Lab 1

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Conditional average treatment effect

For a set of i.i.d subjects i = 1,..., n, we observe a tuple(X_i, Y_i, T_i), comprised of:

- A feature vector $X_i \in \mathbb{R}^p$
- An outcome $Y_i \in \mathbb{R}^p$
- A treatment assignment $T_i \in \{0, 1\}$

Goal is to estimate the **conditional average treatment effect**

$$\tau(x) = \mathbb{E}[Y(1) - Y(0)|X = x]$$

We assume, **unconfoundedness** (Rosenbaum and Robin, 1983)

$$Y_i(0), Y_i(1) \perp\!\!\!\perp T_i|X_i$$

Propensity Score

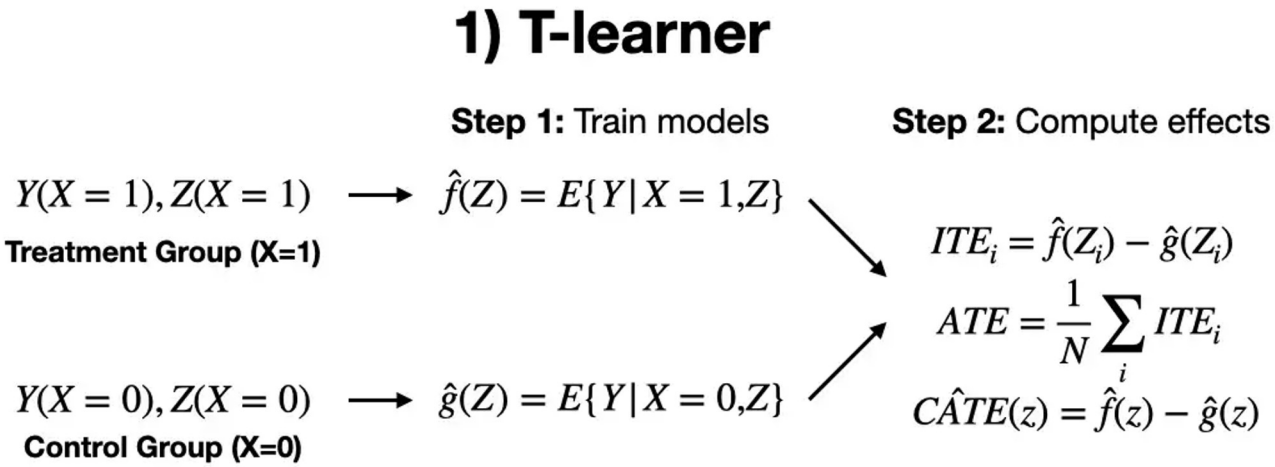
$$e(x) = \mathbb{P}[T_i = 1 | X_i = x]$$

The propensity score measures the probability of being treated conditionally on X_i

- In a randomised trials, the propensity score is constant
- At least qualitatively, the variability of the propensity score gives a measure of how far we are from a randomised trial.

T-learner (or Two Learner)

- Train 2 models to predict outcome, for control group and treatment group.
- Use each model to generate outcome predictions for each sample
- Compute effects (ATE) by averaging over individual treatment effects.



S - Learner (or Single Learner)

- Train one model including treatment variable to estimate outcome
- Predict the effect by calculating if each sample where part of treatment or not.

S-learner

Step 1: Train model

Step 2: Compute effects

$$Y, X, Z \longrightarrow \hat{f}(X, Z) = E\{Y \mid X, Z\} \longrightarrow ITE_i = \hat{f}(X_i = 1, Z_i) - \hat{f}(X_i = 0, Z_i)$$

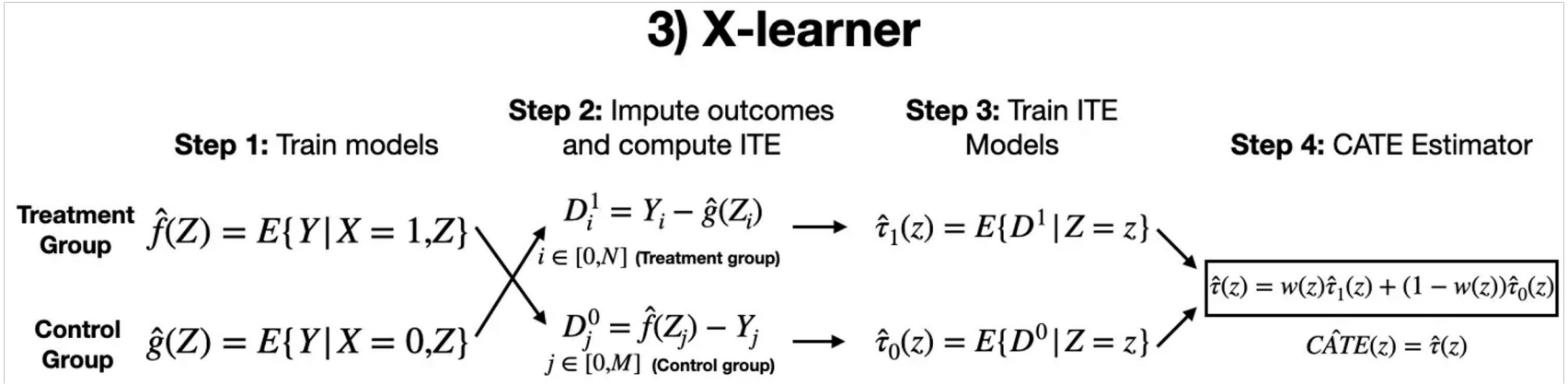
All data for all units

$$ATE = \frac{1}{N} \sum_i ITE_i$$

$$\hat{CATE}(z) = \hat{f}(1, z) - \hat{f}(0, z)$$

X-Learner

- Train 2 models just like a T-learner.
- Use the model to impute unobserved outcome values and calculate ITE
- Train 2 models to estimate ITE for treatment and control groups
- Estimate CATE by weighted average of ITEs using propensity score as weights



Estimating heterogeneous treatment effects

Example:

- **Personalized medicine:** Which form of cancer therapy is most appropriate for specific patient?

Idea: Divide population into subgroups to minimise MSE in treatment effects

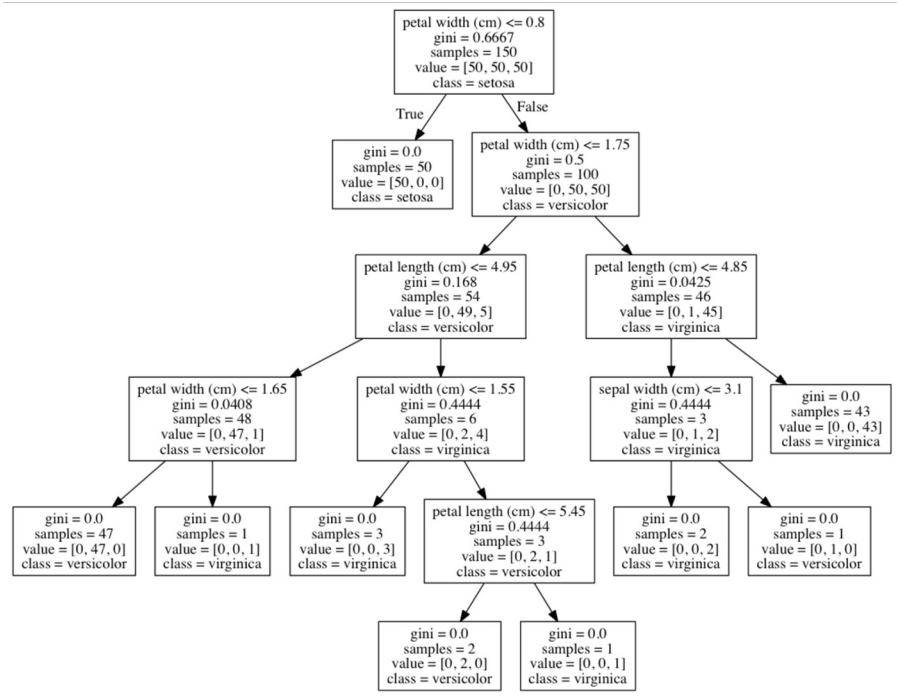
$$\mathbb{E}[(\hat{\tau}(X) - \tau(X))^2]$$

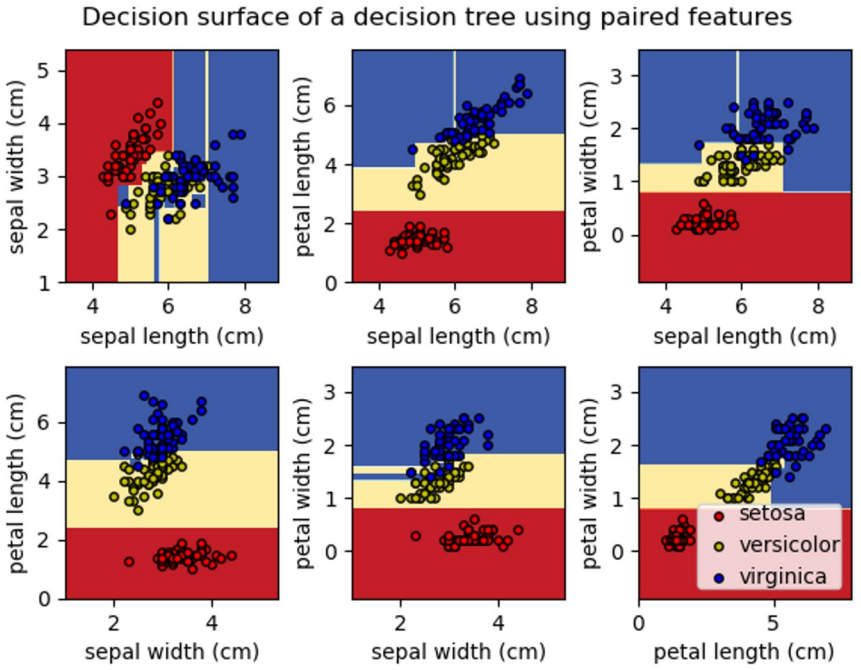
Goal: Estimate treatment effects for subgroups

- How to choose sub groups?
- Sample split: Train on half of the samples and estimate on other half.

Regression or classification trees

Look at all the feature, you try to find a split to create a homogenous group

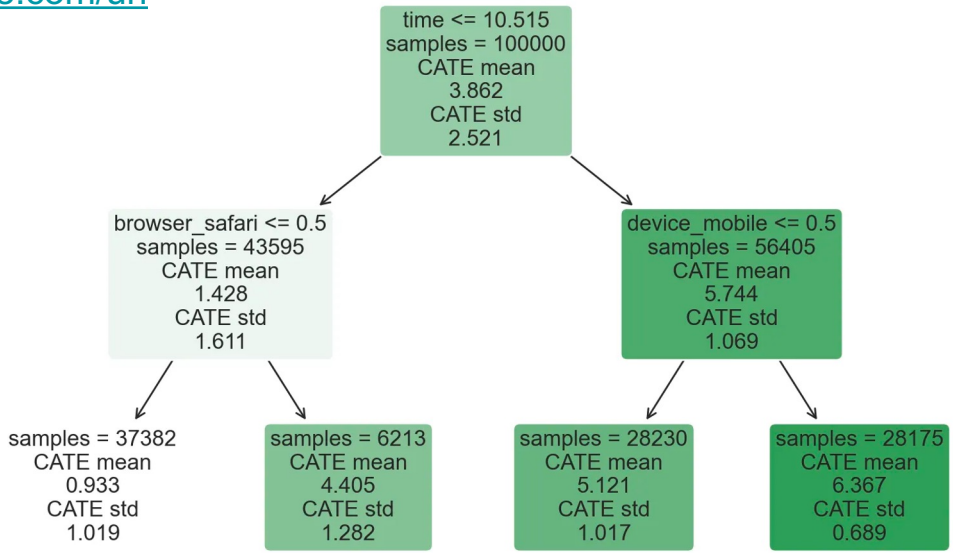




Causal Tree

Dividing into subgroups in a way that it ends up with a leaf similar treatment heterogeneity and different across the leaves.

https://towardsdatascience.com/understanding-causal-trees-920177462149



References:

DoWhy library:

https://www.pywhy.org/dowhy/v0.8/example_notebooks/tutorial-causalinference-machinelearning-using-dowhy-econml.html

Other relevant literatures:

- Identifying subgroups (Athey and Imens, 2016) or other low dimensional parameters
- Testing for heterogeneity across all covariates (List, Shaikh and XU, 2016)
- Robustness to model specification (Athey and Imbens, 2015)
- Imai and Ratkovic (2013) analyse treatment effect heterogeneity with LASSO
- Conditional average treatment effect with theoritical guarantees (Wager and Athey, 2018)
- Identifying individuals with highest estimated treatment effects (Chernozhukov 2018)
- Estimating optimal policies (Zhou et al 2018)
- Contextual bandits for policy learning and data collection (Langford 2016)