Numbers

Types of Numbers:

- Natural Numbers: $\mathbb{N} = \{1, 2, 3, \ldots\}$

- Whole Numbers: $\mathbb{N} \cup \{0\} = \{0, 1, 2, 3, \ldots\}$

- Integers: $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

- Rational Numbers: $\mathbb{Q} = \left\{ \frac{a}{b} \ : \ a, b \in \mathbb{Z} \ , \ b \neq 0 \right\}$. Rational Numbers comprise of fractions (includes all proper and improper fractions and mixed numbers). All terminating decimals (eg, 10.87) and recurring decimals (eg, $0.3\dot{7}\dot{1} = 0.3717171\ldots$) are rational numbers because these can all be expressed as fractions. All integers are rational numbers.

- Irrational Numbers: numbers that cannot be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Eg. π, $\sqrt{2}$, $\sqrt{5}$, $-4\sqrt{7}$, e, $2.75e$, etc. Irrational numbers are non-recurring decimals. Any non-zero rational number multiplied to an irrational number results in an irrational number. For example, $-\frac{3}{4}\pi$ is irrational.

- Perfect Squares: $\{1, 4, 9, 16, 25, \ldots\}$

- Perfect Cubes: $\{1, 8, 27, 64, \ldots\}$

- Prime Numbers: Positive integers at least 2 whose only positive divisors are 1 and itself. $\{2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots\}$

Equality and Inequality Symbols

Symbol	Meaning	Example
$=$	is equal to	$0.1 = \frac{1}{10}$
$\neq$	is not equal to	$0.11 \neq \frac{1}{10}$
$>$	is greater than	$0.1 > 0.01$
$\geqslant$	is greater than or equal to	$a \geqslant 5$
$<$	is less than	$0.05 < 5$
$\leqslant$	is less than or equal to	$b \leqslant 5$

Prime Factorization, HCF, LCM

- Example of prime factorization:

$$
\begin{array}{r|c}
2 & 4356 \\ \hline
2 & 2178 \\ \hline
3 & 1089 \\ \hline
3 & 363 \\ \hline
11 & 121 \\ \hline
 & 11 \\ \hline
\end{array}
$$

  Hence $4356 = 2^2 \times 3^2 \times 11^2$ (in index notation).

- Example of HCF and LCM using prime factorization:

  $4800 = 2^6 \times 3 \times 5^2$

  $5544 = 2^3 \times 3^2 \times 7 \times 11$

  $\text{HCF} = 2^3 \times 3$

  [take common prime factors and lowest power of each]

  $\text{LCM} = 2^6 \times 3^2 \times 5^2 \times 7 \times 11$

  [take all prime factors and highest power of each]

- Examples of square roots and cube roots using prime factorization:

$$
\begin{aligned}
54756 &= 2^2 \times 3^4 \times 13^2 \\
\sqrt{54756} &= 2 \times 3^2 \times 13 = 234 \\
1728 &= 2^6 \times 3^3 \\
\sqrt[3]{1728} &= 2^2 \times 3 = 12
\end{aligned}
$$

Approximation

Significant Figures

Rules of identifying number of significant digits:

1. All non-zero digits are significant.

2. Zeros between non-zero digits are significant.

   Eg. 302 (3 sf)

   Eg. 10.2301 (6 sf)

3. In a whole number, zeros after the last nonzero digit may or may not be significant. It depends on the estimation being made.

Eg.

$7436000 = 7000000$ (1 sf)
$7436000 = 7400000$ (2 sf)
$7436000 = 7440000$ (3 sf)
$7436000 = 7436000$ (4 sf)
$7436000 = 7436000$ (5 sf)
$7436000 = 7436000$ (6 sf)

4. In a decimal number, zeros before the 1st non-zero digit are not significant.

   Eg. 0.004 (1 sf)

   Eg. 0.07008 (4 sf)

5. In a decimal number, zeros after the last non-zero digit are significant.

   Eg. 6.40 (3 sf)

   Eg. 12.000 (5 sf)

   Eg. 20300.000 (8 sf)

   Eg. 0.0700800 (6 sf)

Decimal Place Rounding

Examples:
$0.7374 = 0.74$ (2 dp)
$58.301 = 58.30$ (2 dp)
$207.6296 = 207.630$ (3 dp)
$207.6296 = 207.63$ (2 dp)
$207.977 = 208.0$ (1 dp)
$207.977 = 207.98$ (2 dp)
$18.997 = 19.00$ (2 dp)

Standard Form

$\pm A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

Examples:
$1350000 = 1.35 \times 10^6$
$0.000375 = 3.75 \times 10^{-4}$

Common Prefixes

10^{12}	trillion	tera	T
10^9	billion	giga	G
10^6	million	mega	M
10^3	thousand	kilo	k
10^{-3}	thousandth	milli	m
10^{-6}	millionth	micro	μ
10^{-9}	billionth	nano	n
10^{-12}	trillionth	pico	p

Indices

Rules of Indices:

$$a^0 = 1$$
$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(ab)^n = a^n b^n$$
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$
$$(a^n)^m = a^{nm}$$
$$a^{-n} = \frac{1}{a^n}$$
$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$
$$a^{\frac{1}{n}} = \sqrt[n]{a}$$
$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

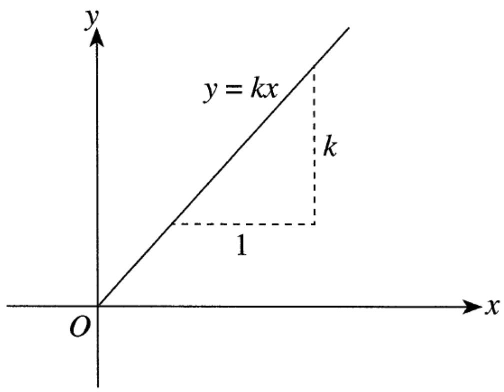
Caution:
1. For indices (powers) that are not integers, then the above rules of indices hold only for $a, b \geq 0$, or only for $a, b > 0$ if some of the indices are negative or there is division by either a or b. The rule $a^0 = 1$ holds only for non-zero values of a.
2. $\sqrt{(-8)^2} = \sqrt{64} = 8$, NOT -8.
3. $\sqrt[3]{-27} = -3$.

Equalities of Indices:
1. If $a^m = a''$, then $m = n$.
2. If $a^m = b^m$, then $a = b$.

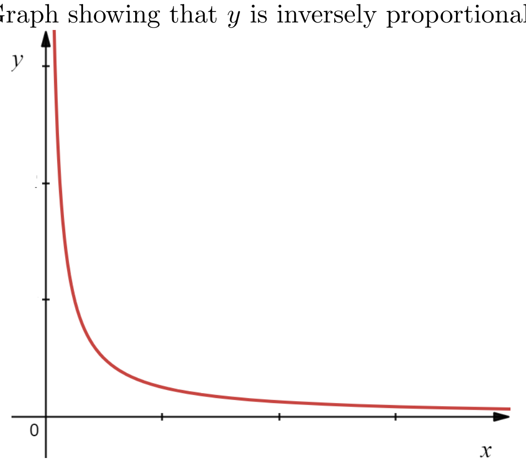
Direct and Inverse Proportion

If y is directly proportional to x, then $y = kx$, where k is a constant and $k \neq 0$. The ratios $\frac{x}{y}$ and $\frac{y}{x}$ are

constant. Furthermore, the graph on y against x (or of x against y) is a straight line through the origin.
Graph showing that y is directly proportional to x:



If y is inversely proportional to x, then $y = \frac{k}{x}$, where k is a constant and $k \neq 0$. The product xy is constant.
Graph showing that y is inversely proportional to x:



Map Scales

- Linear scale:

  $1 : n$ means 1 unit length on map represents n units length on ground.

  Eg. $1 : 5000$ means

  1 cm represents 5000 cm

  which implies 1 cm represents 50 m

  which implies 1 cm represents 0.05 km

- Representative Fraction (RF):

  If the linear scale is $1 : n$, the RF is expressed as $\frac{1}{n}$.

  Eg, if 3 cm represents 6 m, then RF is $\frac{1}{200}$.

- Area Scale:

  If linear scale is $1 : 200000$, then it means

  1 cm represents 200000 cm

  which implies 1 cm represents 0.2 km

  which implies 1^2 cm^2 represents $(0.2)^2$ km^2

  which implies 1 cm^2 represents 0.04 km^2

Percentage, Ratio, Rate

- To express a percentage as a fraction or decimal, divide by 100:

  $x\% = \frac{x}{100}$

  Eg, $23.5\% = \frac{23.5}{100} = \frac{235}{1000} = \frac{47}{200}$

  Eg, $401\% = \frac{401}{100} = 4.01$

- To express any number as a percentage, multiply it by 100%.

  Eg, $0.165 = 0.165 \times 100\% = 16.5\%$.

- Expressing a quantity A as a percentage of a quantity B

  $\frac{A}{B} \times 100\%$

  Eg, Express 63.7 as a percentage of 98.

  Answer: $\frac{63.7}{98} \times 100\% = 65\%$

  In words, we say that 63.7 is 65% of 98.

- Increase or decrease a quantity by a given percentage:

  Eg, Increase 45 by 2.4%:

  Answer: $45 \times \left(1 + \frac{2.4}{100}\right) = 45 \times 1.024 = 46.08$

  Eg, Decrease 45 by 90%:

  Answer: $45 \times \left(1 - \frac{90}{100}\right) = 45 \times 0.1 = 4.5$

- Percentage Increase and Percentage Decrease:

  When a quantity increases, the percentage increase is

$\frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100\%$

When a quantity dereases, the percentage decrease is

$\frac{\text{initial (bigger) value} - \text{final (smaller) value}}{\text{initial value}} \times 100\%$

Percentage increase will always be > 0 if the quantity has increased.

Percentage decrease will always be > 0 if the quantity has decreased.

Percentage change is

$\frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100\%$

regardless of whether the quantity has increased or decreased. Percentage change can be either positive or negative depending on whether the quantity has increased or decreased.