

# Foundations of Modern Optics

Dimitris Papazoglou

[dpapa@materials.uoc.gr](mailto:dpapa@materials.uoc.gr)

Materials Science and Technology Department  
University of Crete



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The background image shows a vast sky filled with various types of clouds. In the foreground, there are large, dark, cumulus-like clouds on both the left and right sides, with their undersides glowing a bright yellow-orange from the low-angle sunlight. Above them, the sky is a deep blue, with wispy cirrus clouds scattered across it.

4

# Detection of radiation

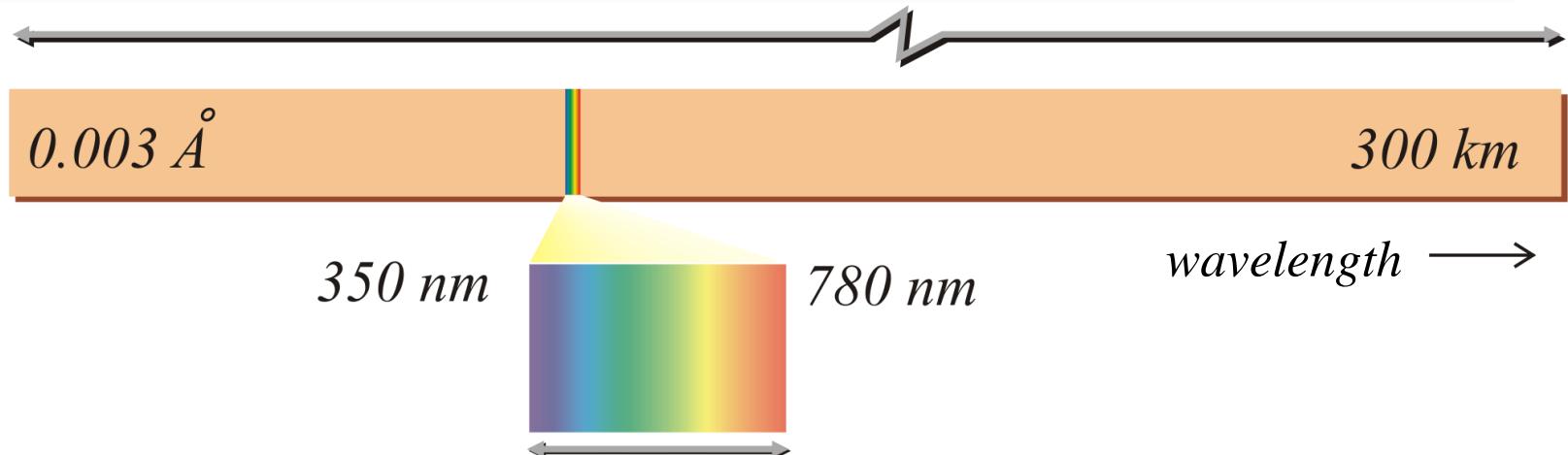
**4.1**

## **Photometry - Radiometry**

# Photometry - Radiometry

**Radiometry:**

*refers to the whole electromagnetic spectrum*



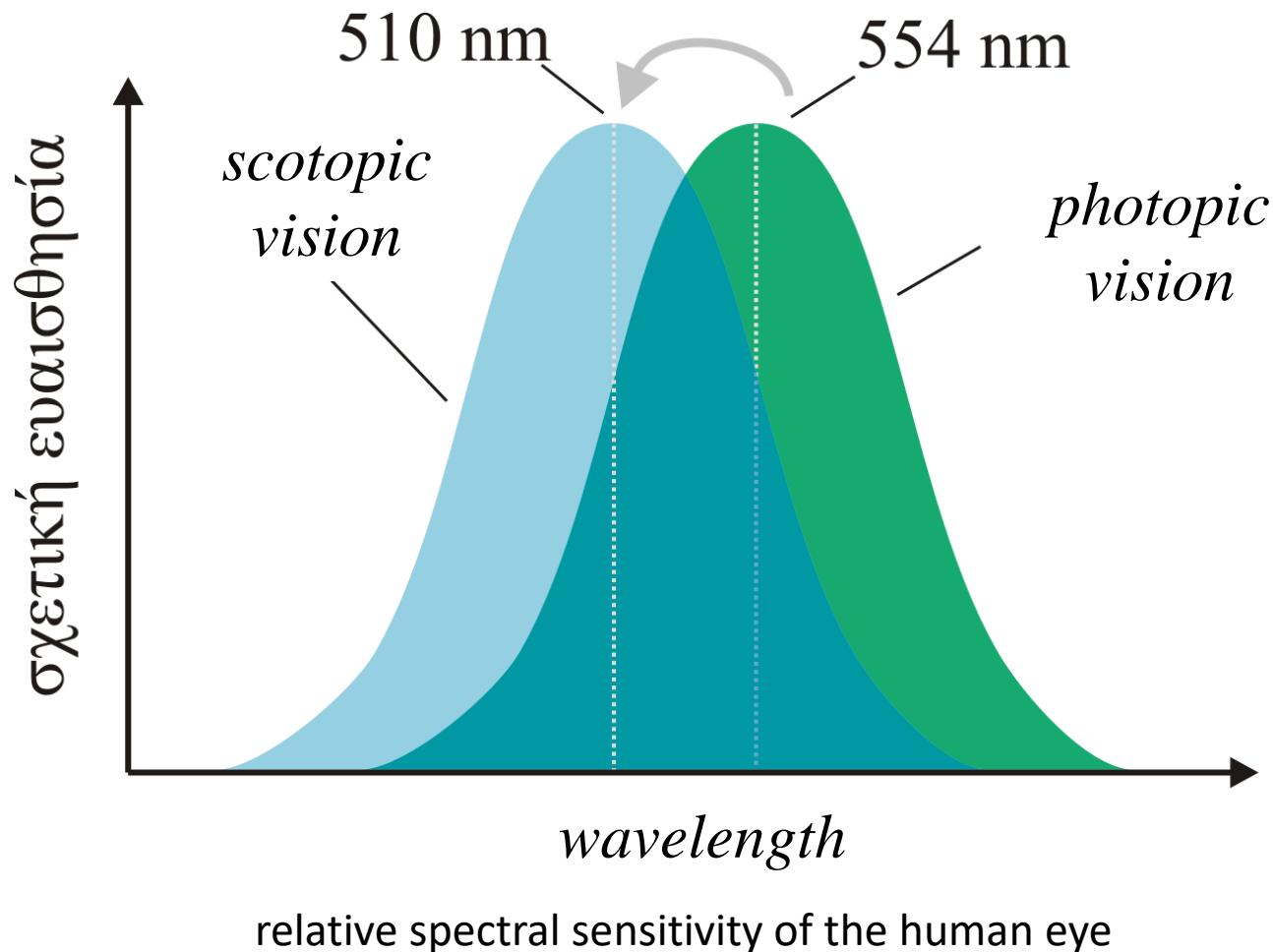
**Photometry:**

*refers only to the visible spectrum*

*In photometry the reference photodetector  
is the **human eye***



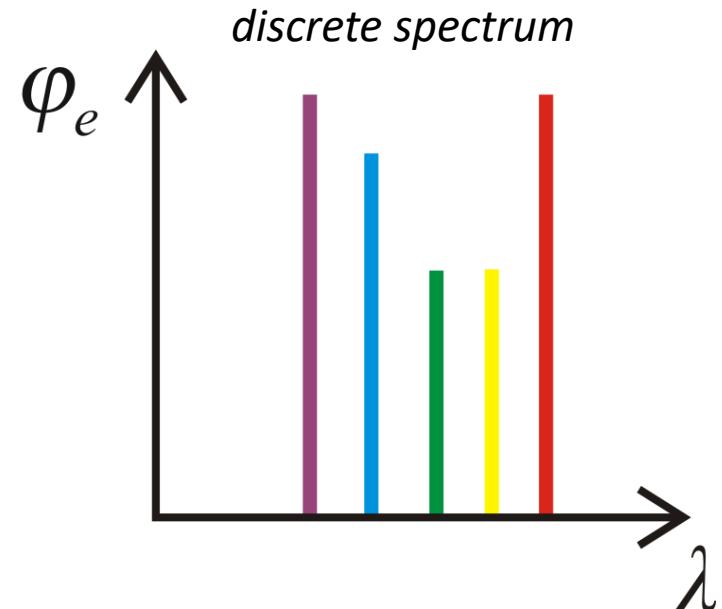
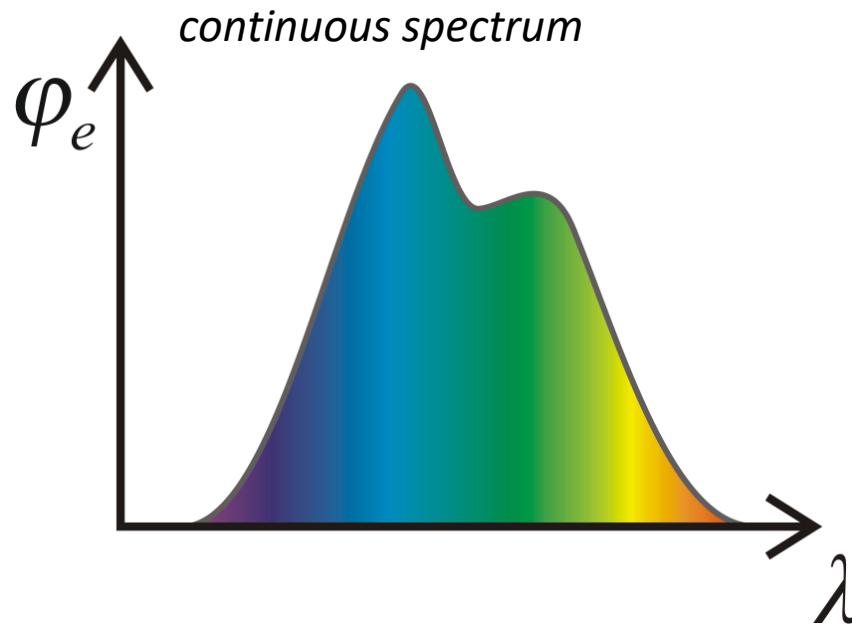
# Spectral sensitivity of the human vision



The relative spectral sensitivity of the **photopic vision** is used as a reference for the calibration of photometric detectors

# Typical emission spectra

A light source can emit a continuous or a discrete spectrum



emitted power

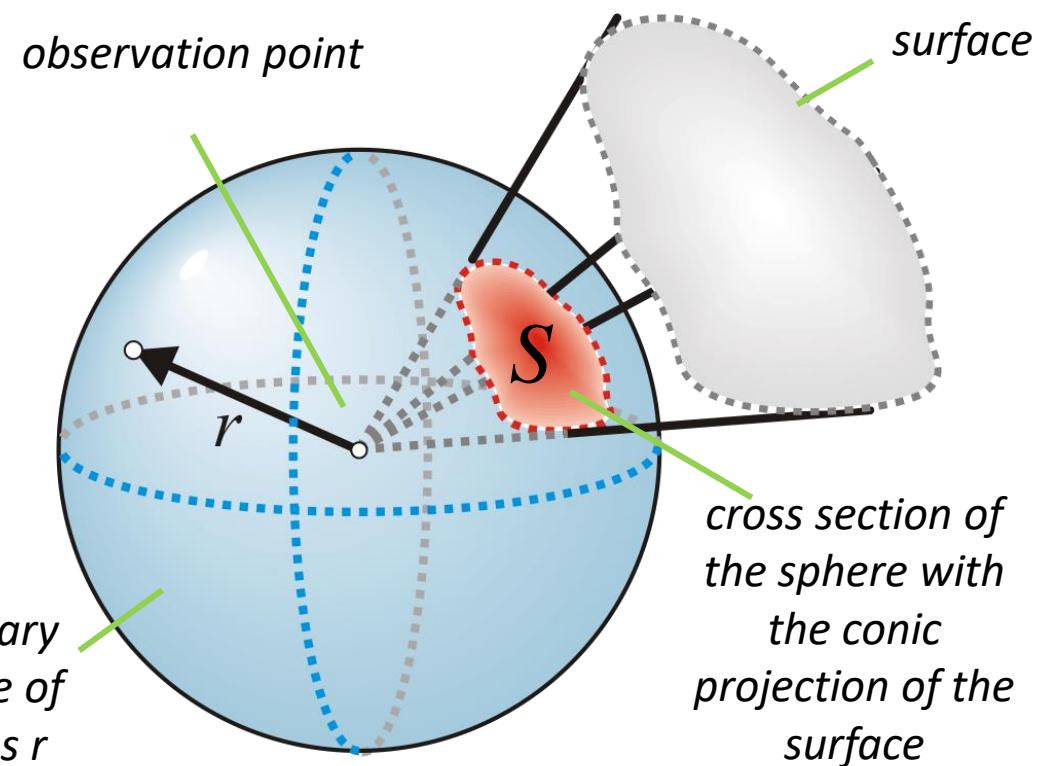
$$\Phi_e = \int_{\lambda_1}^{\lambda_2} \varphi_e(\lambda) d\lambda$$

spectral distribution of  
the emitted power

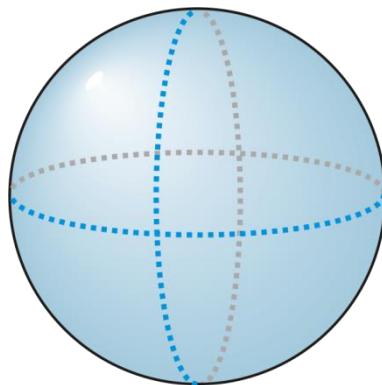
# Solid angle

$$\Omega = \frac{S}{r^2}$$

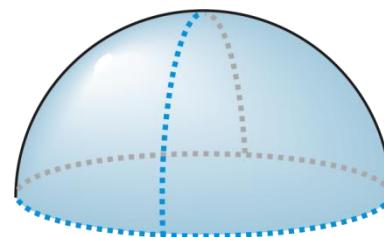
units 1 sr (steradian)



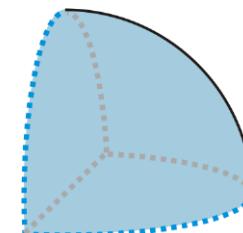
$4\pi$



$2\pi$

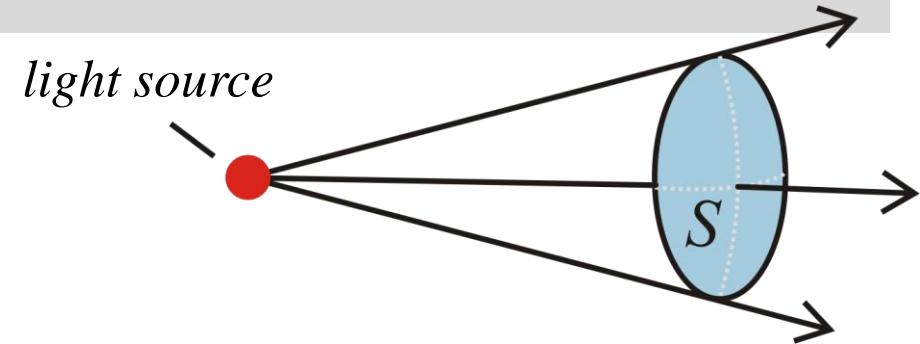


$\pi$



# Flux

expresses the **flow of energy per time unit** through a surface  $S$



*Photometric units*

**Luminous flux**

$$\Phi = \frac{dQ}{dt}$$

**lm (lumen)**

*Radiometric units*

**Radiant flux**

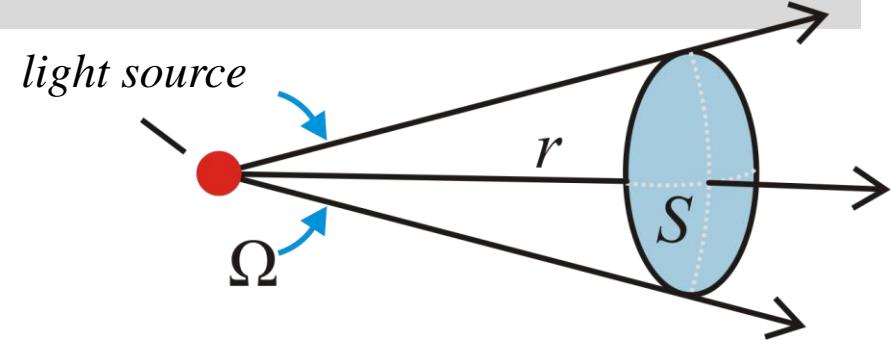
$$\Phi_e = \frac{dQ_e}{dt}$$

**W (Watt)**

*Radiant flux of 1 W of monochromatic radiation at the peak of the eye sensitivity (554 nm) is equivalent to 680 lm*

# Intensity

it expresses the ratio of the **radiant flux** through a surface  $S$  per solid angle



*Photometric units*

**Luminous intensity**

$$I = \frac{d\Phi}{d\Omega}$$

**cd (candela)**

*Radiometric units*

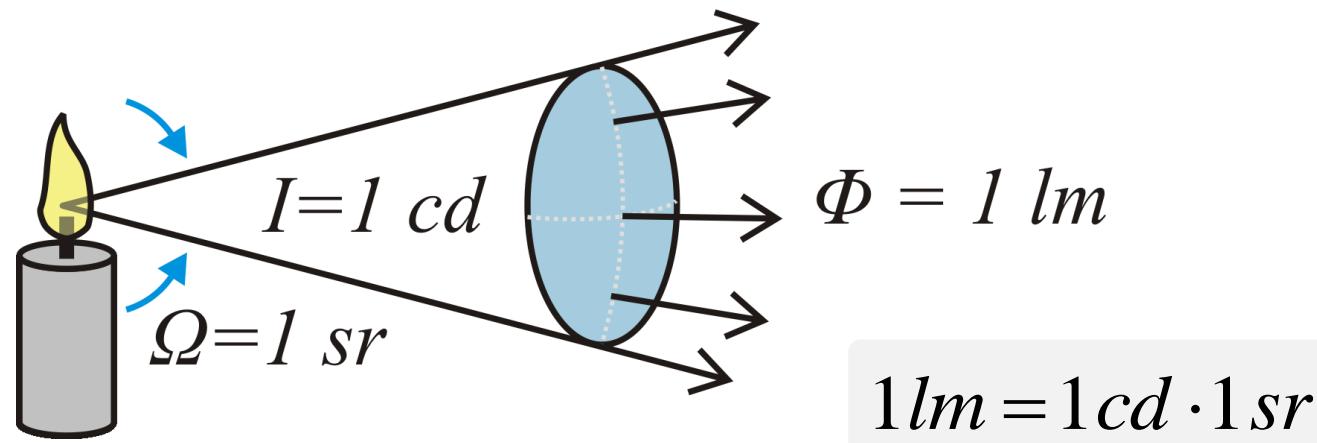
**Radiant intensity**

$$I_e = \frac{d\Phi_e}{d\Omega}$$

**W/sr**

1 cd is defined as the 1/60 of the optical power emitted from a  $1 \text{ cm}^2$  surface of Pt at melting temperature

# Relation between candelas and lumens



## Typical Luminous intensity values

candle	$\sim 1 \text{ cd}$
Incandescent lamp 40 W	40 cd
Incandescent lamp 100 W	130 cd
arc lamp	2000 cd

# Emittance

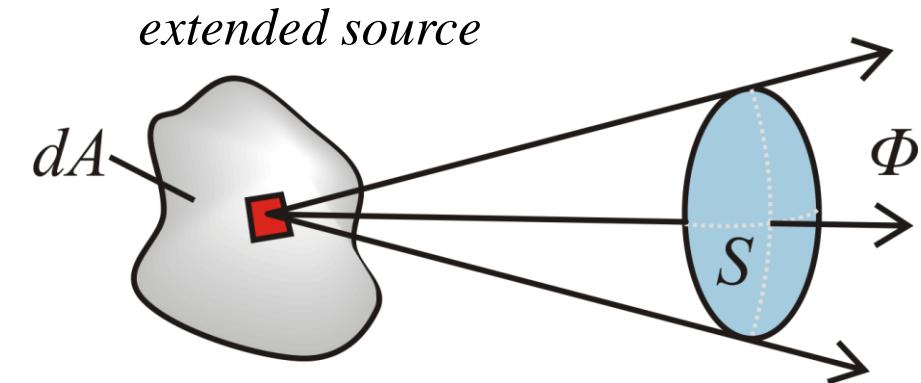
expresses the flow of **radiation per surface area** of the source

*Photometric units*

**Luminous emittance**

$$M = \frac{d\Phi}{dA}$$

**$lm/m^2$**



*Radiometric units*

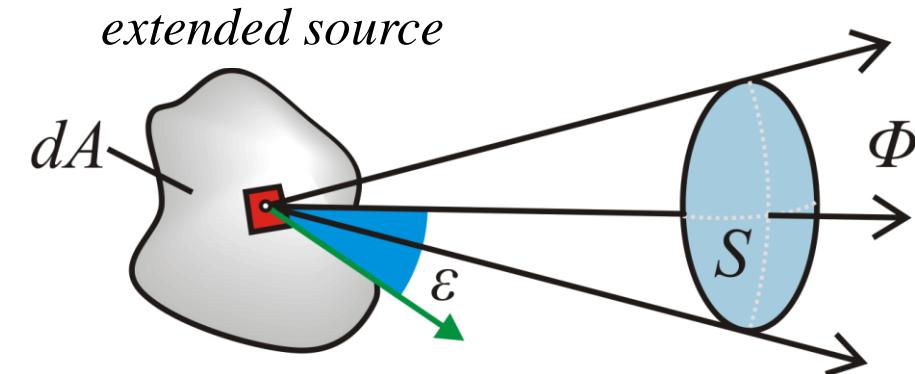
**Radiant emittance**

$$M_e = \frac{d\Phi_e}{dA}$$

**$W/m^2$**

# Radiance

expresses the **intensity of the light source per surface area** for a **specific angle to the surface normal**



*Photometric units*

**Luminance**

$$L = \frac{dI}{dA \cos \varepsilon}$$

**cd/m<sup>2</sup>**

*Radiometric units*

**Radiance**

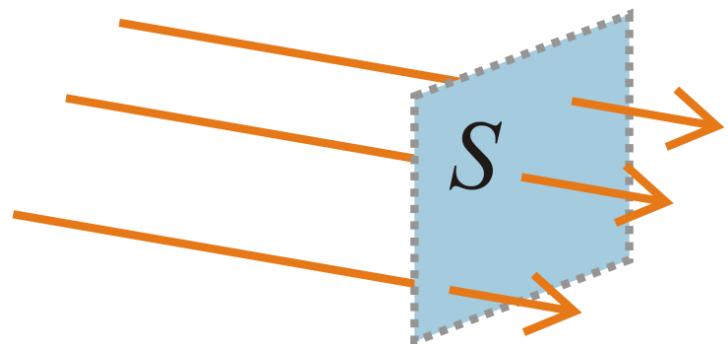
$$L_e = \frac{dI_e}{dA \cos \varepsilon}$$

**W/(sr·m<sup>2</sup>)**

A perfectly diffusing surface emitter exhibits constant Luminance and is called **Lambertian**

# Illumination

expresses the flow of **radiation** through a surface  $S$  per surface area



*Photometric units*

***Illumination***

$$E = \frac{d\Phi}{dS}$$

***lx (lux)***

$$1\text{ lux} = 1\text{ lumen} / \text{m}^2$$



Photometer

*Radiometric units*

***Irradiance***

$$I = \frac{d\Phi_e}{dS}$$

***W/m<sup>2</sup>***

By Thomas Wydra  
[CC BY-SA 3.0 de], Wikimedia Commons

## Typical illumination values

**Illumination from the stars**  $\sim 10^{-4} \text{ lx}$

**Illumination from the full moon**  $0.2 \text{ lx}$

**Illumination of a street at night**  $20 \text{ lx}$

**Typical room illumination**  $100 \text{ lx}$

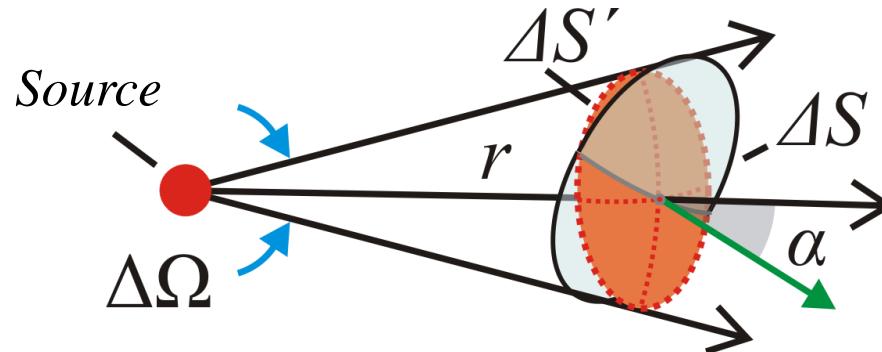
**Illumination for reading**  $500 \text{ lx}$

**Illumination for fine work**  $2000 - 5000 \text{ lx}$

**Typical shiny day**  $\sim 10^4 \text{ lx}$

**Direct solar illumination**  $\sim 10^5 \text{ lx}$

# Basic principles of photometry



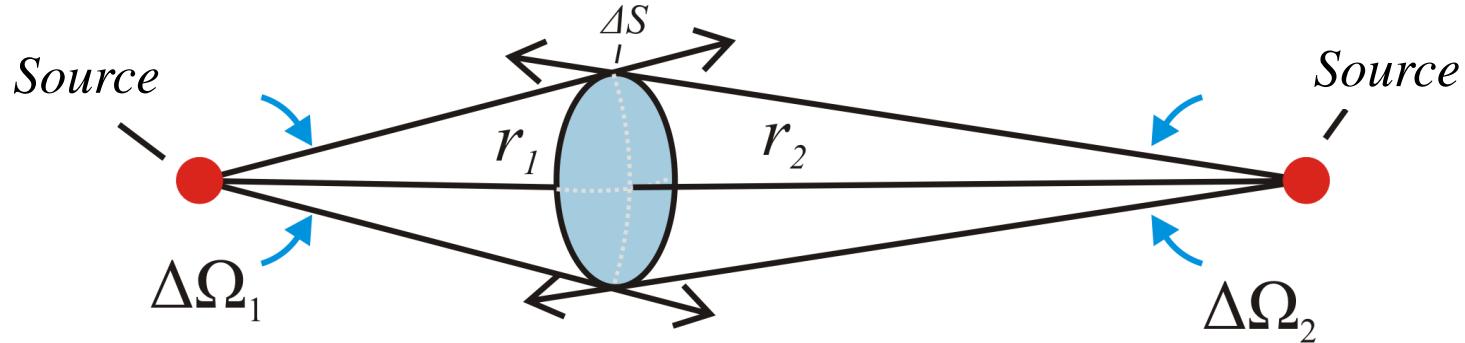
*light source  
intensity*

*illumination*

$$\left. \begin{aligned} E &\equiv \frac{\Delta\Phi}{\Delta S} = \frac{I \Delta\Omega}{\Delta S} \\ \Delta\Omega &\equiv \frac{\Delta S'}{r^2} = \frac{\Delta S \cos \alpha}{r^2} \end{aligned} \right\} \Rightarrow E = \frac{I \cos \alpha}{r^2}$$

*solid angle*

**law of illumination**



$$E_1 = E_2 \Leftrightarrow \frac{I_1}{I_2} = \frac{r_1^2}{r_2^2}$$

*condition for equal illumination*

By using this condition we can measure the intensity of an unknown light source by comparing it to the intensity of a candle

$$I_x = I_{candle} \cdot \frac{r_x^2}{r_{candle}^2} \cong 1 \text{ cd} \cdot \frac{r_x^2}{r_{candle}^2}$$

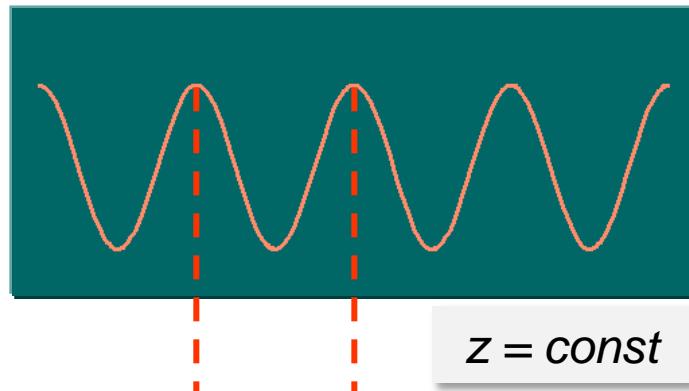
# 4.2

## Detectors



# How fast is light's oscillation ?

The oscillation of the light field is much faster  
from our detector's response time !



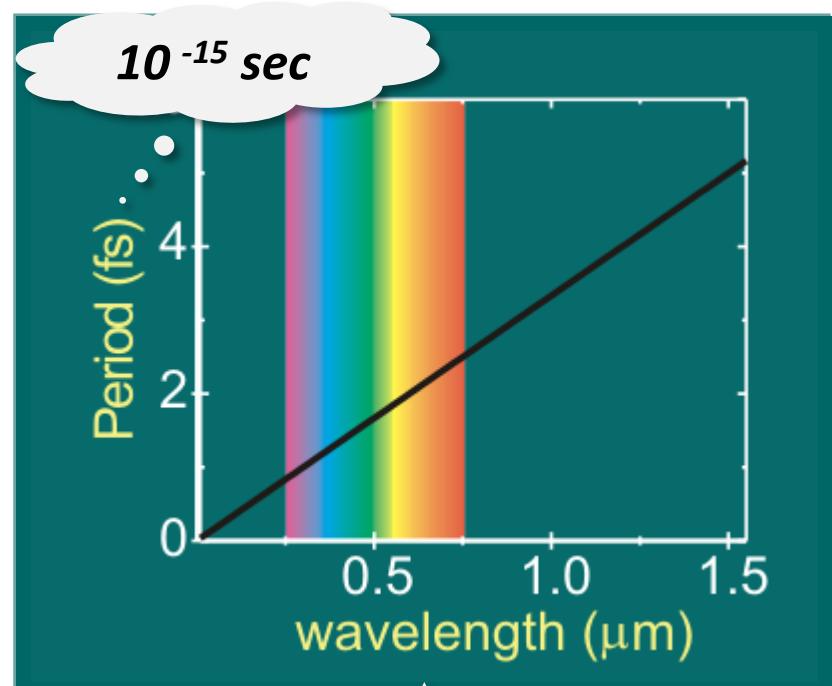
1 period ~ 2 fs

$10^{-15} \text{ sec}$

Period (fs)

4  
2  
0

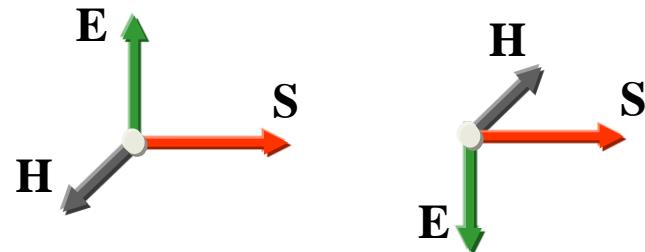
0.5 1.0 1.5  
wavelength ( $\mu\text{m}$ )



# The averaging action

The energy transferred by the field  
is expressed by the **Poynting vector**

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

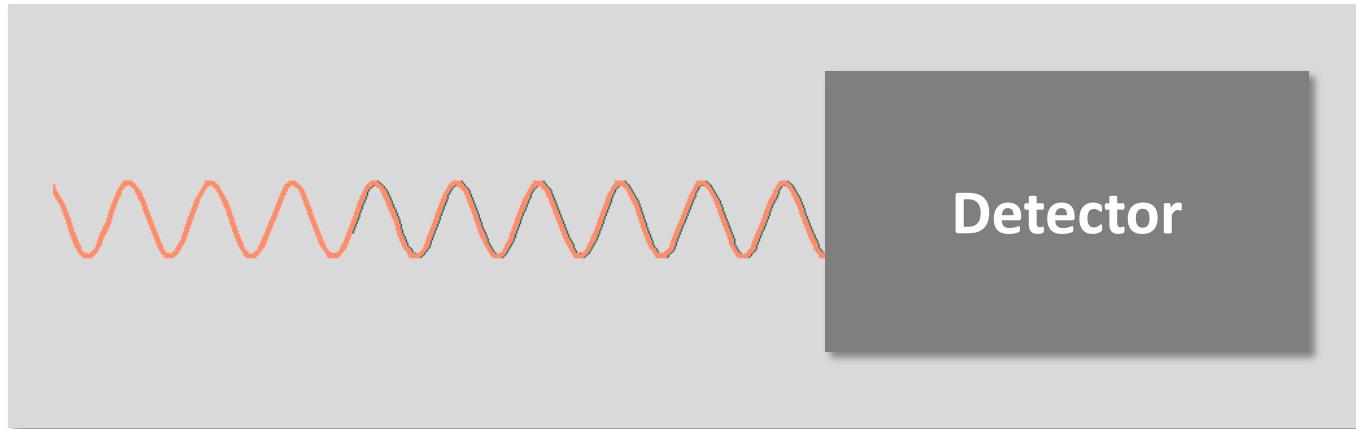


$\mathbf{S}$  oscillates from zero to maximum  
at **twice the frequency** of the light field !

$$I \equiv \langle |\mathbf{S}| \rangle_t = n \varepsilon_0 c \langle |\mathbf{E}|^2 \rangle_t \quad (\text{W/m}^2)$$

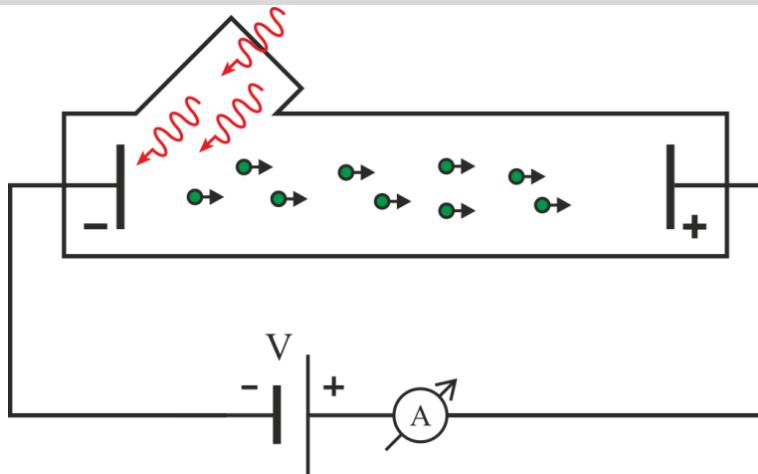
Intensity

# Detecting H/M radiation



By direct measurement we are actually detecting the  
**average energy transferred from the light field**  
**over many oscillation periods**

# Photoelectric effect



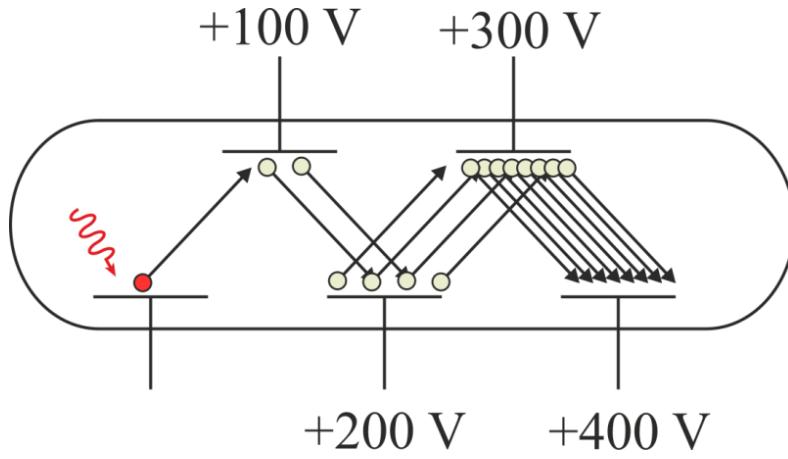
No photoelectrons are emitted,  
for a given metal, **below a minimum frequency  $f$**  of incident radiation.

The **time lag** between the incidence of radiation and the emission of a photoelectron **is very small**, much less than a  $nsec$ .

The maximum kinetic energy of the photoelectrons emitted **depends on the frequency** and is **independent of the intensity** of the incident light.

light consists of particles of energy  $hf$

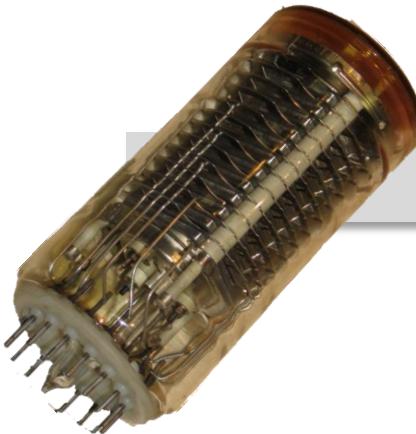
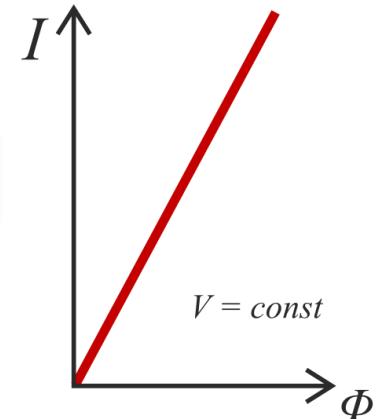
# Typical light detectors: Photomultipliers



Sensitivity  $\sim 100 \text{ mA/W}$

Current amplification  $\sim 10^6$

Operation at UV and VIS



Photomultiplier

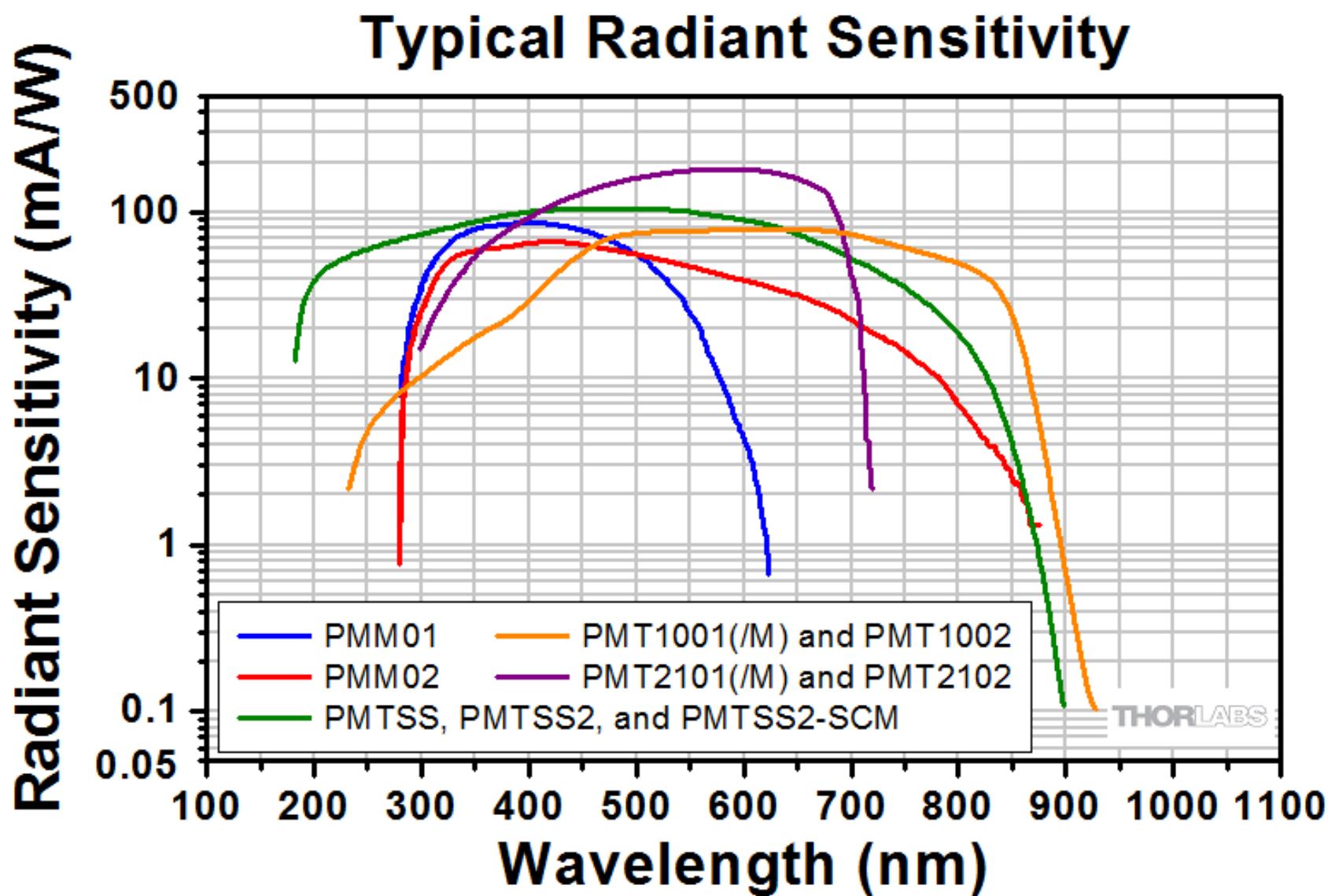
By Poi  
[CC BY], Wikimedia Commons

The most sensitive light detectors !

photomultiplier  
HAMAMATSU R446

By ScAvenger  
[CC BY-SA 4.0], Wikimedia  
Commons





photomultiplier spectral response

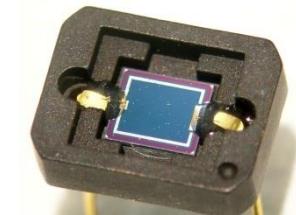
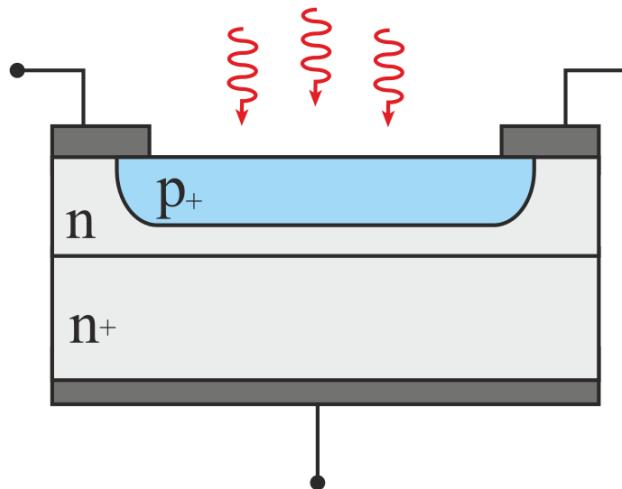
Thorlabs ([www.tohrlabs.com](http://www.tohrlabs.com))

# Typical light detectors: Photodiodes

Sensitivity  $\sim 100 \text{ } \mu\text{A/W}$

Operation at UV, VIS and IR

Pulsed operation up to 10 GHz



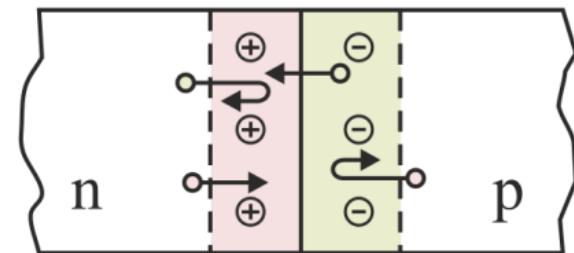
photodiode

By John Maushammer  
[CC BY-SA 2.5], Wikimedia Commons

Photodiodes are based on the operation of **p-n semiconductor junctions**.

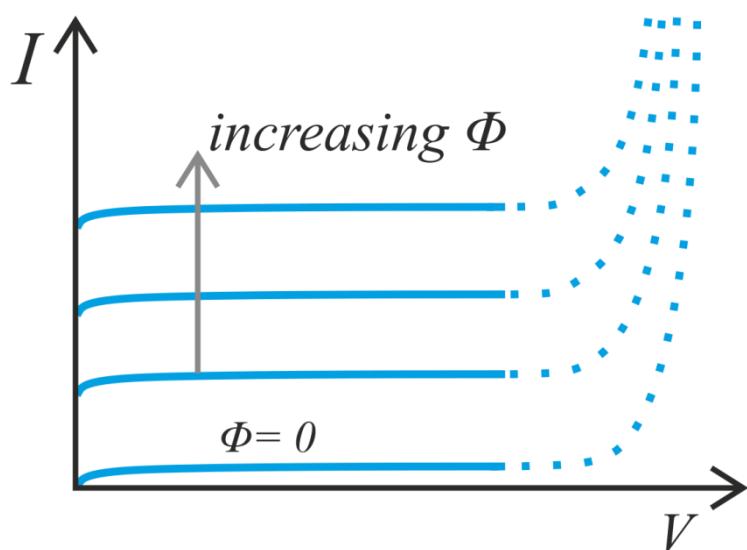
When a photon of sufficient energy strikes the diode, it creates an **electron-hole pair**.

In the depletion region electron and holes move in opposite directions and a photocurrent is produced.

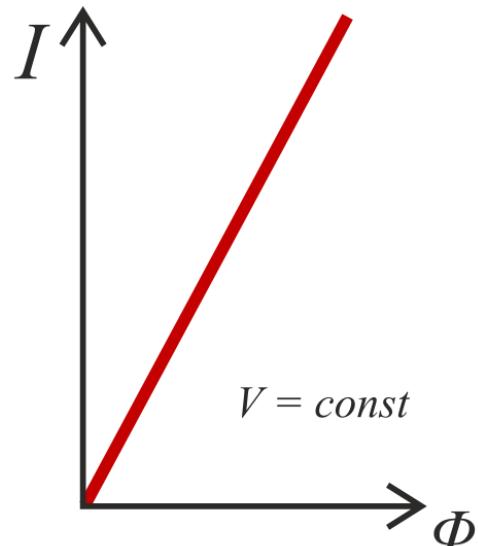


photodiode operation principle

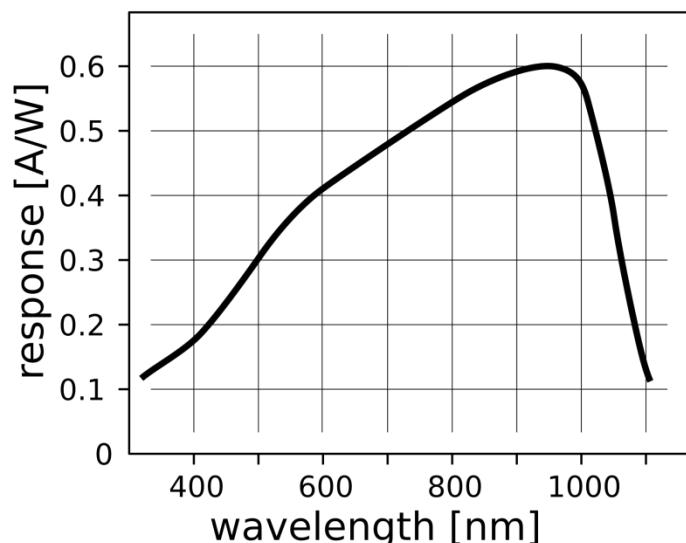
Commonly used, low cost and fast!



I-V characteristic of a photodiode



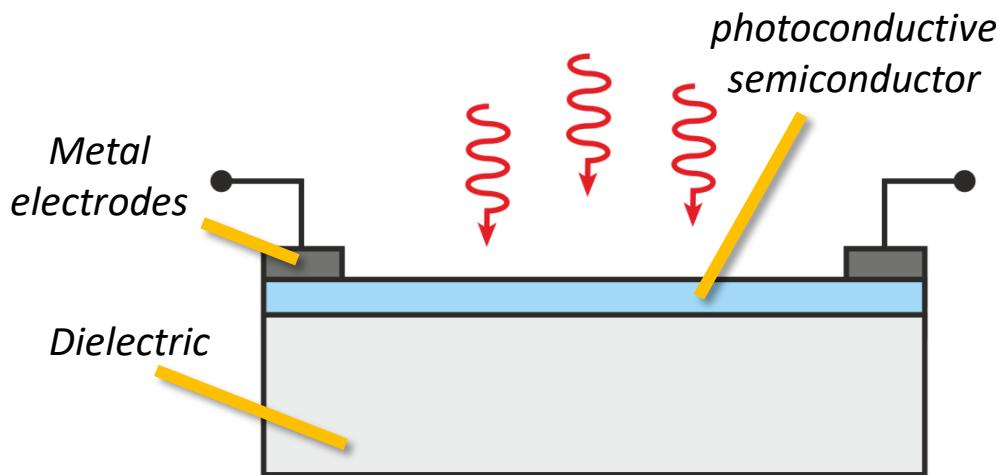
I-Φ characteristic of a photodiode



Spectral Response  
of a silicon photodiode

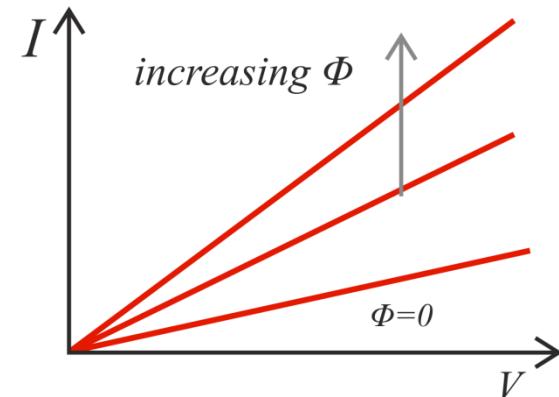
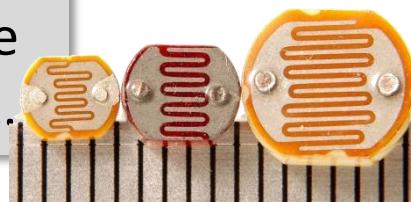
By KaiMartin  
[CC BY-SA 3.0], Wikimedia Commons

# Typical light detectors: Photoresistors

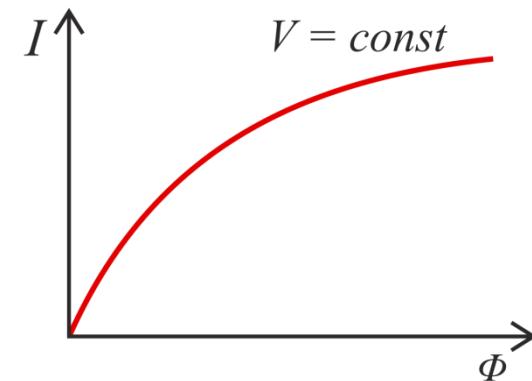


Photoresistors are composed of high resistance semiconductors.

In the dark, their resistance is **several MΩ**, while in the light, it drops **below 1 kΩ**.



I-V characteristic of a photoresistor



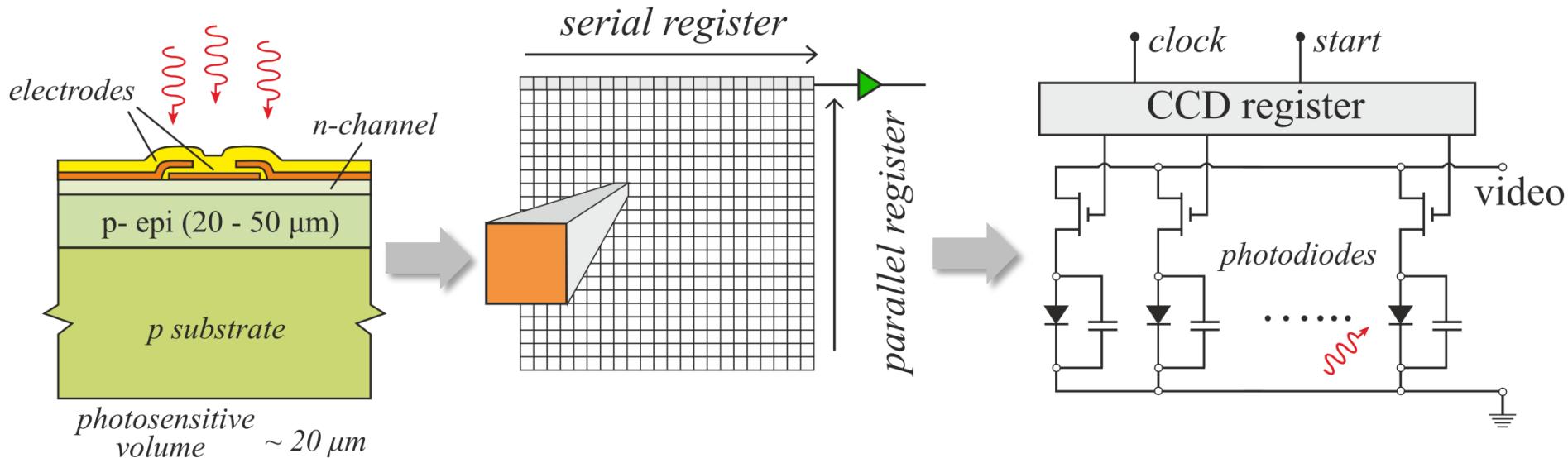
I-Φ characteristic of a photoresistor

photoresistors

By Junkyardsparkle, [CC0], Wikimedia Commons

Very low cost, and slow.

# Typical light detectors: Charge-coupled devices (CCDs)



CCDs are composed of a **photosensitive region** (array of photodiodes) , and a **transmission region** made out of a shift register.

Parallel processing

Operation up to 100 kHz

Operation at VIS and NIR

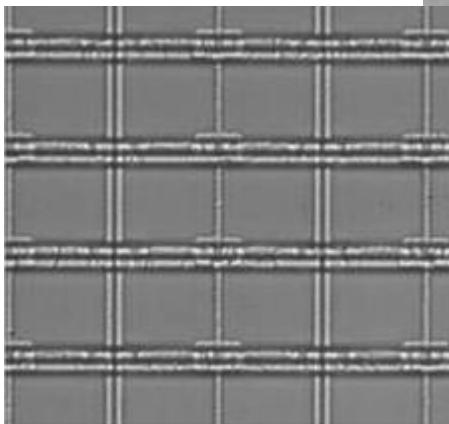
Can provide 2D image sampling !

CCD SONY  
ICX493AQA  
10.14 Mpixels  
APS-C 1.8"



By Andrzej w k 2  
[CC BY-SA 4.0], Wikimedia Commons

SEM image

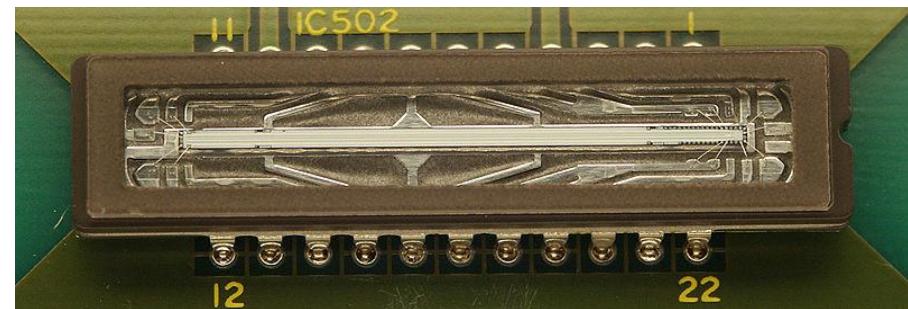


CCD



## CCD vs. CMOS sensors

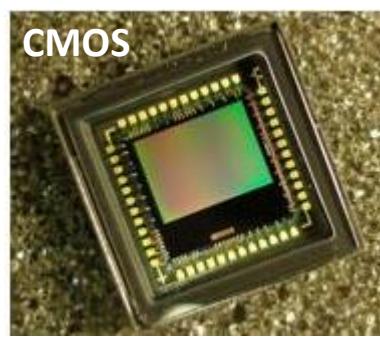
adapted work from image created by Master dpo  
[CC BY-SA 3.0], Wikimedia Commons



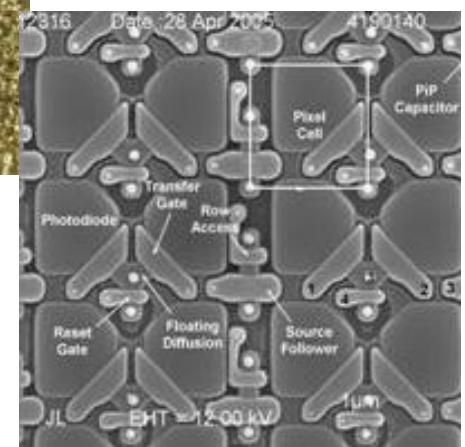
CCD line sensor in a ceramic dual in-line package,  
soldered on a printed circuit board.

By Stefan506  
[CC BY-SA 3.0], Wikimedia Commons

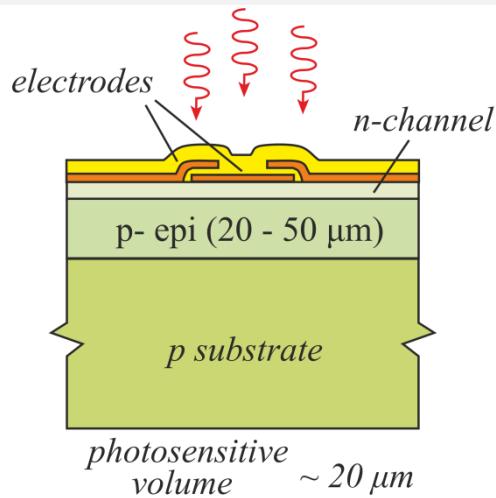
CMOS



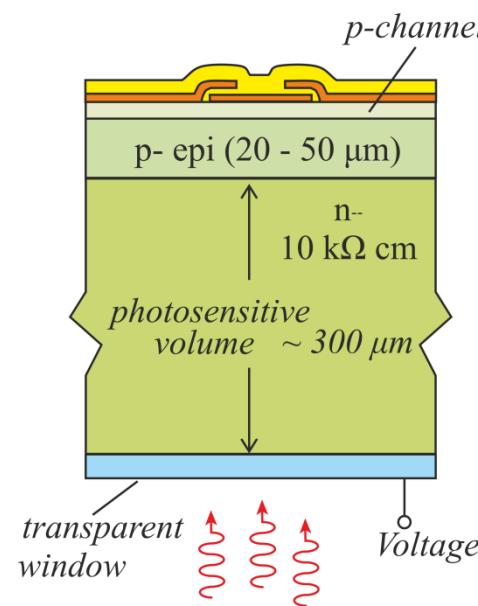
SEM image



## typical CCD pixel

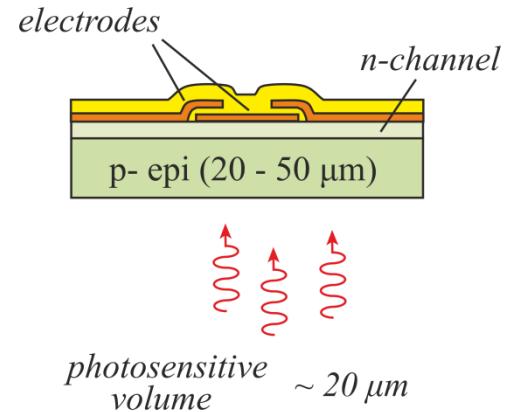


## Back illuminated



- low response in blue due to absorption from the gate electrodes
- low response in IR due to the low thickness of the epitaxial layer
- fringes due to the gate structure

## Thinned, back illuminated



- low cost (we avoid thinning)
- good response in IR (up to 1  $\mu\text{m}$ )
- transparent window with antireflective coating for good response in the blue

- thinning is expensive
- low response in IR due to the low thickness of the epitaxial layer
- fringes due to the gate structure

# **Color CCD sensors**

Color CCD sensors usually use a **Bayer mask** over the CCD.

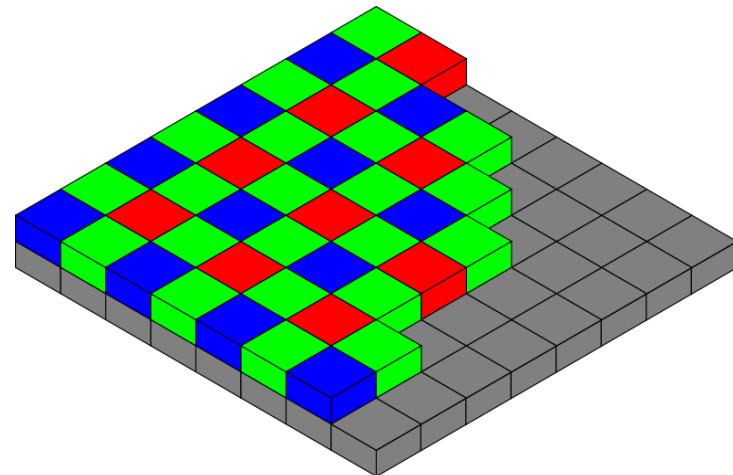
For each 2x2 square of pixels one is filtered red, one blue, and two green

Luminance information is collected at every pixel.

Color resolution is 4 times lower than the luminance resolution.

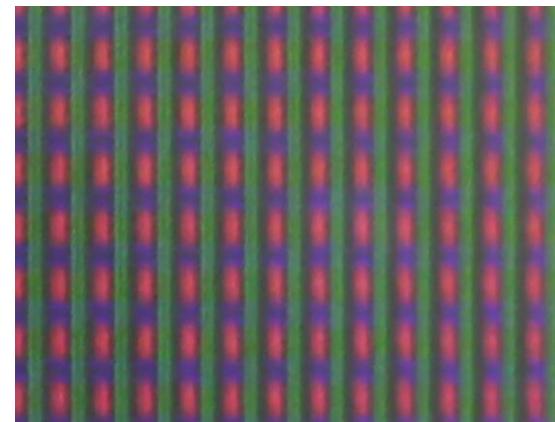
A x80 microscope view of the RGGB Bayer color filter on a 1980's 240 line resolution Sony PAL Camcorder CCD

*By Binarysequence  
[CC BY-SA 3.0], Wikimedia Commons*



A Bayer filter on a CCD sensor

*By Cburnett  
[CC BY-SA 3.0], Wikimedia Commons*





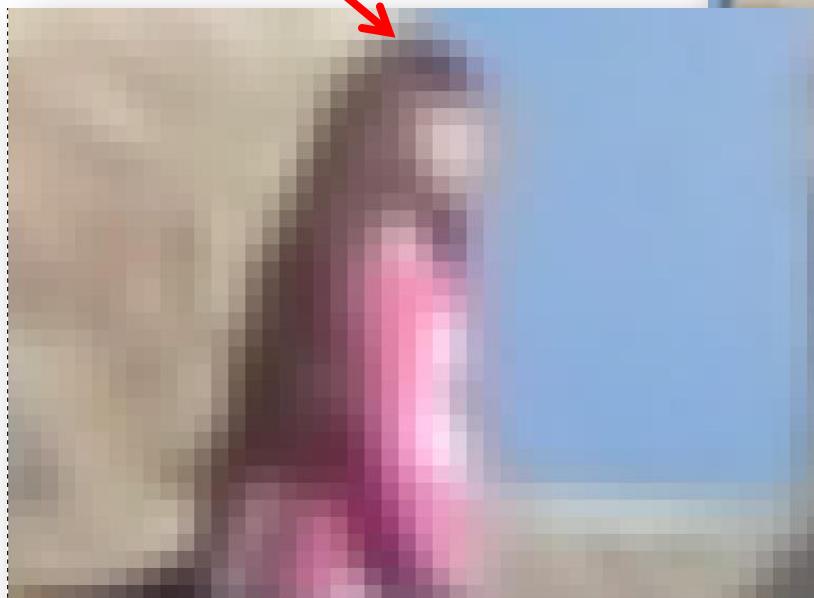
x 4



x 16



color pixel level



x 64

# Digital Cameras

A digital camera produces still, single-frame images that can be stored in digital memory, displayed on a screen and printed on physical media.

It is comprised of:

- **an optical imaging system**
- **an image sensor (CCD or CMOS)**
- **a storage device**
- **a display**

Canon Powershot A95  
compact digital camera

*By Fir0002 (composite version by Matt57) - Own work, GFDL 1.2,  
<https://commons.wikimedia.org/w/index.php?curid=5159773>*



Dimitris Papazoglou, [dpapa@materials.uoc.gr](mailto:dpapa@materials.uoc.gr)



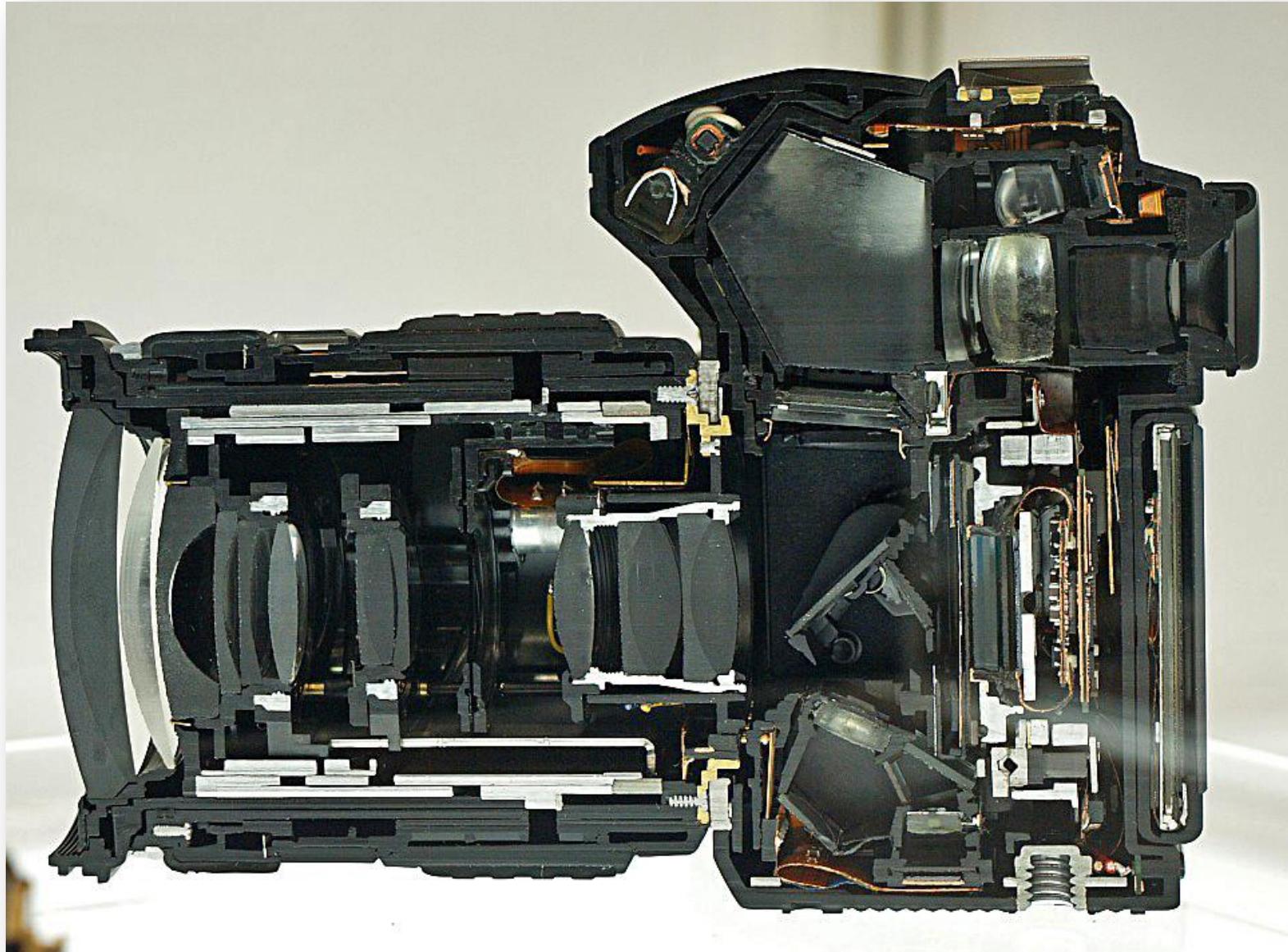
Digital camera Sony DSC-H2.

*By Christian Jansky (User:Tschaensky) –  
Self-photographed, CC BY-SA 3.0,  
<https://commons.wikimedia.org/w/index.php?curid=3161095>*



A CompactFlash (CF)  
card, used to store  
digital photographs

*By Motorrad-67  
<https://commons.wikimedia.org/w/index.php?curid=2845628>*

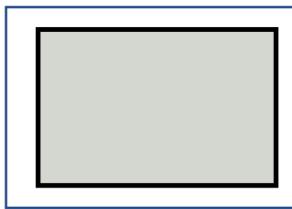


A cutaway of an Olympus E-30 DSLR Camera with Zuiko Digital ED 14-54mm F2.8-3.5 II lens.

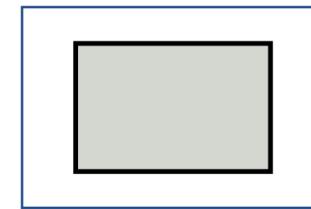
*By Hanabi123, [CC BY-SA 3.0], Wikimedia Commons*



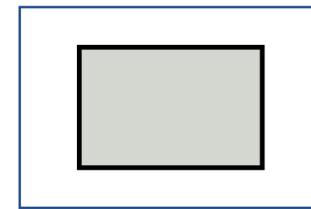
35 mm "full frame"  
36×24 mm  
864 mm<sup>2</sup>



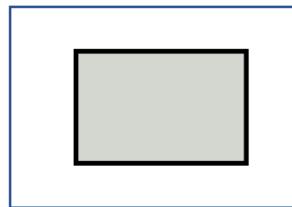
APS-H (Canon)  
28.7×19 mm  
548 mm<sup>2</sup>



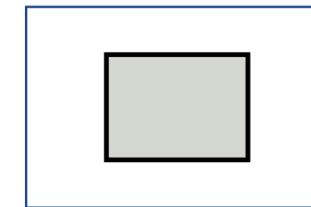
APS-C (Nikon, Sony,  
Pentax, Fuji etc.)  
 $\approx 23.6 \times 15.7$  mm  
 $\approx 370$  mm<sup>2</sup>



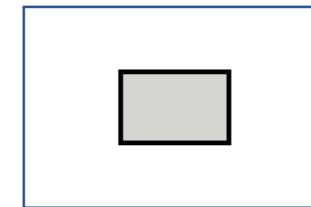
APS-C (Canon)  
22.2×14.8 mm  
329 mm<sup>2</sup>



Foveon (Sigma)  
20.7×13.8 mm  
286 mm<sup>2</sup>



Four Thirds System  
(Olympus, Panasonic)  
17.3×13 mm  
225 mm<sup>2</sup>



1" (Nikon, Sony)  
13.2×8.8 mm  
116 mm<sup>2</sup>



2/3" (Fuji, Nokia)  
8.6×6.6 mm  
58.1 mm<sup>2</sup>



1/1.7"  
7.6×5.7 mm  
43 mm<sup>2</sup>

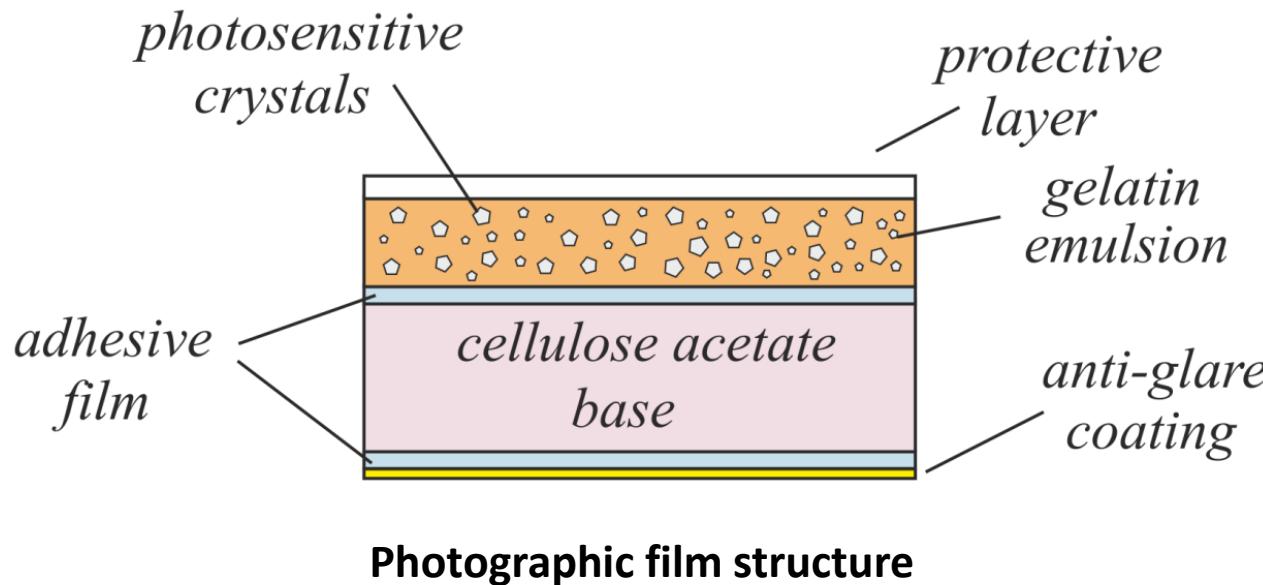


1/2.5"  
5.76×4.29 mm  
25 mm<sup>2</sup>

## Comparison of digital camera image sensor sizes

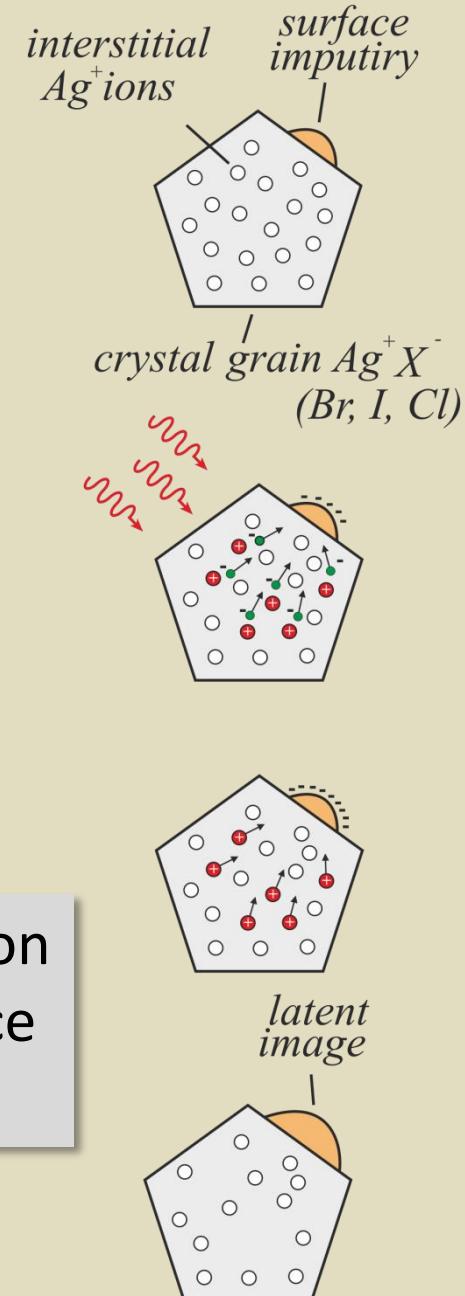
By RenniePet, [PD], Wikimedia Commons

# Typical light detectors: Photographic film

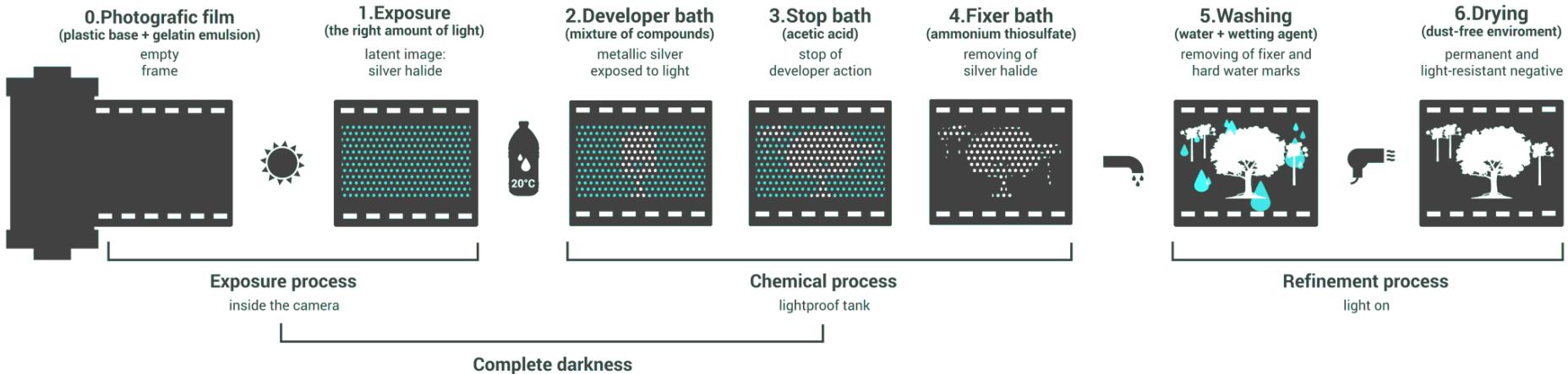


through a complex photochemical reaction  
metallic Ag is concentrated on the surface  
impurities of the  $\text{Ag}^+\text{X}^-$  crystal grains.

Operation: X-Rays to NIR



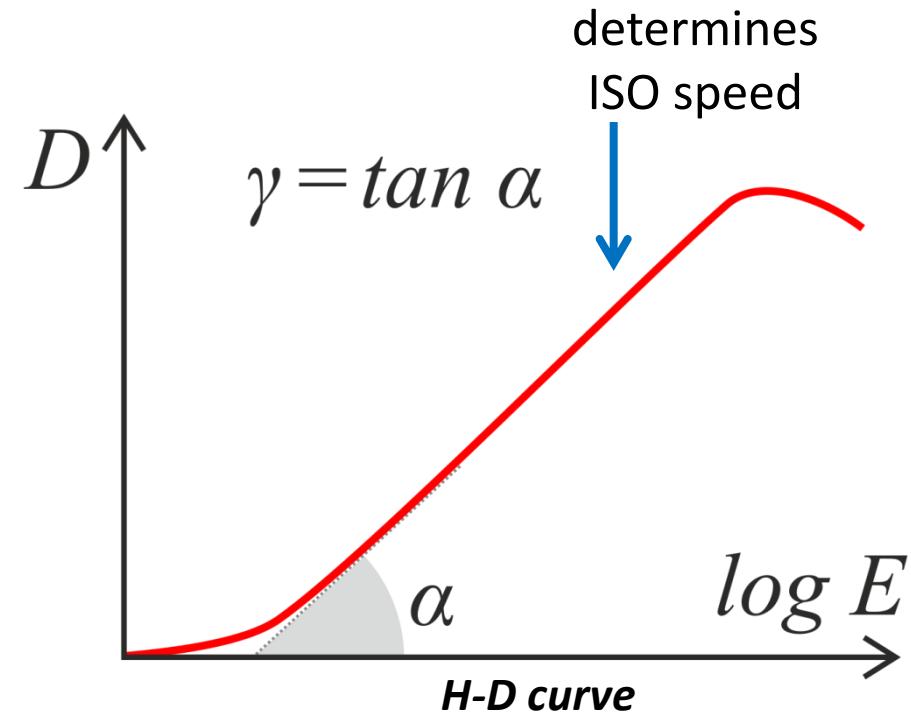
# Photographic film processing



By Marianna Caserta, DensityDesign Research Lab". - Own work, CC BY-SA 4.0,  
<https://commons.wikimedia.org/w/index.php?curid=37058198>

$$D = -\log(I_{out} / I_{in})$$

Optical density



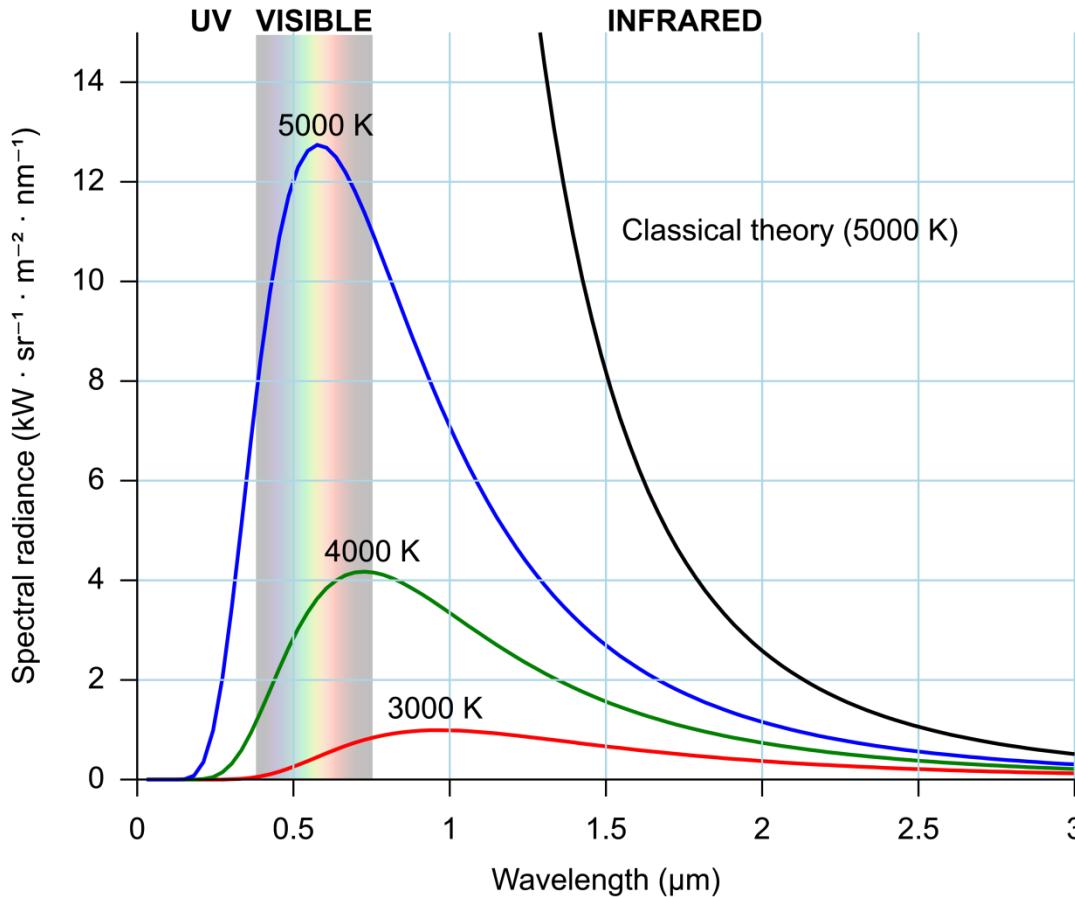
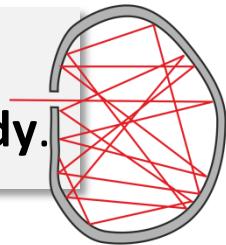


**4.3**

## Sources

# Black Body radiation

An idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence is called **black body**.



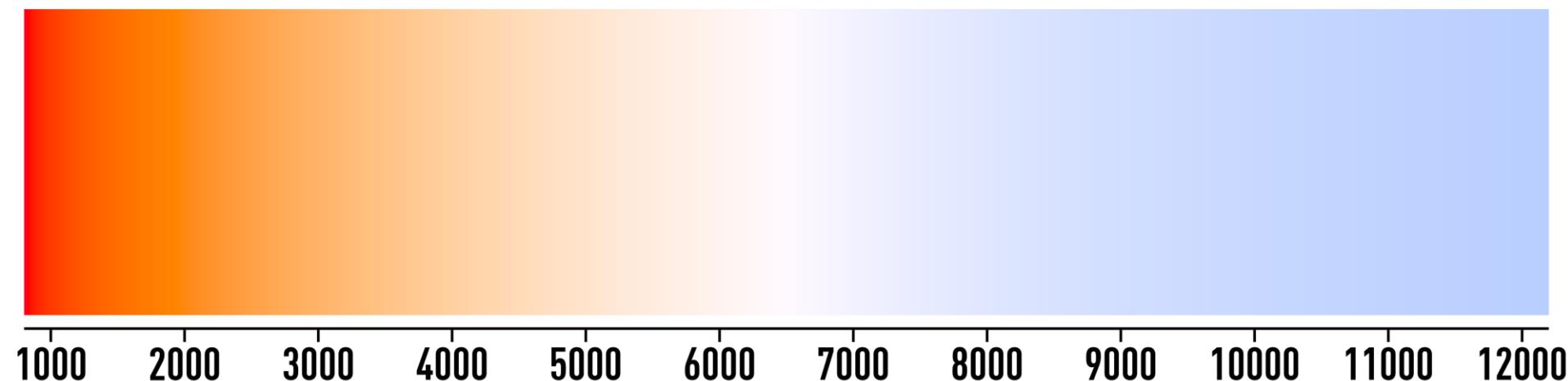
Black-body radiation has a characteristic, continuous frequency spectrum that depends only on the body's temperature.

$$\lambda_{\max} = \frac{2898}{T} \frac{K}{\mu m}$$

Wien law

By Darth Kule , [PD], Wikimedia Commons

## Color of a black body from 800 K to 12200 K.



By Bhutajata - Own work, CC BY-SA 4.0,  
<https://commons.wikimedia.org/w/index.php?curid=44144928>

*radiant  
emittance* →  $I_e = \sigma T^4$

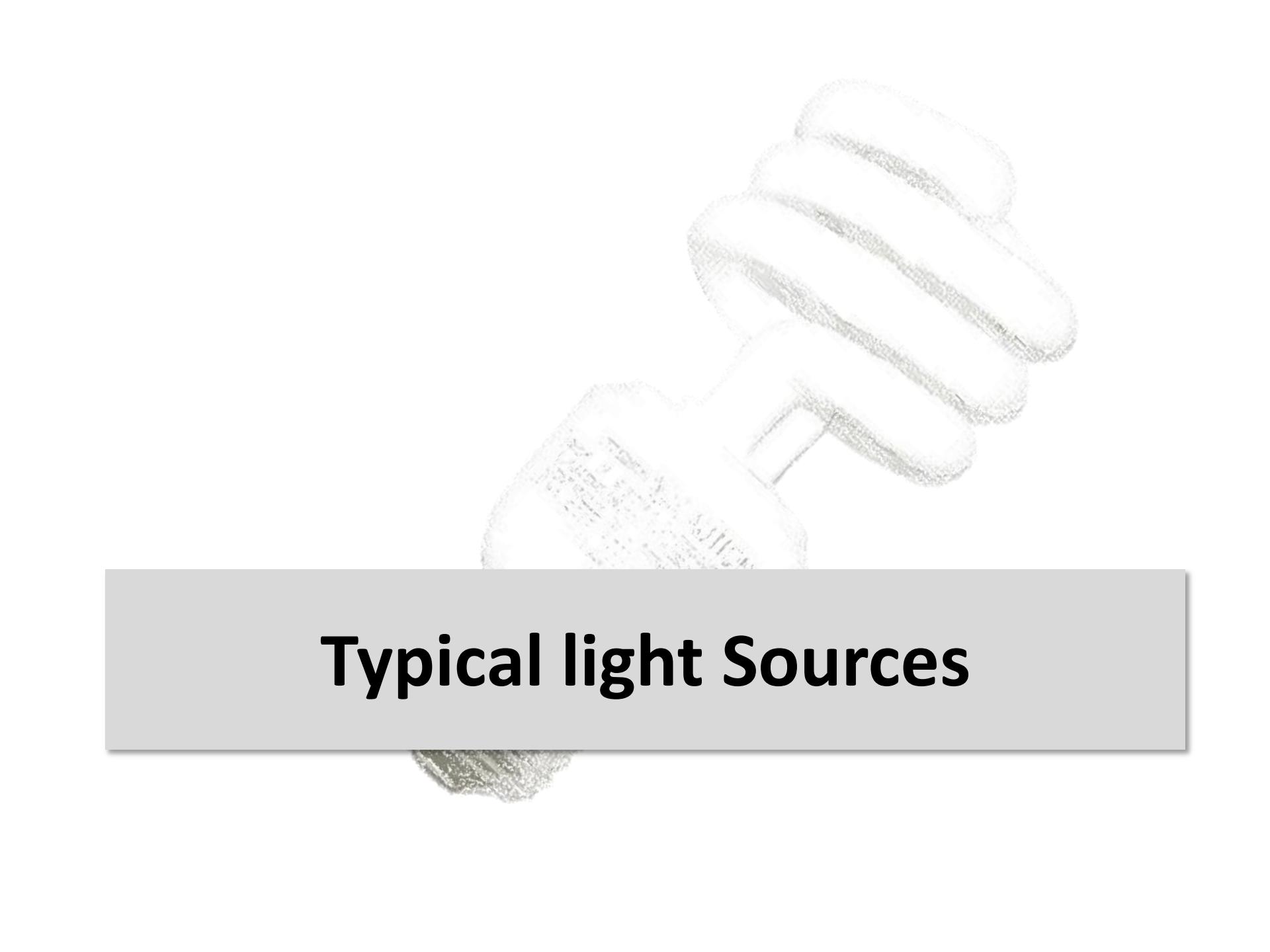
*Stefan–Boltzmann law*

$$\sigma \cong 57 \frac{nW}{m^2 K^4}$$



**Lava (Kilauea Hawaii 2003)**

By Hawaii Volcano Observatory (DAS) , [PD],  
Wikimedia Commons



# Typical light Sources

# Incandescent lamps

A tungsten filament is heated at high temperature up at by passing an electric current through it. It is protected from oxidation with a glass or fused quartz bulb that is filled with inert gas or evacuated.

Continuous spectrum

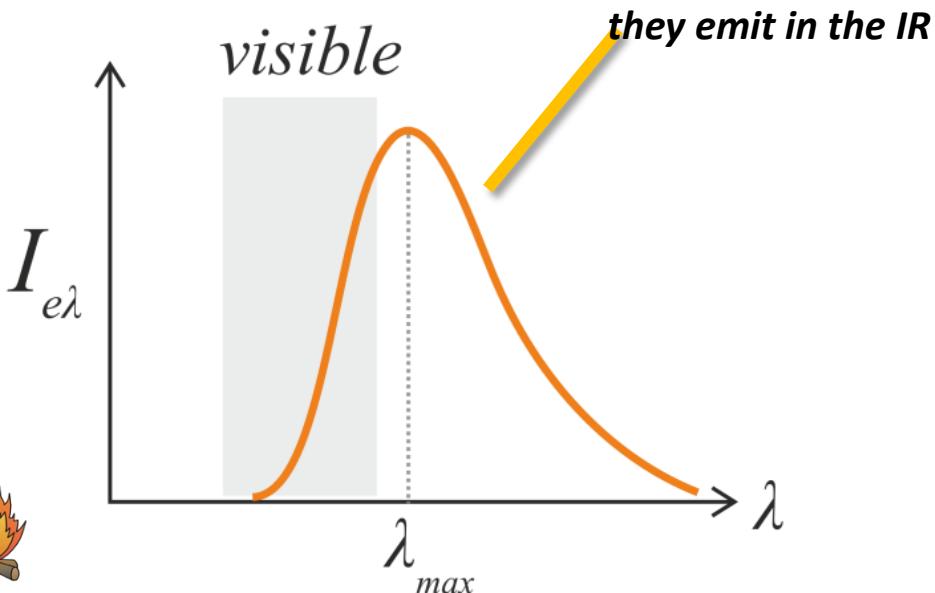
Temperature  $\sim 2500$  K

Improved version: Halogen Lamps

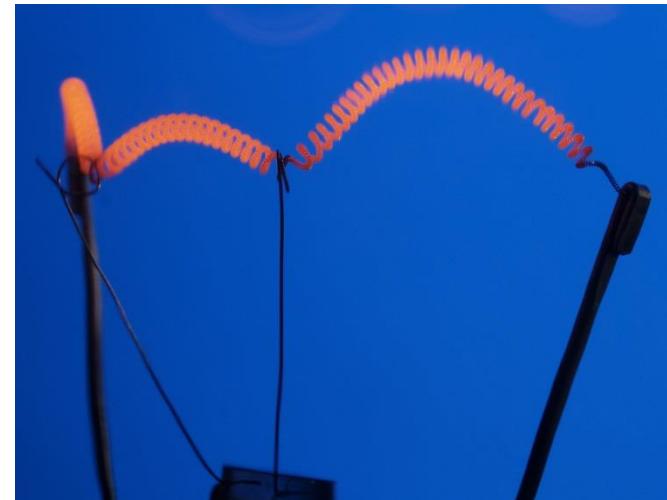


Incandescent lamp

By KMJ CC BY-SA 3.0,  
Wikimedia Commons



"Hot" source



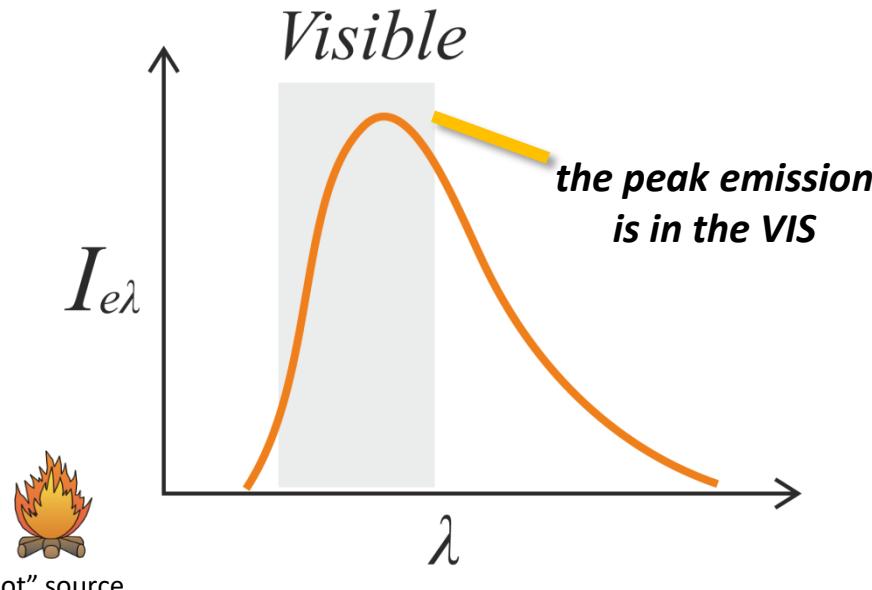
Bulb filament at reduced voltage

By Arnoldius CC-BY-SA-3.0  
Wikimedia Commons

# Arc lamps

In an arc lamp, the electrodes are **carbon rods** in free air.

The rods are first touched together, allowing a relatively low voltage to strike the arc. Then the rods are slowly drawn apart, and electric current heats and maintains an arc across the gap



Continuous spectrum

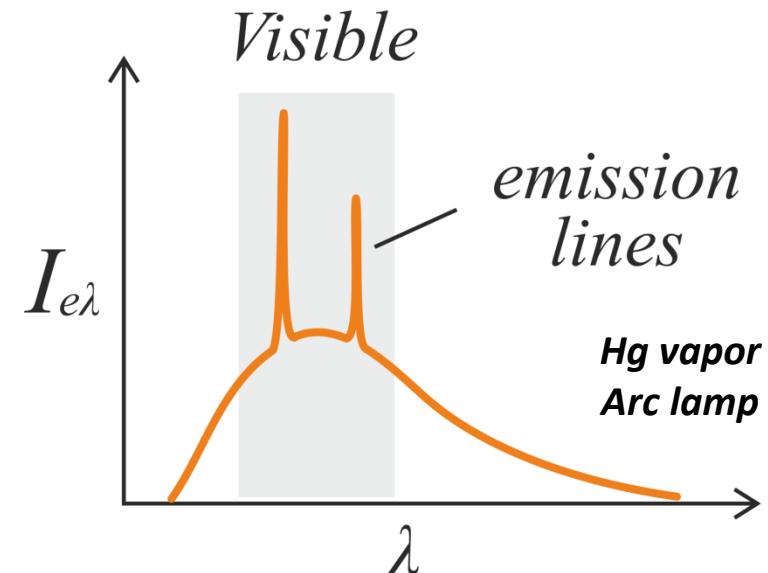
Temperature  $\sim 4000$  K

Excellent white light sources



An arc lamp for IMAX

By Atlant, CC BY 2.5,  
Wikimedia Commons



# Low pressure Gas discharge lamps

In gas discharge lamps light is generated by sending an electric discharge through an ionized gas. Typically, they use a noble gas (Ar, Ne, Kr, Xe) or a mixture of these gases. Some include additional substances, like Hg, Na, and metal halides, which are vaporized during startup to become part of the gas mixture.

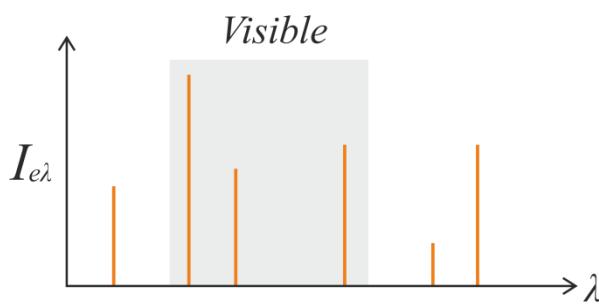
Discrete spectrum

Atomic line emission

Useful in spectroscopy



Gas discharge lamp



$\lambda$ (nm)	color	element	$\lambda$ (nm)	color	element
365.0	UV	Hg	546.07	G	Hg
404.66	V	Hg	587.56	Y	He
435.83		Hg	589.29		Na
479.99	B	Cd	643.85		Cd
486.13		H	656.27	R	H
			706.52		He

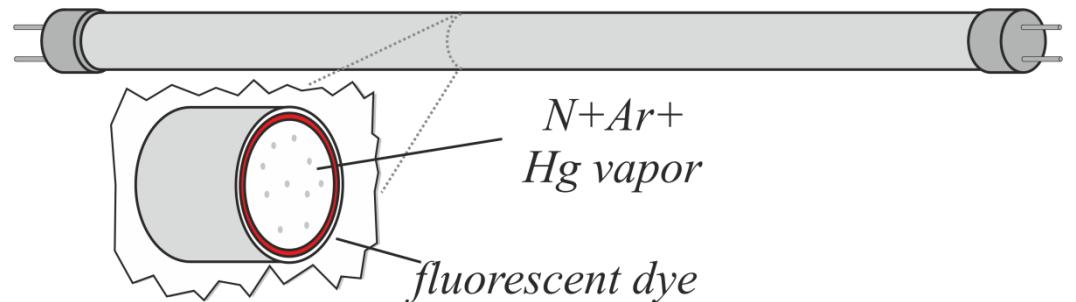


"Cold" source

# Fluorescent Lamps

A fluorescent lamp is a **low-pressure mercury-vapor gas-discharge** lamp that uses fluorescence to produce visible light.

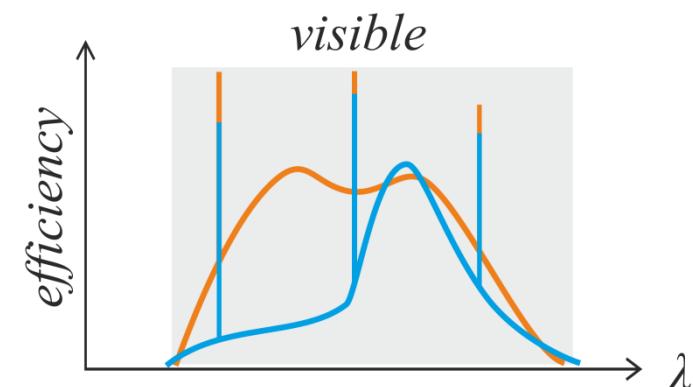
Mercury vapor excited by an electric current in the gas, produces **UV light** that then causes a **fluorescent dye** (phosphor) coating to glow.



Continuous Spectrum

Broad spectrum

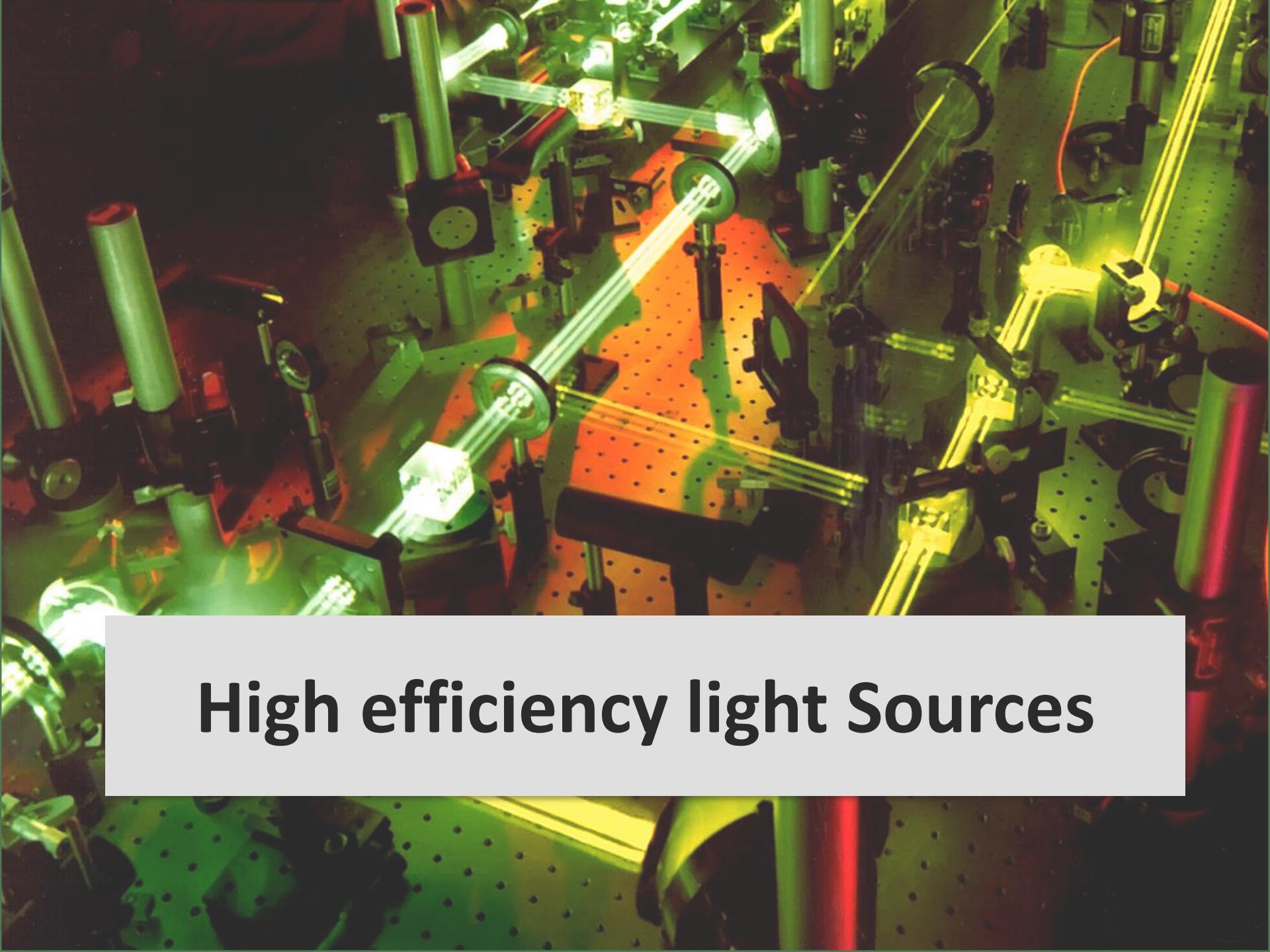
High efficiency



compact  
fluorescent lamp

By Lovelac7 [PD],  
Wikimedia Commons





# High efficiency light Sources

# Light Emitting Diode (LED)

LEDs are **semiconductor light sources**. They consist of a **p-n junction** diode that emits light when activated. When a suitable voltage is applied to the leads, electrons and holes recombine, releasing energy in the form of photons.

Narrow spectrum

Quantum Mechanical

High efficiency

Color tuned by the band gap



**Red, Green, Blue LEDs**

*By PiccoloNamek CC BY-SA 3.0, Wikimedia Commons*



**LEDs are produced in a variety of shapes and sizes**

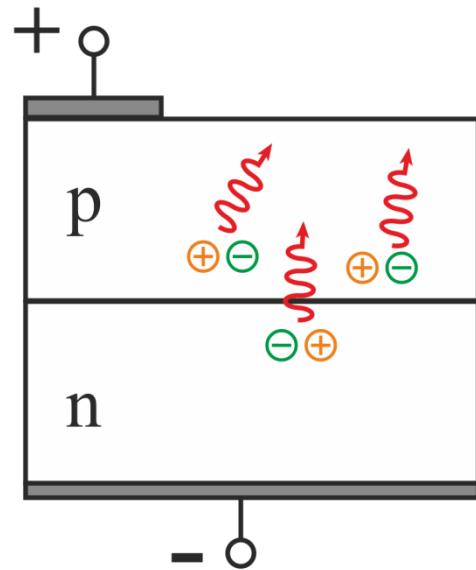
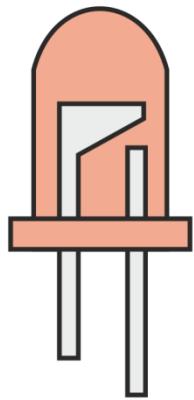


“Cold” source

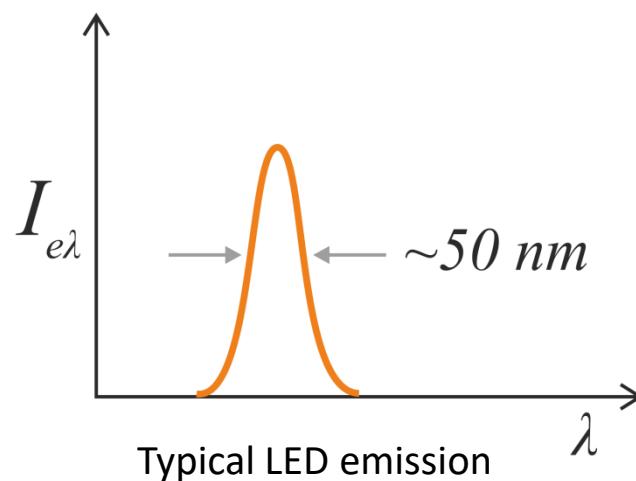
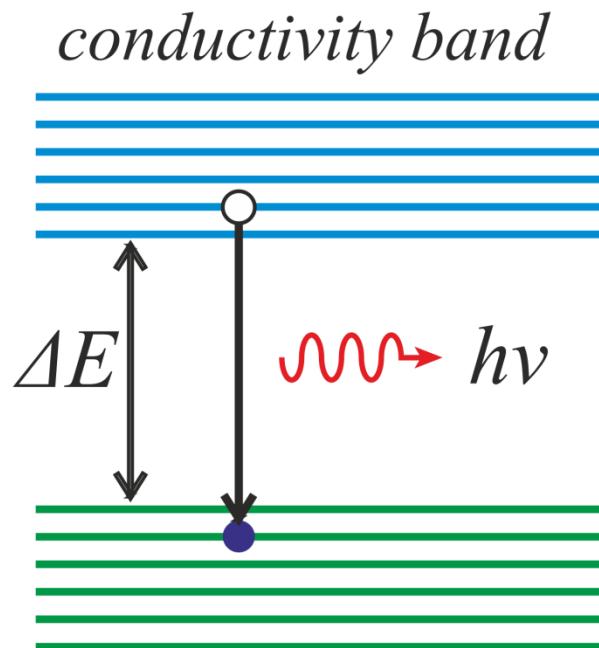


modulated source

*By Afrank99 , CC BY-SA 2.0.,  
Wikimedia Commons*

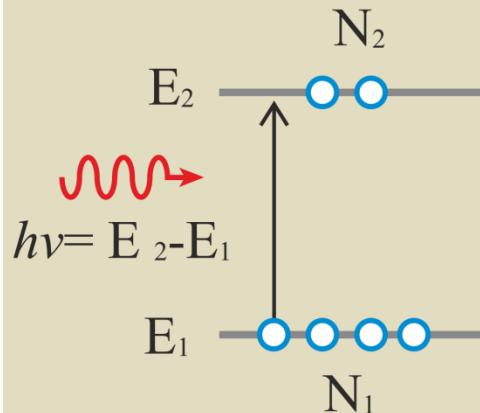


LED Operation principle



# Light Amplification by Stimulated Emission of Radiation (LASER)

## Absorption

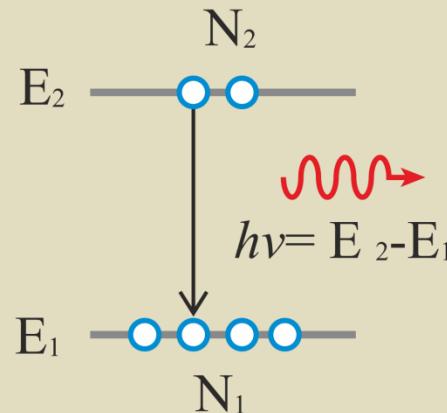


Transition probability:  $B_{12}$

$$\frac{dN_2}{dt} = B_{12} \cdot \rho_v \cdot N_1$$

$$\rho_v = D_v \cdot h \cdot v$$

## Spontaneous emission

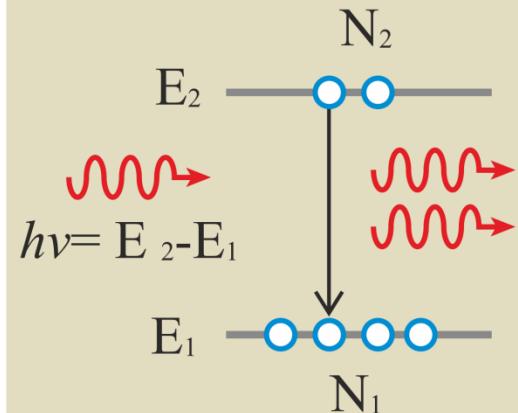


Transition probability:  $A_{12}$

$$\frac{dN_2}{dt} = A_{12} \cdot N_2$$

identical frequency,  
random phase and  
direction

## Stimulated emission



Transition probability:  $B_{21}$

$$\frac{dN_2}{dt} = -B_{21} \cdot \rho_v \cdot N_1$$

identical frequency,  
phase and direction

Light amplification

# The case of thermal equilibrium

$$\frac{A_{21}}{B_{21}} = \frac{8\pi \cdot h \cdot \nu^5}{c^3}$$

$$B_{21} = B_{12}$$

Application to a thermal source

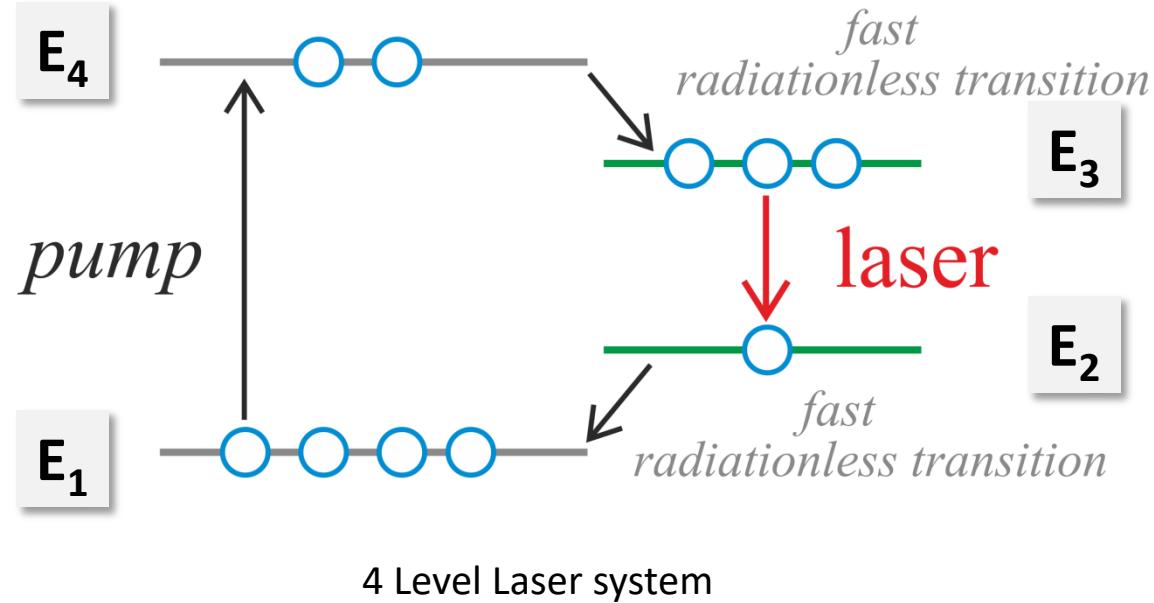
$$T = 2000K, \nu = 500THz \Rightarrow \frac{\rho_\nu B_{21}}{A_{21}} = 6.6 \cdot 10^{-6}$$

In a typical thermal source the ratio of the stimulated to the spontaneous emission rates is very low!

**It is not possible to achieve amplification  
when we are at thermal equilibrium.**

# Beyond equilibrium: Inversion of Population

Population inversion occurs when **more members** of a system are in **higher, excited states** than in lower, unexcited energy states

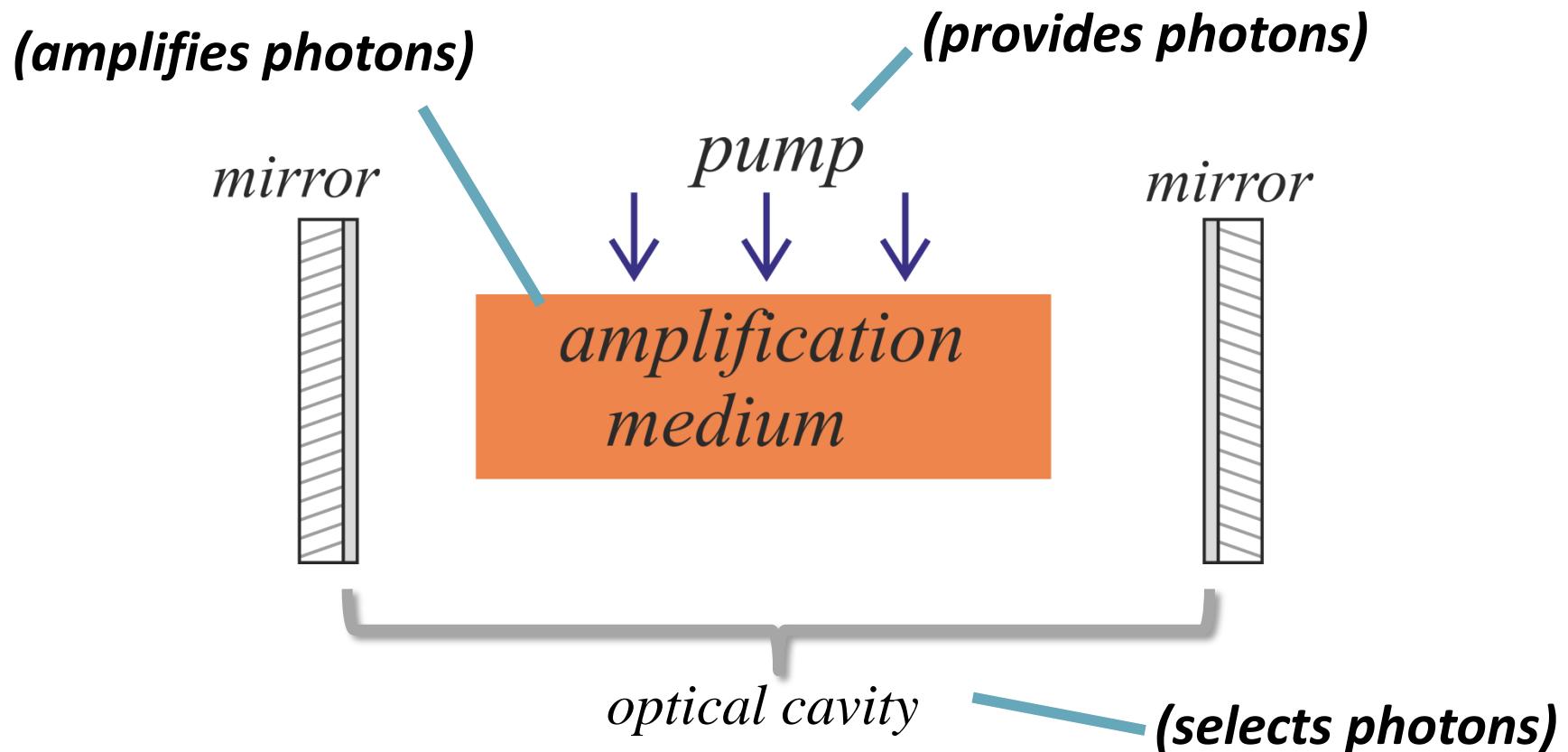


Pumping excites the atoms in the ground state  $E_1$  into  $E_4$ .

From there the atoms decay by a fast, non-radiative transition into the level  $E_3$ . Since the lifetime of the  $E_3 \rightarrow E_2$  transition is long compared to that of  $E_4 \rightarrow E_3$ , a population accumulates in level  $E_3$  (the upper laser level).

into level  $E_2$  (the lower laser level). This level likewise has a fast, non-radiative decay into the ground state.

# LASER sources

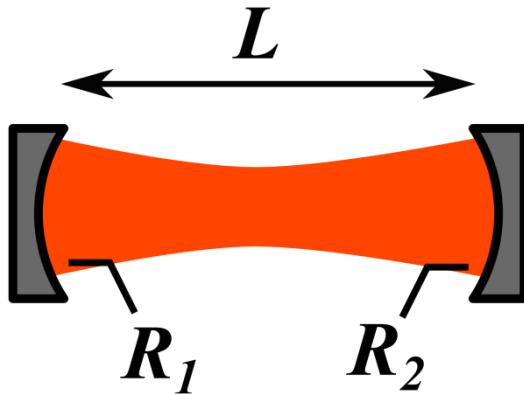


The Pump **provides** the photons

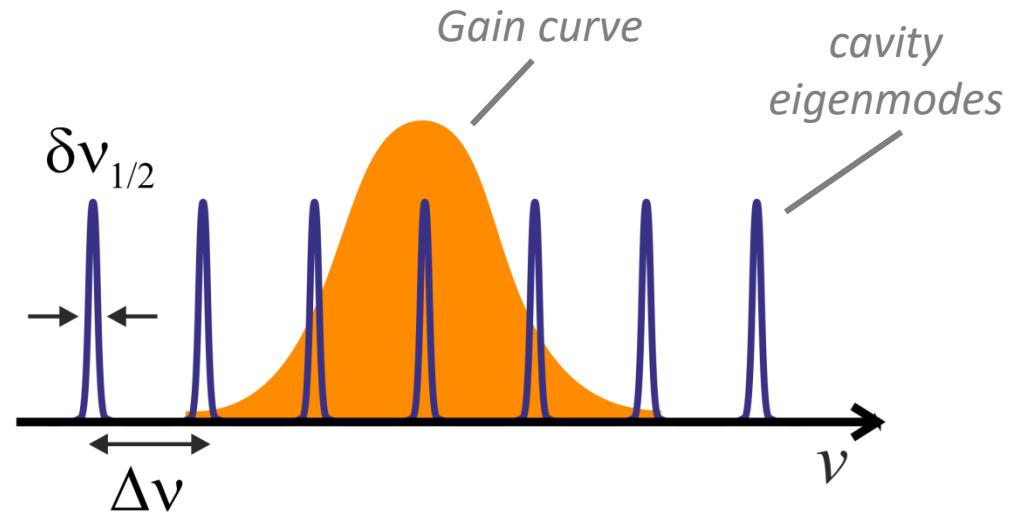
The amplification medium amplifies these photons

The optical cavity **selects** which photons will be **repeatedly amplified** thus determines the LASER output

# Optical cavity



Light confined in a cavity will reflect multiple times, and due to **interference**, only certain patterns and frequencies, the **eigenmodes of the cavity**, will be sustained.

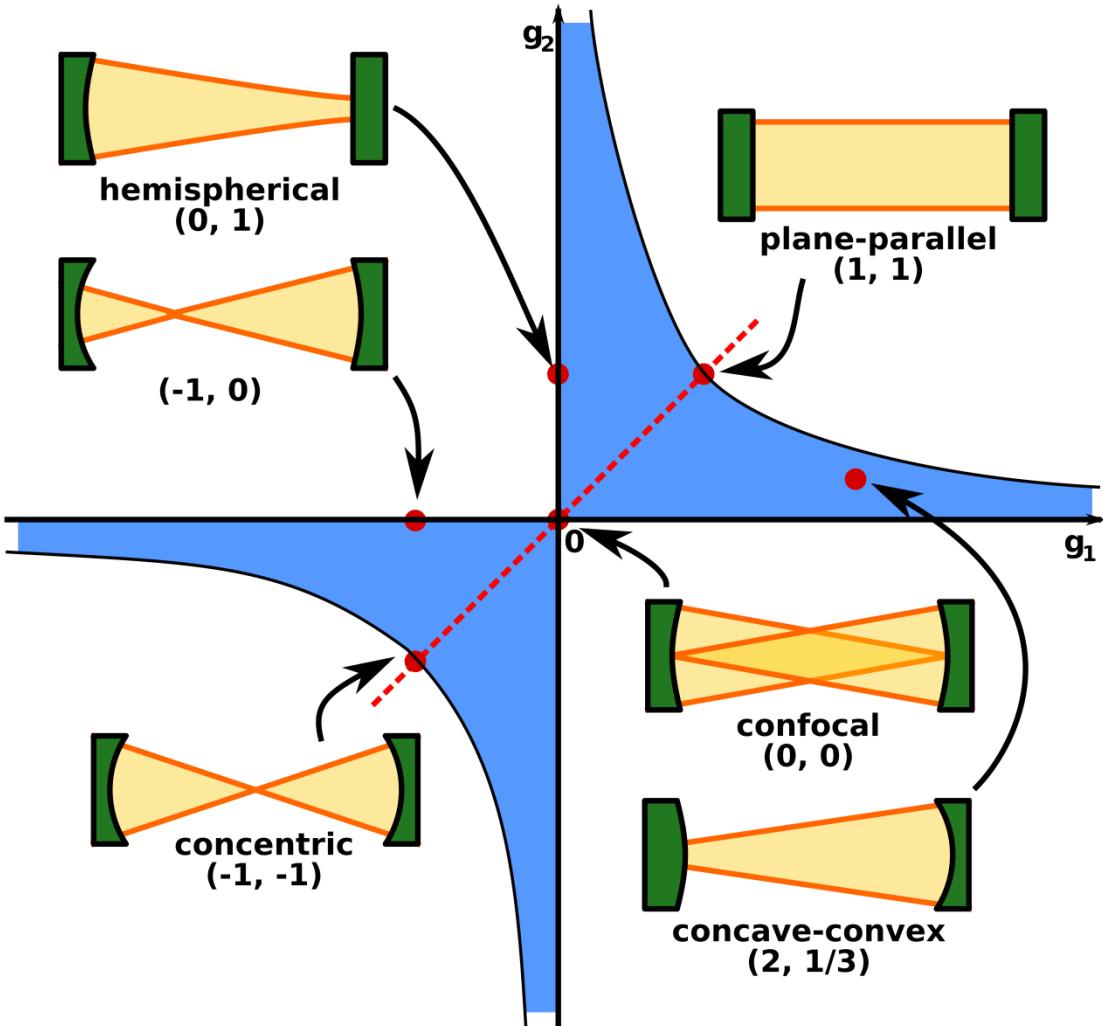


$$\Delta\nu = \frac{c}{2L}$$

$$\Delta\nu_{1/2} = \frac{\Delta\nu}{\pi\sqrt{2F}}$$

$$F \equiv \left( \frac{2r}{1-r^2} \right)^2$$

*finesse*

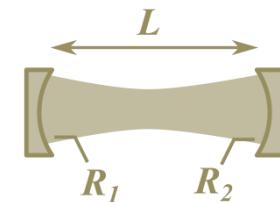


## Stability of an optical resonator

By FDominec, CC BY-SA 3.0,  
<https://commons.wikimedia.org/w/index.php?curid=3017470>

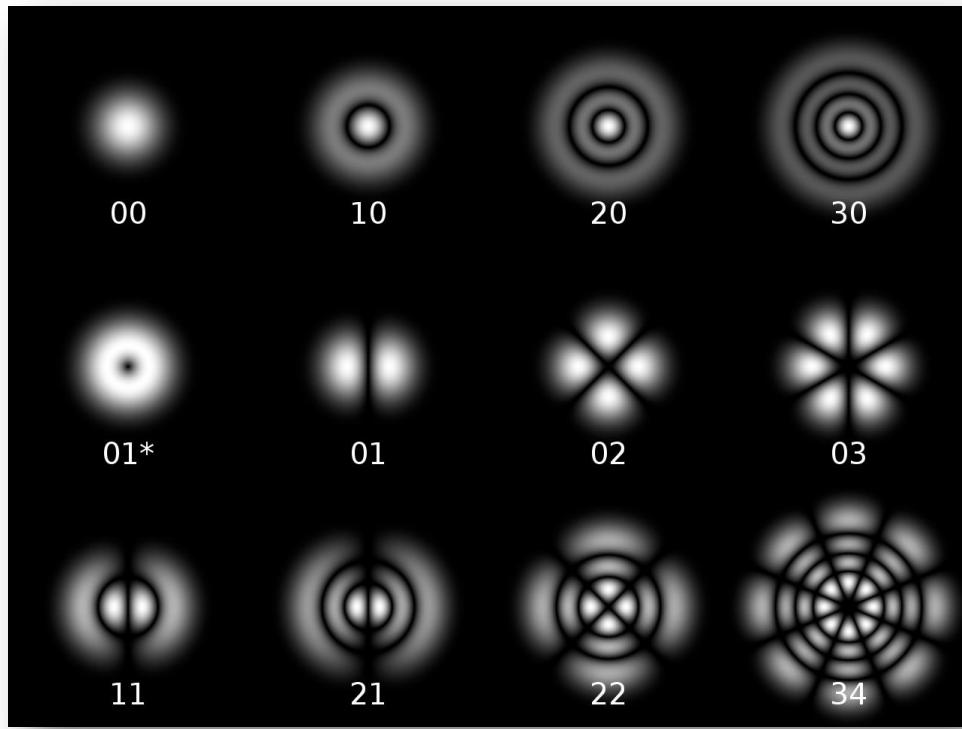
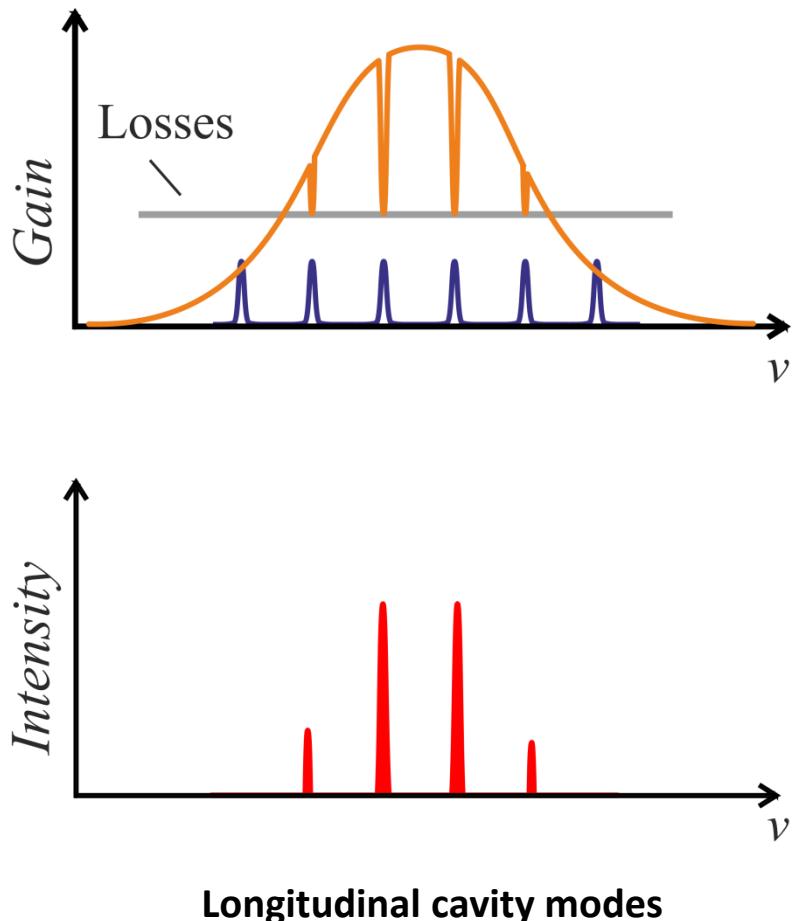
$$g_1 = 1 - \frac{L}{R_1}$$

$$g_2 = 1 - \frac{L}{R_2}$$



Only certain ranges of values for  $R_1$ ,  $R_2$ , and  $L$  produce stable resonators in which periodic refocusing of the intracavity beam is produced

# Longitudinal and transverse cavity modes



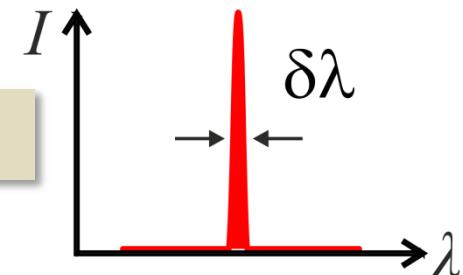
**Transverse cavity modes  
(Laguerre-Gaussian)**

*By DrBob CC BY-SA 3.0  
Wikimedia Commons*

# Laser beam properties

Monochromaticity

$$\frac{\delta\lambda}{\lambda} \propto \frac{\lambda}{L} \sqrt{\frac{2}{F}} < 10^{-5}$$



Directionality

$$\theta \approx \frac{\lambda}{2d} \ll 1 \text{ mrad}$$



Coherence

$$l \approx \frac{c}{\Delta\nu} > 30 \text{ cm}$$

Temporal

Spatial

correlated in phase



correlated  
in phase



These properties result from the cavity and not the amplification!

They are not unique to lasers

The **amplification** in combination to the cavity results extremely high values of **spectral brightness**  
( *power per surface area per solid angle per frequency unit*)

*Spectral  
Brightness*

$$\beta_\nu = \frac{P_\nu}{A \cdot \Delta\Omega \cdot \Delta\nu} \quad \frac{W}{cm^2 sr Hz}$$

*Typical values*

Sun

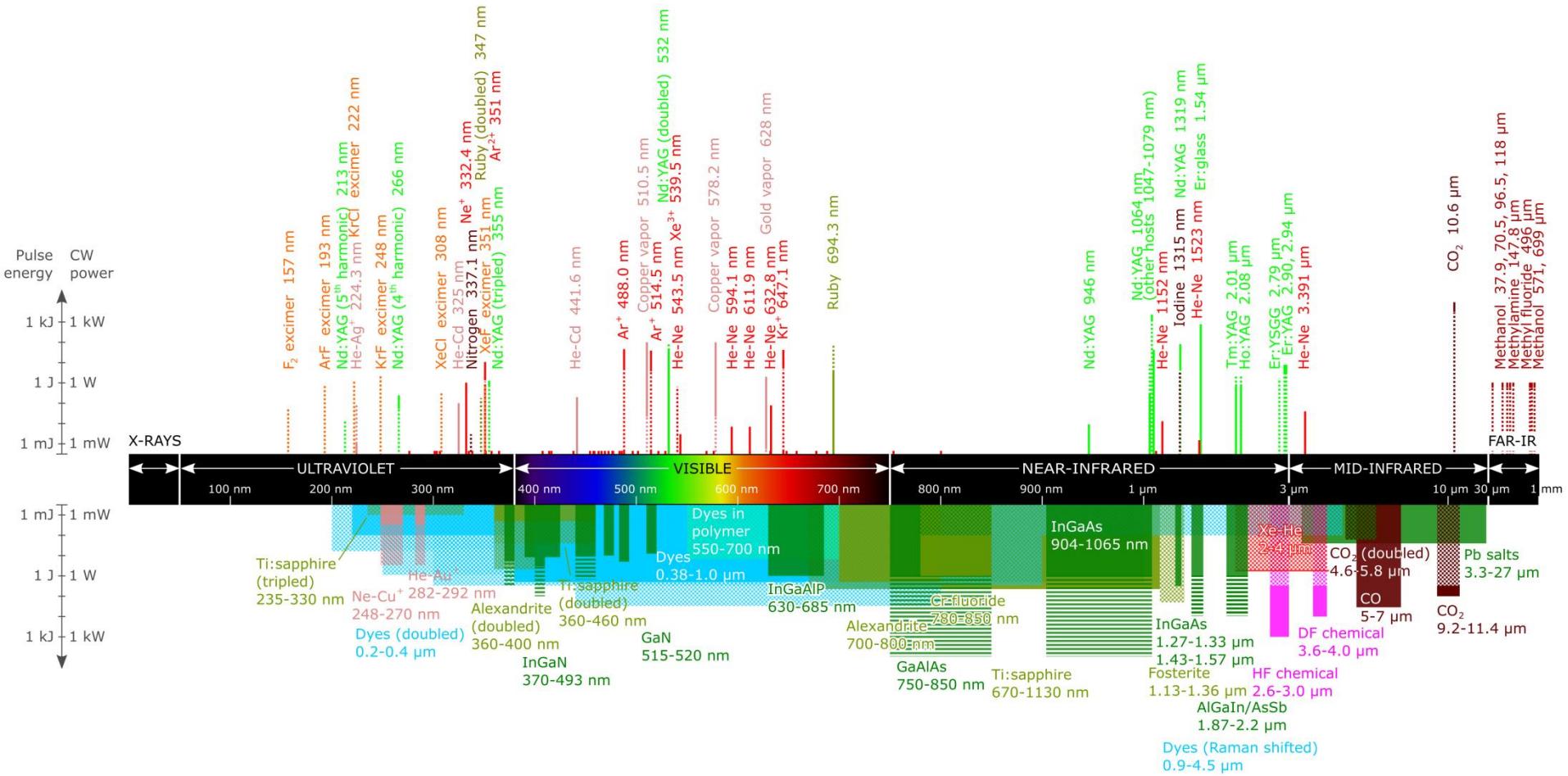
**$1.5 \cdot 10^{-12}$**

He-Ne (1 mW)

**25**

Nd:glass (10 GW)

**$2 \cdot 10^8$**



## Wavelengths of commercially available lasers.

By Danh - Own work.

Uses File:Linear visible spectrum.svg↑ Weber, Marvin J. Handbook of laser wavelengths, CRC Press, 1999. ISBN 0849335086, CC BY-SA 3.0,  
<https://commons.wikimedia.org/w/index.php?curid=8187485>

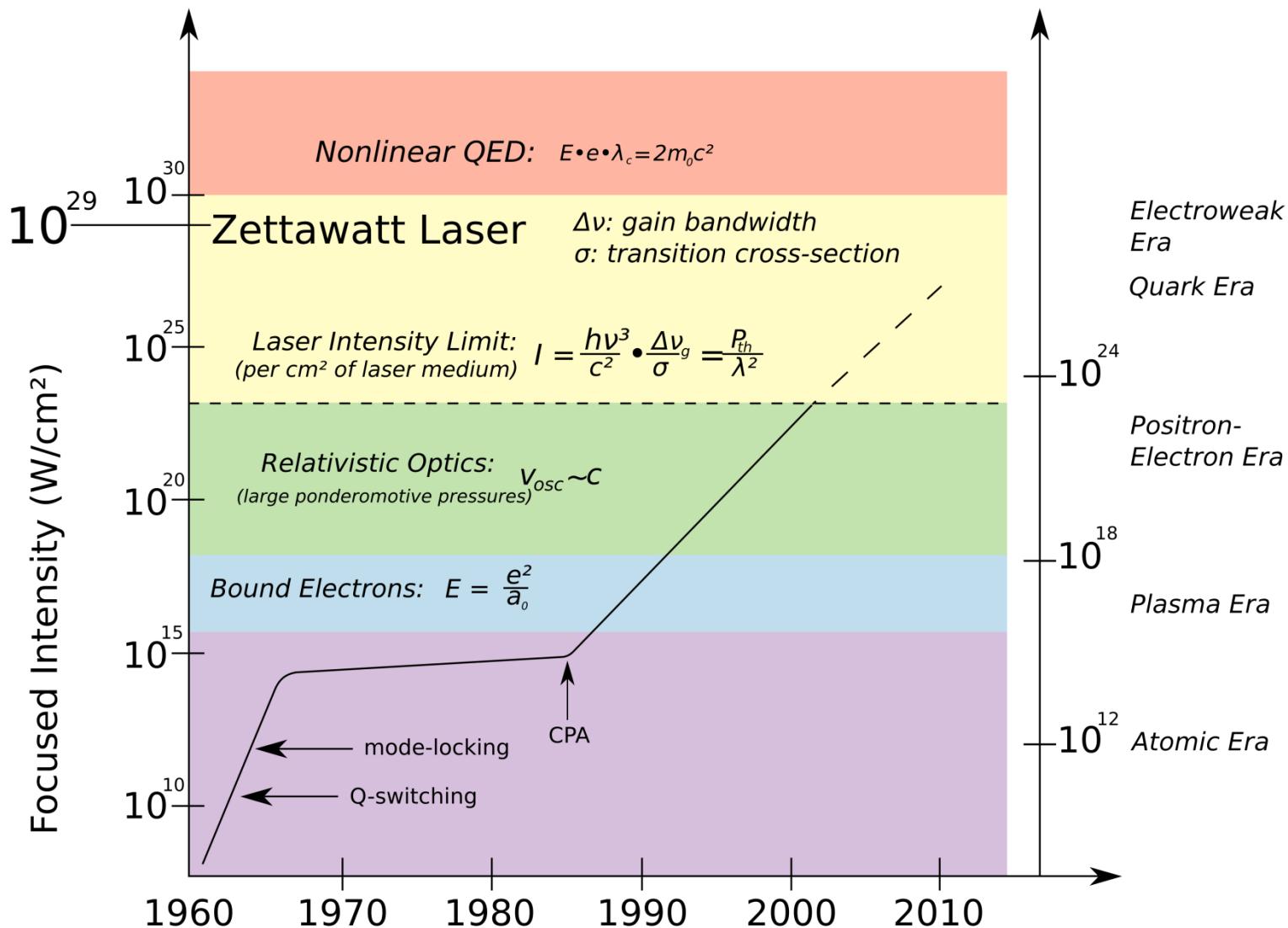
## Wavelength ranges from IR to UV

Continuous wave lasers (CW)

Pulsed lasers

Pulse duration: μsec down to few fs

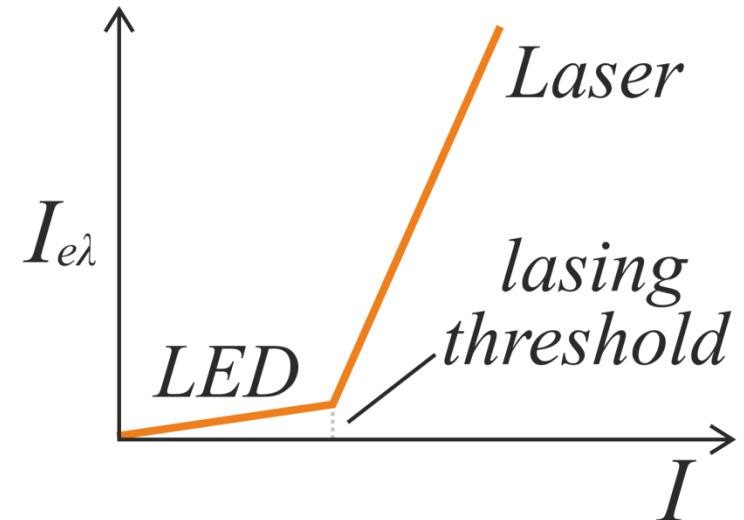
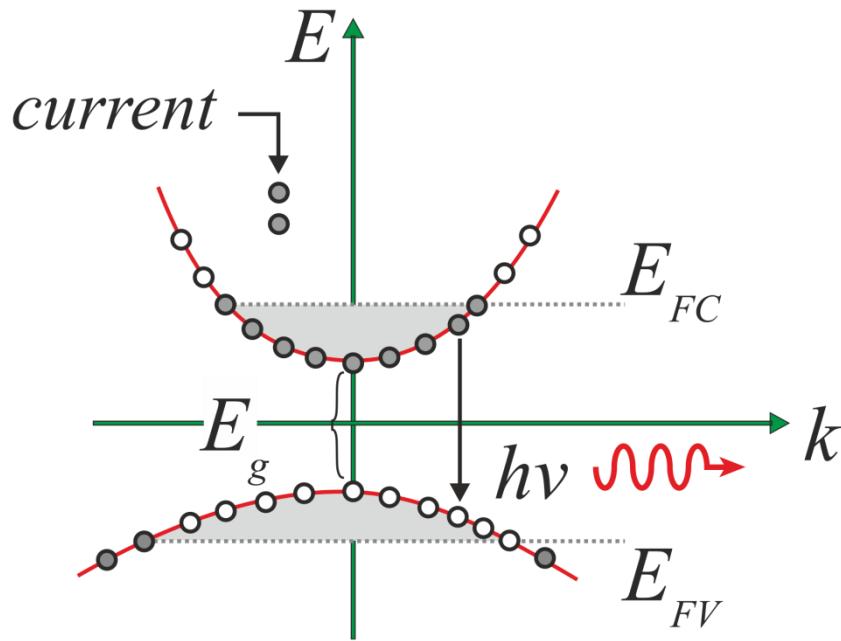
Repetition rate: MHz down to few Hz



Graph showing the history of maximum laser pulse intensity throughout the past 40 years

[PD] Wikimedia Commons

# LASER Diodes



Large carrier density



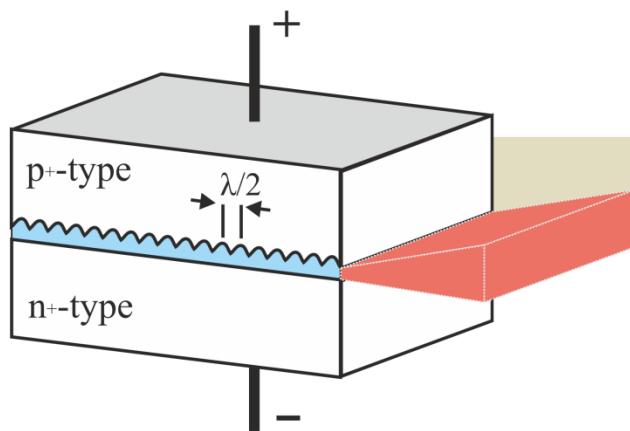
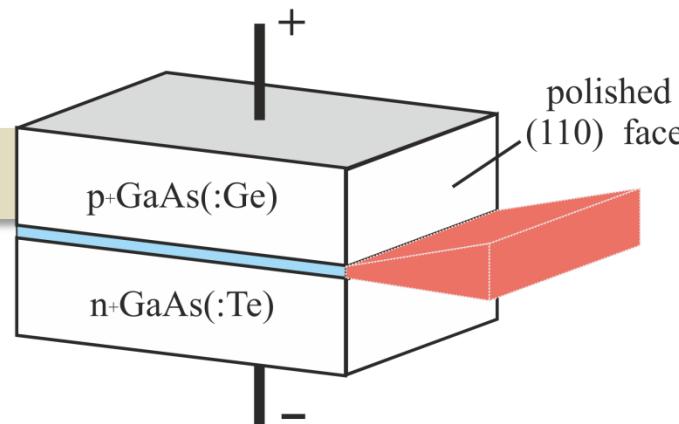
High current density

Population inversion

Stimulated  
electron – hole  
recombination

Lasing

## Simple Diode laser cavity

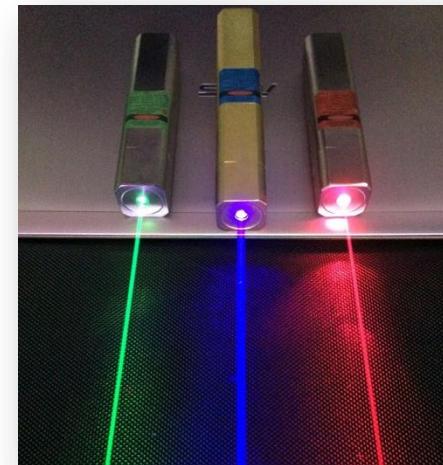


## Distributed feedback Diode laser



packaged laser diode shown with a penny for scale

By Jet Propulsion Laboratory,  
Public Domain, Wikimedia Commons



## Semiconductor lasers (520nm, 445nm, 635nm)

By 彭家杰 - Own work, CC BY-SA 3.0,  
<https://commons.wikimedia.org/w/index.php?curid=31615648>