Simultaneous Linear Equations

Method 1: Elimination
$5x - 2y = 21 \quad$ —(1)
$2x - y = 8 \quad$ —(2)
$(1) \times 2 : 10x - 4y = 42 \quad$ —(3)
$(2) \times 5 : 10x - 5y = 40 \quad$ —(4)
(3) - (4) : $\quad y = 2$
Sub into (1): $5x - 2(2) = 21$
$5x - 4 = 21 \;\Rightarrow\; 5x = 25 \;\Rightarrow\; x = 5$

Method 2: Substitution
$5x - 2y = 21 \quad$ —(1)
$2x - y = 8 \quad$ —(2)
From (2):
$y = 2x - 8 \quad$ —(3)
Sub (3) into (1): $5x - 2(2x - 8) = 21$
$5x - 4x + 16 = 21 \;\Rightarrow\; x - 16 \quad = 21$
$x = 5$
Sub into (2):
$2(5) - y = 8 \;\Rightarrow\; 10 - y = 8 \quad \Rightarrow\; y = 2$

Inequalities

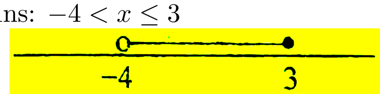
Inequality sign is reversed when both sides are multiplied or divided by a negative number.

Eg. $-3x + 4 \geq 12$
$-3x \geq 12 - 4$
$-3x \geq 8$
$x \leq -\frac{8}{3}$

Eg. $3(x - 1) < 4x + 1 \leq 7 + 2x$

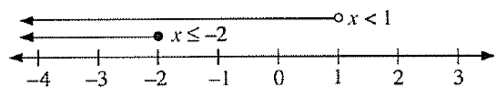
$3(x-1) < 4x + 1$	$4x + 1 \leq 7 + 2x$
$3x - 3 < 4x + 1$	$4x - 2x \leq 7 - 1$
$3x - 4x < 1 + 3$	$2x \leq 6$
$-x < 4$	$x \leq 3$
$x > -4$	

Ans: $-4 < x \leq 3$



Eg. $5x + 4 \leq 3x < 6 - 3x$

$5x + 4 \leq 3x$	$3x < 6 - 3x$
$2x \leq -4$	$6x < 6$
$x \leq -2$	$x < 1$



Ans: $x \leq -2$.

Expansion

Eg.
$2p - 3(p + 1)$
$= 2p - 3p - 3$
$= -p - 3$
Eg.
$5x - (x + 1)(2x - 3)$
$= 5x - \left(2x^2 - 3x + 2x - 3\right)$
$= 5x - 2x^2 + 3x - 2x + 3$
$= -2x^2 + 6x + 3$

Factorization and Identities

Factorization of Quadratic Expressions:
$5x^2 + 9x - 2$

$\times$	$5x$	-1
x	$5x^2$	$-x$
2	$10x$	-2

$\therefore \quad 5x^2 + 9x - 2 = (5x - 1)(x + 2)$

Identities:

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)(a - b) = a^2 - b^2$

Common Factorisation Techniques:

- Common Factors
  Eg. $6a^3b - 2a^2b = 2a^2b(3a - 1)$

- Grouping

  Eg.

  $6p^2 - 3pq - 10ap + 5aq$

  $= 3p(2p - q) - 5a(2p - q)$

  $= (3p - 5a)(2p - q)$

- Using Difference of Two Squares

  Eg. $9a^2 - 1$

  $= (3a)^2 - (1)^2$

  $= (3a + 1)(3a - 1)$

  Eg. $16a^4 - 81$

  $= (4a^2 + 9)(4a^2 - 9)$

  $= (4a^2 + 9)(2a + 3)(2a - 3)$

- Combination of methods:

  Eg. $3x^3 - 12xy^2 = 3x\left(x^2 - 4y^2\right)$ common factor
  $= 3x(x + 2y)(x - 2y)$ then diff. of 2 squares

  Always try common factor first

  Eg.

  $4 - p^2 + 6pq - 9q^2$

  $= 4 - (p^2 - 6pq + 9q^2)$

  $= (2)^2 - (p - 3q)^2$

  $= (2 + (p - 3q))(2 - (p - 3q))$

  $= (2 + p - 3q)(2 - p + 3q)$

Algebraic Fractions

Eg.

$$\frac{x+2}{3} - \frac{x-5}{2} = \frac{2(x+2)}{6} - \frac{3(x-5)}{6}$$
$$= \frac{2(x+2) - 3(x-5)}{6}$$
$$= \frac{2x + 4 - 3x + 15}{6}$$
$$= \frac{19 - x}{6}$$

Eg.

$$\frac{5}{x+1} - \frac{2}{x-3}$$
$$= \frac{5(x-3)}{(x+1)(x-3)} - \frac{2(x+1)}{(x-3)(x+1)}$$
$$= \frac{5(x-3) - 2(x+1)}{(x+1)(x-3)}$$
$$= \frac{5x - 15 - 2x - 2}{(x+1)(x-3)}$$
$$= \frac{3x - 17}{(x+2)(x-5)}$$

Eg.

$$\frac{5}{3x} + \frac{2}{x}$$
$$= \frac{5}{3x} + \frac{6}{3x}$$
$$= \frac{5+6}{3x}$$
$$= \frac{11}{3x}$$

Eg.

$$\frac{3}{(x+2)^2} - \frac{4}{x+2}$$
$$= \frac{3}{(x+2)^2} - \frac{4(x+2)}{(x+2)^2}$$
$$= \frac{3 - 4(x+2)}{(x+2)^2}$$
$$= \frac{3 - 4x - 8}{(x+2)^2}$$
$$= \frac{-4x - 5}{(x+2)^2}$$

Eg.

$$\frac{7}{x^2-9} - \frac{1}{x-3}$$
$$= \frac{7}{(x+3)(x-3)} - \frac{1}{x-3}$$
$$= \frac{7}{(x+3)(x-3)} - \frac{x-3}{(x-3)^2}$$
$$= \frac{7-(x+3)}{(x+3)(x-3)}$$
$$= \frac{7-x-3}{(x+3)(x-3)}$$
$$= \frac{4-x}{(x+3)(x-3)}$$

Eg.

$$\frac{3x+2}{x^2-4} + \frac{1}{x+2} - \frac{2}{x-2}$$
$$= \frac{3x+2}{(x+2)(x-2)} + \frac{1}{x+2} - \frac{2}{x-2}$$
$$= \frac{3x+2+(x-2)-2(x+2)}{(x+2)(x-2)}$$
$$= \frac{3x+2+x-2-2x-4}{(x+2)(x-2)}$$
$$= \frac{2x-4}{(x+2)(x-2)}$$
$$= \frac{2(x-2)}{(x+2)(x-2)}$$
$$= \frac{2}{x+2}$$

Eg.

$$\frac{9}{x-5} + \frac{3}{5-x}$$
$$= \frac{9}{x-5} - \frac{3}{x-5}$$
$$= \frac{9-3}{x-5}$$
$$= \frac{6}{x-5}$$

Eg.

$$\frac{2x}{3y-8x} + \frac{11x}{80x-30y}$$
$$= \frac{2x}{3y-8x} + \frac{11x}{-10(3y-8x)}$$
$$= \frac{2x}{3y-8x} - \frac{11x}{10(3y-8x)}$$
$$= \frac{20x-11x}{10(3y-8x)}$$
$$= \frac{9x}{10(3y-8x)}$$

Eg.

$$\frac{4}{x^2-4} + \frac{1}{2-x}$$
$$= \frac{4}{(x+2)(x-2)} - \frac{1}{x-2}$$
$$= \frac{4-(x+2)}{(x+2)(x-2)}$$
$$= \frac{4-x-2}{(x+2)(x-2)}$$
$$= \frac{2-x}{(x+2)(x-2)}$$
$$= \frac{-x-2)}{(x+2)(x-2)}$$
$$= -\frac{1}{x+2}$$

Eg.

$$\frac{6p^3}{7q} \div \frac{2p}{21q^2}$$
$$= \frac{6p^3}{7q} \times \frac{21q^2}{2p} \quad \text{[do cancelling]}$$
$$= 9p^2q$$

Eg.

$$\frac{4pq^2+4pqr}{9pqr^2+9pq^2r} = \frac{4pq(q+r)}{9pqr(r+q)}$$
$$= \frac{4}{9r}$$

Eg.

$$\frac{5k^2-17k-12}{5k^2-10k-40} = \frac{(5k+3)(k-4)}{5\left(k^2-2k-8\right)}$$
$$= \frac{(5k+3)(k-4)}{5(k-4)(k+2)}$$
$$= \frac{5k+3}{5(k+2)}$$

Eg.

$$\frac{xy-z^2-xz+yz}{y^2-2yz+z^2} \div \frac{11}{2xz+x^2+z^2}$$
$$= \frac{xy-xz+yz-z^2}{y^2-2yz+z^2} \times \frac{2xz+x^2+z^2}{11}$$
$$= \frac{x(y-z)+z(y-z)}{(y-z)^2} \times \frac{(x+z)^2}{11}$$
$$= \frac{(x+z)(y-z)}{(y-z)^2} \times \frac{(x+z)^2}{11}$$
$$= \frac{(x+z)^3}{11(y-z)}$$

Eg.

$$\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}}$$
$$= \left(1-\frac{1}{x}\right) \div \left(1-\frac{1}{x^2}\right)$$
$$= \left(\frac{x-1}{x}\right) \div \left(\frac{x^2-1}{x^2}\right)$$
$$= \frac{x-1}{x} \times \frac{x^2}{x^2-1^2}$$
$$= \frac{x-1}{x} \times \frac{x^2}{(x+1)(x-1)}$$
$$= \frac{x}{x+1}$$

Making Subject of Formula

Eg: Make a the subject

$$y = m(x-a)+b$$
$$y-b = m(x-a)$$
$$m(x-a) = y-b$$
$$x-a = \frac{y-b}{m}$$
$$a = x-\frac{y-b}{m}$$

Eg: Make x the subject

$$ax-by = 3-2x$$
$$ax+2x = 3+by$$
$$x(a+2) = by+3$$
$$x = \frac{by+3}{a+2}$$

Eg: Make d the subject

$$T = 0.25\pi d^2$$
$$\pi d^2 = 4T$$
$$d^2 = \frac{4T}{\pi}$$
$$d = \pm\sqrt{\frac{4T}{\pi}}$$

Note the $\pm$ when taking square-roots in this type of question.

Eg: Make c the subject

$$d = \frac{8-c}{c+7}$$
$$d(c+7) = 8-c$$
$$cd+7d = 8-c$$
$$cd+c = 8-7d$$
$$c(d+1) = 8-7d$$
$$c = \frac{8-7d}{d+1}$$

Eg: Make q the subject

$$5a = \sqrt{\frac{b^2}{q} - \frac{3c}{4}}$$
$$25a^2 = \frac{b^2}{q} - \frac{3c}{4}$$
$$\frac{b^2}{q} = 25a^2 + \frac{3c}{4}$$
$$\frac{b^2}{q} = \frac{100a^2 + 3c}{4}$$
$$\frac{q}{b^2} = \frac{4}{100a^2 + 3c}$$
$$q = \frac{4b^2}{100a^2 + 3c}$$

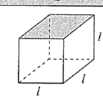
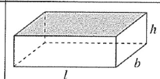
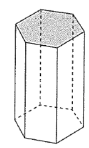
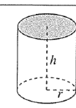
Eg: Make y the subject

$$\frac{x\left(yz - w^2\right)}{2} - \frac{y}{3} = 6y$$
$$3x\left(yz - w^2\right) - 2y = 36y$$
$$3xyz - 3w^2x - 2y = 36y$$
$$3xyz - 38y = 3w^2x$$
$$y(3xz - 38) = 3w^2x$$
$$y = \frac{3w^2x}{3xz - 38}$$

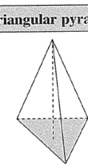
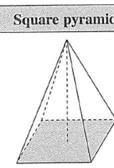
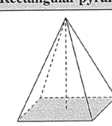
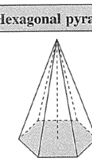
Eg: Make x the subject

$$p\sqrt{x} + q = r + s\sqrt{x}$$
$$p\sqrt{x} - s\sqrt{x} = r - q$$
$$\sqrt{x}(p - s) = r - q$$
$$\sqrt{x} = \frac{r - q}{p - s}$$
$$x = \left(\frac{r - q}{p - s}\right)^2$$

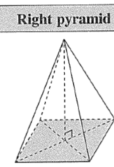
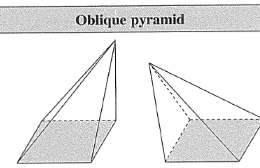
Surface Area and Volume of Solids

Name	Figure	Volume	Total Surface Area
Cube		l^3	$6l^2$
Cuboid		lbh	$2(lb + lh + bh)$
Prism		Area of cross section × height or Area of cross section × length	Total area of lateral faces + 2 × base area = perimeter of base × height + 2 × base area
Closed cylinder		$\pi r^2 h$	$2\pi r^2 + 2\pi rh$

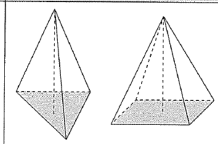
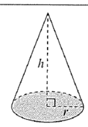
A pyramid is a solid with a polygonal base and triangles as its slanted faces. Each corner point of a pyramid is called a vertex. The vertex of the pyramid that is above the plane of the polygonal base is known as the apex of the pyramid. The apex is also the point where all the slanted triangular faces meet.

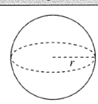
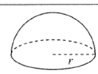
Triangular pyramid	Square pyramid	Rectangular pyramid	Hexagonal pyramid
			

A pyramid whose apex is vertically above the centre of its base is known as a right pyramid. A pyramid whose apex is not vertically above the centre of its base is known as an oblique pyramid.

Right pyramid	Oblique pyramid
	

Surface Area and Volume formulae for Pyramids, Cones, Spheres, Hemispheres:

Name	Figure	Volume	Total Surface Area
Pyramid	 and any other solid with a polygonal base and triangles as its slanted faces	$\frac{1}{3}\times$ base area $\times$ height	Total area of all faces
Cone		$\frac{1}{3}\pi r^2 h$	$\pi r^2 + \pi r l$

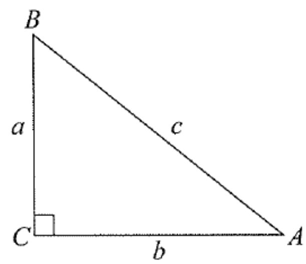
Name	Figure	Volume	Total Surface Area
Sphere		$\frac{4}{3}\pi r^3$	$4\pi r^2$
Hemisphere		$\frac{2}{3}\pi r^3$	$2\pi r^2 + \pi r^2 = 3\pi r^2$

Pythagoras' Theorem and its converse, Trigonometry

Pythagoras' Theorem:
For a right-angled $\triangle ABC$,

$$a^2 + b^2 = c^2$$

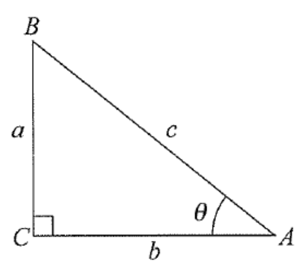
where c is the length of the hypotenuse.



Converse of Pythagoas' Theorem:
If in a $\triangle ABC$, we have $a^2 + b^2 = c^2$, then it follows that $\triangle ABC$ is a right-angled triangle with $\angle BCA = 90°$

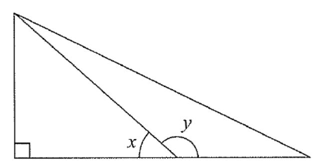
Trigonometric Ratios of an Acute Angle:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

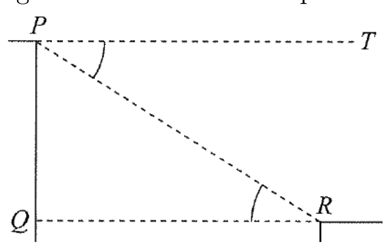


Obtuse Angles:

$$\begin{aligned} &\text{If } x + y = 180° \text{ , then} \\ &\sin y = \sin(180° - x) = \sin x \\ &\cos y = \cos(180° - x) = -\cos x \\ &\tan y = \tan(180° - x) = -\tan x \end{aligned}$$



Angles of Elevation and Depression:



Assume that QR is level ground, and PT is a line parallel to QR.
$\angle PRQ$ is known as the angle of elevation of P from R.
$\angle TPR$ is known as the angle of depression of R from P.
By alternate angles,
Angle of elevation of P from R, $\angle PRQ$ = Angle of depression of R from P, $\angle TPR$.