FAST NUCES, Karachi.

Discrete Structures CS 211, Fall 2018

Assignment No: 1

Due Date: 27 September, 2018

Instructor: Dr. Nouman M Durrani   Maximum Points= 40 (10 Points/Section)

Note: Answer all questions in the space provided. However, Section Propositional Logic's Q 1(iii) should be solved as per instructions.

Propositional Logic

1(i). Let *p* and *q* be the propositions:

p : It is below freezing.   *q* : It is snowing.

Write these propositions using *p* and *q* and logical connectives (including negations).

a) It is below freezing and snowing. ________________

b) It is below freezing but not snowing. ________________

c) It is not below freezing and it is not snowing. ________________

d) It is either snowing or below freezing (or both). ________________

e) If it is below freezing, it is also snowing. ________________

f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing. ________________

g) That it is below freezing is necessary and sufficient for it to be snowing. ________________

1(ii). Let p, q, and r be the propositions

p : You have the flu.

q : You miss the final examination.

r : You pass the course.

Write these propositions using p and q and logical connectives.

a) If you miss the final examination, you will not pass the course. ________________

b) You have flu, or miss the final examination, or also pass the course ________________

1(iii). Construct a truth table for each of these compound propositions (attach solution sheets after page 2).

a) $p \to \neg p$ **b)** $p \leftrightarrow \neg p$ **c)** $p \oplus (p \vee q)$

d) $(p \wedge q) \to (p \vee q)$ **e)** $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$ **f)** $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

1(iv). Let *p* and *q* be the propositions

p :You drive over 65 miles per hour.

q :You get a speeding ticket.

Write these propositions using *p* and *q* and logical connectives (including negations).

a) You do not drive over 65 miles per hour. ____________________

b) You drive over 65 miles per hour, but you do not get a speeding ticket. ____________________

c) You will get a speeding ticket if you drive over 65 miles per hour. ____________________

d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket. ____________________

e) Driving over 65 miles per hour is sufficient for getting a speeding ticket. ____________________

f) You get a speeding ticket, but you do not drive over 65 miles per hour. ____________________

g) Whenever you get a speeding ticket, you are driving over 65 miles per hour. ____________________

1(v). Write each of these statements in the form "if *p*, then *q*" in English.

a) It snows whenever the wind blows from the northeast.

b) The apple trees will bloom if it stays warm for a week.

c) That the Pistons win the championship implies that they beat the Lakers.

d) It is necessary to walk 8 miles to get to the top of Long's Peak.

e) To get tenure as a professor, it is sufficient to be world famous.

f) If you drive more than 400 miles, you will need to buy gasoline.

g) Your guarantee is good only if you bought your CD player less than 90 days ago.

h) Jan will go swimming unless the water is too cold.

2(i). Are the following statements logically equivalent? If yes, show the proof. If no, provide a counterexample. DO NOT use truth table.

a) $(p \wedge q)$ and p

b) $(p \to q) \to r$ and $p \to (q \to r)$

c) $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$

d) $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$

e) $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$

2(ii). The proposition p NAND q is true when either p or q, or both, are false. NAND is denoted by $p \mid q$.

a) Show that $p \mid q$ is logically equivalent to $\neg(p \wedge q)$

b) Show that $p \mid (q \mid r)$ and $(p \mid q) \mid r$ are not equivalent

Predicate and Quantifiers

3(i). Let Q(x) be the statement "x + 1 > 2x." If the domain consists of all integers, what are these truth values?

a) $Q(0)$

b) $\forall x\ Q(x)$

c) $Q(-1)$

d) $\exists x\ \neg Q(x)$

e) $\exists x\ Q(x)$

f) $\forall x\ \neg Q(x)$

3(ii). For each of these arguments determine whether the argument is correct or incorrect and explain why.

a) Every computer science major takes discrete mathematics. Natasha is taking discrete mathematics. Therefore, Natasha is a computer science major.

b) Everyone who eats granola every day is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

c) Quincy likes all action movies. Quincy likes the movie Eight Men Out. Therefore, Eight Men Out is an action movie.

d) All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps.

3(iii). Determine the truth value of each of these statements if the domain consists of all integer numbers. Solve suitable examples for each answer of your claim.

a) $\exists x\ (x^3 = -1)$

b) $\exists x\ (x^4 < x^2)$

c) $\forall x\ ((-x)^2 = x^2)$

d) $\forall x\ (2x > x)$

e) $\forall n\ \exists m\ (n^2 < m)$

f) $\exists n\ \forall m\ (n < m^2)$

g) $\forall n\ \exists m\ (n + m = 0)$

h) $\exists n\ \exists m\ (n^2 + m^2 = 6)$

i) $\exists n\ \exists m\ (n + m = 4 \wedge n - m = 2)$

j) $\forall n\ \forall m\ \exists p\ (p = (m + n)/2)$

Rules of Inference

4(i). For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

a) "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday."

b) "If I eat spicy foods, then I have strange dreams." "I have strange dreams if there is thunder while I sleep." "I did not have strange dreams."

c) "I am either clever or lucky." "I am not lucky." "If I am lucky, then I will win the lottery."

d) "Every computer science major has a personal computer." "Ralph does not have a personal computer." "Ann has a personal computer."

e) "What is good for corporations is good for the United States." "What is good for the United States is good for you." "What is good for corporations is for you to buy lots of stuff."

f) "All rodents gnaw their food." "Mice are rodents." "Rabbits do not gnaw their food." "Bats are not rodents."

4(ii). For each of these arguments, explain which rules of inference are used for each step.

a) “Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job.”

b) “Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program.”

c) “Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean.”

4(iii). Identify the error or errors in this argument that supposedly shows that if $\forall x\,(P(x) \vee Q(x))$ is true then $\forall x\, P(x) \vee \forall x\, Q(x)$ is true.

a) $\forall x\,(P(x) \vee Q(x))$ Premise

b) $P(c) \vee Q(c)$ Universal instantiation from (1)

c) $P(c)$ Simplification from (2)

d) $\forall x\, P(x)$ Universal generalization from (3)

e) $Q(c)$ Simplification from (2)

f) $\forall x\, Q(x)$ Universal generalization from (5)

g) $\forall x\, P(x) \vee \forall x\, Q(x)$ Conjunction from (4) and (6)

4(iv). What rule of inference is used in each of these arguments?

a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees' outside today. Therefore, the pollution is dangerous.

c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

4(v). Specify, rules of inference used in this following argument? "All men are mortal. Socrates is a man. Therefore, Socrates is mortal."

M (x) ="x is mortal"

N(x) ="x is a man"

4(vi). Show that the argument form with premises $(p \wedge t) \to (r \vee s)$, $q \to (u \wedge t)$, $u \to p$, and $\neg s$ and conclusion $q \to r$ is valid

Sets and Sets Operations

1(i). List the members of these sets.

a) $\{x \mid x$ is a real number such that $x^2 = 1\}$ ______________________________

b) $\{x \mid x$ is the square of an integer and $x < 100\}$ ______________________________

c) $\{x \mid x$ is an integer such that $x^2 = 2\}$ ______________________________

1(ii). Use set builder notation to give a description of each of these sets.

a) $\{0, 3, 6, 9, 12\}$ ______________________________

b) $\{-3, -2, -1, 0, 1, 2, 3\}$ ______________________________

1(iii). Determine whether each of these pairs of sets are equal.

a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}$, $\{5, 3, 1\}$ ______________________________

b) $\{\{1\}\}$, $\{1, \{1\}\}$ **c)** $\emptyset$, $\{\emptyset\}$ ______________________________

1(iv). For each of the following sets, determine whether 2 is an element of that set.

a) $\{x \in \mathbf{R} \mid x$ is an integer greater than $1\}$ ____________________

b) $\{x \in \mathbf{R} \mid x$ is the square of an integer$\}$ ____________________

c) $\{2,\{2\}\}$ ____________________

d) $\{\{2\},\{2,\{2\}\}\}$ ____________________

1(v). Determine whether these statements are true or false.

a) $\emptyset \in \{\emptyset\}$ _______ **b)** $\emptyset \in \{\emptyset, \{\emptyset\}\}$ _______ **c)** $\{\emptyset\} \in \{\emptyset\}$ _______ **d)** $\{\emptyset\} \in \{\{\emptyset\}\}$ _________

e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ _______ **f)** $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$ _______ **g)** $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$ __________

2(i). Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find
a) $A \times B \times C$.

b) $(A \times B) \times C$

d) $B \times B \times B$.

e) Explain why $A \times B \times C$ and $(A \times B) \times C$ are not the same.

2(ii). Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find
a) $A \cup B$.

b) $A \cap B$.

c) $A - B$.

d) $B - A$.

2(iii). Draw the Venn diagrams for each of these combinations of the sets A, B, and C.
a) $A \cap (B - C)$

b) $(A \cap B) \cup (A \cap C)$

c) $(A \cap B) \cup (A \cap C')$

3(iii). Let A, B, and C be sets. Show that

a) $(A \cap B) \subseteq A$.

b) $A \subseteq (A \cup B)$.

c) $A - B \subseteq A$. **d)** $A \cap (B - A) = \emptyset$.

d) $(A \cup B) \subseteq (A \cup B \cup C)$.

e) $(A \cap B \cap C) \subseteq (A \cap B)$.

f) $(A - B) - C \subseteq A - C$.

g) $(A - C) \cap (C - B) = \emptyset$.

h) $(B - A) \cup (C - A) = (B \cup C) - A$.

3(ii). Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i,

a) $A_i = \{i, i + 1, i + 2, \ldots\}$.

b) $A_i = \{0, i\}$.

c) $A_i = (0, i)$, that is, the set of real numbers x with $0 < x < i$.

d) $A_i = (i, \infty)$, that is, the set of real numbers x with $x > i$.

3(iii). Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Express each of these sets with bit strings where the i^{th} bit in the string is 1 if i is in the set and 0 otherwise.

a) $\{3, 4, 5\}$

b) $\{1, 3, 6, 10\}$

c) $\{2, 3, 4, 7, 8, 9\}$