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FAST- National University of Computer & Emerging Sciences, Karachi.
Department of Computer Science
Quiz- II, fall 2018

Course Code: CS 211	Course Name: Discrete Structures
Instructors: Mr. Shoaib Raza	
Student Roll No:	Section:

Time Allowed: 50 minutes. Maximum Points: 25 points

Question # 1: (03 points)

Let f: Z x Z → Z be defined by *f* (m, n) = 2m + n. Is the function *f* an injection? Is the function *f* a surjection? Prove it.

Solution:

Let f: Z x Z → Z be defined by *f* (m, n) = 2m+n. The function is not an injection since (0, 2) and (1, 0) both map to 2.

Question # 2: (03 points)

Express gcd (450, 120) as a linear combination of 120 and 450.

Solution:

gcd (450, 120): $30 = 120 \cdot 4 + 450 \cdot (-1)$

Question # 3: (03 points)

Consider the following relation on the set of positive integers. R= {(x, y) | x and y have the same prime divisors} Prove or disprove that the above relation is an Equivalence relation.

Solution:

- R is reflexive since x and x have the same prime divisors for every positive integer x, so (x, x) ∈ R for all x.
- R is symmetric since x and y have the same prime divisors if and only if y and x have the same prime divisors.
- R is transitive, for if x and y have the same prime divisors and y and z have the same prime divisors, then x and z have the same prime divisors. Hence the above relation is equivalence relation.

Question # 4: (03 points)

Encrypt the message NEED HELP by translating the letters into numbers (A=0, B=1. . . Z=25), applying the encryption function *f* (p) = (3p + 7) mod 26, and then translating the numbers back into letters.

Solution: Encrypted form: UTTQ CTOA

Question # 5: (03 points)

Solve the linear congruence 15x ≡ 31 (mod 47).

Solution: x= 24

Question # 6: (03 points)

Determine a relation on {1, 2, 3} that is reflexive and transitive, but not symmetric.

Solution: {(1, 1), (2, 2), (3, 3), (1, 2)}.

Question # 7: (03 points)

Suppose that a computer has only the memory locations 0, 1, 2. . . 19. Use the hashing function h where h(x) = (x + 5) mod 20 to determine the memory locations in which 57, 32, and 97 are stored.

Solution: 57 on 2, 32 on 17 and 97 on 3.

Question # 8: (04 points)

An old woman goes to market and a horse steps on her basket and crushes the eggs. The horse rider offers to pay for the damages and asks her how many eggs she had brought. She does not remember the exact number, but when she had taken them out three at a time, there were 1 egg left. When she took them four at a time, there was one egg left. When she had taken them out five at a time, then too there was one egg left and when she took them seven at a time, there was no egg left. What is the smallest number of eggs she could have had?

Solution:

We will follow the notation used in the proof of the Chinese remainder theorem.
We have $m=m_1 * m_2 * m_3 * m_4 = 420$.
Also, by simple inspection we see that:
$y_1 = 2$ is an inverse for $M_1 = 140$ modulo 3,
$y_2 = 1$ is an inverse for $M_2 = 105$ modulo 4,
$y_3 = 4$ is an inverse for $M_3 = 84$ modulo 5 and
$Y_4 = 2$ is an inverse for $M_3 = 60$ modulo 7.
The solutions to the system are then all numbers x such that
$x = a_1M_1y_1 + a_2M_2y_2 + a_3M_3y_3 = (1 * 140* 2) + (1* 105* 1) + (1* 84* 4) + (0*62*2) = 721 \pmod{420} = 301$.
She could have 301 eggs.

BEST OF LUCK!