

Markov Chain and Absorbing State Analysis

1. Define Markov Chain. Classify the states in Markov Chain and give example for each state.
2. Solve the following
 - (a) Example 17.1-3
 - (b) Problem set 17.1 A Q 1, 2
 - (c) Problem set 17.6 A Q 2
 - (d) According to Kemery, Snell, and Thompson, the Land of Oz is blessed by many things, but not by good weather. They never have two nice days in a row. If they have a nice day, they are just as likely to have snow as rain the next day. If they have snow or rain, they have an even chance of having the same the next day. If there is change from snow or rain, only half of the time is this a change to a nice day. With this information form a Markov chain and classify the states in it. Also construct the graphical representation of it.
 - (e) Each time a certain horse runs in a three-horse race, he has probability $1/2$ of winning, $1/4$ of coming in second, and $1/4$ of coming in third, independent of the outcome of any previous race. Form a Markov chain and classify the states in it. Also construct the graphical representation of it.
 - (f) In the Dark Ages, Harvard, Dartmouth, and Yale admitted only male students. Assume that, at that time, 80 percent of the sons of Harvard men went to Harvard and the rest went to Yale, 40 percent of the sons of Yale men went to Yale, and the rest split evenly between Harvard and Dartmouth; and of the sons of Dartmouth men, 70 percent went to Dartmouth, 20 percent to Harvard, and 10 percent to Yale. Form a Markov chain and classify the states in it. Also construct the graphical representation of it.
 - (g) A man walks along a four-block stretch of Park Avenue. If he is at corner 1, 2, or 3, then he walks to the left or right with equal probability. He continues until he reaches corner 4, which is a bar, or corner 0, which is his home. If he reaches either home or the bar, he stays there.
 - i. Express the problem graphically and in Markov Chain.
 - ii. Classify the states in the Markov chain.
 - iii. Find expected probability and expected duration for each state.
 - (h) For the following given matrices,
 - i. Express the problem graphically.
 - ii. Classify the states in the Markov chain.
 - iii. Find expected probability and expected duration for each state

(a)

	A	B	C	D	E
A	0	1	0	0	0
B	$1/4$	0	$3/4$	0	0
C	0	$1/2$	0	$1/2$	0
D	0	0	$3/4$	0	$1/4$
E	0	0	0	1	0

What is the expected duration and expected probability of state D to switch to an absorbing state?

(b)

	A	B	C	D
A	1/2	1/2	0	0
B	1/2	1/2	0	0
C	0	0	1/4	3/4
D	0	0	0	1

What is the expected duration and expected probability of state C to switch to an absorbing state?

(c)

	0	1	2	3
0	1	0	0	0
1	0.7	0.2	0.1	0
2	0.5	0.1	0.2	0.2
3	0	0	0	1

What is the expected duration and expected probability of state 1 to switch to an absorbing state?