**The Isolated Human Lens is Unaccommodated:**

**Undermining the Basis for Helmholtz’s Theory of Accommodation**

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The generally accepted Helmholtz theory predicts that during accommodation the ciliary muscle contracts causing all the zonules to relax and the lens to round-up with an increase in central optical power.1 Therefore, fundamental to the foundation of the Helmholtz theory is that the isolated lens with no zonular tension must have maximum optical power.

Interestingly, Helmholtz clinically examined the right eyes of three females aged 25 to 30 years with an ophthalmometer and found the anterior lens radii of curvatures for far vision of 11.9 mm, 8.8 mm and 10.4 mm and stated that two dead lenses with radii of 10.2 mm and 8.9 mm “*agrees well with measurements on the living eyes*.”1 Consistent with this finding, Stadfeldt2 in 1896 measured *in vivo* the anterior radius of curvature of 11 unaccommodated human crystalline lenses using an ophthalmophakometer. He found that the mean central radius of curvature was 10.5 mm. After removal of 6 of these crystalline lenses, the mean central anterior radius did not decrease, but actually increased to 11.4 mm. Using a topographer, Schachar3 similarly found that the mean ± SD radius of curvature of *in vitro* lenses from donors with a mean age of 33.6 years was 10.5 ± 0.6 mm. These experiments used the same basic reliable technique as keratometry to determine radius of curvature by measuring the size of reflections from the central optical zone of the lens.

During accommodation under normal lighting conditions, the pupil generally constricts and for accommodation ≥ 6 diopters the clinically measured pupil has a diameter less than 4 mm.4 Since the true pupil size at the surface of the lens is 13% smaller,5 the functional optical zone of the anterior lens surface is < 3.5 mm during ≥ 6 diopters of accommodation. Since the lens is an oblate spheroid with its minor axis coincident with its optical axis, radius of curvature decreases with distance from the optical axis6 as observed *in vivo* when unaccommodated.7 However, during accommodation the peripheral lens surface flattens at distances > 2 mm from the optic axis causing spherical aberration to move in the negative direction. With accommodation ≥ 4 diopters spherical aberration becomes negative making the peripheral radius of curvature greater than the central radius of curvature.8

Urs et al. 9,10 performed detailed shadow-photogrammetric measurements of isolated human lenses and developed an age-related Fourier equation for the whole lens sagittal profile. Since the Fourier equation is a periodic function, the local radius of curvature is markedly variable. This makes the surface of the lens irregular, which is not consistent with the optical qualities of the human lens.11

To provide a more realistic equation for the lens profile, the curve fit, bending energy, waviness and radius of curvature variance of the Fourier,9 Chien11, Forbes12 (a polynomial specific for aspheric lenses) and ellipse equations were evaluated.

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Using each of these equations, the isolated lens accommodative state was determined by comparing the functional central 3 mm diameter optical zone lens power to reported *in vivo* measurements.

**Methods**

The x-y coordinates of fresh isolated lens sagittal profiles from a miniature digital shadow-photogrammetric system obtained within a median of 26.8 hours postmortem from young donors aged 20 to 30 years were kindly supplied by R.C. Augusteyn and A Mohamed.13 Using the x-y coordinates, the coefficients for the Chien and Forbes curves were calculated. The radius of curvature (ROC),difference of the least squares fit, bending energy (E),14,15 waviness (W),16-18 and radius of curvature variance (RoCV)19 of the Chien,11 Fourier,9 Forbes12 and ellipse were compared for the central 6 mm diameter optical zone by applying the following equations:

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| --- | --- |
|  | () |
|  | () |
|  | (7) |
|  | (8) |

where,

*s = arc length, k* = curvature, and = mean RoC.

In addition, the anterior and posterior RoCs and the optical power (COP) were calculated for central optical zone diameters of 1 mm, 2 mm, 3 mm, 4 mm, and 6 mm with the following formula:20,21

|  |  |
| --- | --- |
|  | (9) |

where,

*na* = 1.336 and *nl* = 1.42 are the indices of refractions of the aqueous humor/vitreous and lens, respectively; *ra* and *rp* , which is negative, are the anterior and posterior central lens surface, RoCs respectively; and *t* = central lens thickness.­­

**Results**

The curve parameters and optical powers of the isolated lenses from donors aged 20 to 30 years are given in Tables 1a and 1b. Plots of the fits, RoCs, and first derivative of curvature for the anterior and posterior surfaces for the four equations are shown in Figs. 1-3.

To assess the accommodative state of fresh isolated lenses, their RoCs and COPs at the 3 mm optical zone were compared to *in vivo* measurements using the following formulas:22-24

|  |  |
| --- | --- |
|  | (10) |
|  | (11) |
|  | () |

where,

*rav and rpv = in vivo* anterior and posterior lens RoCs, respectively, *tv*= *in vivo* central lens thickness, *a* = age (years), and *AA* = accommodative amplitude (diopters).

The lens donors aged 20 to 30 years had lower limit accommodative amplitude range of 9.7 to 7.1 diopters, respectively.25 If their isolated lens was in the maximumly accommodated state, the central anterior and posterior RoCs, and COP (Eqs. 9-12) for a 3 mm diameter optical zone should be for the 20 y/o ≤ 5.84mm, ≥ -4.70 mm, ≥ 31.6 diopters and for the 30 y/o ≤ 6.86 mm, ≥ -4.92 mm, ≥ 28.7 diopters, respectively, Table 2.

**Table 1a. Comparison of Isolated Lens Curve Parameters and Optical Powers**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Parameter** | | **Chien**  **Fourier** | | | | | | | | | | **Fourier** | | | | | | | | | |
| **Age** (years) | | **20** | **21** | **22** | **24** | **25** | **26** | **27** | **28** | **29** | **30­­­­** | **20** | **21** | **22** | **24** | **25** | **26** | **27** | **28** | **29** | **30­­­­** |
| **Fit** (nm) | **A** | **406.33** |  |  |  |  |  |  |  |  |  | **1.0431e6** |  |  |  |  |  |  |  |  |  |
| **P** | **35.3053** |  |  |  |  |  |  |  |  |  | **6.24e4** |  |  |  |  |  |  |  |  |  |
| **E** | **A** | **0.0309** |  |  |  |  |  |  |  |  |  | **0.0323** |  |  |  |  |  |  |  |  |  |
| **P** | **0.0475** |  |  |  |  |  |  |  |  |  | **0.0721** |  |  |  |  |  |  |  |  |  |
| **W** | **A** | **0.0535** |  |  |  |  |  |  |  |  |  | **0.2757** |  |  |  |  |  |  |  |  |  |
| **P** | **0.0230** |  |  |  |  |  |  |  |  |  | **2.3039** |  |  |  |  |  |  |  |  |  |
| **RoCV** () | **A** | **4.9812** |  |  |  |  |  |  |  |  |  | **6.4460** |  |  |  |  |  |  |  |  |  |
| **P** | **0.1467** |  |  |  |  |  |  |  |  |  | **116.8911** |  |  |  |  |  |  |  |  |  |
| **t** (mm) | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| *Central Optical Zone Diameter*(mm) | | | | | | | | | | | | | | | | | | | | | |
| **ARoC** (mm) | **1** | **11.4408** |  |  |  |  |  |  |  |  |  | **7.0934** |  |  |  |  |  |  |  |  |  |
| **2** | **10.6318** |  |  |  |  |  |  |  |  |  | **6.4643** |  |  |  |  |  |  |  |  |  |
| **3** | **9.4247** |  |  |  |  |  |  |  |  |  | **9.4466** |  |  |  |  |  |  |  |  |  |
| **4** | **7.9762** |  |  |  |  |  |  |  |  |  | **10.6643** |  |  |  |  |  |  |  |  |  |
| **6** | **4.8641** |  |  |  |  |  |  |  |  |  | **6.0067** |  |  |  |  |  |  |  |  |  |
| **PRoC** (mm) | **1** | **6.5320** |  |  |  |  |  |  |  |  |  | **6.6775** |  |  |  |  |  |  |  |  |  |
| **2** | **6.6881** |  |  |  |  |  |  |  |  |  | **2.9592** |  |  |  |  |  |  |  |  |  |
| **3** | **6.8800** |  |  |  |  |  |  |  |  |  | **3.0850** |  |  |  |  |  |  |  |  |  |
| **4** | **6.9460** |  |  |  |  |  |  |  |  |  | **8.3204** |  |  |  |  |  |  |  |  |  |
| **6** | **5.2445** |  |  |  |  |  |  |  |  |  | **4.6333** |  |  |  |  |  |  |  |  |  |
| **COP** (diopters) | **1** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **2** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **3** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **4** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **6** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **E** = Bending energy; **W** = waviness; **RoCV** = Radius of curvature variance; ***t*** = central lens thickness; **ARoC** = Anterior radius of curvature; **PRoC** = Anterior radius of curvature; **COP** = Central Lens optical power | | | | | | | | | | | | | | | | | | | | | |

**Table 1b. Comparison of Isolated Lens Curve Parameters and Optical Powers**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Parameter** | | **Forbes** | | | | | | | | | | | **Ellipse** | | | | | | | | | |
| **Age** (years) | | **20** | **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30­­­­** | **20** | **21** | **22** | **24** | **25** | **26** | **27** | **28** | **29** | **30­­­­** |
| **Fit** (nm) | **A** | **47.90** |  |  |  |  |  |  |  |  |  |  | **963.8** |  |  |  |  |  |  |  |  |  |
| **P** | **8.08e3** |  |  |  |  |  |  |  |  |  |  | **55.9418** |  |  |  |  |  |  |  |  |  |
| **E** | **A** | **0.3535** |  |  |  |  |  |  |  |  |  |  | **0.0228** |  |  |  |  |  |  |  |  |  |
| **P** | **0.2498** |  |  |  |  |  |  |  |  |  |  | **0.0712** |  |  |  |  |  |  |  |  |  |
| **W** | **A** | **2.5971** |  |  |  |  |  |  |  |  |  |  | **0.0605** |  |  |  |  |  |  |  |  |  |
| **P** | **0.4223** |  |  |  |  |  |  |  |  |  |  | **0.0401** |  |  |  |  |  |  |  |  |  |
| **RoCV** () | **A** | **-** |  |  |  |  |  |  |  |  |  |  | **6.6116** |  |  |  |  |  |  |  |  |  |
| **P** | **-** |  |  |  |  |  |  |  |  |  |  | **0.8396** |  |  |  |  |  |  |  |  |  |
| ***t*** (mm) | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| *Central Optical Zone Diameter*(mm) | | | | | | | | | | | | | | | | | | | | | | |
| **ARoC** (mm) | **1** | **6.6756** |  |  |  |  |  |  |  |  |  |  | **13.0120** |  |  |  |  |  |  |  |  |  |
| **2** | **39.5817** |  |  |  |  |  |  |  |  |  |  | **12.2519** |  |  |  |  |  |  |  |  |  |
| **3** | **9.8378** |  |  |  |  |  |  |  |  |  |  | **11.0196** |  |  |  |  |  |  |  |  |  |
| **4** | **3.6533** |  |  |  |  |  |  |  |  |  |  | **9.3692** |  |  |  |  |  |  |  |  |  |
| **6** | **6.3692** |  |  |  |  |  |  |  |  |  |  | **5.1799** |  |  |  |  |  |  |  |  |  |
| **PRoC** (mm) | **1** | **6.0661** |  |  |  |  |  |  |  |  |  |  | **6.9318** |  |  |  |  |  |  |  |  |  |
| **2** | **3.6880** |  |  |  |  |  |  |  |  |  |  | **6.6690** |  |  |  |  |  |  |  |  |  |
| **3** | **3.7027** |  |  |  |  |  |  |  |  |  |  | **6.2386** |  |  |  |  |  |  |  |  |  |
| **4** | **7.6981** |  |  |  |  |  |  |  |  |  |  | **5.6524** |  |  |  |  |  |  |  |  |  |
| **6** | **6.4863** |  |  |  |  |  |  |  |  |  |  | **4.0879** |  |  |  |  |  |  |  |  |  |
| **COP** (diopters) | **1** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **2** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **3** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **4** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **6** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **E** = Bending energy; **W** = waviness; **RoCV** = Radius of curvature variance; ***t*** = central lens thickness;**ARoC** = Anterior radius of curvature; **PRoC** = Anterior radius of curvature; **CoP** = Central Lens optical power | | | | | | | | | | | | | | | | | | | | | | |

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| ***In Vivo*** | | | | | | | | | | ***Isolated Lens*** | | | |
| Age (years) | AA\*  (diopters) | **Accommodated** | | | | **Unaccommodated** | | | |
| *rav*  (mm) | *rpv*  (mm) | *tv*  (mm) | *COPv*  (diopters) | *rav*  (mm) | *rpv*  (mm) | *tv*  (mm) | *COPv*  (diopters) | *ra*  (mm) | *rp*  (mm) | *t*  (mm) | *COP*  (diopters) |
| 20 | 9.7 | 5.84 | -4.70 | 3.88 | 31.6 | 11.76 | -6.44 | 3.41 | 20.0 |  |  |  |  |
| 21 | 9.4 | 5.97 | -4.73 | 3.88 | 31.2 | 11.70 | -6.45 | 3.41 | 20.0 |  |  |  |  |
| 22 | 9.2 | 6.03 | -4.74 | 3.89 | 31.0 | 11.65 | -6.46 | 3.46 | 20.0 |  |  |  |  |
| 24 | 8.7 | 6.23 | -4.78 | 3.91 | 30.4 | 11.53 | -6.49 | 3.51 | 20.0 |  |  |  |  |
| 25 | 8.4 | 6.35 | -4.81 | 3.92 | 30.1 | 11.48 | -6.50 | 3.70 | 20.1 |  |  |  |  |
| 26 | 8.2 | 6.42 | -4.82 | 3.93 | 29.9 | 11.42 | -6.51 | 3.70 | 20.0 |  |  |  |  |
| 27 | 7.9 | 6.54 | -4.85 | 3.93 | 29.5 | 11.36 | -6.52 | 3.58 | 20.0 |  |  |  |  |
| 28 | 7.6 | 6.67 | -4.88 | 3.94 | 29.2 | 11.30 | -6.54 | 3.60 | 20.0 |  |  |  |  |
| 29 | 7.3 | 6.79 | -4.90 | 3.95 | 28.9 | 11.25 | -6.55 | 3.63 | 20.1 |  |  |  |  |
| 30 | 7.1 | 6.86 | -4.92 | 3.96 | 28.7 | 11.19 | -6.56 | 3.65 | 20.1 |  |  |  |  |
| AA\* = lower limit of accommodative amplitude;*ra, rav , tv, COPv,* and *ra, ra, t, COP = in vivo* and isolated lens (Chien curve values) anterior, posterior radii of curvatures, central thickness and optical power, respectively | | | | | | | | | | | | | |

**Table 2.** *In Vivo* LensCompared to the Isolated Lens for 3 mm Central Optical Zone Diameter

The finding that the < 4 mm optical zone radii of curvatures are greater than the peripheral radii of curvatures is additional evidence that the isolated lens is not in the maximally accommodated state.

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