# Financial Mathematics

- The formula to calculate the simple interest is $I = \frac{PRT}{100}$, where

  $I$ = total simple interest earned or payable,

  $P$ = principal amount (original or starting amount),

  $R$ = rate of interest per annum (eg. if 4.5% per annum, then $R = 4.5$),

  $T$ = number of years (eg. if 18 months, then $T = 1.5$).

- The formula to calculate compound interest is $A = P\left(1 + \frac{r}{100}\right)^n$, where

  $A$ = total amount after $n$ units of time,

  $P$ = principal amount (original or starting amount),

  $r$ = rate of interest (eg. if 2.4%, then $r = 2.4$),

  $n$ = number of units of time.

  Example: 3 years, 4.5% per annum:

  $A = P\left(1 + \frac{4.5}{100}\right)^3$

  Example: Suppose the year is divided into $m$ equal sub-intervals. For quarterly compounding $m = 4$. For monthly compounding, $m = 12$. For semi-annual compounding, $m = 2$. Then if $r$ denotes interest rate **per annum** and $n$ denotes number of **years**,

  $A = P\left(1 + \frac{r/m}{100}\right)^{nm} = P\left(1 + \frac{r}{100m}\right)^{nm}$

- Profit = Selling Price - Cost Price

- If sold at below cost, then:

  Loss = Cost Price - Selling Price

- % Profit or Loss = $\frac{\text{Profit or Loss}}{\text{Cost price}} \times 100\%$

  Cost price is regarded as 100%

- In absense of GST, **marked price** refers to **orginal selling price**

  Discount = Marked Price - Discounted Sales Price

- % Discount = $\frac{\text{Discount}}{\text{Marked Price}} \times 100\%$

  $= \frac{\text{Marked Price - Discounted Sales Price}}{\text{Marked Price}} \times 100\%$

  Marked price is regarded as 100%

- In presence of GST, marked price **includes** GST unless otherwise stated.

  If GST is 9%, and the selling price before GST is regarded as 100% then the marked price after GST is 109%.

  Any discount imposed is on the marked price inclusive of GST.

- Hire Purchase:

  An initial down payment is made.

  Simple interest is added to the **remaining amount**, that is the amount left over after the down payment. Regard the **principal amount** in the simple interest calculation as the **amount left over after the down payment**.

  Then the total balance to be paid **inclusive of simple interest** is divided by the number of instalment periods to obtain the instalment amount per time period.

- Income Tax example 1:

  Use the tax rates given below to calculate the amount of tax Matthew and Tom have to pay.

  (a) Matthew's annual chargeable income was \$28500. Tom's annual chargeable income was \$37400.

  (b) Irene's income tax for last year was \$499.60. How much was her annual chargeable income last year?

| Chargeable income (\$) | Rate (%) | Tax Payable (\$) |
|---|---|---|
| On the first 20000 | 0 | 0 |
| On the next 10000 | 2 | 200 |
| On the first 30000 | - | 200 |
| On the next 10000 | 3.5 | 350 |

  (a)

  Matthew:

  Tax on the first \$20000 = \$0

  Tax on the next \$8500 = $\frac{2}{100} \times$ \$8500 = \$170

Total tax $= \$0 + \$170 = \$170$

$\therefore$ Matthew has to pay \$170 of tax.

Tom:

Tax on the first \$30000 $= \$200$

Tax on the next \$7400 $= \frac{3.5}{100} \times \$7400 = \$259$

Total tax $= \$200 + \$259 = \$459$

$\therefore$ Tom has to pay \$459 of tax.

(b)

Irene:

Tax on the first \$30000 $= \$200$

Tax on the remainder $= \$499.60 - \$200$
$= \$299.60$

Let $\$x$ be the remainder.

$$\frac{3.5}{100} \times \$x = \$299.60$$
$$\$x = \frac{100}{3.5} \times \$299.60$$
$$= \$8560$$

Annual chargeable income last year
$= \$30000 + \$8560$
$= \$38560$

- Income Tax example 2:

  The table below shows the rates of income tax for 2018.

  (a) Theresa's chargeable income for the year ended 2018 was \$108000. Calculate the amount of income tax she has to pay for 2018 .

  (b) Theresa received a pay drop in 2019. The tax rate in 2019 is the same as 2018. Calculate her chargeable income in 2019 if her income tax payable in 2019 is \$480.

| Chargeable Income | Rate (%) | Gross Tax Payable (\$) |
|---|---|---|
| First \$20000 | 0 | 0 |
| Next \$10000 | 2 | 200 |
| First \$30000 | - | 200 |
| Next \$10000 | 3.50 | 350 |
| First \$40000 | - | 550 |
| Next \$40000 | 7 | 2800 |
| First \$80000 | - | 3350 |
| Next \$40000 | 11.5 | 4600 |

(a)

\$108000 − \$80000 = \$28000

Tax on first \$80000 = \$3350

Tax on next \$28000 $= \frac{11.5}{100} \times \$28000 = \$3220$

Total tax $= \$3350 + \$3220 = \$6570$

(b)

If income tax payable in 2019 is \$480, then she earns at least \$30000 but below \$40000

Tax on the remainder above \$30000
$= \$480 - \$200$
$= \$280$

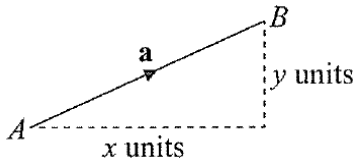
Let $\$x$ be the remainder.

$$\frac{3.50}{100} \times \$x = \$280$$
$$\$x = \frac{100}{3.50} \times \$280$$
$$= \$8000$$

Total chargeable income
$= \$30000 + \$8000$
$= \$38000$

# Vectors

- The vector below can be denoted by $\overrightarrow{AB}$.



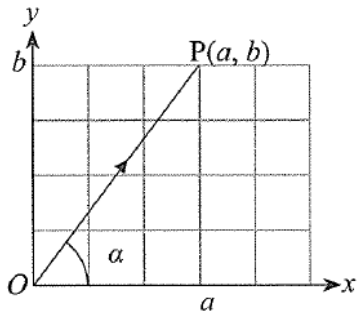

- The magnitude/length of the vector is denoted by $|\overrightarrow{AB}|$ or $\left|\binom{x}{y}\right|$.

  If $\mathbf{a} = \binom{x}{y}$, then its magnitude is given by: $|\mathbf{a}| = \sqrt{x^2 + y^2}$ (using Pythagoras' Theorem)

- Position vectors:

  A directed line segment $\overrightarrow{OP}$ indicates the position of a point $P$ taken with reference to the origin $O$.

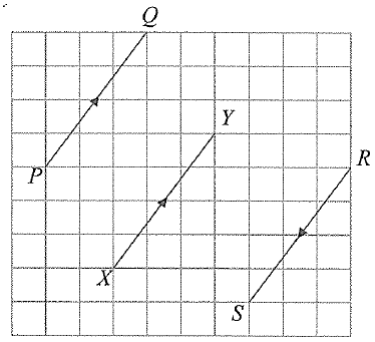
  Vectors with the origin as their initial point are known as position vectors.




  If the point $P$ has coordinates $(a, b)$, then the position vector of $P$ is $\overrightarrow{OP} = \binom{a}{b}$.

  The magnitude or length of $OP$ is given by $|\overrightarrow{OP}| = \sqrt{a^2 + b^2}$.

- In the diagram below,




  $\overrightarrow{PQ} = \overrightarrow{XY}$

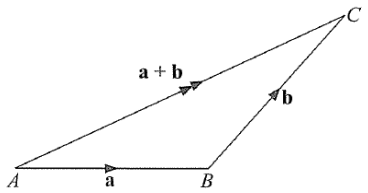
  $\overrightarrow{RS}$ is the negative vector of $\overrightarrow{PQ}$.

  $\overrightarrow{PQ} = -\overrightarrow{RS}$

  Note that $|\overrightarrow{PQ}| = |\overrightarrow{RS}|$.

  Zero or null vectors are vectors whose magnitude is zero. For example: $\overrightarrow{PQ} + \overrightarrow{RS} = \mathbf{0}$.

- Sum of Two Vectors



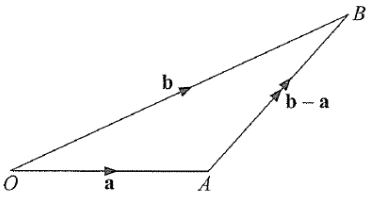

  $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

- Difference of Two Vectors

  $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ (using vector addition)

  $= -\overrightarrow{OA} + \overrightarrow{OB}$ (using negative vector $\overrightarrow{AO} = -\overrightarrow{OA}$ )

  $= \overrightarrow{OB} - \overrightarrow{OA}$




- Translation:

  If an object point $P(a, b)$, which can be expressed as $\overrightarrow{OP} = \binom{a}{b}$, undergoes a translation $T = \binom{h}{k}$,

then the image point of $P$, i.e. $Q$ can be found by:

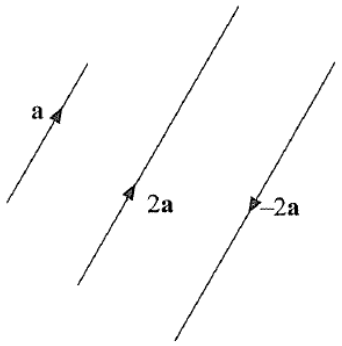
$$\begin{aligned}\overrightarrow{OQ} &= \overrightarrow{OP} + \begin{pmatrix} h \\ k \end{pmatrix} \\ &= \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} h \\ k \end{pmatrix} \\ &= \begin{pmatrix} a+h \\ b+k \end{pmatrix}\end{aligned}$$

- Vector $\mathbf{a}$ is parallel to vector $\mathbf{b} \Leftrightarrow \mathbf{a} = k\mathbf{b}$, where $k$ is a scalar.

  If $\mathbf{a} = k\mathbf{b}$, where $k$ is a scalar, then

  (i) $|\mathbf{a}| = |k||\mathbf{b}|$

  (ii) If $k$ is positive, $\mathbf{a}$ and $\mathbf{b}$ are in the same direction. If $k$ is negative, $\mathbf{a}$ and $\mathbf{b}$ are in opposite directions.




- Suppose $\mathbf{a}$ and $\mathbf{b}$ are non-parallel vectors, and $h, k, m$ and $n$ are scalars.

  (i) If $h\mathbf{a} = k\mathbf{b}$, then $h = 0$ and $k = 0$.

  (ii) If $n\mathbf{a}+m\mathbf{b} = h\mathbf{a}+k\mathbf{b}$, then $n = h$ and $m = k$.

- Points $A, B$ and $C$ are collinear, i.e. they lie on a straight line $\Leftrightarrow \overrightarrow{AB} = k\overrightarrow{BC}$

  $\Leftrightarrow \overrightarrow{AB} = h\overrightarrow{AC}$

  $\Leftrightarrow \overrightarrow{BC} = m\overrightarrow{AC}$