Discrete Structures – Practice Problems -I

Instructions: Provide all steps necessary to solve the problem. *Unless otherwise stated, your answer must be exact and reasonably simplified*. Additionally, clearly indicate the value or expression that is your final answer. Calculators are NOT allowed.

1. Find the truth table of the compound proposition $(p \vee q) \rightarrow (p \wedge \neg r)$.

2. Give the converse, the contrapositive, and the inverse of the statement "If it rains today, then I will drive to work."

3. Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent using the laws of logical equivalences. Be sure to cite each law whenever used.

4. Use the table of logical equivalences to simplify the compound proposition $[(p \vee q) \wedge \neg p] \rightarrow q$. Be sure to justify your answers.

5. Let $P(m,n)$ be the statement "$m \mid n$," where the domain for both variables consists of all positive integers. [By "$m \mid n$", which we say as "m divides n", we mean that $n = km$ for some integer k.] Determine the truth values of each of these statements.

(a) $P(4,5)$	***(b)*** $P(2,4)$	***(c)*** $\forall m \forall n\, P(m,n)$
(d) $\exists m \forall n\, P(m,n)$	***(e)*** $\exists n \forall m\, P(m,n)$	***(f)*** $\forall n\, P(1,n)$

6. Consider the compound proposition $(\forall m \exists n\, [P(m,n)]) \rightarrow (\exists n \forall m [P(m,n)])$ where both m and n are integers. Determine the truth value of the proposition if
   (a) $P(m,n)$ is the statement "$m < n$".
   (b) $P(m,n)$ is the statement "$m \mid n$".

7. Suppose that the variable x represents students, $F(x)$ means "x is a freshman," and $M(x)$ means "x is a math major". For each of the three statements ***(a)***, ***(b)***, and ***(c)***, determine which of the symbolic statements are equivalent. (Note: Each statement may have multiple answers.)

I. $\forall x\, [M(x) \rightarrow \neg F(x)]$	**II.** $\neg \exists x\, [M(x) \wedge \neg F(x)]$	**III.** $\forall x\, [F(x) \rightarrow \neg M(x)]$	**IV.** $\forall x\, [M(x) \rightarrow F(x)]$
V. $\exists x\, [F(x) \wedge M(x)]$	**VI.** $\neg \forall x\, [\neg F(x) \vee \neg M(x)]$	**VII.** $\forall x\, [\neg (M(x) \wedge \neg F(x))]$	**VIII.** $\forall x\, [\neg M(x) \vee \neg F(x)]$
IX. $\neg \exists x\, [M(x) \wedge \neg F(x)]$	**X.** $\neg \exists x\, [M(x) \vee F(x)]$	**XI.** $\neg \forall x\, [F(x) \rightarrow \neg M(x)]$	

   (a) Some freshmen are math majors. **Answer:** ______________________

   (b) Every math major is a freshman. **Answer:** ______________________

   (c) No math major is a freshman. **Answer:** ______________________

8. Determine whether the following argument is valid.

$$\begin{array}{ll} & p \rightarrow r \\ & q \rightarrow r \\ & \neg(p \vee q) \\ & \text{--------} \\ \therefore & \neg r \end{array}$$

- If the argument if valid, provide a valid proof of the result (that is, use the laws of logical equivalences and the rules of inference to demonstrate that the conclusion is valid).
- If the argument is not valid, provide specific truth values of p, q, and r in which the premises are true, but the conclusion is false.

<u>Exercises from the text.</u> I would STONGLY recommend that you try as many of these problems as you can. Any of these problems (or ones similar to them) could appear on the exam.