Interdisciplinary postgraduate program «Optics and Vision»

Foundations of Modern Optics

Dimitris Papazoglou

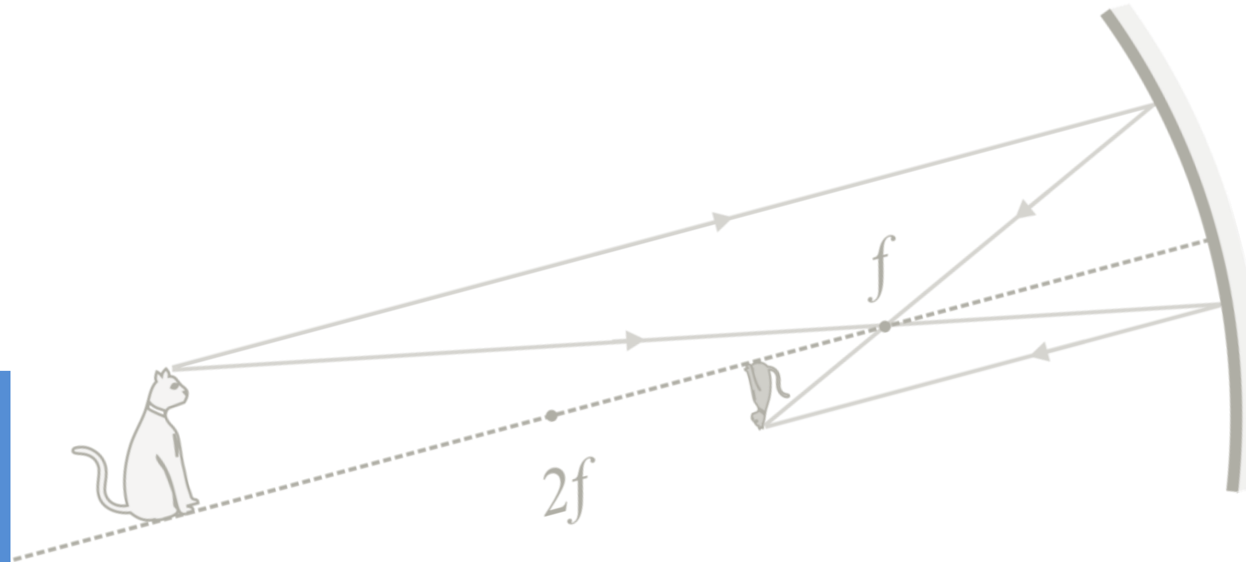
dpapa@materials.uoc.gr

Materials Science and Technology Department

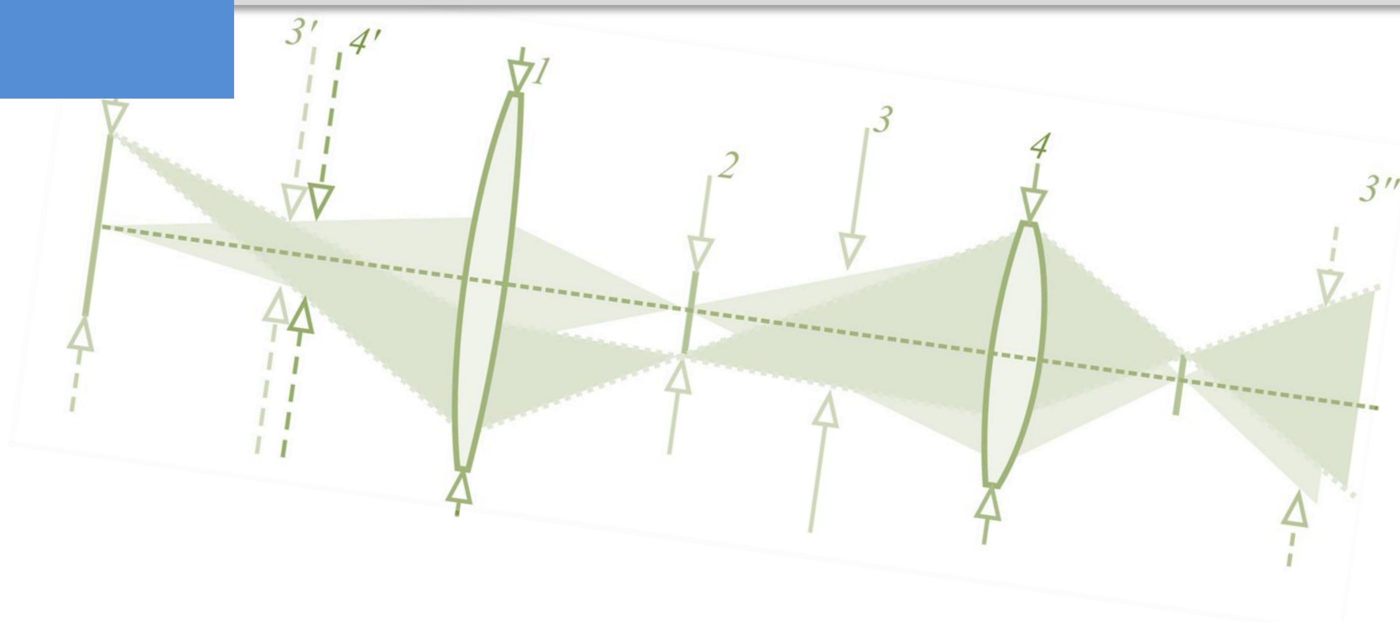
University of Crete

Licenses

3 Imaging



3.1 Geometrical Optics



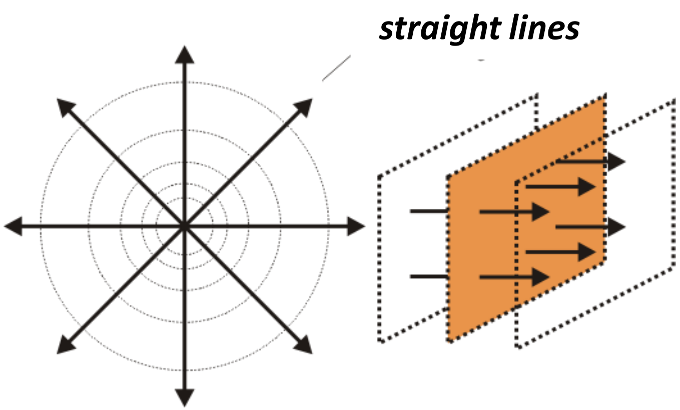
Optical Rays

Optical Ray:
An optical ray is a **curve** that describes the propagation of energy

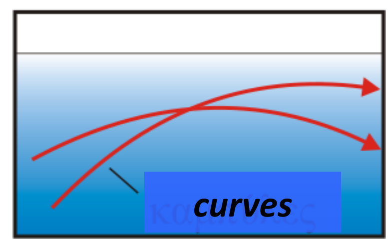
Isotropic medium

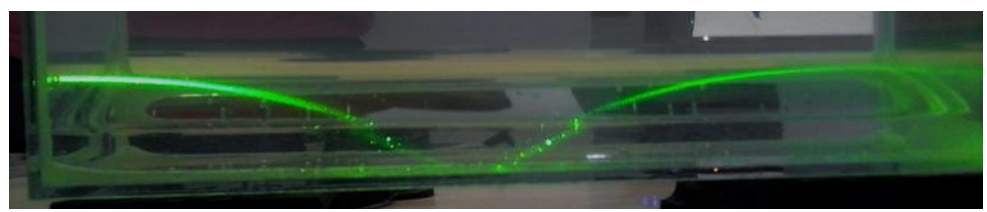
- optical rays are orthogonal to the wavefront
- optical rays are parallel to the wavevector $\boldsymbol{k}$

Homogeneous isotropic media



Inhomogeneous isotropic media





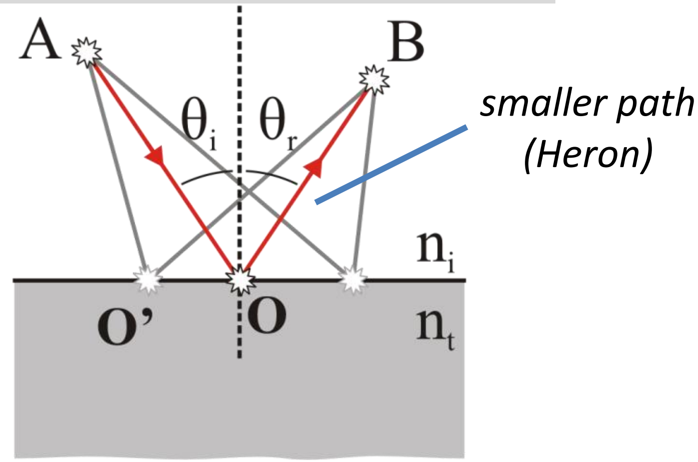
Very simple description

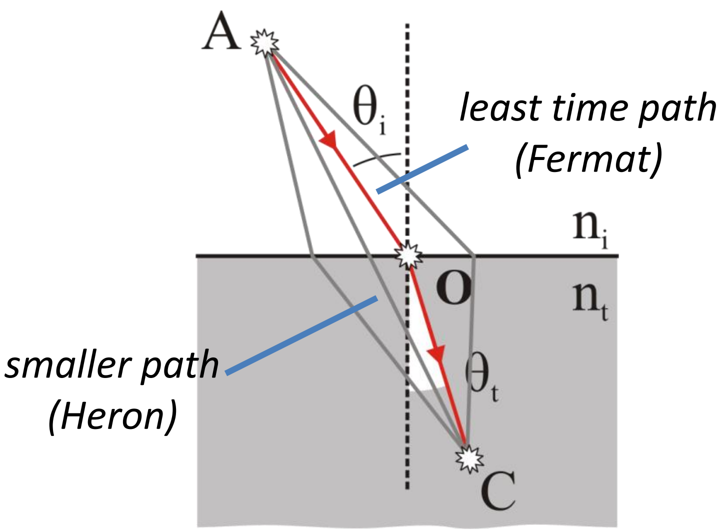
ignores the wave nature of light

How do optical rays propagate?

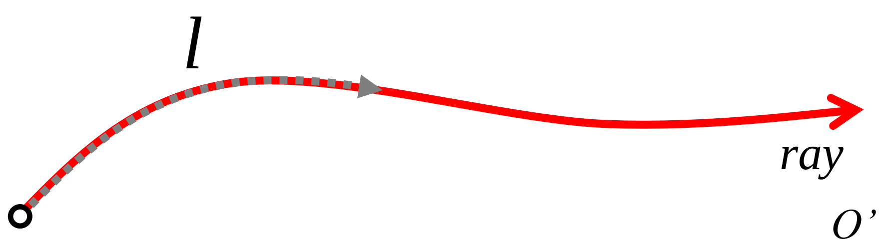
Heron from Alexandria *(~ 50 A.C.)*
«The path that light follows from one point to the other is the **smaller**»

Interprets reflection but fails to interpret refraction





Fermat *(1657 A.C.)*
«The path that light follows between two points is the one traversed in the ***minimum time****!»*



geometrical path

$$t = \int \frac{dl}{\upsilon} = \frac{1}{c}\underbrace{\int n\,dl}_{\text{optical path}}$$

Geometrical optics and Maxwell equations

Is geometrical optics a good approximation of the Maxwell equations?

To answer the question, we need to implement a simplification process of the Maxwell equations.

$$\mathbf{E} = \mathbf{e}(\mathbf{r})e^{ik_o\mathcal{L}(\mathbf{r})}e^{-i\omega t}, \quad \mathbf{H} = \mathbf{h}(\mathbf{r})e^{ik_o\mathcal{L}(\mathbf{r})}e^{-i\omega t}$$

$\mathcal{L}(\mathbf{r})$ Geometrical optical path function *(Real number)*

$\mathbf{e}(\mathbf{r}), \mathbf{h}(\mathbf{r})$ Vector amplitudes *(complex number)*

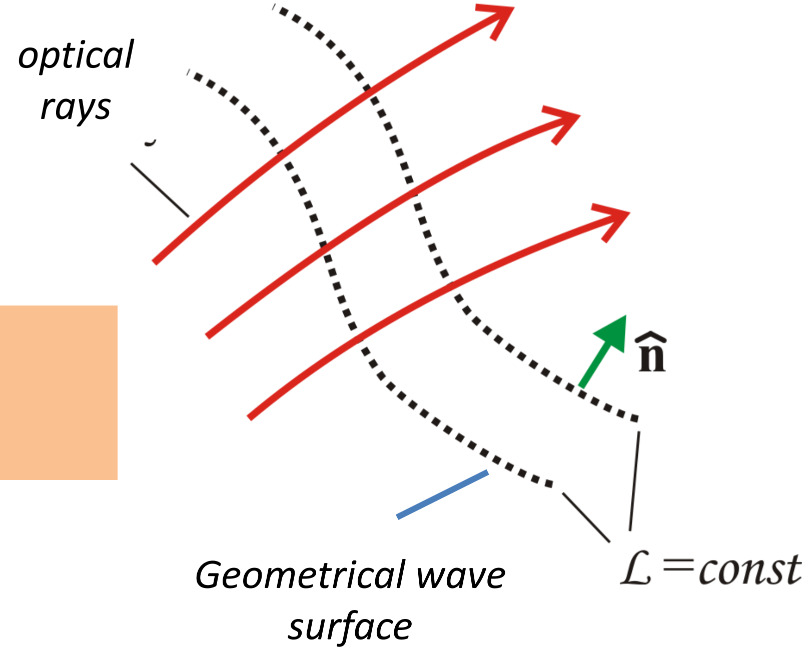
We also assume that there are no free charges and currents $\rho = 0, \quad \mathbf{j} = \mathbf{0}$

Eikonal equation

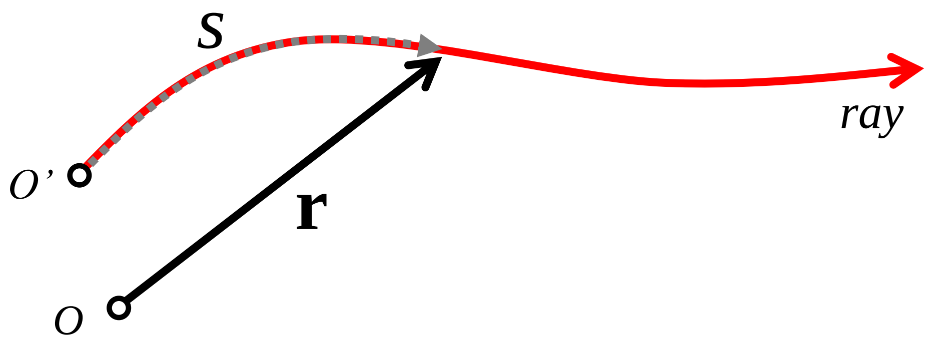
$$[\nabla \mathcal{L}(\mathbf{r})]^2 = \left(\frac{\partial \mathcal{L}(\mathbf{r})}{\partial x}\right)^2 + \left(\frac{\partial \mathcal{L}(\mathbf{r})}{\partial y}\right)^2 + \left(\frac{\partial \mathcal{L}(\mathbf{r})}{\partial z}\right)^2 = n^2(\mathbf{r})$$

H/M radiation flux

$$\langle \mathbf{S} \rangle_t = \upsilon \langle w_e \rangle \hat{\mathbf{n}} \qquad \hat{\mathbf{n}} \equiv \frac{\nabla \mathcal{L}(\mathbf{r})}{|\nabla \mathcal{L}(\mathbf{r})|}$$



optical rays are perpendicular to the geometrical wave surfaces!

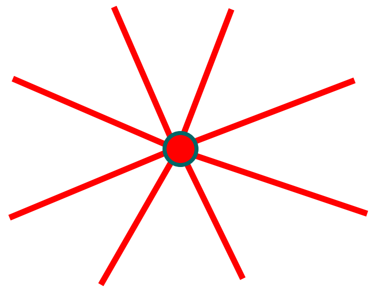


$$\frac{d\mathbf{r}}{ds} = \hat{\mathbf{n}} = \frac{\nabla \mathcal{L}(\mathbf{r})}{|\nabla \mathcal{L}(\mathbf{r})|} \Rightarrow n(\mathbf{r})\frac{d\mathbf{r}}{ds} = \nabla \mathcal{L}(\mathbf{r}) \Rightarrow \frac{d}{ds}\left[n(\mathbf{r})\frac{d\mathbf{r}}{ds}\right] = \nabla n(\mathbf{r})$$

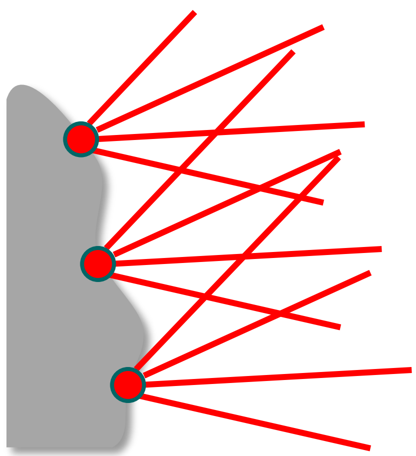
Ray equations

The concept of imaging

Point source

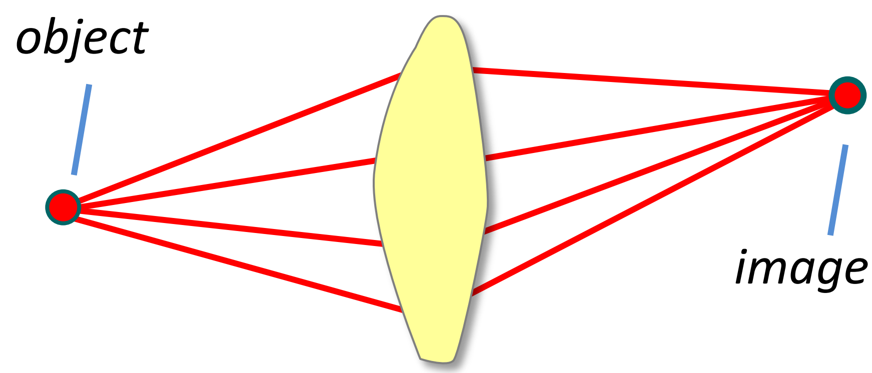


Extended source



Stigmatic imaging

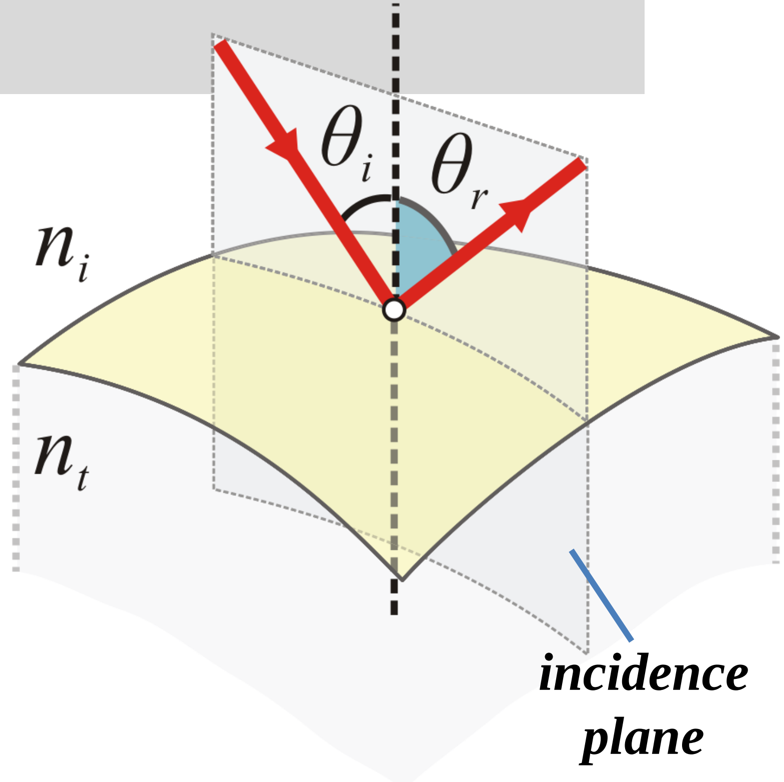
To properly direct all the rays originating from a **single point** of the object to a **single point** of the image



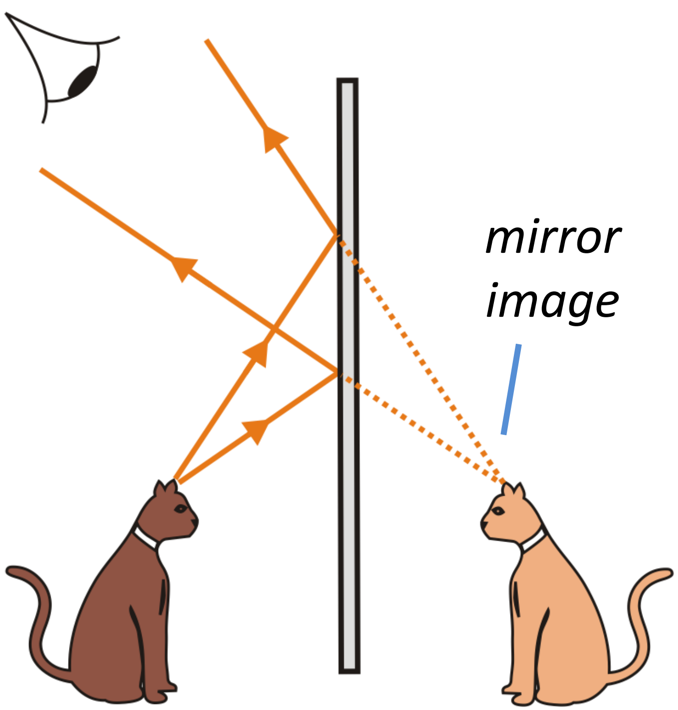
Imaging system

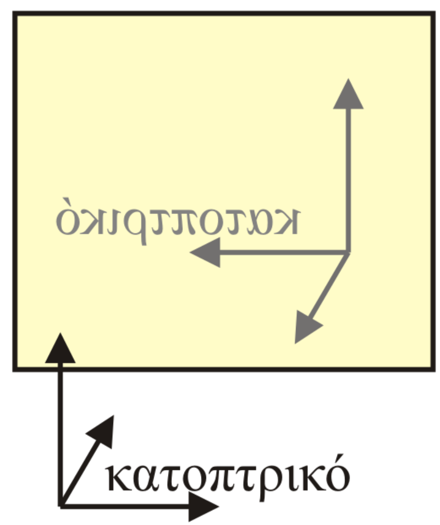
Specular reflection

In specular reflection the reflected ray lies on the incidence plane



$$\theta_r = \theta_i$$

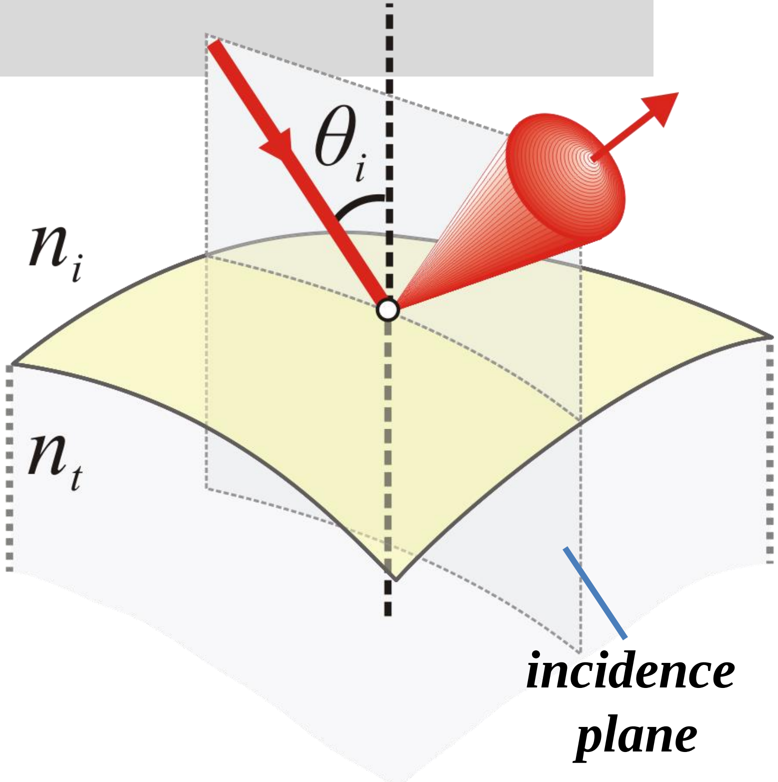


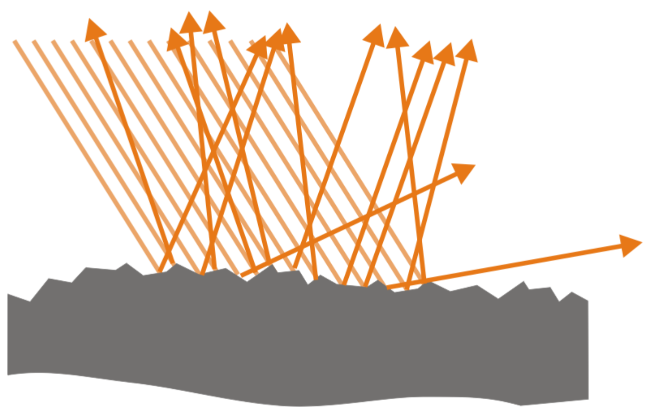


Diffuse reflection

In diffuse reflection, due to the microstructure of the surface, the reflected beam does not lie on the incidence plane

$$\theta_r \neq \theta_i$$





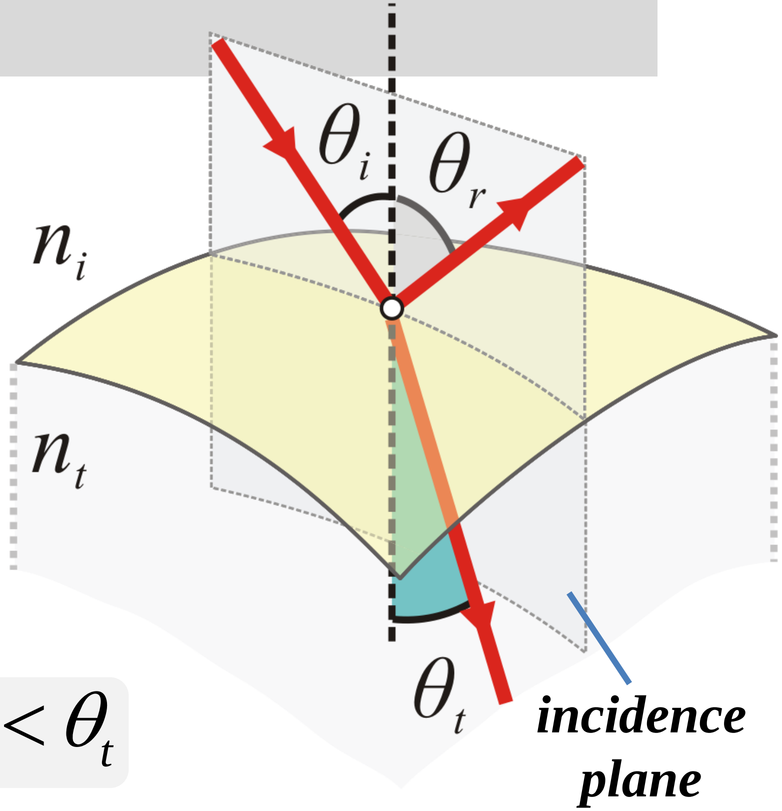
typical microstructure of reflective surface in diffuse reflection

Refraction

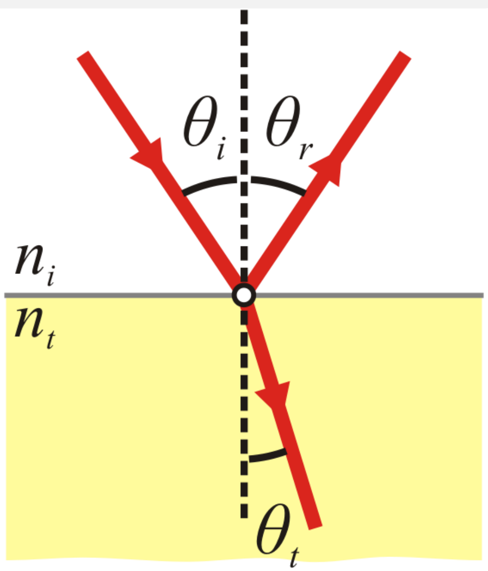
Snell's law

$$n_i \sin\theta_i = n_t \sin\theta_t$$

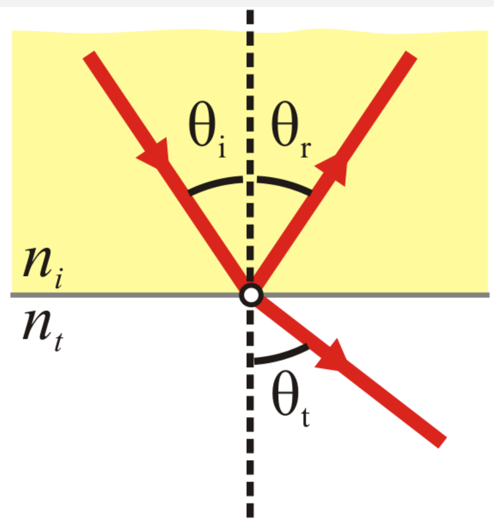
the refracted ray lies on the plane of incidence



$$n_i < n_t \Rightarrow \theta_i > \theta_t$$



$$n_i > n_t \Rightarrow \theta_i < \theta_t$$



Special case: Total Internal reflection (TIR)

Refraction from an optically dense to an optically rare medium $n_i > n_t \Rightarrow \theta_i < \theta_t$

critical angle

$$\theta_t = 90^o \Rightarrow \boxed{\sin\theta_c = \frac{n_t}{n_i}}$$

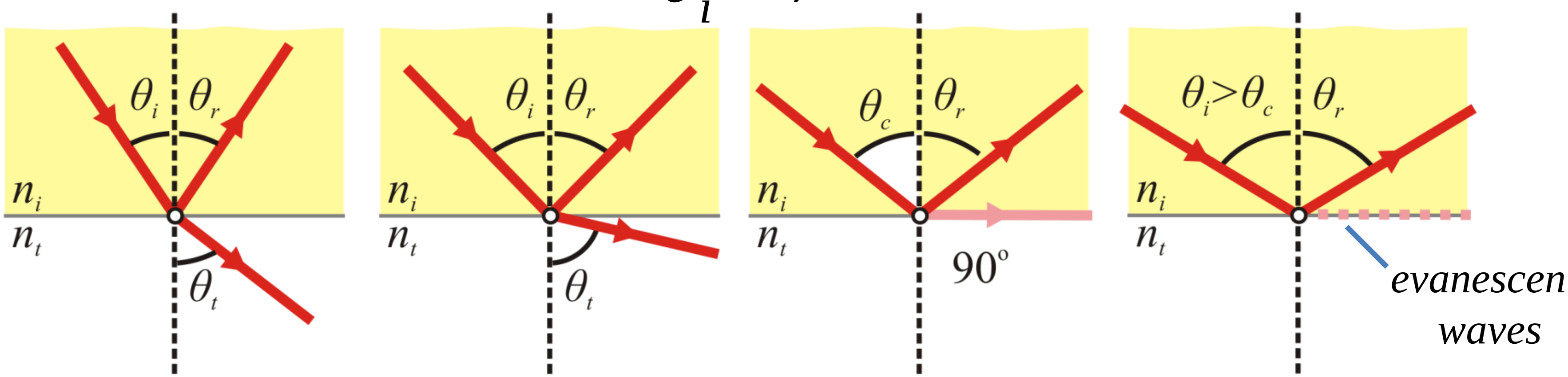
Typical values

$n_i = 1.5\ (glass)$

$n_t = 1\ \ (air)$

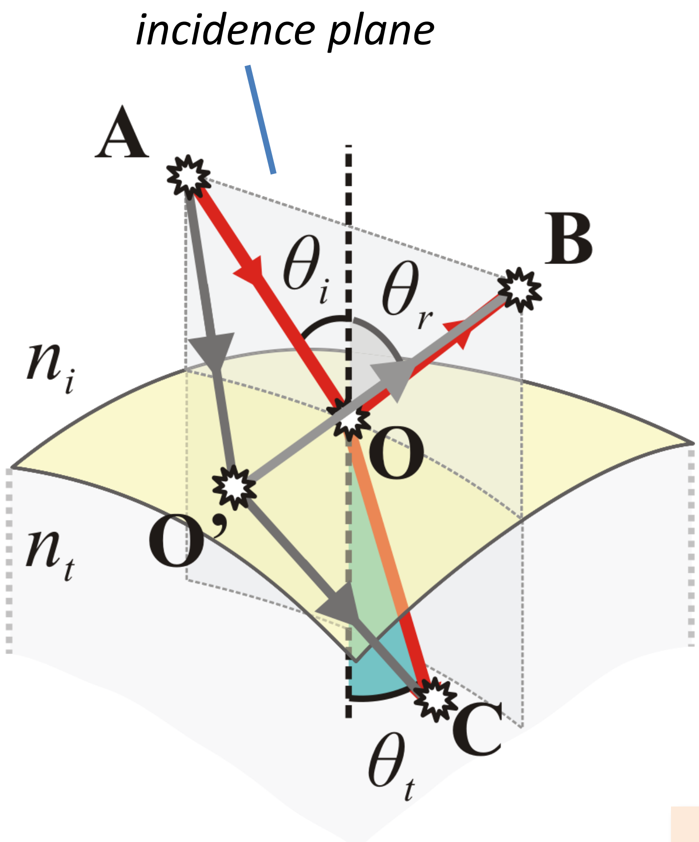
$\theta_c \cong 41.8^o$

$\theta_i \rightarrow$



for incidence angles $\vartheta_i > \vartheta_c$, 100% of the energy is reflected

Reflection, refraction and Fermat's principle



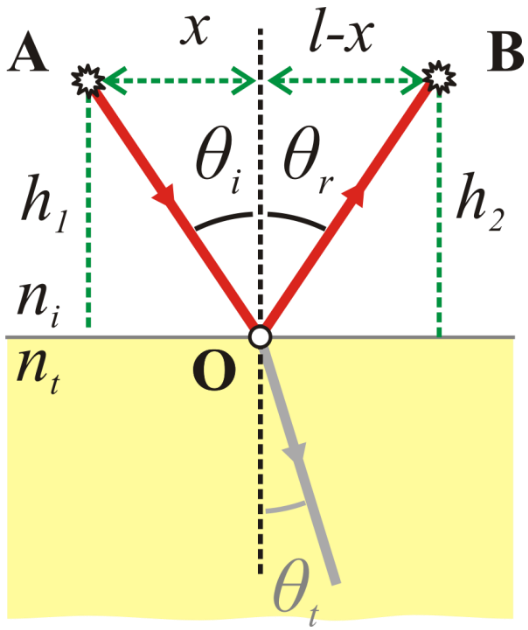
reflected

$$n_i(AO)+n_i(OB)<n_i(AO')+n_i(O'B)$$

refracted

$$n_i(AO)+n_t(OC)<n_i(AO')+n_t(O'C)$$

the reflected and the refracted rays lie on the plane of incidence!



reflection

$$(OPL) = n_i(AO) + n_i(OB)$$
$$= n_i[\sqrt{x^2 + h_1^2} + \sqrt{(l-x)^2 + h_2^2}]$$

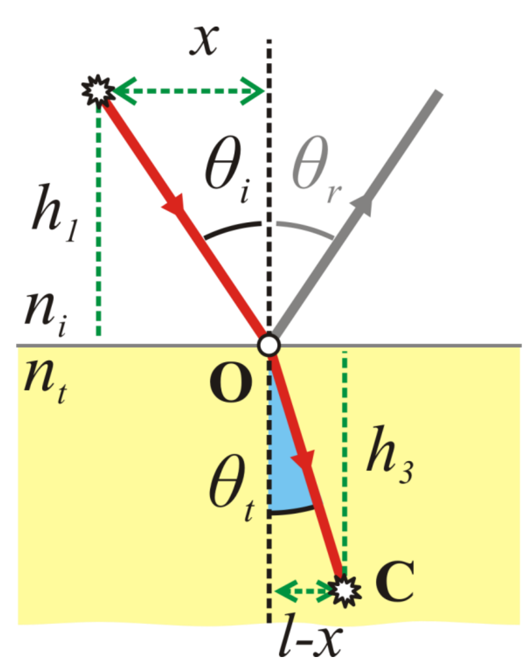
$$(OPL)_{\min} \Rightarrow \sin\theta_i = \sin\theta_r$$

Refraction

$$(OPL) = n_i(AO) + n_t(OC) =$$
$$= n_i\sqrt{x^2 + h_1^2} + n_t\sqrt{(l-x)^2 + h_2^2}$$

$$(OPL)_{\min} \Rightarrow n_i \sin\theta_i = n_t \sin\theta_t$$

Snell's law !



What about the energy of the reflected/ refracted beams?

The geometrical optics can not provide any information about the energy that is reflected or transmitted through a transparent interface !

Maxwell equations

$$\nabla \cdot \mathbf{D} = \rho, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$$

Boundary conditions

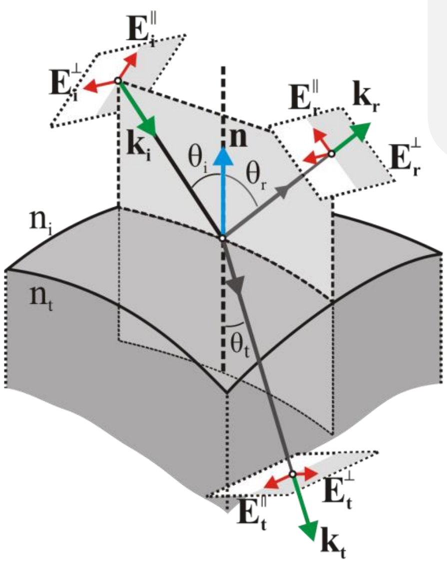
$$(\mathbf{D}_i + \mathbf{D}_r - \mathbf{D}_t)\cdot \hat{\mathbf{n}} = 0, \quad (\mathbf{B}_i + \mathbf{B}_r - \mathbf{B}_t)\cdot \hat{\mathbf{n}} = 0,$$

$$(\mathbf{E}_i + \mathbf{E}_r - \mathbf{E}_t)\times \hat{\mathbf{n}} = 0, \quad (\mathbf{H}_i + \mathbf{H}_r - \mathbf{H}_t)\times \hat{\mathbf{n}} = 0$$

Plane of incidence

$$r_\perp \equiv \left(\frac{E_r^0}{E_i^0}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp \equiv \left(\frac{E_t^0}{E_i^0}\right)_\perp = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

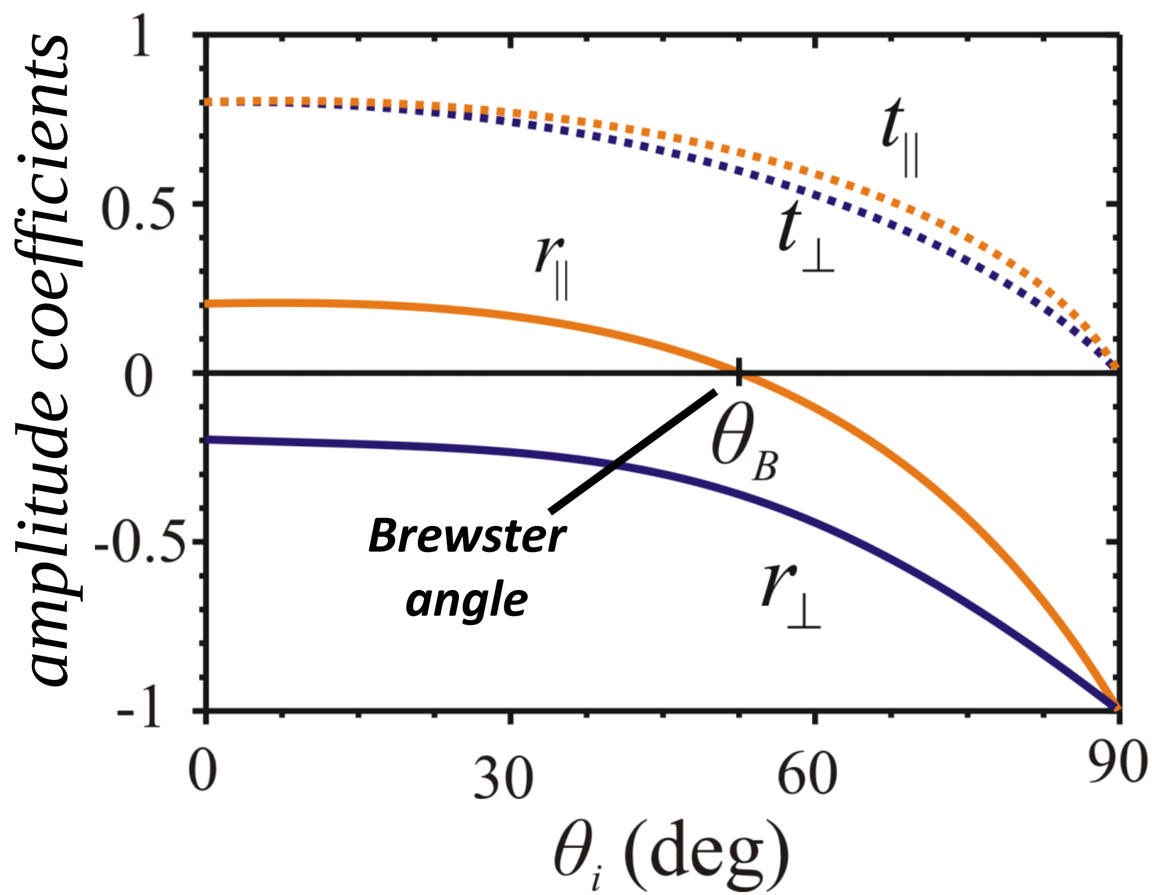


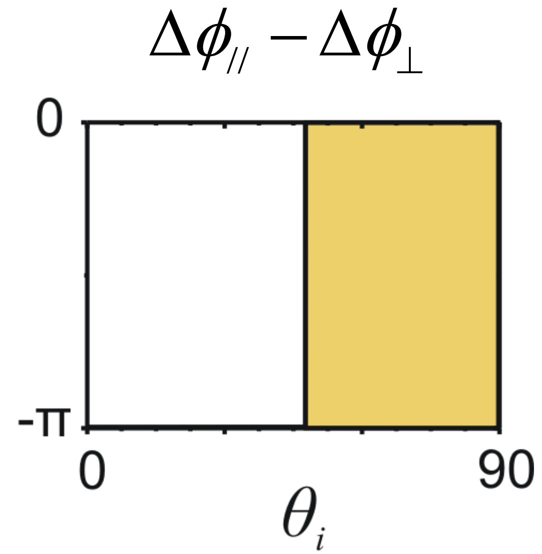
transverse plane

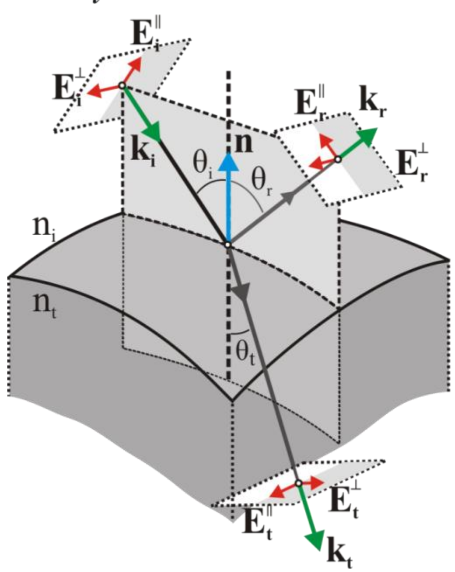
$$r_\parallel \equiv \left(\frac{E_r^0}{E_i^0}\right)_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$t_\parallel \equiv \left(\frac{E_t^0}{E_i^0}\right)_\parallel = \frac{2 n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t}$$

Examples: $n_t > n_i$



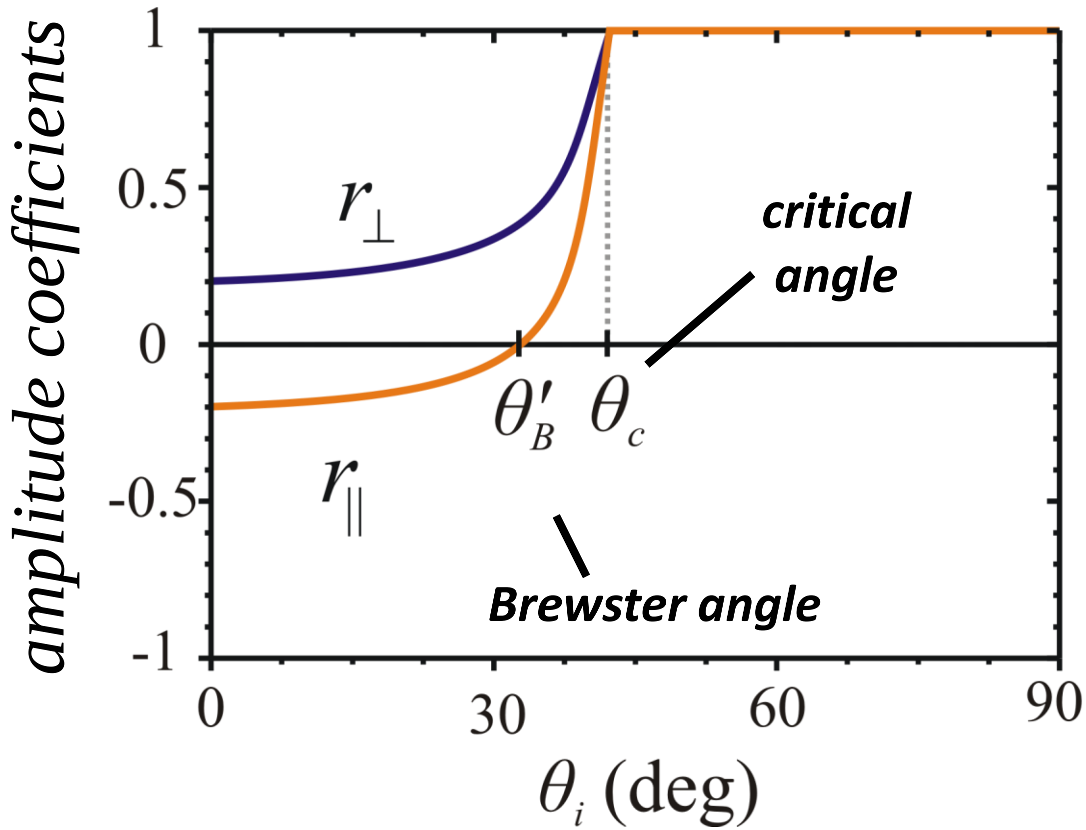


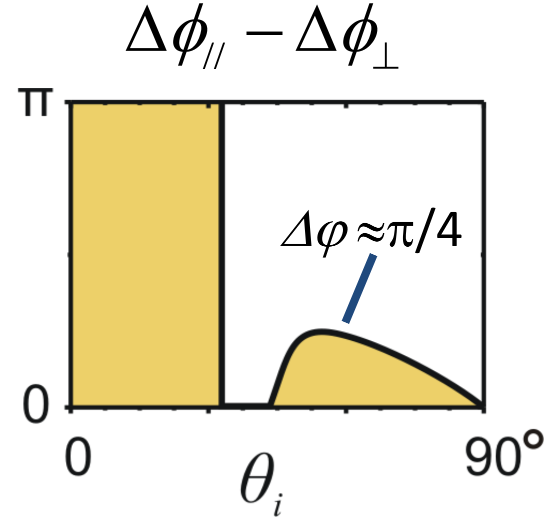


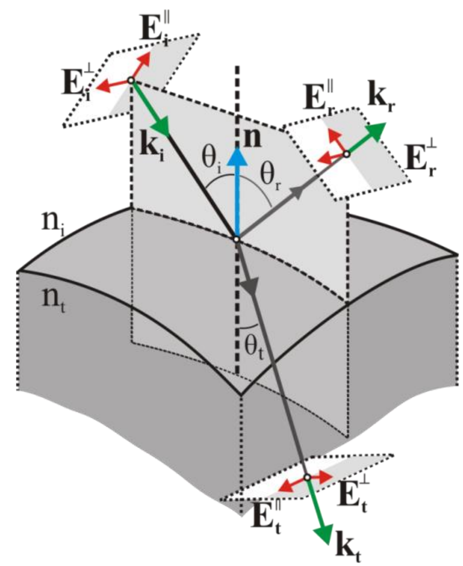
$$\tan\theta_B = \frac{n_t}{n_i}$$

At Brewster angle, the reflected beam is polarized perpendicularly to the plane of incidence !

Παραδείγματα: $n_i > n_t$







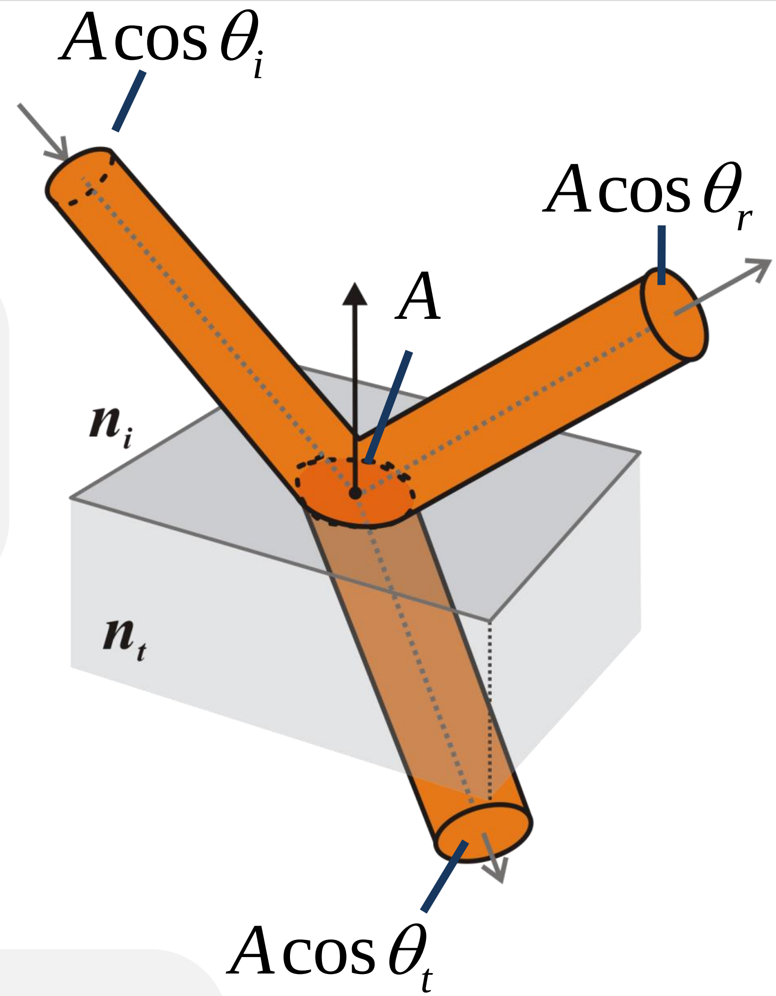
$$\tan\theta'_B = \frac{n_i}{n_t} \Rightarrow \theta'_B + \theta_B = 90^0$$

$$\sin\theta_c = \frac{n_t}{n_i}$$

Reflectivity, Transmissivity

Reflectivity $$R \equiv \frac{\text{reflected power}}{\text{incident power}}$$

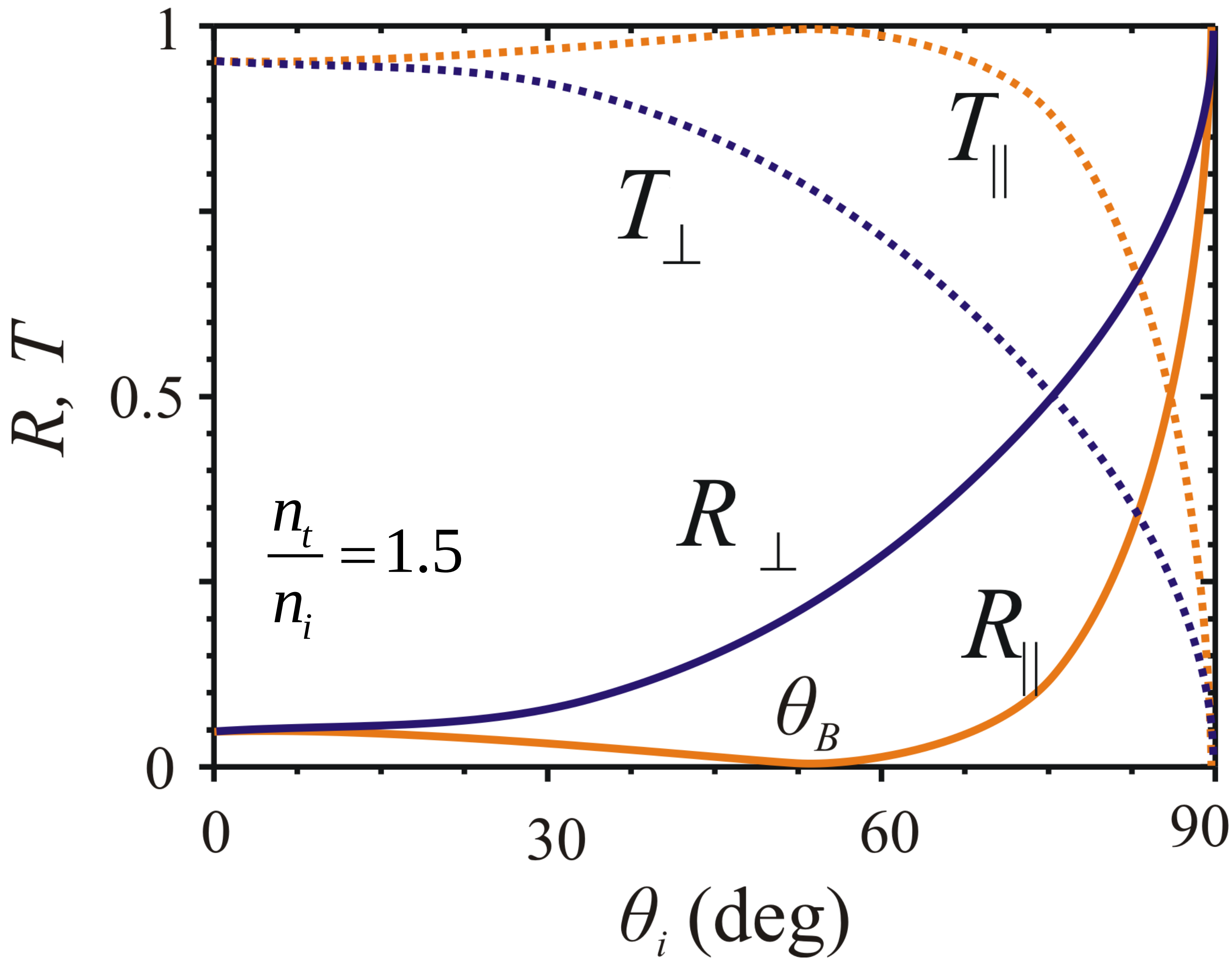
$$R \equiv \frac{I_r(A\cos\theta_r)}{I_i(A\cos\theta_i)} = \frac{I_r}{I_i} = \frac{\frac{1}{2}n_i\varepsilon_o c\left|E_r^0\right|^2}{\frac{1}{2}n_i\varepsilon_o c\left|E_i^0\right|^2} = r^2$$



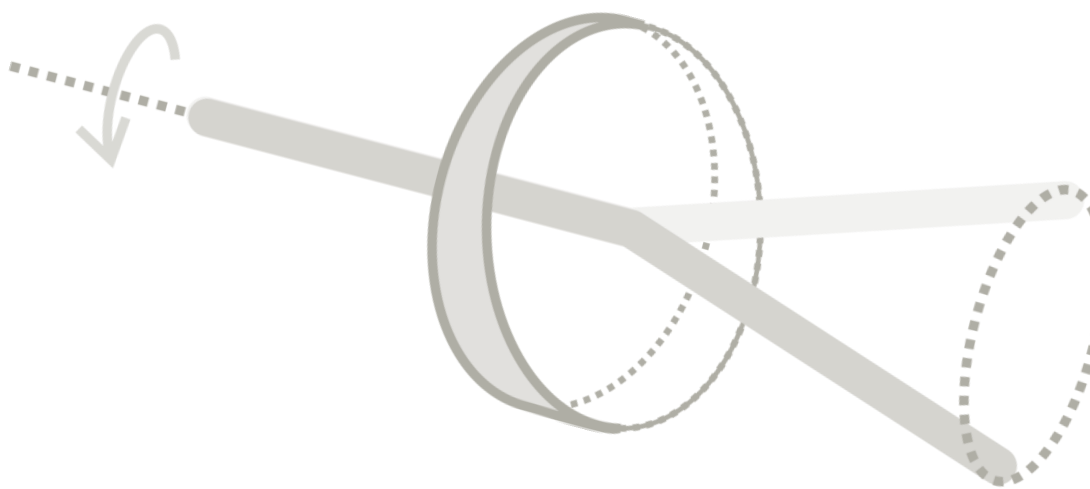
Transmisivity $$T \equiv \frac{\text{refracted power}}{\text{incident power}}$$

$$T = \frac{I_t(A\cos\theta_t)}{I_i(A\cos\theta_i)} = \frac{\frac{1}{2}n_t\varepsilon_o c\left|E_t^0\right|^2}{\frac{1}{2}n_i\varepsilon_o c\left|E_i^0\right|^2}\frac{\cos\theta_t}{\cos\theta_i} = \left(\frac{n_t\cos\theta_t}{n_i\cos\theta_i}\right)t^2$$

examples: Reflectivity, Transmissivity



Prisms



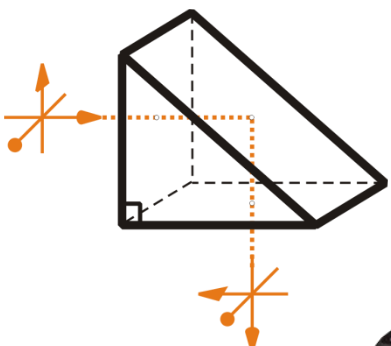
Dimitris Papazoglou dpapa@materials.uoc.gr

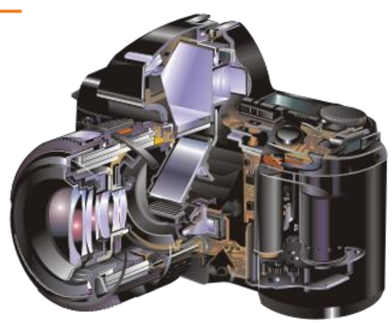
Prisms

Reflection prisms

Can replace mirrors
ensuring stability

they operate using total internal reflection

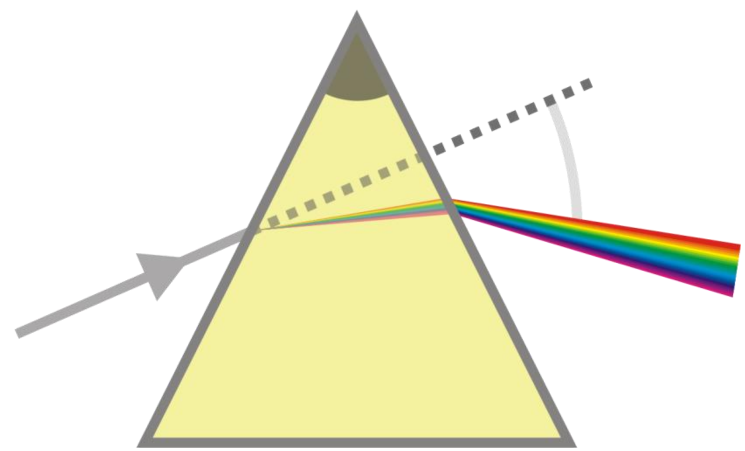




Dispersion prisms

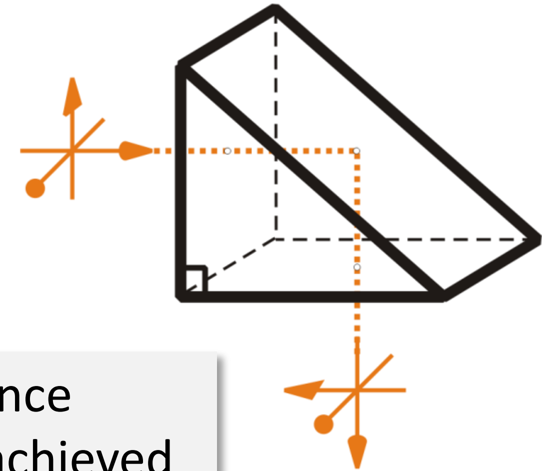
Can be used as spectral analyzers

they operate using dispersion

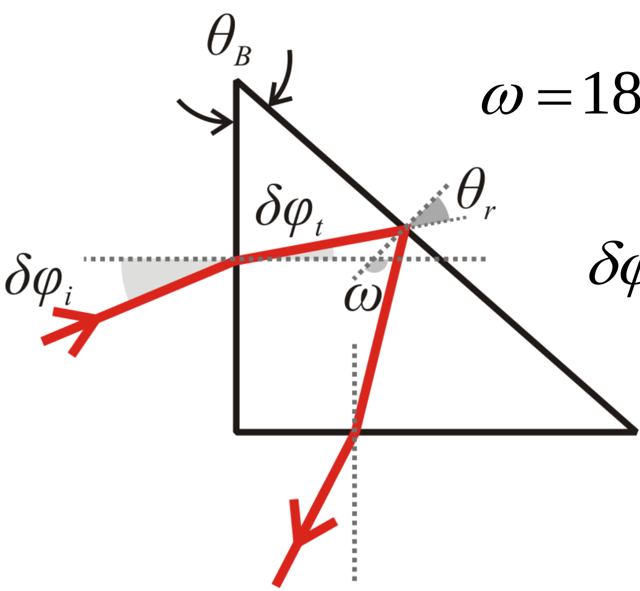


Right angle prism

replaces the plane mirror,
rotates the optical axis by 90° and inverts the image



if the incoming beam is not perpendicular to the entrance surface of the prism, total internal reflection might not be achieved



$$\omega = 180^\circ - \theta_B \Rightarrow \theta_r = \theta_B - \delta\varphi_t, \qquad \theta_r \geq \sin^{-1}\left(\frac{n_i}{n_t}\right) \equiv \theta_{cr}$$

$$\delta\varphi_t = \sin^{-1}\left(\frac{n_i}{n_t}\sin\delta\varphi_i\right) \Rightarrow \sin^{-1}\left(\frac{n_i}{n_t}\sin\delta\varphi_i\right) \leq \theta_B - \theta_{cr}$$

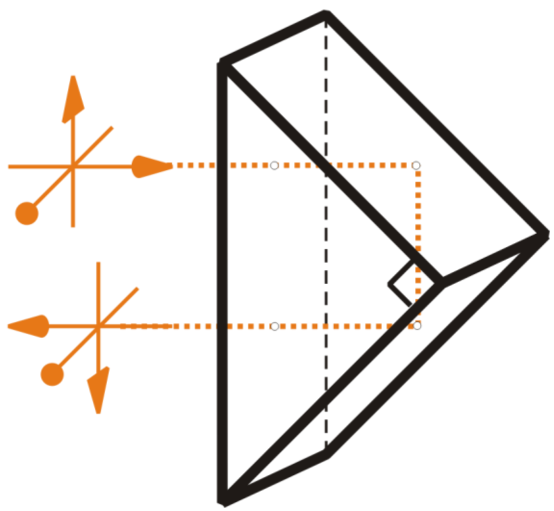
$$\Delta\varphi_t^{BK10} = 8^o\,28'$$

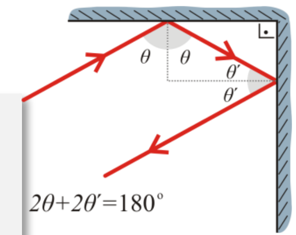
$$\Delta\varphi_t^{K8} = 5^o\,40'$$

BK10 θ_{cr} =39°36’
K8 θ_{cr} =41°16’

typ. $n = 1.5 \Rightarrow \theta_{cr} \cong 41.8^o \Rightarrow \Delta\varphi_t^{typ} = 4^o\,47'$

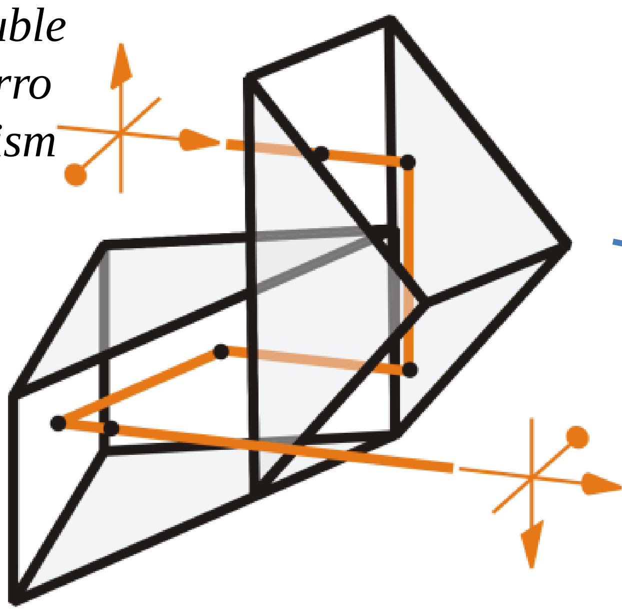
Porro prism





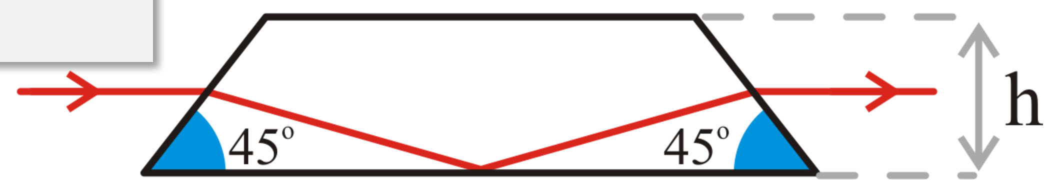
It replaces the retro-reflector, irrespective of the angle at which a beam enters the prism, the outgoing beam is parallel and displaced to it.
Turns the optical axis by 180 °.
Causes partial reversal of the image.

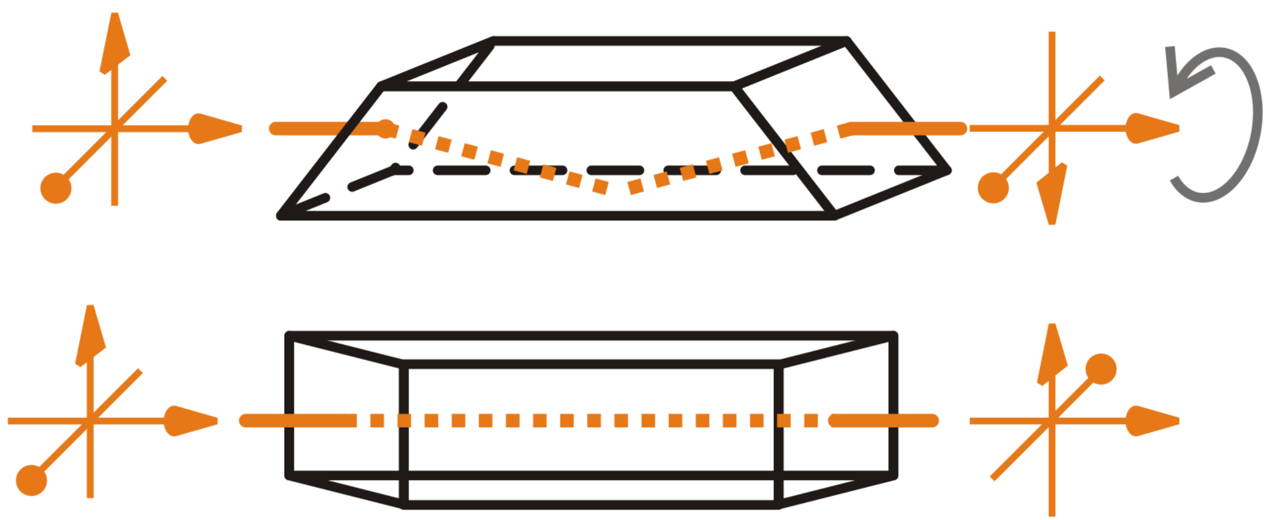
Double Porro prism



Dove prism

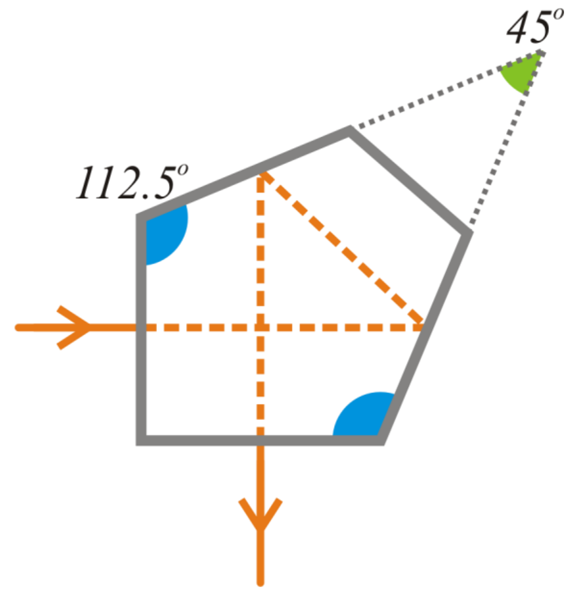
Generates a mirror image.
It works only with a collimated beam.
By rotating it around its optical axis the image rotates at a twice angle.

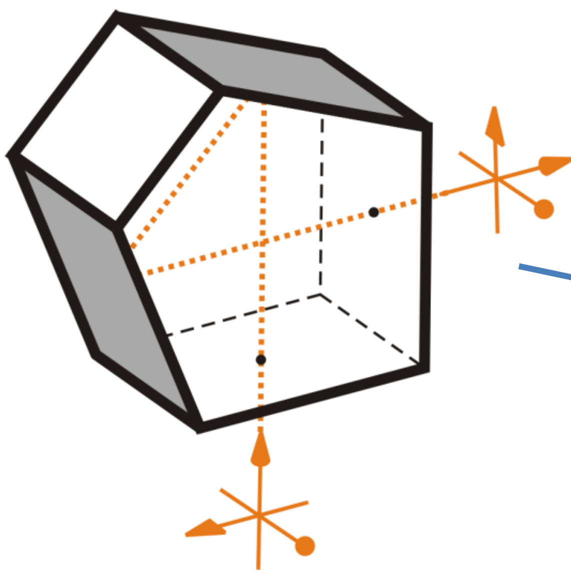


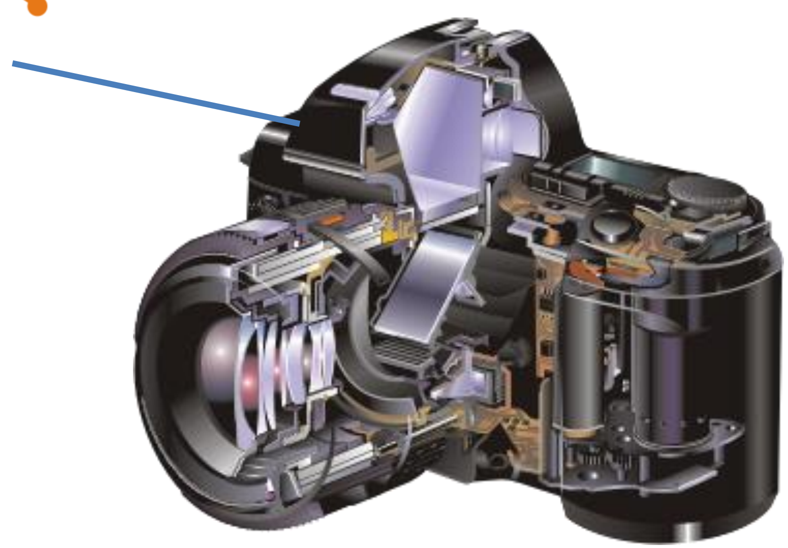


Pentaprism

Rotates the optical axis by 90º.
The deflection is stable and independent of the orientation of the prism.
Creates an erect image.

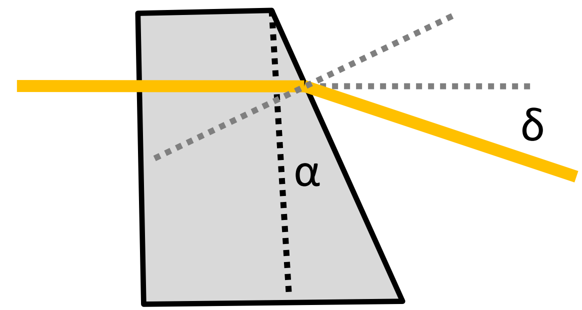






Wedge prism

In an optical wedge the prism angle α is very small



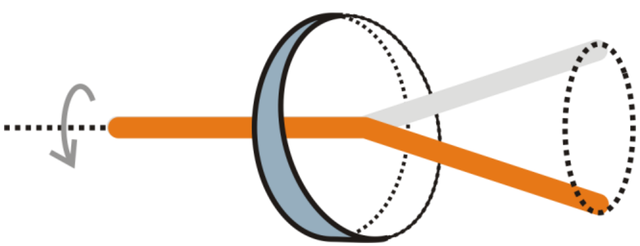
$$\left.\begin{aligned} n\sin\alpha = \sin\theta_t &\Rightarrow n\alpha \approx \theta_t \\ \delta = \theta_t - \alpha & \end{aligned}\right\} \Rightarrow \boxed{\delta \approx (n-1)a}$$

deviation angle

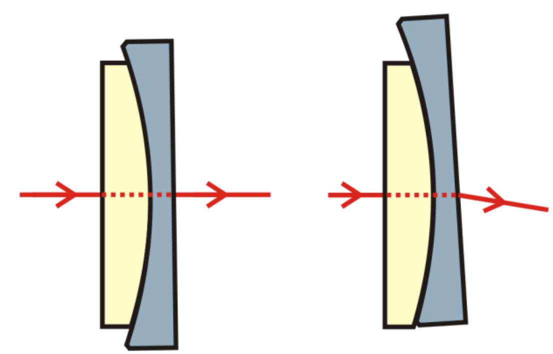
Typical values

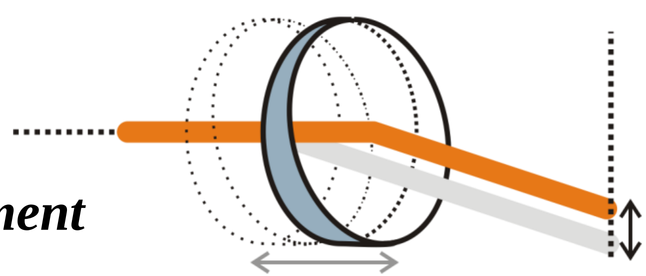
$$\left.\begin{aligned} n &\cong 1.5 \\ a &\leq 4^o \end{aligned}\right\} \Rightarrow \delta \leq 2^o$$

beam rotation



variable wedge



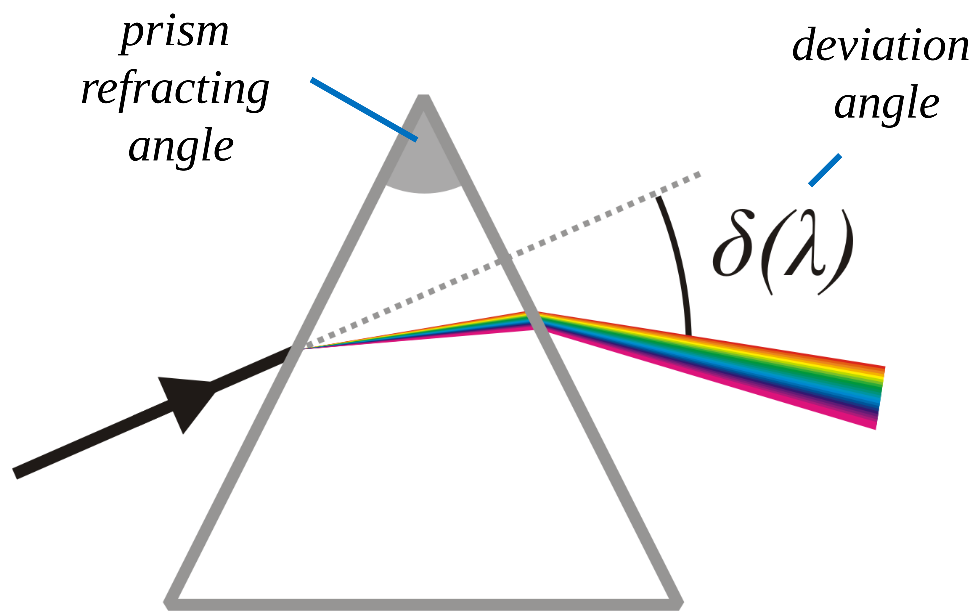


beam displacment

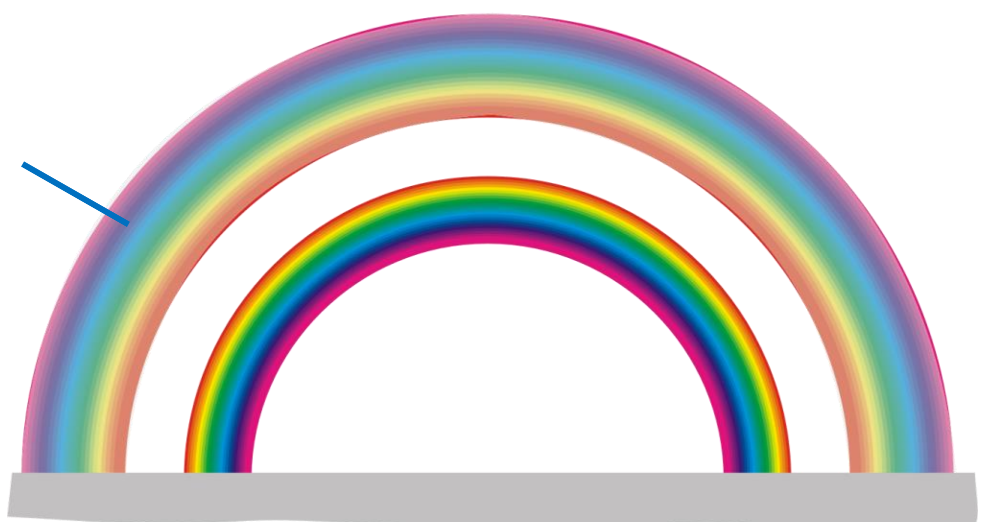
Dimitris Papazoglou dpapa@materials.uoc.gr

Dispersion prisms

Due to the dispersion, as white light passes through a prism, it is analyzed into its color components

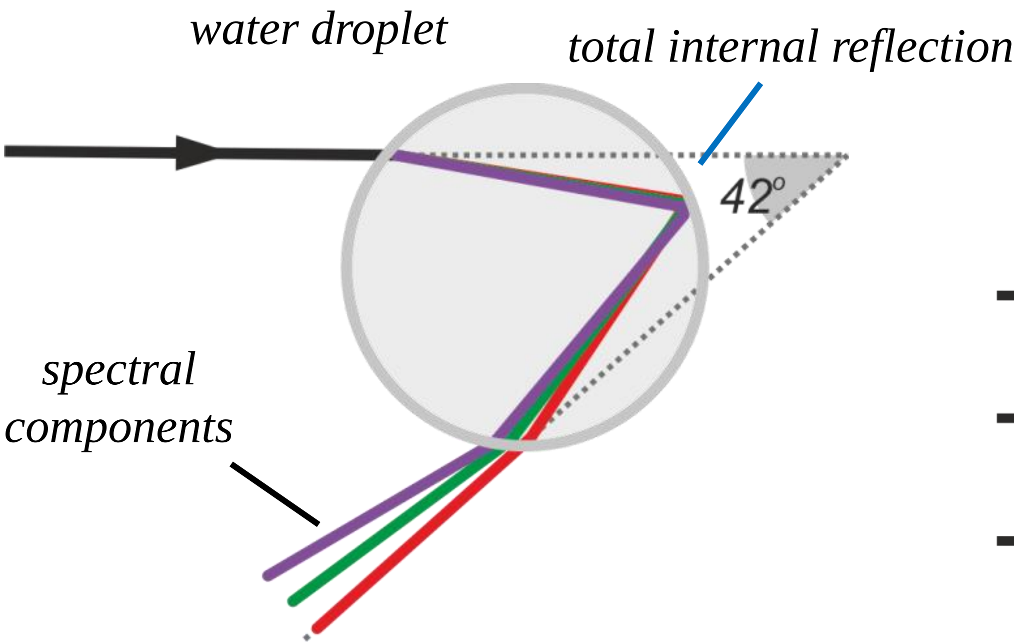


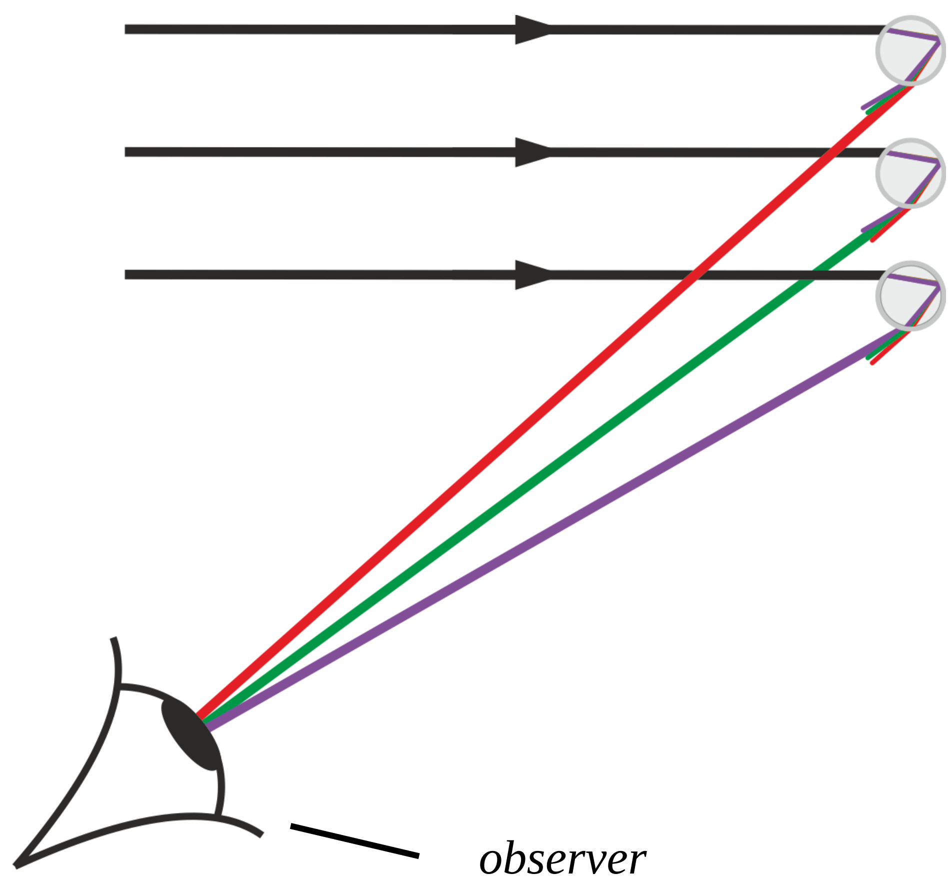
prism diopters: 100 tanδ

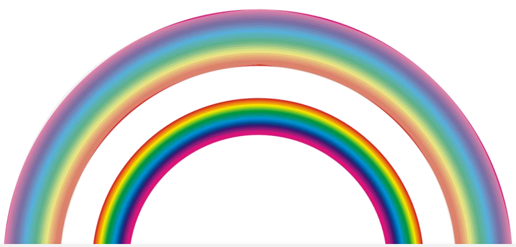


dispersion effect is responsible for the rainbow

Rainbow



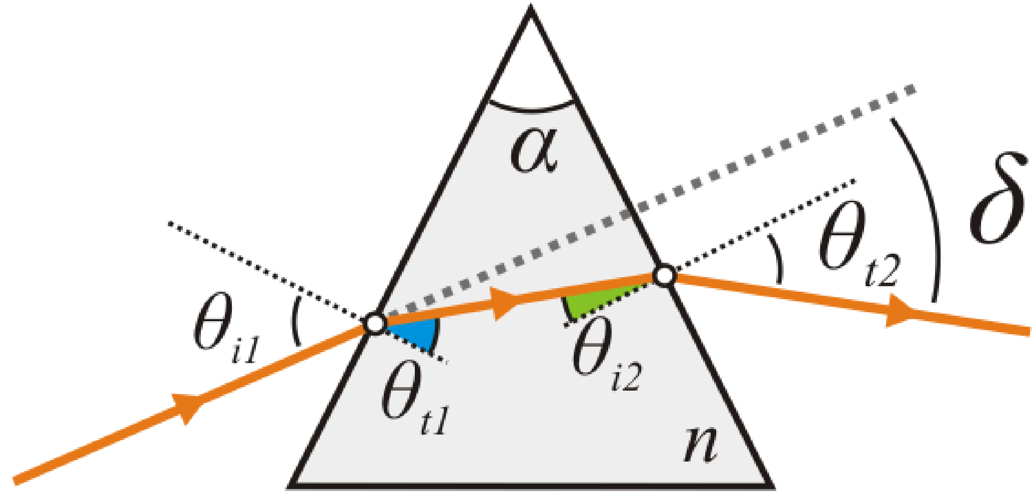




refraction from many droplets results to the formation of the rainbow

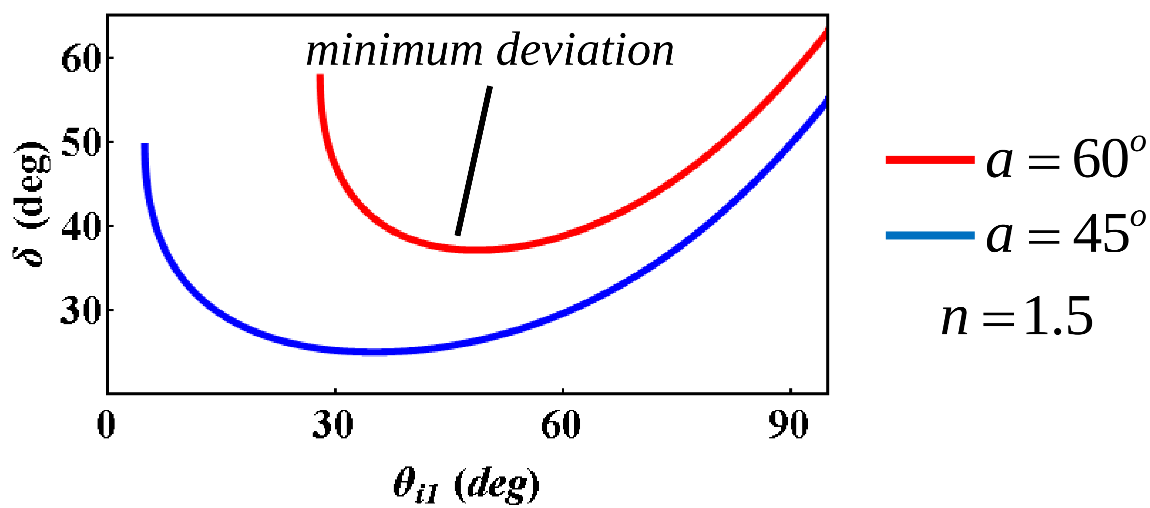
Dispersion prisms

$$\left.\begin{aligned}\delta &= (\theta_{i1}-\theta_{t1})+(\theta_{t2}-\theta_{i2})\\ a &= \theta_{t1}+\theta_{i2}\end{aligned}\right\}\Rightarrow$$

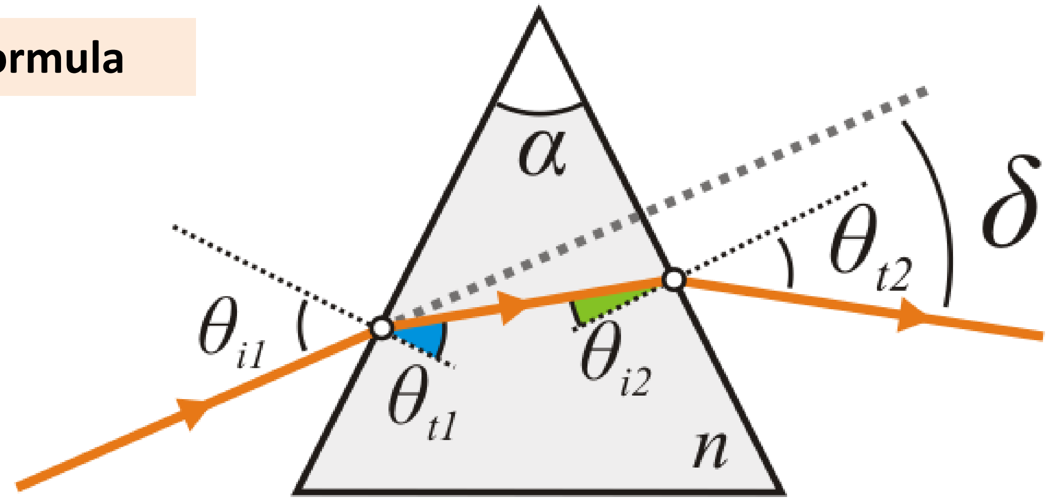


$$\delta = \theta_{i1}+\theta_{t2}-a$$

$$\delta = \theta_{i1}+\sin^{-1}\left[\sin\alpha\sqrt{n^2-\sin^2\theta_{i1}}-\sin\theta_{i1}\cos\alpha\right]-a$$



analytical derivation of the deviation formula



$$\left.\begin{aligned} \sin\theta_{i_1} &= n\sin\theta_{t_1} \\ n\sin\theta_{i_2} &= \sin\theta_{t_2} \\ a &= \theta_{t_1} + \theta_{i_2} \\ \delta &= \theta_{i1} + \theta_{t2} - a \end{aligned}\right\} \Rightarrow$$

$$\theta_{t_2} = \sin^{-1}[n\sin\theta_{i_2}] = \sin^{-1}[n\sin(a-\theta_{t_1})]$$

$$= \sin^{-1}[n(\sin a\cos\theta_{t_1} - \cos a\sin\theta_{t_1})]$$

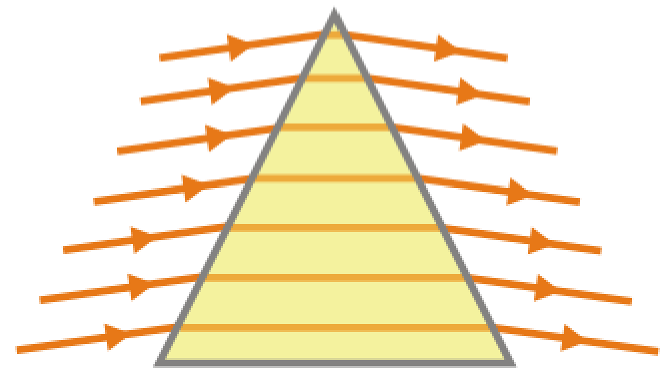
$$= \sin^{-1}[n\sin a\sqrt{1-\sin^2\theta_{t_1}} - \cos a\sin\theta_{i_1})]$$

$$= \sin^{-1}[\sin a\sqrt{n^2-\sin^2\theta_{i_1}} - \cos a\sin\theta_{i_1})] \Rightarrow$$

$$\boxed{\delta = \theta_{i1} + \sin^{-1}\left[\sin\alpha\sqrt{n^2-\sin^2\theta_{i1}} - \sin\theta_{i1}\cos\alpha\right] - a}$$

At minimum deviation:

$$\theta_{t_1} = \theta_{i_2} \Rightarrow \theta_{i_1} = \theta_{t_2}$$



Minimum deviation conditions

$$\left.\begin{aligned} \delta_{\min} = 2\theta_{i_1} - \alpha \Rightarrow \theta_{i_1} = \frac{\delta_{\min} + \alpha}{2} \\ a = 2\theta_{t_1} \Rightarrow \theta_{t_1} = \frac{a}{2} \Rightarrow \sin\theta_{i_1} = n\sin\frac{a}{2} \end{aligned}\right\} \Rightarrow n\sin\frac{a}{2} = \sin\left(\frac{\delta_{\min} + \alpha}{2}\right) \Rightarrow$$

$$n = \frac{\sin\left(\frac{\delta_{\min} + \alpha}{2}\right)}{\sin\frac{a}{2}}$$

Derivation of the minimum deviation condition

$$\boxed{\theta_{t_1} = \theta_{i_2} \Rightarrow \theta_{i_1} = \theta_{t_2}}$$

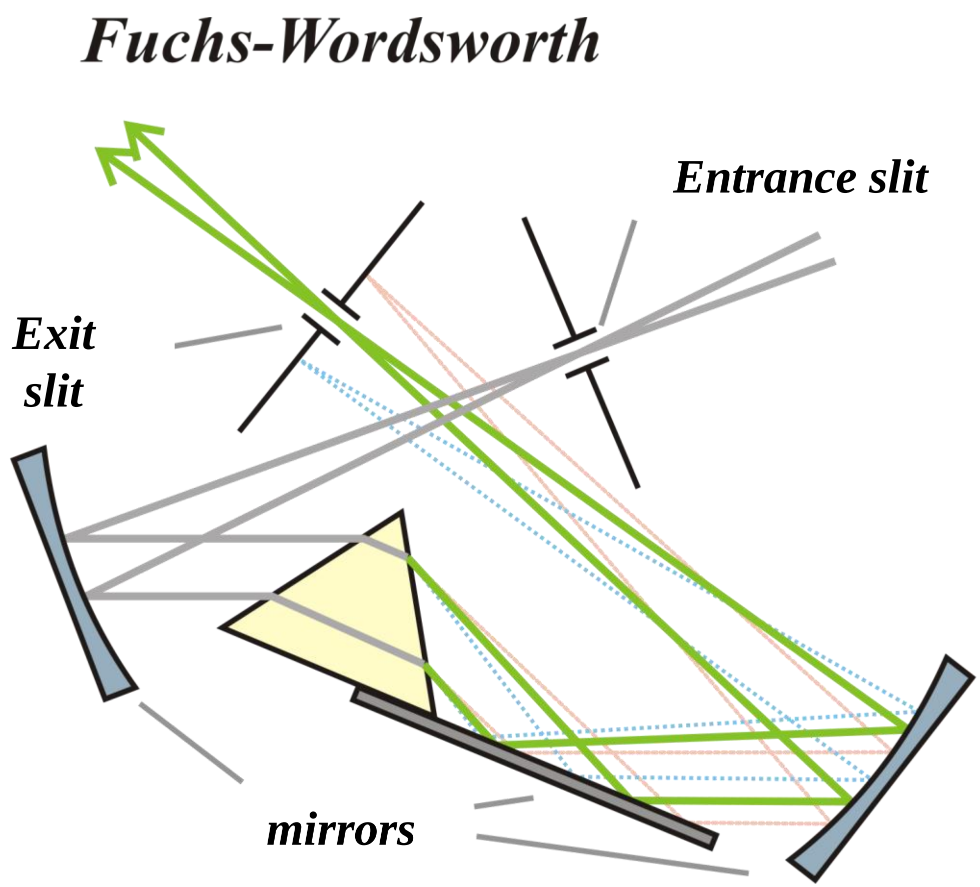
$$\left.\begin{aligned}
&\delta = \theta_{i_1} + \theta_{t_2} - \alpha \Rightarrow \frac{d\delta}{d\theta_{i_1}} = 1 + \frac{d\theta_{t_2}}{d\theta_{i_1}} \overset{\min}{\equiv} 0 \Rightarrow \frac{d\theta_{t_2}}{d\theta_{i_1}} = -1 \\
&a = \theta_{t_1} + \theta_{i_2} \Rightarrow d\theta_{t_1} = -d\theta_{i_2} \\
&\sin\theta_{i_1} = n\sin\theta_{t_1} \Rightarrow \cos\theta_{i_1}\, d\theta_{i_1} = n\cos\theta_{t_1}\, d\theta_{t_1} \\
&n\sin\theta_{i_2} = \sin\theta_{t_2} \Rightarrow \cos\theta_{t_2}\, d\theta_{t_2} = n\cos\theta_{i_2}\, d\theta_{i_2}
\end{aligned}\right\} \Rightarrow$$

$$\frac{\cos\theta_{i_1}}{\cos\theta_{t_2}}\frac{d\theta_{i_1}}{d\theta_{t_2}} = \frac{\cos\theta_{t_1}}{\cos\theta_{i_2}}\frac{d\theta_{t_1}}{d\theta_{i_2}} \Rightarrow \frac{\cos\theta_{i_1}}{\cos\theta_{t_2}} = \frac{\cos\theta_{t_1}}{\cos\theta_{i_2}} \Rightarrow$$

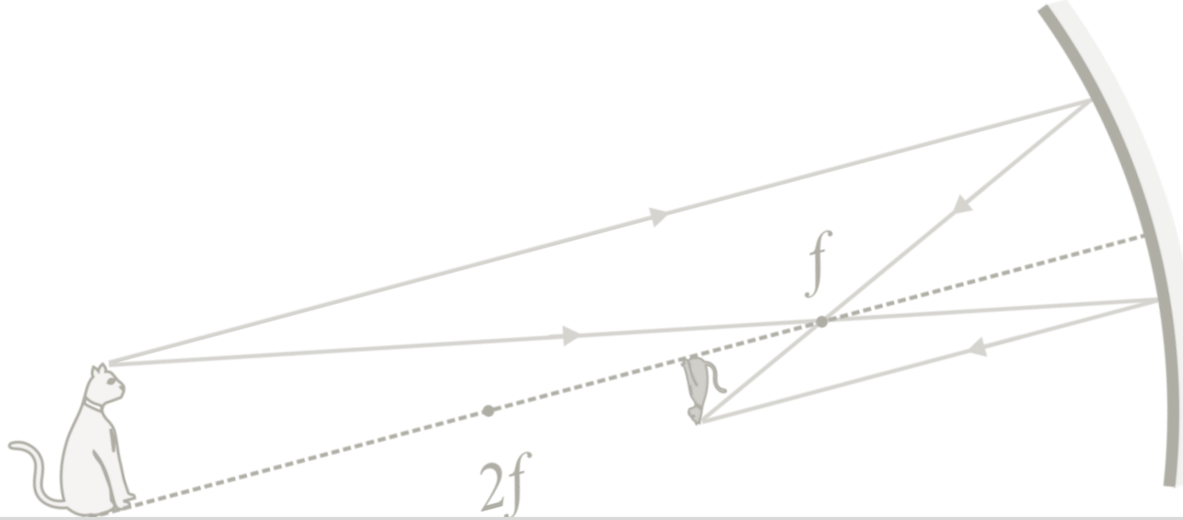
$$\frac{1-\sin^2\theta_{i_1}}{1-\sin^2\theta_{t_2}} = \frac{\overbrace{1-n^2\sin^2\theta_{t_1}}^{Snell}}{1-n^2\sin^2\theta_{i_2}} = \frac{1-\sin^2\theta_{t_1}}{1-\sin^2\theta_{i_2}} \Rightarrow \boxed{\theta_{t_1} = \theta_{i_2} \Rightarrow \theta_{i_1} = \theta_{t_2}}$$

Monochromators

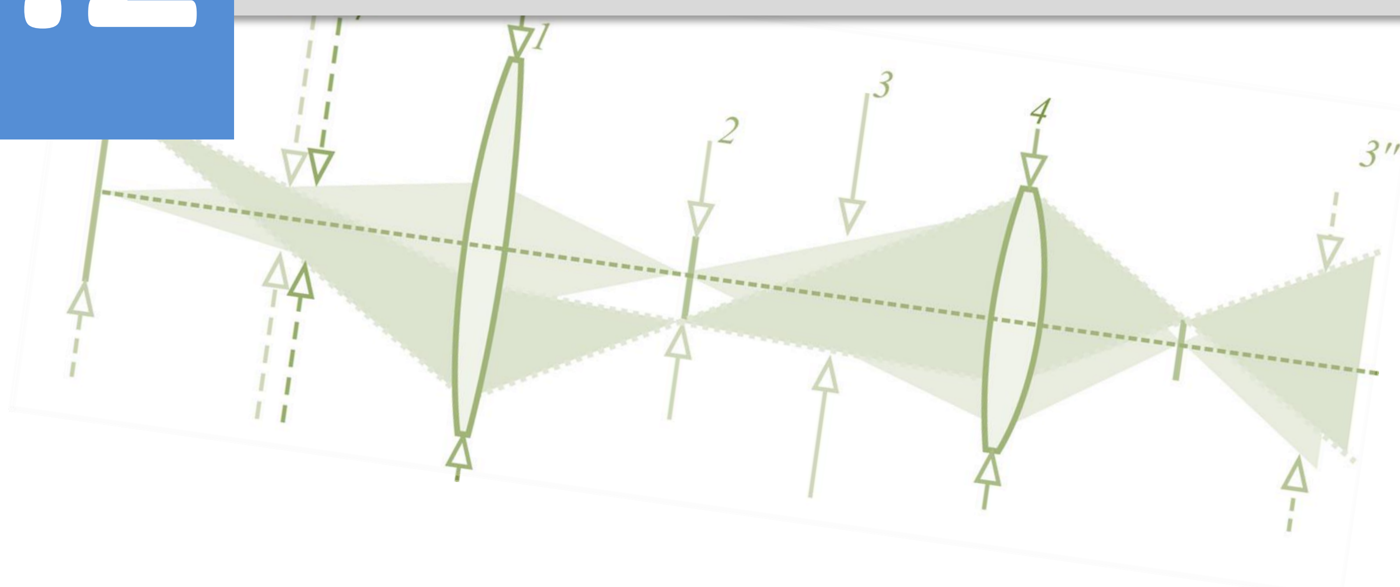
Monochromators are composite optical systems designed to **isolate** specific regions of the optical radiation spectrum emitted by a source.



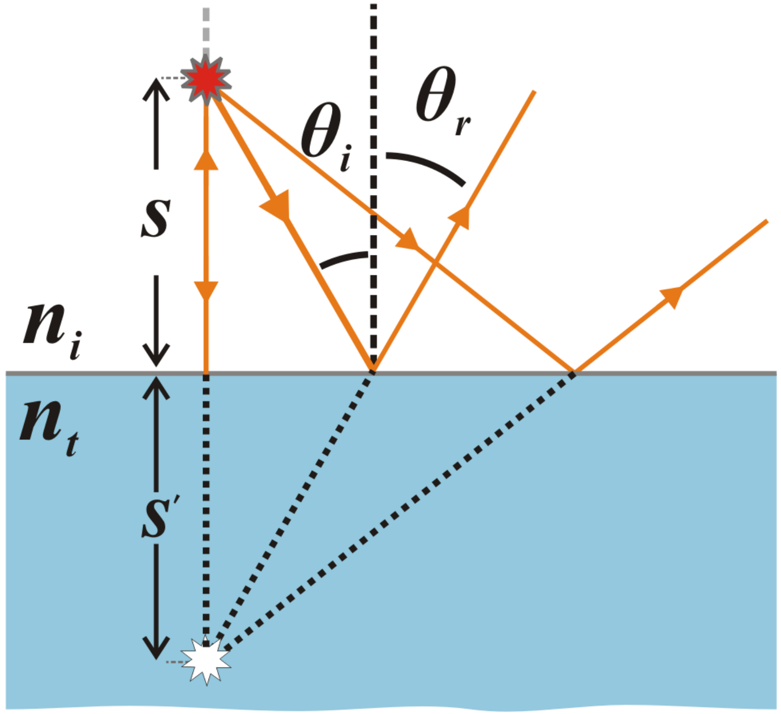
The study of extended spectral areas of a source is performed using a spectrometer.



3.2 Simple optical systems



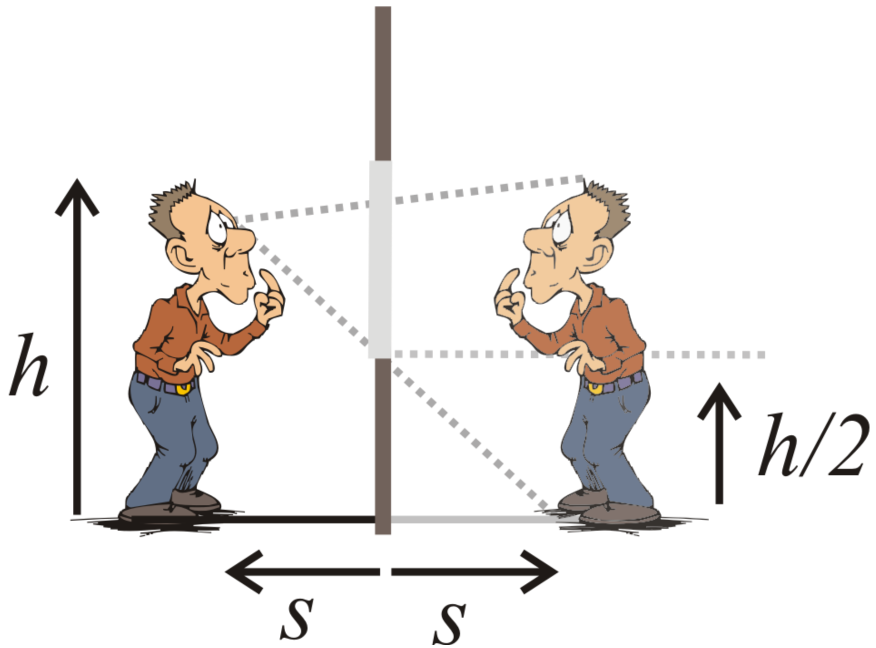
Reflection from a plane surface



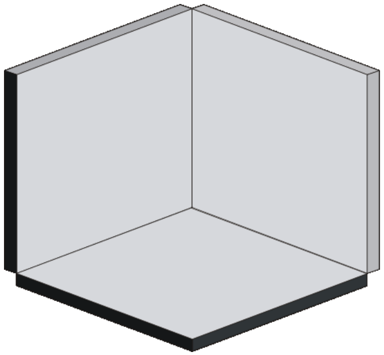
$$\theta_i = \theta_r \Rightarrow s' = -s$$

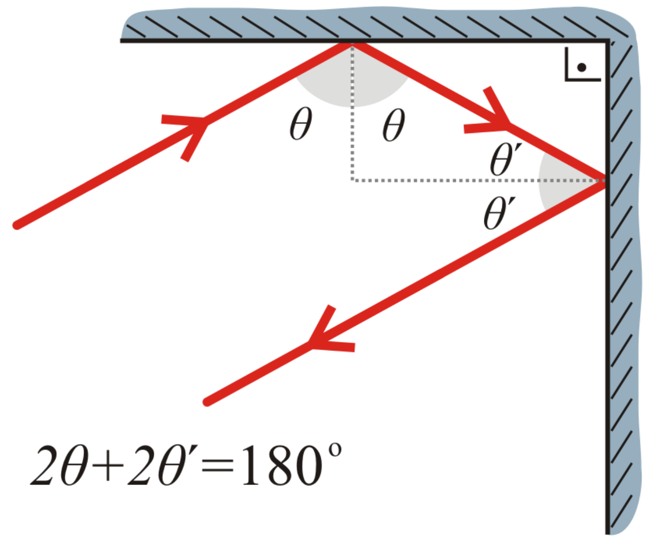
stigmatic imaging!

Application:
what should be the height of the mirror so that the observer is able to see his whole body?



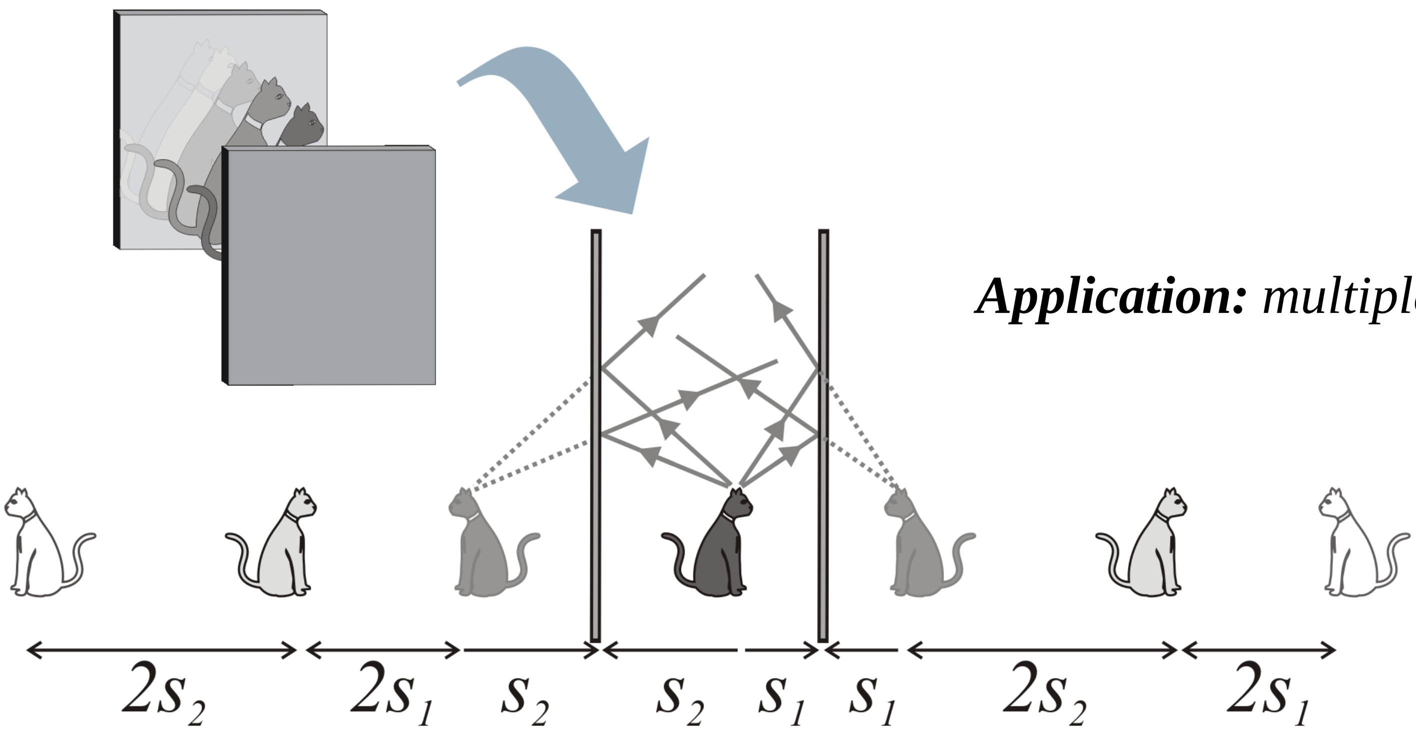
Application: *retro-reflector*



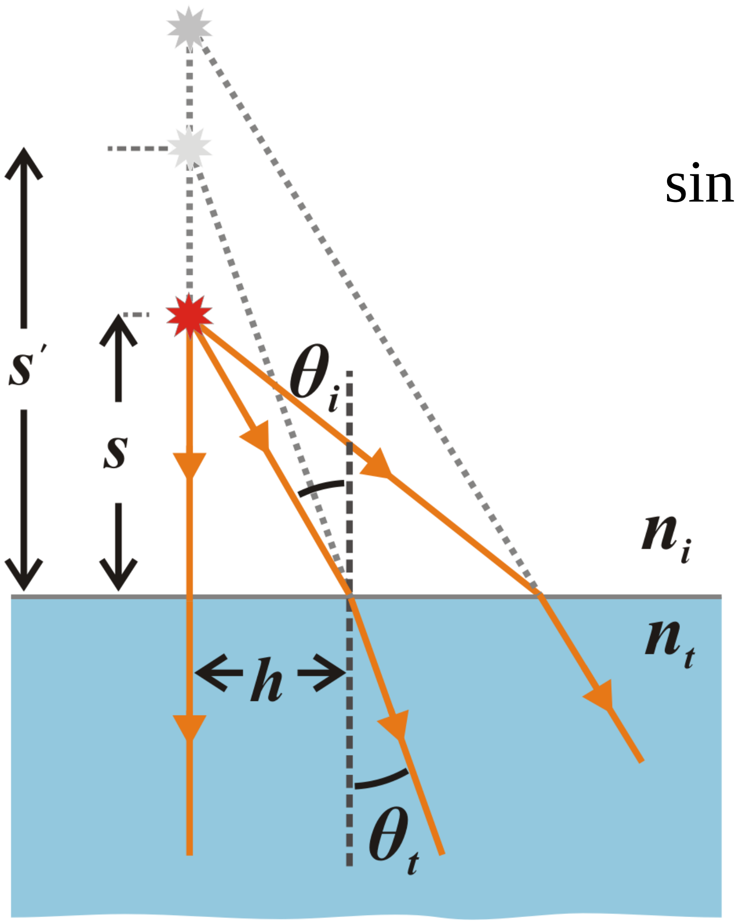


$$2\theta+2\theta'=180^{\circ}$$

Application: *multiple reflections*



Refraction from a plane surface



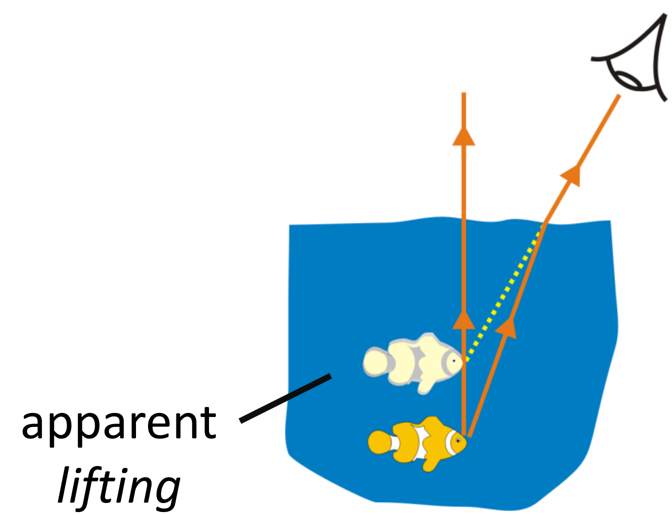
$$\left.\begin{array}{c} n_i \sin\theta_i = n_t \sin\theta_t \\ \sin\theta_i = \dfrac{h}{\sqrt{s^2+h^2}}, \quad \sin\theta_t = \dfrac{h}{\sqrt{s'^2+h^2}} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \boxed{s' = \frac{n_t}{n_i}\sqrt{s^2 + \left(1 - \frac{n_i^2}{n_t^2}\right)h^2}}$$

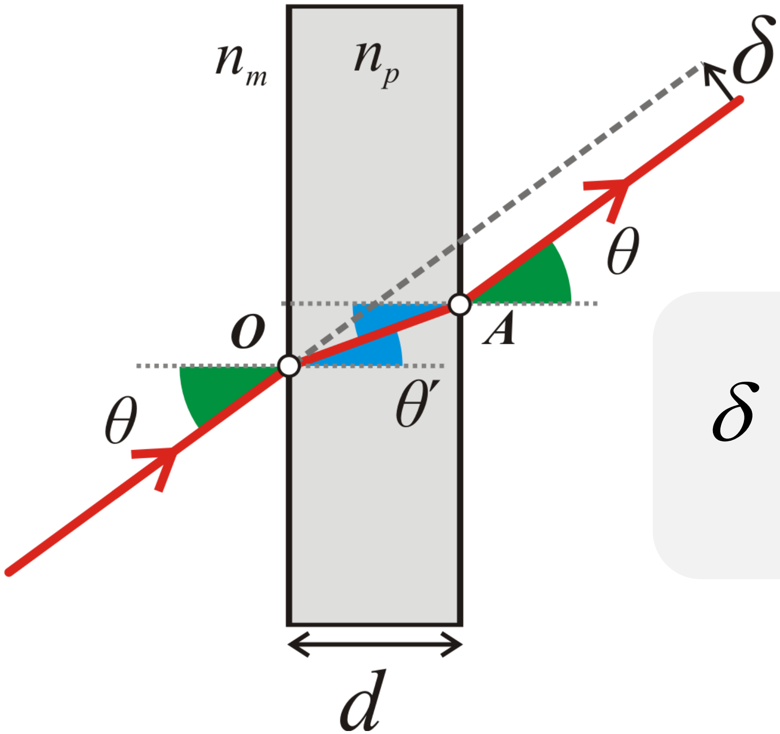
imaging is not stigmatic since the image positions depends on h

Paraxial case:

$$\boxed{h \cong 0 \Rightarrow s' \cong \frac{n_t}{n_i} s}$$



Transparent plate *(parallel deviation of rays)*



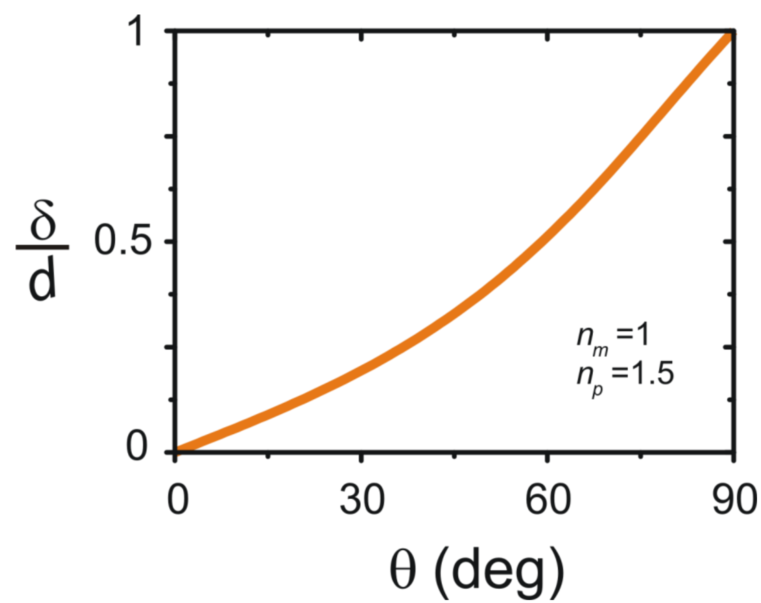
$$\delta = (OA)\sin(\theta - \theta') \Rightarrow$$

$$\delta = d(\sin\theta - \cos\theta\tan\theta') \Rightarrow$$

$$\delta = d\sin\theta\,(1 - \frac{n_m\cos\theta}{\sqrt{n_p^2 - n_m^2\sin^2\theta}})$$

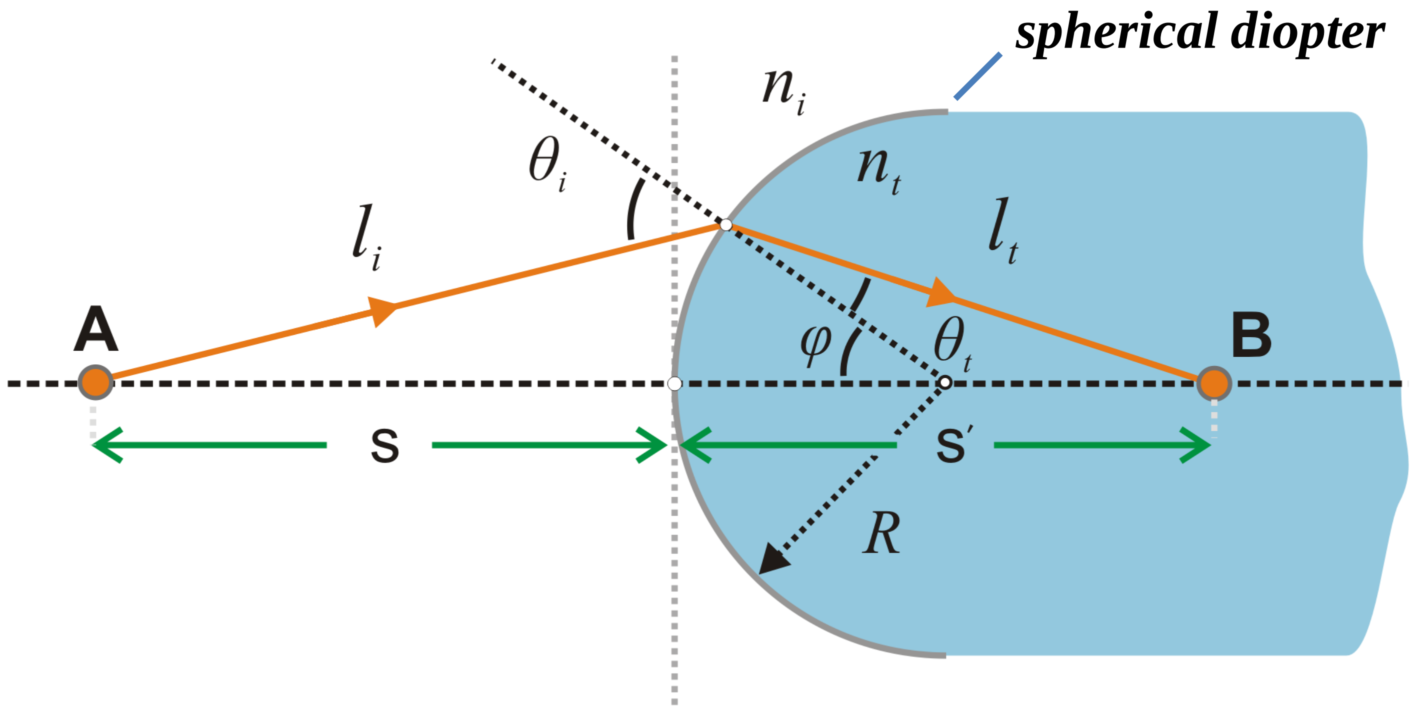
paraxial case:

$$\theta \cong 0 \Rightarrow \delta \cong d(\frac{n_p - 1}{n_p})\theta$$



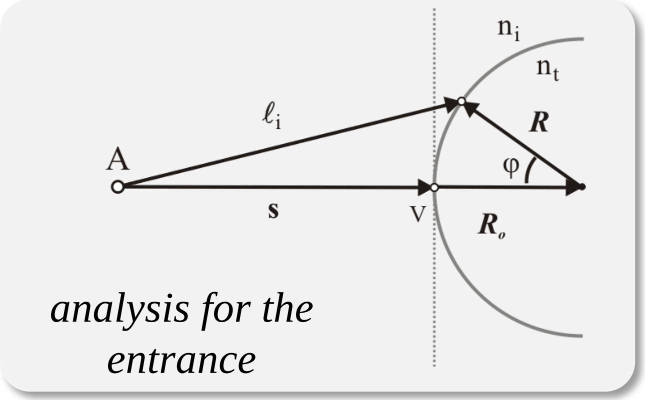
Diopters and Lenses

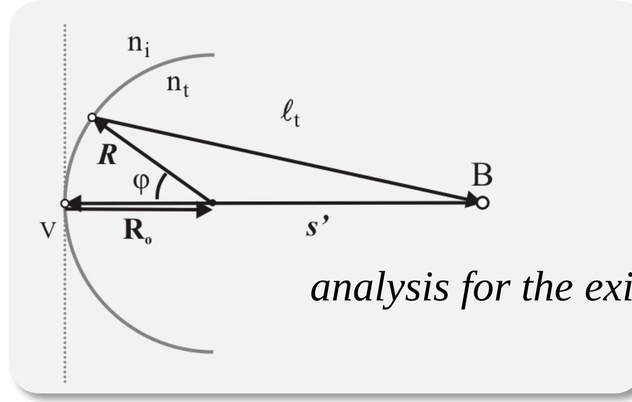
Refraction from a spherical surface



In order to calculate the estimate the relation between the position of the image (B) and the position of the object (A) for each ray refracted by the spherical diopter, we will use Fermat's principle.

$$(OPL) \equiv L = n_i l_i + n_t l_t$$





$$\left.\begin{array}{l}\mathbf{l}_i = (\mathbf{s}+\mathbf{R}_o)+\mathbf{R} \Rightarrow l_i = \sqrt{\mathbf{l}_i^{\,2}} = \sqrt{R^2+(s+R)^2-2R(s+R)\cos\varphi} \\ \mathbf{l}_t = (\mathbf{s}'+\mathbf{R}_o)+\mathbf{R} \Rightarrow l_t = \sqrt{\mathbf{l}_t^{\,2}} = \sqrt{R^2+(s'-R)^2+2R(s'-R)\cos\varphi}\end{array}\right\} \Rightarrow$$

Fermat's principle

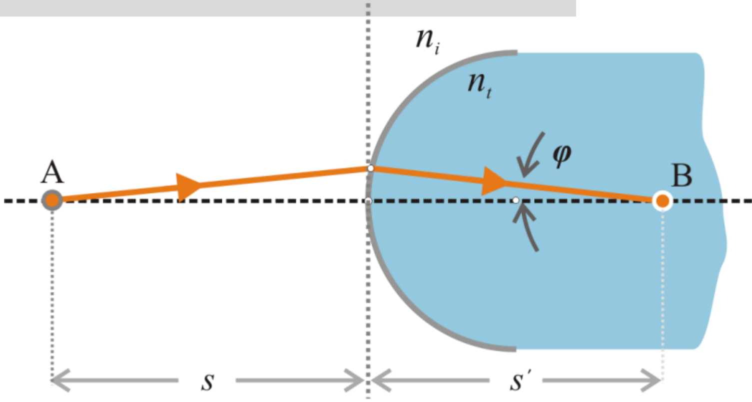
$$L = n_i l_i + n_t l_t \Rightarrow \frac{dL}{d\varphi} = n_i \frac{2R(s+R)\sin\varphi}{2l_i} + n_t \frac{-2R(s'-R)\sin\varphi}{2l_t} \equiv 0 \Rightarrow$$

$$\frac{n_i(s+R)}{l_i} - \frac{n_t(s'-R)}{l_t} = 0 \Rightarrow \boxed{s' = \frac{n_i}{n_t}(s+R)\frac{l_t}{l_i} + R}$$

Imaging is not stigmatic since each ray corresponds to a different image!

Dimitris Papazoglou dpapa@materials.uoc.gr

Paraxial approximation ($\varphi \approx 0$)



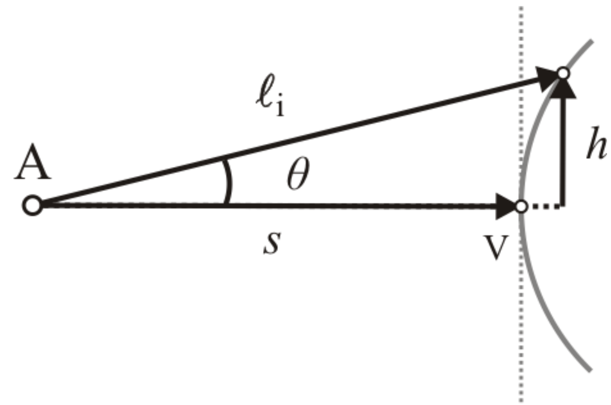
$$\varphi \cong 0 \Rightarrow \cos\varphi \cong 1 \Rightarrow l_i \cong s,\ l_t \cong s'$$

$$\frac{n_i(s+R)}{l_i} - \frac{n_t(s'-R)}{l_t} = 0 \Rightarrow \frac{n_i(s+R)}{s} - \frac{n_t(s'-R)}{s'} = 0 \Rightarrow$$

$$\frac{n_i}{s} + \frac{n_t}{s'} = \frac{n_t - n_i}{R}$$

Imaging is now stigmatic!

Typical validity limits of paraxial approximation

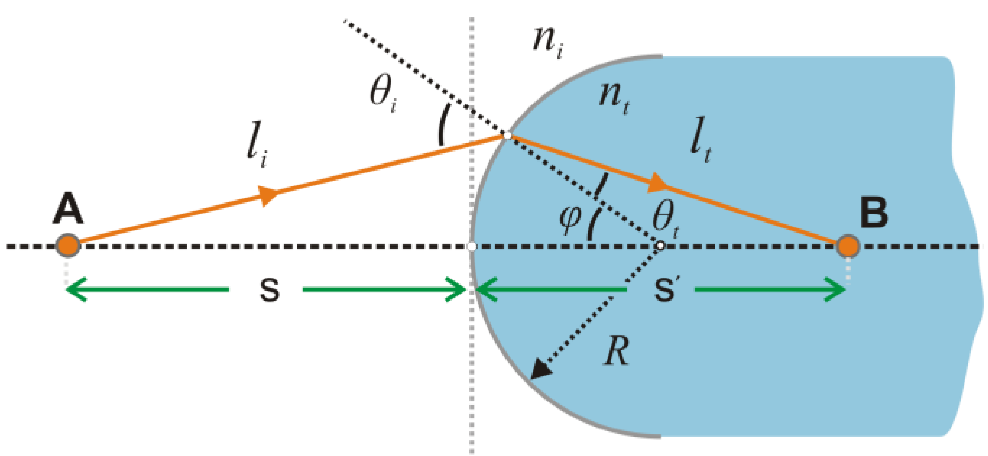


$$\frac{l_i}{s} \cong \sqrt{1+(\frac{h}{s})^2} \Rightarrow l_i \cong s \;\; \forall \frac{l_i}{s} \le 1.1 \Rightarrow h \le \frac{s}{2.2} \Rightarrow \theta \le 24^o$$

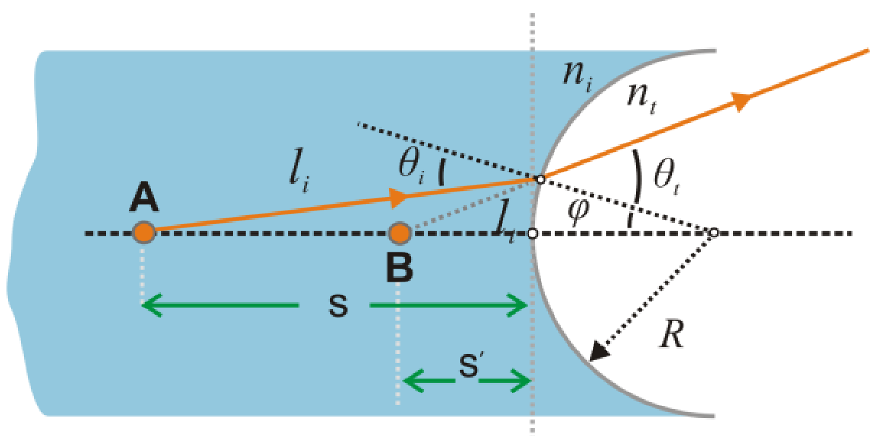
$$\frac{l_i}{s} = 1+m, \;\; (m<1) \Rightarrow \frac{h}{s} \le \sqrt{2m} \Rightarrow \begin{cases} 10\% \rightarrow 24^o & 0.4\,NA \\ 5\% \rightarrow 17^o & 0.29\,NA \\ 1\% \rightarrow 8^o & 0.14\,NA \\ 0.1\% \rightarrow 2.5^o & 0.04\,NA \end{cases}$$

Assumptions for spherical diopters

$n_i < n_t$	Sign	
	+	**-**
s	**Left of the diopter** *(real object)*	**Right of the diopter** *(virtual object)*
s'	**Right of the diopter** *(real image)*	**Left of the diopter** *(virtual image)*
R	**Convex (**	**Concave)**



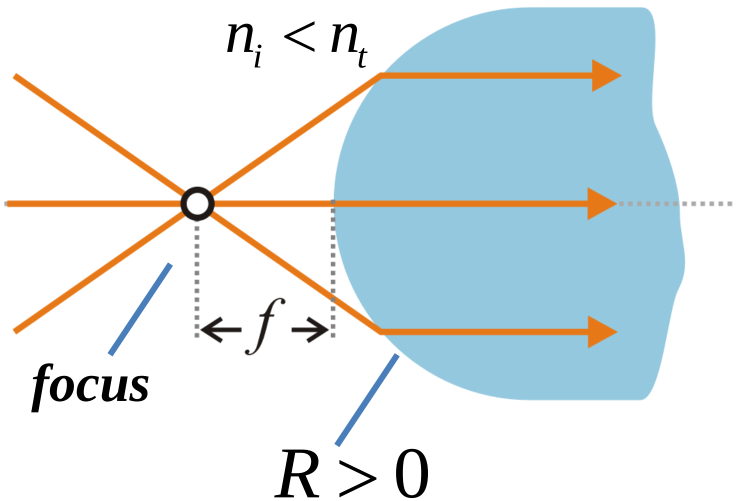
	Sign	
$n_i > n_t$	**+**	**-**
s	**Left of the diopter** *(real object)*	**Right of the diopter** *(virtual object)*
s'	**Right of the diopter** *(real image)*	**Left of the diopter** *(virtual image)*
R	**Convex** (	**Concave**)



Foci position

$$\frac{n_i}{s} + \frac{n_t}{s'} = \frac{n_t - n_i}{R}$$

convex diopter



$$s' \to \infty \Rightarrow s = \boxed{f = \frac{n_i R}{n_t - n_i} > 0}$$

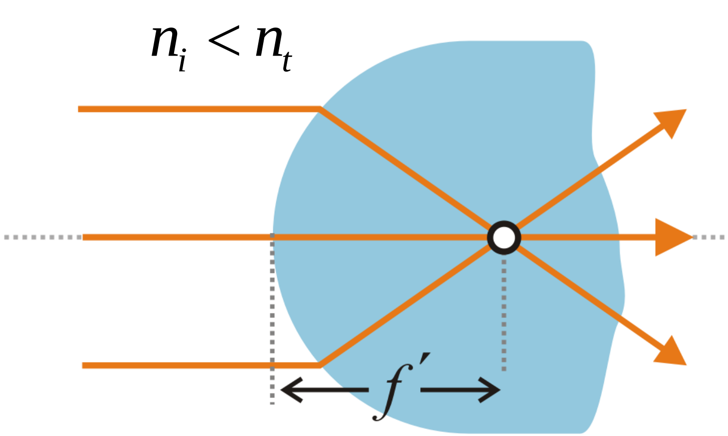
Typical values

$n_i = 1\ (air),\ n_t = 1.5\ (glass) \Rightarrow \boxed{f \cong 2R > 0}$

$$s \to \infty \Rightarrow s' = \boxed{f' = \frac{n_t R}{n_t - n_i} > 0}$$

Typical values

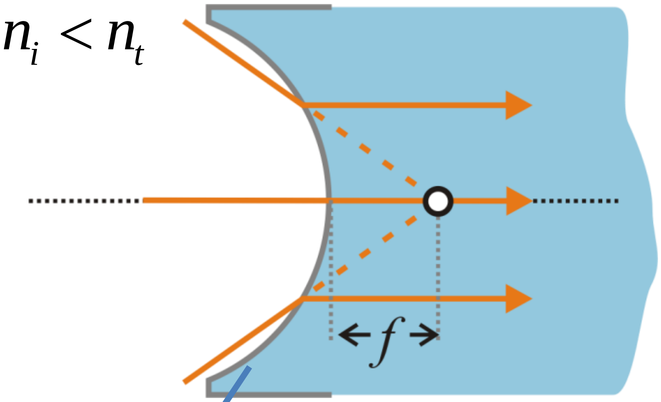
$n_i = 1\ (air),\ n_t = 1.5\ (glass) \Rightarrow \boxed{f' \cong 3R > 0}$



$f' \neq f$

Dimitris Papazoglou dpapa@materials.uoc.gr

concave diopter



$$s' \to \infty \Rightarrow s = \boxed{f = \frac{n_i R}{n_t - n_i} < 0}$$

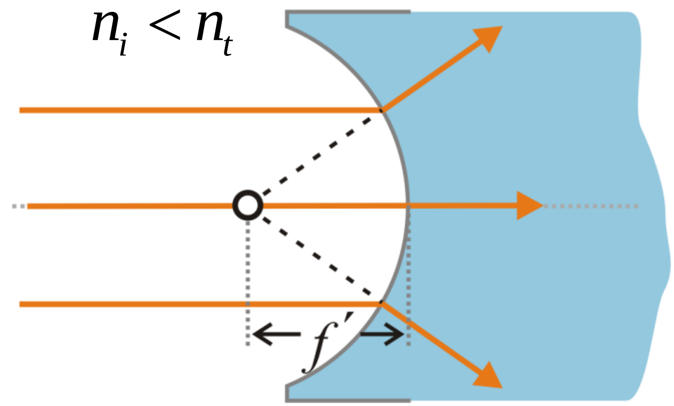
Typical value

$$n_t = 1 \ (air), \ n_t = 1.5 \ (glass) \Rightarrow \boxed{f \cong 2R < 0}$$

$$s \to \infty \Rightarrow s' = \boxed{f' = \frac{n_t R}{n_t - n_i} < 0}$$

Typical value

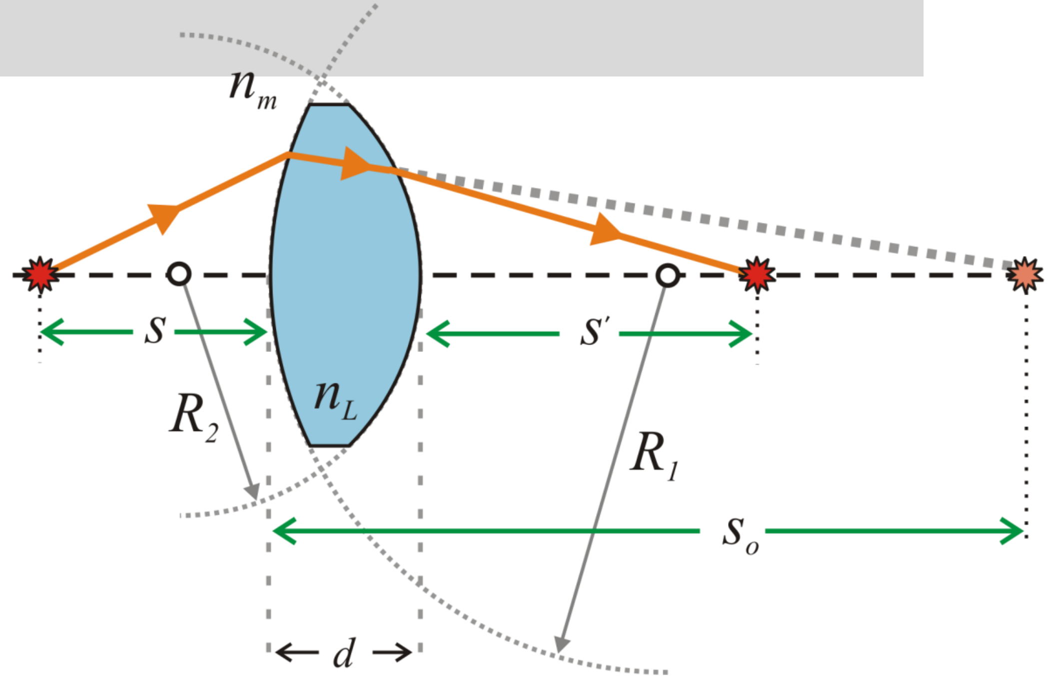
$$n_t = 1 \ (air), \ n_t = 1.5 \ (glass) \Rightarrow \boxed{f' \cong 3R < 0}$$



$$f' \neq f$$

Lenses

A lens consists of a sequence of diopters

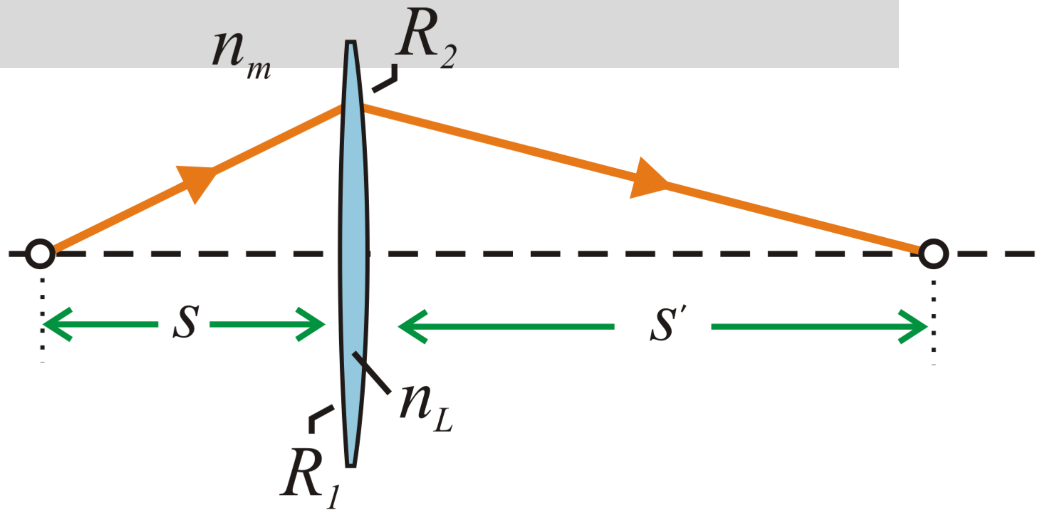


$$\left.\begin{aligned} \frac{n_m}{s}+\frac{n_L}{s_o} &= \frac{n_L-n_m}{R_1} \\ \frac{n_L}{-(s_o-d)}+\frac{n_m}{s'} &= \frac{n_m-n_L}{R_2} \end{aligned}\right\} \Rightarrow$$

gets to zero when d→0

$$\frac{1}{s}+\frac{1}{s'}=\frac{n_L-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{n_L}{n_m}\frac{d}{(s_o-d)s_o}$$

Thin lenses



$$d \to 0 \Rightarrow$$

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad \Leftrightarrow \quad (s-f)(s'-f)=f^2$$

Gauss ***Newton***

$$\frac{1}{f} \equiv \left(\frac{n_L}{n_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

lens maker's formula

Gauss-Newton formulation equivalence

Newton

$$\boxed{(s-f)\cdot(s'-f)=f^2} \Leftrightarrow$$

$$s\cdot s'-(s+s')f+f^2=f^2 \Leftrightarrow$$

$$f=\frac{s\cdot s'}{(s+s')} \Leftrightarrow \boxed{\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}}$$

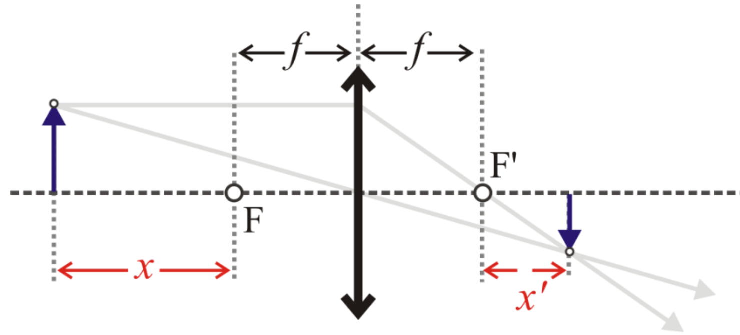
Gauss

Normalization of the variables

Newton

$$\left.\begin{array}{c} x \cdot x' = f^2 \\ \zeta = \dfrac{x}{f},\ \zeta' = \dfrac{x'}{f} \end{array}\right\} \Rightarrow \boxed{\zeta \cdot \zeta' = 1}$$

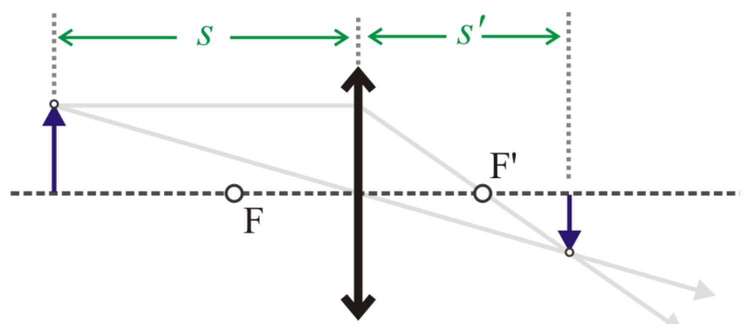
we measure in focal distances



Gauss

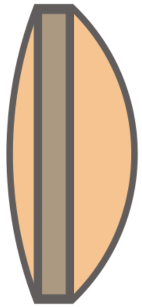
$$\left.\begin{array}{c} \dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \\ \xi = \dfrac{s}{f},\ \xi' = \dfrac{s'}{f} \end{array}\right\} \Rightarrow \boxed{\dfrac{1}{\xi} + \dfrac{1}{\xi'} = 1}$$

we measure in focal distances



Thin lens types

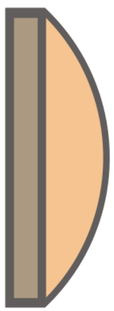
Converging $f > 0$



Bi-convex (BCX)

$R_1 > 0,$

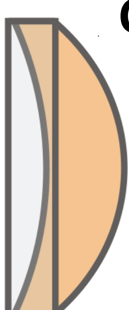
$R_2 < 0$



Plano-convex (PCX)

$R_1 = \infty,$

$R_2 < 0$

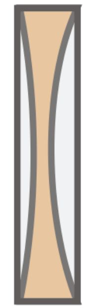


Convex Meniscus

$R_1 < 0,$

$R_2 < 0$

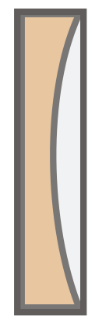
Diverging $f < 0$



Bi-concave (BCV)

$R_1 < 0,$

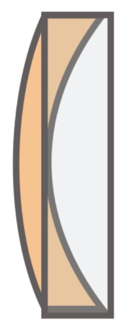
$R_2 > 0$



Plano-concave (PCV)

$R_1 = \infty,$

$R_2 > 0$



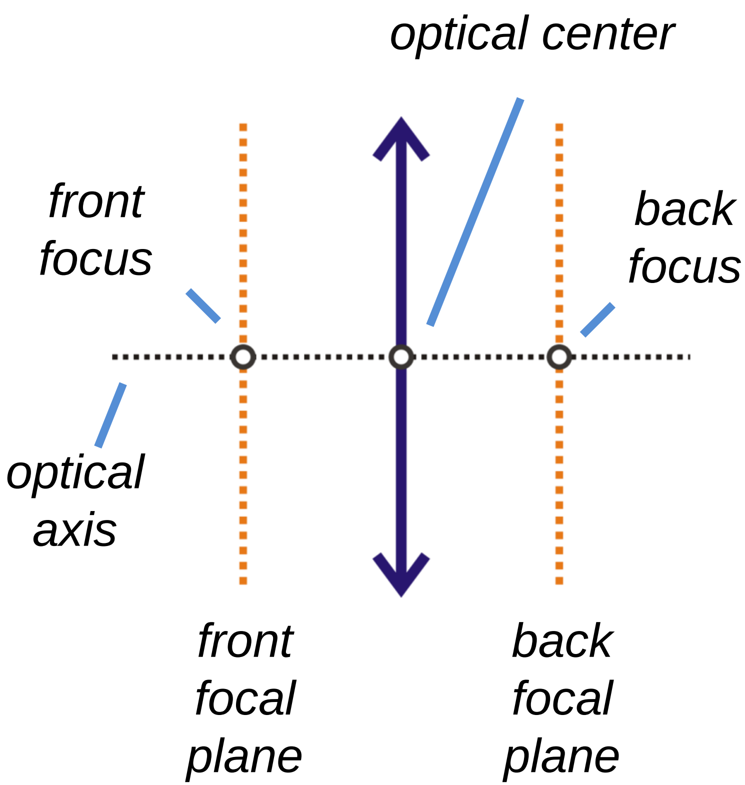
Convex Meniscus

$R_1 > 0,$

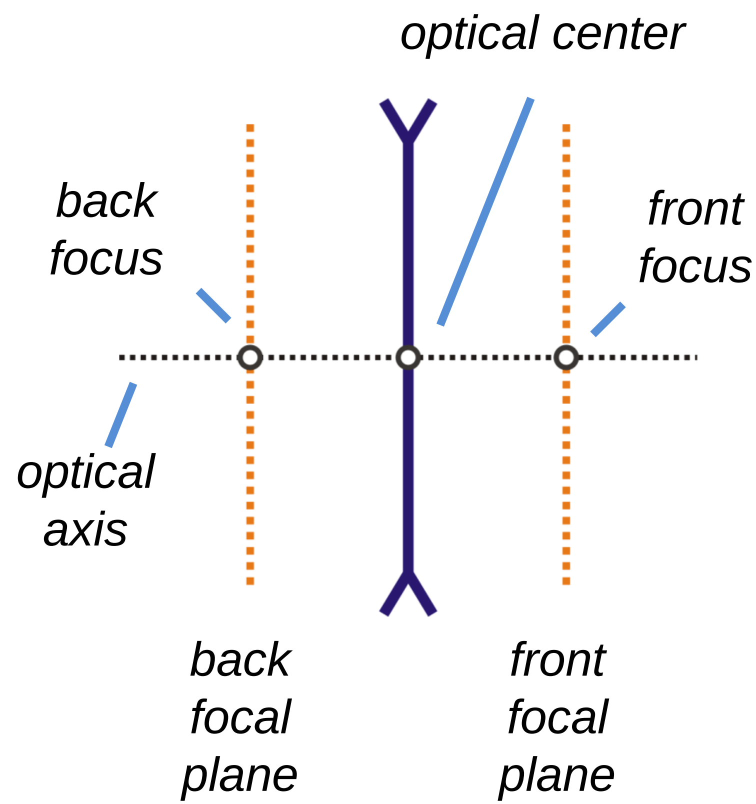
$R_2 > 0$

Cardinal points of a lens

Converging

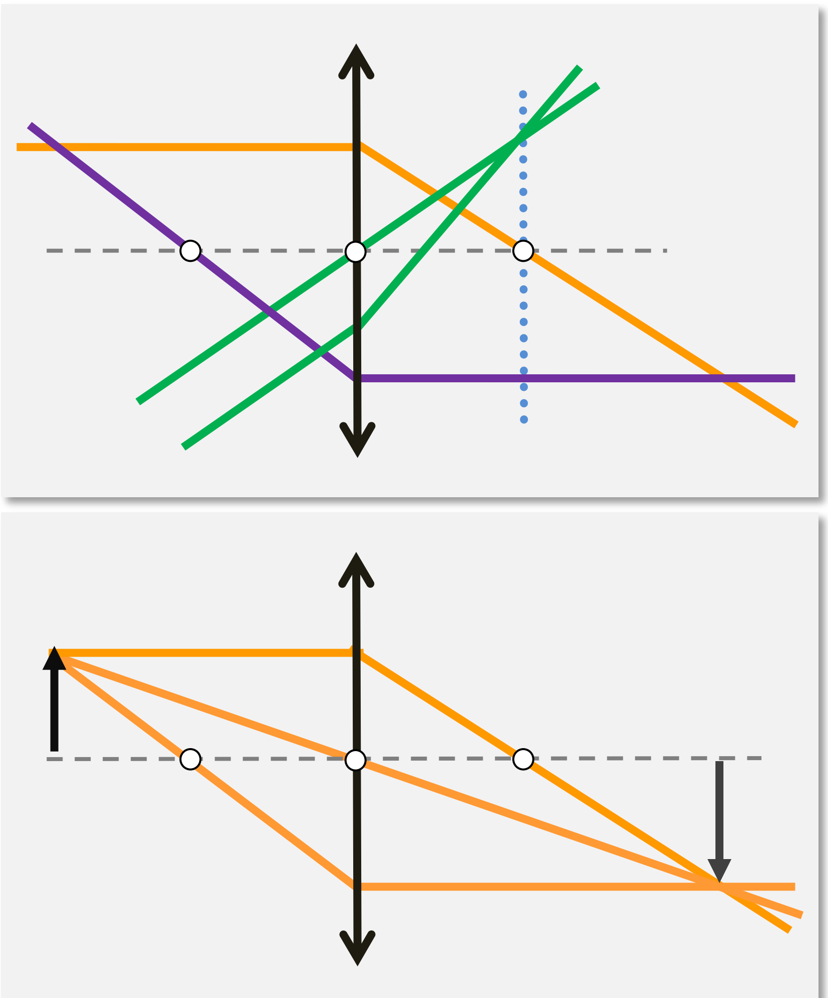


Diverging

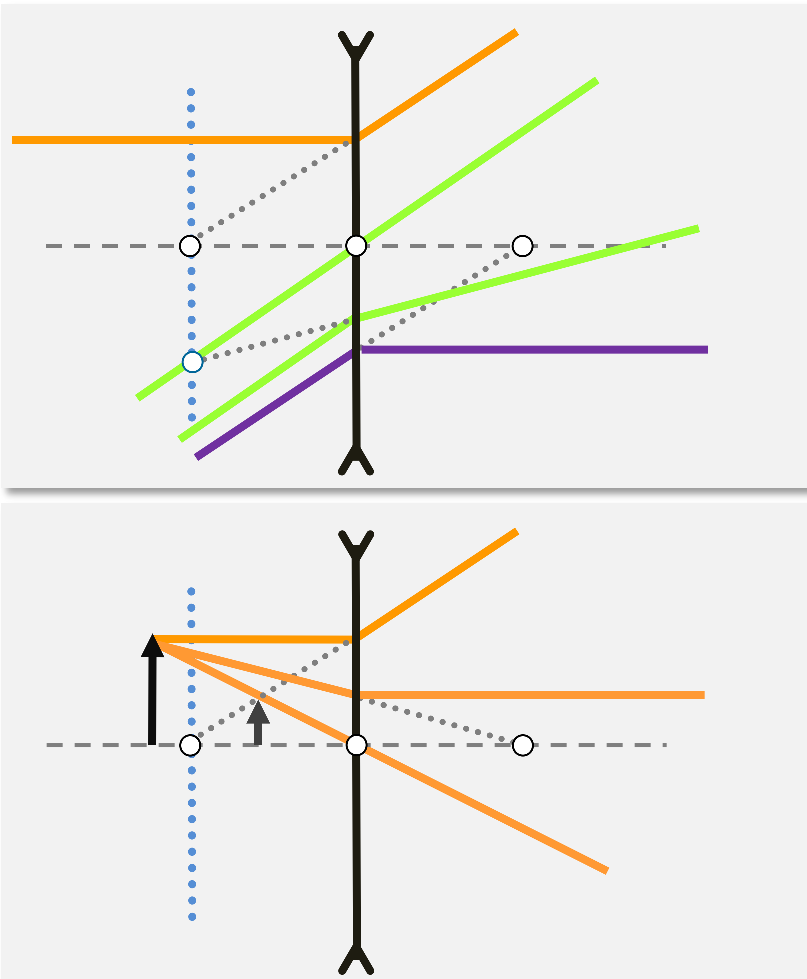


Imaging using a thin lens

Converging lens

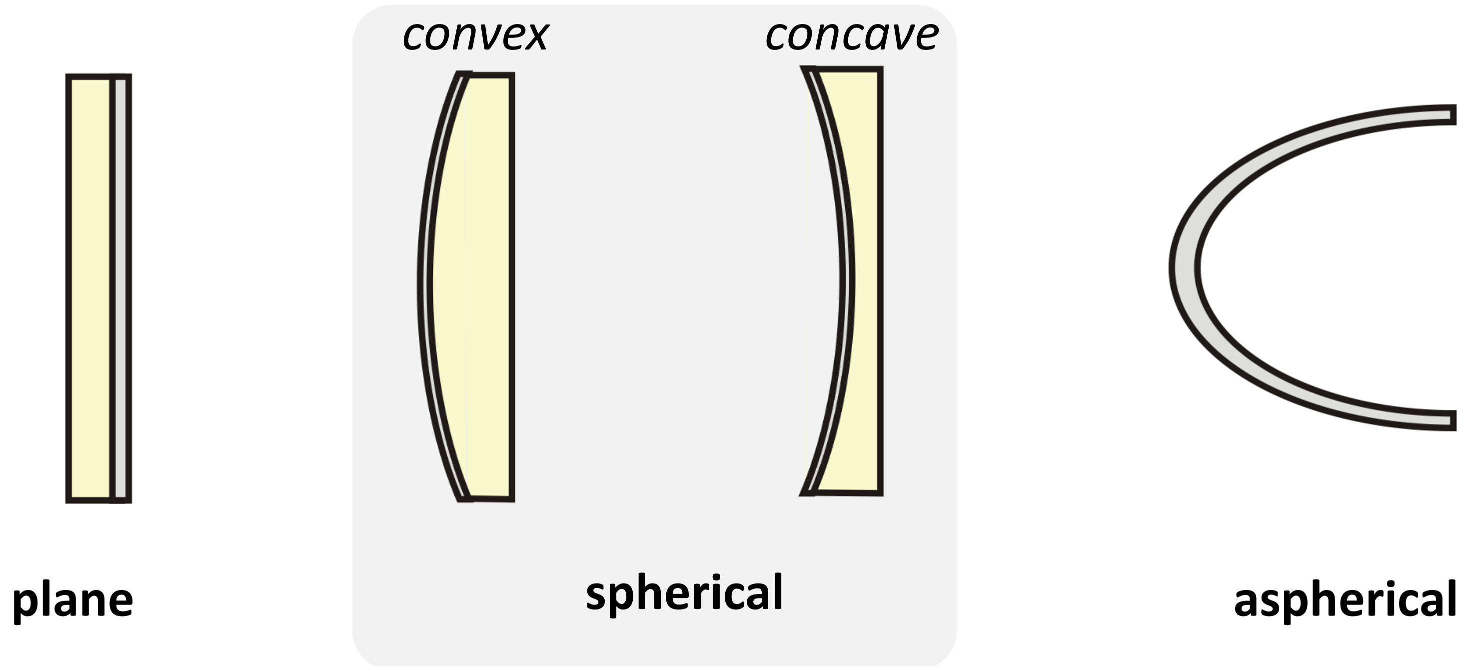


Diverging lens



Mirrors

Their function is based on light reflection !

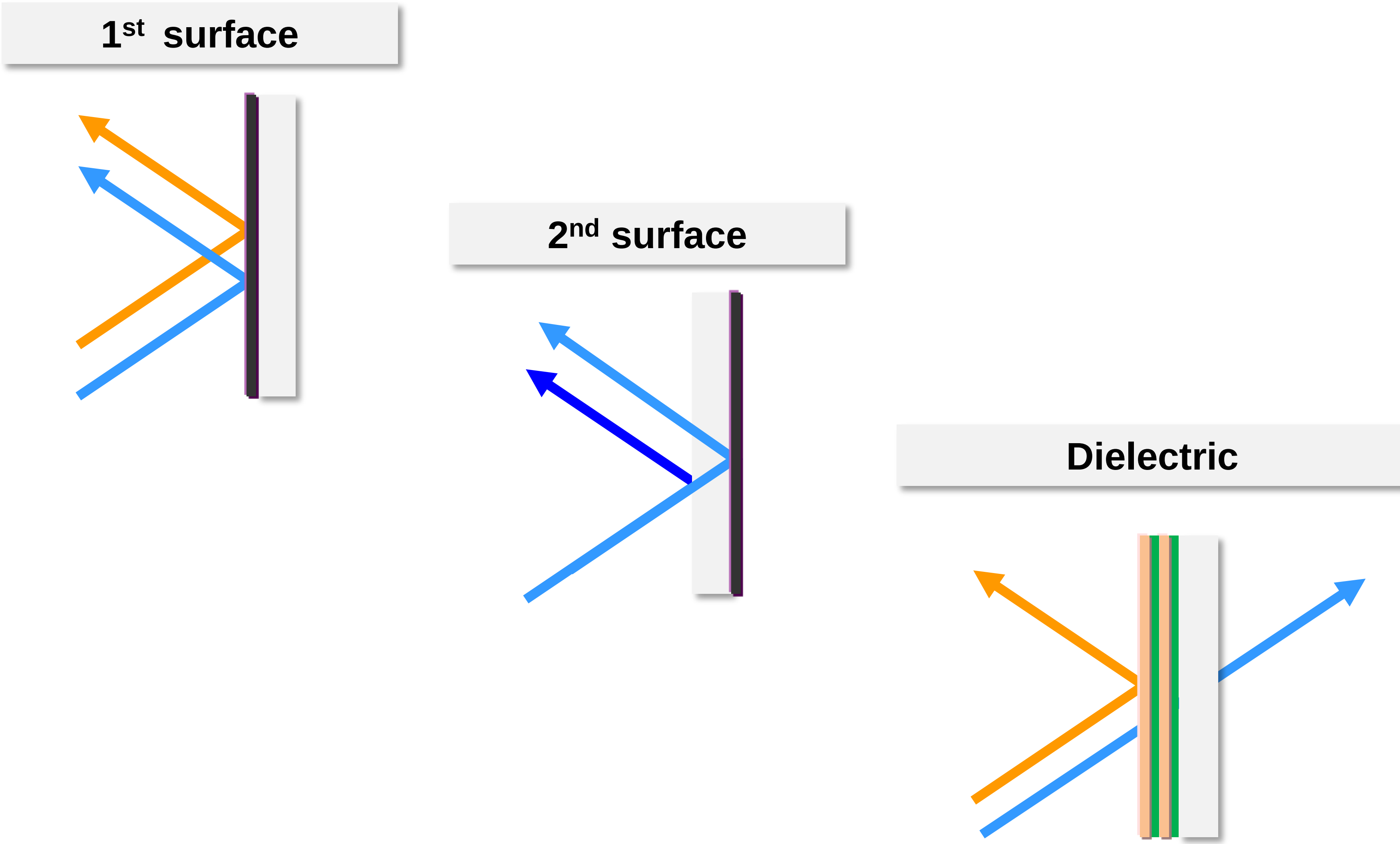


- ✓ **They operate in a broad spectral range**
- ✓ **Easier to manufacture (only one useful surface)**

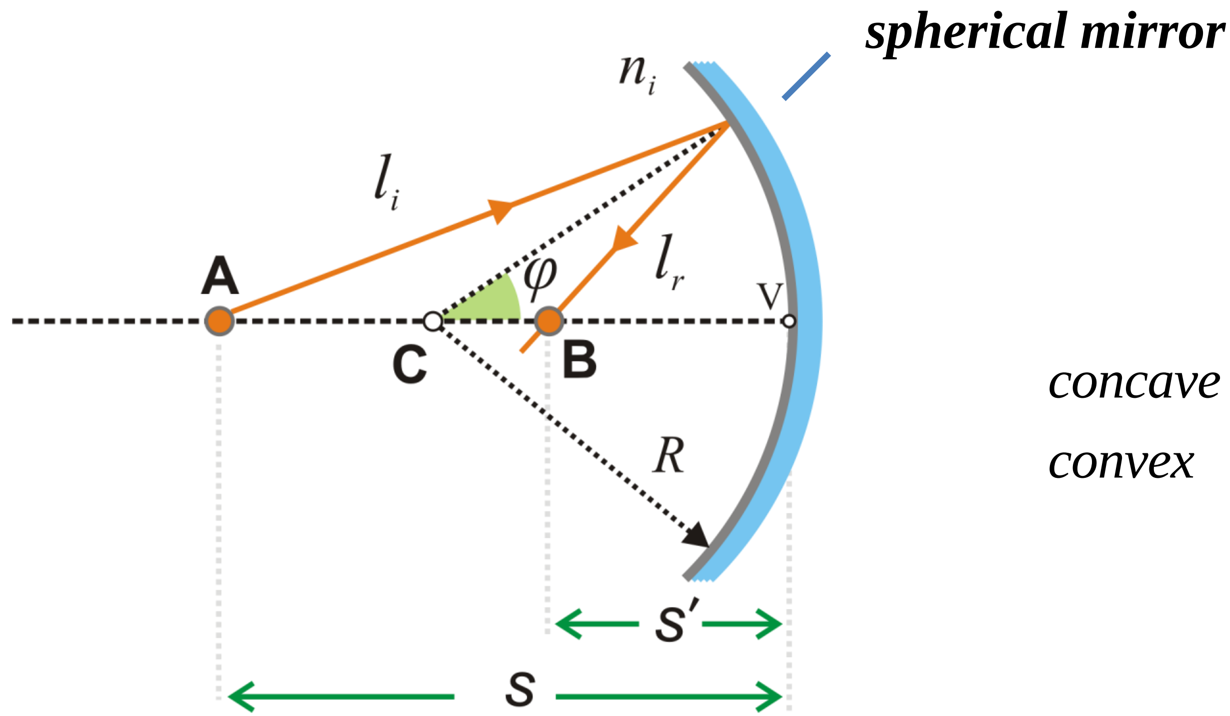
  It's easier to manufacture larger-sized mirrors than lenses

- **They reverse the direction of the beam**

Mirror types



Reflection from a spherical surface

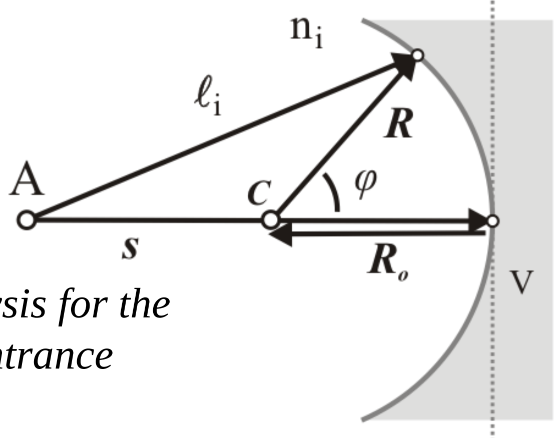


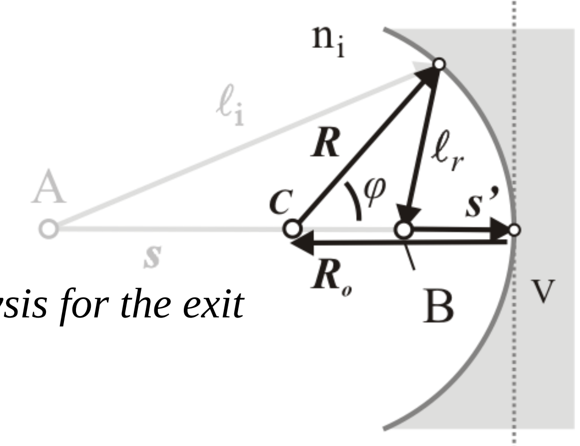
$concave \quad R < 0$

$convex \quad R > 0$

In order to calculate the estimate the relation between the position of the image (B) and the position of the object (A) for each ray reflected by the spherical mirror, we will use Fermat's principle

$$(OPL) \equiv L = n_i l_i + n_i l_r$$





$$\left.\begin{array}{l} \mathbf{l}_i = (\mathbf{s}+\mathbf{R}_o)+\mathbf{R} \Rightarrow l_i = \sqrt{\mathbf{l}_i^{\,2}} = \sqrt{R^2+[s-(-R)]^2+2R[s-(-R)]\cos\varphi} \\ \mathbf{l}_r = (\mathbf{s}'+\mathbf{R}_o)+\mathbf{R} \Rightarrow l_r = \sqrt{\mathbf{l}_r^{\,2}} = \sqrt{R^2+[s'-(-R)]^2+2R[s'-(-R)]\cos\varphi} \end{array}\right\} \Rightarrow$$

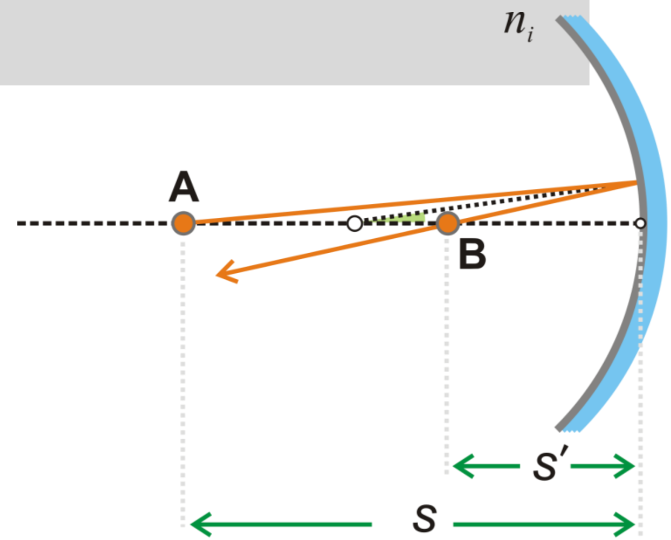
Fermat's principle

$$L = n_i l_i + n_i l_r \Rightarrow \frac{dL}{d\varphi} = n_i \frac{2R(s+R)\sin\varphi}{2l_i} + n_i \frac{2R(s'+R)\sin\varphi}{2l_r} \equiv 0 \Rightarrow$$

$$\frac{(s+R)}{l_i} + \frac{(s'+R)}{l_r} = 0 \Rightarrow \boxed{s' = -(s+R)\frac{l_r}{l_i} - R}$$

Imaging is not stigmatic since each ray corresponds to a different image!

Paraxial approximation ($\varphi \approx 0$)



$$\varphi \cong 0 \Rightarrow \cos\varphi \cong 1 \Rightarrow s \approx l_i, \quad s' \approx l_r$$

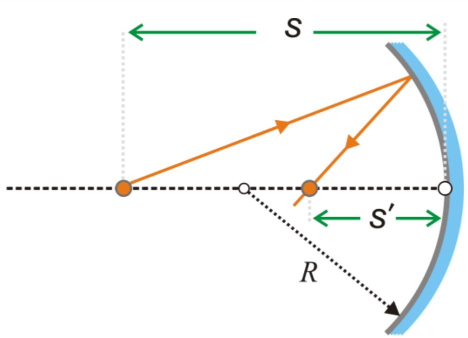
$$s' = -(s+R)\frac{s'}{s} - R \Rightarrow 1 = -1 - \frac{R}{s} - \frac{R}{s'} \Rightarrow$$

$$\boxed{\frac{1}{s} + \frac{1}{s'} = -\frac{2}{R} \equiv \frac{1}{f}}$$

Imaging is now stigmatic!

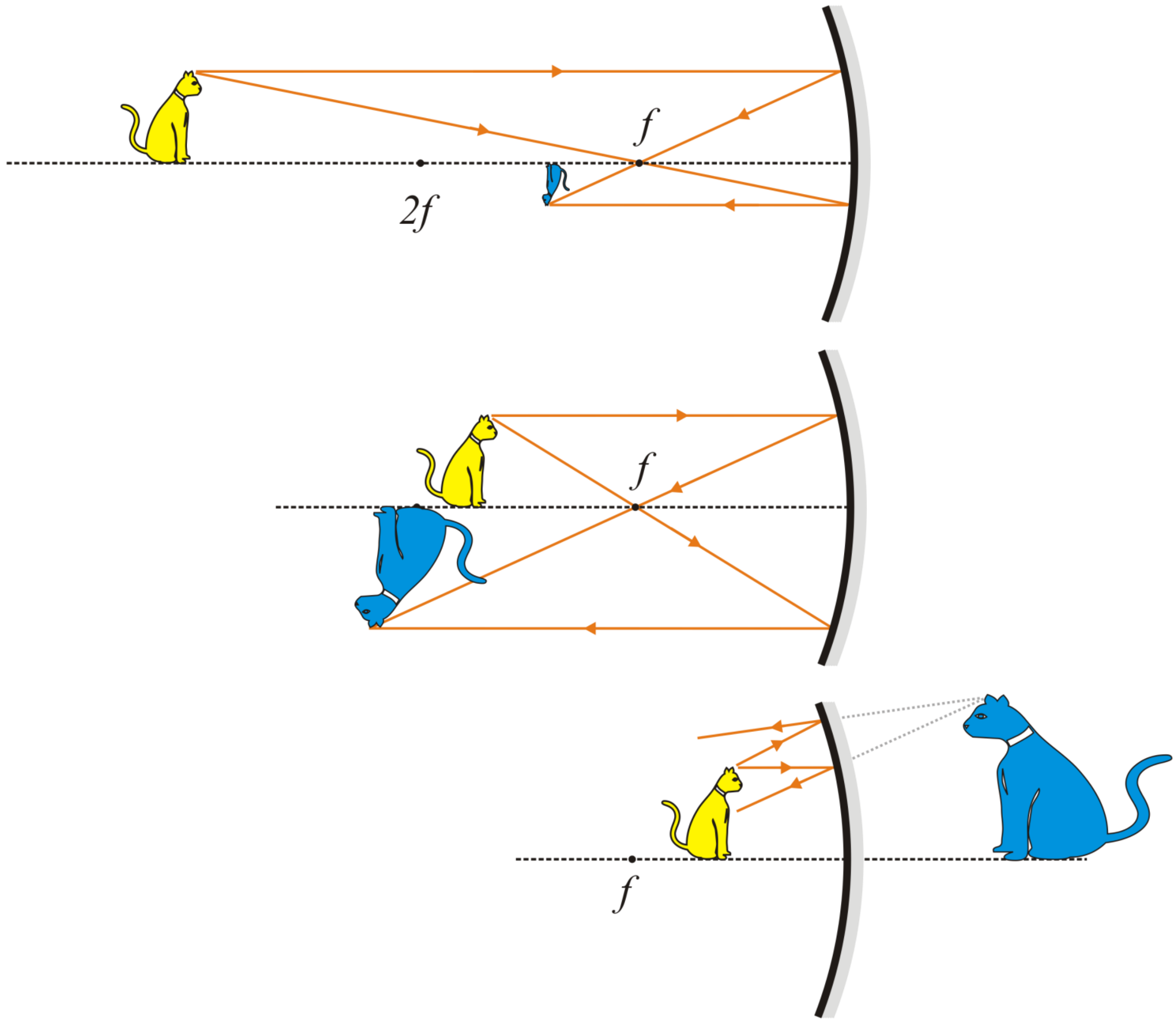
Assumptions for spherical mirrors

	Πρόσημο	
	+	**-**
s	**Left of the mirror** *(real object)*	**Right of the mirror** *(virtual object)*
s'	**Left of the mirror** *(real image)*	**Right of the mirror** *(virtual image)*
R	**convex (**	**Concave)**
f	**Concave)**	**convex (**



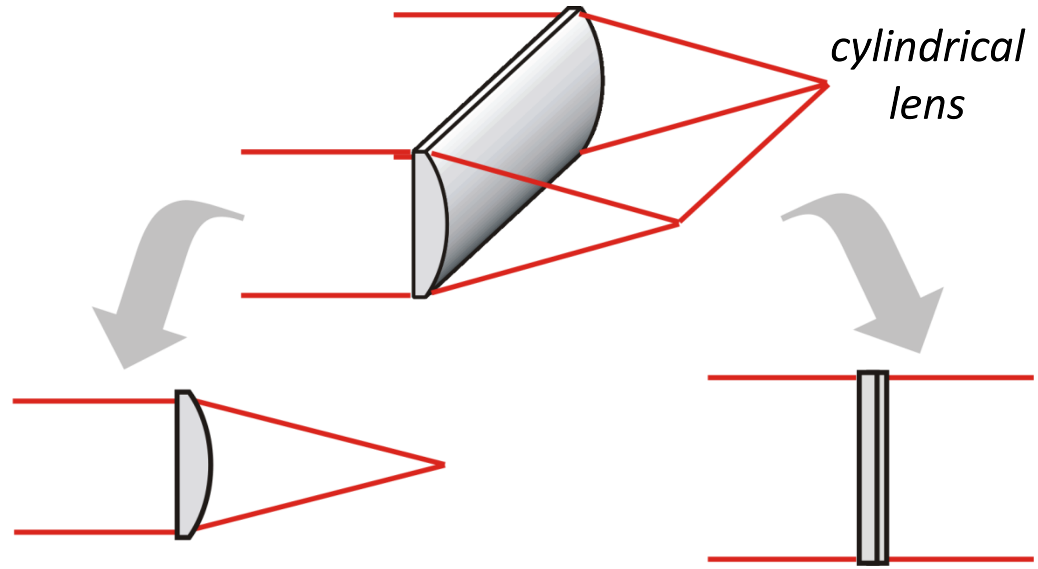
Dimitris Papazoglou dpapa@materials.uoc.gr

Examples of imaging using mirrors



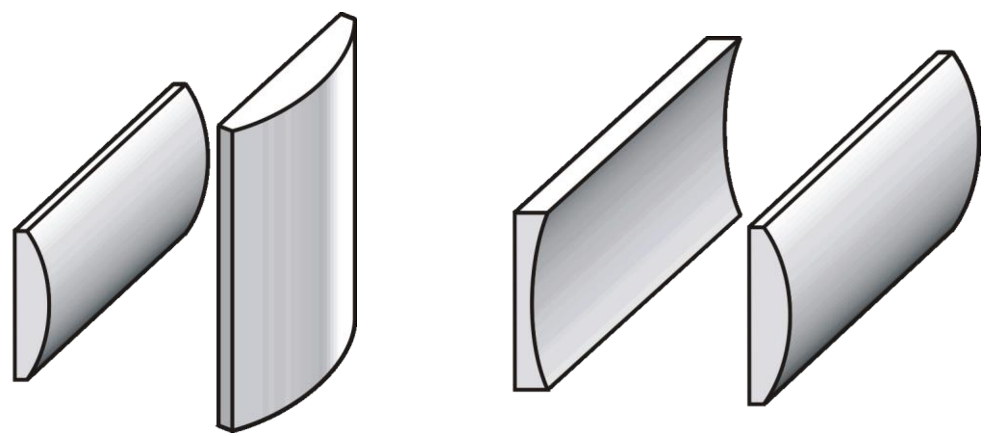
Special lenses: Anamorphic lenses

an optical system is anamorphic when it's optical power is different in two perpendicular to each other meridian planes



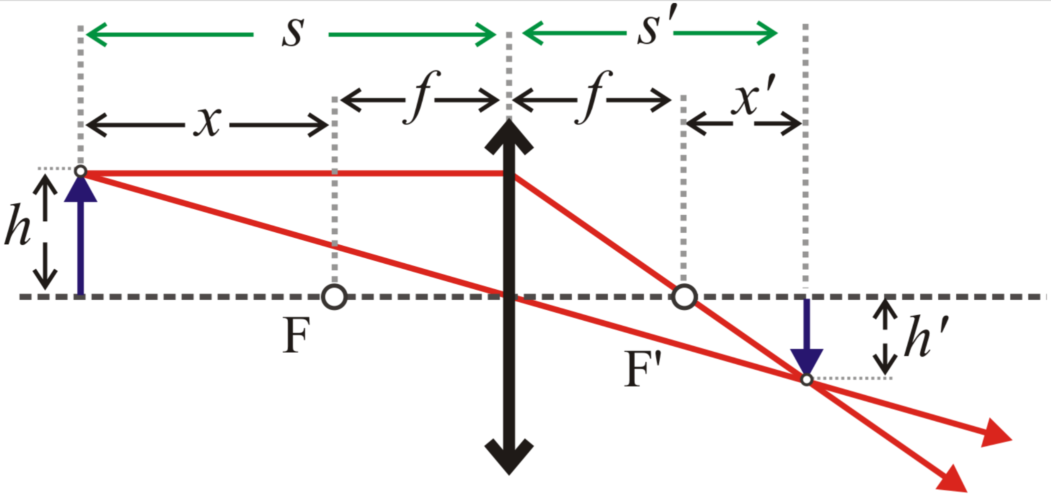
$$A = \frac{f_{//}}{f_{\perp}}$$

anamorphic factor



anamorphic optical systems

Magnification



Transverse magnification: *ratio of the transverse dimensions of image to object*

$$M_T = \frac{h'}{h} = -\frac{s'}{s} = -\frac{f}{x} = -\frac{x'}{f}$$

>0 erect image
<0 reversed image

Longitudinal magnification
ratio of the image displacement for a infinitesimal displacement of the object

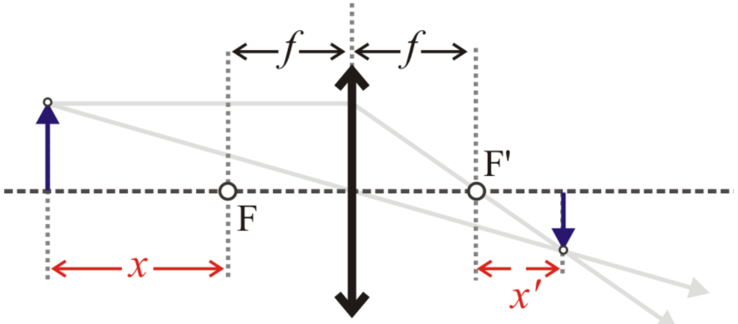
$$M_L \equiv \frac{dx'}{dx} = -\frac{f^2}{x^2} = -M_T^2$$

Magnification in normalized coordinates

Newton

$$\left.\begin{array}{l} M_T \equiv \dfrac{h'}{h} = -\dfrac{f}{x} = -\dfrac{x'}{f} \\[2ex] M_L \equiv \dfrac{dx'}{dx} = -\dfrac{f^2}{x^2} \\[2ex] \zeta = \dfrac{x}{f}, \zeta' = \dfrac{x'}{f} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \boxed{M_T = -\zeta' = -\dfrac{1}{\zeta}} \\[2ex] \boxed{M_L = -\zeta'^2 = -\dfrac{1}{\zeta^2}} \end{array}\right.$$

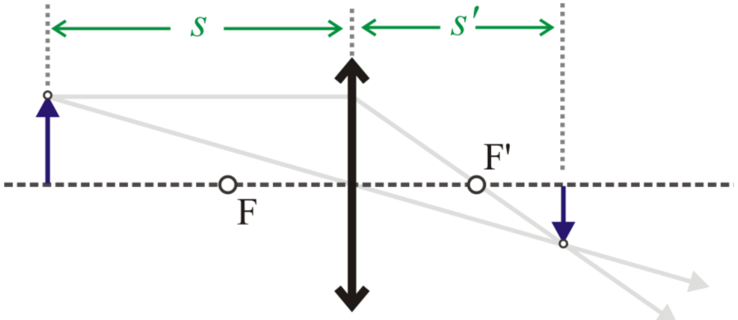
simplification of the formulation



Gauss

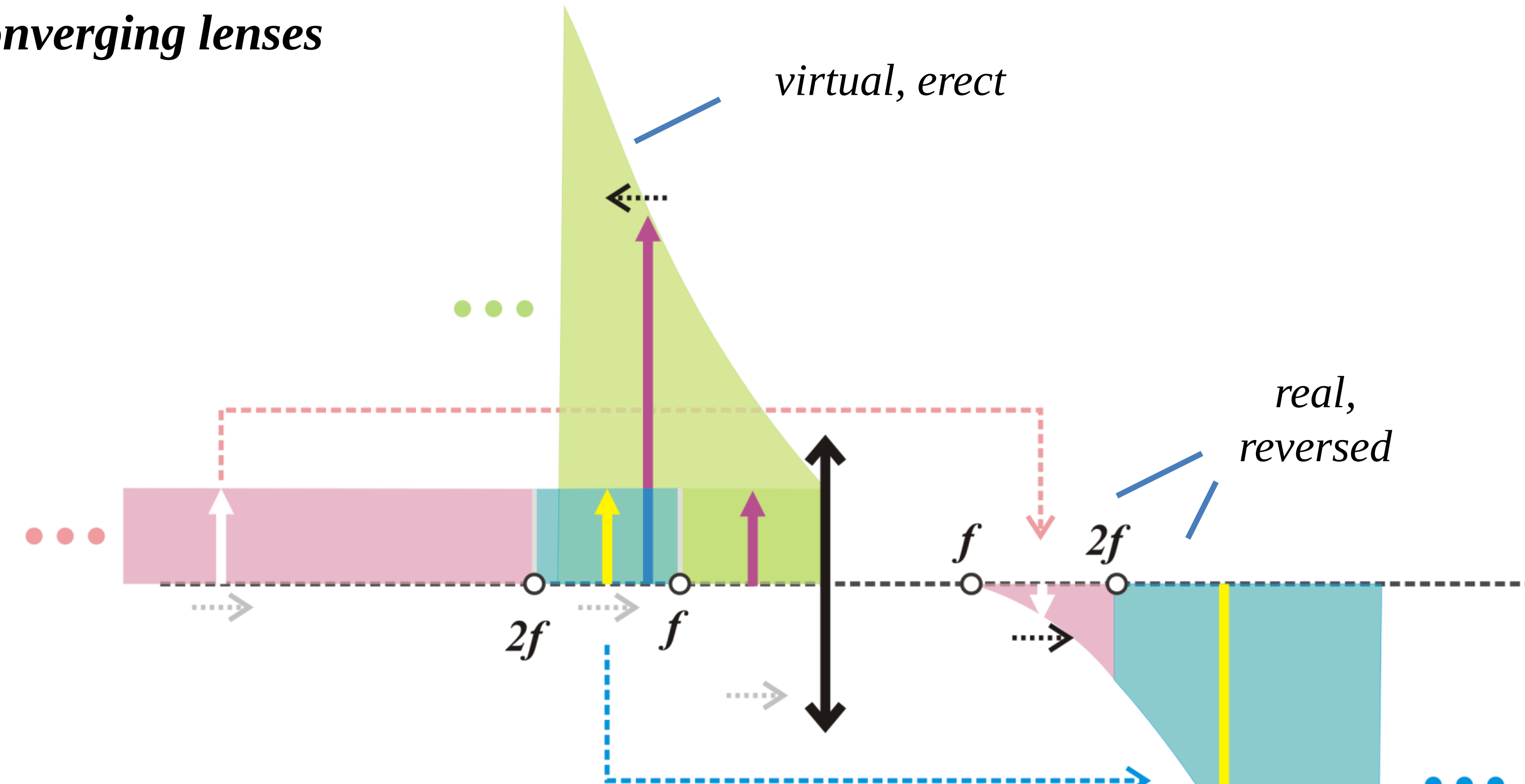
there is no simplification in the formulation

$$\left.\begin{aligned} M_T &\equiv \frac{h'}{h} = -\frac{s'}{s} \\ M_L &\equiv \frac{dx'}{dx} = M_T^2 \\ \xi &= \frac{s}{f},\ \xi' = \frac{s'}{f} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} M_T &= -\frac{\xi'}{\xi} \\ M_L &= \frac{\xi'^2}{\xi^2} \end{aligned}\right.$$

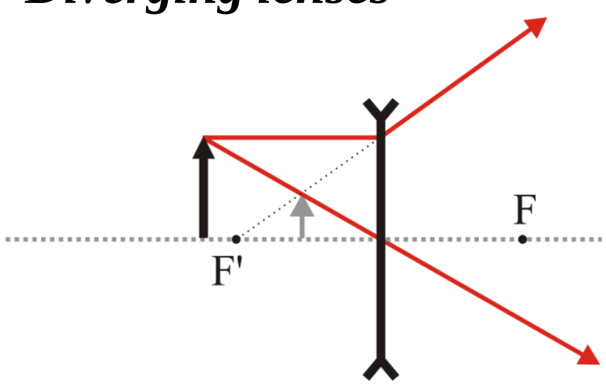


Magnification and image position in thin lens imaging

Converging lenses

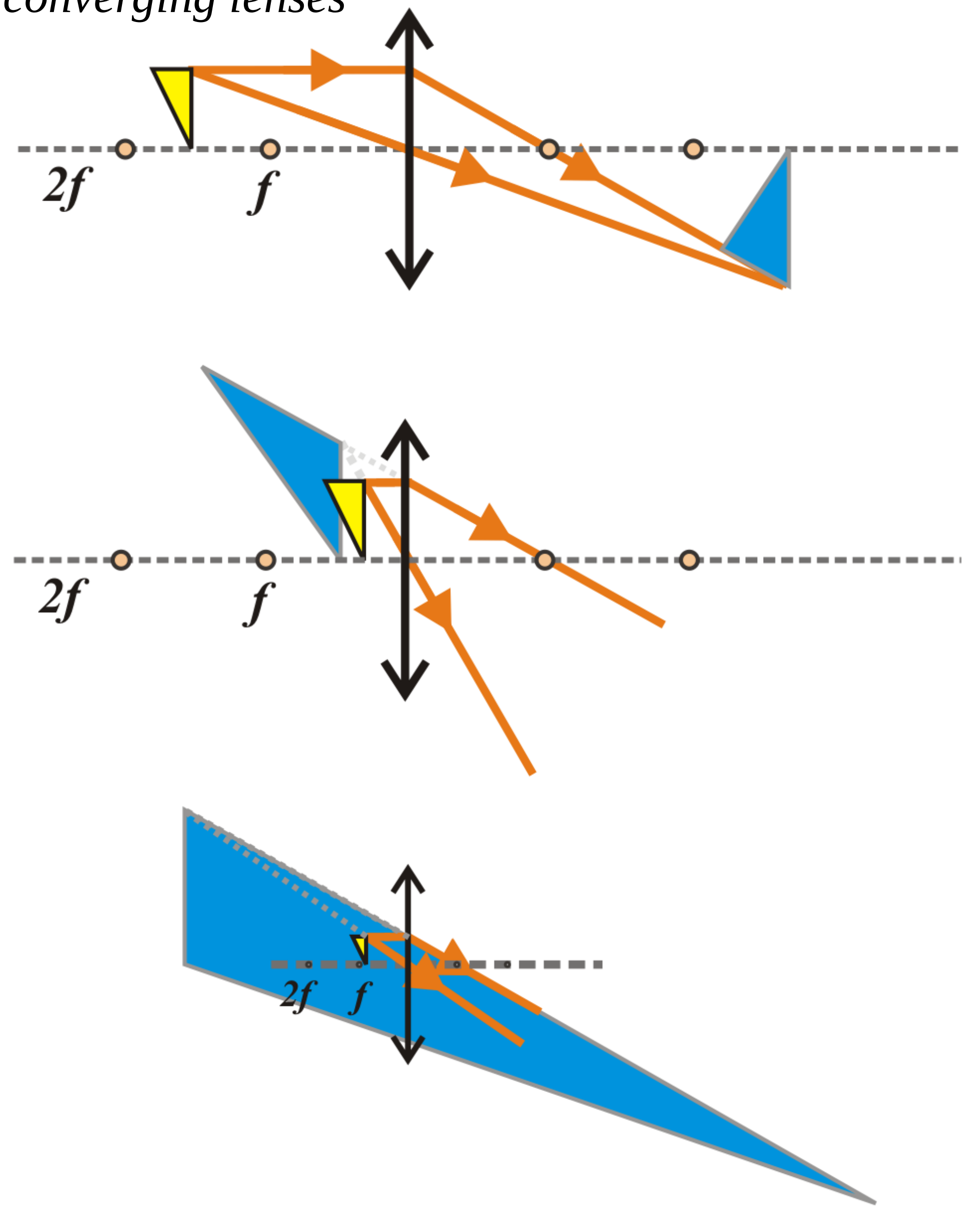


Diverging lenses

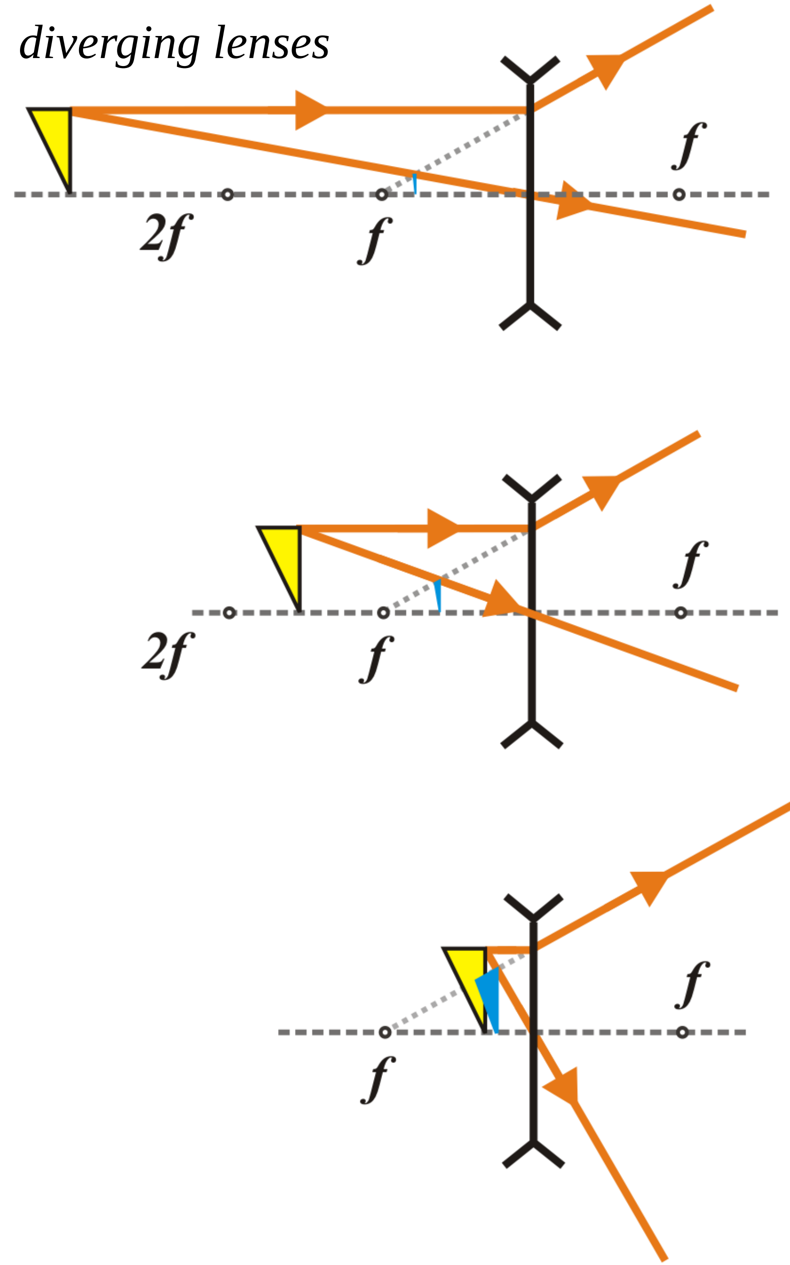


How are 3D objects imaged?

converging lenses

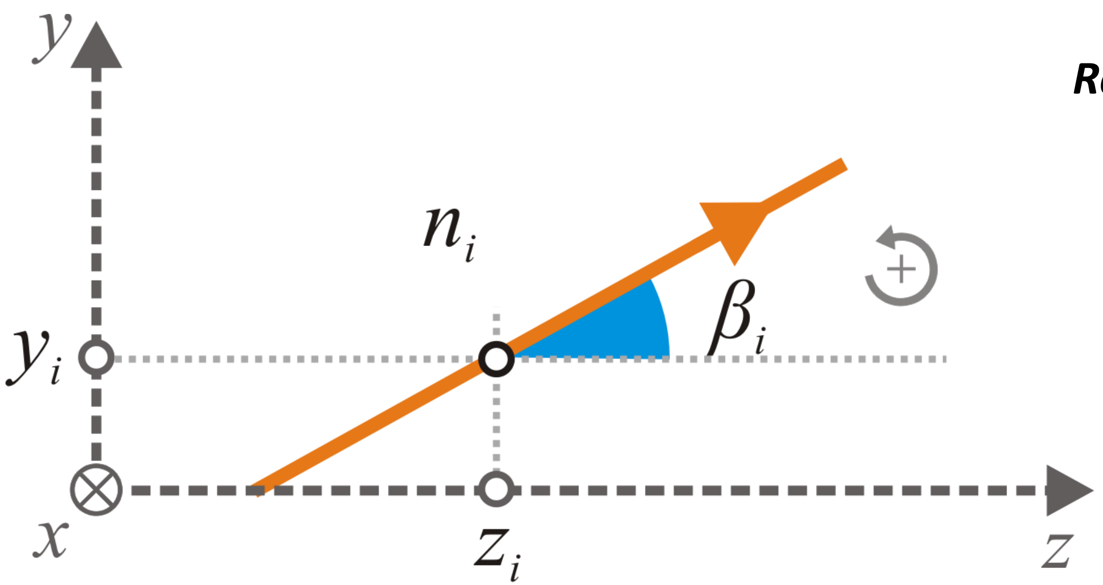


diverging lenses



3.3 Matrix Theory

Ray vector



Ray vector

$$\begin{bmatrix} y_i \\ n_i \beta_i \end{bmatrix}$$

Numerical aperture

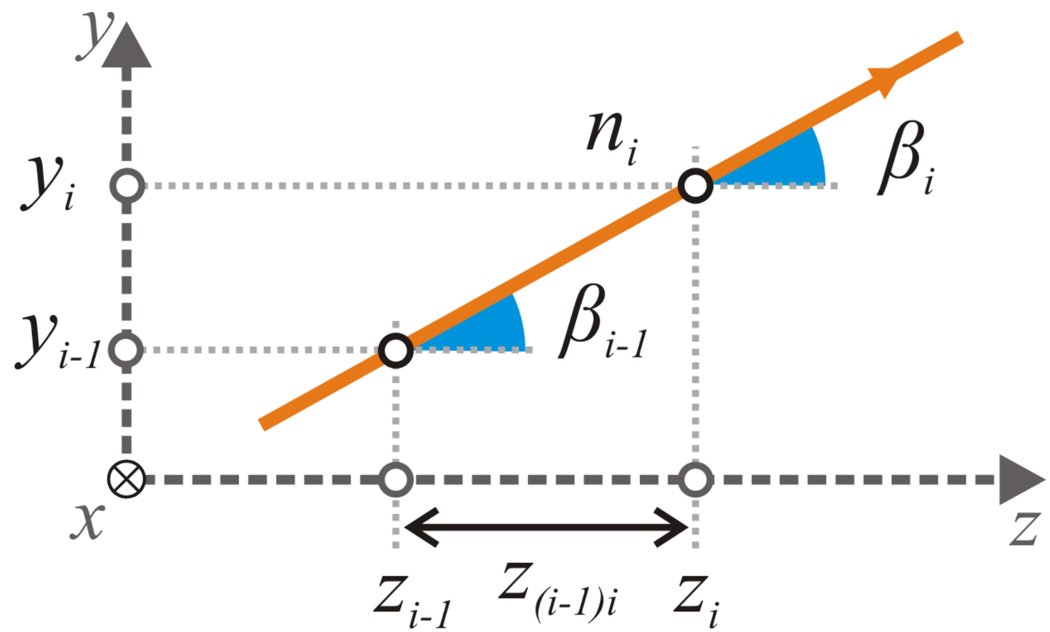
Paraxial approximation $\quad \sin\beta_i \cong \tan\beta_i \cong \beta_i$

$$\begin{bmatrix} y_i \\ n_i\,\beta_i \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \cdot \begin{bmatrix} y_{i-1} \\ n_{i-1}\,\beta_{i-1} \end{bmatrix}$$

new ray vector *transformation matrix* *previous ray vector*

Optical displacement transformation matrix

$$y_i \approx y_{i-1} + z_{(i-1)i}\,\beta_{i-1}\,,$$
$$\beta_i = \beta_{i-1}$$



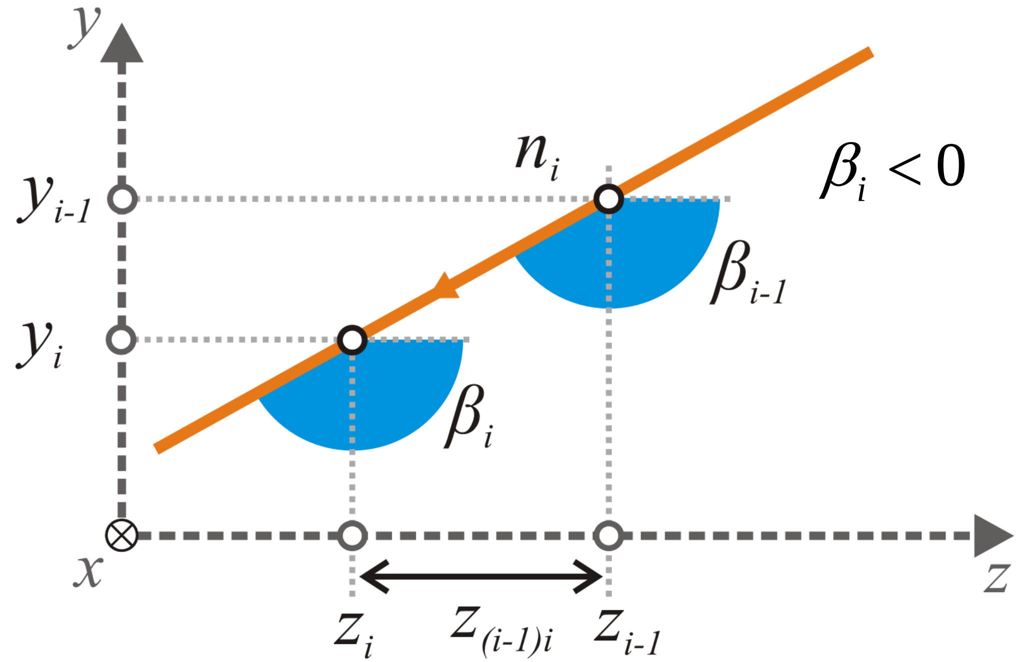
$$y_i \approx y_{i-1} + \frac{z_i - z_{i-1}}{n_{i-1}} n_{i-1}\beta_{i-1}$$
$$\beta_i = \beta_{i-1}$$

transformation matrix

$$\begin{bmatrix} y_i \\ n_i\,\beta_i \end{bmatrix} = \begin{bmatrix} 1 & z_{(i-1)i}/n_{i-1} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_{i-1} \\ n_{i-1}\,\beta_{i-1} \end{bmatrix}$$

- z optical displacement transformation matrix

$$y_{i-1} - y_i \approx -(z_{i-1} - z_i)\,\beta_i$$
$$\beta_i = \beta_{i-1}$$



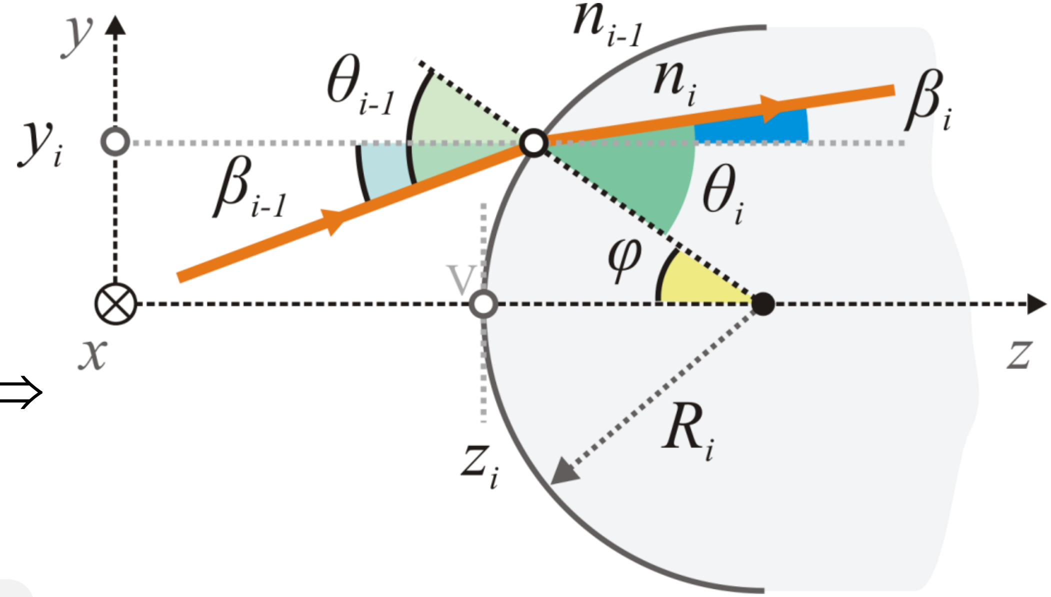
$$y_i \approx y_{i-1} + \frac{z_i - z_{i-1}}{(-n_{i-1})} n_{i-1}\beta_{i-1}$$
$$\beta_i = \beta_{i-1}$$

transformation matrix

$$\begin{bmatrix} y_i \\ n_i\,\beta_i \end{bmatrix} = \begin{bmatrix} 1 & z_{(i-1)i}/(-n_{i-1}) \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_{i-1} \\ n_{i-1}\,\beta_{i-1} \end{bmatrix}$$

Dimitris Papazoglou dpapa@materials.uoc.gr

Refraction transformation matrix



$$\left.\begin{aligned} \theta_{i-1} \approx \beta_{i-1} + \frac{y_i}{R_i},\ \ \theta_i \approx \beta_i + \frac{y_i}{R_i}, \\ n_i\,\theta_i \approx n_{i-1}\,\theta_{i-1} \end{aligned}\right\} \Rightarrow$$

$$y_i = y_{i-1}$$

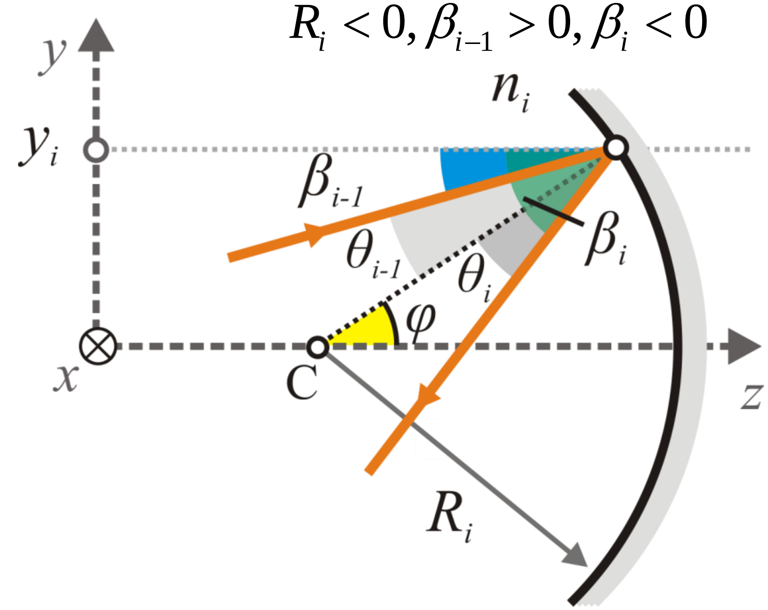
$$n_i\,\beta_i \approx -\frac{n_i - n_{i-1}}{R_i}\,y_{i-1} + n_{i-1}\,\beta_{i-1}$$

transformation matrix

$$\begin{bmatrix} y_i \\ n_i\,\beta_i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -(n_i - n_{i-1})/R_i & 1 \end{bmatrix} \cdot \begin{bmatrix} y_{i-1} \\ n_{i-1}\,\beta_{i-1} \end{bmatrix}$$

Reflection transformation matrix

$$\left.\begin{array}{c} y_i = y_{i-1}, \quad \theta_i = \theta_{i-1} \\ \varphi \cong -\dfrac{y_i}{R_i} \\ \left.\begin{array}{l} \theta_i = \varphi - \beta_{i-1} \\ \beta_i = -(\varphi + \theta_i) \end{array}\right\} \Rightarrow \beta_i = -2\varphi + \beta_{i-1} \end{array}\right\} \Rightarrow$$



$$y_i = y_{i-1}$$

$$n_i \beta_i \cong \frac{2 n_i y_i}{R_i} + n_i \beta_{i-1}$$

transformation matrix

$$\begin{bmatrix} y_i \\ n_i\,\beta_i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2n_i/R_i & 1 \end{bmatrix} \cdot \begin{bmatrix} y_{i-1} \\ n_{i-1}\,\beta_{i-1} \end{bmatrix}$$

Generalized transformation matrices

Displacement $\pm z$

$$\begin{bmatrix} 1 & Z_{(i-1)i} \\ 0 & 1 \end{bmatrix}$$

reduced length

$$Z_{(i-1)i} \equiv \frac{z_{(i-1)i}}{n_{i-1}}$$

Refraction / Reflection

$$\begin{bmatrix} 1 & 0 \\ -P_i & 1 \end{bmatrix}$$

optical power *(diopters or m^{-1})*

$$P_i \equiv \frac{(n_i - n_{i-1})}{R_i}$$

When a ray propagates towards in the negative direction of the z-axis (e.g., after reflection) we assume that the refractive index ***n*** is **negative**

Dimitris Papazoglou dpapa@materials.uoc.gr

Properties of transformation matrices

The determinant of all displacement and refraction transformation matrices is **equal to one**

$$\begin{vmatrix} 1 & Z_{(i-1)i} \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ -P_i & 1 \end{vmatrix} = 1$$

In a complex optical system the total transformation matrix can be calculated by **iteratively applying** displacement and refraction transformation matrices to the input ray vector.

$$M_{tot} = \begin{bmatrix} M_{11}^{tot} & M_{12}^{tot} \\ M_{21}^{tot} & M_{22}^{tot} \end{bmatrix} = [M_N] \cdot [M_{N-1}] \ldots [M_2] \cdot [M_1]$$

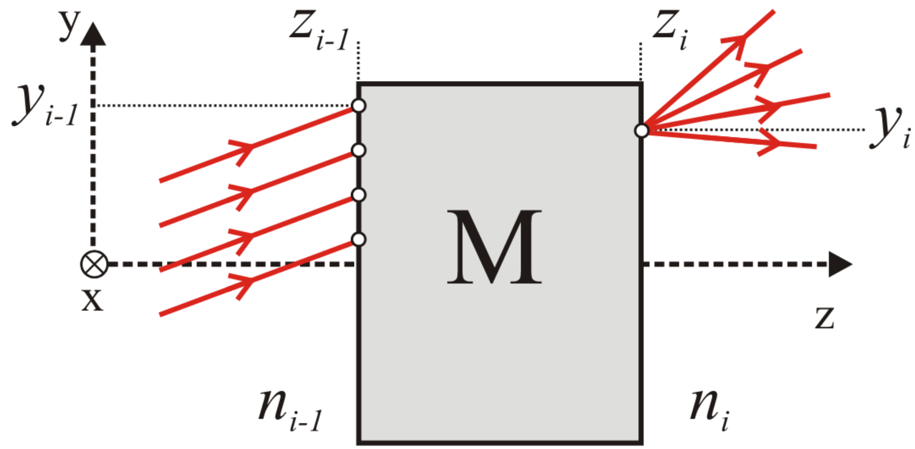
determinant equal to one

N^{th} element that the ray propagates through

1^{st} element that the ray intersects

Boundary conditions of transformation matrices

$$M_{11} \equiv 0 \Rightarrow M = \begin{bmatrix} 0 & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \Rightarrow$$



the ray height in the exit does not depend on the ray height in the entrance

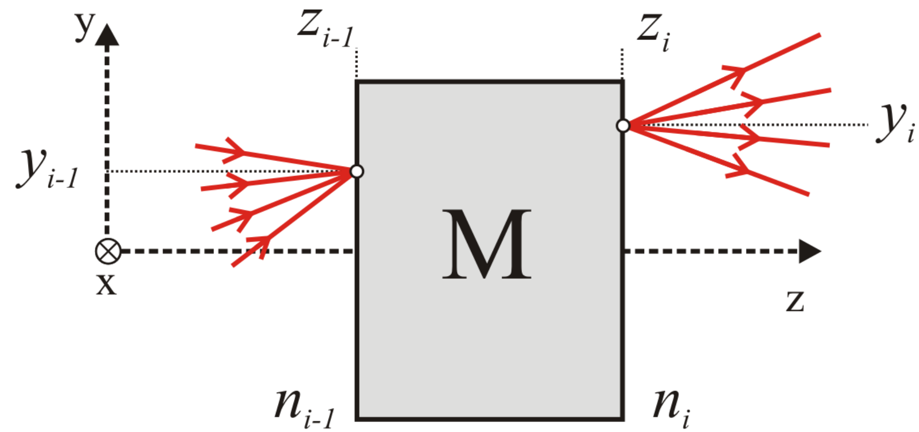
$$y_i = 0 \cdot y_{i-1} + M_{12} \cdot n_{i-1}\beta_{i-1}$$

$$n_i\beta_i = M_{21} \cdot y_{i-1} + M_{22} \cdot n_{i-1}\beta_{i-1}$$

When matrix element M_{11} is zero **parallel rays in the entrance** exit from the **same point in the exit**

$M_{11} = 0 \Leftrightarrow$ the exit of the optical system is a focal plane

$$M_{12} \equiv 0 \Rightarrow M = \begin{bmatrix} M_{11} & 0 \\ M_{21} & M_{22} \end{bmatrix} \Rightarrow$$



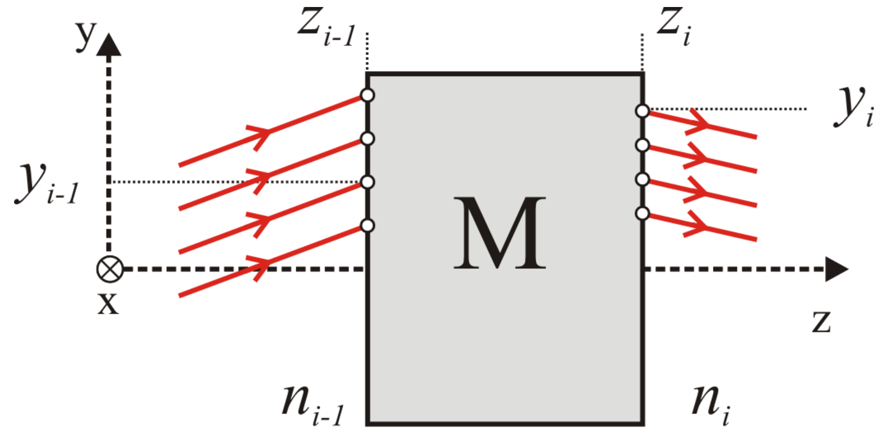
the ray height in the exit does not depend on the ray inclination in the entrance

$$y_i = M_{11} \cdot y_{i-1} + 0 \cdot n_{i-1}\beta_{i-1}$$
$$n_i\beta_i = M_{21} \cdot y_{i-1} + M_{22} \cdot n_{i-1}\beta_{i-1}$$

When matrix element M_{12} is zero rays that enter from **the same point in the entrance** exit from the **same point in the exit**

$M_{12} = 0 \Leftrightarrow$ the entrance and the exit of the optical system are conjugate planes (object / image)

$$M_{21} \equiv 0 \Rightarrow M = \begin{bmatrix} M_{11} & M_{12} \\ 0 & M_{22} \end{bmatrix} \Rightarrow$$



$$y_i = M_{11} \cdot y_{i-1} + M_{12} \cdot n_{i-1}\beta_{i-1}$$

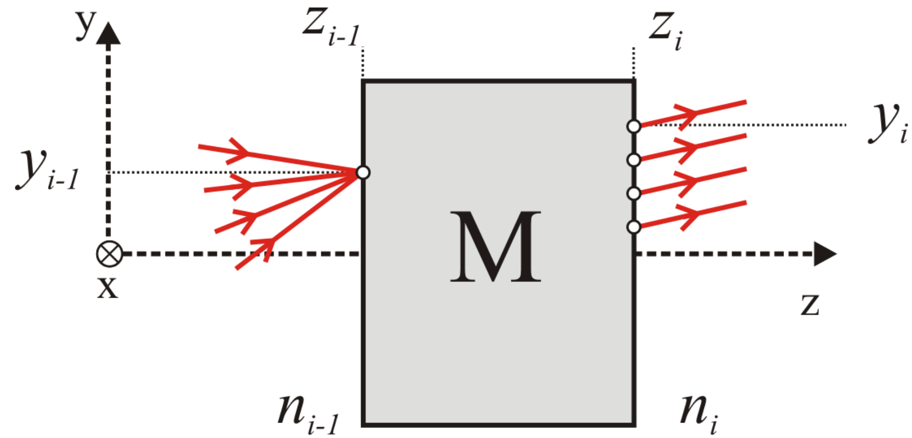
$$n_i\beta_i = \boxed{0 \cdot y_{i-1}} + M_{22} \cdot n_{i-1}\beta_{i-1}$$

the ray inclination in the exit is independent to the ray height in the entrance

When matrix element M_{21} is zero **parallel rays in the entrance** emerge also **parallel from the exit**

$M_{21} = 0 \Leftrightarrow$ **The optical system is telescopic/telecentric**

$$M_{22} \equiv 0 \Rightarrow M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & 0 \end{bmatrix} \Rightarrow$$



$$y_i = M_{11} \cdot y_{i-1} + M_{12} \cdot n_{i-1}\beta_{i-1}$$
$$n_i\beta_i = M_{21} \cdot y_{i-1} + 0 \cdot n_{i-1}\beta_{i-1}$$

the ray inclination in the exit is independent to the ray inclination in the entrance

When matrix element M_{22} is zero rays that enter from **the same point in the entrance emerge parallel** from the exit

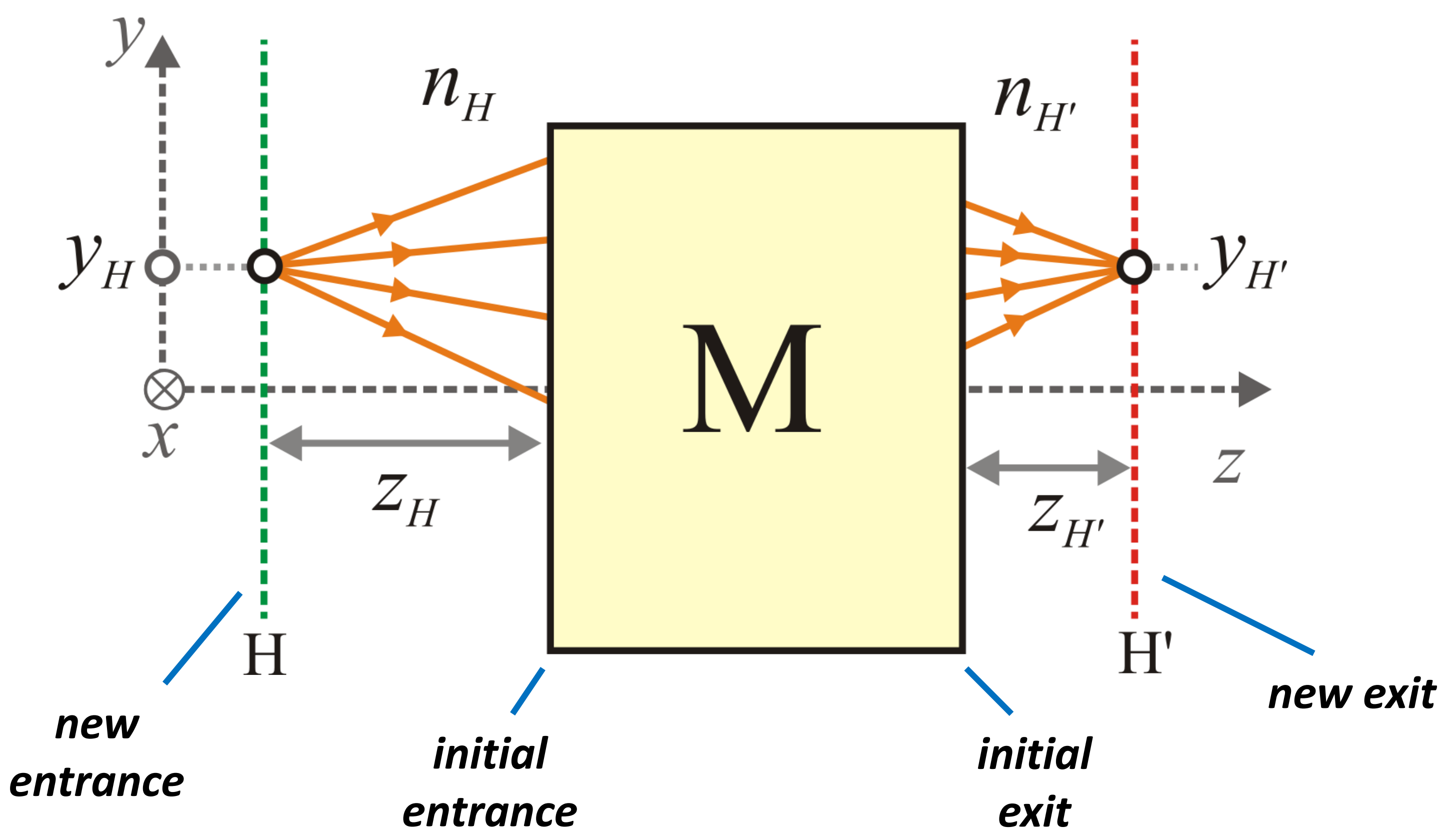
$M_{22} = 0 \Leftrightarrow$ **The entrance of the optical system is a focal plane**

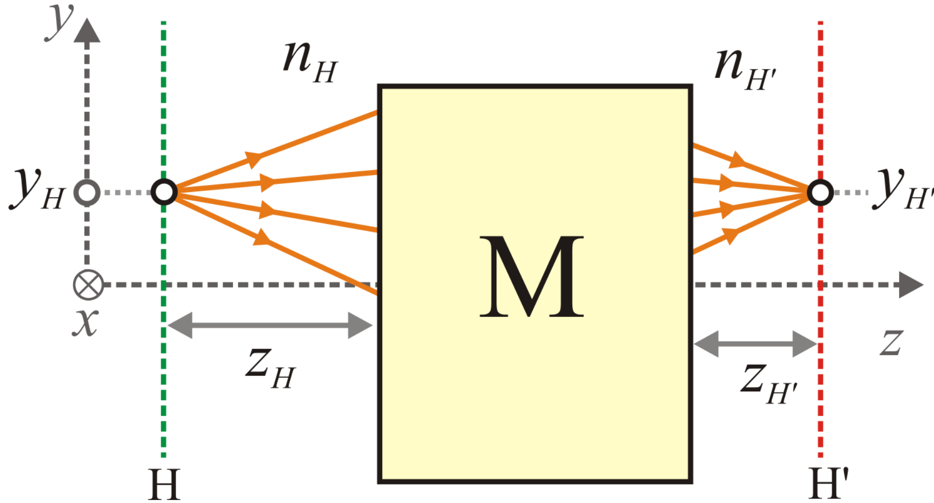
Complex optical systems described as a simple diopter

Is it possible to describe an arbitrarily complex optical system using a simple refraction transformation matrix?

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \overset{?}{\rightleftarrows} \begin{bmatrix} 1 & 0 \\ P & 1 \end{bmatrix}$$

Similarly we can ask if a complex lens comprised by many optical elements can behave as a simple diopter we construct a new “optical system” with a **new entrance (H)** and a **new exit (H’)** located respectively at a distance z_H and $z_{H'}$ from the initial ones. These new planes are called **Principal Planes**





reduced length

reduced length

$$\begin{bmatrix} y_{H'} \\ n_{H'}\,\beta_{H'} \end{bmatrix} = \begin{bmatrix} 1 & Z_{H'} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & Z_H \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_H \\ n_H\,\beta_H \end{bmatrix}$$

we demand:

$$\begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix} \equiv \begin{bmatrix} 1 & Z_{H'} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & Z_H \\ 0 & 1 \end{bmatrix}$$

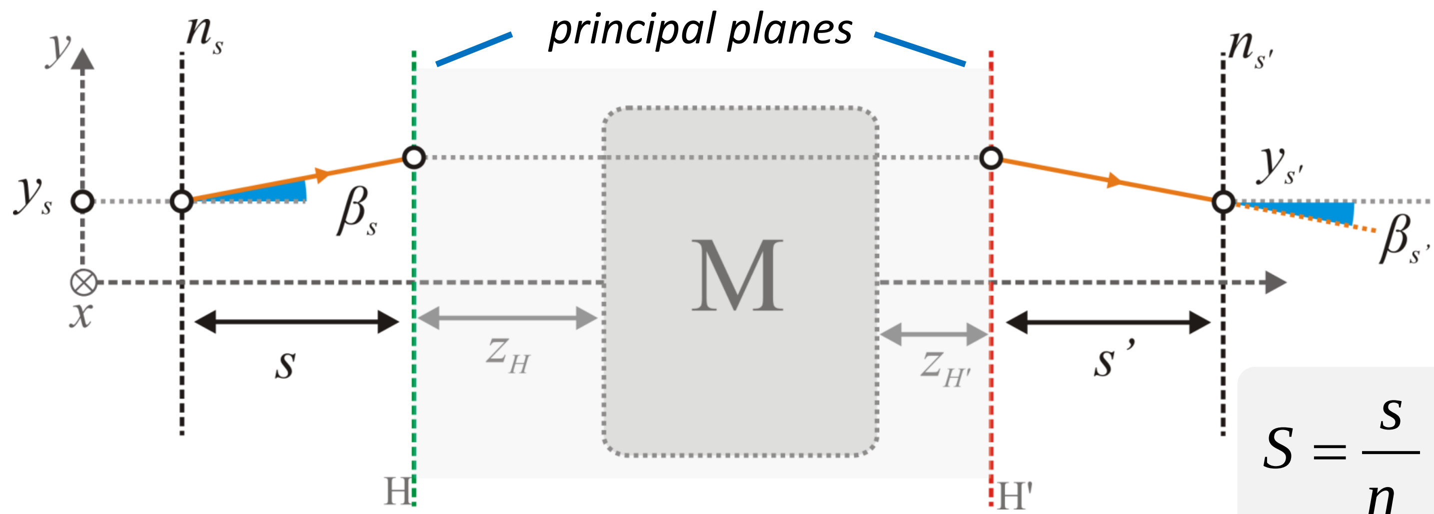
$$\begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix} \equiv \begin{bmatrix} 1 & Z_{H'} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & Z_H \\ 0 & 1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix} \equiv \begin{bmatrix} M_{11} + Z_{H'} M_{21} & Z_H M_{11} + M_{12} + Z_{H'}\left(Z_H M_{21} + M_{22}\right) \\ M_{21} & M_{22} + Z_H M_{21} \end{bmatrix} \Rightarrow$$

$$P \equiv -M_{21}$$

$$Z_H \equiv \frac{1 - M_{22}}{M_{21}}, \quad Z_{H'} \equiv \frac{1 - M_{11}}{M_{21}}$$

Imaging using the concept of principal planes



$$S=\frac{s}{n_s}\,,\qquad S'=\frac{s'}{n_{s'}}$$

$$\begin{bmatrix} y_{s'} \\ n_{s'}\,\beta_{s'} \end{bmatrix}=\begin{bmatrix} 1 & S' \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & S \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} y_s \\ n_s\,\beta_s \end{bmatrix}=\begin{bmatrix} 1-S'P & S'+S\left(1-S'P\right) \\ -P & 1-S\,P \end{bmatrix}\cdot\begin{bmatrix} y_s \\ n_s\,\beta_s \end{bmatrix}$$

conjugate planes $\Leftrightarrow M_{12}=0$

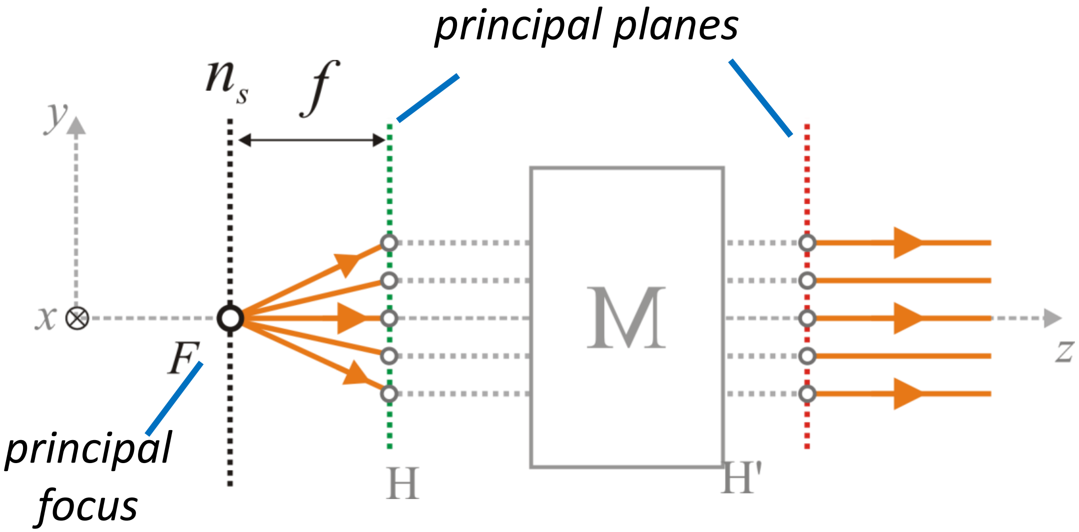
$$S'+S\left(1-S'P\right)=0\Rightarrow$$

Generalized Gauss imaging formula

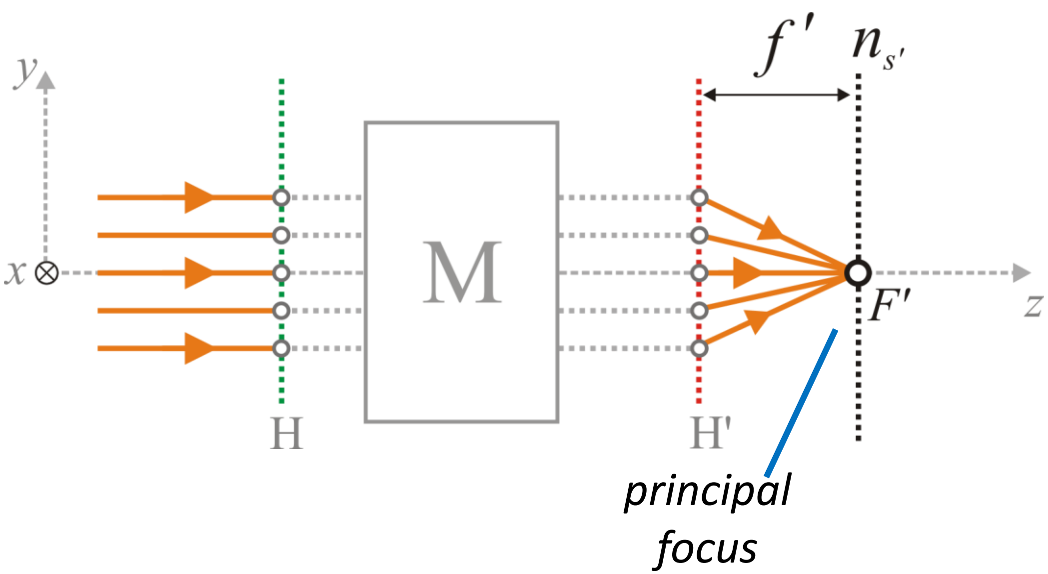
$$\frac{n_s}{s}+\frac{n_{s'}}{s'}=P$$

Principal foci

$$\frac{n_s}{s} + \frac{n_{s'}}{s'} = P$$



$$s' \to \infty \Rightarrow f = \frac{n_s}{P}$$



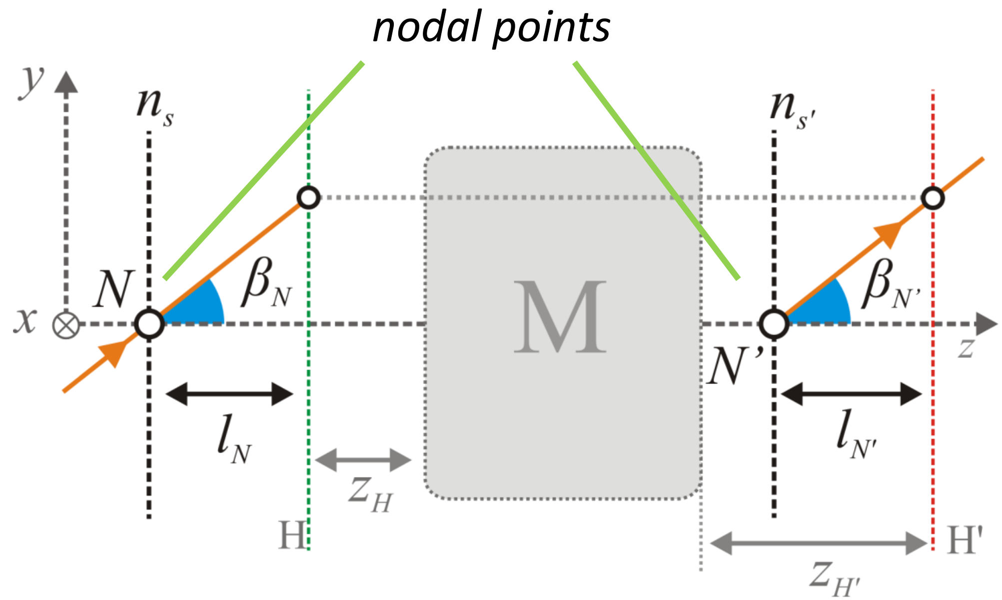
$$s \to \infty \Rightarrow f' = \frac{n_{s'}}{P}$$

in general: $f \neq f'$

Nodal points

$$\begin{bmatrix} 0 \\ n_{s'}\beta_{N'} \end{bmatrix} = \begin{bmatrix} 1-\frac{l_{N'}}{n_{s'}}P & \frac{l_{N'}}{n_{s'}}+\frac{l_N}{n_s}\left(1-\frac{l_{N'}}{n_{s'}}P\right) \\ -P & 1-\frac{l_N}{n_s}P \end{bmatrix} \cdot \begin{bmatrix} 0 \\ n_s\beta_N \end{bmatrix} =$$

$$= \begin{bmatrix} \left\{\frac{l_{N'}}{n_{s'}}+\frac{l_N}{n_s}\left(1-\frac{l_{N'}}{n_{s'}}P\right)\right\} n_s\beta_N \\ (1-\frac{l_N}{n_s}P)n_s\beta_N \end{bmatrix} \Rightarrow$$



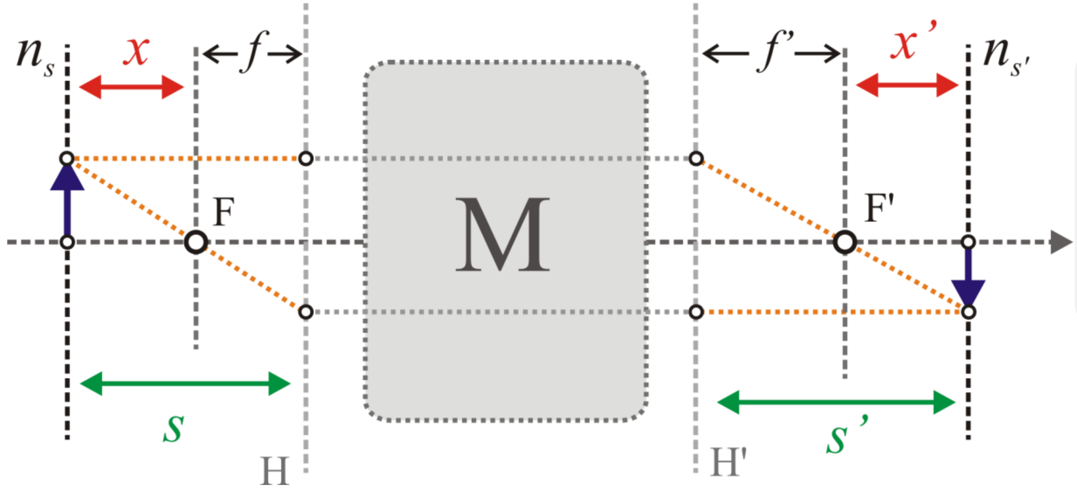
$$\Rightarrow \begin{cases} \left. \begin{array}{c} n_{s'}\beta_{N'} = (1-\frac{l_N}{n_s}P)n_s\beta_N \\ \beta_{N'} = \beta_N \end{array} \right\} \Rightarrow \\ \left[\frac{l_{N'}}{n_{s'}}+\frac{l_N}{n_s}\left(1-\frac{l_{N'}}{n_{s'}}P\right)\right] n_s\beta_N = 0 \Rightarrow \end{cases}$$

$$l_N = \frac{n_s - n_{s'}}{P}$$

$$l_{N'} = -\frac{n_s - n_{s'}}{P} = -l_N$$

Nodal points N, N' are always on the optical axis, either left of right from the principal planes. They are located on the primary planes H, H' when n_s=$n_{s'}$

Generalized Newton imaging formula



$$f = \frac{n_s}{P}, \qquad f' = \frac{n_{s'}}{P}$$

$$s = f + x, \quad s' = f' + x'$$

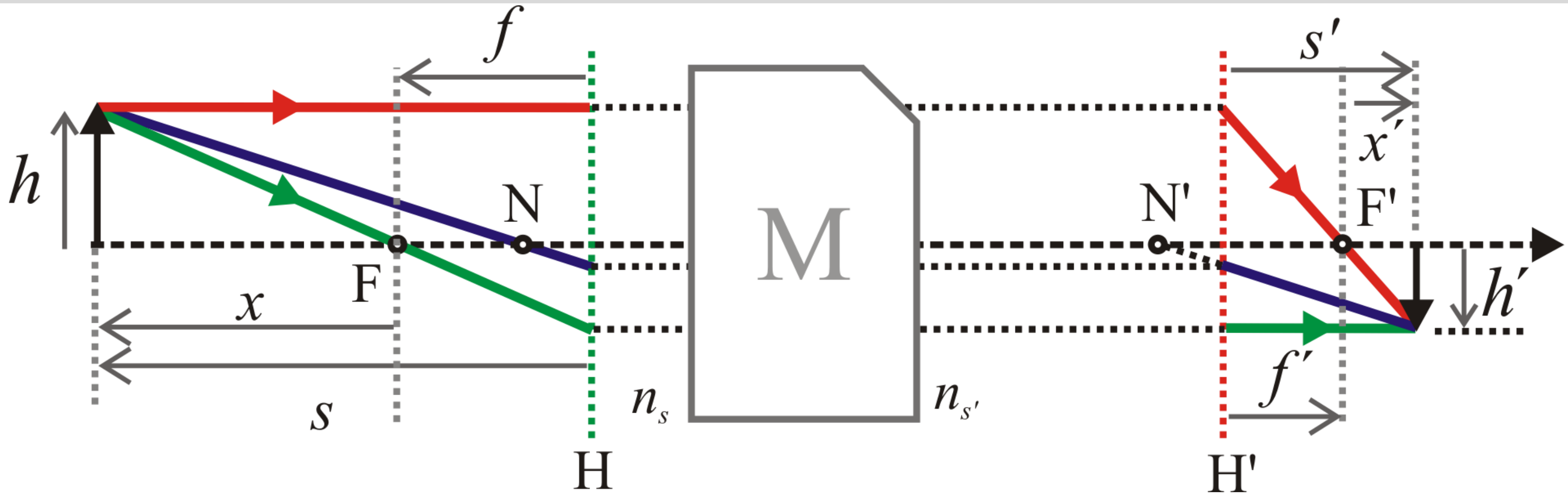
$$\frac{n_s}{s} + \frac{n_{s'}}{s'} = P \Leftrightarrow \frac{n_s}{f+x} + \frac{n_{s'}}{f'+x'} = P \Leftrightarrow n_s(f'+x') + n_{s'}(f+x) = P(f'+x')(f+x) \Leftrightarrow$$

$$n_s f' + n_s x' + n_{s'} f + n_{s'} x = P(ff' + x'f + xf' + xx') \Leftrightarrow$$

$$n_s\left(\frac{n_{s'}}{P}\right) + n_s x' + n_{s'}\left(\frac{n_s}{P}\right) + n_{s'} x = P\left(\frac{n_s}{P}\right)\left(\frac{n_{s'}}{P}\right) + Px'\left(\frac{n_s}{P}\right) + Px\left(\frac{n_{s'}}{P}\right) + Pxx' \Leftrightarrow \frac{n_s n_{s'}}{P} = Pxx' \Leftrightarrow$$

$$x \cdot x' = \frac{n_s n_{s'}}{P^2} = f \cdot f'$$

Generalized Magnification



Transverse Magnification

$$M_T \equiv \frac{h'}{h} = -\frac{f}{x} = -\frac{x'}{f'}$$

Longitudinal Magnification

$$M_L \equiv \frac{ds'}{ds} = -\frac{f'}{f} \cdot M_T^2 = -\frac{n_{s'}}{n_s} \cdot M_T^2$$

Special case

$$n_S = n_{s'} \Rightarrow \begin{cases} M_T = -\dfrac{s'}{s} \\ M_L = -M_T^2 \end{cases}$$

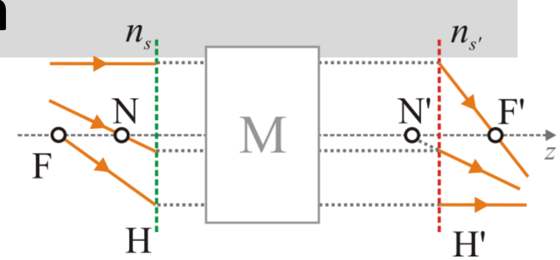
Analytic derivation of the generalized Longitudinal Magnification relation

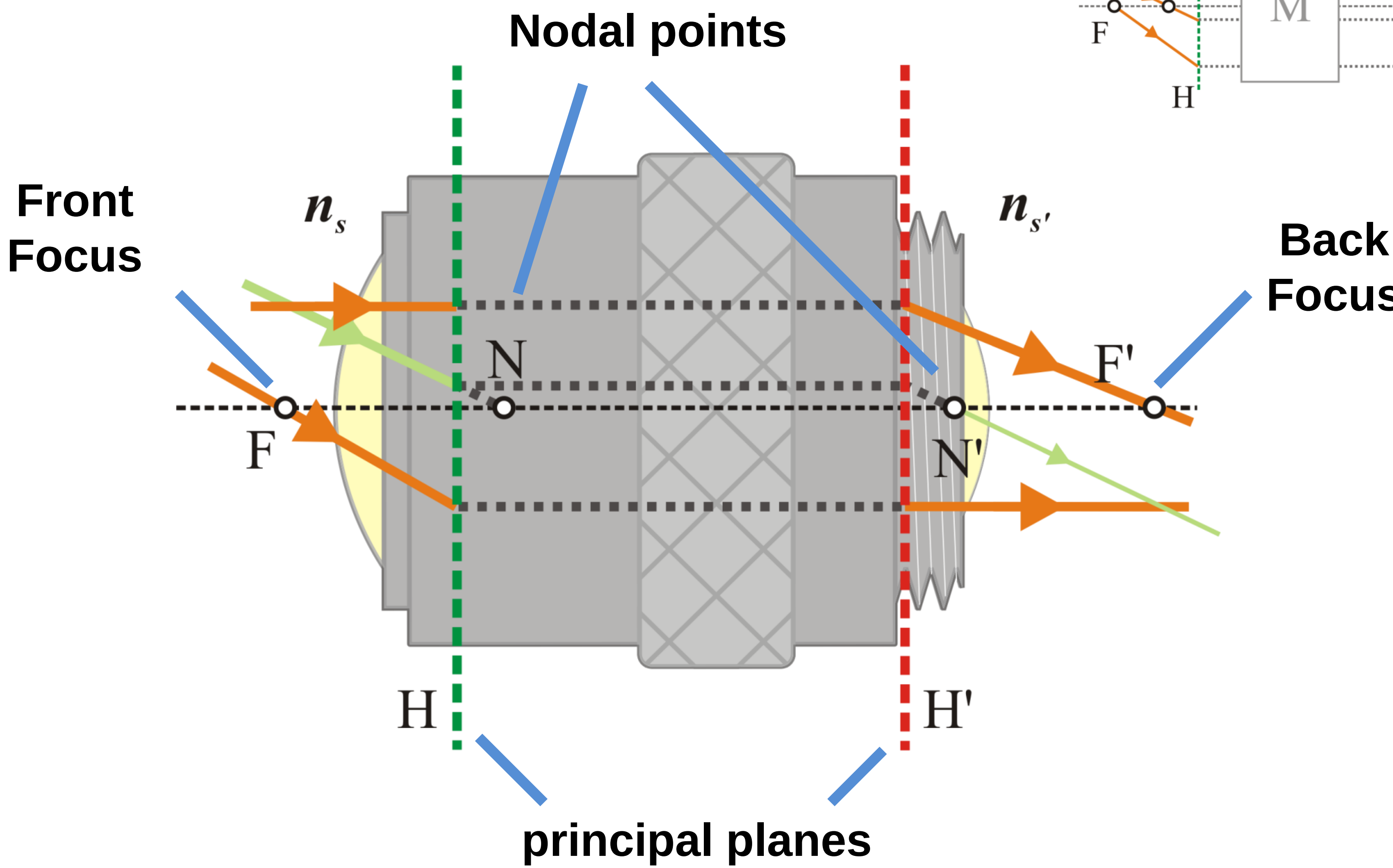
$$\left.\begin{array}{l} M_L \equiv \dfrac{ds'}{ds} = \dfrac{dx'}{dx} \\ x \cdot x' = f \cdot f' \end{array}\right\} \Rightarrow$$

$$M_L = -\frac{f \cdot f'}{x^2} = -\frac{x \cdot x'}{x^2} = -\frac{x'}{x} = -\frac{f'}{f} \cdot \frac{f}{x} \cdot \frac{x'}{f'} =$$

$$= -\frac{f'}{f} \cdot (-M_T) \cdot (-M_T) = \boxed{-\frac{f'}{f} \cdot M_T^2 = -\frac{n_{s'}}{n_s} \cdot M_T^2}$$

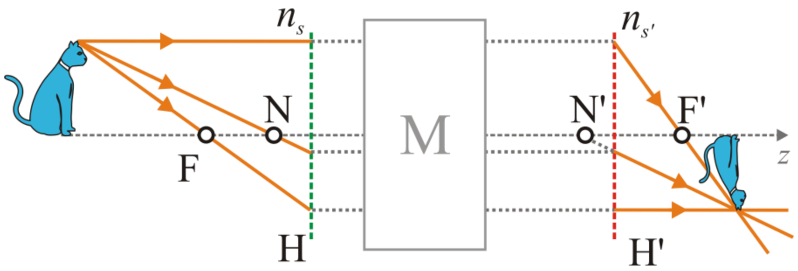
Cardinal (or primary) points of an optical system

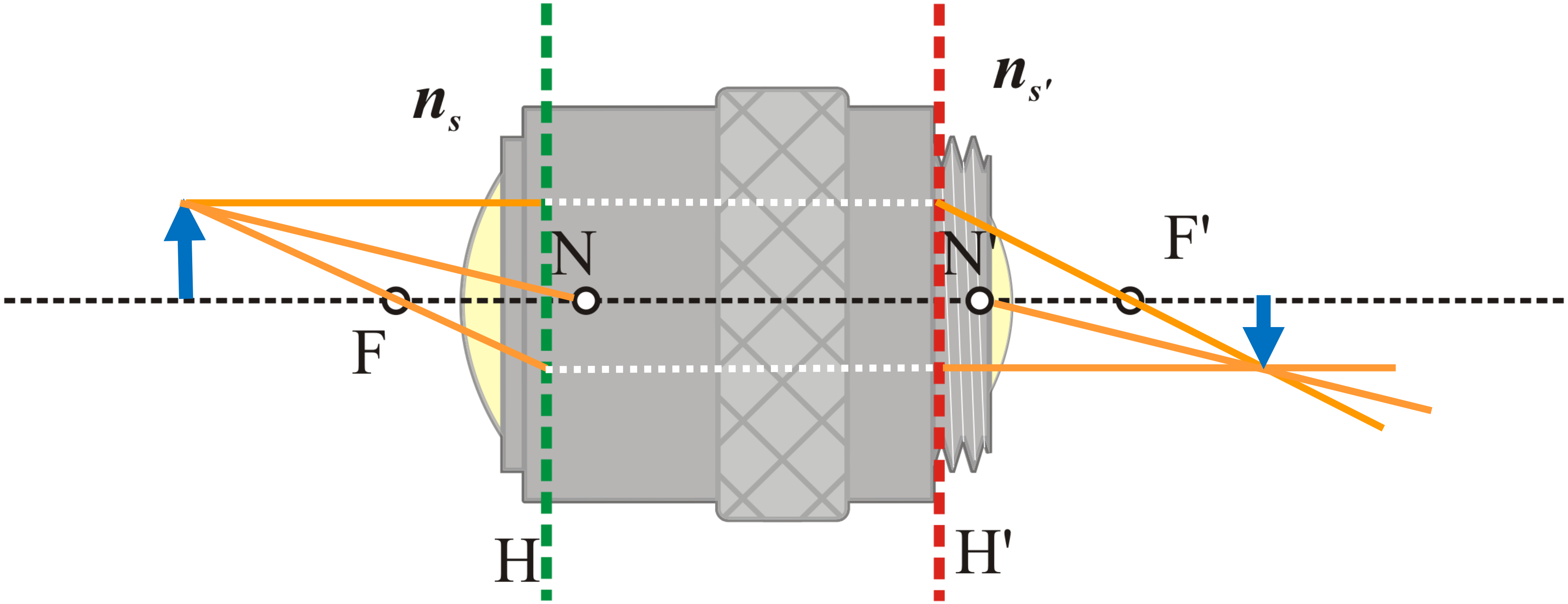




The **paraxial** imaging properties of any lens system are fully described by it's **cardinal points**

Raytracing using cardinal points

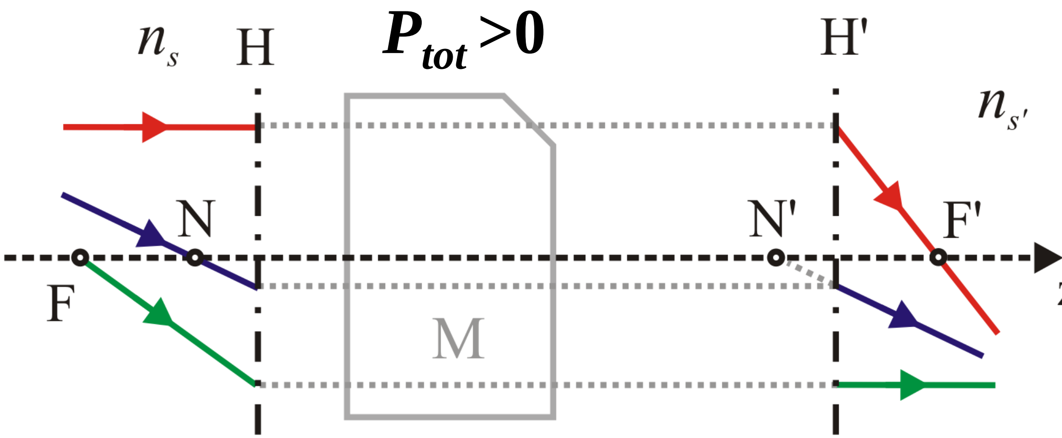


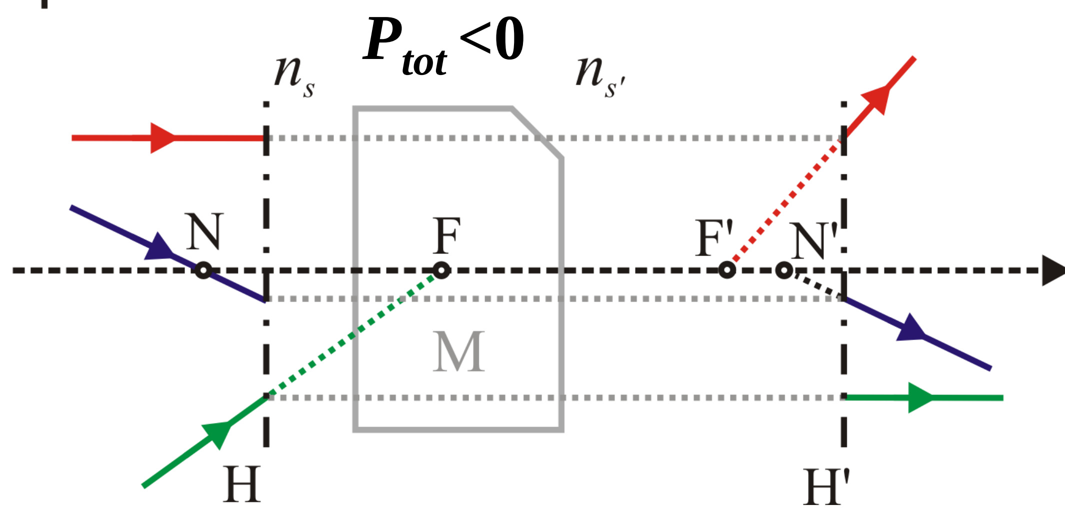


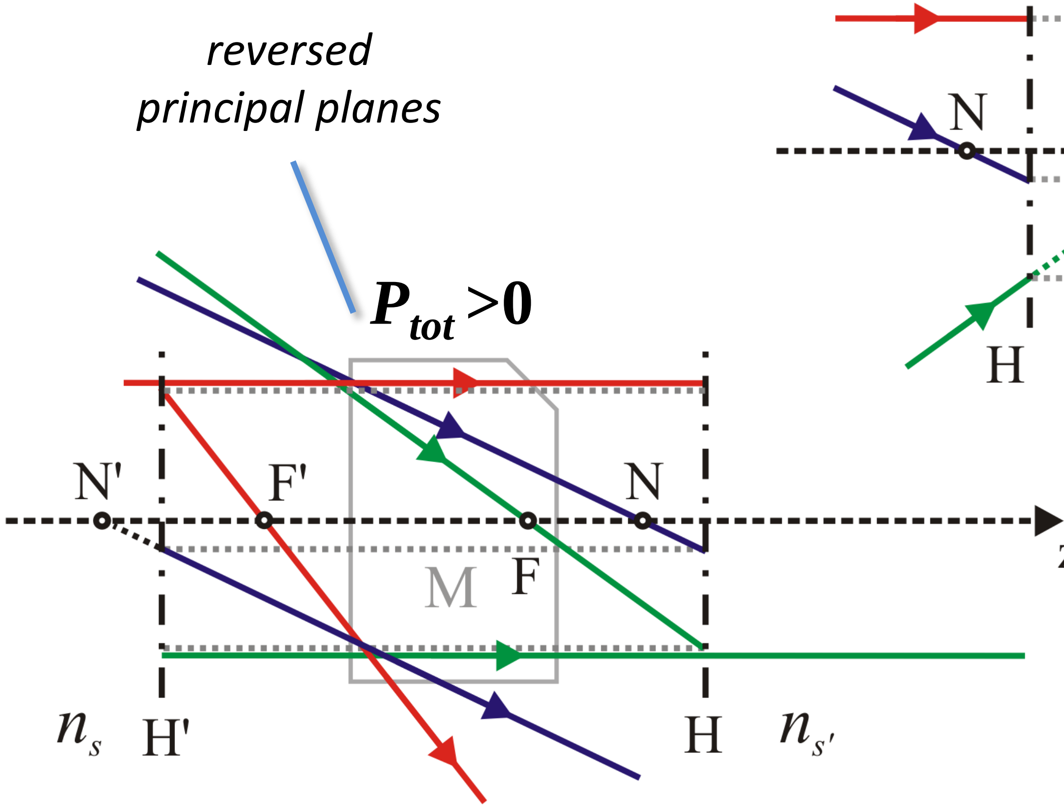
All imaging relationships that apply to thin lenses are also valid
to lens systems, as long as we measure the distances from the principal planes

Dimitris Papazoglou dpapa@materials.uoc.gr

Raytracing
primary rays



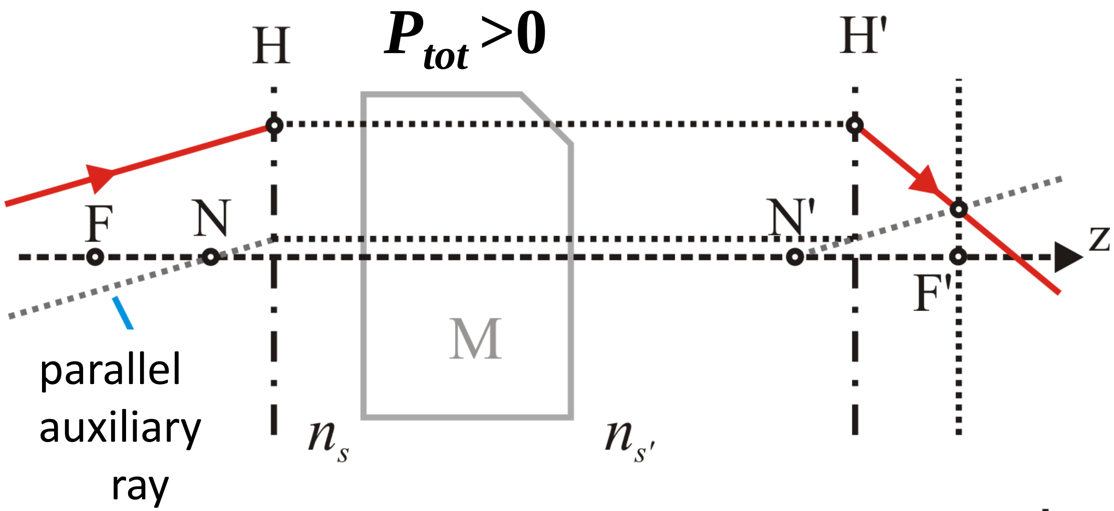


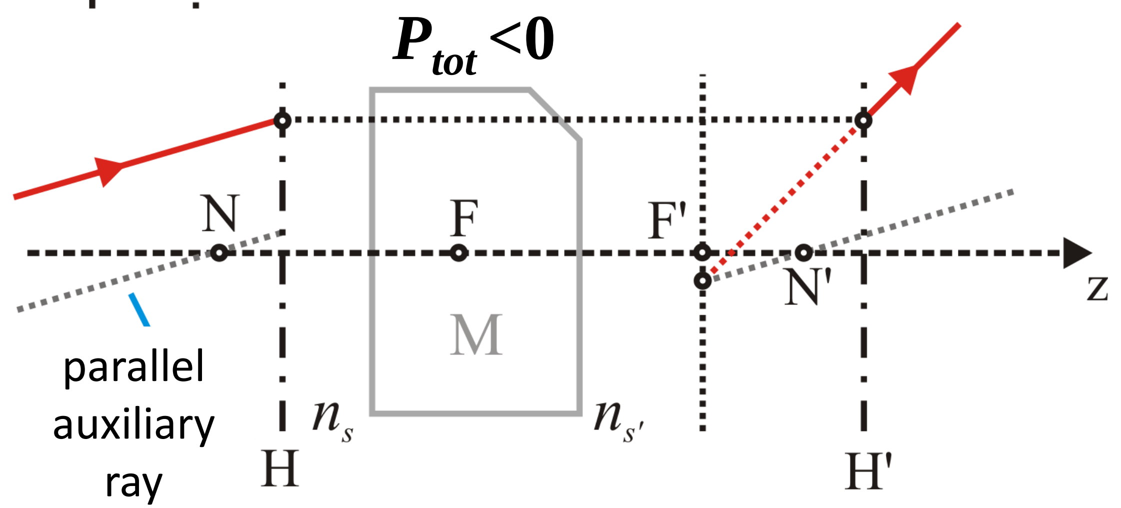


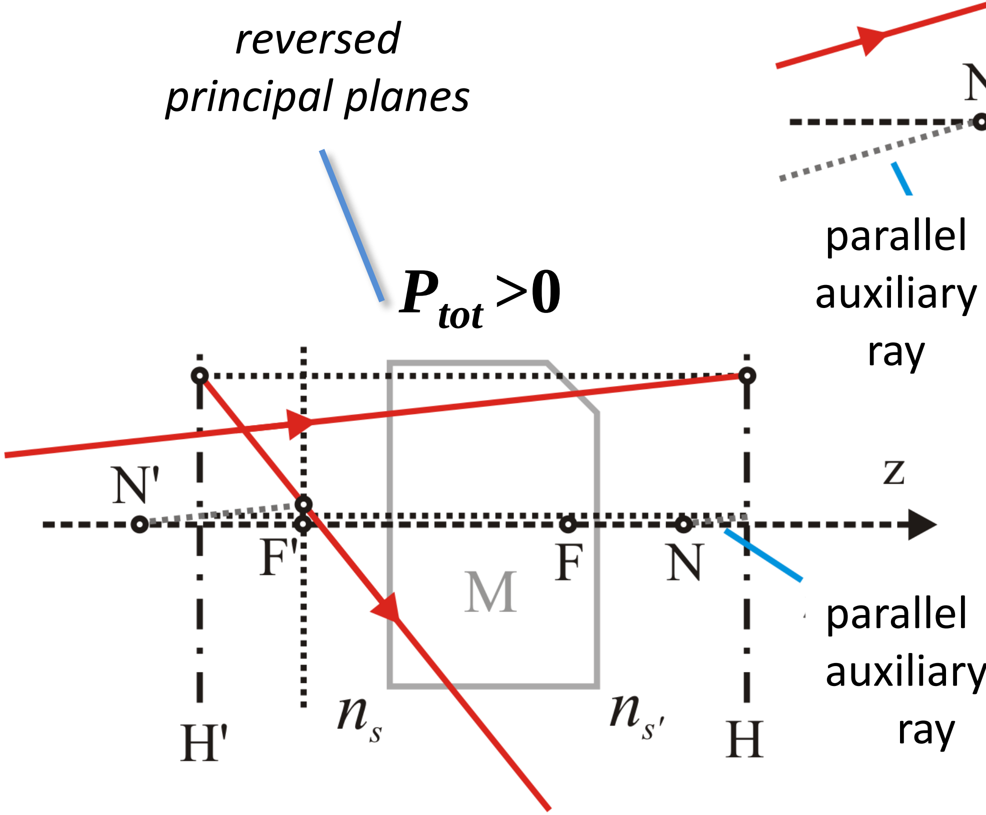
H, H'	*principal planes*
F, F'	*principal foci*
N, N'	*Nodal points*

Dimitris Papazoglou dpapa@materials.uoc.gr

Raytracing
random ray

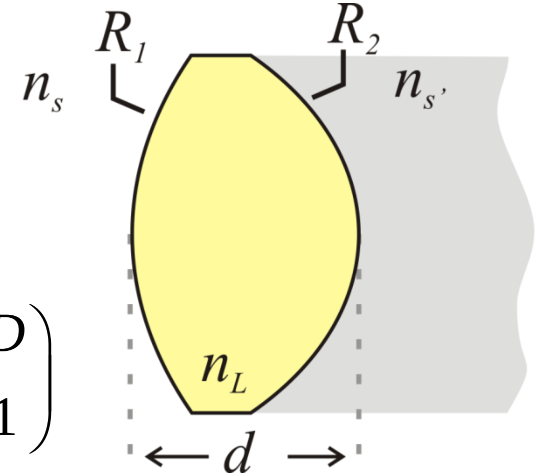






H, H'	*principal planes*
F, F'	*principal foci*
N, N'	*Nodal points*

Application: Thick lens



$$P_1 = \frac{n_L - n_s}{R_1}, \quad P_2 = \frac{n_{s'} - n_L}{R_2}, \quad D = \frac{d}{n_L}$$

$$M = \begin{pmatrix} 1 & 0 \\ -P_2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & D \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -P_1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -P_2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 - D \cdot P_1 & D \\ -P_1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - D \cdot P_1 & D \\ -P_{tot} & 1 - D \cdot P_2 \end{pmatrix}$$

Optical power

$$P_{tot} = P_1 + P_2 - D P_1 P_2$$

principal planes

$$Z_H \equiv \frac{1 - M_{22}}{M_{21}} = -\frac{d}{n_L} \frac{P_2}{P_{tot}}$$

$$Z_{H'} \equiv \frac{1 - M_{11}}{M_{21}} = -\frac{d}{n_L} \frac{P_1}{P_{tot}}$$

focal distances (principal)

$$f \equiv \frac{n_s}{P_{tot}} \qquad f' \equiv \frac{n_{s'}}{P_{tot}}$$

nodal points

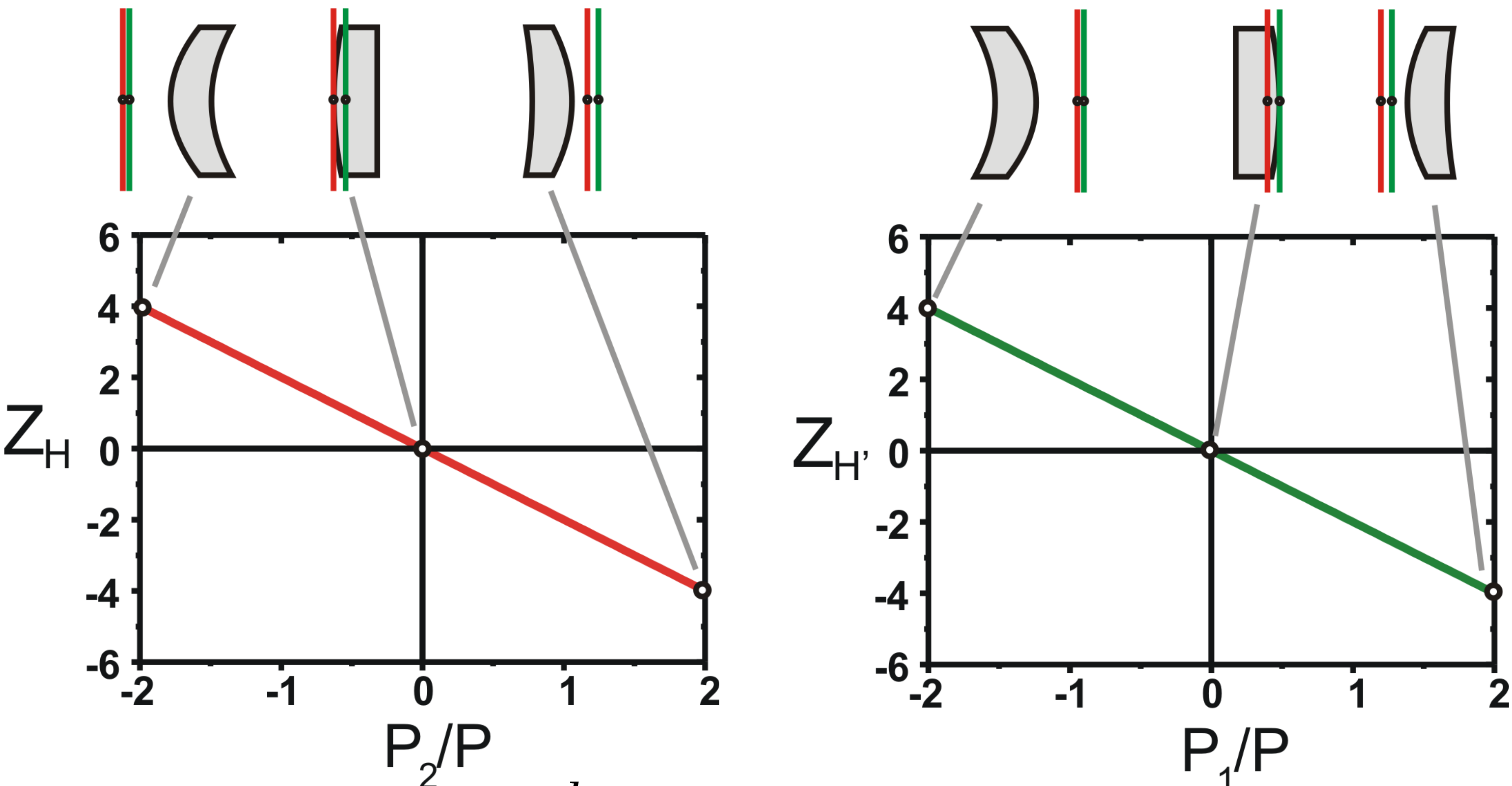
$$l_N = -l_{N'} \equiv \frac{n_s - n_{s'}}{P_{tot}}$$

Dimitris Papazoglou dpapa@materials.uoc.gr

Thick lens: positions of the principal planes

$$Z_H \equiv \frac{1-M_{22}}{M_{21}} = -\frac{d}{n_L}\frac{P_2}{P_{tot}}$$

$$Z_{H'} \equiv \frac{1-M_{11}}{M_{21}} = -\frac{d}{n_L}\frac{P_1}{P_{tot}}$$



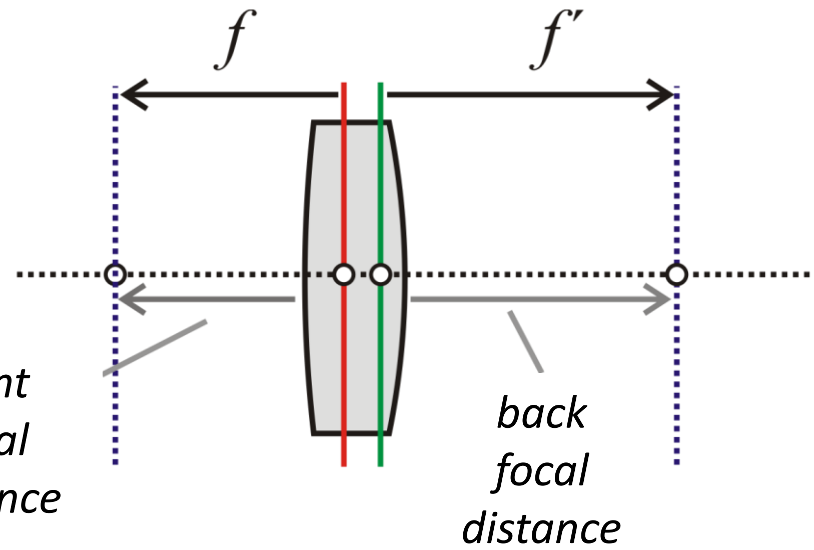
$$\frac{d}{n_L} = 2\,mm, \quad P = 1D$$

Dimitris Papazoglou dpapa@materials.uoc.gr

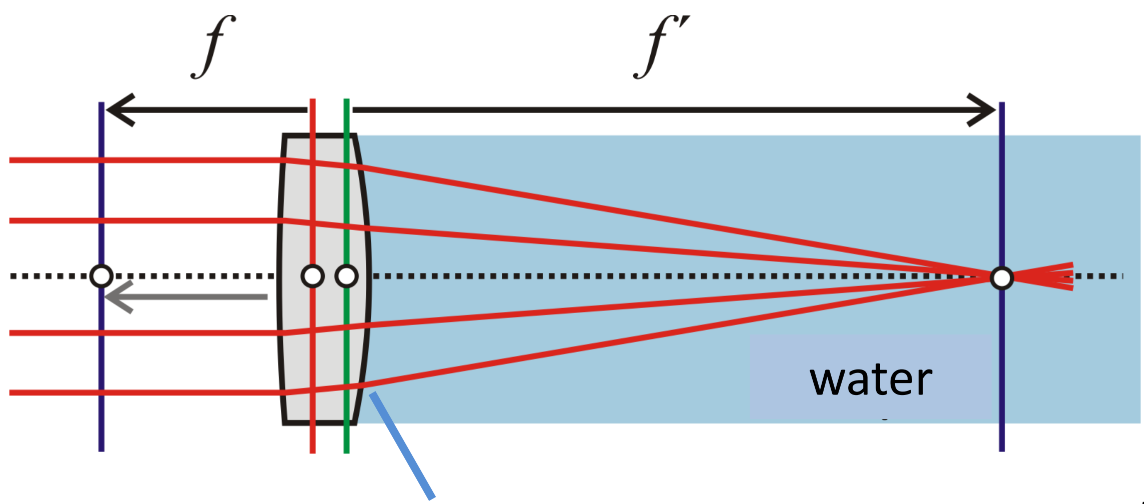
Thick lens: principal foci

$$f \equiv \frac{n_s}{P_{tot}}$$

$$f' \equiv \frac{n_{s'}}{P_{tot}}$$



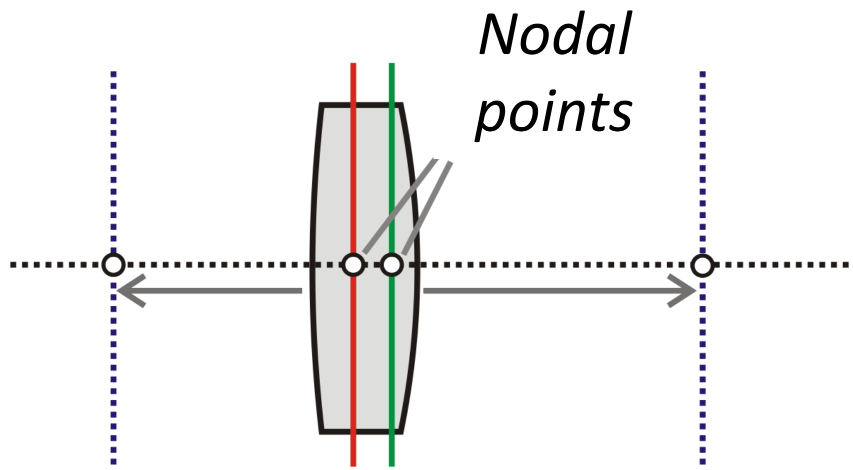
the effective focal distance depends on the surrounding medium refractive index.



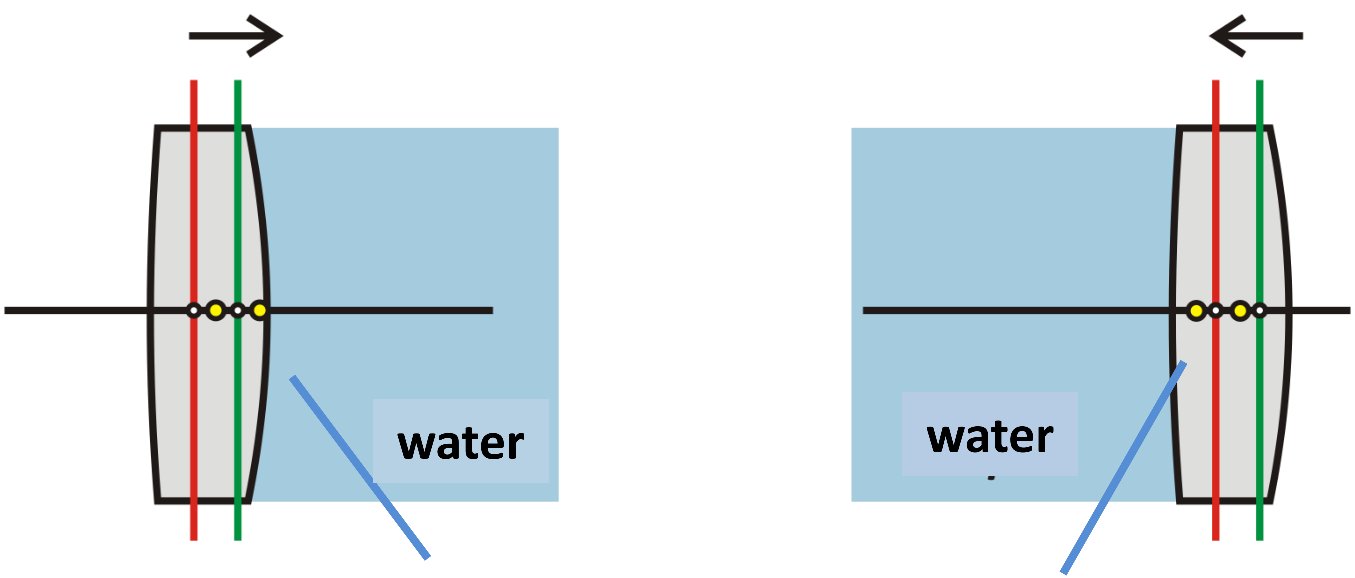
Attention! the refractive power of the 2nd diopter also changes

Thick lens: Nodal points

$$l_N = -l_{N'} \equiv \frac{n_s - n_{s'}}{P_{tot}}$$



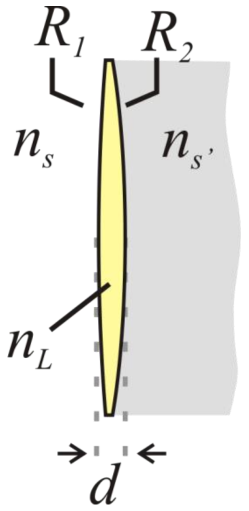
Nodal points do not coincide with the principal planes
when the media before and after the lens are different !



Nodal points move towards the optically denser medium!

Special case: thin lens d→ 0

$$P_1 = \frac{n_L - n_s}{R_1},\ \ P_2 = \frac{n_{s'} - n_L}{R_2},\ \ D = \frac{d}{n_L} \approx 0$$



$$M = \begin{pmatrix} 1 - D\cdot P_1 & D \\ -P_{tot} & 1 - D\cdot P_2 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ -P_{tot} & 1 \end{pmatrix}$$

Optical power

$$P_{tot} = P_1 + P_2$$

Principal planes

$$Z_H \equiv \frac{1 - M_{22}}{M_{21}} \approx 0$$

$$Z_{H'} \equiv \frac{1 - M_{11}}{M_{21}} \approx 0$$

focal distances (principal)

$$f \equiv \frac{n_s}{P_{tot}}$$

$$f' \equiv \frac{n_{s'}}{P_{tot}}$$

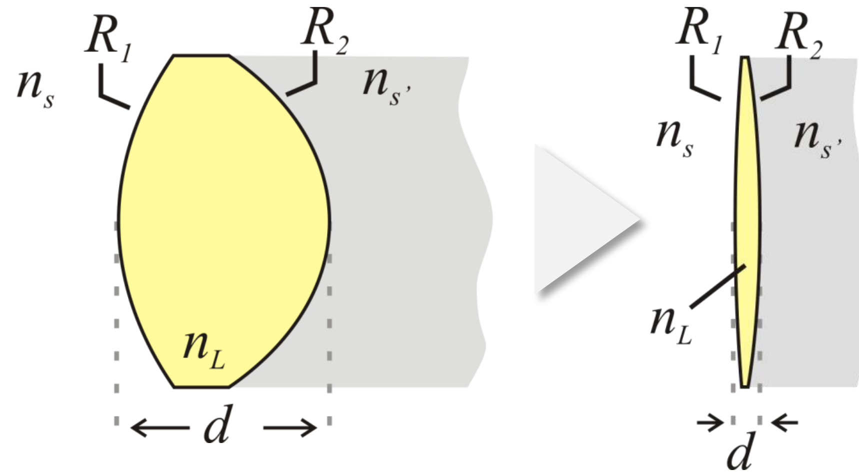
nodal points

$$l_N = -l_{N'} \equiv \frac{n_s - n_{s'}}{P_{tot}}$$

When a lens can be considered as thin?

$$P_1 = \frac{n_L - n_s}{R_1}, \quad P_2 = \frac{n_{s'} - n_L}{R_2}, \quad D = \frac{d}{n_L}$$

$$M = \begin{pmatrix} 1 - D \cdot P_1 & D \\ -(P_1 + P_2 - D P_1 P_2) & 1 - D \cdot P_2 \end{pmatrix}$$



$$\left.\begin{aligned} P_{tot} \cong P_1 + P_2 &\Rightarrow D \ll \left|\frac{1}{P_1} + \frac{1}{P_2}\right| \\ 1 - D \cdot P_1 \cong 1 &\Rightarrow D \ll \left|\frac{1}{P_1}\right| \\ 1 - D \cdot P_2 \cong 1 &\Rightarrow D \ll \left|\frac{1}{P_2}\right| \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} d &\ll \left|\frac{n_L}{n_L - n_s} R_1\right| \\ d &\ll \left|\frac{n_L}{n_{s'} - n_L} R_2\right| \end{aligned}\right.$$

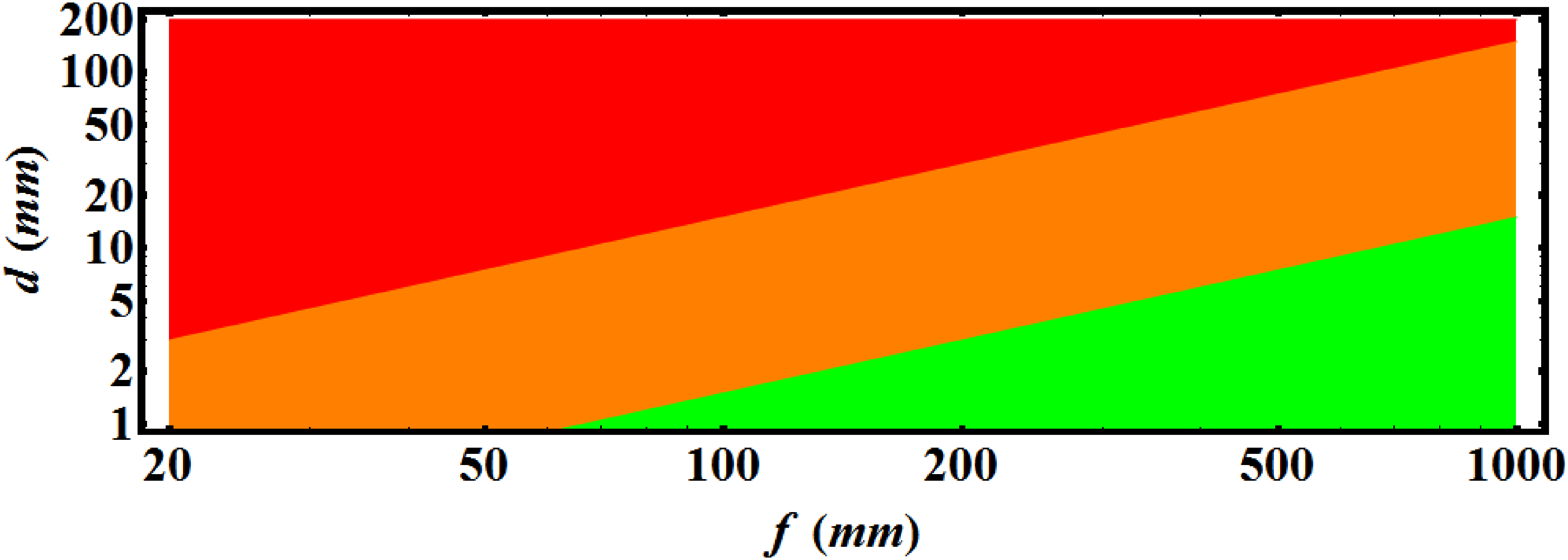
Typical thickens values for thin lenses

Glass lens surrounded by air:

$$n_L \cong 1.5,\ n_s = n_{s'} = 1 \Rightarrow d \ll 3|R_1| \wedge d \ll 3|R_2|$$

Plano-convex lens ($R_2 \to \infty$) of focal distance f

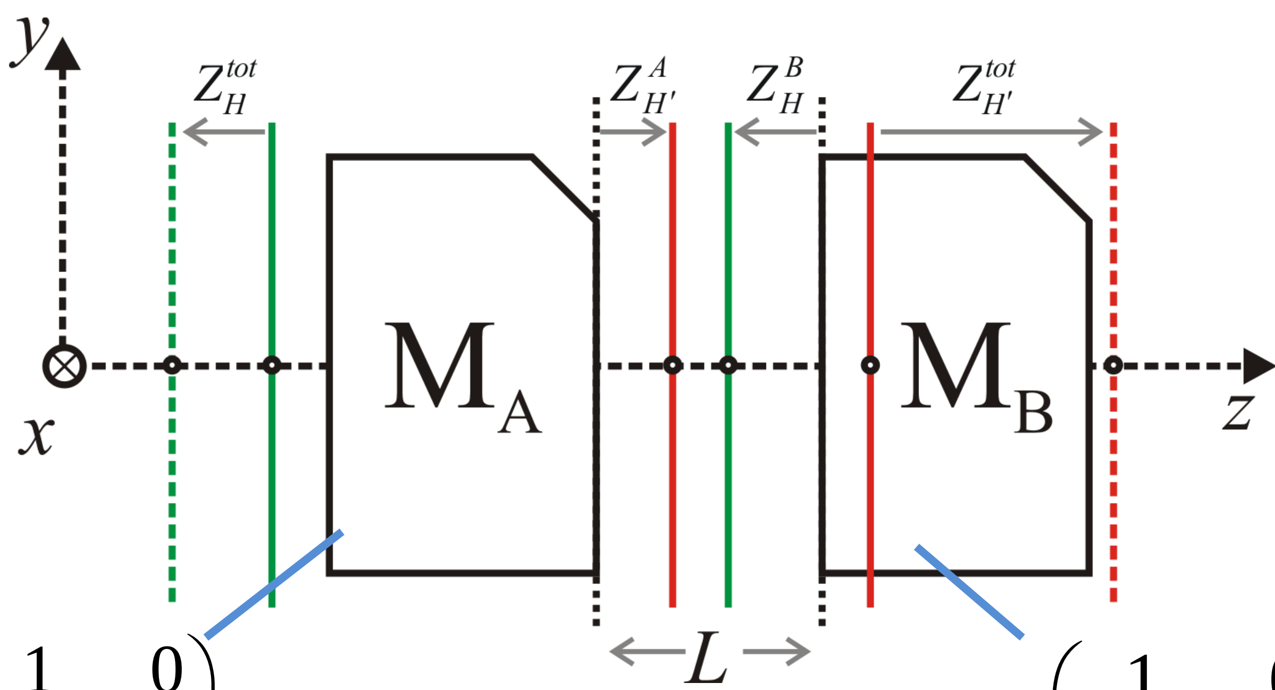
$$f \cong 2R_1 \Rightarrow d \ll 1.5|f| \qquad i.e.\ \ d < 0.015|f|$$



Matrix Theory
in optical engineering

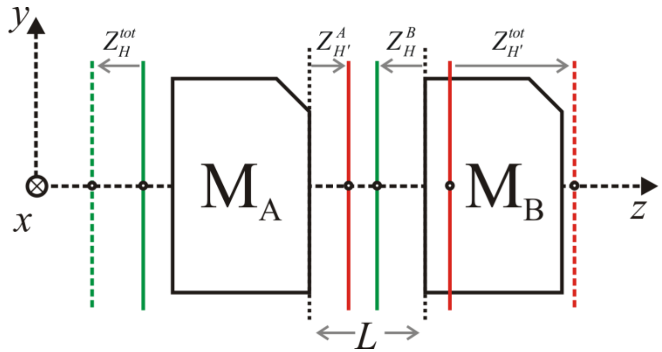
Optical system composition

How can we describe the sequence of two optical systems $\mathbf{M_A}$, $\mathbf{M_B}$ separated by distance L (exit-entrance distance)?



$$\begin{pmatrix} 1 & 0 \\ -P_1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ -P_2 & 1 \end{pmatrix}$$



$$M_{tot} = \begin{pmatrix} 1 & 0 \\ -P_2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L-(Z_{H'}^A + Z_H^B) \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -P_1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 1-[L-(Z_{H'}^A + Z_H^B)]P_1 & L-(Z_{H'}^A + Z_H^B) \\ -\{P_1 + P_2 - [L-(Z_{H'}^A + Z_H^B)]P_1P_2\} & 1-[L-(Z_{H'}^A + Z_H^B)]P_2 \end{pmatrix}$$

$$P_{tot} = P_1 + P_2 - [L-(Z_{H'}^A + Z_H^B)]P_1P_2,$$

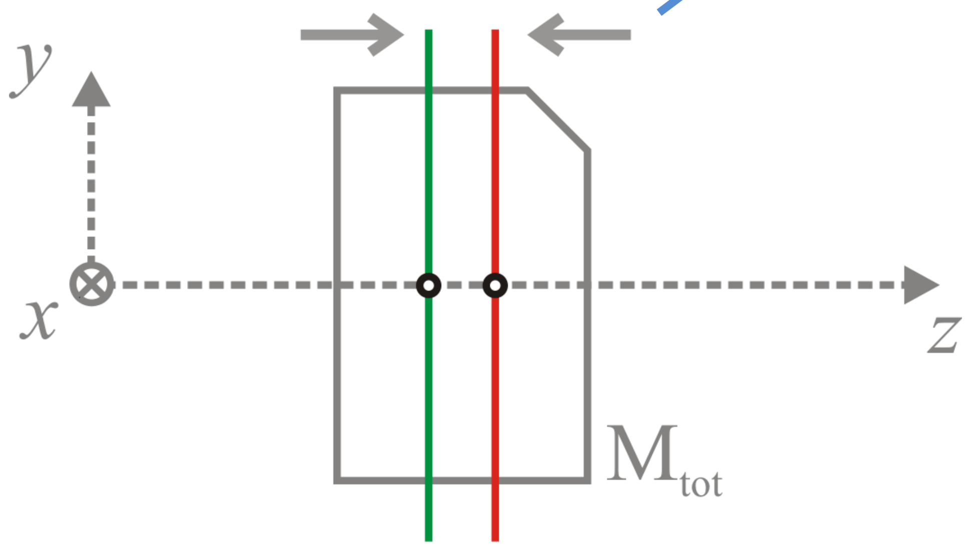
$$Z_H^{tot} = -[L-(Z_{H'}^A + Z_H^B)]\frac{P_2}{P_{tot}},$$

$$Z_{H'}^{tot} = -[L-(Z_{H'}^A + Z_H^B)]\frac{P_1}{P_{tot}}$$

Example of optical system composition: Normal lens

$$\left.\begin{aligned} & P_{tot}, P_1, P_2 > 0 \\ & L > (Z_{H'}^{A} + Z_{H}^{B}) \end{aligned}\right\} \Rightarrow Z_{H}^{tot} < 0,\ Z_{H'}^{tot} < 0$$

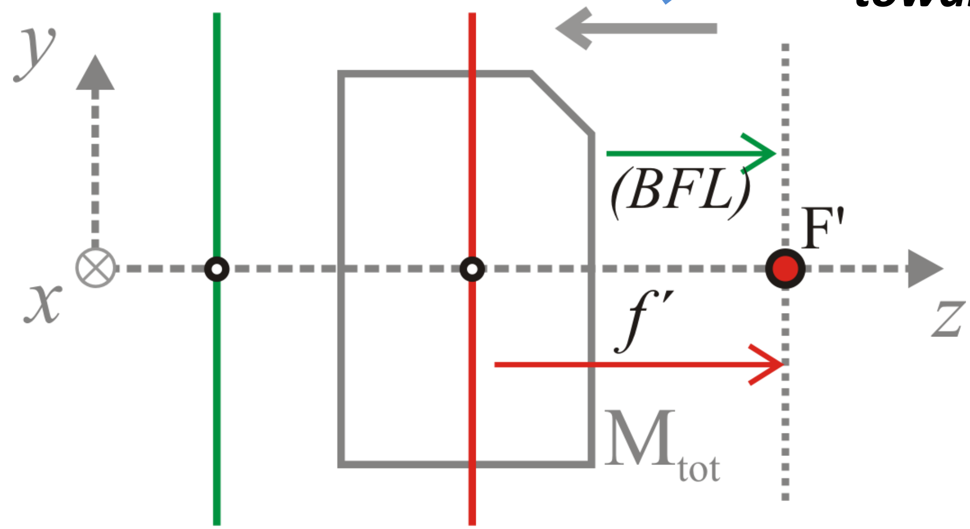
The composite system principal planes shift inwards



Example of optical system composition: Telephoto lens

$$\left.\begin{array}{l} P_{tot}, P_1 > 0,\ P_2 < 0 \\ L > (Z_{H'}^{A} + Z_{H}^{B}) \end{array}\right\} \Rightarrow Z_{H}^{tot} > 0,\ Z_{H'}^{tot} < 0$$

The composite system principal planes shift towards the object

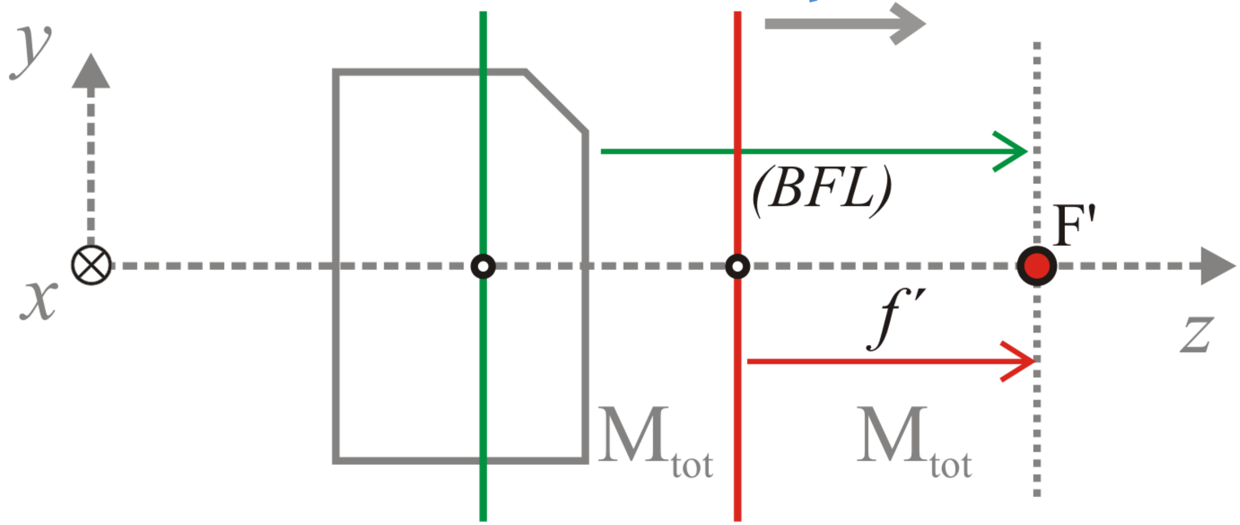


In a telephoto lens the effective focal distance *f′* is **larger** than the back focal distance (BFL)

Example of optical system composition: Inverse Telephoto lens

$$\left.\begin{array}{l} P_{tot}, P_2 > 0, P_1 < 0 \\ L > (Z_{H'}^{A} + Z_{H}^{B}) \end{array}\right\} \Rightarrow Z_{H}^{tot} < 0,\ Z_{H'}^{tot} > 0$$

The composite system principal planes shift towards the image

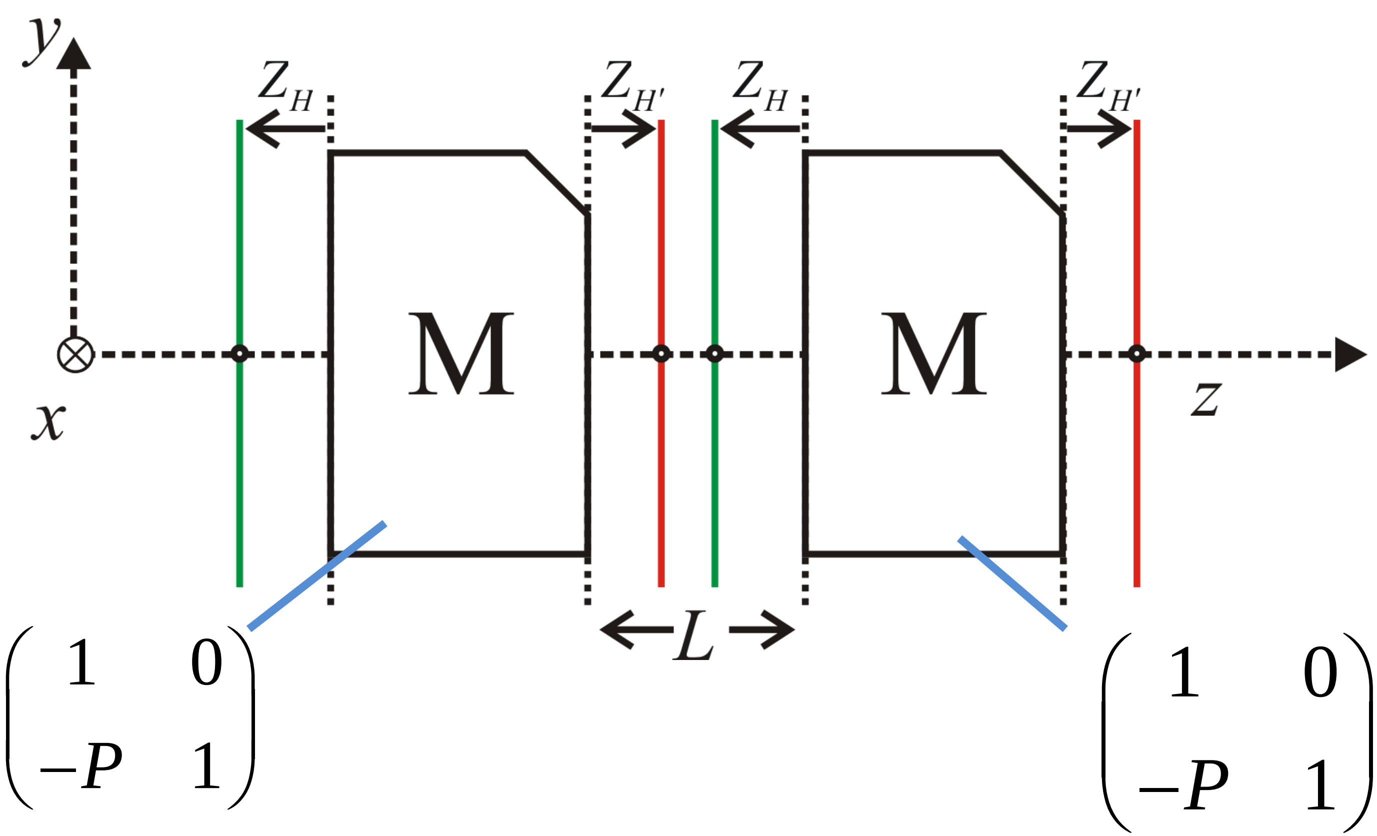


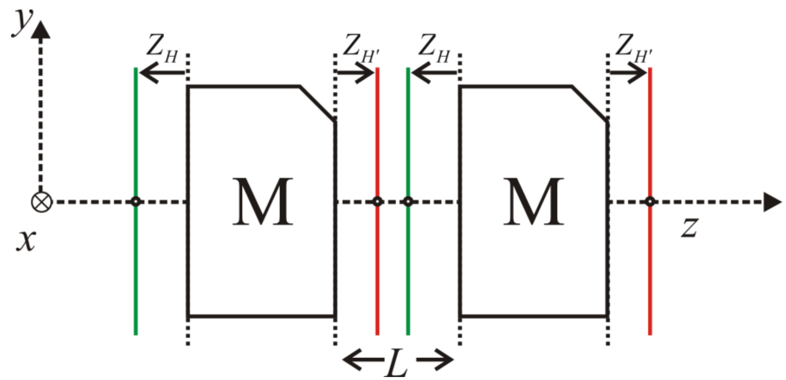
In an inverse telephoto lens the effective focal distance f' is **smaller** than the back focal distance (BFL)

Dimitris Papazoglou dpapa@materials.uoc.gr

Composition of equal optical systems

How can we describe the sequence of two equal optical systems $\mathbf{M}$ separated by distance L (exit-entrance distance)?





$$M_{tot} = \begin{pmatrix} 1-[L-(Z_{H'}+Z_H)]P & L-(Z_{H'}+Z_H) \\ -\{2P-[L-(Z_{H'}+Z_H)]P^2\} & 1-[L-(Z_{H'}+Z_H)]P \end{pmatrix}$$

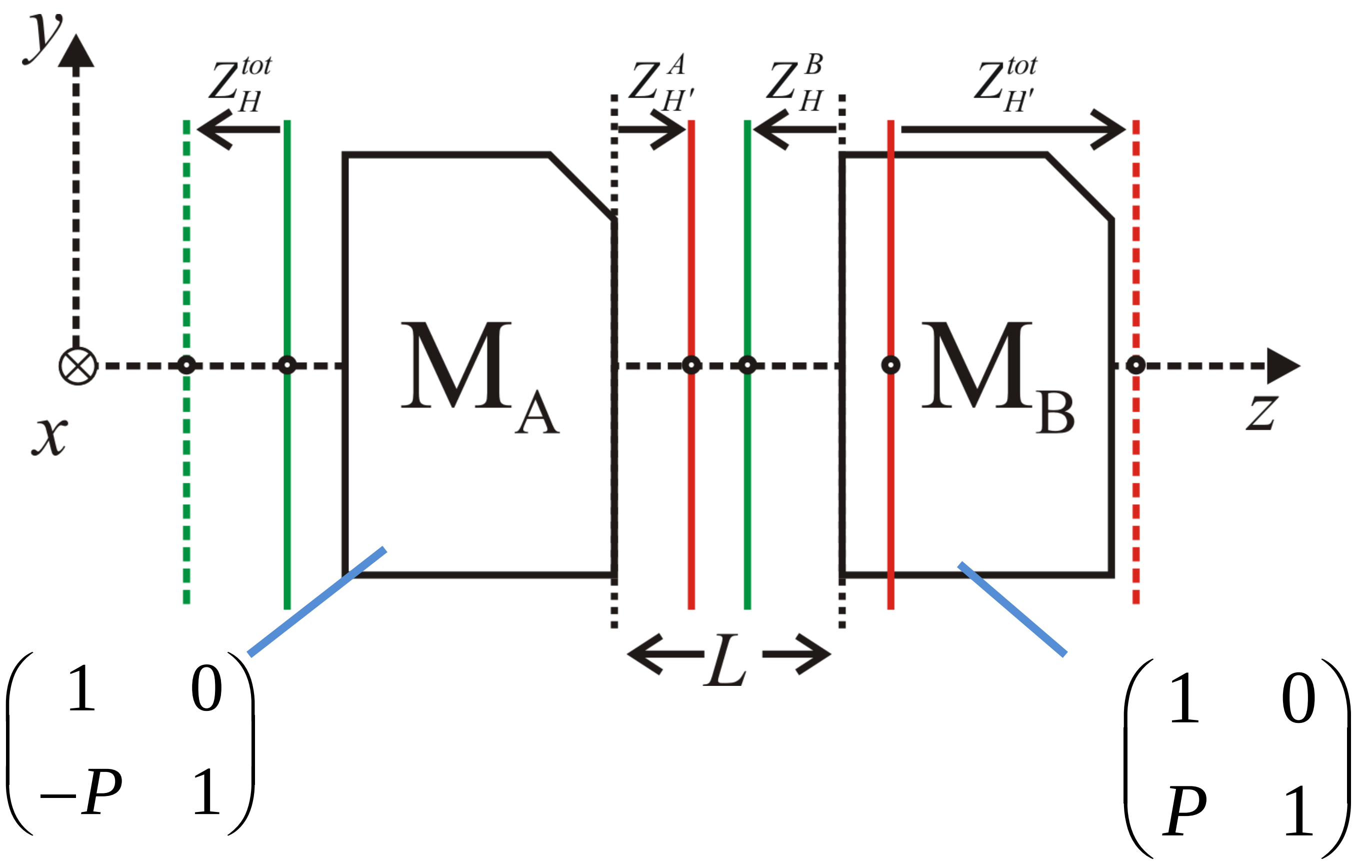
$$P_{tot} = 2P-[L-(Z_{H'}+Z_H)]P^2,$$

$$Z_H^{tot} = Z_{H'}^{tot} = -\frac{L-(Z_{H'}+Z_H)}{2-[L-(Z_{H'}+Z_H)]P}$$

in a optical system composed by two equal sub-systems
the principal planes are located symmetrically.

Composition of optical systems of opposite optical power

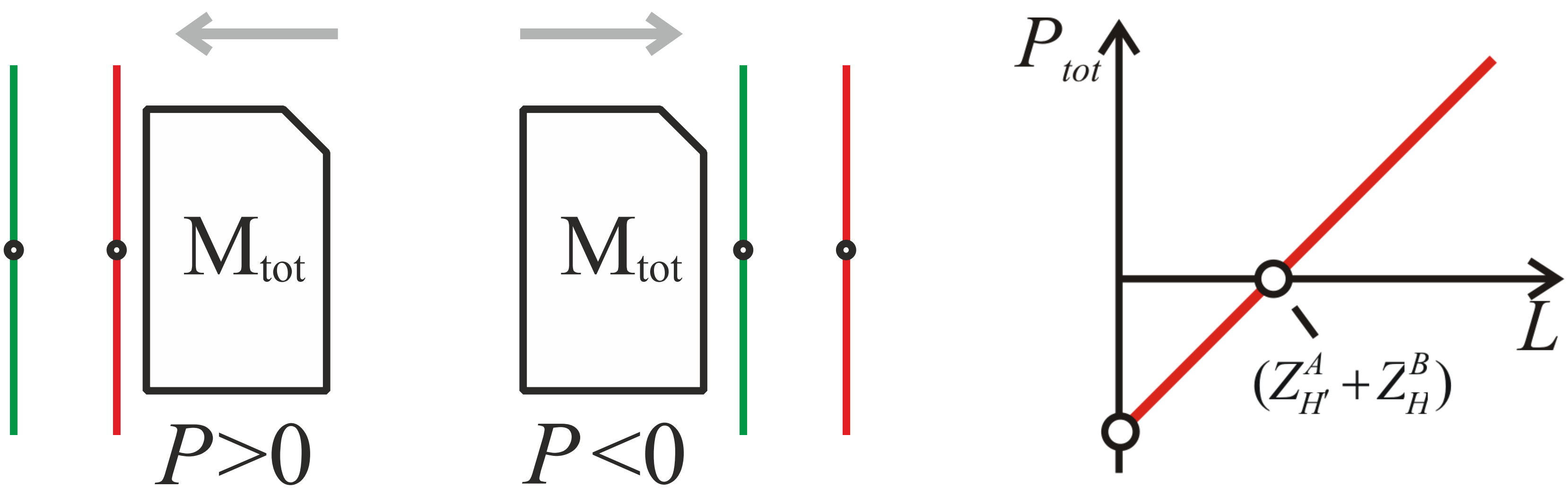
How can we describe the sequence of two optical systems $\mathbf{M_A}$, $\mathbf{M_B}$ of opposite optical power separated by distance L (exit-entrance distance)?



$$M_{tot} = \begin{pmatrix} 1-[L-(Z_{H'}^{A}+Z_{H}^{B})]P & L-(Z_{H'}^{A}+Z_{H}^{B}) \\ -[L-(Z_{H'}^{A}+Z_{H}^{B})]P^{2} & 1+[L-(Z_{H'}^{A}+Z_{H}^{B})]P \end{pmatrix}$$

$$P_{tot} = [L-(Z_{H'}^{A}+Z_{H}^{B})]P^{2},$$

$$Z_{H}^{tot} = \frac{1}{P}, \; Z_{H'}^{tot} = -\frac{1}{P}$$



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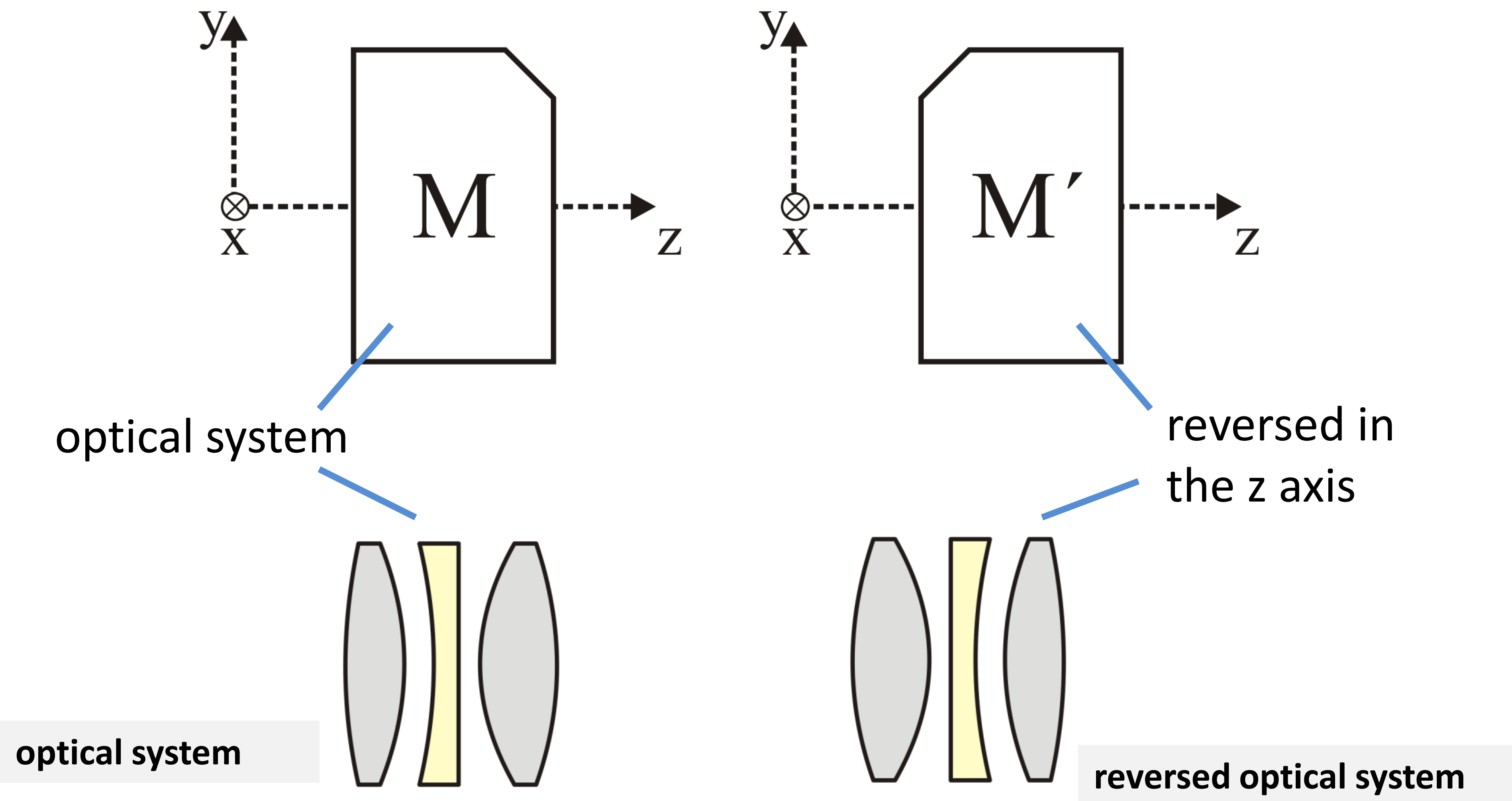
Reversing an optical system

How do we describe the reverse of an optical system $\mathbf{M}$;



when we reverse (flip) an optical system the total optical power **remains the same** while the principal planes **interchange**.

optical system

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \rightleftarrows \begin{pmatrix} M_{22} & M_{12} \\ M_{21} & M_{11} \end{pmatrix}$$

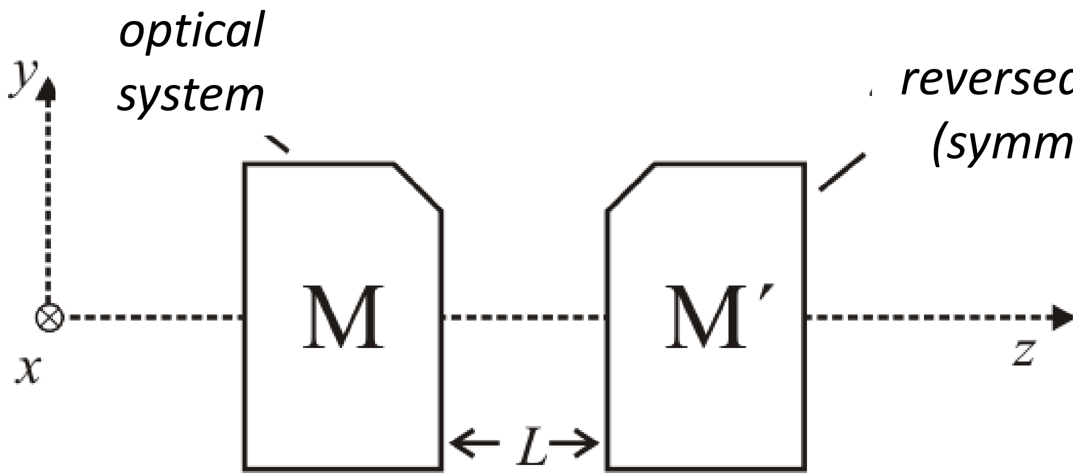
reversed optical system

Attention:
By inversing the optical system matrix we do not reverse the system!

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}^{-1} = \begin{pmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{11} \end{pmatrix}$$

$$\begin{bmatrix} y_i \\ n_i\beta_i \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \cdot \begin{bmatrix} y_{i-1} \\ n_{i-1}\beta_{i-1} \end{bmatrix} \Rightarrow \begin{bmatrix} y_{i-1} \\ n_{i-1}\beta_{i-1} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} y_i \\ n_i\beta_i \end{bmatrix}$$

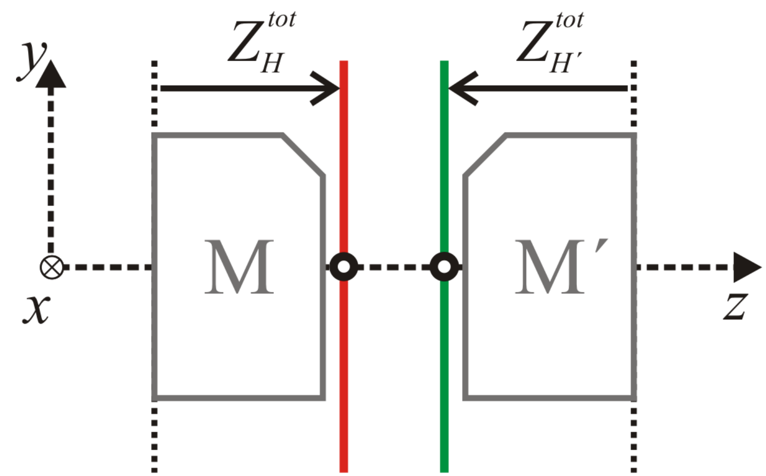
Symmetrical optical systems



$$M_{tot} = \begin{pmatrix} M_{22} & M_{12} \\ M_{21} & M_{11} \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

$$P_{tot} = -M_{21}(2M_{11} + L\,M_{21}),$$

$$Z_H^{tot} = Z_{H'}^{tot} = -\frac{2M_{12} + L\,M_{22}}{2M_{11} + L\,M_{21}}$$



in a symmetrical optical system the principal planes are located symmetrically.

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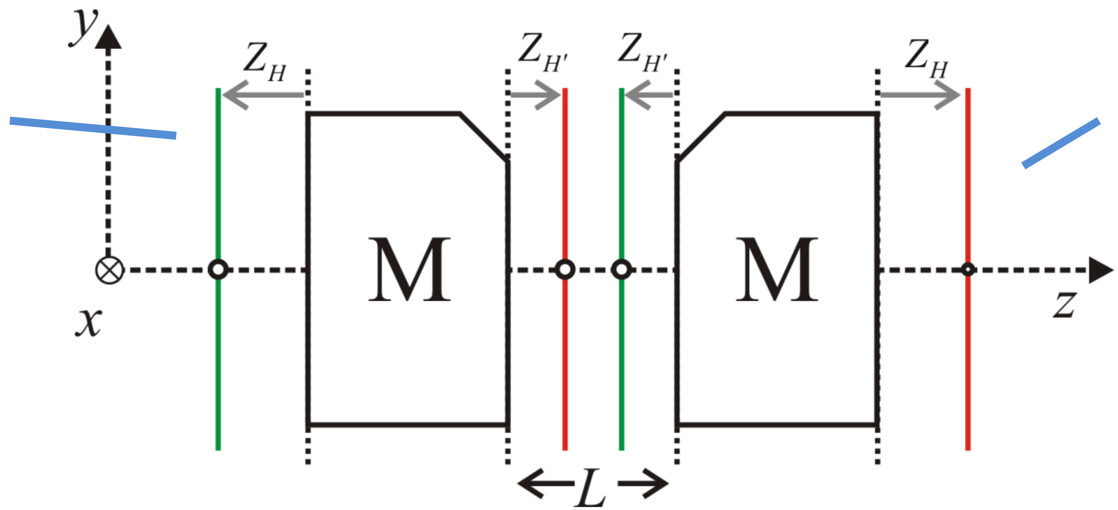
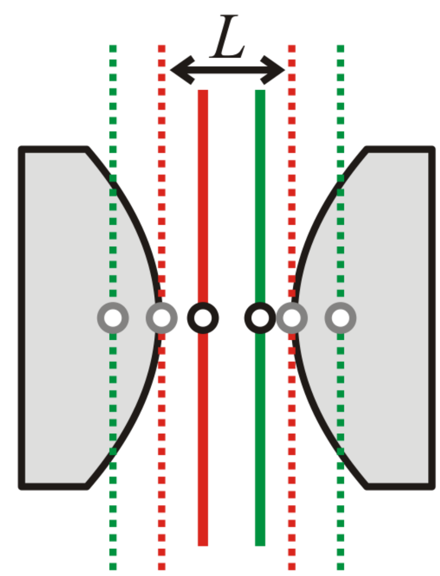
Symmetrical optical systems; alternative description

$$\begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix}$$

$$M_{tot} = \begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L-2Z_{H'} \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix}$$



$$P_{tot} = 2P - (L - 2Z_{H'})P^2,$$

$$Z_H^{tot} = Z_{H'}^{tot} = -\frac{L - 2Z_{H'}}{2 - (L - 2Z_{H'})P}$$

3.4 Image Illumination

Apertures in optical systems

An optical system **receives only a portion of the energy** emitted from every point of the object and can **image only a portion of the object space.**

Apertures in optical systems can influence:

the total image illumination

the field of the object (angular of linear) that is imaged

the distribution of image illumination

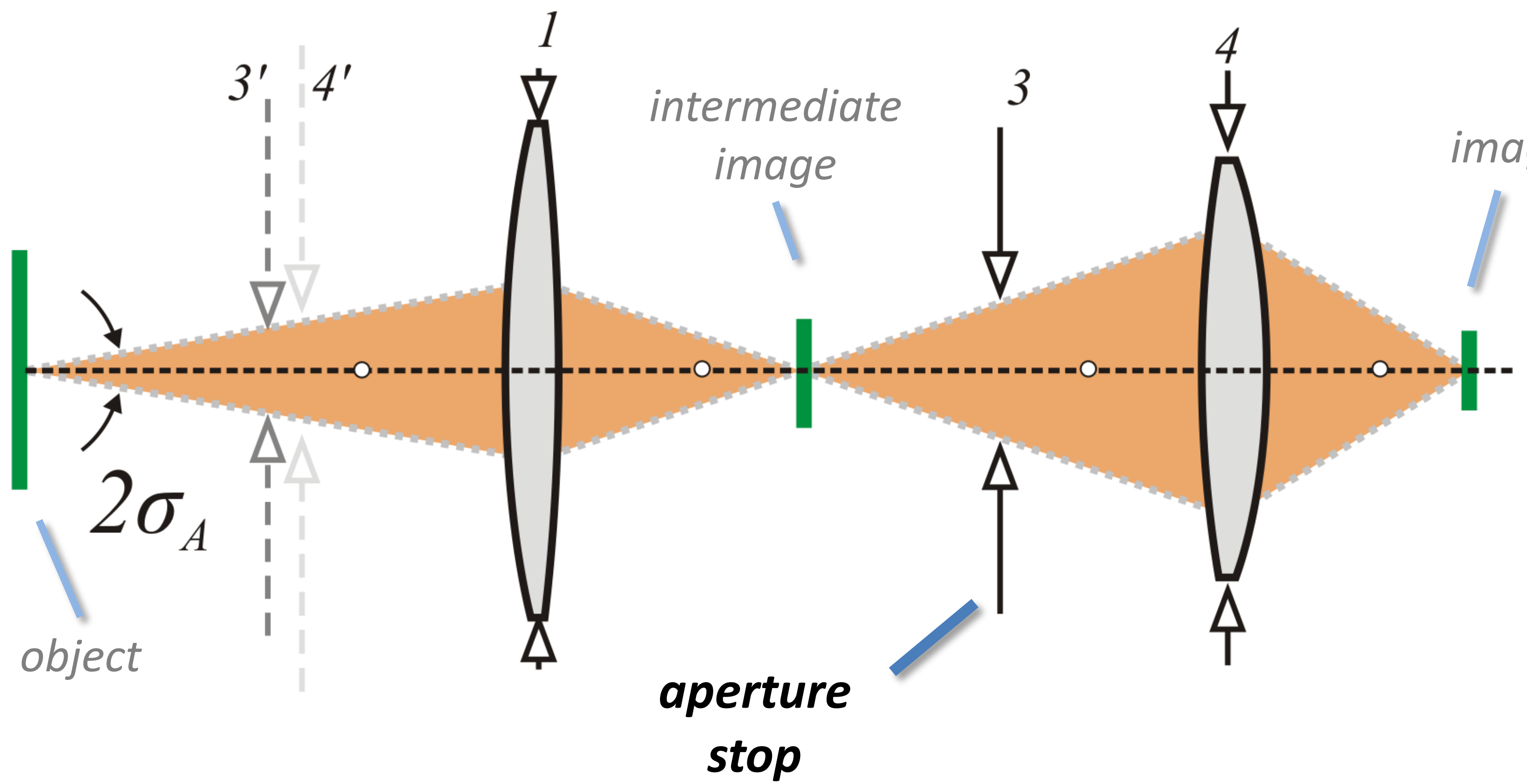
the contrast and other qualitative features of the image

Aperture stop

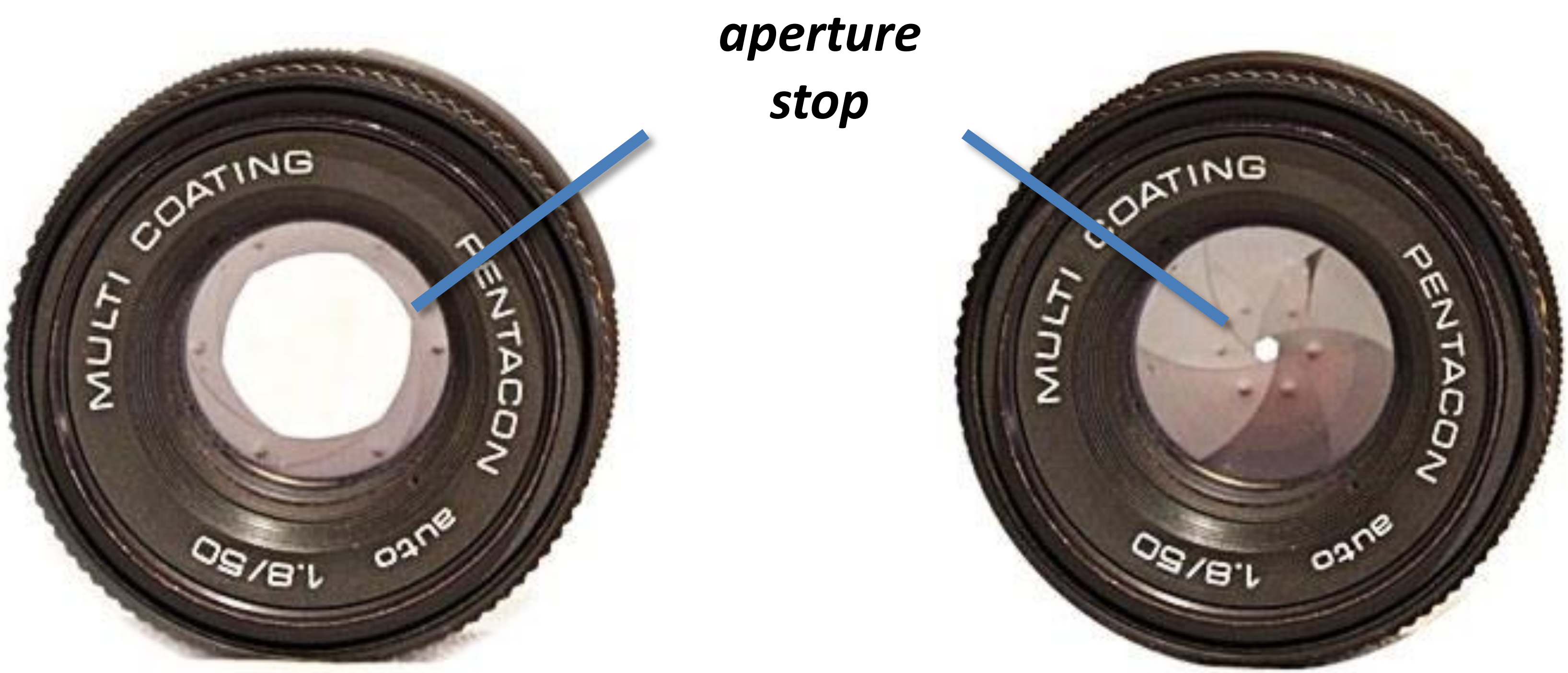
it limits the angle of acceptance of rays that are emitted from an axial object

controls the image illumination

Optical system example



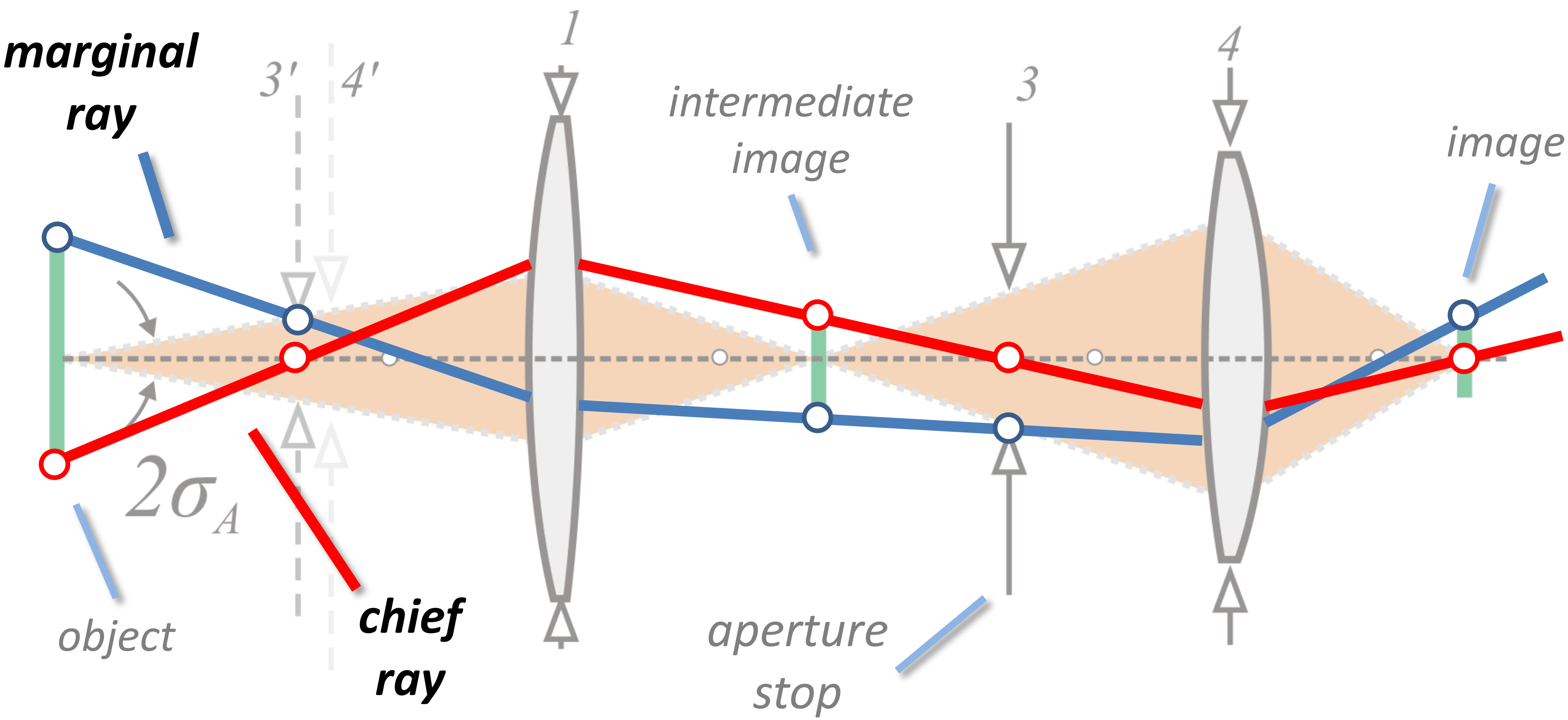
The aperture stop can be identified by imaging all the apertures of the system in the object space.



Aperture stop of a photographic lens in an open and closed state

Chief and marginal rays

*A ray that passes **through the center** of the aperture stop is called **chief ray**.*



*A ray that passes **through the edge** of the aperture stop is called **marginal ray**.*

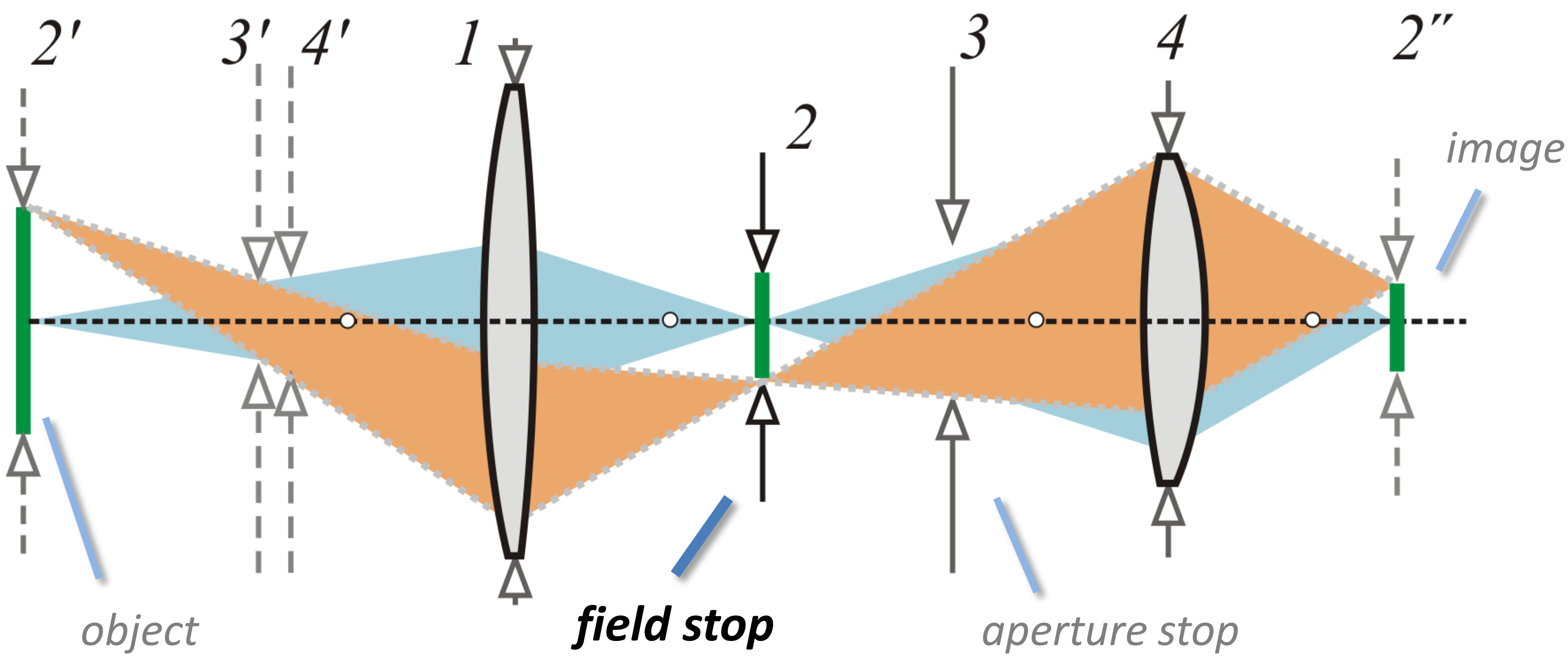
Field stop

it limits the field
(angular or linear) of the object

controls the size of the image space

it is always located on a plane conjugate to the object plane

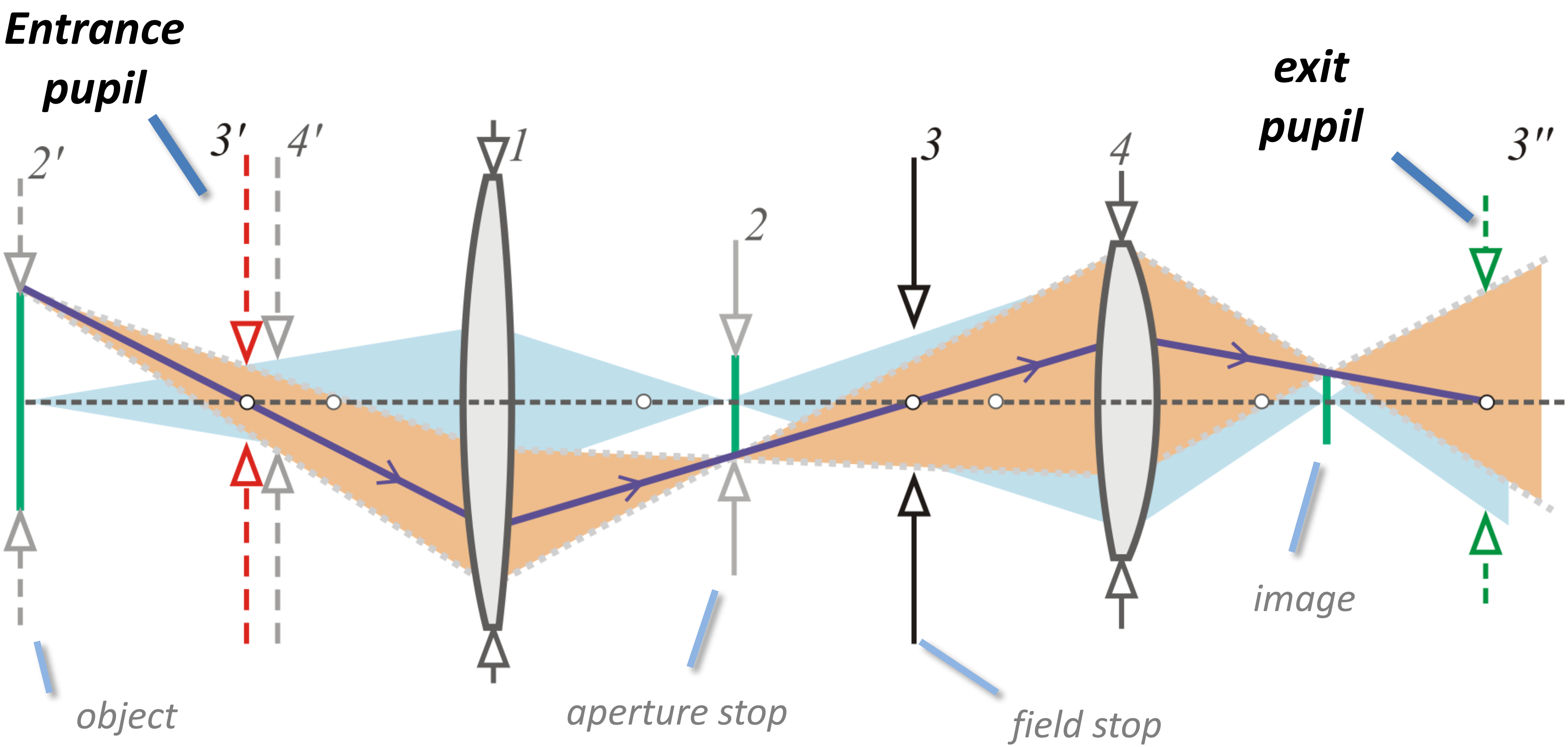
optical system example



Entrance pupil and exit pupil

Entrance pupil — image of the aperture stop in the object space

image of the aperture stop in the image space — **Exit pupil**



Entrance pupil and image illumination

photographic lens

f

$(\#F)$

(for a fully open aperture)

$(\#F)$ *number:*
it is the ratio of the effective focal distance to the entrance pupil diameter

$$(\#F) \equiv \frac{f}{D}$$

Image illumination is inversely proportional $(\#F)^2$

$$I \propto \frac{1}{(\#F)^2}$$

Dimitris Papazoglou, *dpapa@materials.uoc.gr*

Typical (#F) values:

1.8	**2**	**2.8**	**4**	**5.6**	**8**	**11**	**16**	**22**	**32**	**...**

$\times\sqrt{2}$

1 stop: Illumination reduces in half

$(\#F)$

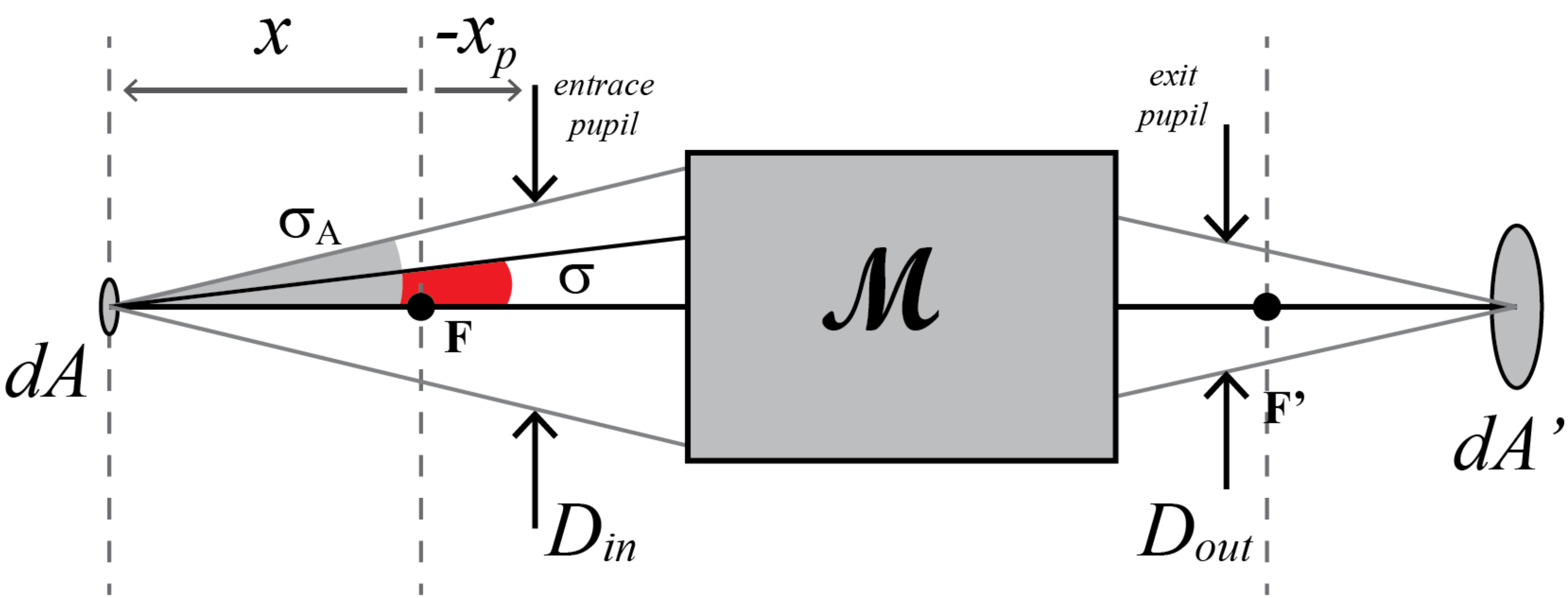
ring for changing the #F number

$(\#F)=8$

photographic lens

User:MarkSweep/ Wikimedia Commons / Public Domain

Analytic evaluation of Image illumination in an optical system



We assume that the object is a Lambertian source

radiance

$$L = \frac{dI}{dA\cos\sigma}, \quad dI = \frac{d\Phi_{in}}{d\Omega},$$

$$d\Omega = \frac{(2\pi R\sin\sigma)(R\cdot d\sigma)}{R^2} = 2\pi\sin\sigma\cdot d\sigma$$

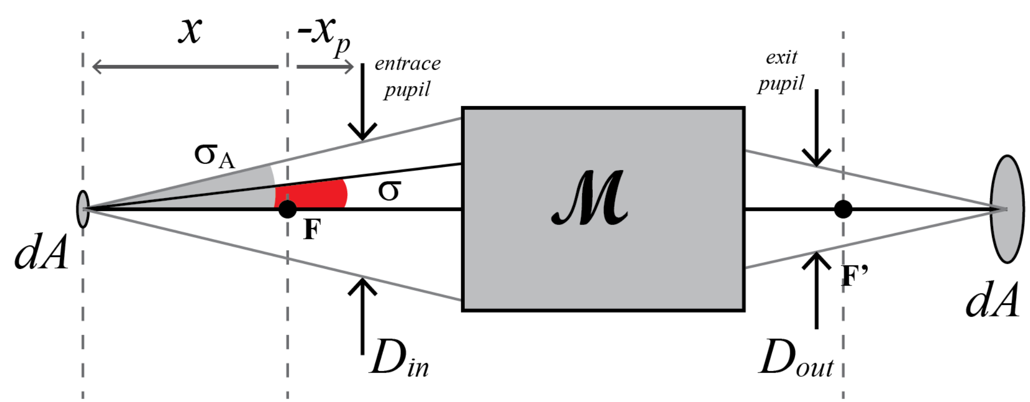
$$\Rightarrow \boxed{d\Phi_{in} = 2\pi L dA\sin\sigma\cos\sigma\cdot d\sigma}$$

emission from dA at an angle σ over a solid angle $d\sigma$

$$\Phi_{in} = 2\pi L dA \int_0^{\sigma_A} \sin\sigma \cos\sigma \cdot d\sigma$$

$$\boxed{= \pi L \sin^2 \sigma_A \cdot dA}$$

emission from dA over a solid angle σ_A



$$\sin\sigma_A = \frac{D_{in}/2}{\sqrt{(x-x_p)^2 + D_{in}^2/4}} \Rightarrow \boxed{\sin\sigma_A \cong \frac{D_{in}}{2(x-x_p)}}, \quad (x-x_p) \gg D_{in}$$

$$\boxed{\Phi_{in} \cong \pi L \frac{D_{in}^2}{(x-x_p)^2} \cdot dA}$$

$$dA' = M_T^2 \cdot dA \Rightarrow \boxed{\Phi_{out} = T\Phi_{in} \cong T\pi L \frac{D_{in}^2}{(x-x_p)^2} \cdot \frac{1}{M_T^2} dA'}$$

$$\left.\begin{array}{c} E \equiv \dfrac{\Phi_{out}}{dA'} \cong T\pi L \dfrac{D_{in}^2}{4(x-x_p)^2} \cdot \dfrac{1}{M_T^2} \\ M_T = \dfrac{f}{x} \end{array}\right\} \Rightarrow E \cong (\frac{\pi}{4}T) \cdot L \cdot (\frac{x}{x-x_p})^2 \cdot (\frac{D_{in}}{f})^2$$

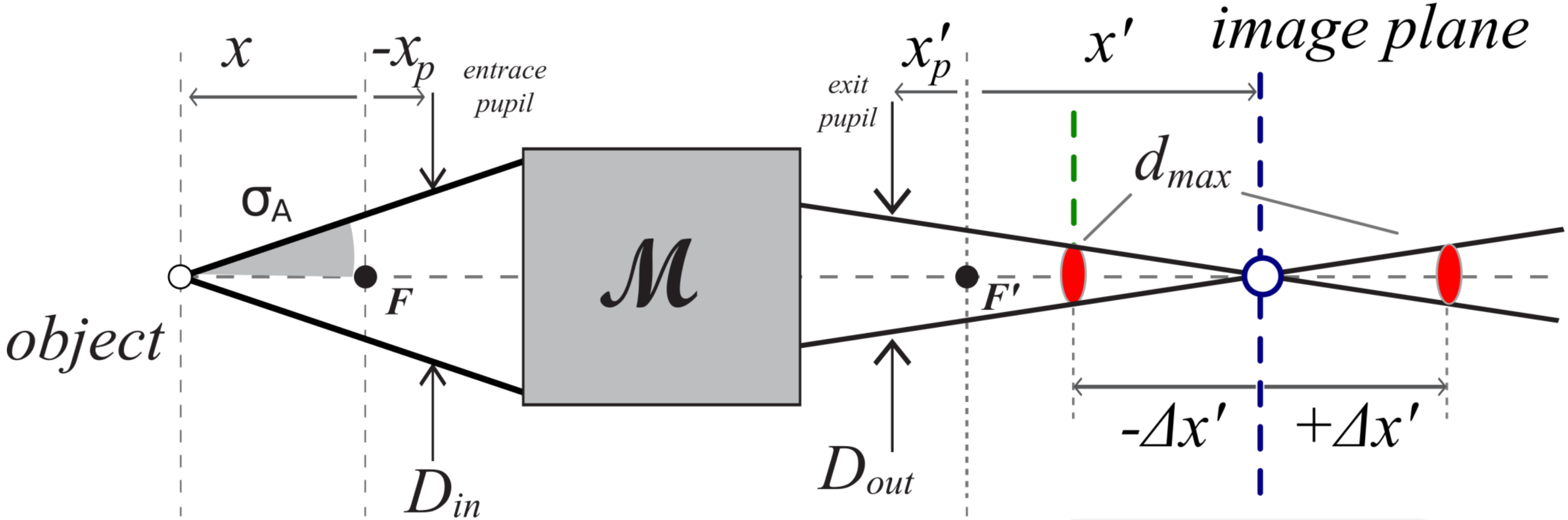
image illumination

$$\# F \equiv \frac{f}{D_{in}}$$

$$E \cong (\frac{\pi}{4}T) \cdot L \cdot \frac{1}{(\# F)^2}, \quad (x \gg x_p)$$

image illumination when object is sufficiently far away

Depth of focus

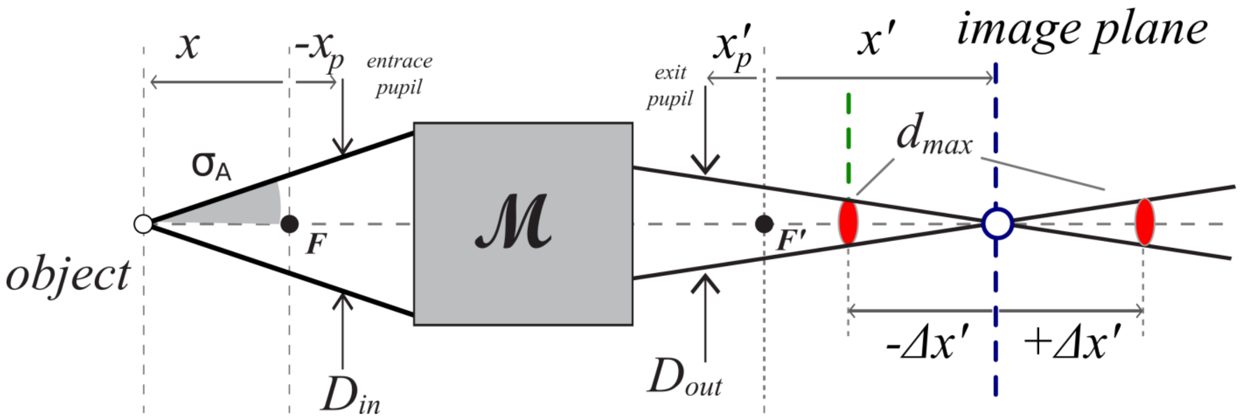


Depth of focus describes the **tolerance of the image plane placement** in relation to the optical system

for a $\pm\Delta x'$ shift of the sensor the image can be still acceptable

condition

$$circle\ of\ confusion \le d_{\max}$$

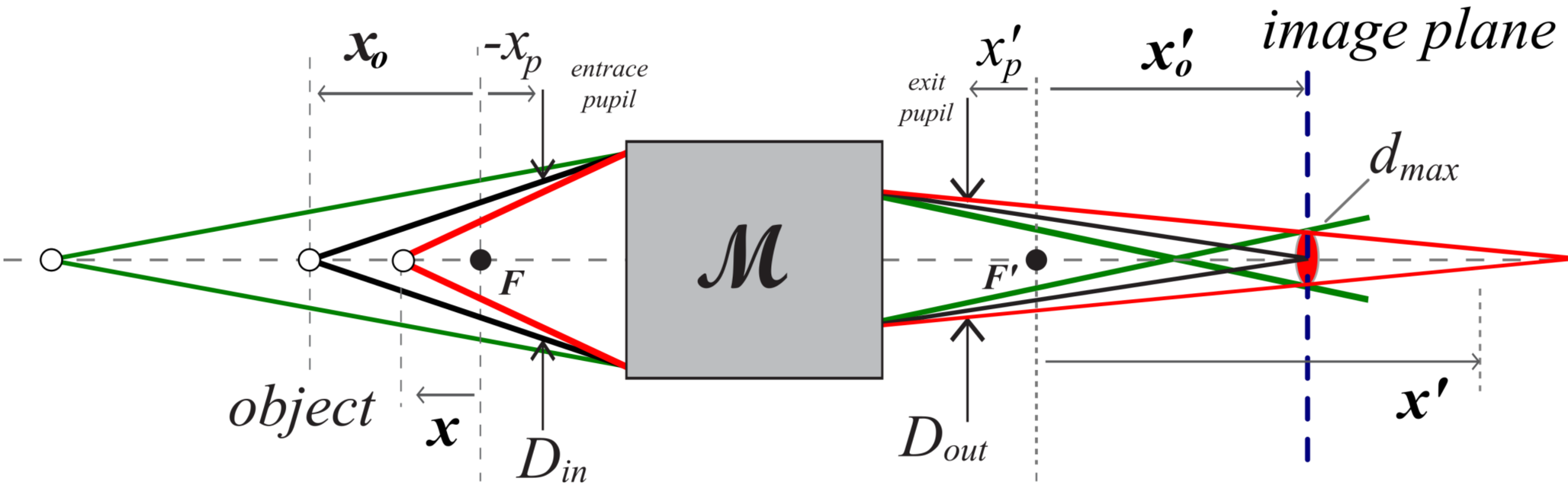


$$\left.\begin{array}{l} 2\Delta x' = 2(x' - x'_p)\dfrac{d_{\max}}{D_{out}} \\ \dfrac{D_{out}}{D_{in}} = -\dfrac{f}{x_p}, \quad x \cdot x' = f^2 \end{array}\right\} \Rightarrow 2\Delta x' = 2\left(\frac{f^2}{x_p} - \frac{f^2}{x}\right)\frac{x_p d_{\max}}{D_{in} \cdot f} \Rightarrow$$

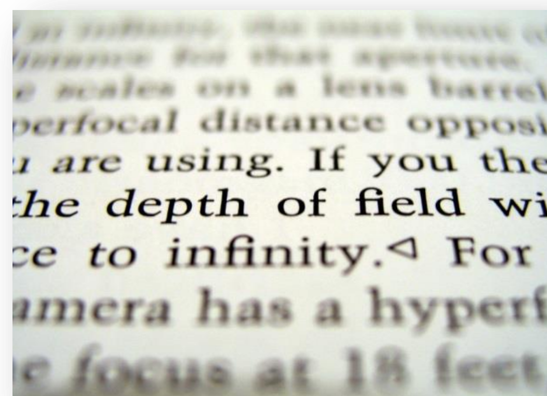
$$2\Delta x' = 2\frac{x - x_p}{x}\underbrace{\frac{f}{D_{in}}}_{(\#F)} d_{\max} \cong 2(\# F) d_{\max} \qquad (x \gg x_p)$$

the depth of focus
is proportional to the F-Number

Depth of field (DOF)



Depth of field describes the **tolerance of the object position** in relation to the optical system



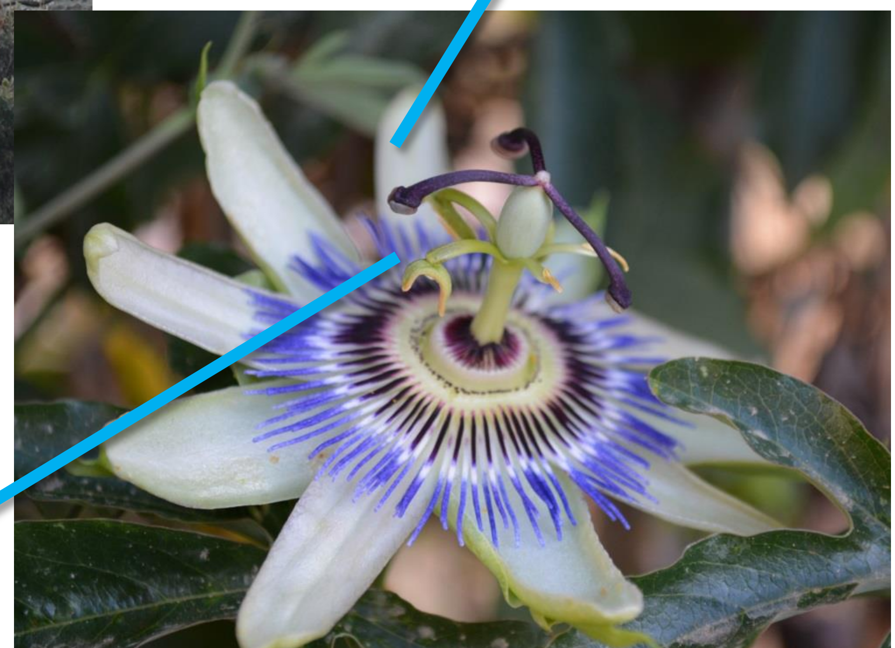
example of depth of field

user: PiccoloNamek/ WikiMedia Commons/CC BY-SA 3.0 *background (out of focus)*

in focus

DOF

out of focus



in focus

photos by Dimitris Papazoglou

$$\left.\begin{array}{l} \dfrac{\pm(x'-x'_o)}{d_{\max}} = \dfrac{x'_o - x'_p}{D_{out}} \\ \dfrac{D_{out}}{D_{in}} = -\dfrac{f}{x_p}, \quad x\cdot x' = f^2 \end{array}\right\} \Rightarrow \pm\left(\frac{f^2}{x} - \frac{f^2}{x_o}\right) = \left(\frac{f^2}{x_o} - \frac{f^2}{x_p}\right)\frac{x_p d_{\max}}{D_{in}\cdot f} = \frac{x_o - x_p}{x_o}(\#F)d_{\max} \Rightarrow$$

$$\boxed{x = \frac{x_o f^2}{f^2 \pm (x_o - x_p)(\#F)d_{\max}} \cong \frac{x_o f^2}{f^2 \pm x_o(\#F)d_{\max}} \Rightarrow} \quad (x_o \gg x_p)$$

DOF scale detail on a Nikon lens

Wikipedia/ Public Domain

$$\left\{\begin{array}{l} x_{Far} \cong \dfrac{x_o f^2}{f^2 - x_o(\#F)d_{\max}} \\ x_{Near} \cong \dfrac{x_o f^2}{f^2 + x_o(\#F)d_{\max}} \end{array}\right.$$

farthest object adequately focused

nearest object adequately focused

Hyperfocal distance

The *hypefocal* (H_f) distance is the object distance **beyond which all objects** are adequately **focused**

Using the DOF terminology when the object is located at the *hyperfocal* distance $x_{Far} \to \infty$

$$x_o = H_f \Rightarrow x_{Far} \to \infty \Rightarrow f^2 - H_f (\#F) d_{\max} = 0 \Rightarrow$$

Mobile phone camera. (Large *#F* thus very small H_f)

$$H_f = \frac{f^2}{(\#F)\,\mathrm{d}_{\max}}$$

everything from $\boldsymbol{H_f/2}$ to **infinity** is adequately **focused**

$$x_{Near} \cong \frac{H_f f^2}{f^2 + H_f (\#F) d_{\max}} = \frac{H_f}{2}$$

Dimitris Papazoglou, *dpapa@materials.uoc.gr*

DOF using Hyperfocal distance

$$\left.\begin{aligned} x_{Far} &\cong \frac{x_o H_f}{H_f - x_o} \\ x_{Near} &\cong \frac{x_o H_f}{H_f + x_o} \end{aligned}\right\} \Rightarrow (DOF) \equiv x_{Far} - x_{Near} = \frac{2x_o^2 H_f}{H_f^2 - x_o^2}$$

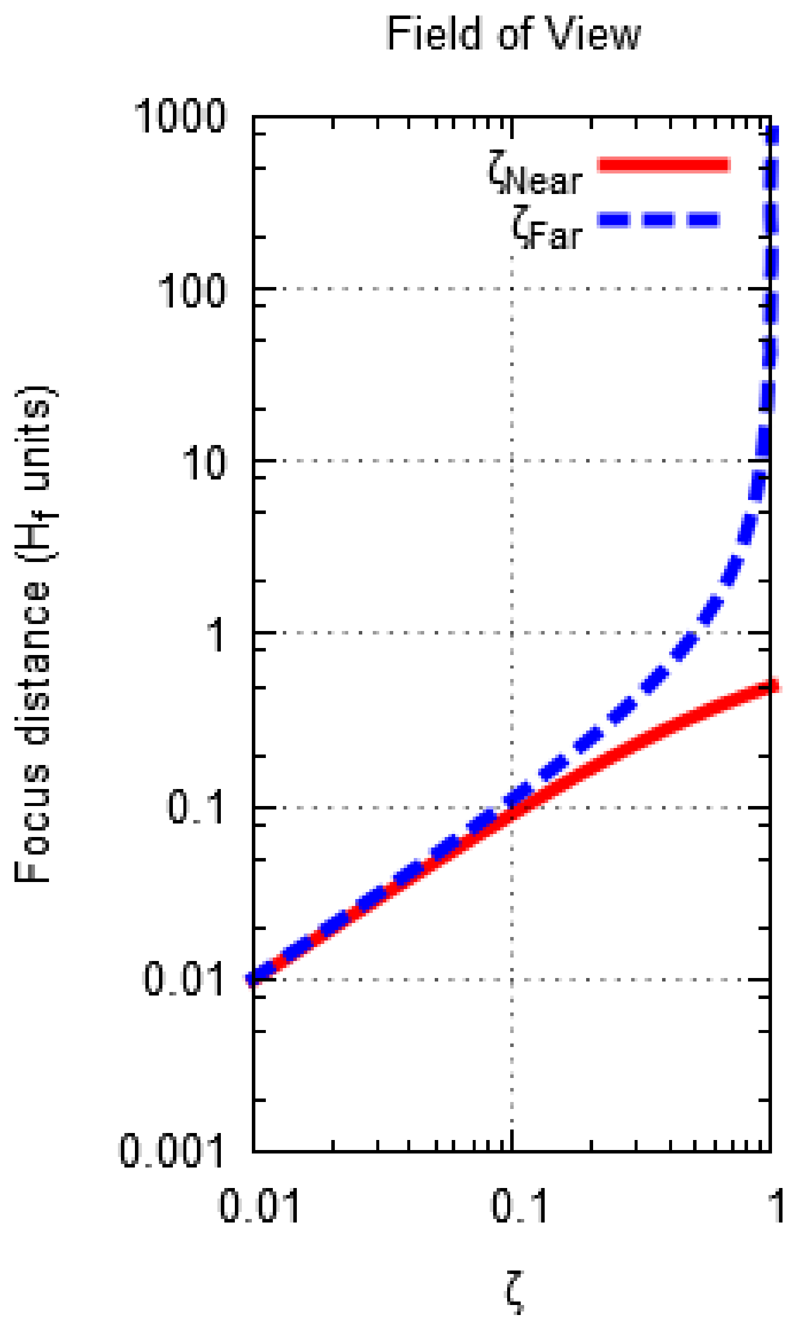
Normalizing all distances using H_f

$$\zeta \equiv \frac{x_o}{H_f}, \ \zeta_{Far} \equiv \frac{x_{Far}}{H_f}, \ \zeta_{Near} \equiv \frac{x_{Near}}{H_f}, \quad D_f \equiv \frac{(DOF)}{H_f}$$

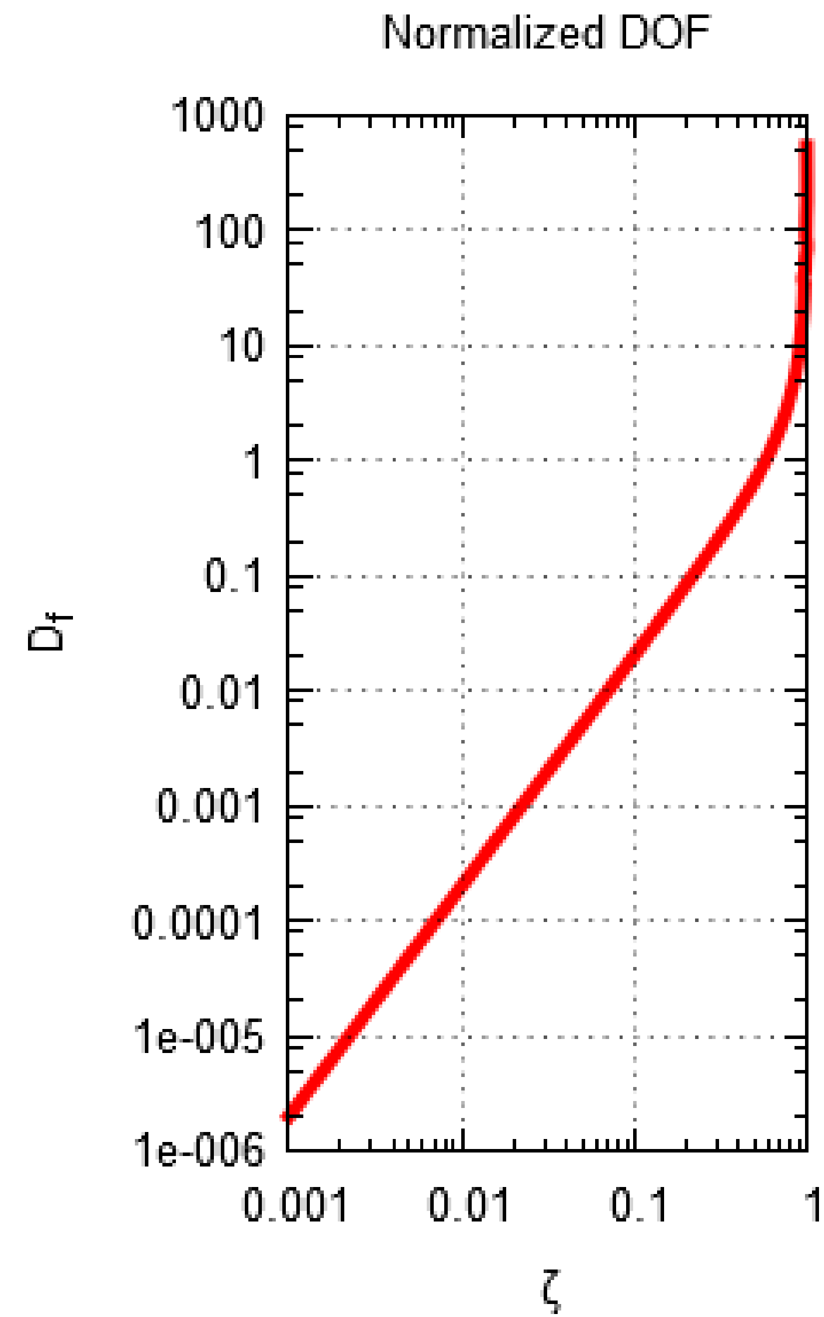
$$\zeta_{Near} \cong \frac{\zeta}{1+\zeta}$$

$$\zeta_{Far} \cong \frac{\zeta}{1-\zeta}$$

$$D_f = \frac{2\zeta^2}{1-\zeta^2}$$

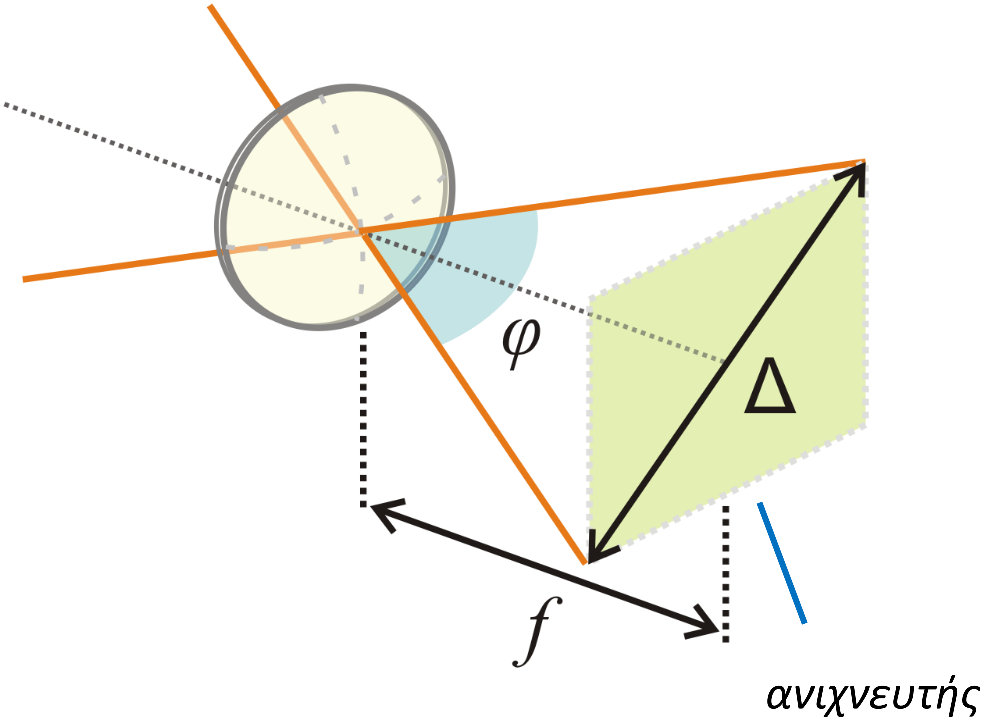


Field of View
Focus distance (H_f units)
ζ_{Near}
ζ_{Far}
1000
100
10
1
0.1
0.01
0.001
0.01
0.1
1
ζ



Normalized DOF
D_f
1000
100
10
1
0.1
0.01
0.001
0.0001
1e-005
1e-006
0.001
0.01
0.1
1
ζ

Angular field



$$\varphi = 2\tan^{-1}\frac{\Delta}{2f}$$

$\varphi < 40^o$	*telephoto*
$40^o \leq \varphi \leq 60^o$	*normal*
$\varphi > 60^o$	*wide angle*

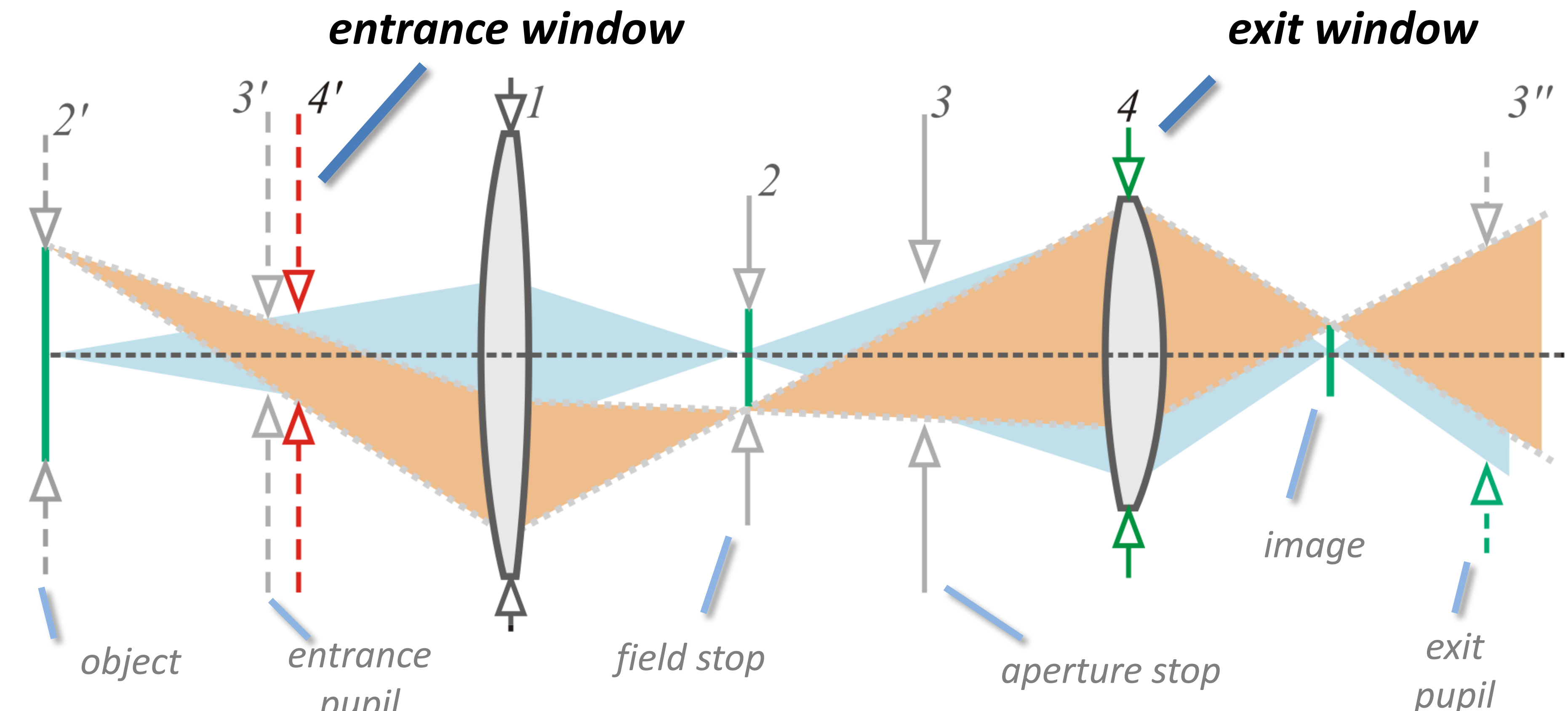
example of an angular field change in a zoom telephoto (by changing the effective focal distance)

User: User:Patche99z/ Wikimedia Commons / Public Domain

Entrance window, Exit window

vignetting aperture → it controls the distribution of image illumination

Entrance window ← image in the object / image space → Exit window



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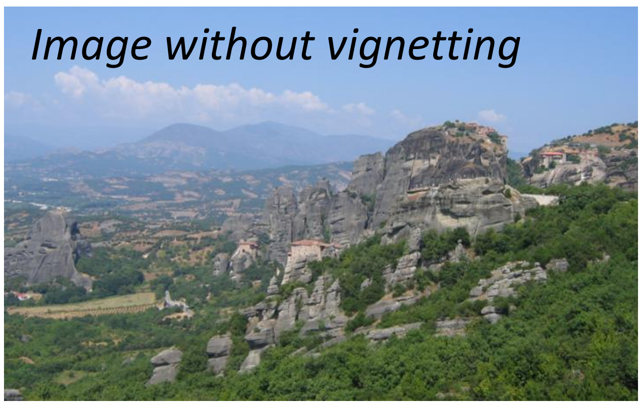
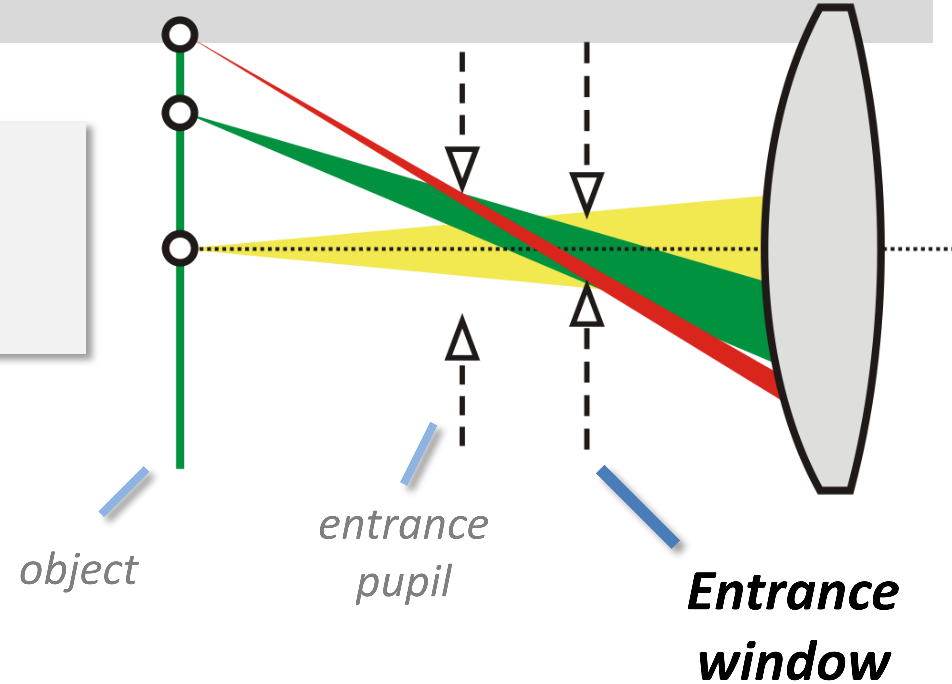
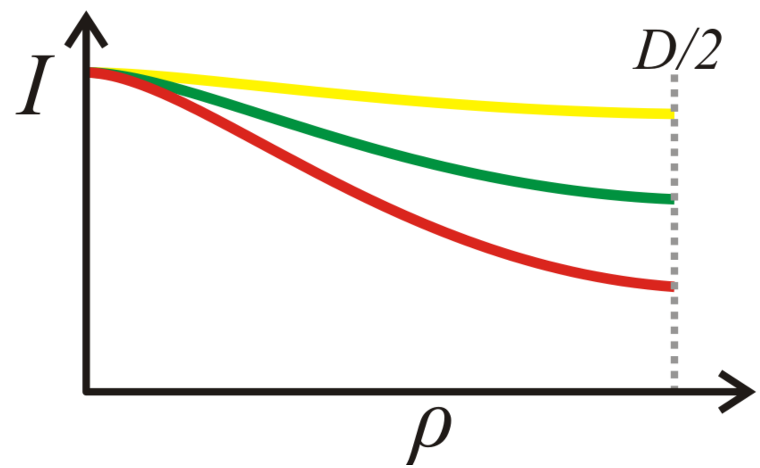
Vignetting

only a portion *of rays emitted from off-axis object points can reach the image*





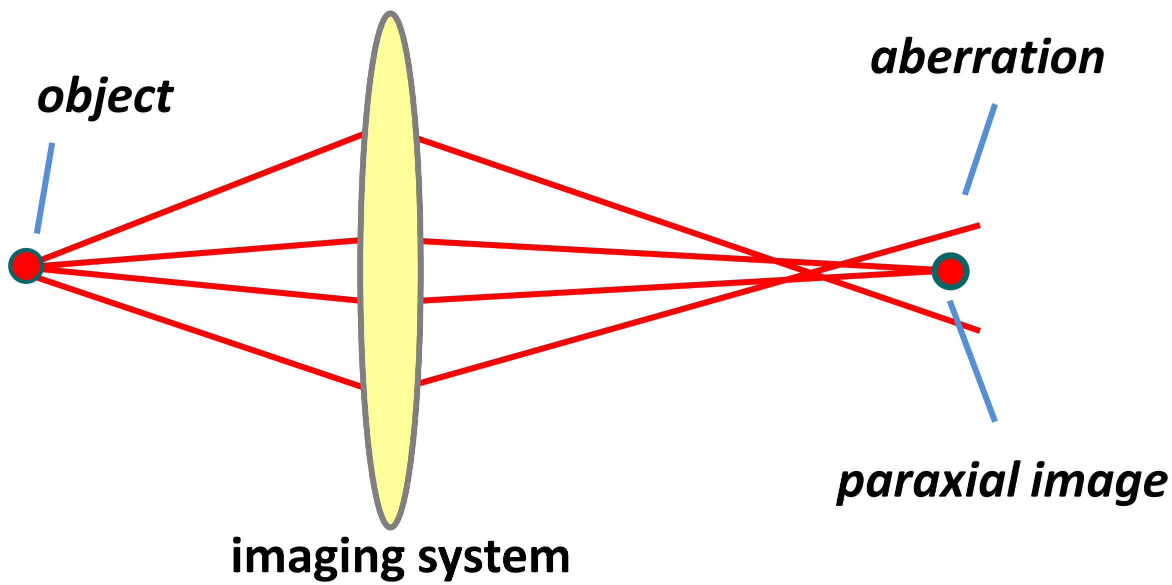


Vignetting can reach up to 65%

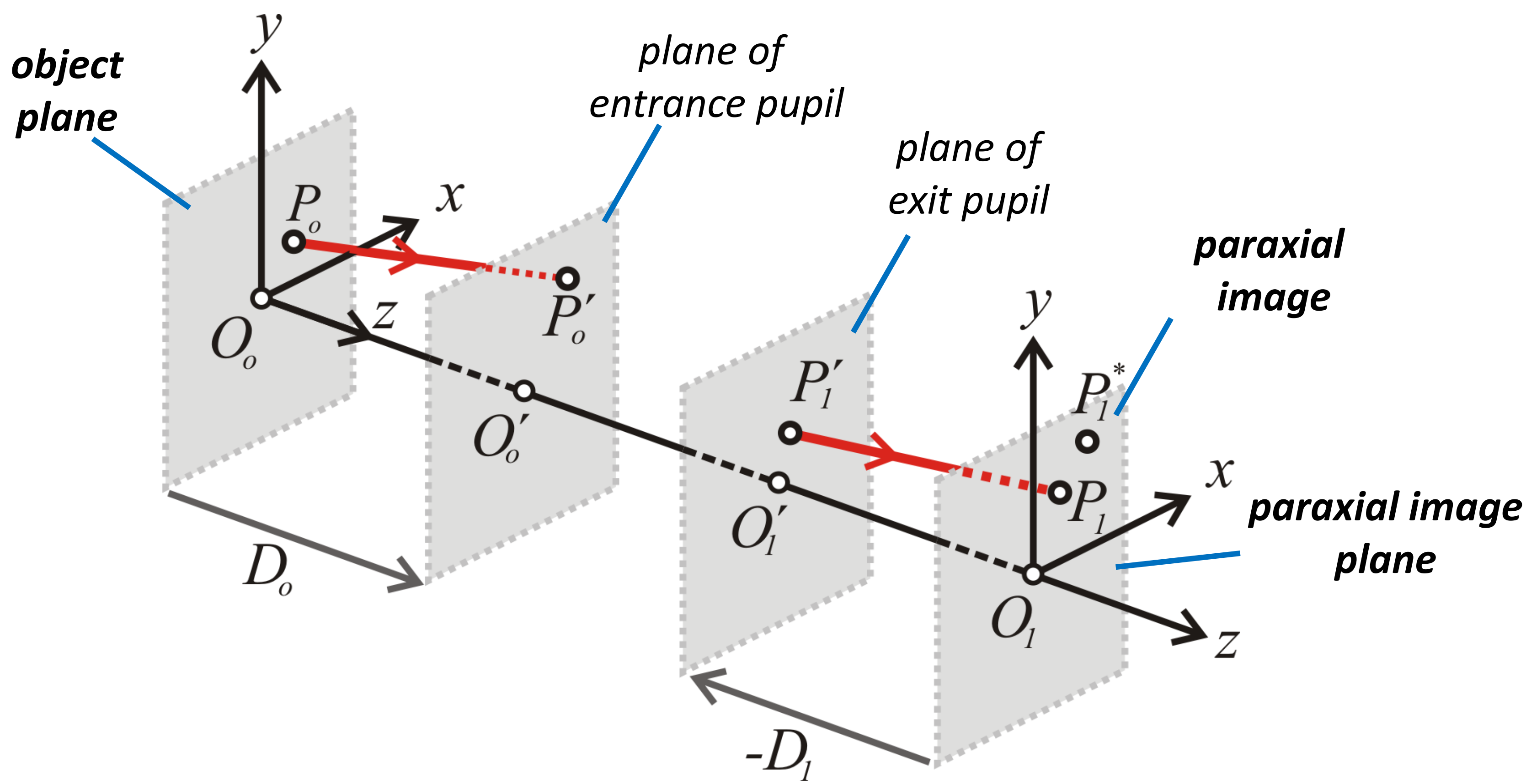
3.5 Optical aberrations

What do we mean by "optical aberration"

The «**failure**» of an optical imaging system to redirect all the optical rays from an object point to a single image point is called **optical aberration**



Ray Aberration

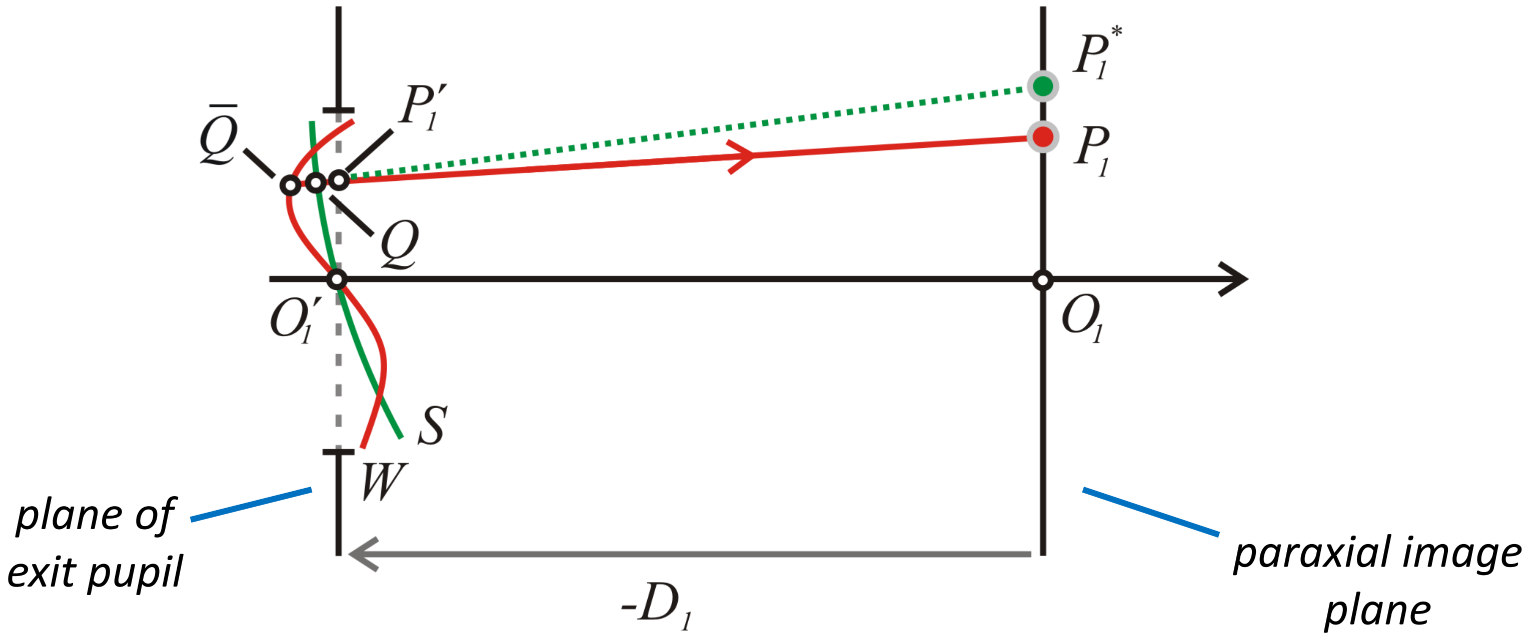


Ray aberration

$$\delta = \overrightarrow{P_1^* P_1}$$

Ray aberration is a vector that quantifies the "failure" of the optical image to redirect the ray towards the paraxial image

Wavefront aberration

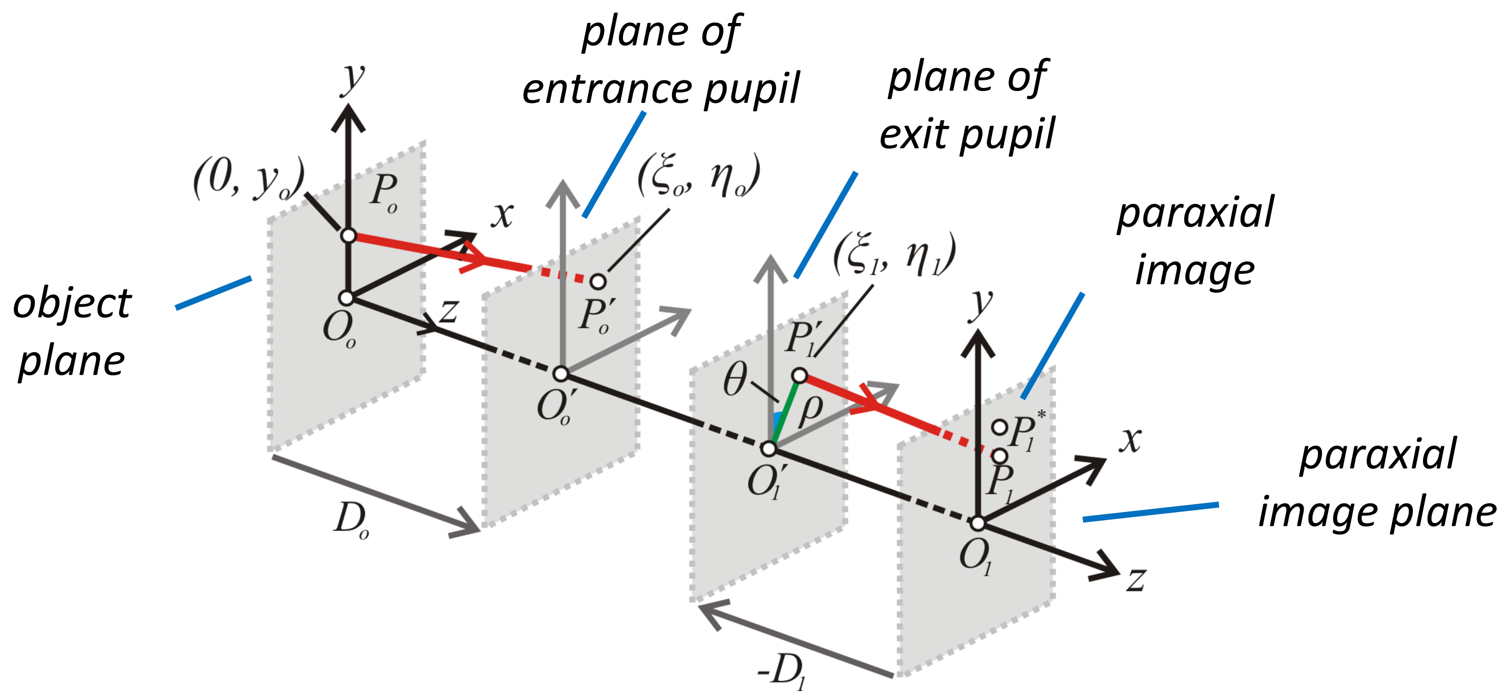


$$\Phi = (\bar{Q}Q)_{OPL}$$

wavefront aberration

In a typical optical system wavefront aberration can reach a tens of wavelengths λ, while in hi-end optical instrumentation it should be less than $\lambda/4$!

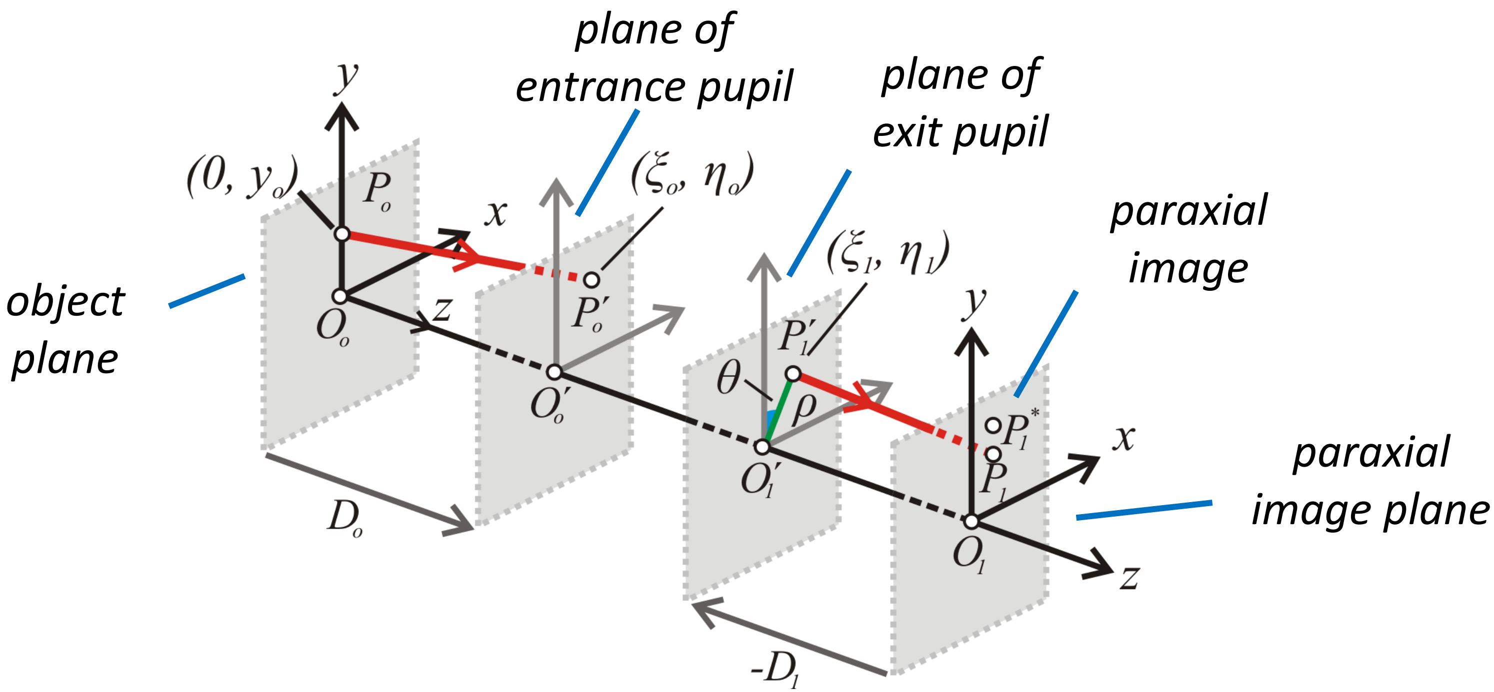
Primary monochromatic optical aberrations



$$\boldsymbol{\delta} \equiv \Delta x(\rho, \theta, y_o)\hat{\mathbf{x}} + \Delta y(\rho, \theta, y_o)\hat{\mathbf{y}}$$

Optical aberrations can be described as a Taylor series of the parameters ρ, θ, y_o.
In a first approximation, if we keep the first non-linear terms,
we get the **5 primary optical aberrations**
The higher order terms are referred as higher order aberrations.

Seidel approximation, Ray aberration

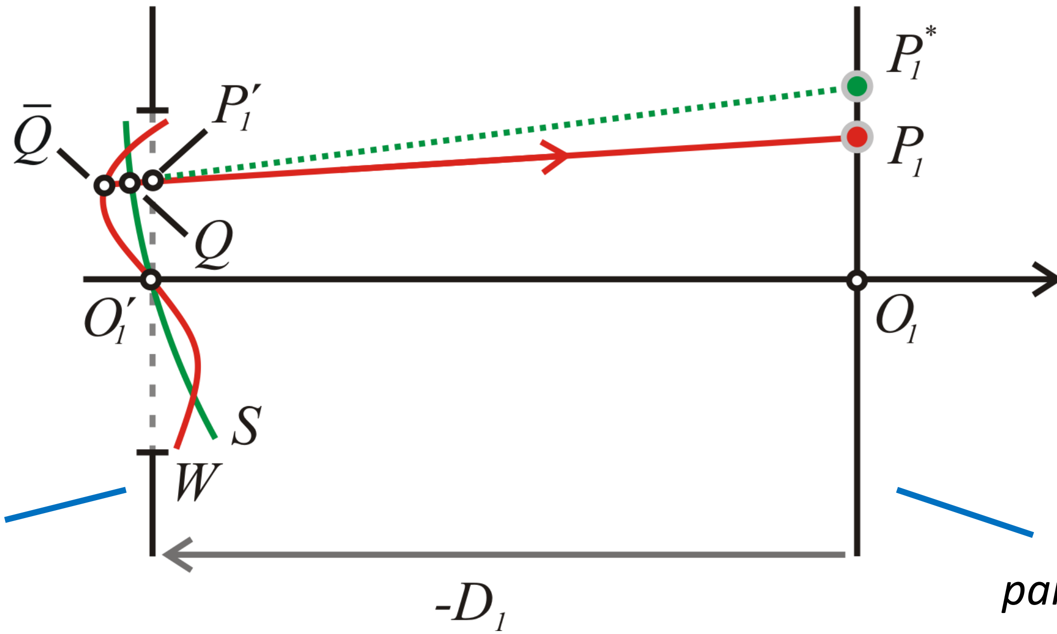


Seidel 3[rd] order terms of the Ray aberration

$$\Delta^{(3)}x = S_1\rho^3 \sin\theta - 2S_2 y_o \rho^2 \cos\theta \sin\theta + S_4 y_o^2 \rho \sin\theta$$

$$\Delta^{(3)}y = S_1\rho^3 \cos\theta - S_2 y_o \rho^2 (1+\cos^2\theta) + (2S_3 + S_4) y_o^2 \rho \cos\theta - S_5 y_o^3$$

Seidel approximation, wavefront error

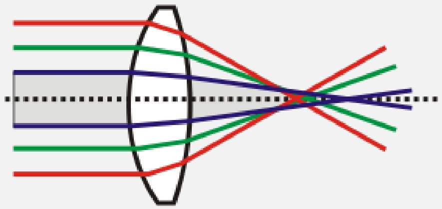


$$\Phi = (\bar{Q}Q)_{OPL} = \Phi^{(0)} + \underbrace{\Phi^{(4)}} + \underbrace{\ldots + \Phi^{(2\kappa)} + \ldots}$$

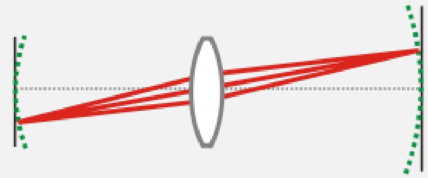
Primary 4th order Seidel terms
(can be analytically described)

higher order aberrations
(can be estimated using numerical methods)

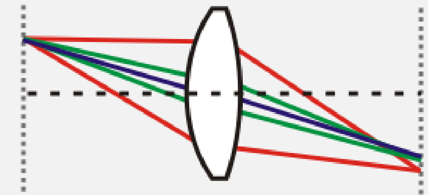
Spherical lenses: primary Seidel aberrations



spherical aberration

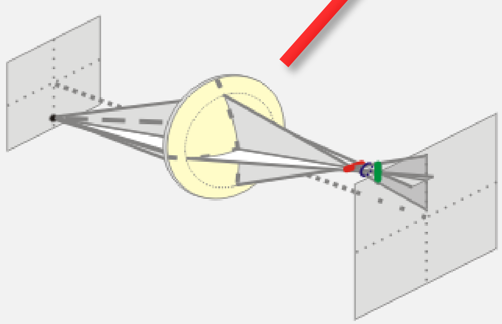


field curvarture

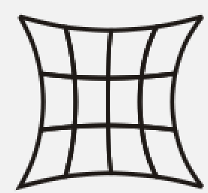


coma

$$\Phi^{(4)} = -\frac{1}{4}B\rho^4 - \frac{1}{2}(2C\cos^2\theta + D)\, r_o^2\rho^2 + E\, r_o^3\rho\cos\theta + F r_o\rho^3\cos\theta$$



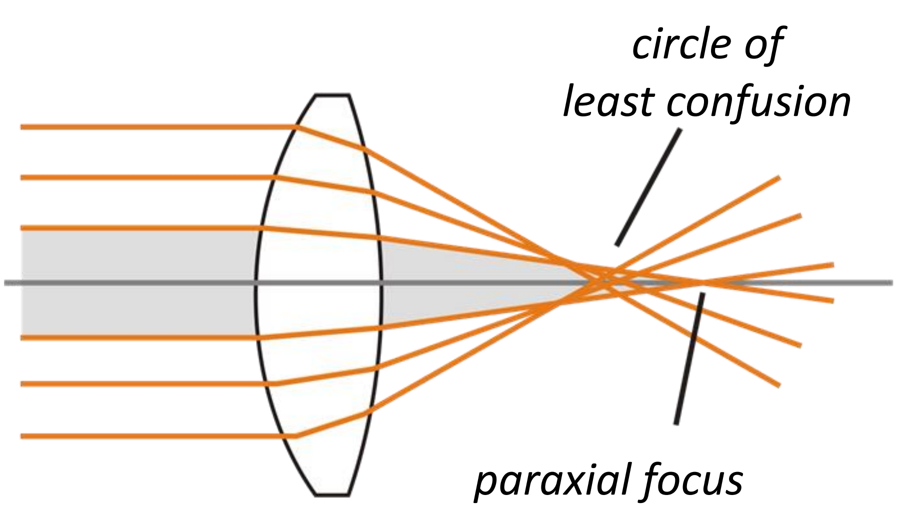
Astigmatism

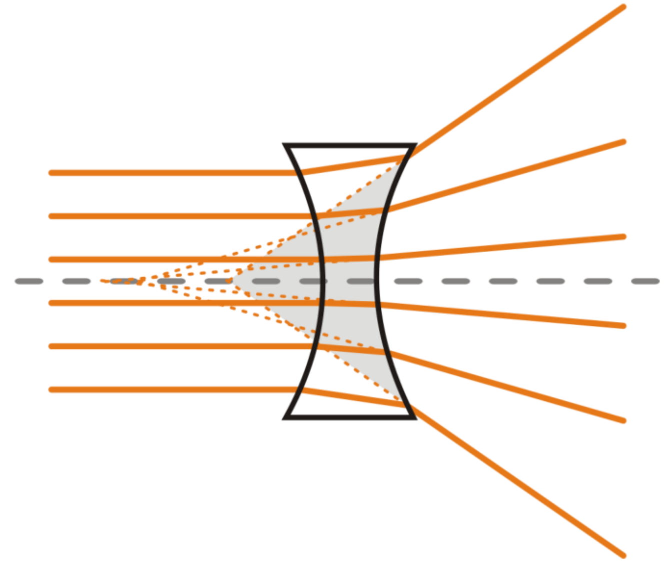


distortion

Dimitris Papazoglou dpapa@materials.uoc.gr

Spherical aberration (*SA*)



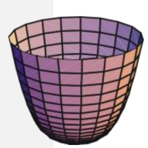


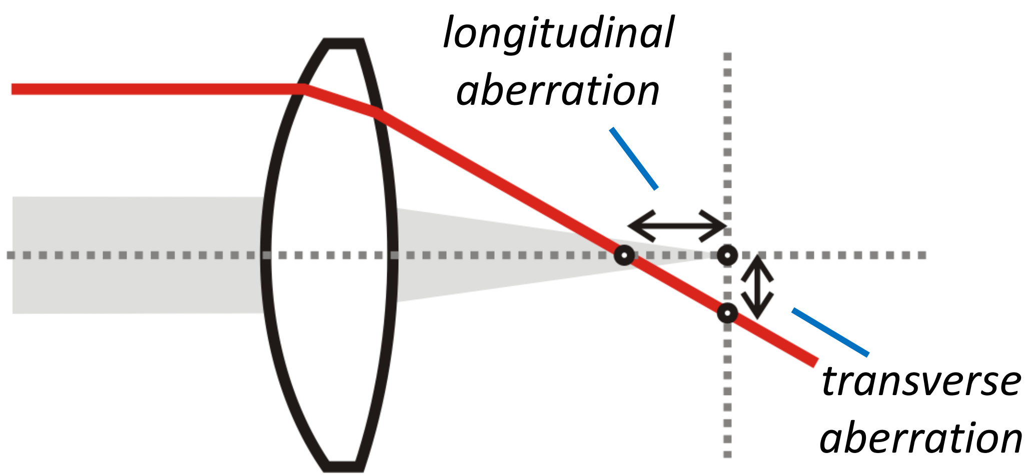
Seidel

$$\Delta^{(3)} x = S_1 \rho^3 \sin\theta$$

$$\Delta^{(3)} y = S_1 \rho^3 \cos\theta$$

$$\Phi^{(4)} = -\frac{1}{4} B \rho^4$$

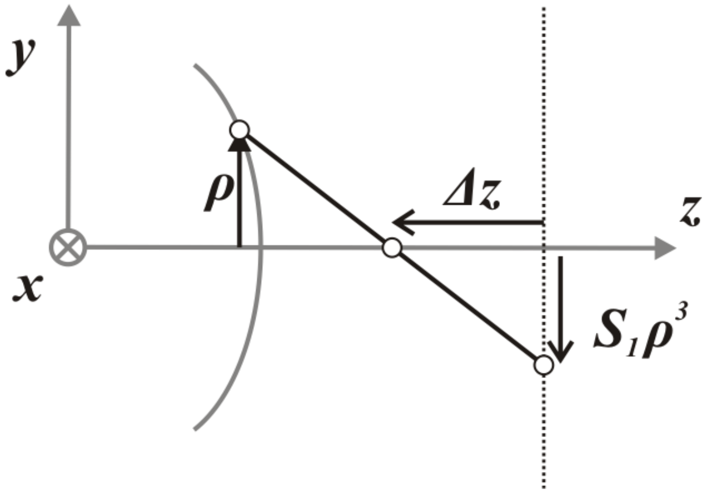


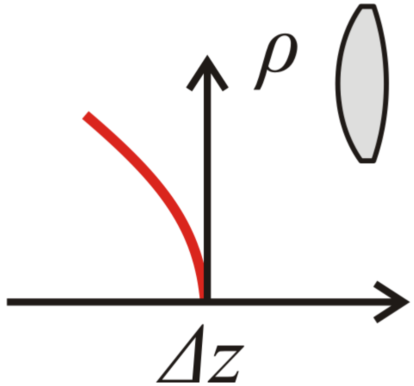


It manifests even in paraxial objects, does not affect the beam's symmetry

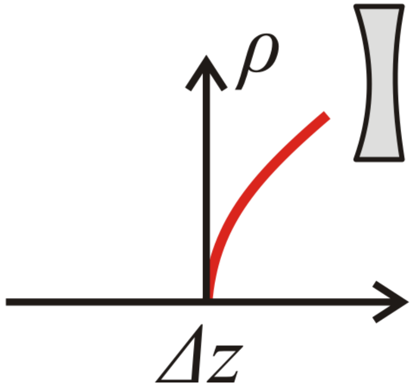
Dimitris Papazoglou dpapa@materials.uoc.gr

Estimation: *Longitudinal SA*





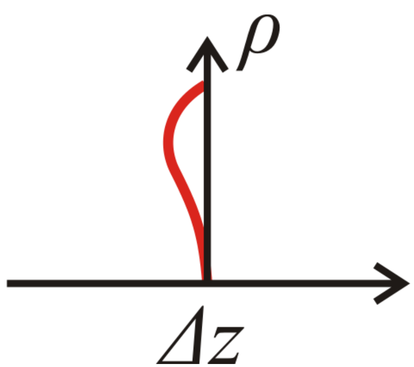
negative spherical aberration



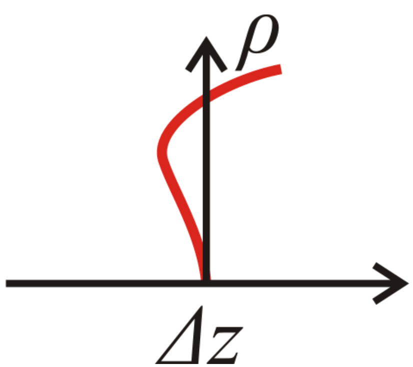
positive spherical aberration

$$\frac{S_1\rho^3}{\Delta z} = \frac{\rho}{f+\Delta z} \Rightarrow$$

$$\Delta z = \left(\frac{S_1\rho^2}{1-S_1\rho^2}\right) f$$

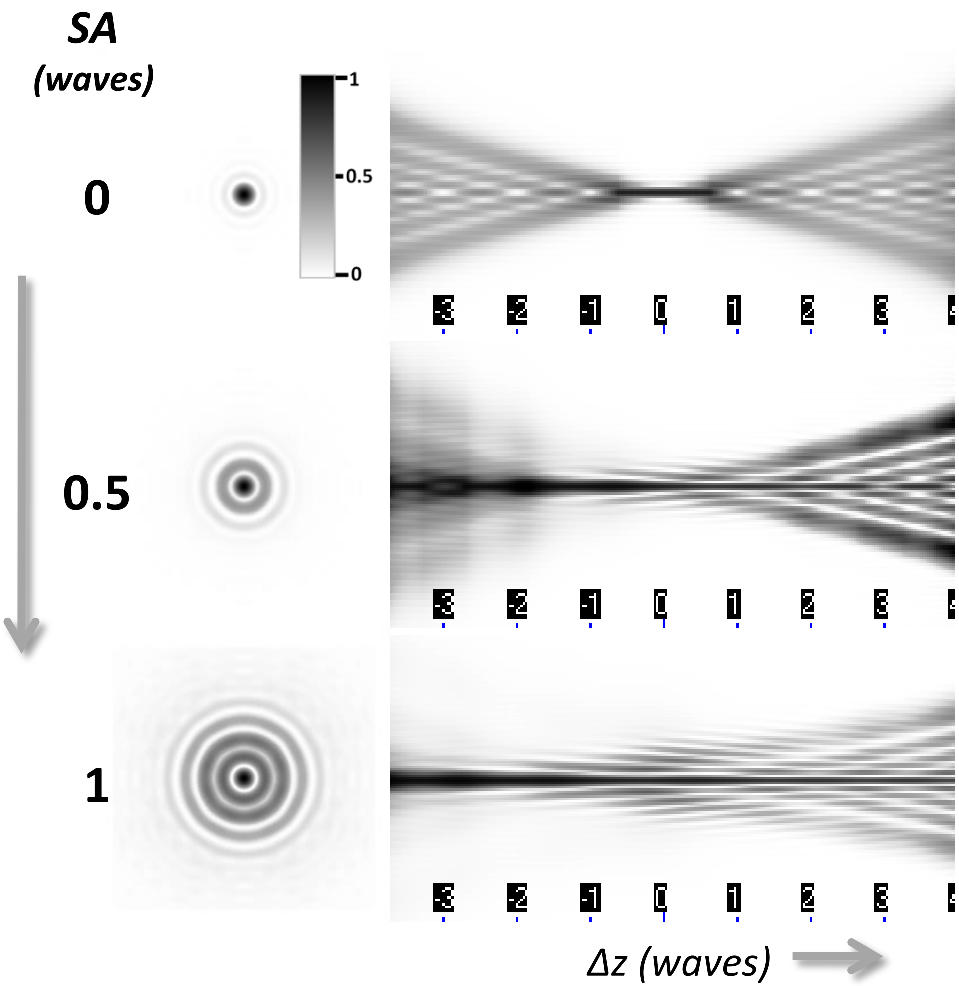


corrected spherical aberration



over-corrected spherical aberration

SA effect on focus

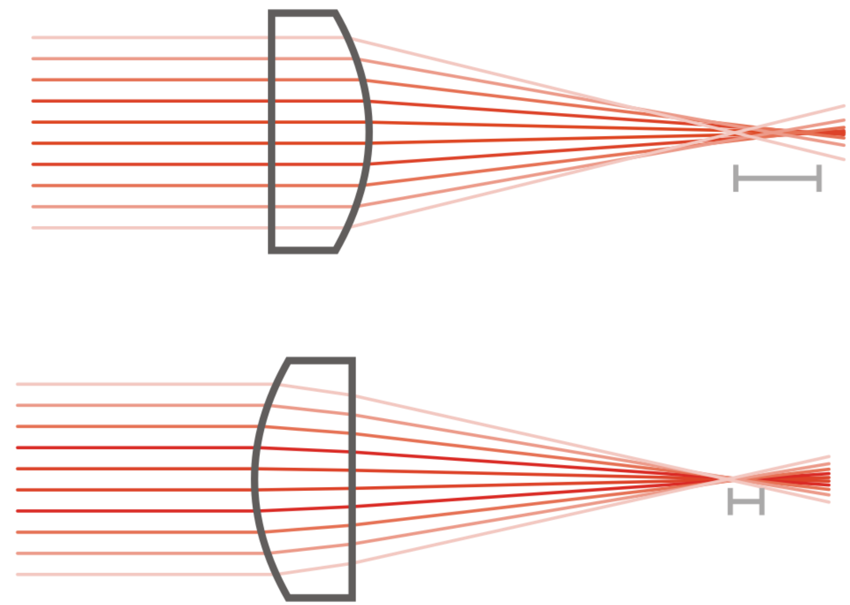


focus

transverse section along the propagation direction

simulations preformed using Aberrator (Developed by Cor Berrevoets)

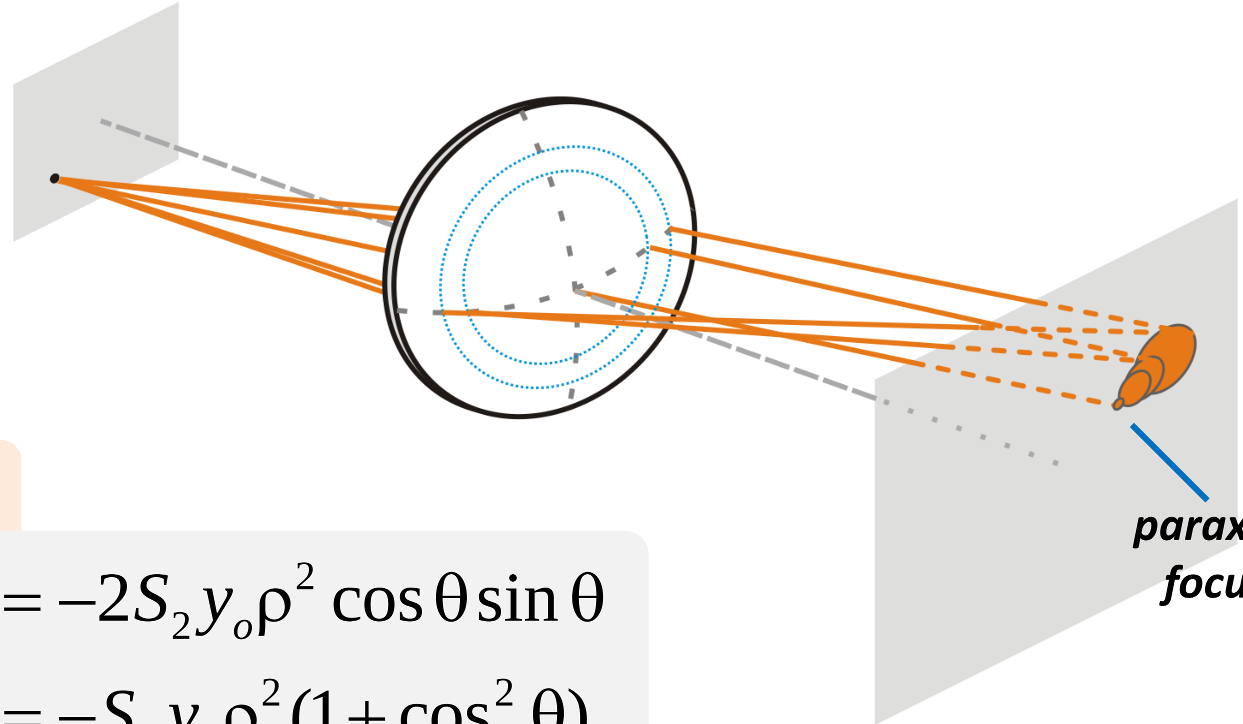
Reducing SA



reducing the spherical aberration by inverting the lens (raytracing results)

spherical aberration is reduced when we have symmetry between the object and the image

Coma aberration (CA)

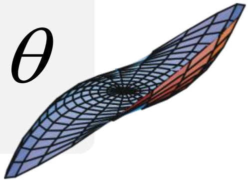


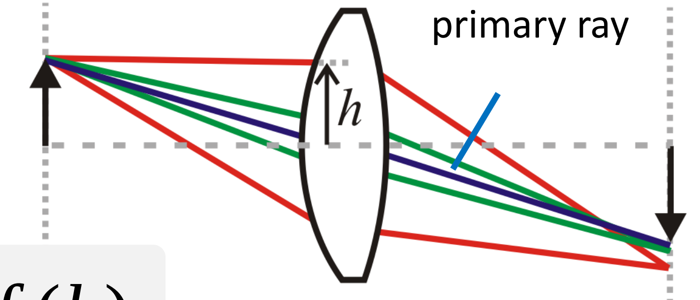
Seidel

$$\Delta^{(3)}x = -2S_2 y_o \rho^2 \cos\theta \sin\theta$$

$$\Delta^{(3)}y = -S_2 y_o \rho^2 (1+\cos^2\theta)$$

$$\Phi^{(4)} = F\, r_o \rho^3 \cos\theta$$

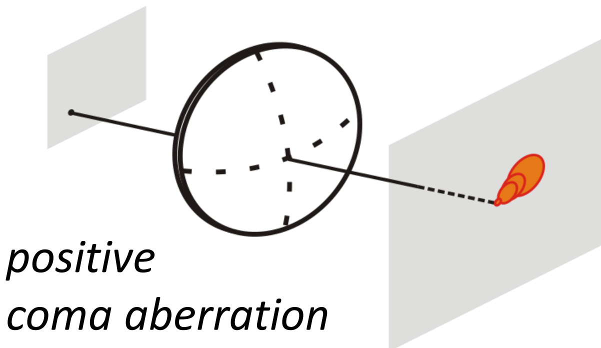


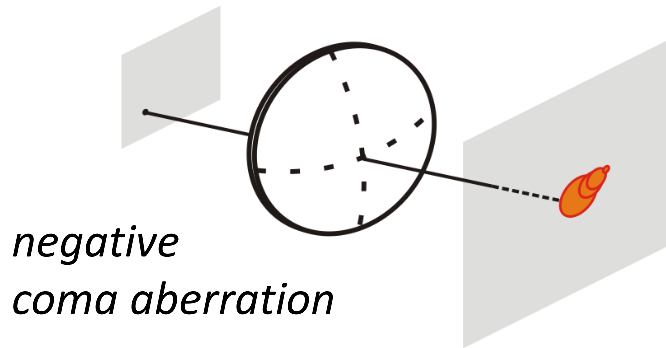


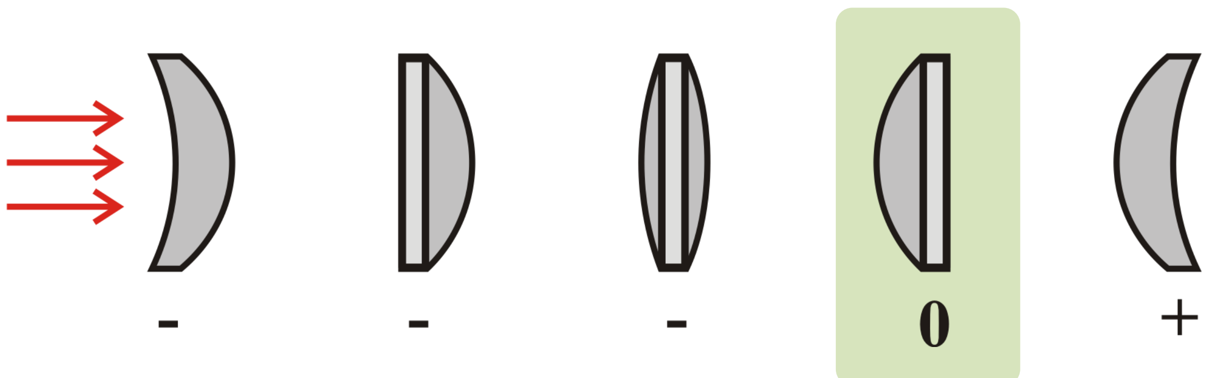
$$M_T = f(h)$$

transverse magnification

breaks the beam's symmetry around the primary ray

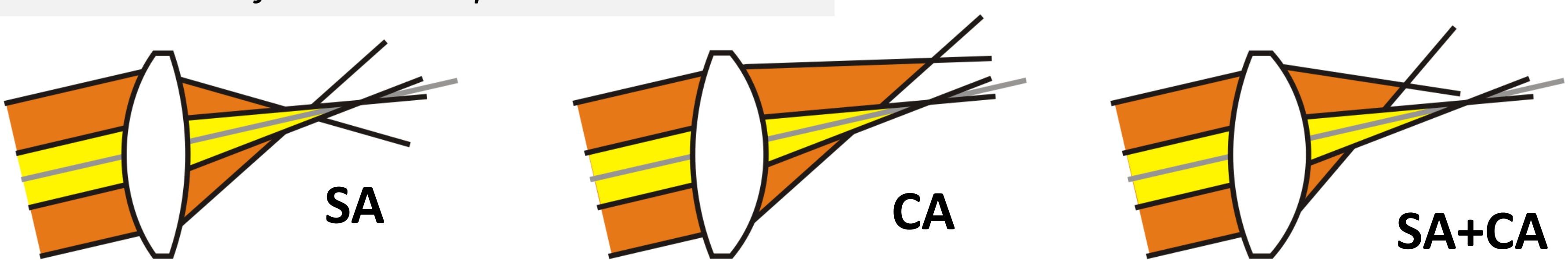






lens shape dependence of coma aberration

Combination of coma and spherical aberration



in the presence of spherical aberration coma aberration depends on the aperture stop position

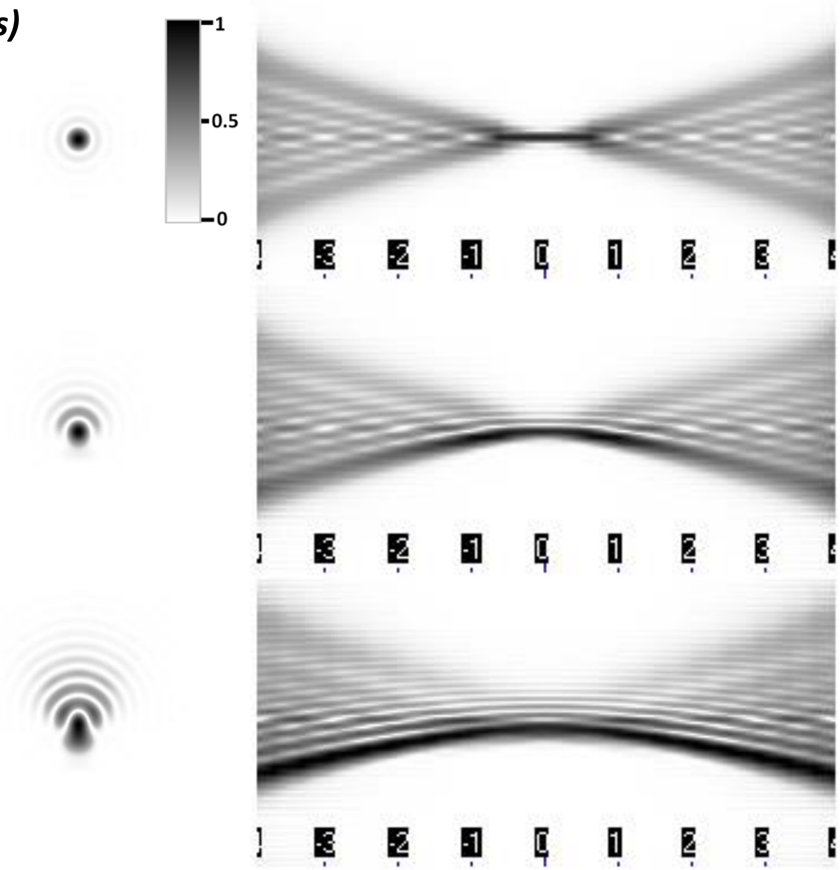
CA effect on the focus

CA (waves)

0

0.5

1



Δz (waves)

focus

transverse section along the propagation direction

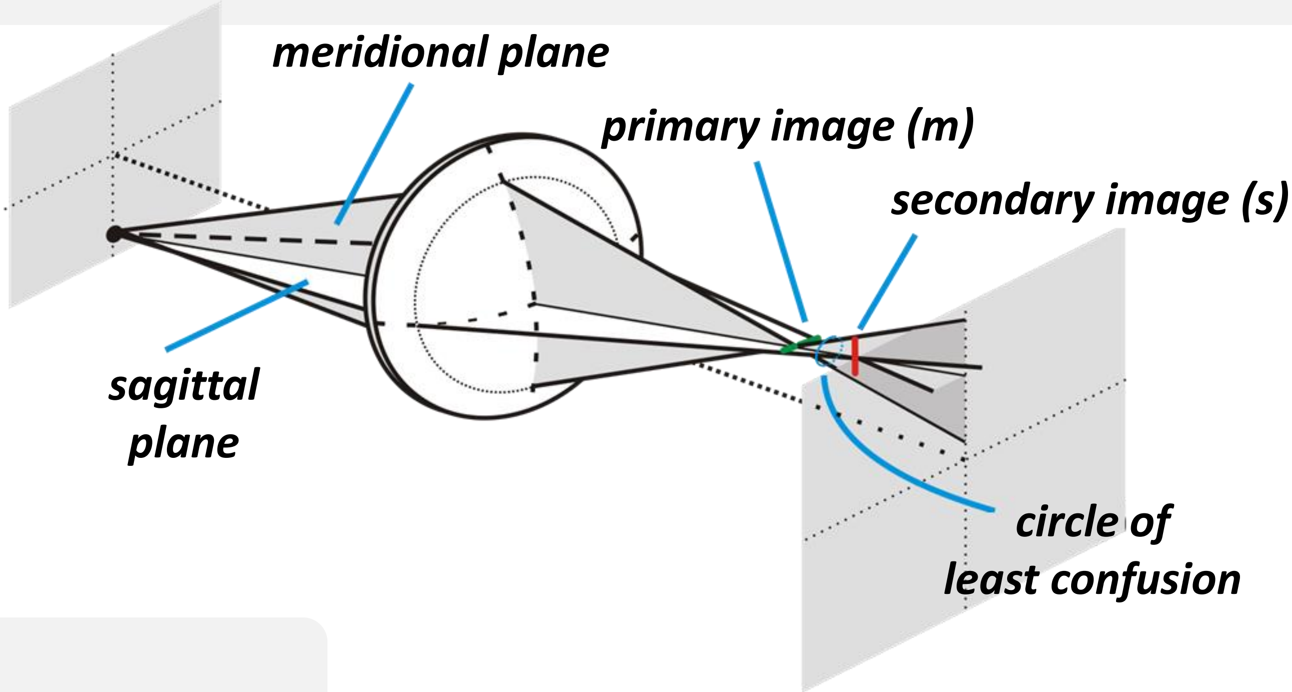
combination of CA & SA at the focus

CA (waves)	***SA (waves)***
1	**0.5**
1.5	**0.5**
2	**0.5**
2.5	**0.5**

simulations preformed using Aberrator (Developed by Cor Berrevoets)

Dimitris Papazoglou dpapa@materials.uoc.gr

Astigmatism

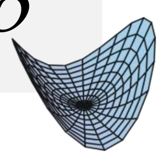


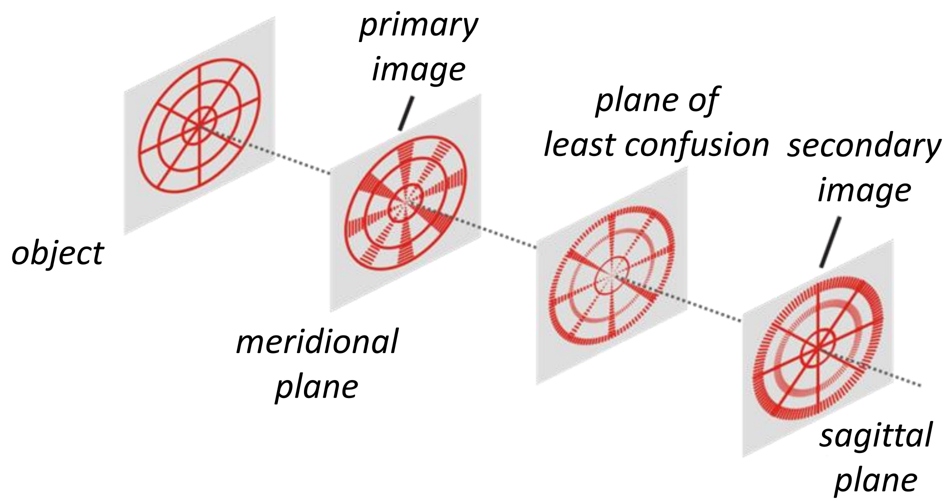
Seidel

$$\Delta^{(3)} x = 0$$

$$\Delta^{(3)} y = 2S_3 y_o^2 \rho \cos\theta$$

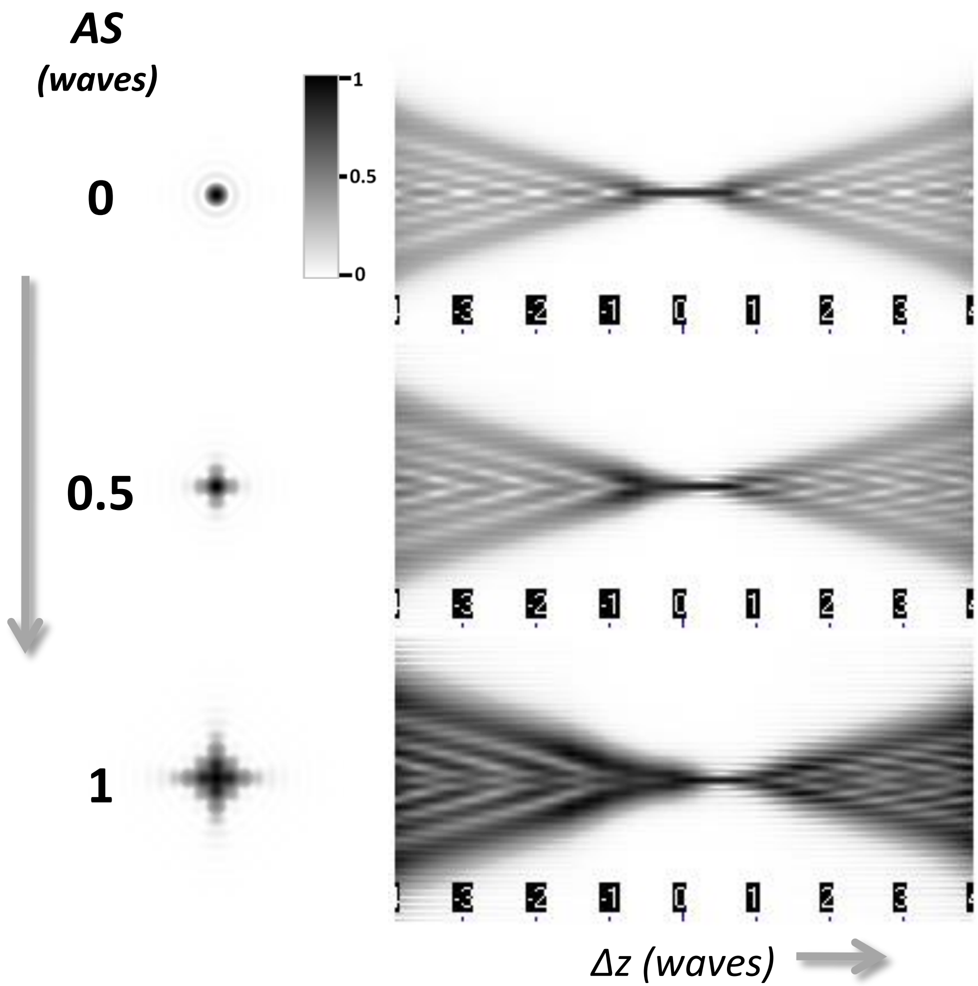
$$\Phi^{(4)} = -C\cos^2\theta\, r_o^2 \rho^2$$





originates from the asymmetry between the meridional and sagittal plane.

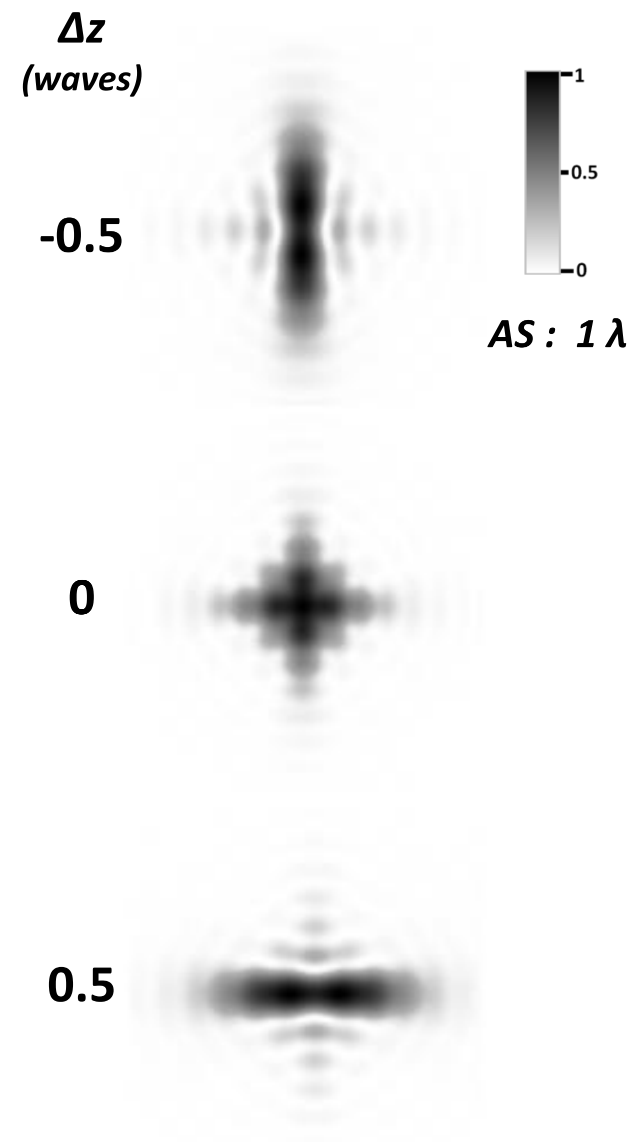
AS effect on focus



focus

transverse section along the propagation direction

Defocusing and astigmatism



Aberrator *simulations preformed using Aberrator (Developed by Cor Berrevoets)*

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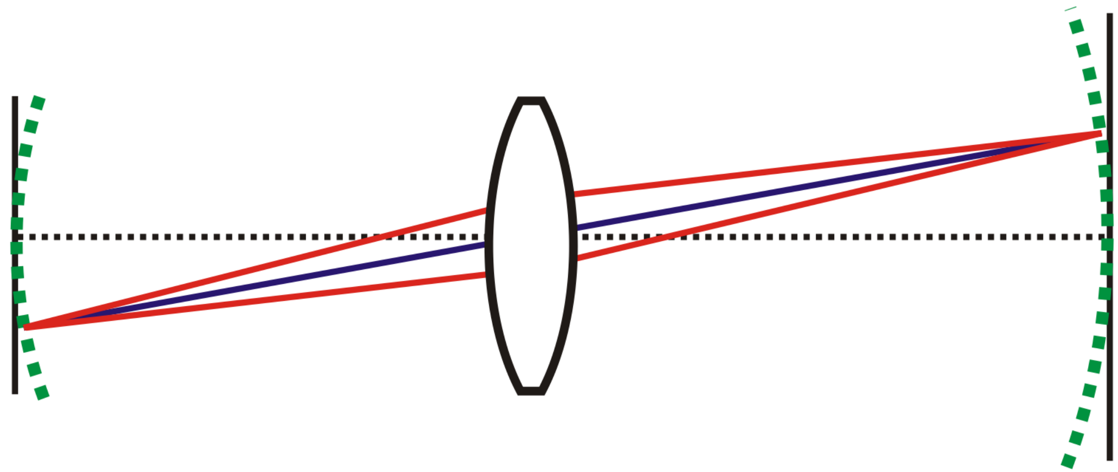
Field curvature (FC)

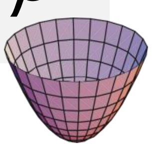


Seidel

$$\Delta^{(3)} x = S_4 y_o^2 \rho \sin\theta$$

$$\Delta^{(3)} y = S_4 y_o^2 \rho \cos\theta$$

$$\Phi^{(4)} = -\frac{1}{2} D r_o^2 \rho^2$$



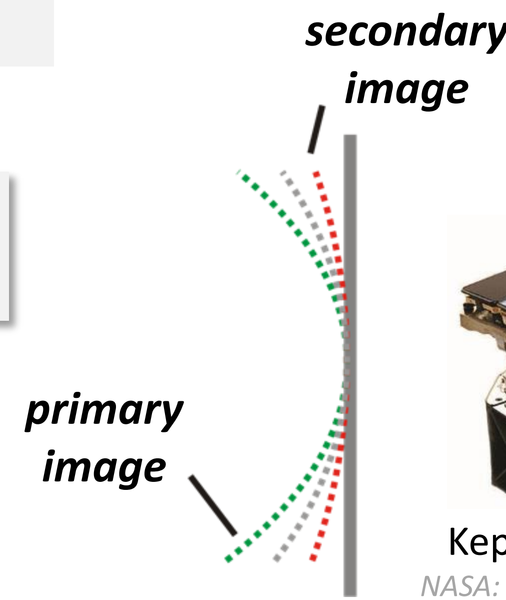
The symmetry of the spherical lenses leads us to spherical conjugate surfaces.

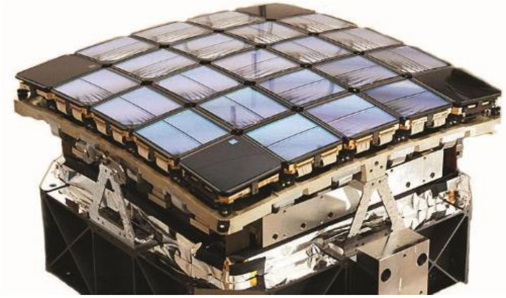
A plane in the object space is imaged stigmatically on a curved surface in the image space (Petzval surface)

field curvature and astigmatism

in the presence of astigmatism the Petzval surface splits into two surfaces:

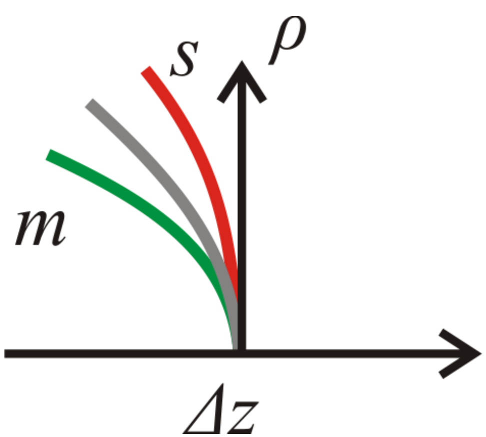
primary image (*meridional*)
secondary image (*sagittal*)



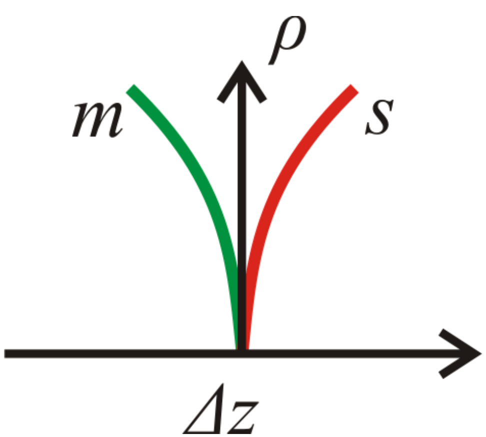


Kepler Spacecraft Focal Plane

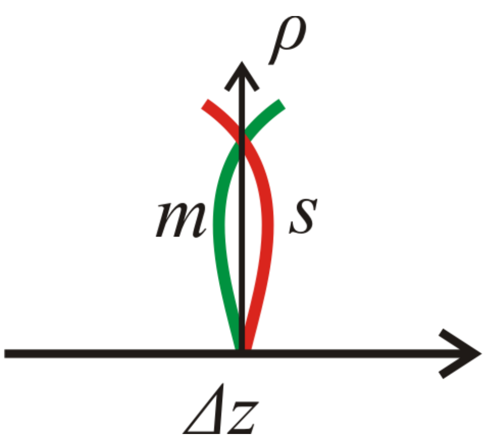
NASA: Wikimedia Commons/Public Domain



astigmatism and field curvature



corrected field curvature

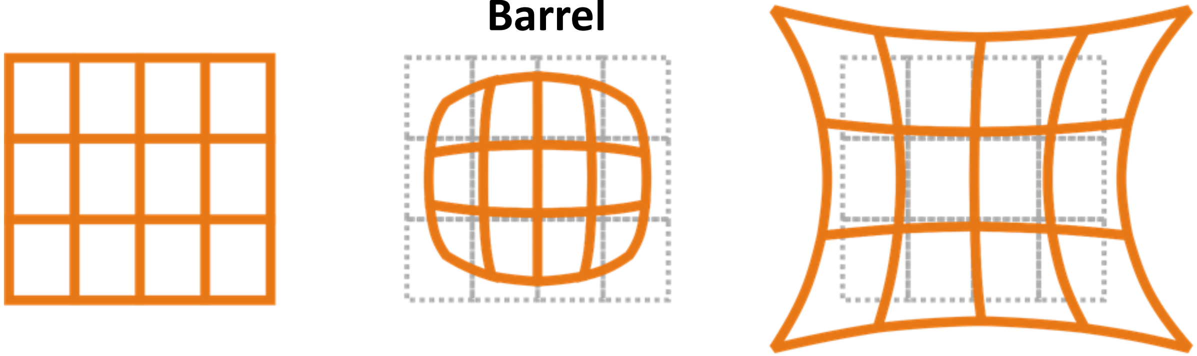


corrected field curvature and astigmatism

Distortion (DS)

Barrel

Pincushion



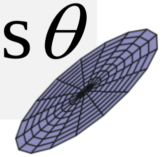
The transverse magnification M_T is a function of the object height.

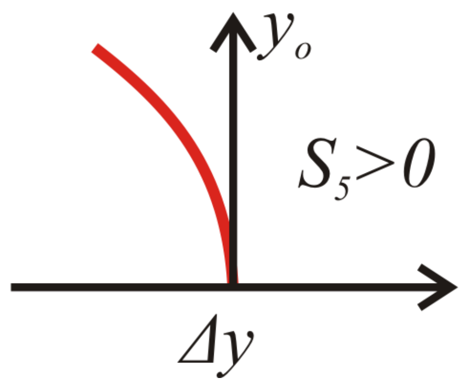
Seidel

$$\Delta^{(3)} x = 0$$

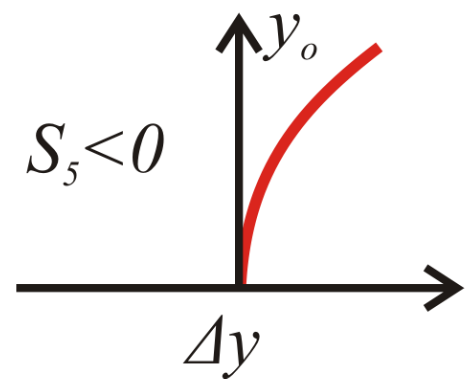
$$\Delta^{(3)} y = -S_5 y_o^3$$

$$\Phi^{(4)} = E\, r_o^3 \rho \cos\theta$$





Barrel distortion

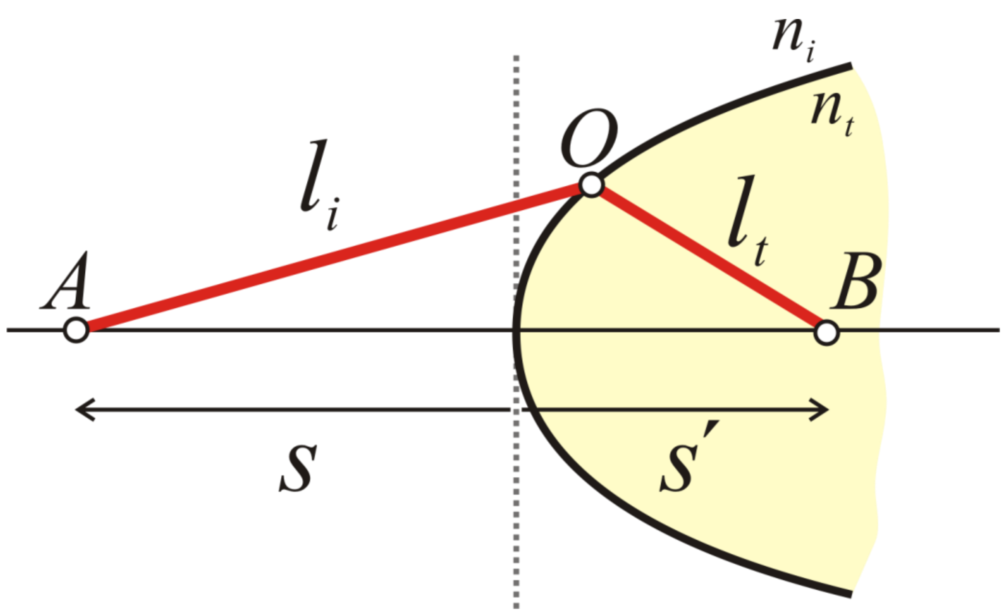


pincushion distortion

Does not affect the image resolution but it “geometrically” distorts it!

Dimitris Papazoglou dpapa@materials.uoc.gr

Corrected aspheric systems



an aspheric lens is designed to stigmatically image a finite object area to a finite image area

$$(OPL)_{\mathrm{AOB}} = n_i l_i + n_t l_t \Rightarrow$$

object position

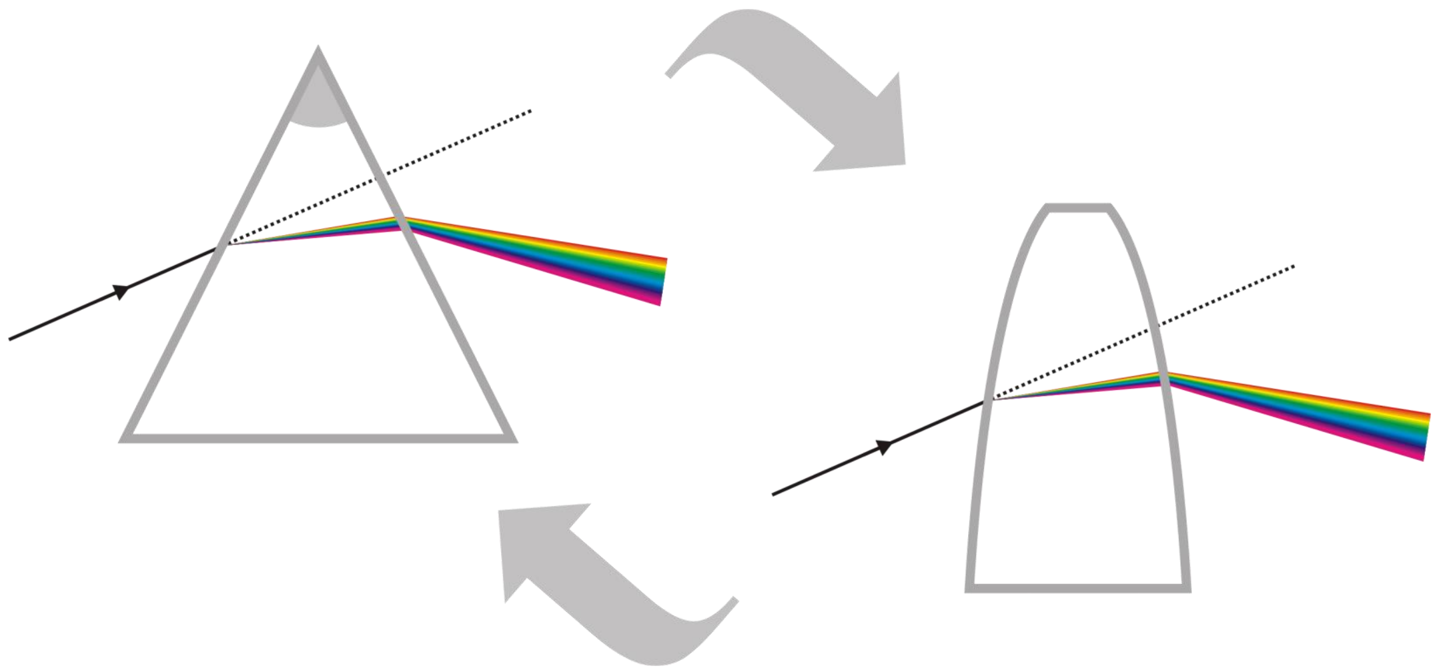
image position

$$n_i l_i + n_t l_t \equiv const = n_i s + n_t s'$$

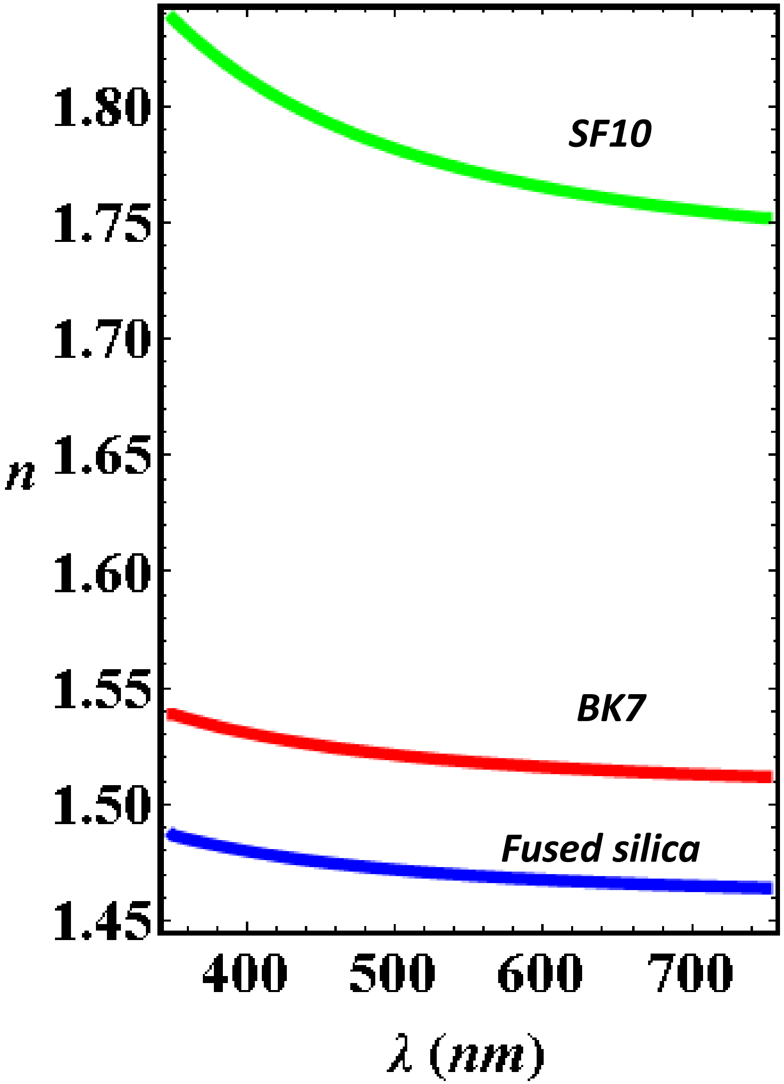
Design condition of an aspheric lens

Chromatic aberrations

Light dispersion though optical media leads to Chromatic aberrations



Dispersion – Abbe number



Refractive index dispersion for typical optical glasses

Cauchy formula

$$n(\lambda) \cong B + \frac{C}{\lambda^2}$$

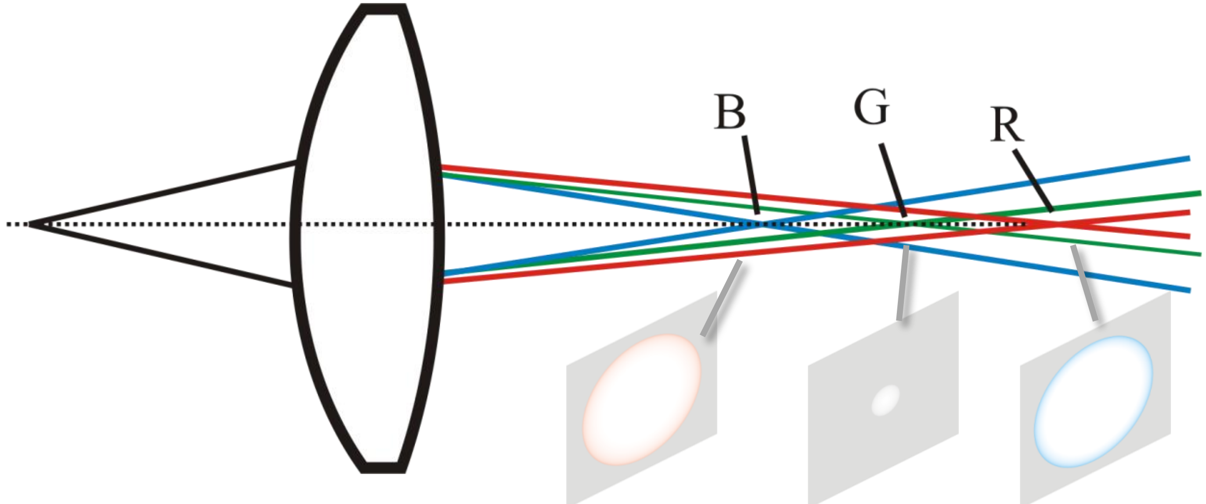
Typical values of B, C constants		
material	**B**	**C (μm^2)**
Fused silica	**1.4580**	**0.00354**
Borosilicate glass BK7	**1.5046**	**0.00420**
Hard crown glass K5	**1.5220**	**0.00459**
Barium crown glass BaK4	**1.5690**	**0.00531**
Barium flint glass BaF10	**1.6700**	**0.00743**
Dense flint glass SF10	**1.7280**	**0.01342**

Abbe number is a measure of material dispersion

$$V \equiv \frac{n(\lambda_Y) - 1}{n(\lambda_B) - n(\lambda_R)}$$

$$\lambda_Y \equiv 589nm, \quad \lambda_R \equiv 656nm, \quad \lambda_B \equiv 486nm$$

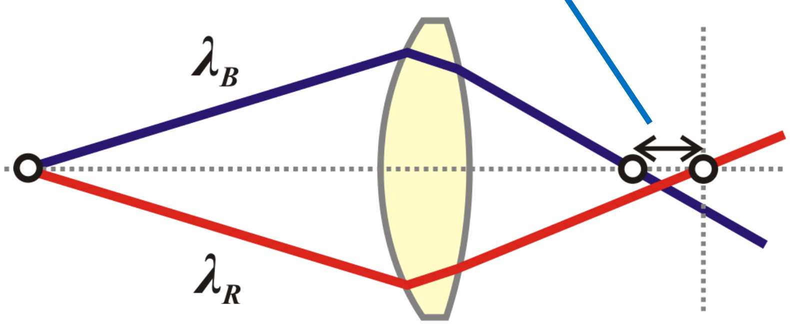
Paraxial chromatic aberration



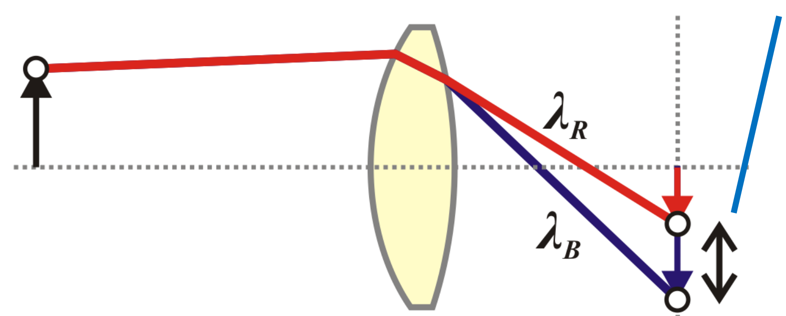
$$\frac{1}{s}+\frac{1}{s'}=(\frac{n_L(\lambda)}{n_m(\lambda)}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\equiv\frac{1}{f(\lambda)}$$

depends on the wavelength

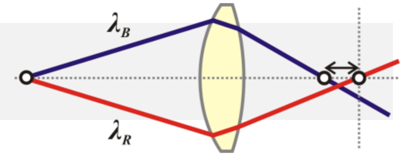
longitudinal chromatic aberration



transverse chromatic aberration



Longitudinal chromatic aberration



$$\frac{1}{f(\lambda)} \equiv [n(\lambda)-1]\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow \left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \frac{1}{n(\lambda_Y)-1}\cdot\frac{1}{f(\lambda_Y)} \Rightarrow$$

$$\frac{1}{s'(\lambda)} = \frac{n(\lambda)-1}{n(\lambda_Y)-1}\cdot\frac{1}{f(\lambda_Y)} - \frac{1}{s} \Rightarrow -\frac{\Delta s'}{s'(\lambda_Y)^2} \cong \frac{\overbrace{n(\lambda_B)-n(\lambda_R)}^{V^{-1}}}{n(\lambda_Y)-1}\cdot\frac{1}{f(\lambda_Y)} \Rightarrow$$

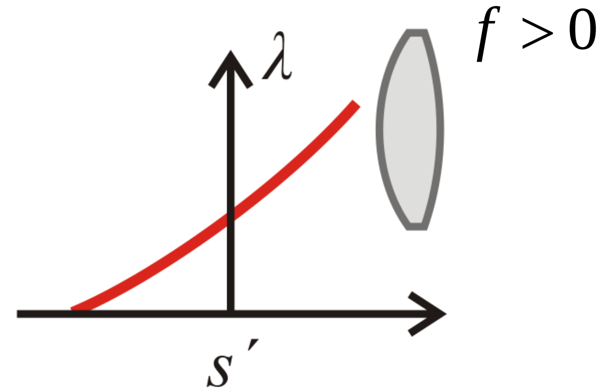
$$\frac{\Delta s'}{s'(\lambda_Y)} \cong -\frac{1}{V}\frac{s'(\lambda_Y)}{f(\lambda_Y)}$$

object at infinity

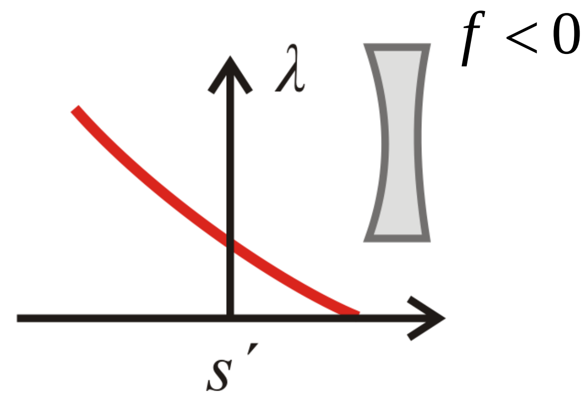
$$s \to \infty \Rightarrow s' = f(\lambda)$$

$$\frac{\Delta s'}{s'(\lambda_Y)} = \frac{\Delta f}{f(\lambda_Y)} \cong -\frac{1}{V}$$

relative change of focal distance



negative longitudinal chromatic aberration



positive longitudinal chromatic aberration

Transverse magnification and chromatic aberration

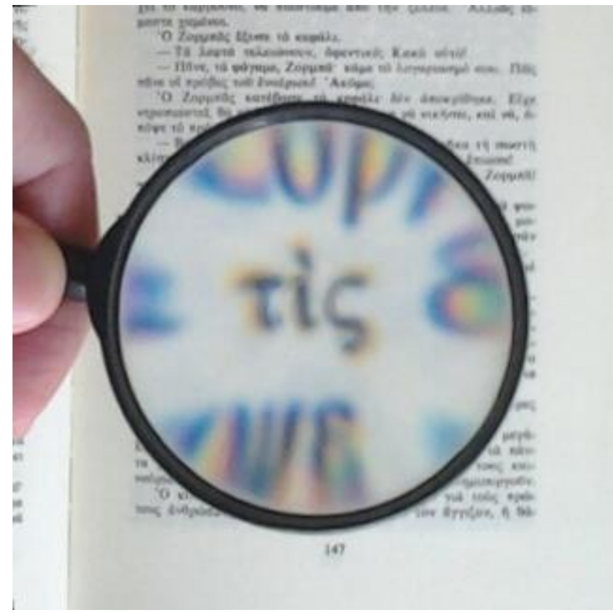
$$M_T \equiv -\frac{s'}{s} \Rightarrow$$

$$\Delta M_T = -\frac{\Delta s'}{s} \cong \frac{1}{s} \cdot \frac{s'(\lambda_Y)^2}{V \cdot f(\lambda_Y)} \Rightarrow$$

$$\frac{\Delta M_T}{M_T^Y} \cong -\frac{s'(\lambda_Y)}{V \cdot f(\lambda_Y)}$$

$$\boxed{= \frac{1}{V} \frac{s}{f(\lambda_Y) - s}}$$

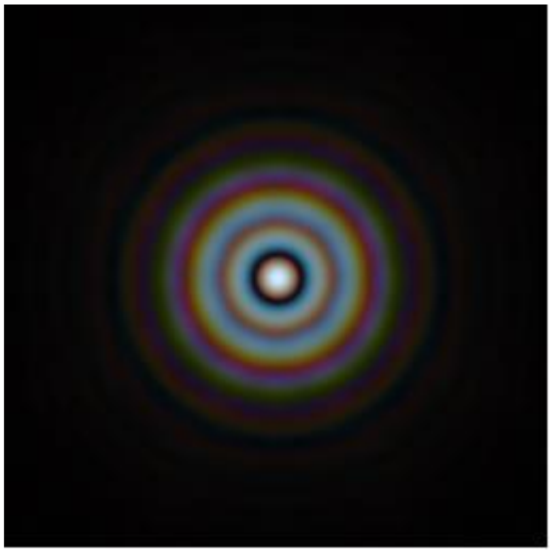
The transverse magnification is a function of the wavelength



Chromatic aberration
(combined with distortion)

Dispersion and optical aberrations

Optical aberrations manifest beyond the paraxial approximation and depend on the wavelength



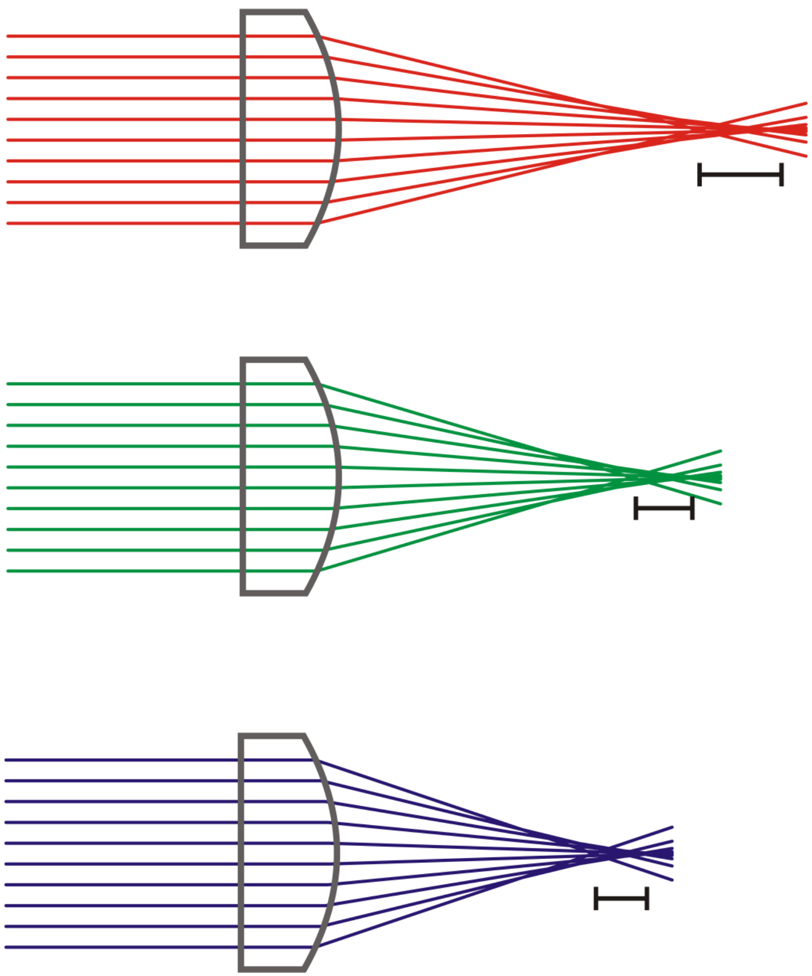
spherical
and chromatic aberration

coma
and chromatic aberration

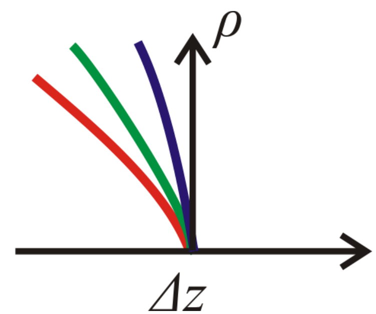
Coma, spherical and chromatic
aberrations

simulations preformed using Aberrator
(Developed by Cor Berrevoets)

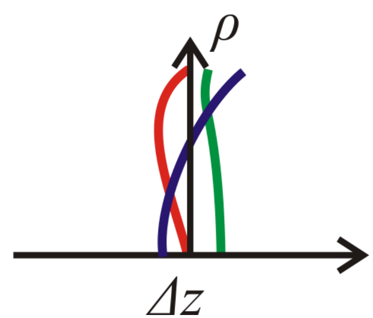
Sphero-chromatic aberration



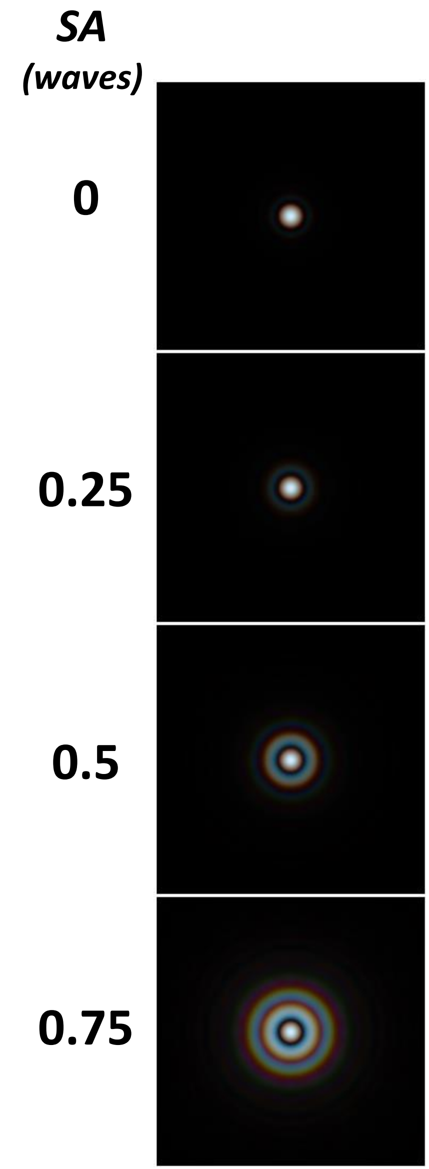
spherochromatic aberration
(Raytracing results)



sphero-chromatic aberration



corrected sphero-chromatic aberration

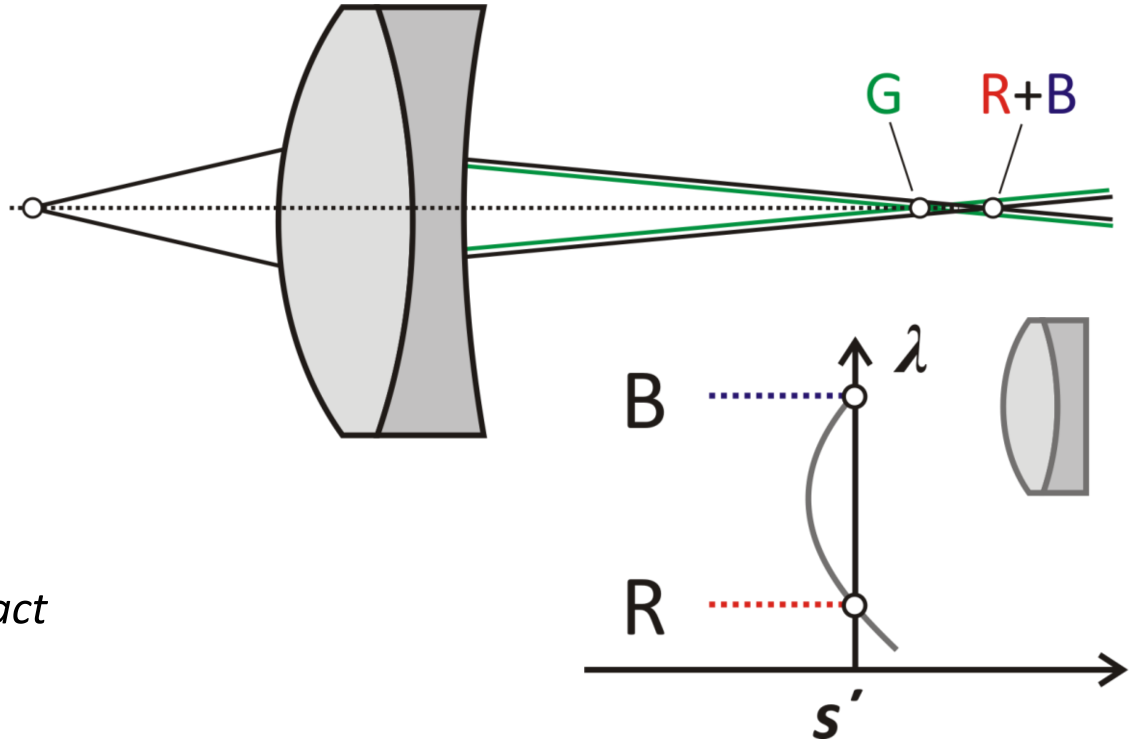


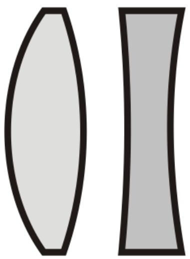
Aberrator

simulations preformed using Aberrator (Developed by Cor Berrevoets)

Achromatic lenses

An achromatic lens is designed so that the red and blue focus coincide





More degrees of freedom when the lenses are not in contact

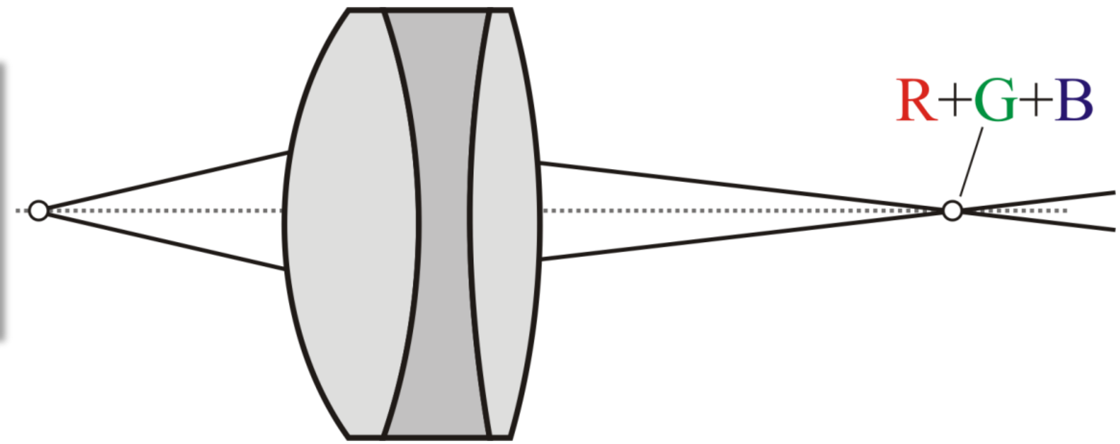
Achromaticity condition for thin lenses in contact

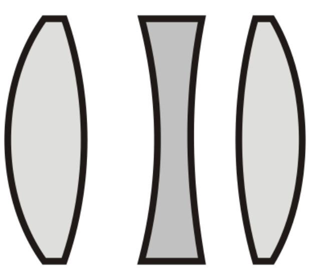
$$V_1 f_1^Y + V_2 f_2^Y = 0$$

Abbe number

Apo-chromatic lenses

An apo-chromatic lens is designed so that the red, green and blue focus coincide.





More degrees of freedom when the lenses are not in contact