4 Optimal Control of Pitch/Travel without Feedback

4.1 State-space formulation

$$
\begin{aligned}
\dot{x} &= \begin{bmatrix} \dot{\lambda} \\ \dot{r} \\ \dot{p} \\ \ddot{p} \end{bmatrix} = A_c x + B_c u \\
A_c &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & K_2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -K_1 K_p & -K_1 K_{pd} \end{bmatrix} B_c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_1 K_{pp} \end{bmatrix}
\end{aligned} \tag{7}
$$

This is a model of the pitch and travel of the helicopter and the pitch controller of the basic control layer. Optimization layer and derivation is not inlcuded.

4.2 Discretization

$$
\begin{aligned}
\frac{x_{k+1} - x_k}{\Delta t} &= A_c x_k + B_c u_k \\
x_{k+1} &= (I + A_c \Delta t) x_k + B_c \Delta t u_k = A x_k + B u_k
\end{aligned} \tag{8}
$$

4.3 Computation of optimal trajectory

$$
\begin{aligned}
x_0 &= \begin{bmatrix} \lambda_0 = \pi \\ 0 \\ 0 \\ 0 \end{bmatrix}, x_f = \begin{bmatrix} \lambda_f = 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
|p_k| &\leq \frac{30\pi}{180}, |p_c| \leq \frac{30\pi}{180} \\
\phi &= \sum_{i=1}^{N} (\lambda_i - \lambda_f)^2 + q p_{ci}^2, q \geq 0 \\
\Delta t &= 0.25s, N = 100
\end{aligned} \tag{9}
$$

An optimal trajectory can be generated by minimizing the cost function for some scalar weight $q \geq 0$, while implementing the system dynamics and other user-defined limitations as linear constraints. Note that this cost function does not take into consideration that λ_i plus some multiple of 2π describes the same physical orientation of the helicopter. For example, if the reference is 0 and $\lambda_i = 2\pi$, it will be regarded as a large error, even though the helicopter is infact in the desired orientation. A more optimal scheme would take this into consideration.