf{D} = 0, & i\mathbf{k}\times\mathbf{E} = i\omega\mathbf{B} \\ i\mathbf{k}\cdot\mathbf{B} = 0, & i\mathbf{k}\times\mathbf{H} = -i\omega\mathbf{D} \end{array}$$

$$\mathbf{k}\cdot\mathbf{D} = 0 \Rightarrow \mathbf{k}\perp\mathbf{D}$$

$$\mathbf{k}\cdot\mathbf{B} = 0 \Rightarrow \mathbf{k}\perp\mathbf{B}$$

$$\mathbf{k}\times\mathbf{E} = \omega\mathbf{B} \Rightarrow \mathbf{k}\perp\mathbf{B}, \ \mathbf{E}\perp\mathbf{B}$$

$$\mathbf{k}\times\mathbf{H} = -\omega\mathbf{D} \Rightarrow \mathbf{k}\perp\mathbf{D}, \ \mathbf{H}\perp\mathbf{D}$$

in isotropic materials

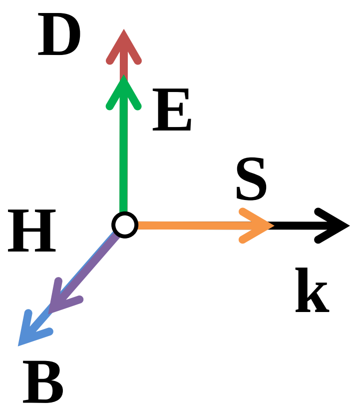
$$\mathbf{D} // \mathbf{E}, \quad \mathbf{B} // \mathbf{H}$$

we then get:

$$\mathbf{k}\perp\mathbf{D} \Rightarrow \mathbf{k}\perp\mathbf{E}$$

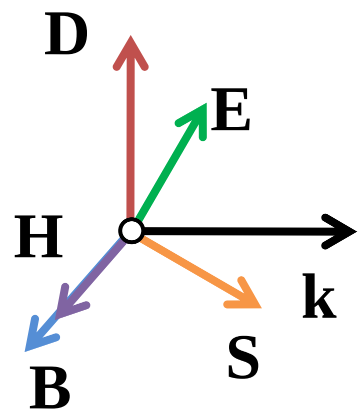
$$\mathbf{k}\perp\mathbf{B} \Rightarrow \mathbf{k}\perp\mathbf{H}$$

ISOTROPIC MATERIALS



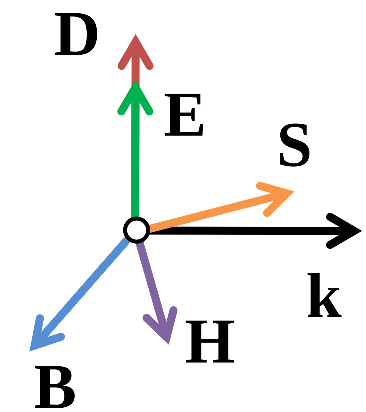
$\mathbf{D} /\!/ \mathbf{E}, \ \mathbf{B} /\!/ \mathbf{H}$

ANISOTROPOIC MATERIALS



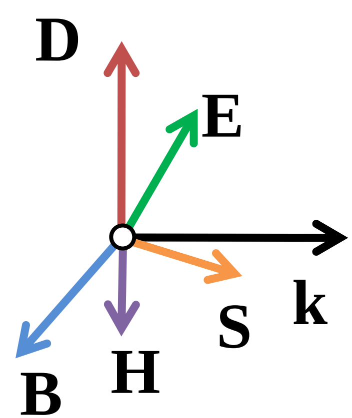
$\mathbf{B} /\!/ \mathbf{H}$

Electrically anisotropic
Magnetically isotropic



$\mathbf{D} /\!/ \mathbf{E}$

Electrically isotropic
Magnetically anisotropic



Electrically anisotropic
Magnetically anisotropic

$\mathbf{k} \perp \mathbf{D}, \ \mathbf{k} \perp \mathbf{B}, \ \mathbf{E} \perp \mathbf{B}, \ \mathbf{H} \perp \mathbf{D}$

Pointing vector in isotropic materials (harmonic fields)

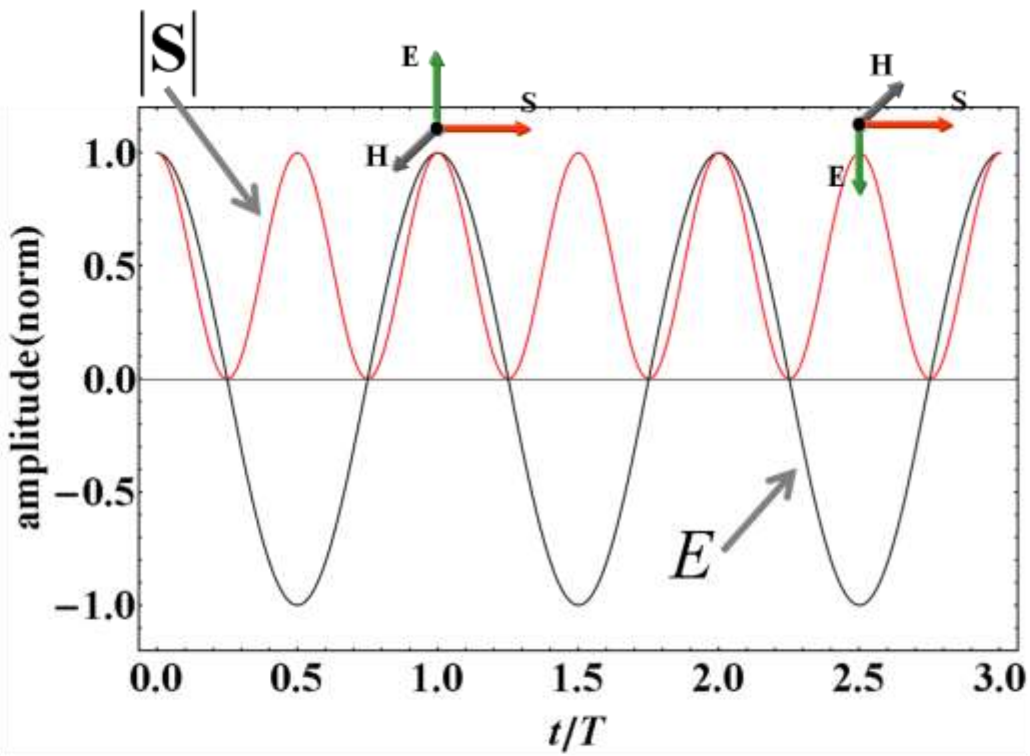
isotropic materials

$$\mathbf{D} // \mathbf{E},\ \ \mathbf{B} // \mathbf{H} \Rightarrow \begin{cases} \mathbf{k} \perp \mathbf{D} \Rightarrow \mathbf{k} \perp \mathbf{E} \\ \mathbf{k} \perp \mathbf{B} \Rightarrow \mathbf{k} \perp \mathbf{H} \end{cases}$$

Plane harmonic waves

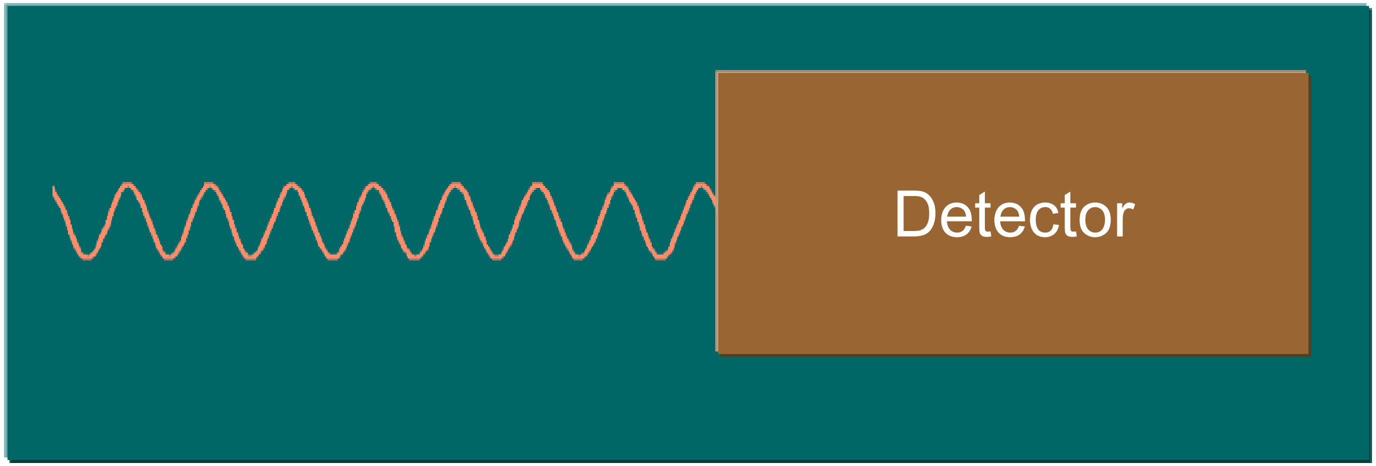
$$\mathbf{A} = \mathbf{A}_o e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

$$|\mathbf{S}| = n\,\varepsilon_o\, c\,|\mathbf{E}|^2$$



The H/M field of the optical wave oscillates much faster than the response time of our detector!

How do we detect H/M radiation?



The oscillation period of the Poynting vector is <2 fs in the visible range.

No detector is so fast.

In fact, we measure the **average** value of the energy transferred from the optical wave over **many oscillation periods**.

Intensity

The average, over time, value of the Poynting vector magnitude is called intensity

$$I \equiv \left\langle \left|\mathbf{S}\right| \right\rangle_t = n\,\varepsilon_o\,c \left\langle \left|\mathbf{E}\right|^2 \right\rangle_t$$ **(*W/m²*)**

For harmonic waves:

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_o(\mathbf{r})\cos(\omega t) \Rightarrow$$

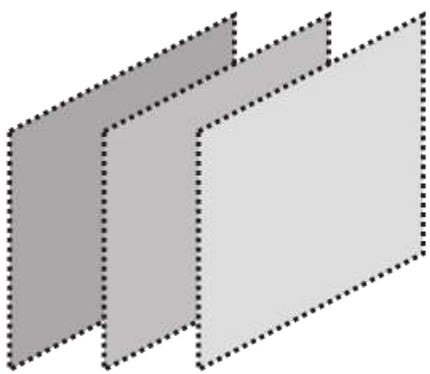
$$\left\langle \left|\mathbf{E}\right|^2 \right\rangle_t = \left|\mathbf{E}_o\right|^2 \left\langle \cos^2(\omega t) \right\rangle_t = \left|\mathbf{E}_o\right|^2 \left\langle \frac{1}{2}[1+\cos(2\omega t)] \right\rangle_t = \frac{1}{2}\left|\mathbf{E}_o\right|^2 \Rightarrow$$

$$I = \frac{1}{2} n\,\varepsilon_o\,c \left|\mathbf{E}_o\right|^2$$

Intensity

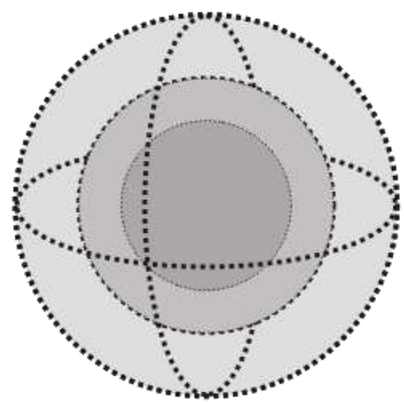
$$I = \frac{1}{2} n \varepsilon_o c \left|\mathbf{E}_o\right|^2$$

$$I = const$$



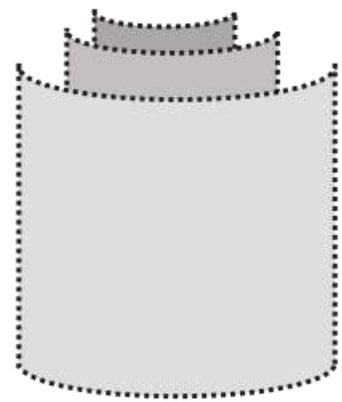
Plane

$$I \propto \frac{1}{r^2}$$



Spherical

$$I \propto \frac{1}{r}$$



Cylindrical

Spectrum of the electromagnetic radiation

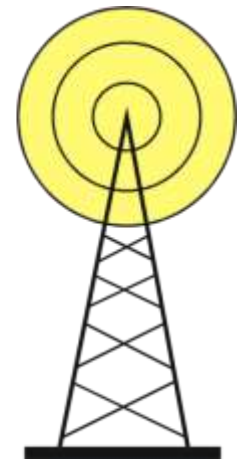
E/M radiation sources

Electrodynamics

Accelerating charges

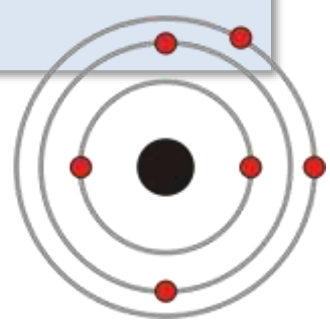
Alternating currents

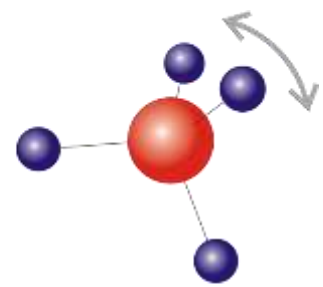
There is no lower frequency limit



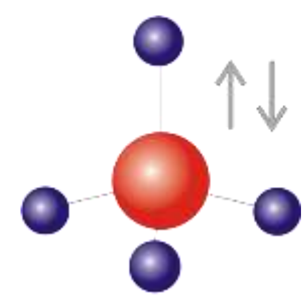
Quantum mechanical

Energy transitions



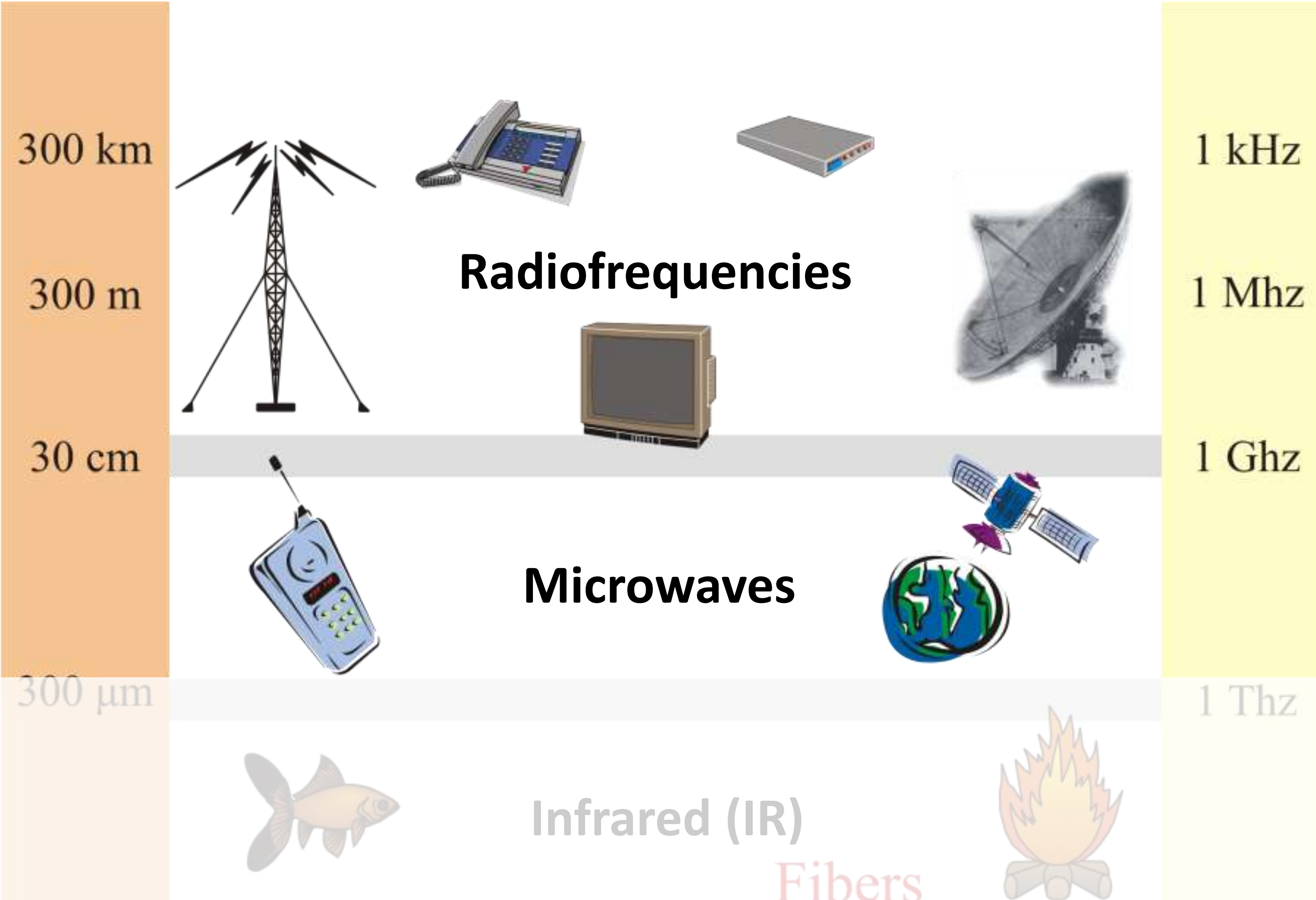


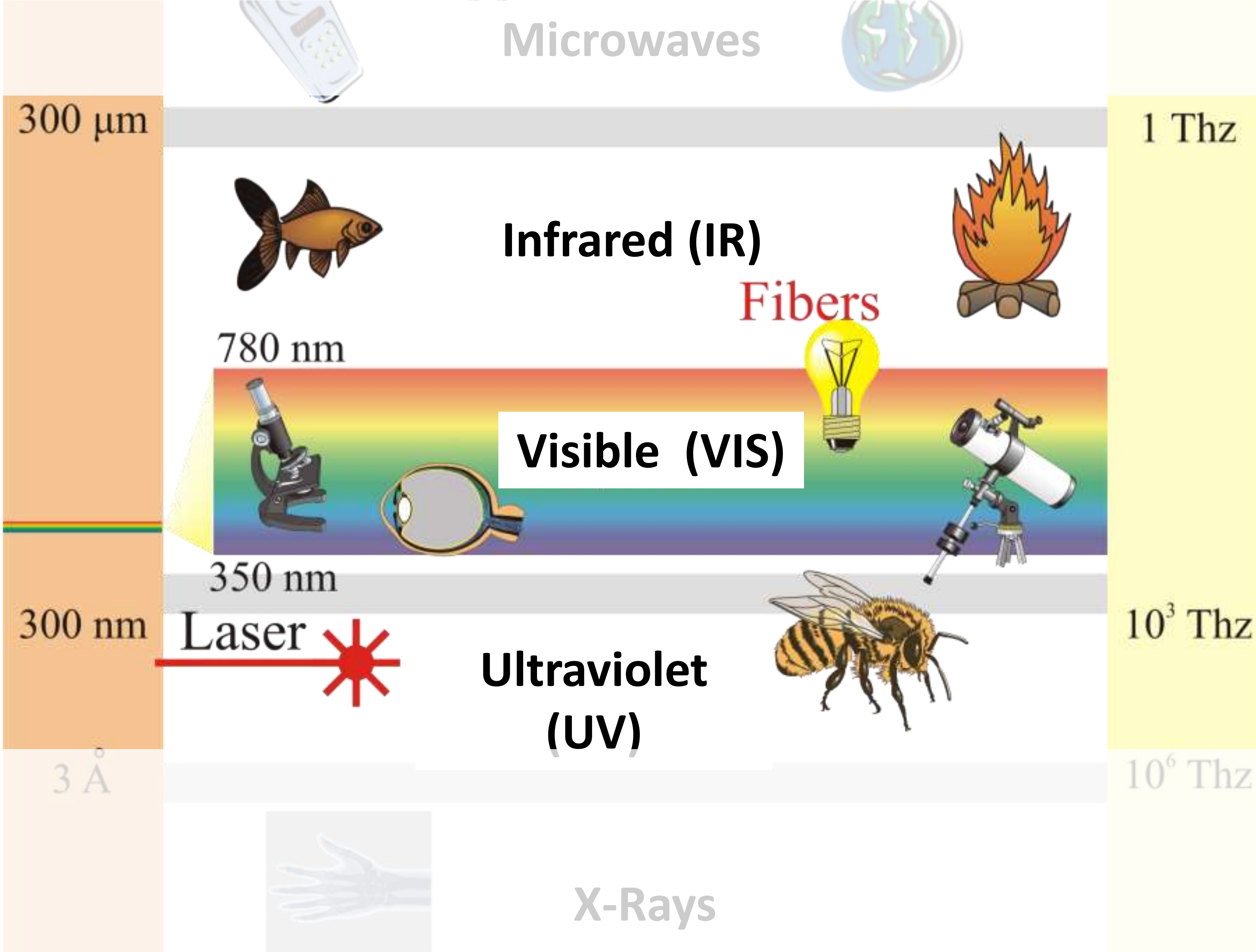
rotations

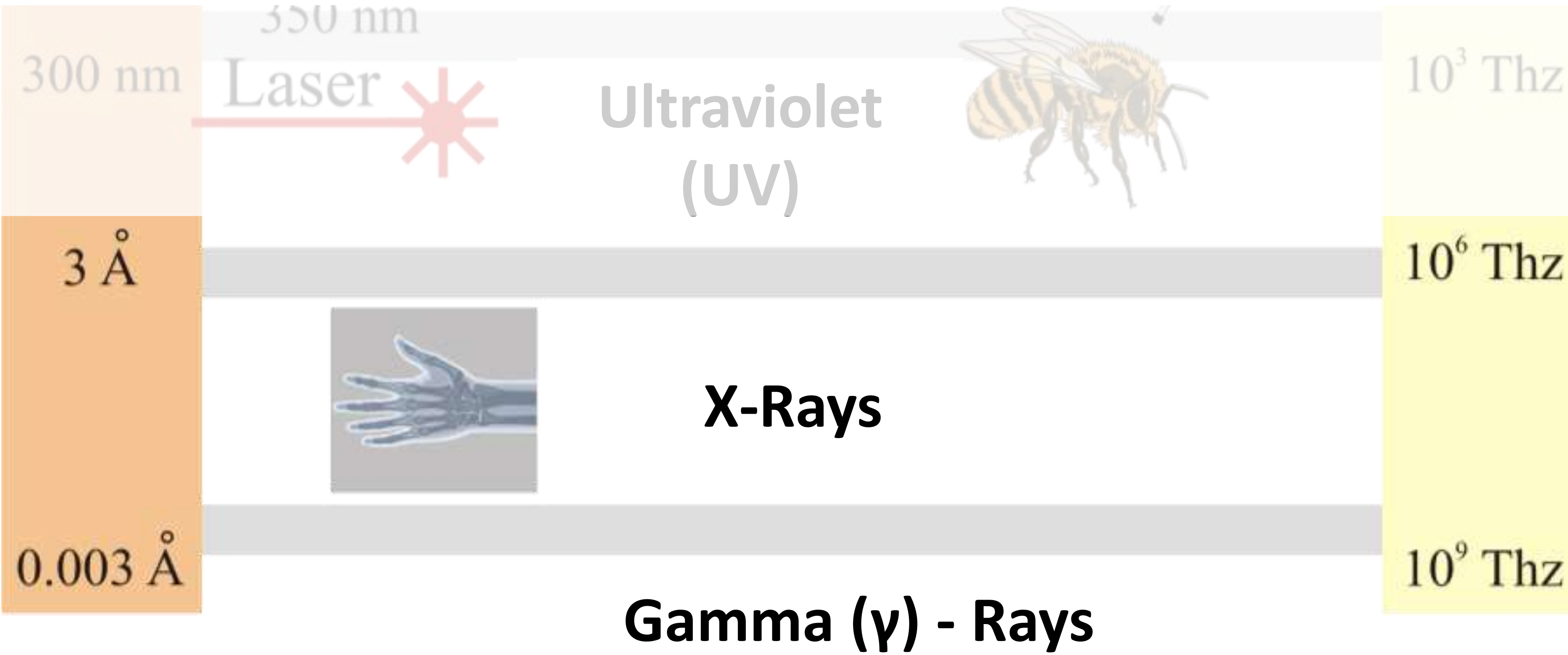


vibrations

Electromagnetic radiation spectrum



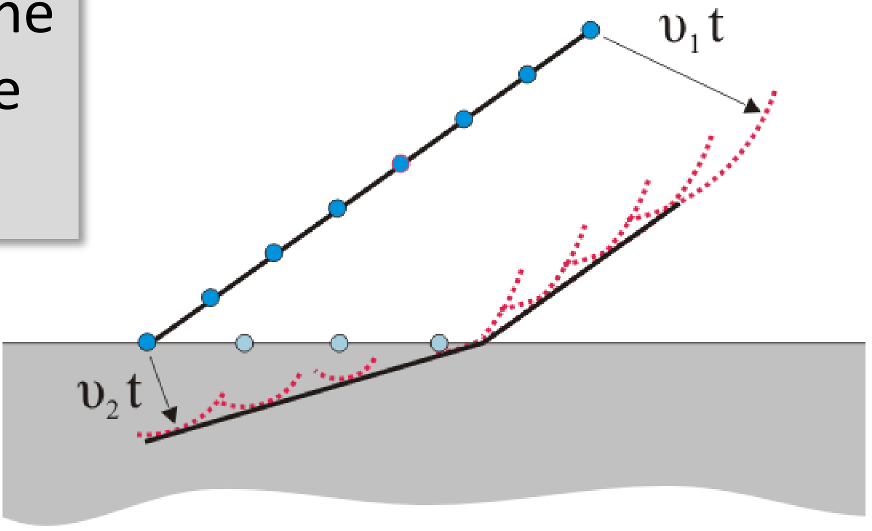

Microwaves
300 μm
1 Thz
Infrared (IR)
Fibers
780 nm
Visible (VIS)
350 nm
300 nm
10^3 Thz
Laser
Ultraviolet
(UV)
3 Å
10^6 Thz
X-Rays


350 nm
300 nm
Laser
Ultraviolet
(UV)
10³ Thz
3 Å
10⁶ Thz
X-Rays
0.003 Å
10⁹ Thz
Gamma (γ) - Rays

Optical properties

Refractive index

The refractive index is correlated to the **propagation velocity** of an iso-phase surface of an E/M wave



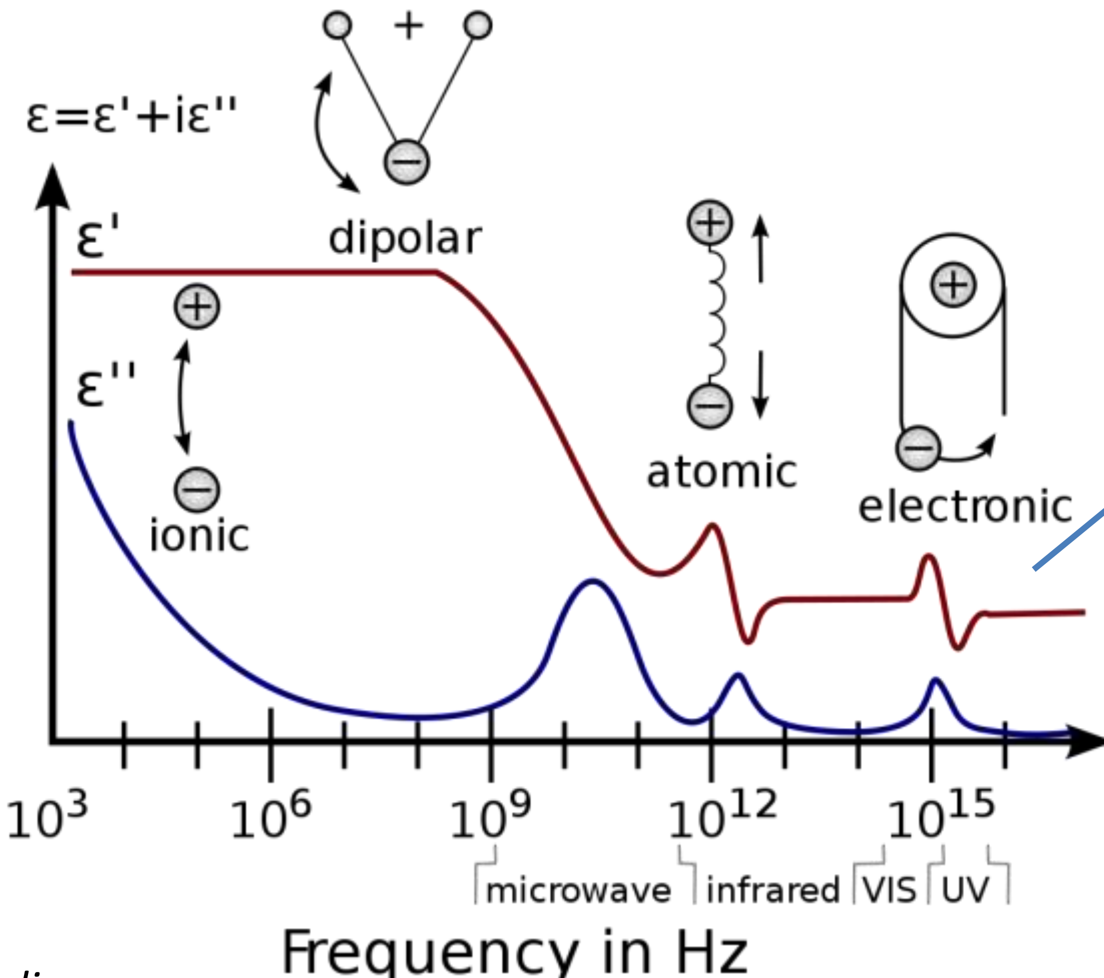
refractive index

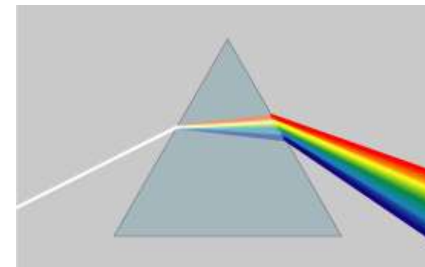
$$n \equiv \frac{c}{\upsilon} = \sqrt{\varepsilon_r \mu_r} \cong \sqrt{\varepsilon_r} \quad (\mu_r \approx 1)$$

depends on frequency

Dispersion

The velocity of E/M radiation in a dielectric medium depends on frequency

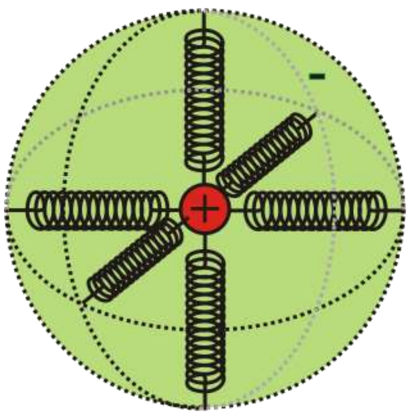




Souce: wikipedia

Dispersion: Classical approach

- The dielectric is not continuous but consists of a large number of atoms that can be polarized.
- The variable electric field $\boldsymbol{E(t)}$ of the E/M wave "drives" the atoms into a forced oscillation.
- Each oscillator has a natural resonance frequency ω_0



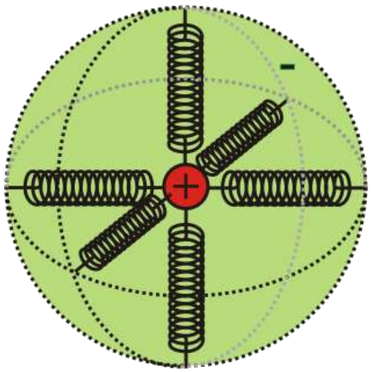
$$\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P}$$

$$\mathbf{P} = \varepsilon_o \chi_e \mathbf{E}$$

electric susceptibility
can be a tensor in anisotropic materials

Dispersion: Classical approach

Isotropic material



This model is mathematically equivalent to a forced oscillator

displacement

harmonic field

$$m_e \frac{d^2x}{dt^2} = -k\,x + q_e E_o e^{i\omega t}$$

*restoring force **

$$* \quad \left. \begin{aligned} m_e \frac{d^2x}{dt^2} &= -k\,x \\ x(t) &= x_o e^{i\omega_o t} \end{aligned} \right\} \Rightarrow -m_e x_o \omega_o^2 e^{i\omega_o t} = -k\,x_o e^{i\omega_o t} \Rightarrow \boxed{k = m_e \omega_o^2}$$

Απλοποιημένο μοντέλο: Απουσία απωλειών

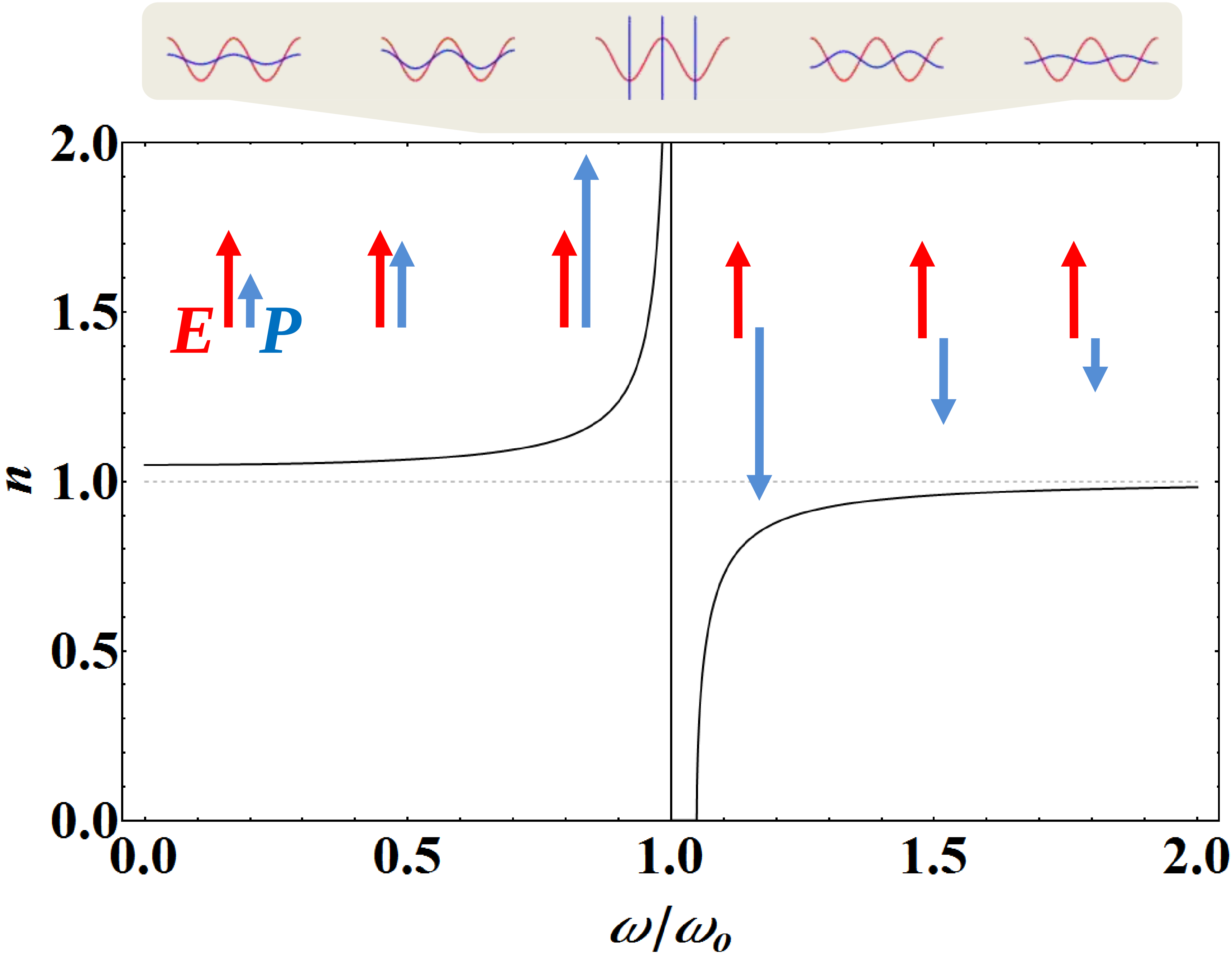
$$\left.\begin{aligned} m_e\frac{d^2x}{dt^2} &= -m_e\omega_o^2\,x + q_eE_oe^{-i\omega t} \\ x(t) &= x_oe^{-i\omega t} \end{aligned}\right\}\Rightarrow$$

$$-m_e\omega^2x_oe^{-i\omega t} = -m_e\omega_o^2x_oe^{-i\omega t} + q_eE_oe^{-i\omega t} \Rightarrow$$

$$\boxed{x_o = \frac{q_e}{m_e}\frac{1}{\omega_o^2-\omega^2}E_o}$$

$$\left.\begin{aligned} &\boxed{P(t) = Nq_e\,x(t)} \Rightarrow P(t) = \frac{Nq_e^2}{m_e}\frac{1}{\omega_o^2-\omega^2}E_oe^{-i\omega t} \\ &D(t) = \varepsilon_oE(t) + P(t) = \varepsilon_o\varepsilon_rE(t) \end{aligned}\right\}\Rightarrow$$

$$\boxed{\varepsilon_r = 1 + \frac{Nq_e^2}{\varepsilon_om_e}\frac{1}{\omega_o^2-\omega^2}}$$



E
P
n
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
ω/ω_o

Διασπορά παρουσία απωλειών

$$\left.\begin{aligned} m_e \frac{d^2 x}{dt^2} &= -m_e \omega_o^2 x - \boxed{m_e \gamma \frac{dx}{dt}} + q_e E_o e^{-i\omega t} \\ x(t) &= x_o e^{-i\omega t} \end{aligned}\right\} \Rightarrow$$

$$-m_e \omega^2 x_o e^{-i\omega t} = -m_e \omega_o^2 x_o e^{-i\omega t} + i m_e \gamma\, \omega x_o e^{-i\omega t} + q_e E_o e^{-i\omega t} \Rightarrow$$

$$\boxed{x_o = \frac{q_e}{m_e} \frac{1}{\omega_o^2 - \omega^2 - i\gamma\omega} E_o}$$

$$\left.\begin{aligned} \boxed{P(t) = N q_e\, x(t)} \Rightarrow P(t) &= \frac{N q_e^2}{m_e} \frac{1}{\omega_o^2 - \omega^2 - i\gamma\omega} E_o e^{-i\omega t} \\ D(t) = \varepsilon_o E(t) + P(t) &= \varepsilon_o \varepsilon_r E(t) \end{aligned}\right\} \Rightarrow$$

$$\boxed{\varepsilon_r = 1 + \frac{N q_e^2}{\varepsilon_o m_e} \frac{1}{\omega_o^2 - \omega^2 - i\gamma\omega}}$$

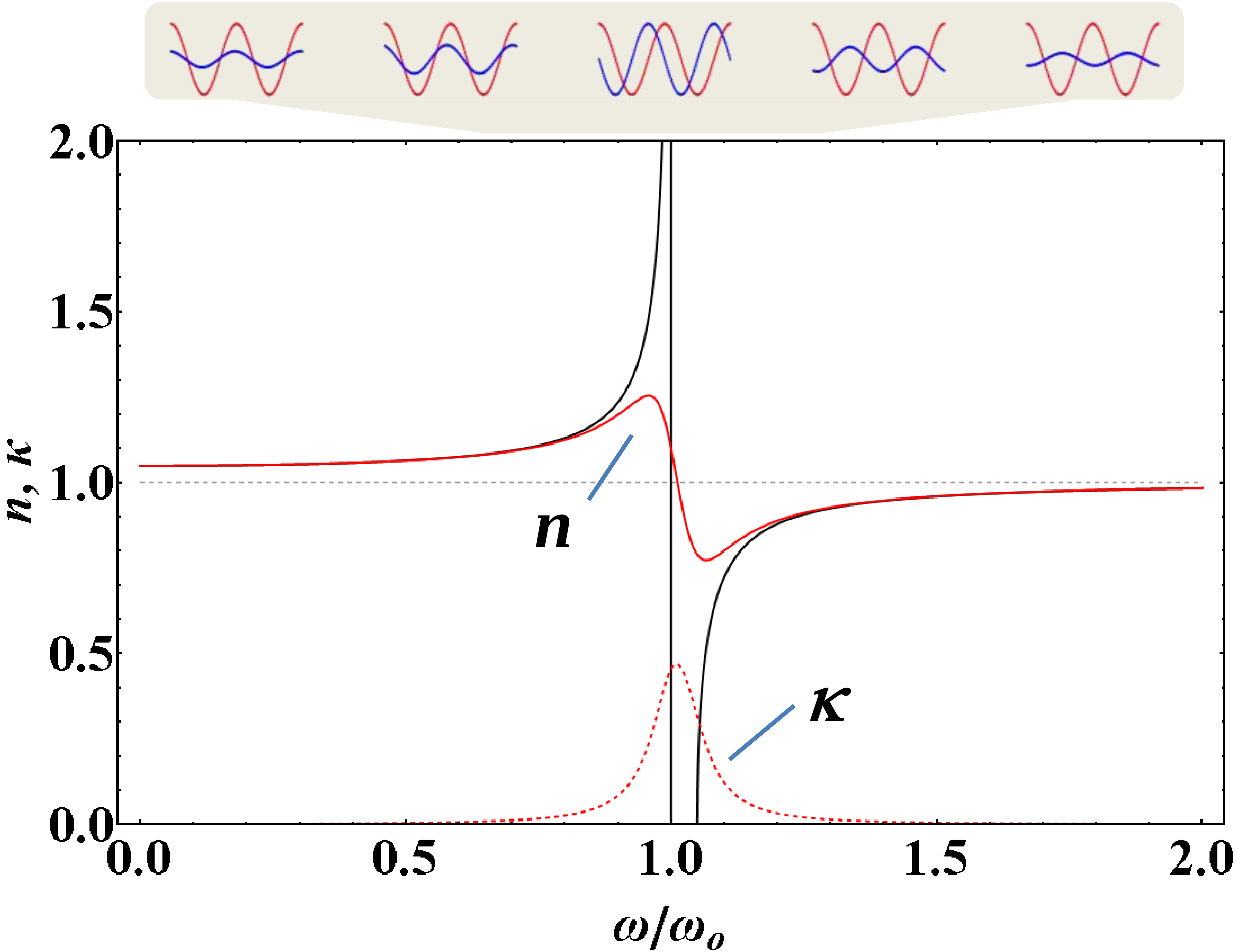
$$\varepsilon_r = 1 + \frac{Nq_e^2}{\varepsilon_o m_e} \frac{1}{\omega_o^2 - \omega^2 - i\gamma\omega} = \varepsilon' + i\varepsilon''$$

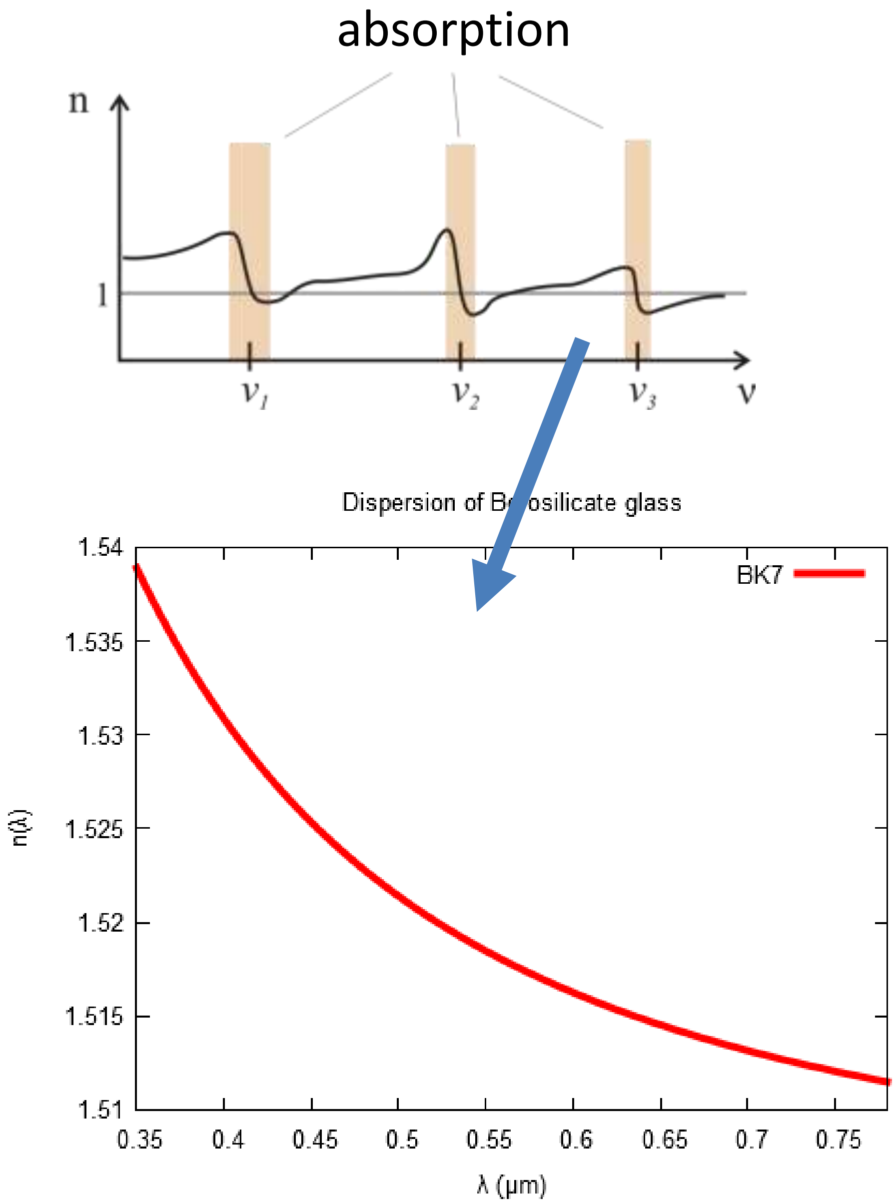
Dielectric constant can have an imaginary part

Therefore, the refractive index in the presence of absorption is complex

$$\varepsilon_r = (n + i\kappa)^2 \Rightarrow \begin{cases} \varepsilon' = n^2 - \kappa^2 \\ \varepsilon'' = 2n\kappa \end{cases}$$

$$n = \sqrt{\frac{1}{2}\sqrt{(\varepsilon'^2 + \varepsilon''^2)} + \varepsilon'}, \quad \kappa = \sqrt{\frac{1}{2}\sqrt{(\varepsilon'^2 + \varepsilon''^2)} - \varepsilon'}$$


2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
n, κ
ω/ω_o
n
κ



Normal dispersion in the visible

Abbe number

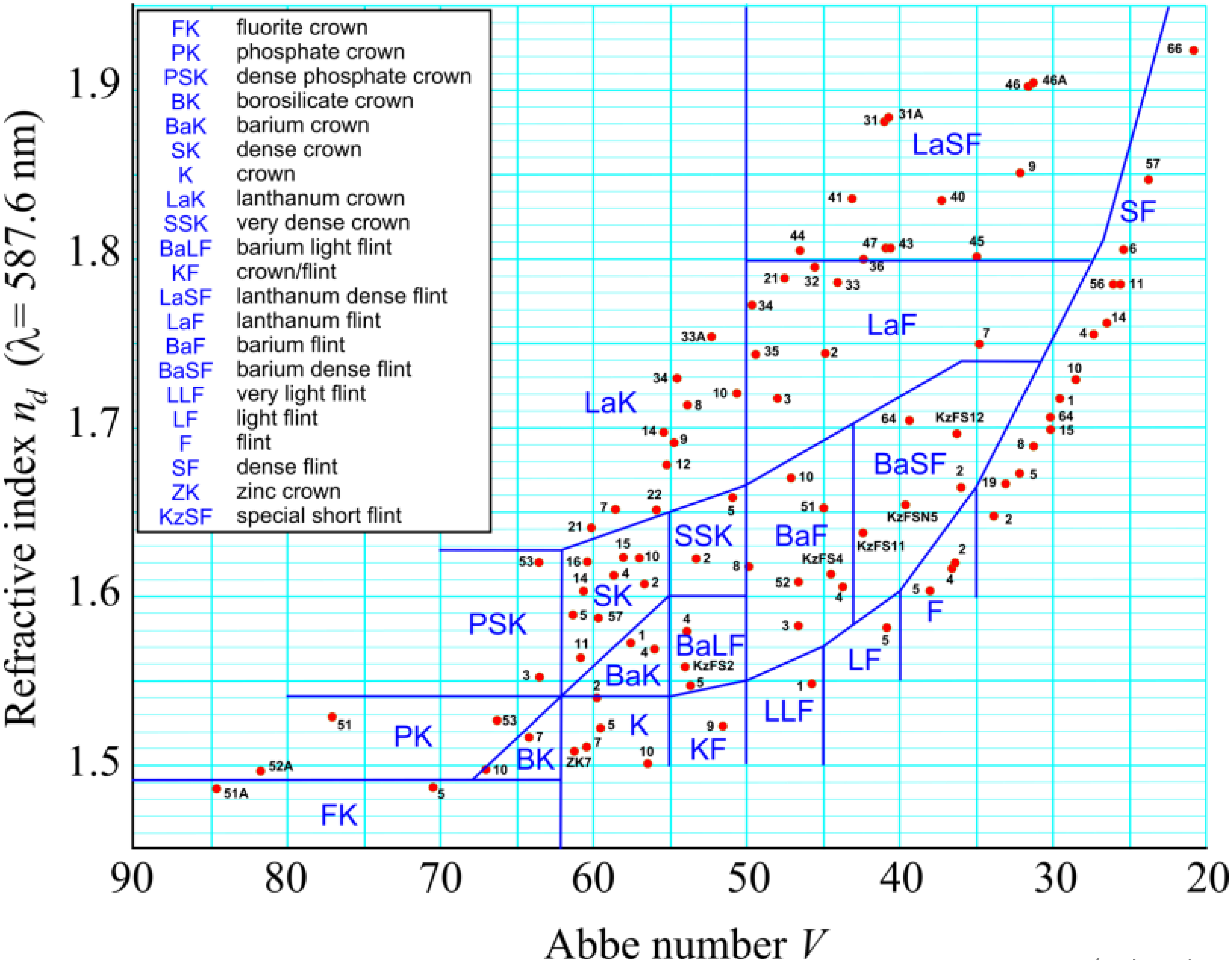
Glass dispersion is categorized by the Abbe number:

$$V \equiv \frac{n(\lambda_Y)-1}{n(\lambda_B)-n(\lambda_R)}$$

$$\lambda_B \equiv 486\, nm$$

$$\lambda_Y \equiv 589\, nm$$

$$\lambda_R \equiv 656\, nm$$



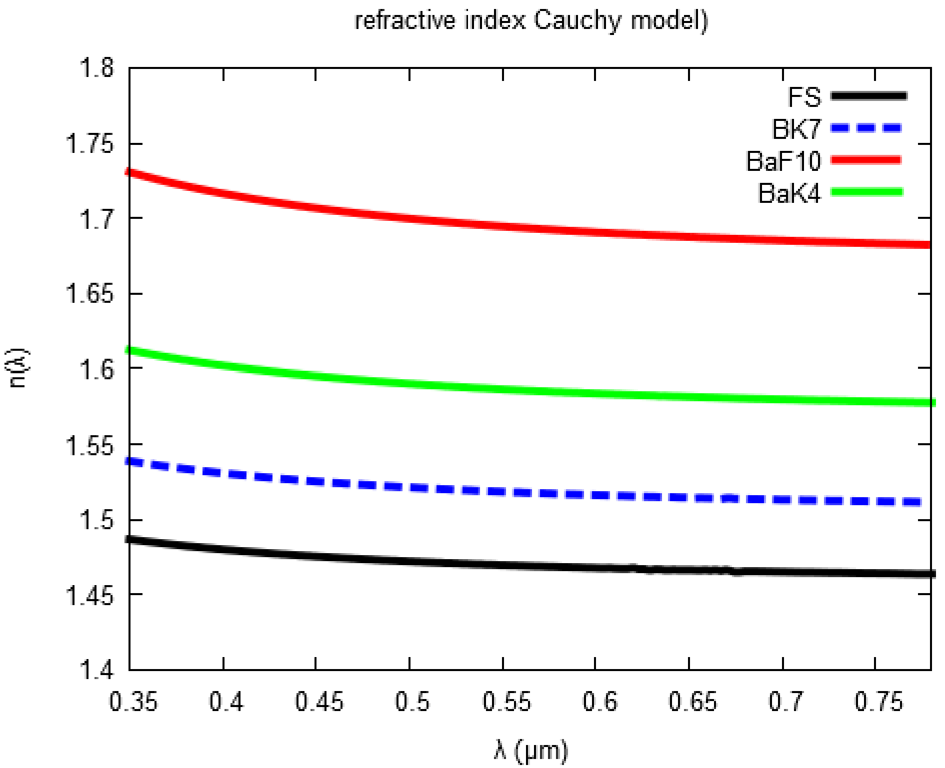
Eric Bajart/Wikipedia

Modeling the refractive index

Cauchy's equation

$$n(\lambda) = B + \sum_n \frac{C_n}{\lambda^{2n}} \cong B + \frac{C}{\lambda^2}$$

good model in the visible

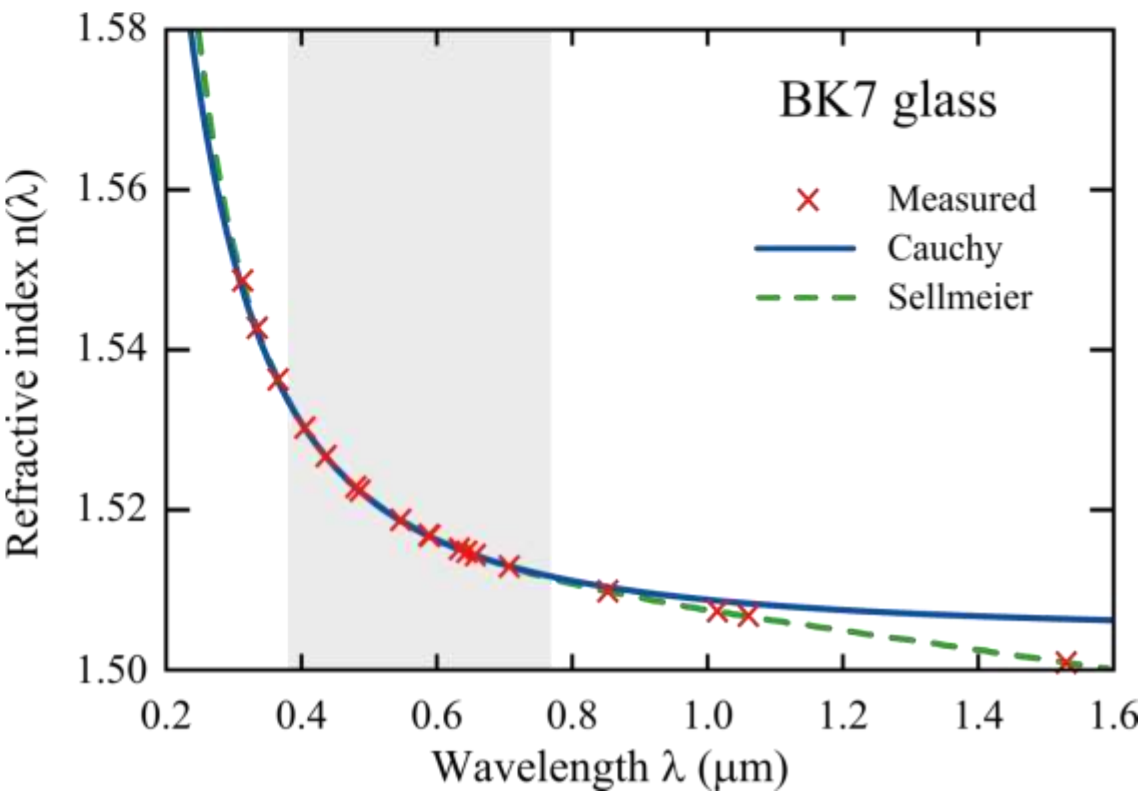


Cauchy constants for various glasses		
Material	**B**	**C (μm^2)**
Fused silica	1.4580	0.00354
Borosilicate glass BK7	1.5046	0.00420
Barium crown glass BaK4	1.5690	0.00531
Barium flint glass BaF10	1.6700	0.00743
Dense flint glass SF10	1.7280	0.01342

Modeling the refractive index

Sellmeier equation

$$n^2(\lambda) = 1 + \sum_i \frac{B_i \lambda^2}{\lambda^2 - C_i} \cong 1 + \frac{B_1 \lambda^2}{\lambda^2 - C_1} + \frac{B_2 \lambda^2}{\lambda^2 - C_2} + \frac{B_3 \lambda^2}{\lambda^2 - C_3}$$



Bob Mellish/Wikipedia

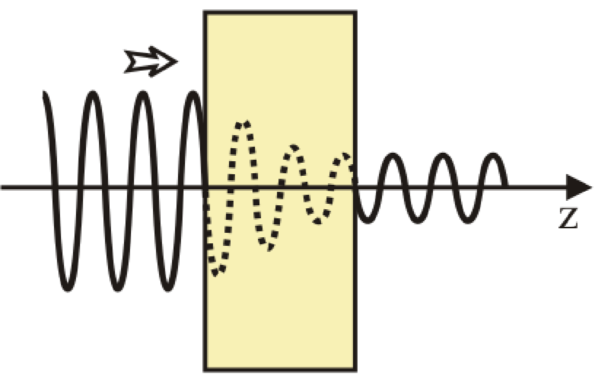
BK7	Value
B_1	1.03961212
B_2	0.231792344
B_3	1.01046945
C_1	$6.00069867 \times 10^{-3}$ μm^2
C_2	$2.00179144 \times 10^{-2}$ μm^2
C_3	1.03560653×10^{2} μm^2

http://refractiveindex.info/

Imaginary part of the refractive index and absorption

$$\left.\begin{aligned} \mathbf{E}(\mathbf{r},t) &= \mathbf{E}_o e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \\ \mathbf{k} &= \tilde{n}\,k_o\hat{\mathbf{n}} \\ \tilde{n} &= n + i\kappa \end{aligned}\right\} \Rightarrow \mathbf{E}(\mathbf{r},t) = \mathbf{E}_o e^{i(\tilde{n}k_o\hat{\mathbf{n}}\cdot\mathbf{r}-\omega t)} \Rightarrow$$

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_o e^{-\kappa k_o\hat{\mathbf{n}}\cdot\mathbf{r}} e^{i(nk_o\hat{\mathbf{n}}\cdot\mathbf{r}-\omega t)}$$



$$\hat{\mathbf{n}} = \hat{\mathbf{z}} \Rightarrow \mathbf{E}(z,t) = \mathbf{E}_o e^{-\kappa k_o z} e^{i(nk_o z-\omega t)} \Rightarrow$$

$$I(z) = I(0)e^{-2\kappa k_o z}$$

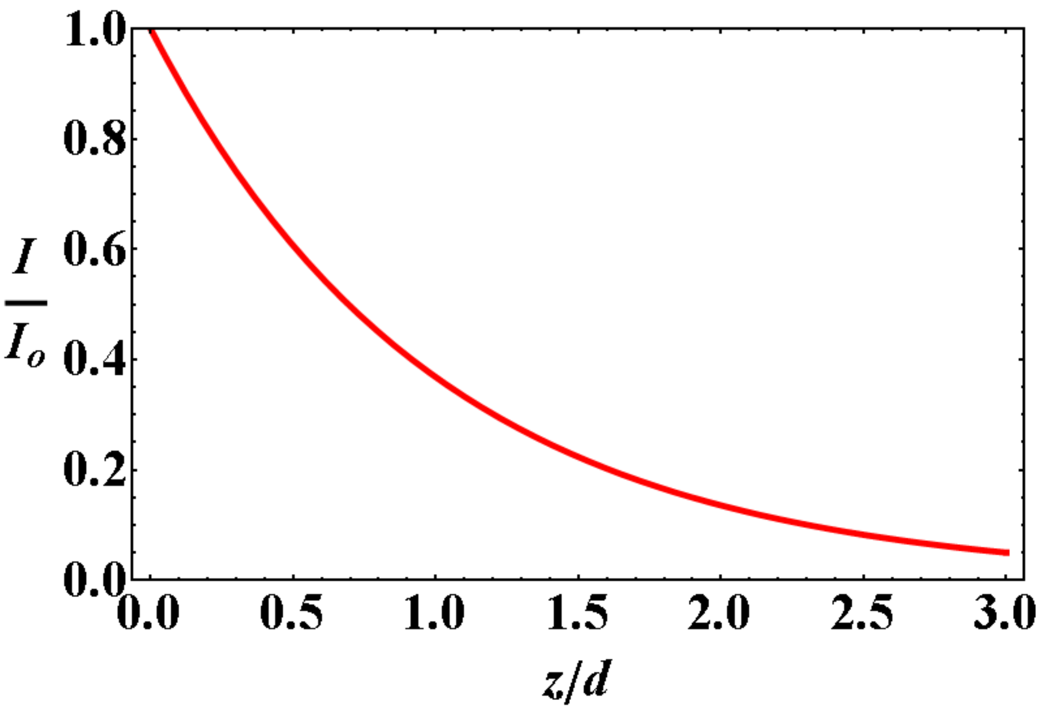
$$I(z) = I(0)e^{-az}$$

$$a = 2\kappa k_o = \frac{4\pi\kappa}{\lambda_o}\ (cm^{-1})$$

absorption coefficient

Penetration depth d:

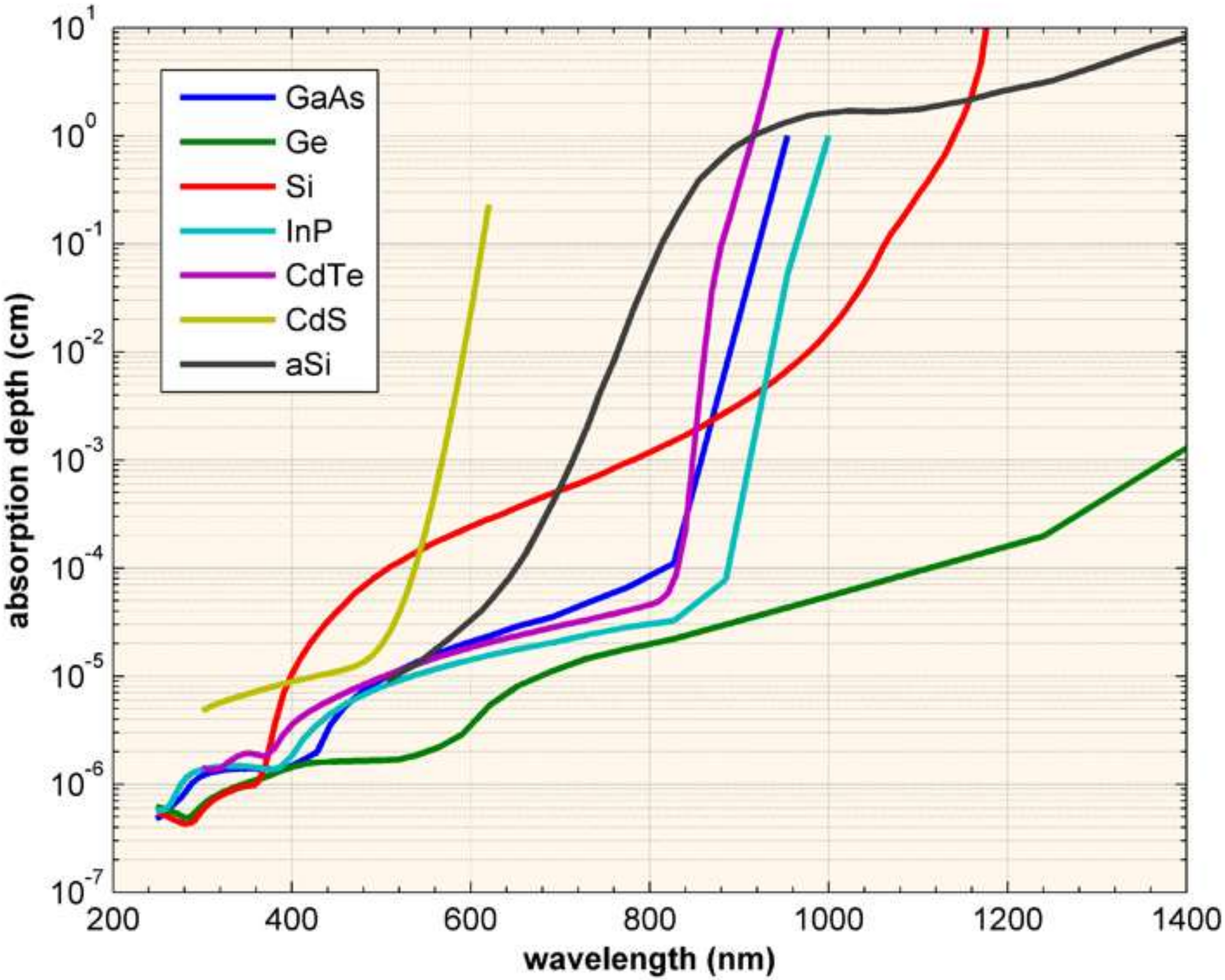
the depth at which the intensity of the radiation inside the material falls to 1/e (about 37%) of its original value just beneath the surface.



$$d \equiv \frac{1}{a} = \frac{\lambda_o}{4\pi\kappa}$$

$$water \ \ (@550\,nm) \qquad a \sim 5\cdot 10^{-4}\,cm^{-1} \Rightarrow d \sim 20\,m$$

$$Al \ \ (@550\,nm) \qquad a \sim 1.5\cdot 10^{6}\,cm^{-1} \Rightarrow d \sim 6.6\,nm$$



Source: http://pveducation.org

Polarization basics

Polarization of light

Polarization refers to the orientation of the electric field at a plane transverse to the propagation direction

Polarization degree

Non-polarized
(natural)

Partially polarized

Polarized

Polarization state

Linearly polarized

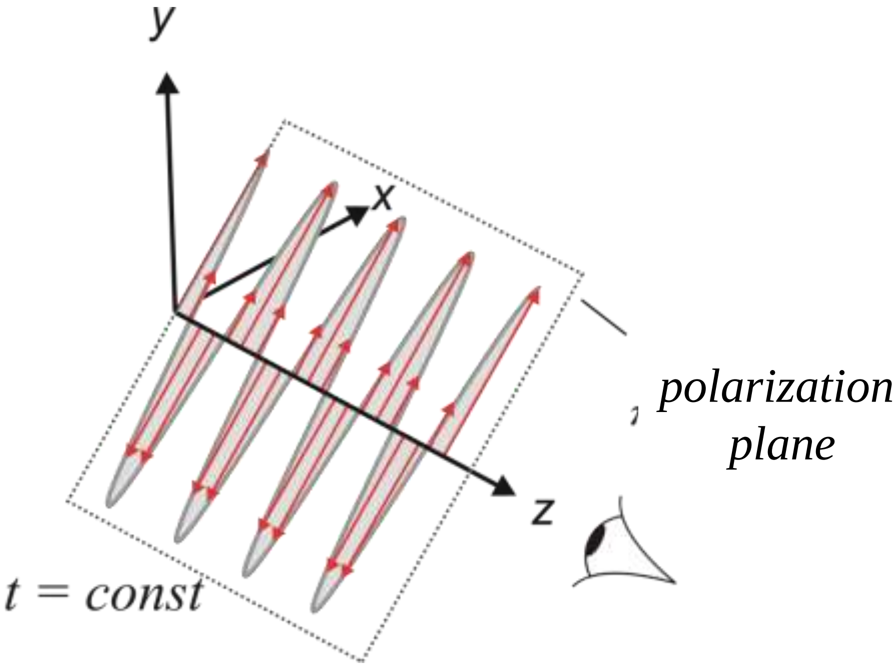
*The electric field **E** oscillates on a constant plane.*

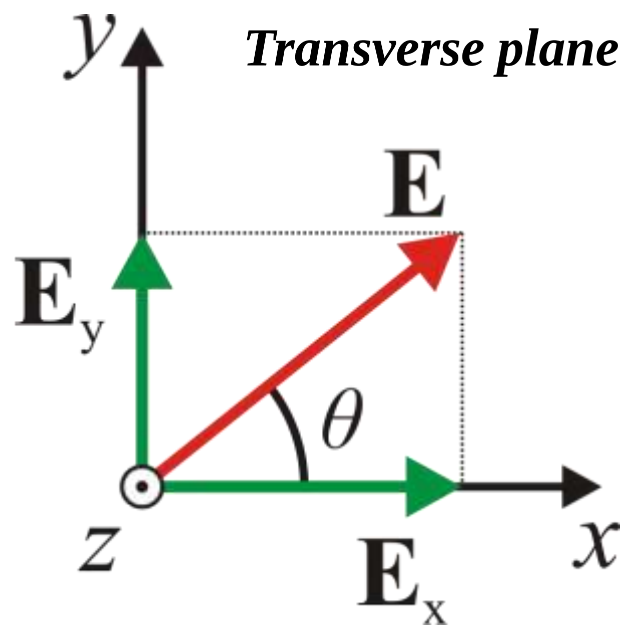
Elliptically polarized

*The electric field **E** rotates describing an ellipse*

Circularly polarized

Linear Polarization



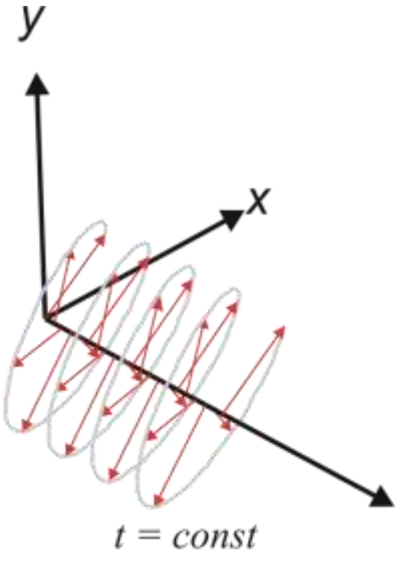


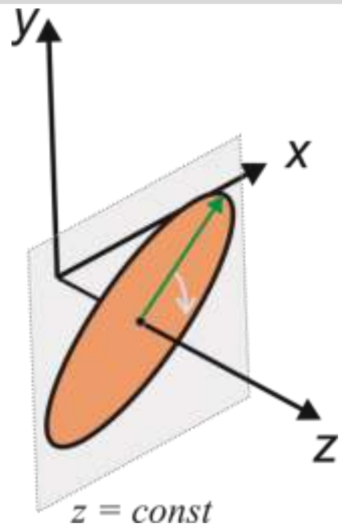
$$\mathbf{E} = \mathbf{E}_x + \mathbf{E}_y = (E_x^o \hat{\mathbf{x}} + E_y^o \hat{\mathbf{y}}) \cdot e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} =$$

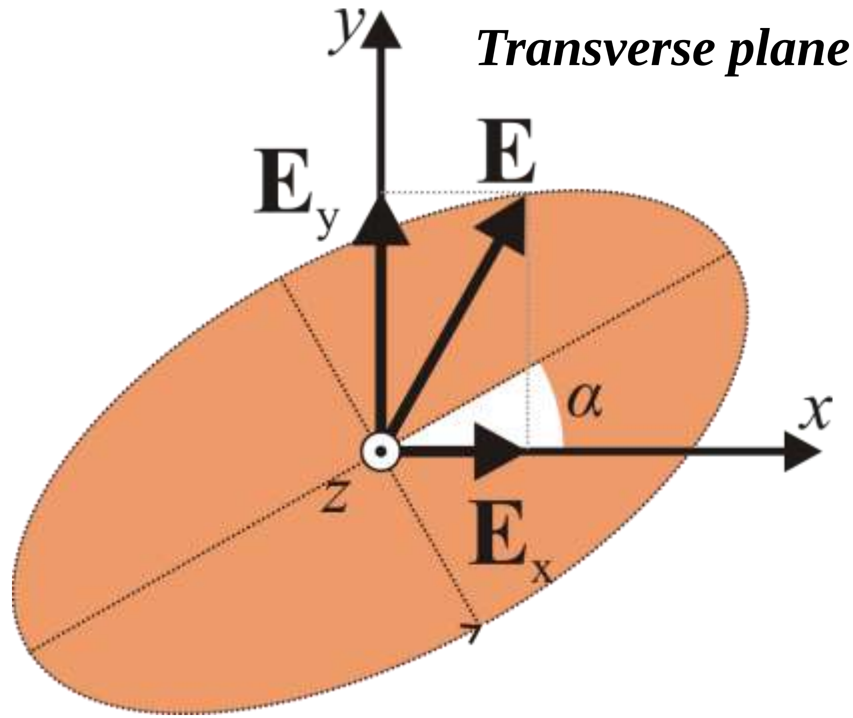
$$= E_o(\cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}}) \cdot e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

The polarization plane is the plane defined by **E**, ***z***

Elliptical polarization





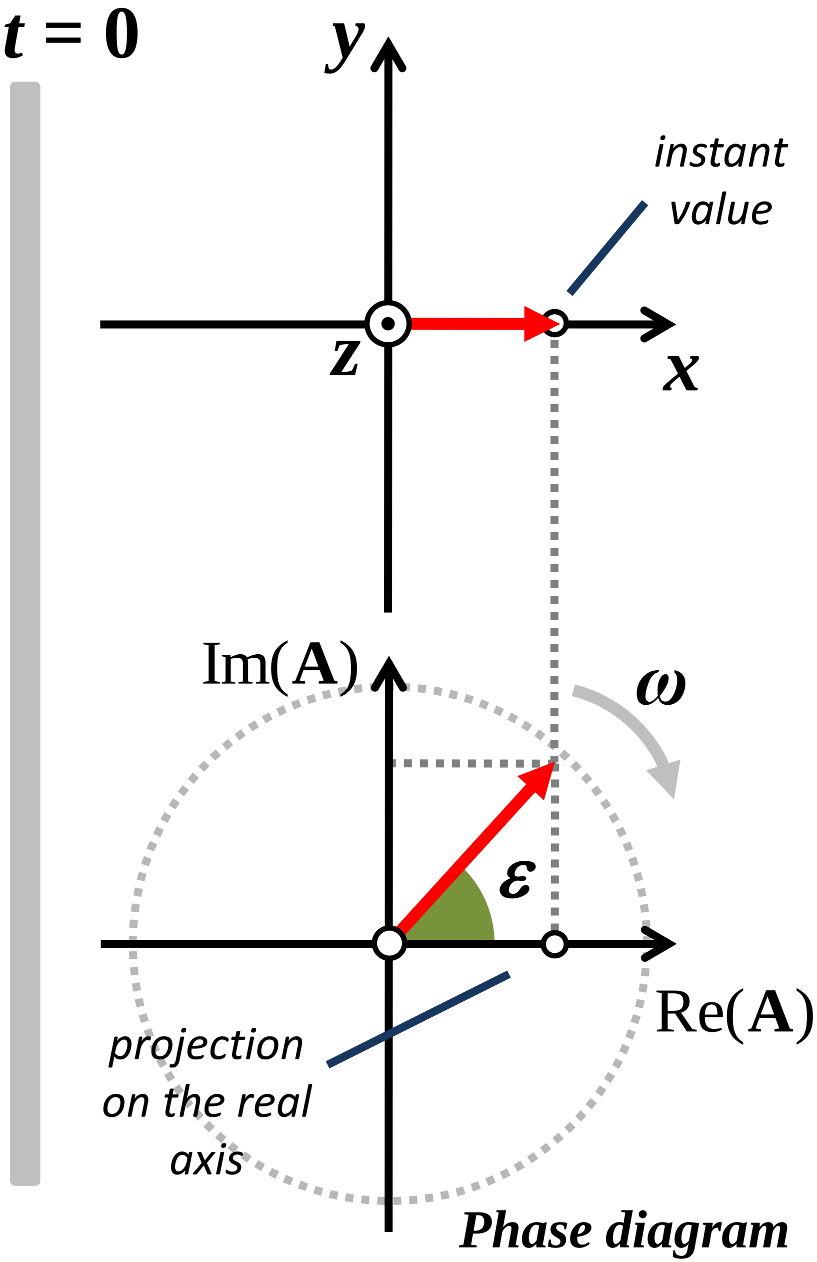


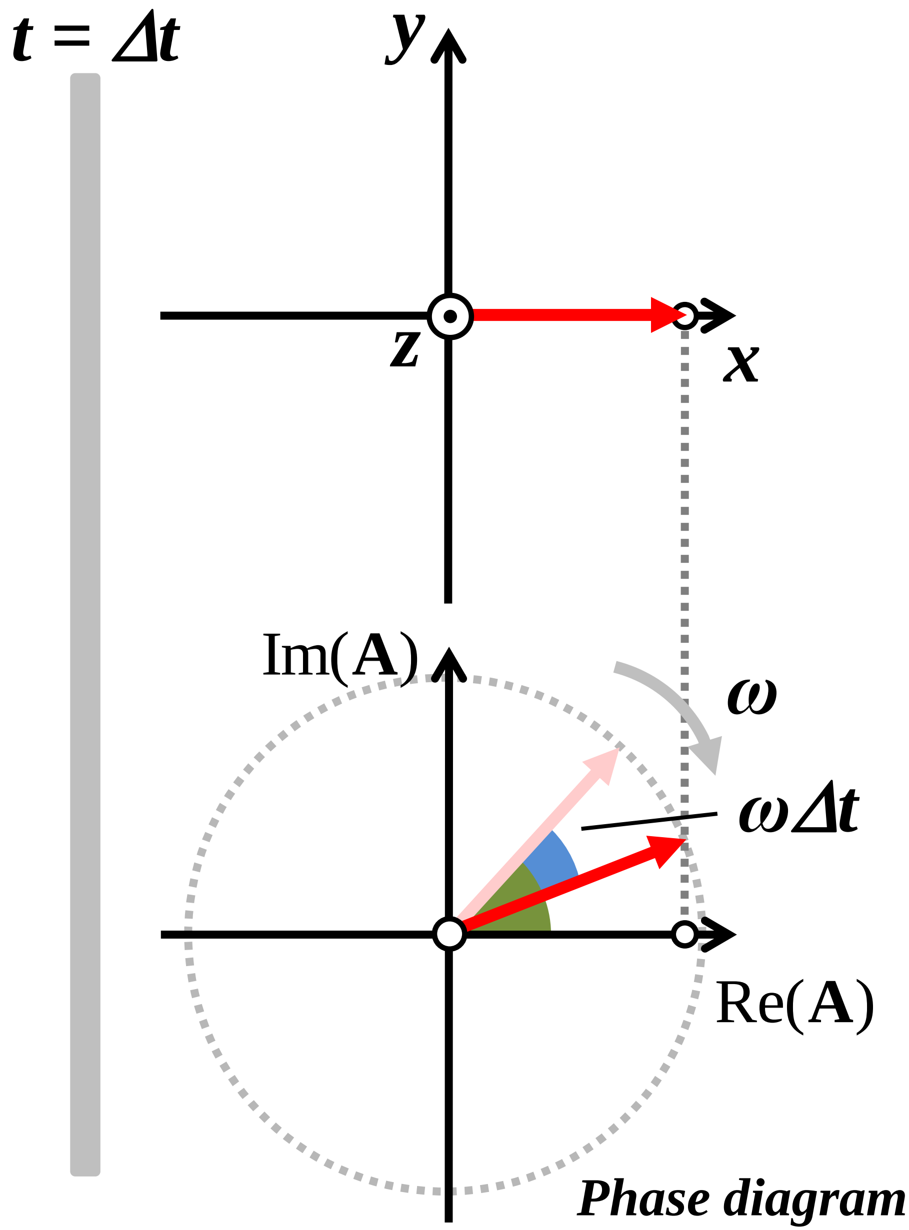
$$\left.\begin{aligned}\mathbf{E}_x &= E_x^o e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\hat{\mathbf{x}} \\ \mathbf{E}_y &= E_y^o e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t+\varepsilon)}\hat{\mathbf{y}}\end{aligned}\right\} \Rightarrow \mathbf{E} = (E_x^o\hat{\mathbf{x}} + E_y^o e^{i\varepsilon}\hat{\mathbf{y}})e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

The field vector $\mathbf{E}$ describes an ellipse in the transverse plane

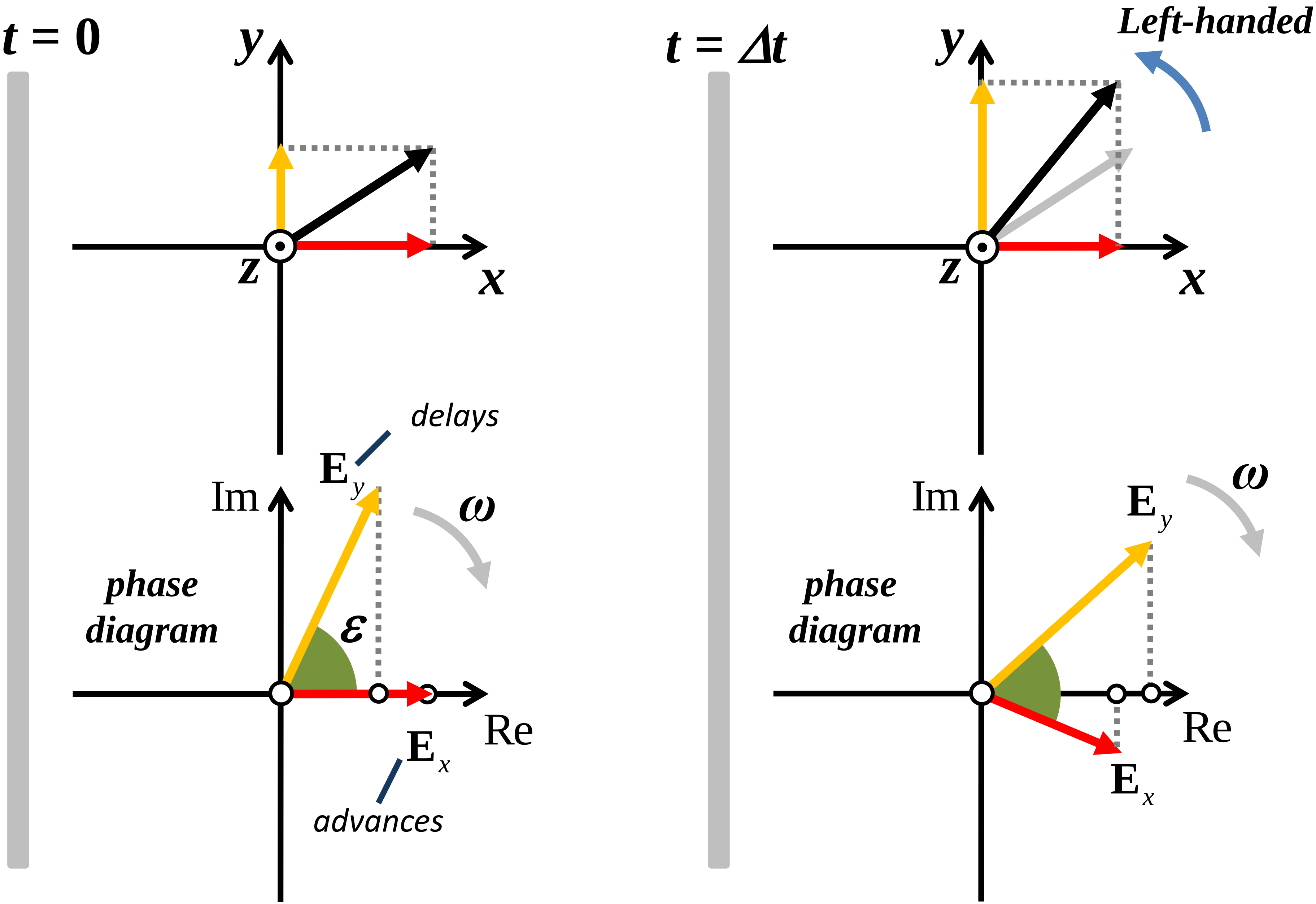
rotation handedness : Phase diagram

$$\mathbf{A} = A_o \hat{\mathbf{x}} e^{-i\omega t} e^{i\varepsilon}$$

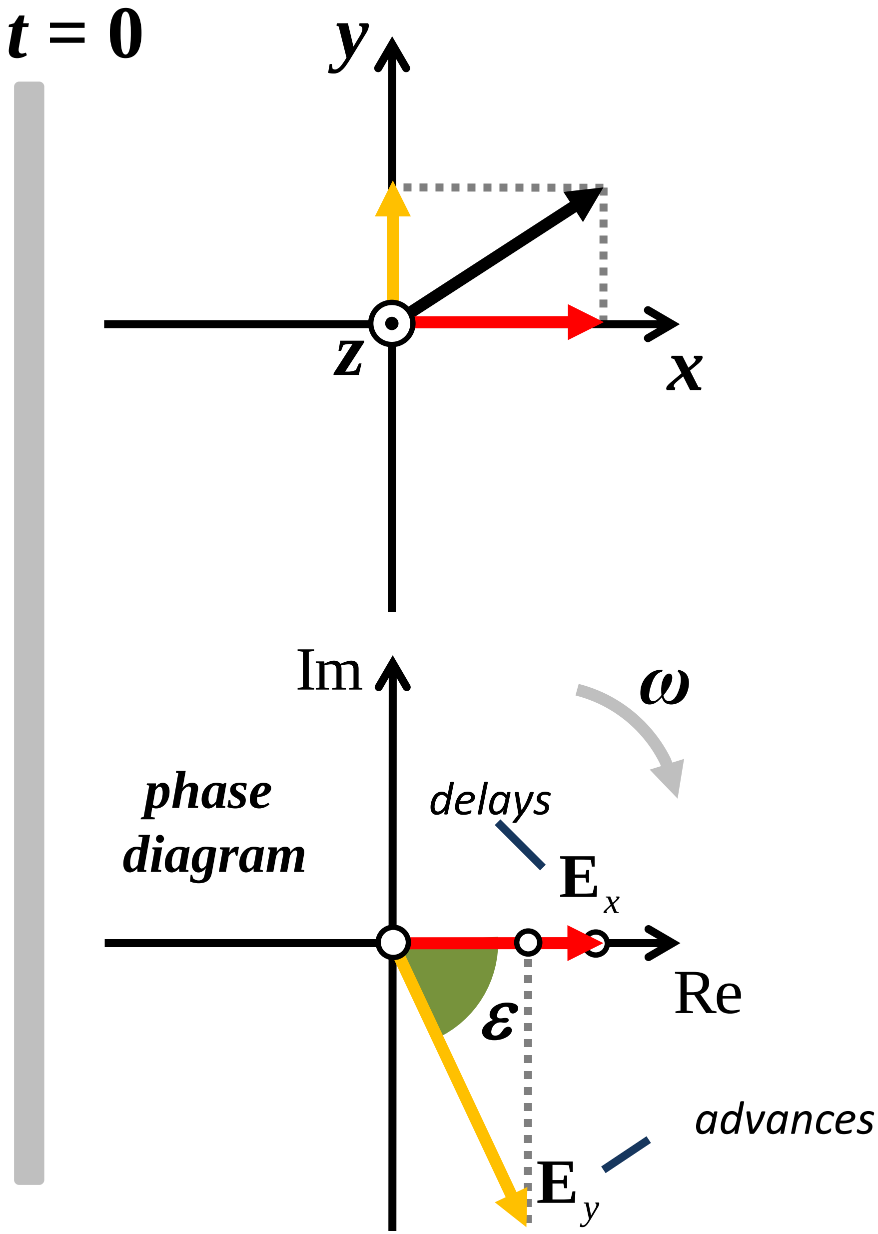


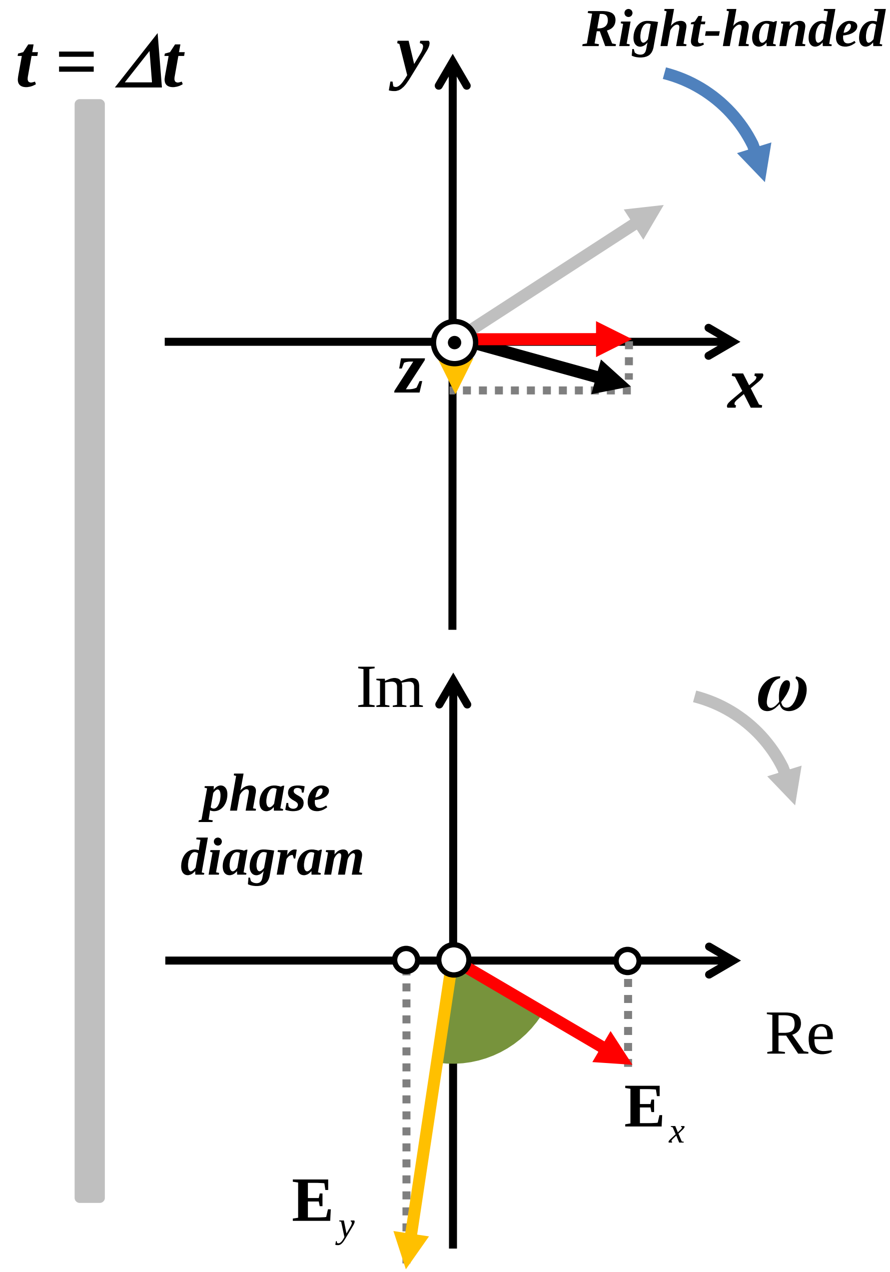


$\sin\varepsilon > 0$



$\sin \varepsilon < 0$





Circularly polarized light

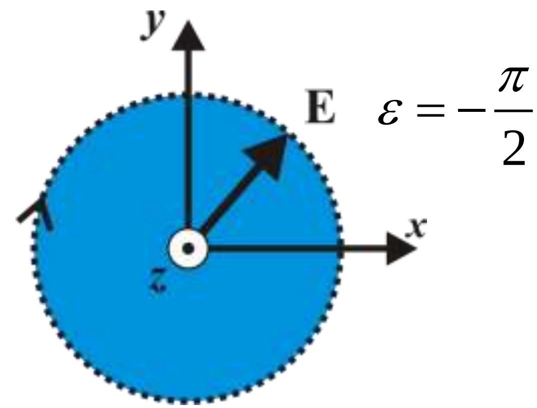
Special case of elliptically polarized:

$$E_x^o = E_y^o = E_o, \quad \varepsilon = \pm\frac{\pi}{2}$$

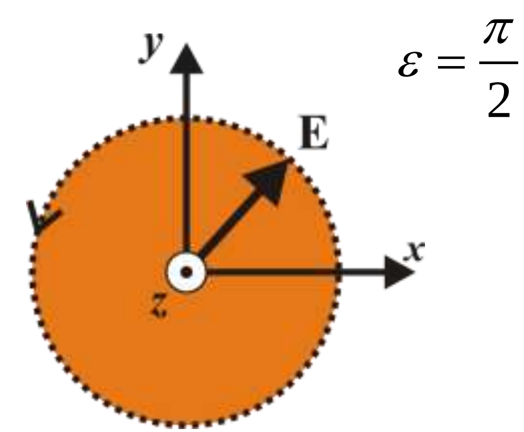
$$\left.\begin{aligned} \mathbf{E}_x &= E_o e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\hat{\mathbf{x}}, \\ \mathbf{E}_y &= E_o e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t\pm\frac{\pi}{2})}\hat{\mathbf{y}} \end{aligned}\right\} \Rightarrow$$

$$\mathbf{E} = E_o(\hat{\mathbf{x}} + e^{\pm i\frac{\pi}{2}}\hat{\mathbf{y}})e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$= E_o(\hat{\mathbf{x}} \pm i\,\hat{\mathbf{y}})e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$



right-handed circularly polarized

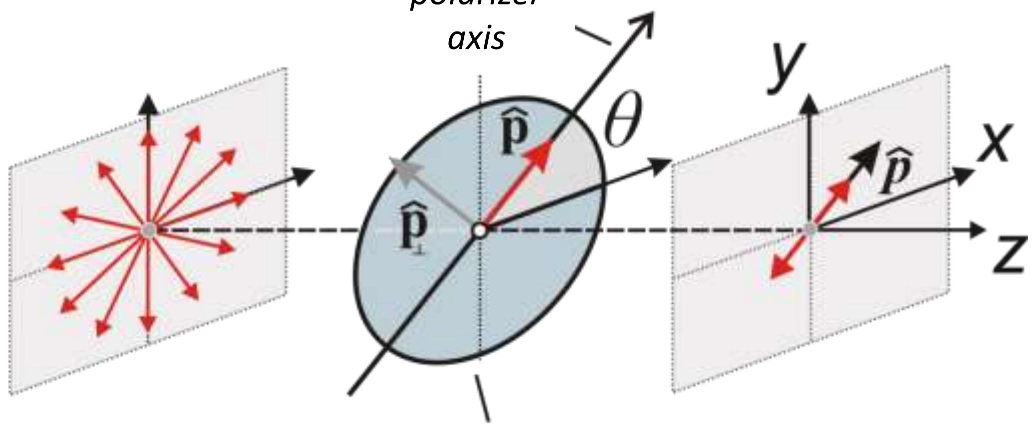


left-handed circularly polarized

The vector $\mathbf{E}$ describes a circle on the transverse plane

Polarizer

An optical element that selectively absorbs the $\mathbf{E}$ field component that is vertical to its axis.



$$\mathbf{E}_{out} = a_{//}(\mathbf{E}_{in} \cdot \hat{\mathbf{p}})\hat{\mathbf{p}} + a_\perp(\mathbf{E}_{in} \cdot \hat{\mathbf{p}}_\perp)\hat{\mathbf{p}}_\perp$$

component that propagates through

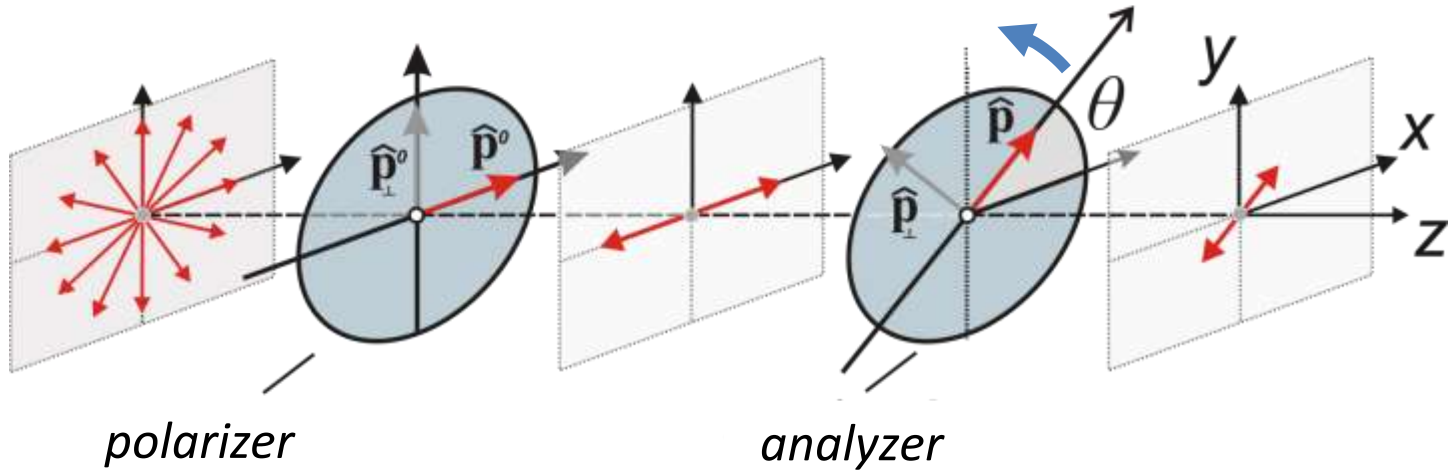
leak

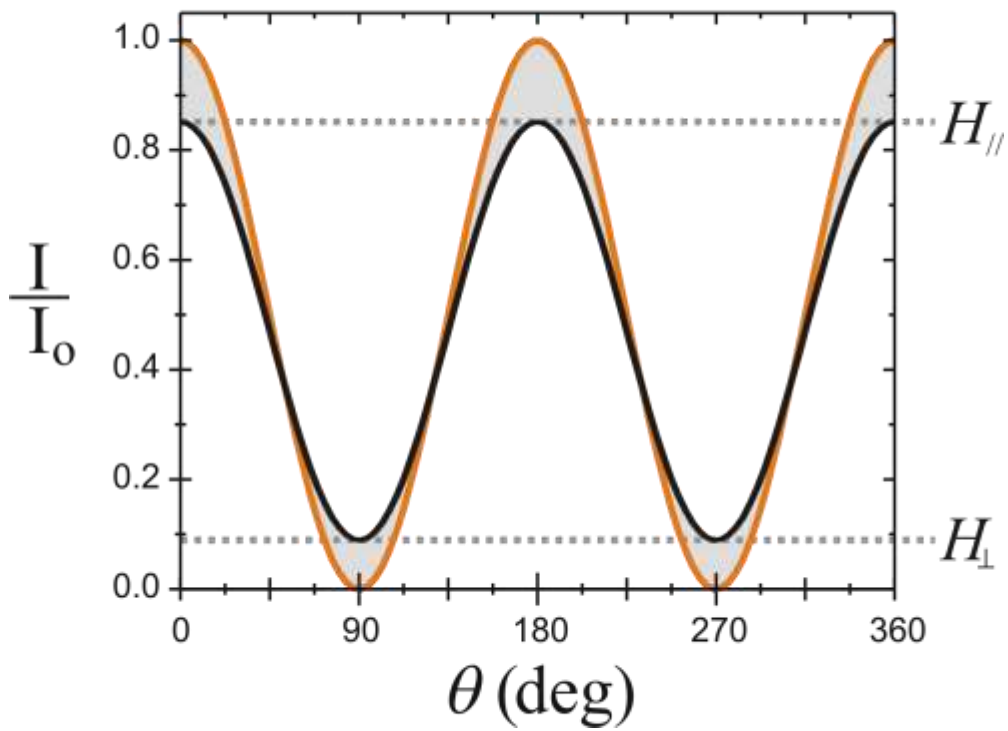
$$a_{//} < 1$$
$$a_\perp \ll a_{//}$$

typical values

Ideal polarizer:

$$\left.\begin{array}{l} a_{//} = 1 \\ a_\perp = 0 \end{array}\right\} \Rightarrow \mathbf{E}_{out} = (\mathbf{E}_{in} \cdot \hat{\mathbf{p}})\hat{\mathbf{p}}$$





Malus Law

$$I(\theta) = I_o \cos^2 \theta$$

Ideal polarizer

Non-ideal polarizer

$$I(\theta) = I_o(H_\perp + H_{//} \cos^2 \theta)$$