TTK4135 Optimization and Control
Lab Report

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Abstract

This document outlines a few important aspects of the lab report. It contains some advice on both content and layout. The Latex source for this document is also published, and you can use it as a template of sorts for your own report.

When you write your own report, this section (the abstract) should contain a *very* short summary of what the lab is about and what you have done.

Contents

1 Introduction

Your introduction should contain an overview of the work you were assigned, as well as a few sentences putting the work into a larger perspective. You should also give a quick description of how the report is organized (as is done below).

You should of course put most of the work into doing good work in the lab and then presenting it in the report. When presenting your work in the report, both content and presentation/layout matters. Since your only way of communicating your good effort in the lab is through writing about it here, the way you write about it is essential. This means that even if you have the very best controller but describe it poorly, you will probably not be rewarded for the good results. A plot showing perfect control is worth very little if it is not accompanied by a clear description of what it represents.

Layout is naturally less important than content, but it still matters. You can think of report writing like selling an apartment; when you present your apartment for potential buyers you will of course clean the apartment and make it good looking. How clean the apartment is does of course not determine its value, but it is still important since it influences the subjective value your buyers will put on the apartment.

1.1 Software

You are of course free to use whatever software you want for report writing. You can also submit a handwritten report, although this is probably not a great idea if your handwriting can be hard to read.

You can also use Word or a similar word processor. However, it is next to impossible to achieve decent layout with Word. The support for vector graphics (discussed later) is extremely poor, and text tends to look pretty bad (bad support for kerning and ligatures). Furthermore, math is both time consuming and difficult to input, and tends to look very ugly. In general, a report written in Word looks like a draft.

It is strongly recommended to use Latex. Unless you tweak the layout too much, your report will almost certainly look very good. Although it can take a bit of effort to get started, it is also much quicker to use than Word and similar programs. The support for math and vector graphics is also great.

If you are new to Latex, you can have a look at the source for this document to get started. You can also look at the presentation by Berland (2010) (in Norwegian) or consult Oetiker et al. (2011). Another good reason to learn Latex is that you probably don't want to write your master's thesis in something like Word, doing so would likely be very frustrating. Being reasonably fluent in Latex before you get that far will make your thesis work much smoother.

Some of you are probably fluent in Latex and might plan to write the report using it. Please resist the temptation (if any) to change the fonts, make super fancy headers (they are not necessary for a report like this), change the margins, change the paragraph indentation and/or spacing, and similar things.

A great tool for collaborating on Latex documents is ShareLaTeX at `https://www.sharelatex.com/`; if you use this you won't have to install anything on your computer. Texmaker at `http://www.xm1math.net/texmaker/` is a good cross-platform editor. Some people like Lyx, which is a Latex editor that behaves a little bit like Word.

1.2 Other Comments

If you have problems with Latex, the solution is usually just a few Google searches away.

You can write the report in Norwegian or English. Writing in English is encouraged and is great practice, but entirely optional. *Do not interpret any of the advice or suggestions here as requirements.*

This report is organized as follows: Section 2 contains a few remarks on report writing and some random Latex advice. An example of a table can be foundin Section 3, along with two remarks on report writing. Section 4 contains some advice on using plots from MATLAB. A few suggestions for making illustrations are given in Section 5; a matrix equation can also be found here. Section 6 has a few comments on references and floats in Latex. The closing discussion and concluding remarks are in Sections 7 and 8, respectively. Appendix A contains a MATLAB file while Appendix B shows an example Simulink diagram. The Bibliography can be found at the end, on page 15.

2 Problem Description

In this section you should describe the lab setup and discuss the model. If you want, you can copy the source code for the model equations:

$$\ddot{e} + K_3 K_{ed} \dot{e} + K_3 K_{ep} e = K_3 K_{ep} e_c \tag{1}$$

$$\ddot{p} + K_1 K_{pd} \dot{p} + K_1 K_{pp} p = K_1 K_{pp} p_c \tag{2}$$

$$\dot{\lambda} = r \tag{3}$$

$$\dot{r} = -K_2 p \tag{4}$$

Since these equations belong together, it's a good idea to number them like this:

$$\ddot{e} + K_3 K_{ed} \dot{e} + K_3 K_{ep} e = K_3 K_{ep} e_c \tag{5a}$$

$$\ddot{p} + K_1 K_{pd} \dot{p} + K_1 K_{pp} p = K_1 K_{pp} p_c \tag{5b}$$

$$\dot{\lambda} = r \tag{5c}$$

$$\dot{r} = -K_2 p \tag{5d}$$

You can then both reference individual equations ("the elevation equation (5a)") or reference the entire model ("the linear model (5)"). Regardless of your choice of software, never hard-code a reference, always use dynamic references.

You could also align the equations like this:

$$\begin{aligned} \ddot{e} + K_3 K_{ed} \dot{e} + K_3 K_{ep} e &= K_3 K_{ep} e_c && (6a) \\ \ddot{p} + K_1 K_{pd} \dot{p} + K_1 K_{pp} p &= K_1 K_{pp} p_c && (6b) \\ \dot{\lambda} &= r && (6c) \\ \dot{r} &= -K_2 p && (6d) \end{aligned}$$

You can consult Downes (2002) for more about writing math.

If you decide to include a figure, that's great. You can in general copy figures from the textbook, the assignement text, or other places. However: ALWAYS CITE THE SOURCE.

Table 1: Parameters and values.

Symbol	Parameter	Value	Unit
l_a	Distance from elevation axis to helicopter body	0.63	m
l_h	Distance from pitch axis to motor	0.18	m
K_f	Force constant motor	0.25	N/V
J_e	Moment of inertia for elevation	0.83	$\mathrm{kg\,m^2}$
J_t	Moment of inertia for travel	0.83	$\mathrm{kg\,m^2}$
J_p	Moment of inertia for pitch	0.034	$\mathrm{kg\,m^2}$
m_h	Mass of helicopter	1.05	kg
m_w	Balance weight	1.87	kg
m_g	Effective mass of the helicopter	0.05	kg
K_p	Force to lift the helicopter from the ground	0.49	N

3 Repetition/Introduction to Simulink/QuaRC

4 Optimal Control of Pitch/Travel without Feedback

4.1 State-space formulation

4.2 Discretization

4.3 Computation of optimal trajectory

An optimal trajectory can be generated by minimizing the cost function

$$\phi = \sum_{i=1}^{N} (\lambda_i - \lambda_f)^2 + q p_{ci}^2 \tag{7}$$

for some scalar weight $q \geq 0$ over the finite horizon of states and inputs

$$z = (x_1 \; x_2 \; \dots \; x_N \; u_1 \; u_2 \; \dots \; u_N)^T \tag{8}$$

The discrete-time system dynamics are implemented as equality constraints of the form $A_{\text{eq}} z = B_{\text{eq}}$, where A_{eq} and B_{eq} are given by the left- and right-hand side of the N constraints

$$\begin{aligned} x_1 - Bu_0 &= Ax_0 \\ x_2 - Ax_1 - Bu_1 &= 0 \\ &\vdots \\ x_N - Ax_{N-1} - Bu_{N-1} &= 0 \end{aligned}$$

We would also like to constrain the system state and input to be within a range

$$x^{\text{min}} \leq x_{t+1} \leq x^{\text{max}} \tag{9}$$

$$u^{\text{min}} \leq u_t \leq u^{\text{max}} \tag{10}$$

for $t = 0 \dots N-1$. Applying these constraints to all states and inputs in the solution horizon, we have

$$\begin{bmatrix} I \\ -I \end{bmatrix} z \leq \begin{bmatrix} \{x_{t+1}^{max}\} \\ \{u_t^{max}\} \\ \{x_{t+1}^{min}\} \\ \{u_t^{min}\} \end{bmatrix}_{t=0..N-1} \tag{11}$$

which can be implemented as an inequality constraint of the form $A_{\text{iq}} z \leq B_{\text{iq}}$.

Note that the cost function (7) does not take into consideration that λ_i plus some multiple of 2π describes the same physical orientation of the helicopter. For example, if the reference is 0 and $\lambda_i = 2\pi$, it will be regarded as a large error, even though the helicopter is infact in the desired orientation. A more optimal scheme would take this into consideration.

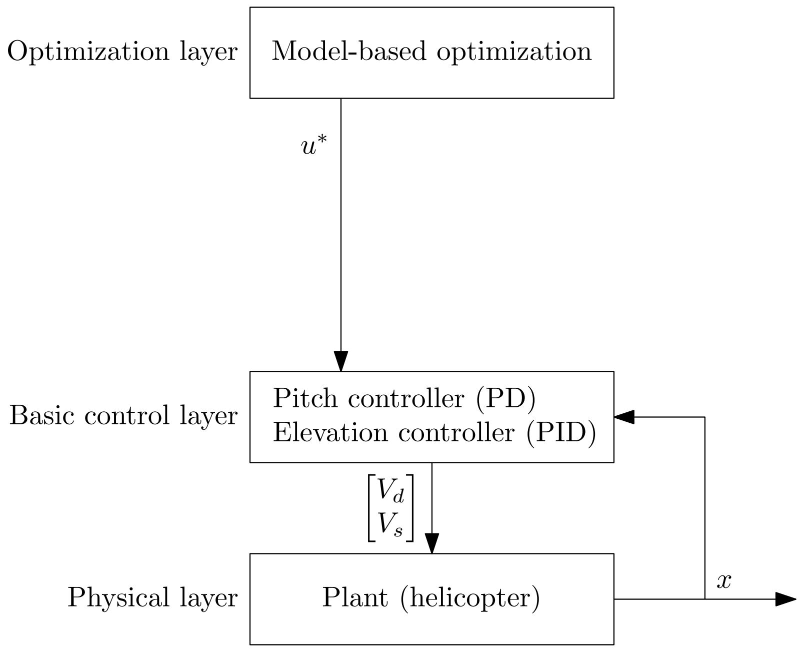

Figure 1: A figure created with Ipe.

5 Optimal Control of Pitch/Travel with Feedback (LQ)

You are as mentioned welcome to use the figures from the assignment text if you want to (cite the source!). You can also draw your own (cite the source if it is heavily based on someone else's.). Figure 1 was created quickly with Ipe. Inkscape is a good alternative for more advanced illustrations. Some people prefer the Latex package TikZ (`http://texample.net/tikz/examples/`), but this takes a little effort to learn.

Here is a matrix equation you can use as a template:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -b & 0 & 0 & 0 \\ -a & 1 & 0 & 0 & 0 & -b & 0 & 0 \\ 0 & -a & 1 & 0 & 0 & 0 & -b & 0 \\ 0 & 0 & -a & 1 & 0 & 0 & 0 & -b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ u_0 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} ax_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

6 Optimal Control of Pitch/Travel and Elevation with and without Feedback

If you claim something you read in the textbook (Nocedal and Wright, 2006), you should cite it. Also, remember to reference all figures in the text. Figures have a number and should be referenced by that number (again, always use dynamic references). They also tend to float around, meaning they generally don't appear where you ask them to in the text. This is fine, do not try to force a figure (or a table) to appear in a particular place. As long a you refer to it, it's easy to find. No figure should be included without being referenced in the text.

If you look at the source code for including figures, you can see that I've used the optional option `[htb]`. This tells Latex where you wish the figure to appear, in prioritized order. h means "Here", t means "Top of this page", b means "Bottom of this page", and p (not used here) means "on a Page with only floats (such as figures and tables)". Note that your wish might not be granted, and this is because Latex actually optimizes the placement of figures. If you start forcing figures to be in specific places, it often leads to really strange layout somewhere else in the document.

Generally, let Latex handle the documentation layout.

6.1 Results and Discussion

Each of the four problems should have their own discussion of results.

Remember: all plots and results need a description, explanation, and discussion.

7 Discussion

A section like this does not have to be long, but write a few short paragraphs that show you understand what you have been doing and how the different results relate to each other.

8 Conclusion

Again, this does not have to be long, but try to write a few reasonable closing remarks.

A MATLAB Code

This section should contain your MATLAB code. DO NOT attach files posted online (that you didn't write). Note that the method used to input code below does not look as pretty when the lines are too long.

A.1 plot_constraint.m

```
% Plot a figure with some Latex in the labels
l = linspace(70,170)*pi/180;
a = 0.2;
b = 20;
l_b = 2*pi/3;

e = a*exp(-b*(l-l_b).^2);

l_deg = l*180/pi;
e_deg = e*180/pi;

figure(1)
plot(l_deg,e_deg, 'LineWidth', 2)

handles(1) = xlabel('$\lambda$/degrees');
handles(2) = ylabel('$e$/degrees');
set(handles, 'Interpreter', 'Latex');
```

B Simulink Diagrams

This section should contain your Simulink diagrams. Just like the plots, these should be in vector format, like in Figure 2. Make them tidy enough to understand.

B.1 A Simulink Diagram

Figure 2 shows a Simulink diagram.

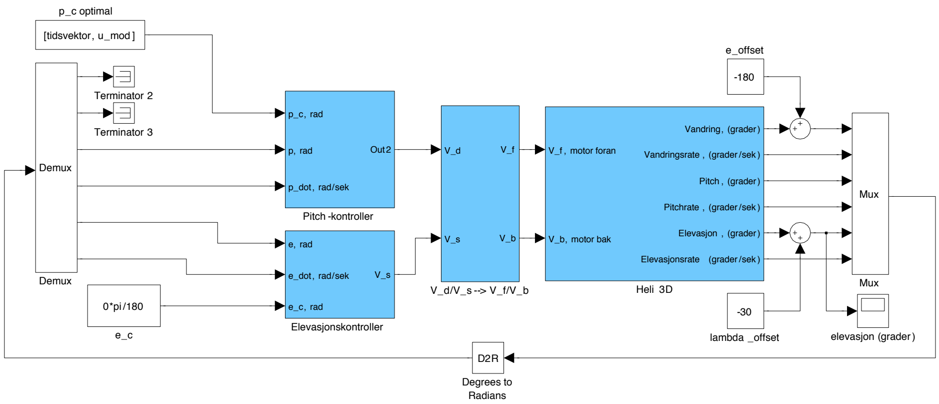

Figure 2: A Simulink diagram.

Bibliography

Berland, H. (2010). *En introduksjon til Latex*. `http://www.pvv.ntnu.no/~berland/latex/latexintro4up.pdf`.

Downes, M. (2002). *Short Math Guide for LATEX*. `ftp://ftp.ams.org/pub/tex/doc/amsmath/short-math-guide.pdf`.

Nocedal, J. and Wright, S. J. (2006). *Numerical Optimization*. Springer, second edition.

Oetiker, T., Partl, H., Hyna, I., and Schlegl, E. (2011). *The Not So Short Introduction to LATEX 2e*. `http://tobi.oetiker.ch/lshort/lshort.pdf`.