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Comparison of Transient Performance in the Control of Soft Tissue Grasping

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***Abstract*— In robot-assisted surgery, surgical tools interact with tissues that have nonlinear mechanical properties. For situations where a pre-specified trajectory of tool positions (or applied forces) is desired, there are many controller designs that might be used. Four candidates are comparatively evaluated here, via computer simulation involving a nonlinear model of soft tissue behavior during grasping actions. The parameters for this model were obtained experimentally (in earlier work). The four candidate controllers are: (1) a well-tuned PID controller; (2) feedback linearization in combination with deadbeat control; (3) an optimal open-loop control law obtained via minimization of a quadratic cost function; and (4) a model predictive controller. Simulation trials are used to compare the transient performance of these candidate controllers under different assumptions regarding input and output noises. The conditions where each of the candidates is best are characterized.**

***Index Terms*— Robot-Assisted Surgery, Transient Control, Trajectory Following, Soft Tissue Grasping, PID control, Feedback Linearization, Deadbeat Control, Model Predictive Control.**

I. Introduction

Commercially available robot-assisted surgery systems are essentially open loop devices. The surgeon provides the control function [1]. It might be useful to have the system incorporate feedback control to improve the precision and accuracy of end effector positions and applied forces, in the face of disturbances (including respiratory and other patient motions). A more ambitious goal would be the automatic execution of tasks such as grasping, cutting and suturing.

There are several factors that complicate the application of automatic control to robotic surgery. These include:

a) The nonlinear (and time-varying) properties of soft tissues;
b) Sensor, actuator and other system noises and disturbances; and
c) The combination of primitive operations to accomplish more complex surgical actions is a hybrid system (discrete events plus continuous dynamics) that incorporates controlled, but partially random transitions between different modes.

In this study, we focus on one specific primitive operation – the grasping of soft tissue, where a trajectory of desired tool positions is specified. Four different control architectures are evaluated via computer simulation using a nonlinear model of the tissue. The parameters for this model are based upon values obtained in earlier work [2]. The four candidate controllers are: (1) a well-tuned PID controller; (2) feedback linearization in combination with deadbeat control; (3) an optimal open-loop control law obtained via minimization of a quadratic cost function; and (4) a model predictive controller. We examine which of four candidate controllers results in the the best transient performance when a somewhat realistic model of the load and a reference trajectory that is appropriate for actual surgery is used, and where constraints on control effort (reflecting motor limitations) are imposed.

In section II the soft tissue model and the desired grasping trajectory are described. In section III a metric is proposed to evaluate the transient trajectory tracking performance. In section IV the four candidate control methods are described. In section V simulation results are presented and compared.

II. Background

A. Mathematical Model of Soft Tissue During Grasping

A wide variety of living soft tissues have an exponential-like biomechanical response to applied force [3]. Fig. 1 depicts a model for soft tissue under the operation of grasping (*i. e.*, squeezing) by a mechanical device. There is a static exponential relation between the force and position. This nonlinear mass-spring-damper model is described by the following differential equation [4]:

$$u = m\,\frac{d^2p}{dt^2} + d\,\frac{dp}{dt} + \alpha\,(e^{\beta\,p} - 1) \tag{1}$$

where

u	is the force applied to tissue;
p	is the position of robot end effector, incontact with the tissue;
m	is the lumped mass of the robot end effector and tissue;
d	is the viscosity of tissue;
$\alpha\,,\beta$	are parameters related to the stiffness of the tissue.

In the above model, it is assumed the that robot is a mass attached to the tissue. All of the parameters are assumed to be known. There are many possible refinements to this type of soft tissue model, reviewed in [3]; but this is a single model which captures several essential properties.

Eq. (1) can be formulated as the following single input, single output system:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{2}$$

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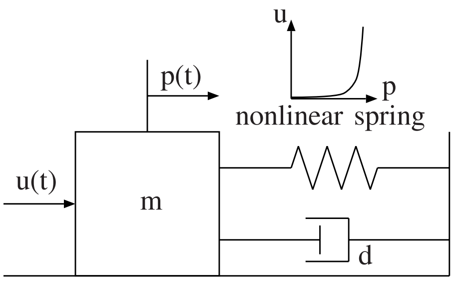

Fig. 1. Nonlinear model of soft tissue during grasping/squeezing

where

$$f(x) = \begin{pmatrix} x_2 \\ -\frac{d}{m}\, x_2 - \frac{\alpha}{m}\, e^{\beta\, x_1} + \frac{\alpha}{m} \end{pmatrix}$$
$$g(x) = \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix}$$
$$h(x) = x_1, \qquad (3)$$

with system state $x = (x_1, x_2)$, where

x_1 is the position of robot end effector and soft tissue;
x_2 is the velocity of robot end effector and soft tissue.

B. Desired Grasping Trajectory

In prior work [2], [5], experimental measurements of position and forces were obtained during approximately 30 hours of animal abdominal surgery. In these experiments, the mean grasp time was 3 seconds, and 90% of the grasps were of 10 seconds or less in duration.

For the work reported here, we represent the desired reference trajectory as an increasing ramp displacement into the nonlinear compressive tissue, followed by a holding of this displacement, such that the steady state grasp position (*i. e.* $t \in [1, 9]$ in Fig. 2) requires 50% of peak control output, and then a ramping of displacement back to the initial position. The peak value is chosen so as to scale the control effort to realistic design constraints of surgical instruments.

Fig. 2 illustrates the soft tissue displacement-force relationship during the grasping operation and shows the desired ramp-hold-ramp trajectory, which the candidate controllers seek to make the system follow. This trajectory has smooth corners appropriate to the low-pass behavior of the the robotic end effector.

III. Performance Metric

The control effort is applied to the soft-tissue through the robot arm, so as to make the displacement follow the desired reference trajectory. There are several performance goals:

a) The control efforts should be small;
b) The state (*i. e.* position and velocity) deviations from the trajector should be small;
c) Only a finite time horizon is of interest.

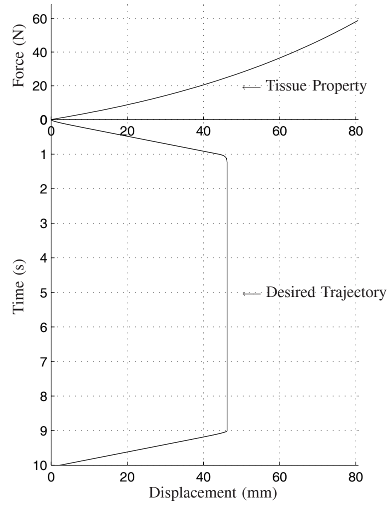

Fig. 2. Soft tissue displacement-force relationship (top trace) and the desired ramp-hold-ramp reference trajectory (bottom trace)

These objectives can be represented by the following quadratic cost function:

$$J = \int_{t_0}^{t_f} [(x - x_r)^T Q (x - x_r) + (u - u_r)^T R (u - u_r)] dt \qquad (4)$$

where Q is a positive semi-definite weighting matrix for state deviations, R is a positive definite weighting matrix for control effort, and x_r represents the desired position and velocity. Here u_r is the steady-state input that brings the system to a constant displacement of x_{1r}:

$$u_r = \alpha\, (e^{\beta\, x_{1r}} - 1). \qquad (5)$$

IV. Candidate Control Methods

Four different control methods are investigated. One is an open loop controller, obtained by numerically solving the optimization problem of minimizing the given cost function (using Pontryagin's minimum principle). Two methods are feedback controllers. One (PID control) requires only minimal knowlege of the system; the other (feedback linearization combined with deadbeat control) explicitly uses the nonlinear model of the load. The fourth method (Model Predictive Control) applies an open loop control for a fixed intervals, updated with periodic measurements of the state.

A. PID Control

For PID control, tuning is accomplished by an iterative search of the controller parameter space using particle swarm optimization (PSO) [6]. In each iterative searching step, the system (with the PID controller included) is simulated in the presence of a small amount of input and output noise. An alternative tuning method would be an exhaustive search of the parameter space, but this requires a much greater computational effort. Comparison of both methods (over a limited search domain) yields almost equivalent tuning parameter values.

B. Feedback Linearization Combined with Deadbeat Control

To achieve the desired dynamics between the reference input and system output, a control law can be built by combining the ideas of feedback linearization and deadbeat control [7]. Using methods of nonlinear control [8]–[10], the following *exact feedback linearization* control law is obtained:

$$u = mv + dx_2 + \alpha e^{\beta x_1} - \alpha. \tag{6}$$

This control transforms the nonlinear system (3) into a linear system, by canceling all of the nonlinear terms. The result is the following relationship between the system reference input v and output $y = x_1$:

$$\ddot{y} = v. \tag{7}$$

The derivation of this control law appears in the Appendix. Deadbeat control is then applied to this linear system. The combined control law for the system (3) is:

$$u = m(a_1 x_1 + a_2 x_2 + a_3 r) + dx_2 + \alpha e^{\beta x_1} - \alpha \tag{8}$$

which yields a 2$^{\text{nd}}$ order linear system,

$$\begin{aligned} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ a_1 & a_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ a_3 \end{pmatrix} r(t) \\ y &= x_1 \end{aligned} \tag{9}$$

where $r(t)$ is the ramp-hold-ramp trajectory depicted in the Fig. 2.

Eq. (9) describes a SISO system with the following transfer function:

$$T(s) = \frac{Y}{R} = \frac{a_3}{s^2 - a_2 s - a_1} \tag{10}$$

Designing a second order deadbeat controller as in [7], the following relationships are obtained:

$$\begin{aligned} a_1 &= -\left(\frac{4.82}{t_s}\right)^2 \\ a_2 &= -1.82 \times \frac{4.82}{t_s} \\ a_3 &= -a_1 \end{aligned} \tag{11}$$

where t_s is the desired settling time a step response.

C. Optimal Open Loop Control

The optimal trajectory tracking problem for the soft-tissue grasping system is mathematically formulated as:

$$\begin{aligned} \min_u \quad & J \\ s.t. \quad & \dot{x} = f(x) + g(x)u \end{aligned} \tag{12}$$

where J is described in Eq. (4) and the system given by Eq. (3). The Hamiltonian [11] of this nonlinear optimal control problem is:

$$\begin{aligned} H(x,u,\lambda) = {} & (x - x_r)^T Q (x - x_r) + (u - u_r)^T R (u - u_r) \\ & + \lambda^T (f(x) + g(x)u) \end{aligned} \tag{13}$$

Here Q = diag[q_1 0; 0 q_2] and R = ρ, where q_1, q_2, ρ are constants. This method can be extended to the case where Q and R are time varying. The Hamiltonian is thus

$$\begin{aligned} H(x,u,\lambda) = {} & q_1(x_1 - x_{1r})^2 + q_2(x_2 - x_{2r})^2 + \rho(u - u_r)^2 \\ & + \lambda_1 x_2 + \lambda_2\left(-\frac{d}{m}x_2 - \frac{\alpha}{m}e^{\beta x_1} + \frac{\alpha}{m} + \frac{u}{m}\right) \end{aligned} \tag{14}$$

Based on Pontryagin's minimum principle, the necessary conditions for optimality are given by the differential equations,

$$\frac{\partial H}{\partial \lambda} = \dot{x} = f(x) + g(x)u \tag{15}$$

$$\frac{\partial H}{\partial x} = -\dot{\lambda} \tag{16}$$

$$\frac{\partial H}{\partial u} = 0 \tag{17}$$

with boundary conditions,

$$\begin{aligned} x(t_0) &= x_0 \\ x(t_f) &= x_f \end{aligned} \tag{18}$$

In detail,

$$\dot{x}_1 = x_2 \tag{19}$$

$$\dot{x}_2 = -\frac{d}{m}x_2 - \frac{\alpha}{m}e^{\beta x_1} + \frac{\alpha}{m} + \frac{u}{m} \tag{20}$$

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial x_1} = -2q_1(x_1 - x_{1r}) + \frac{\alpha\beta}{m}\lambda_2 e^{\beta x_1} \tag{21}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial x_2} = -2q_2(x_2 - x_{2r}) - \lambda_1 + \frac{d}{m}\lambda_2 \tag{22}$$

$$0 = \frac{\partial H}{\partial u} = 2\rho(u - u_r) + \frac{\lambda_2}{m} \tag{23}$$

From the above, the optimal control problem can be numerically solved by collocation methods [12]. In this derivation of the optimal open loop controller, no constraint on the input values is imposed.

D. Model Predictive Control

In practice, the open loop approach may not be practical for robot-assisted surgery because the entire reference trajectory might not be available *a priori*, and the computation time for obtaining the control inputs for a long time horizon might be too high for real-time use. Model Predictive Control (MPC) is a suboptimal and practical implementation of the open loop optimal control problem [13], [14]. The problem is successively solved for short intervals of time. Thus the requirement for knowledge of the entire reference trajectory is relaxed.

For each controller time interval, the MPC solves a nonlinear optimal control problem which has the same Hamiltonian as in Eq. (13) and the same differential algebraic equations (but with different time horizons and boundary values). Suppose that the current time is t_c, the time horizon for this step is [t_c, t_c+t_p]. The initial value $x(t_c)$ is fixed, and can be acquired by either measurements or calculations. The final value $x(t_c + t_p)$ is floating. Without loss of generality, for a terminal cost function $\Phi(x)$ Pontryagin's minimum principle gives the following boundary conditions,

$$\lambda(t_c + t_p) = \frac{\partial \Phi}{\partial x}|_{x(t_c+t_p)} \tag{24}$$

With above boundary conditions and the differential algebraic equations given in section IV-C, for each time interval the optimization problem can be numerically solved. The prediction horizon t_p and the applied control horizon t_a can be adjusted in the MPC algorithm, which allows modification to different situations.

V. SIMULATIONS AND COMPARISONS

The four candidate controllers are tested in computer simulation. In each case, the controller is applied to the nonlinear load, and the resulting performance is evaluated using the performance metric specified by Eq. (4). We choose $t_0 = 0s$ and $t_f = 20s$, which is an appropriate duration for a basic grasping task during robotic surgery. For the soft-tissue model, we choose the following parameter values: $m = 2kg$, $d = 0.5kg/s$, $\alpha = 25kg/s^2$ and $\beta = 15$. The tissue stiffness parameters α and β used are chosen based upon experimental results obtained in our laboratory for the liver [2]. The cost function parameters are chosen to be Q = I (the identity matrix) and R = $2.1456 \cdot 10^{-6}$. These values keep the maximum deviations of position, velocity and control equally weighted. The maximum control input is $50N$, and the corresponding maximum position and velocity are $73.24mm$ and $73.24mm/s$, so

$$\rho = (\frac{73.24 \cdot 10^{-3}}{50})^2 = 2.1456 \cdot 10^{-6}. \tag{25}$$

The system initial conditions are set to $x(0) = [0, 0]$ and the terminal conditions are $x(20) = [0, 0]$. The PID control parameters are $K_p = 720$, $K_i = 3100$ and $K_d = 8$.

Sensor and actuator noises are inevitably present in a soft tissue manipulation system. In the absence of specific knowledge about these noises, in the following simulations the zereo mean noises are chosen to be additive, white, Gaussian and uncorrelated with each other. Different levels of noise variance, as a percentage of the maximum force (for the input) or the maximum position (for the output) are examined. The desired settling time t_s for the feedback linearization controller with deadbeat control is set to 0.1 second; the prediction horizon t_p for the MPC and the applied control horizon t_a are both set to 1 second. These values are arbitarily chosen.

A. No Noise

As expected, in the ideal *no noise* case, the open loop optimal controller achieves the best performance (total cost is 6.931×10^{-7}). Model predictive control is slightly worse (total cost is 7.333×10^{-7}), followed by feedback linearization with deadbeat control (total cost is 9.145×10^{-5}) and PID control is the worst (total cost is 5.850×10^{-4}). All but the PID controller track the desired position and velocity fairly well. However the PID controller takes about 1 second to reach the constant values and shows about 50% velocity oscillation during the ramping phases.

B. With Added Input Noise

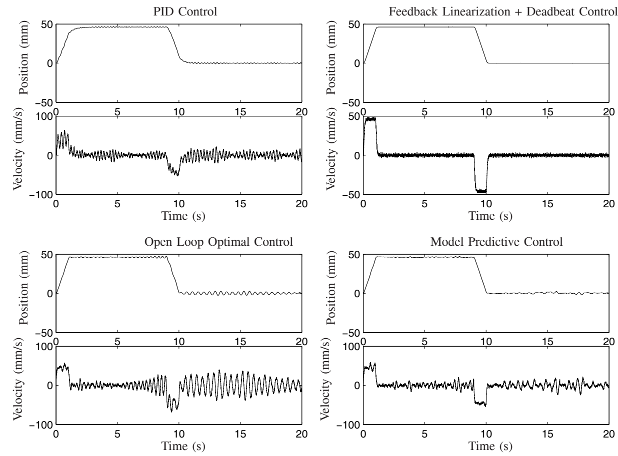

Fig. 3. Performance of candidate controllers with 5% added input noise. Total costs are PID = 1.678×10^{-3}, FLDB = 3.769×10^{-4}, OLOPT = 4.228×10^{-3}, and MPC = 1.033×10^{-3}.

Fig. 3 depicts the performance of the candidate controllers when we add 5% input noise. The feedback linearization with deadbeat control is the best and the open loop optimal control is the worst, in terms of the total cost. All but the feedback linearization with deadbeat control show significant oscillations in the velocity.

C. With Added Output Noise

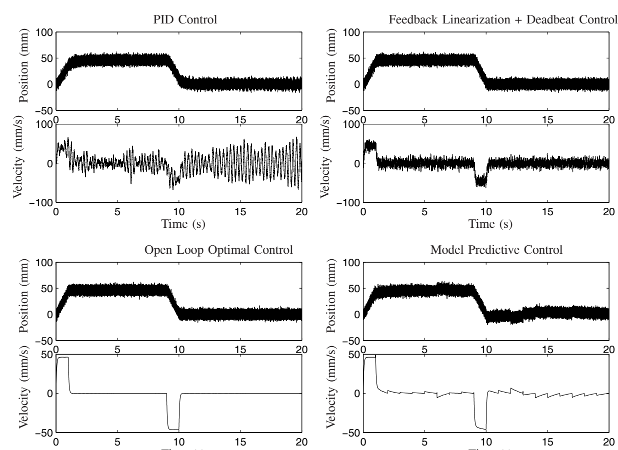

Fig. 4. Performance of candidate controllers with 5% added output noise. Total costs are PID = 2.910×10^{-2}, FLDB = 1.050×10^{-2}, OLOPT = 2.686×10^{-4}, and MPC = 6.165×10^{-4}.

Fig. 4 shows the performance of the candidate controllers when we add 5% output noise. The open loop optimal control is the best, since it has no feedback mechanism that can be affected by the output noise. However, the impacts of the noise clearly appear in the position tracking for all of the controllers. At this noise level, instability of velocity shows up using the PID control.

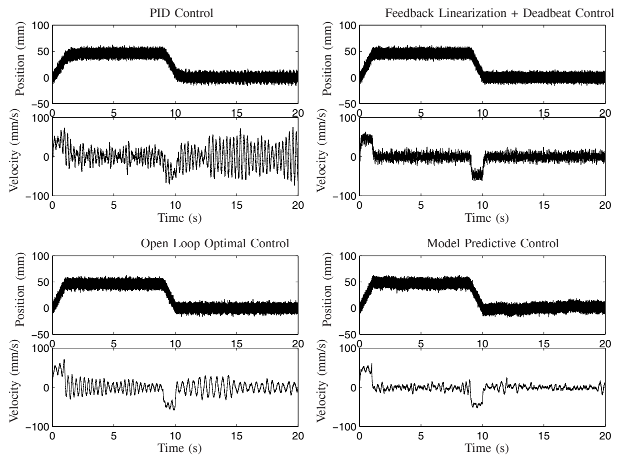

Fig. 5. Performance of candidate controllers with 5% added input and output noise. Total costs are PID $= 3.140\times10^{-2}$, FLDB $= 1.080\times10^{-2}$, OLOPT $= 3.635 \times 10^{-3}$, and MPC $= 1.334 \times 10^{-3}$.

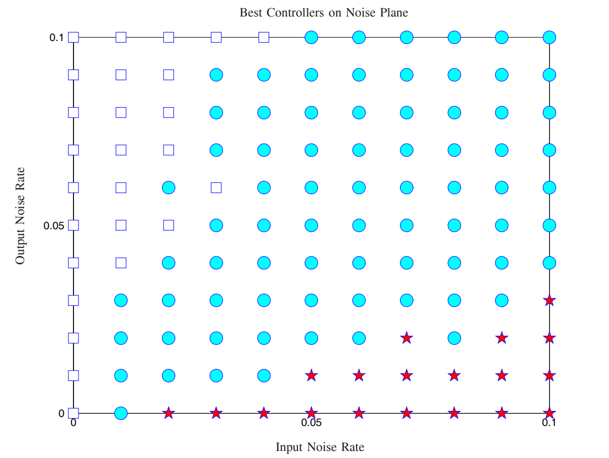

Fig. 6. The best control method on the input/output noise plane; □ area is open-loop optimal control; ● area is MPC (open-loop feedback control); ★ area is feedback linearization with deadbeat control (closed-loop control).

D. With Both Input and Output Noise Added

Fig. 5 depicts the performance of the candidate controllers when there is both 5% added input and output noise. The MPC has the best overall performance. The impacts of the input and output noises appear in both the position tracking and the velocity tracking for all the controllers.

E. Comparisons and Conclusions

The relative performance of the different controllers will depend upon the amount of noise present, the specific nonlinear dynamics of the load, and the tuning parameters of the controller. We have found that there appears to be a regular pattern to the relative performance of the different controllers. In Fig. 6, the best controller is indicated at different levels of input and output noise.

Without noise, the open-loop optimal control performs the best. With significant input noise, the closed-loop controllers are best since the feedback mechanism can mitigate the noise impacts. There is an in-between region, in the input noise *vs.* output noise plane, where the MPC is best. However because of the random nature of the added noise, for any given trial any of the candidates could be the best. Thus there are outlier points in the figure and the exact boundary of the regions changes slightly for each simulation run.

By tuning the prediction horizon t_p and the applied control horizon t_a, the performance of MPC can be systematically improved. By enlarging t_p, the performance of MPC approaches that of the open loop optimal control; by reducing t_a, the frequency of checking the newest system states has been increased and the feedback loop (and hence noise resistance) is strengthened. However this is achieved at a cost of increased computations.

These comparative simulation trials demonstrate that trajectory tracking control is potentially feasible for the grasping/squeezing of soft tissue application. MPC performed best for the widest range of noise configurations. If there are significant time dependent changes to the model parameters, then real-time identification of them might be required to achieve good transient performance. This issue is not addressed in the simulations reported here. Future steps in this work will include experimental trials (using butcher products at first), and extension of the controllers to take uncertainties of the soft tissue model into account, using nonlinear system identification methods.

APPENDIX

Two basic concepts in the analysis of a nonlinear system are *relative degree* and *zero dynamics* [8]–[10]. Both the relative degree and zero dynamics are intrinsic properties of the nonlinear system. They cannot be changed by using different feedback linearization inputs.

In a linear system, relative degree refers to the polynomial order difference between the denominator and numerator of the system transfer function (*i. e.*, the excess of poles over zeros). Similarly, in a nonlinear system relative degree is the number of differentiations of the output needed to acquire a linear integral relationship between system input u and output y.

For the SISO nonlinear system described by Eq. (2), the relative degree is γ at $x_0 \in U$ if

$$\begin{aligned} L_g L_f^i h(x) &\equiv 0 \qquad \forall x \in U, i = 0, ..., \gamma - 2 \\ L_g L_f^{\gamma-1} h(x_0) &\neq 0 \end{aligned} \tag{26}$$

Here, $L_f^i h(x)$ is the i^{th} order Lie derivative of $h(x)$ with respect to $f(x)$. For system described by Eq. (3), the relative degree is $\gamma = 2$ because

$$L_f h(x) = \frac{\partial h}{\partial x} f(x) = x_2 \tag{27}$$

$$L_g h(x) = \frac{\partial h}{\partial x} g(x) = 0 \tag{28}$$

$$L_g L_f h(x) = \frac{\partial L_f h}{\partial x} g(x) = \frac{1}{m} \neq 0. \tag{29}$$

The relative degree $\gamma = 2$ is confirmed by Eqs. (28, 29). This is true $\forall x \in R$.

Zero dynamics in nonlinear systems is a generalization of the concept of zeros in linear systems. Zero dynamics can be defined as the internal dynamics when the system output is kept at zero by the feedback linearization input. The zero output indicates that the zero dynamics are unobservable and critical for overall system stability.

The soft-tissue grasp system is a nonlinear 2^{nd} order system. It has a relative degree of 2, which is the same as the order of the system dynamics. So the nonlinear system is fully linearizable by the feedback linearization and there are no zero dyamics.

For soft-tissue grasping system (3), applying the feedback linearization control law,

$$u = \frac{v - L_f^2 h(x)}{L_g L_f h(x)} = mv + dx_2 + \alpha e^{\beta x_1} - \alpha \tag{30}$$

yields a linear integral relation between the system reference input v and output y,

$$\ddot{y} = v. \tag{31}$$

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