

## Asymptotics Drill HW

### 1. Ordering Run Times

1)  $n^2$

- a) Doubling:  $(2n)^2/n^2 = 4$  : algorithm increases by a factor of 4
- b) Adding 1:  $(n+1)^2/n^2 = (n^2 + 2n + 1)/n^2 = 1$  (as  $n$  gets arbitrarily big and  $n^2$  in the denominator grows, the terms  $2n$  and  $1$  in numerator become negligible) : algorithm basically remains the same

2)  $n^3$

- a) Doubling:  $(2n)^3/n^3 = 8$  : algorithm increases by a factor of 8
- b) Adding 1:  $(n+1)^3/n^3 = (n^3 + 3n^2 + 3n + 1)/n^3 = 1$  (as  $n$  gets arbitrarily big, the terms  $3n^2$ ,  $3n$ , and  $1$  in numerator become negligible) : algorithm basically remains the same

3)  $100n^2$

- a) Doubling:  $100(2n)^2/100n^2 = 4$  : algorithm increases by a factor of 4
- b) Adding 1:  $100(n+1)^2/100n^2 = (100n^2 + 200n + 200)/100n^2 = 1$  (as  $n$  gets arbitrarily big, the terms  $200n$  and  $200$  in numerator become negligible) : algorithm basically remains the same

4)  $n \log n$

- a) Doubling:  $2n(\log 2n)/n \log n = 2$  (as  $n$  becomes arbitrarily big,  $\log 2n/\log n$  approaches 1) : algorithm increases by a factor of 2
- b) Adding 1:  $(n+1)(\log(n+1))/n \log n = 1$  (as  $n$  becomes arbitrarily big, the equation approaches 1)

5)  $2^n$

- a) Doubling:  $2^{2n}/2^n = 2^n$  : algorithm increases by a factor of  $2^n$
- b) Adding 1:  $2^{n+1}/2^n = 2$  : algorithm increases by a factor of 2

### 2. Really Understanding Order of Growth

$10^{10}$  operations per second \* 60 seconds \* 60 minutes =  $3.6 * 10^{13}$  operations per hour

Solve for  $n$  value which will produce less than  $3.6 * 10^{13}$  steps:

1)  $n^2 = 3.6 * 10^{13} : n < 6000000$

2)  $n^3 = 3.6 * 10^{13} : n < 33019.27249$

3)  $100n^2 = 3.6 * 10^{13} : n < 600000$

4)  $n \log n = 3.6 * 10^{13} : n < 2.8893 * 10^{12}$

5)  $2^n = 3.6 * 10^{13} : n = \log(3.6 * 10^{13}) : n < 13.5563$

6)  $2^{2n} = 3.6 * 10^{13} : n < 6.77815$