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1.0 Data Structures

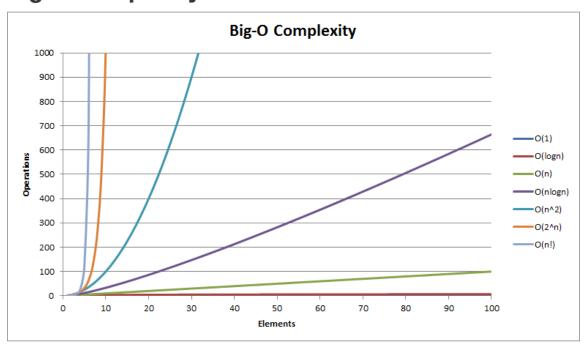
1.1 Overview

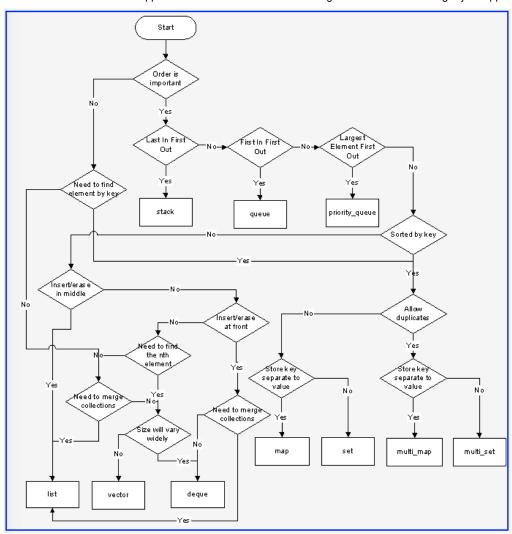


Data Structures

| Data Structure | Time Complex | Time Complexity | | | | | Space Complexity | | |
|--------------------|--------------|-----------------|-----------|-----------|-----------|-----------|------------------|-----------|-------------|
| | Average | | | Worst | | | Worst | | |
| | Indexing | Search | Insertion | Deletion | Indexing | Search | Insertion | Deletion | |
| Basic Array | 0(1) | 0(n) | - | - | 0(1) | 0(n) | - | - | 0(n) |
| Dynamic Array | 0(1) | 0(n) | 0(n) | 0(n) | 0(1) | 0(n) | 0(n) | 0(n) | 0(n) |
| Singly-Linked List | 0(n) | 0(n) | 0(1) | 0(1) | 0(n) | 0(n) | 0(1) | 0(1) | 0(n) |
| Doubly-Linked List | 0(n) | 0(n) | 0(1) | 0(1) | 0(n) | 0(n) | 0(1) | 0(1) | 0(n) |
| Skip List | O(log(n)) | O(log(n)) | O(log(n)) | 0(log(n)) | 0(n) | 0(n) | 0(n) | 0(n) | O(n log(n)) |
| Hash Table | - | 0(1) | 0(1) | 0(1) | - | 0(n) | 0(n) | 0(n) | 0(n) |
| Binary Search Tree | O(log(n)) | O(log(n)) | O(log(n)) | 0(log(n)) | 0(n) | 0(n) | 0(n) | 0(n) | 0(n) |
| Cartresian Tree | - | 0(log(n)) | O(log(n)) | 0(log(n)) | - | 0(n) | 0(n) | 0(n) | 0(n) |
| B-Tree | O(log(n)) | O(log(n)) | O(log(n)) | 0(log(n)) | O(log(n)) | O(log(n)) | O(log(n)) | 0(log(n)) | 0(n) |
| Red-Black Tree | O(log(n)) | O(log(n)) | O(log(n)) | 0(log(n)) | O(log(n)) | O(log(n)) | 0(log(n)) | O(log(n)) | 0(n) |
| Splay Tree | - | O(log(n)) | O(log(n)) | O(log(n)) | - | O(log(n)) | O(log(n)) | O(log(n)) | 0(n) |
| AVL Tree | O(log(n)) | 0(log(n)) | 0(log(n)) | O(log(n)) | O(log(n)) | 0(log(n)) | O(log(n)) | O(log(n)) | 0(n) |

Big-O Complexity Chart





1.2 Vector std::vector

Use for

- Simple storage
- Adding but not deleting
- Serialization
- Quick lookups by index
- Easy conversion to C-style arrays
- Efficient traversal (contiguous CPU caching)

Do not use for

- Insertion/deletion in the middle of the list
- Dynamically changing storage
- Non-integer indexing

Time Complexity

| Operation | Time Complexity |
|--------------|-----------------|
| Insert Head | 0(n) |
| Insert Index | 0(n) |

| Operation | Time Complexity | | |
|--------------|-----------------|--|--|
| Insert Tail | 0(1) | | |
| Remove Head | 0(n) | | |
| Remove Index | 0(n) | | |
| Remove Tail | 0(1) | | |
| Find Index | 0(1) | | |
| Find Object | 0(n) | | |

Example Code

```
std::vector<int> v;
//-----
// General Operations
//-----
// Insert head, index, tail
v.insert(v.begin(), value);
                                  // head
v.insert(v.begin() + index, value);  // index
                                  // tail
v.push_back(value);
// Access head, index, tail
int head = v.front(); // head
int value = v.at(index); // index
int tail = v.back();  // tail
unsigned int size = v.size();
// Iterate
for(std::vector<int>::iterator it = v.begin(); it != v.end(); it++) {
   std::cout << *it << std::endl;</pre>
// Remove head, index, tail
v.erase(v.begin());
                          // head
v.erase(v.begin() + index);  // index
v.pop_back();
                           // tail
// Clear
v.clear();
```

1.3 Deque std::deque

Use for

- Similar purpose of std::vector
- Basically std::vector with efficient push_front and pop_front

Do not use for

• C-style contiguous storage (not guaranteed)

Notes

- Pronounced 'deck'
- Stands for Double Ended Queue

Example Code

```
std::deque<int> d;
//-----
// General Operations
//-----
// Insert head, index, tail
                                   // head
d.push_front(value);
d.insert(d.begin() + index, value);
                                   // index
d.push_back(value);
                                   // tail
// Access head, index, tail
int head = d.front();  // head
int value = d.at(index); // index
int tail = d.back(); // tail
// Size
unsigned int size = d.size();
// Iterate
for(std::vector<int>::iterator it = d.begin(); it != d.end(); it++) {
   std::cout << *it << std::endl;</pre>
// Remove head, index, tail
                           // head
d.pop_front();
d.erase(d.begin() + index);  // index
d.pop_back();
                           // tail
// Clear
d.clear();
```

1.4 List std::list and std::forward_list

Use for

- Insertion into the middle/beginning of the list
- Efficient sorting (pointer swap vs. copying)

Do not use for

• Direct access

Time Complexity

| Operation | Time Complexity | | |
|--------------|-----------------|--|--|
| Insert Head | 0(1) | | |
| Insert Index | 0(n) | | |
| Insert Tail | 0(1) | | |
| Remove Head | 0(1) | | |
| Remove Index | 0(n) | | |
| Remove Tail | 0(1) | | |
| Find Index | 0(n) | | |
| Find Object | 0(n) | | |

Example Code

```
std::list<int> 1;
//-----
// General Operations
//-----
// Insert head, index, tail
1.push_front(value);
                                    // head
                                    // index
1.insert(1.begin() + index, value);
1.push_back(value);
                                    // tail
// Access head, index, tail
int head = 1.front();
                                                          // head
int value = std::list<int>::iterator it = 1.begin() + index;
                                                          // index
int tail = 1.back();
                                                          // tail
// Size
unsigned int size = 1.size();
// Iterate
for(std::list<int>::iterator it = 1.begin(); it != 1.end(); it++) {
   std::cout << *it << std::endl;</pre>
// Remove head, index, tail
                            // head
1.pop_front();
1.erase(1.begin() + index); // index
                            // tail
1.pop_back();
// Clear
1.clear();
//-----
// Container-Specific Operations
//-----
// Splice: Transfer elements from list to list
      splice(iterator pos, list &x)
       splice(iterator pos, list &x, iterator i)
       splice(iterator pos, list &x, iterator first, iterator last)
//
1.splice(l.begin() + index, list2);
// Remove: Remove an element by value
1.remove(value);
// Unique: Remove duplicates
1.unique();
// Merge: Merge two sorted lists
1.merge(list2);
// Sort: Sort the list
1.sort();
// Reverse: Reverse the list order
1.reverse();
```

1.5 Map std::map and std::unordered_map

Use for

- Key-value pairs
- Constant lookups by key

- Searching if key/value exists
- Removing duplicates
- std::map
 - o Ordered map
- std::unordered_map
 - Hash table

Do not use for

Sorting

Notes

- Typically ordered maps (std::map) are slower than unordered maps (std::unordered_map)
- Maps are typically implemented as binary search trees

Time Complexity

std::map

| Operation | Time Complexity |
|-------------------|-----------------|
| Insert | 0(log(n)) |
| Access by Key | O(log(n)) |
| Remove by Key | O(log(n)) |
| Find/Remove Value | O(log(n)) |

std::unordered_map

| Operation | Time Complexity |
|-------------------|-----------------|
| Insert | 0(1) |
| Access by Key | 0(1) |
| Remove by Key | 0(1) |
| Find/Remove Value | |

Example Code

```
std::map<std::string, std::string> m;

//-------
// General Operations
//-----
// Insert
m.insert(std::pair<std::string, std::string>("key", "value"));

// Access by key
std::string value = m.at("key");

// Size
unsigned int size = m.size();

// Iterate
for(std::map<int>::iterator it = m.begin(); it != m.end(); it++) {
    std::cout << *it << std::endl;
}

// Remove by key</pre>
```

```
m.erase("key");
// Clear
m.clear();
//------
// Container-Specific Operations
//------
// Find if an element exists by key
bool exists = (m.find("key") != m.end());
// Count the number of elements with a certain key
unsigned int count = m.count("key");
```

1.6 Set std::set

Use for

- Removing duplicates
- Ordered dynamic storage

Do not use for

- Simple storage
- Direct access by index

Notes

• Sets are often implemented with binary search trees

Time Complexity

| Operation | Time Complexity | | |
|-----------|-----------------|--|--|
| Insert | 0(log(n)) | | |
| Remove | O(log(n)) | | |
| Find | O(log(n)) | | |

Example Code

```
s.clear();
//-----
// Container-Specific Operations
//-----
// Find if an element exists
bool exists = (s.find(20) != s.end());
// Count the number of elements with a certain value
unsigned int count = s.count(20);
```

1.7 Stack std::stack

Use for

- First-In Last-Out operations
- Reversal of elements

Time Complexity

| Operation | Time Complexity | | |
|-----------|-----------------|--|--|
| Push | 0(1) | | |
| Рор | 0(1) | | |
| Тор | 0(1) | | |

Example Code

1.8 Queue std::queue

Use for

- First-In First-Out operations
- Ex: Simple online ordering system (first come first served)
- Ex: Semaphore queue handling
- Ex: CPU scheduling (FCFS)

Notes

• Often implemented as a std::deque

Example Code

1.9 Priority Queue std::priority_queue

Use for

- First-In First-Out operations where priority overrides arrival time
- Ex: CPU scheduling (smallest job first, system/user priority)
- Ex: Medical emergencies (gunshot wound vs. broken arm)

Notes

• Often implemented as a std::vector

Example Code

```
std::priority_queue<int> p;

//------
// General Operations
//-----
// Insert
p.push(value);

// Access
int top = p.top(); // 'Top' element

// Size
unsigned int size = p.size();

// Remove
p.pop();
```

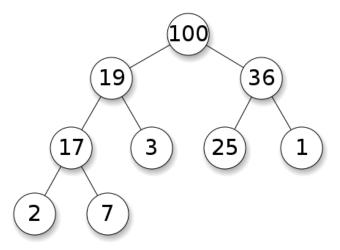
1.10 Heap std::priority_queue

Notes

• A heap is essentially an instance of a priority queue

- A min heap is structured with the root node as the smallest and each child subsequently smaller than its parent
- A max heap is structured with the root node as the largest and each child subsequently larger than its parent
- A min heap could be used for Smallest Job First CPU Scheduling
- A max heap could be used for Priority CPU Scheduling

Max Heap Example (using a binary tree)

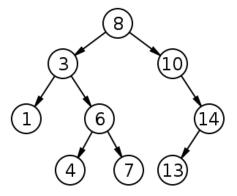


2.0 Trees

2.1 Binary Tree

- A binary tree is a tree with at most two (2) child nodes per parent
- Binary trees are commonly used for implementing O(log(n)) operations for ordered maps, sets, heaps, and binary search trees
- Binary trees are **sorted** in that nodes with values greater than their parents are inserted to the **right**, while nodes with values less than their parents are inserted to the **left**

Binary Search Tree



2.2 Balanced Trees

- Balanced trees are a special type of tree which maintains its balance to ensure O(log(n)) operations
- When trees are not balanced the benefit of log(n) operations is lost due to the highly vertical structure
- Examples of balanced trees:
 - o AVL Trees
 - o Red-Black Trees

2.3 Binary Search

Idea:

- 1. If current element, return
- 2. If less than current element, look left
- 3. If more than current element, look right
- 4. Repeat

Data Structures:

- Tree
- Sorted array

Space:

• 0(1)

Best Case:

• 0(1)

Worst Case:

• 0(log n)

Average:

• 0(log n)

Visualization:



2.4 Depth-First Search

Idea:

- 1. Start at root node
- 2. Recursively search all adjacent nodes and mark them as searched
- 3. Repeat

Data Structures:

- Tree
- Graph

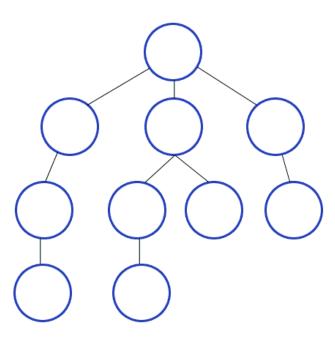
Space:

• O(V), V = number of verticies

Performance:

• O(E), E = number of edges

Visualization:



2.5 Breadth-First Search

Idea:

- 1. Start at root node
- 2. Search neighboring nodes first before moving on to next level

Data Structures:

- Tree
- Graph

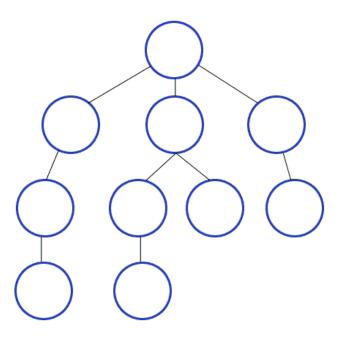
Space:

 \bullet O(V), V = number of verticies

Performance:

• O(E), E = number of edges

Visualization:



3.0 NP Complete Problems

3.1 NP Complete

- NP Complete means that a problem is unable to be solved in polynomial time
- NP Complete problems can be verified in polynomial time, but not solved

3.2 Traveling Salesman Problem

3.3 Knapsack Problem

4.0 Algorithms

4.1 Insertion Sort

Idea

- 1. Iterate over all elements
- 2. For each element:
 - o Check if element is larger than largest value in sorted array
- 3. If larger: Move on
- 4. If smaller: Move item to correct position in sorted array

Details

- Data structure: Array
- Space: 0(1)
- Best Case: Already sorted, 0(n)

- Worst Case: Reverse sorted, 0(n^2)
- Average: 0(n^2)

Advantages

- Easy to code
- Intuitive
- Better than selection sort and bubble sort for small data sets
- Can sort in-place

Disadvantages

• Very inefficient for large datasets

Visualization

6 5 3 1 8 7 2 4

4.2 Selection Sort

Idea

- 1. Iterate over all elements
- 2. For each element:
 - o If smallest element of unsorted sublist, swap with left-most unsorted element

Details

- Data structure: Array
- Space: 0(1)
- Best Case: Already sorted, 0(n^2)
- Worst Case: Reverse sorted, 0(n^2)
- Average: 0(n^2)

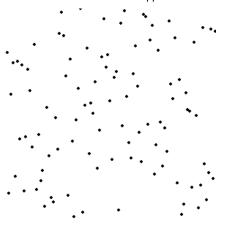
Advantages

- Simple
- Can sort in-place
- Low memory usage for small datasets

Disadvantages

• Very inefficient for large datasets

Visualization



4.3 Bubble Sort

Idea

- 1. Iterate over all elements
- 2. For each element:
 - Swap with next element if out of order
- 3. Repeat until no swaps needed

Details

- Data structure: Array
- Space: 0(1)
- Best Case: Already sorted 0(n)
- Worst Case: Reverse sorted, 0(n^2)
- Average: 0(n^2)

Advantages

• Easy to detect if list is sorted

Disadvantages

- Very inefficient for large datasets
- Much worse than even insertion sort

Visualization

6 5 3 1 8 7 2 4

4.4 Merge Sort

Idea

- 1. Divide list into smallest unit (1 element)
- 2. Compare each element with the adjacent list
- 3. Merge the two adjacent lists
- 4. Repeat

Details

- Data structure: Array
- Space: O(n) auxiliary
- Best Case: O(nlog(n))
- Worst Case: Reverse sorted, O(nlog(n))
- Average: O(nlog(n))

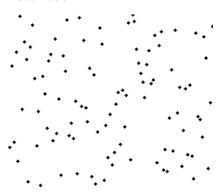
Advantages

- High efficiency on large datasets
- Nearly always O(nlog(n))
- Can be parallelized
- Better space complexity than standard Quicksort

Disadvantages

- Still requires O(n) extra space
- Slightly worse than Quicksort in some instances

Visualization



6 5 3 1 8 7 2 4

4.5 Quicksort

Idea

- 1. Choose a pivot from the array
- 2. Partition: Reorder the array so that all elements with values *less* than the pivot come before the pivot, and all values *greater* than the pivot come after
- 3. Recursively apply the above steps to the sub-arrays

Details

• Data structure: Array

• **Space**: 0(n)

• Best Case: O(nlog(n))

• Worst Case: All elements equal, 0(n^2)

• Average: O(nlog(n))

Advantages

- Can be modified to use O(log(n)) space
- Very quick and efficient with large datasets
- Can be parallelized
- Divide and conquer algorithm

Disadvantages

- Not stable (could swap equal elements)
- Worst case is worse than Merge Sort

Optimizations

- Choice of pivot:
 - o Choose median of the first, middle, and last elements as pivot
 - o Counters worst-case complexity for already-sorted and reverse-sorted

Visualization

