

# COMP232 - Mathematics for Computer Science

## Tutorial 2

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## Exercise 5

Use the truth table to verify the distributive law

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r).$$

<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>
<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>

## Exercise 6

Use the truth table to verify the first De Morgan law

$$\neg(p \wedge q) \equiv \neg p \vee \neg q.$$

<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>

## Exercise 8

Use De Morgan's laws to find the negation of each of the following statements.

- a) Kwame will take a job in industry or go to graduate school.  
Kwame will **not** take a job in industry **and** will **not** go to graduate school.
- b) Yoshiko knows Java and calculus.  
Yoshiko **doesn't** know Java **or** **doesn't** know calculus.
- c) James is young and strong.  
James is **not** young **or** is **not** strong
- d) Rita will move to Oregon or Washington  
Rita will **not** move to Oregon **and** will **not** move to Washington

## Exercise 9.a

Show that each of these conditional statements is a tautology by using truth table  $(p \wedge q) \rightarrow p$ .

<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>

## Exercise 9.b

Show that each of these conditional statements is a tautology by using truth table  $p \rightarrow (p \vee q)$ .

<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>

## Exercise 9.c

Show that each of these conditional statements is a tautology by using truth table  $\neg p \rightarrow (p \rightarrow q)$ .

<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>



## Exercise 9.d

Show that each of these conditional statements is a tautology by using truth table  $(p \wedge q) \rightarrow (p \rightarrow q)$ .

$T$	$T$	$T$	$T$
$T$	$F$	$F$	$T$
$F$	$T$	$T$	$T$
$F$	$F$	$T$	$T$

## Exercise 9.e

Show that each of these conditional statements is a tautology by using truth table  $\neg(p \rightarrow q) \rightarrow p$ .

<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>

## Exercise 9.f

Show that each of these conditional statements is a tautology by using truth table  $\neg(p \rightarrow q) \rightarrow \neg q$ .

<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>

## Exercise 18

Show that  $p \rightarrow q$  and  $\neg q \rightarrow \neg p$  are logically equivalent.

$T$	$T$	$T$	$T$
$T$	$F$	$F$	$F$
$F$	$T$	$T$	$T$
$F$	$F$	$T$	$T$

## Exercise 20

Show that  $\neg(p \oplus q)$  and  $p \leftrightarrow q$  are logically equivalent.

<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i>
<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>

## Exercise 26

Show that  $\neg p \rightarrow (q \rightarrow r)$  and  $q \rightarrow (p \vee r)$  are logically equivalent.

<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>
<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>

## Exercise 41

Find a compound proposition involving the propositional variables  $p$ ,  $q$  and  $r$  that is true when exactly two of  $p$ ,  $q$  and  $r$  are true and is false otherwise.

Hint: Form a disjunction of conjunctions.

Hint: Include a conjunction for each combination of values for which the compound proposition is true.

Hint: Each conjunction should include each of three propositional variables or its negations.

Answer:

$$(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$$