

# Numbers

## Types of Numbers:

- Natural Numbers:  $\mathbb{N} = \{1, 2, 3, \dots\}$
- Whole Numbers:  $\mathbb{N} \cup \{0\} = \{0, 1, 2, 3, \dots\}$
- Integers:  $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$
- Rational Numbers:  $\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$ . Rational Numbers comprise of fractions (includes all proper and improper fractions and mixed numbers). All terminating decimals (eg, 10.87) and recurring decimals (eg,  $0.3\dot{7}1 = 0.3717171\dots$ ) are rational numbers because these can all be expressed as fractions. All integers are rational numbers.
- Irrational Numbers: numbers that cannot be expressed in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are integers and  $b \neq 0$ . Eg.  $\pi$ ,  $\sqrt{2}$ ,  $\sqrt{5}$ ,  $-4\sqrt{7}$ ,  $e$ ,  $2.75e$ , etc. Irrational numbers are non-recurring decimals. Any non-zero rational number multiplied to an irrational number results in an irrational number. For example,  $-\frac{3}{4}\pi$  is irrational.
- Perfect Squares:  $\{1, 4, 9, 16, 25, \dots\}$
- Perfect Cubes:  $\{1, 8, 27, 64, \dots\}$
- Prime Numbers: Positive integers at least 2 whose only positive divisors are 1 and itself.  $\{2, 3, 5, 7, 11, 13, 17, 19, 23, \dots\}$

## Equality and Inequality Symbols

Symbol	Meaning	Example
=	is equal to	$0.1 = \frac{1}{10}$
$\neq$	is not equal to	$0.11 \neq \frac{1}{10}$
>	is greater than	$0.1 > 0.01$
$\geq$	is greater than or equal to	$a \geq 5$
<	is less than	$0.05 < 5$
$\leq$	is less than or equal to	$b \leq 5$

## Prime Factorization, HCF, LCM

- Example of prime factorization:

2	4356
2	2178
3	1089
3	363
11	121
	11

Hence  $4356 = 2^2 \times 3^2 \times 11^2$  (in index notation).

- Example of HCF and LCM using prime factorization:

$$4800 = 2^6 \times 3 \times 5^2$$

$$5544 = 2^3 \times 3^2 \times 7 \times 11$$

$$\text{HCF} = 2^3 \times 3$$

[take common prime factors and lowest power of each]

$$\text{LCM} = 2^6 \times 3^2 \times 5^2 \times 7 \times 11$$

[take all prime factors and highest power of each]

- Examples of square roots and cube roots using prime factorization:

$$54756 = 2^2 \times 3^4 \times 13^2$$

$$\sqrt{54756} = 2 \times 3^2 \times 13 = 234$$

$$1728 = 2^6 \times 3^3$$

$$\sqrt[3]{1728} = 2^2 \times 3 = 12$$

## Approximation

### Significant Figures

Rules of identifying number of significant digits:

1. All non-zero digits are significant.
2. Zeros between non-zero digits are significant.  
Eg. 302 (3 sf)  
Eg. 10.2301 (6 sf)
3. In a whole number, zeros after the last nonzero digit may or may not be significant. It depends on the estimation being made.

Eg.

$$7436000 = 7000000 \text{ (1 sf)}$$

$$7436000 = 7400000 \text{ (2 sf)}$$

$$7436000 = 7440000 \text{ (3 sf)}$$

$$7436000 = 7436000 \text{ (4 sf)}$$

$$7436000 = 7436000 \text{ (5 sf)}$$

$$7436000 = 7436000 \text{ (6 sf)}$$

4. In a decimal number, zeros before the 1<sup>st</sup> non-zero digit are not significant.

Eg.  $0.004$  (1 sf)

Eg.  $0.07008$  (4 sf)

5. In a decimal number, zeros after the last non-zero digit are significant.

Eg.  $6.40$  (3 sf)

Eg.  $12.000$  (5 sf)

Eg.  $20300.000$  (8 sf)

Eg.  $0.0700800$  (6 sf)

### Decimal Place Rounding

Examples:

$$0.7374 = 0.74 \text{ (2 dp)}$$

$$58.301 = 58.30 \text{ (2 dp)}$$

$$207.6296 = 207.630 \text{ (3 dp)}$$

$$207.6296 = 207.63 \text{ (2 dp)}$$

$$207.977 = 208.0 \text{ (1 dp)}$$

$$207.977 = 207.98 \text{ (2 dp)}$$

$$18.997 = 19.00 \text{ (2 dp)}$$

### Standard Form

$\pm A \times 10^n$ , where  $1 \leq A < 10$  and  $n$  is an integer.

Examples:

$$1350000 = 1.35 \times 10^6$$

$$0.000375 = 3.75 \times 10^{-4}$$

### Common Prefixes

$10^{12}$	trillion	tera	T
$10^9$	billion	giga	G
$10^6$	million	mega	M
$10^3$	thousand	kilo	k
$10^{-3}$	thousandth	milli	m
$10^{-6}$	millionth	micro	$\mu$
$10^{-9}$	billionth	nano	n
$10^{-12}$	trillionth	pico	p

### Indices

Rules of Indices:

Assume that  $a, b, m, n$  are non-zero.

$$a^0 = 1$$

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(a^n)^m = a^{nm}$$

$$a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

Caution:

- For indices (powers) that are not integers, the above rules of indices hold only for  $a, b > 0$ .
- Likewise, if some of the indices are negative or there is division by either  $a$  or  $b$ , then the above rules hold only for  $a, b > 0$ .
- $\sqrt{(-8)^2} = \sqrt{64} = 8$ , NOT  $-8$ .
- $\sqrt[3]{-27} = -3$ .

Equalities of Indices:

- If  $a^m = a^n$ , then  $m = n$ .
- If  $a^m = b^m$ , then  $a = b$ .

## Percentage, Ratio, Rate

- To express a percentage as a fraction or decimal, divide by 100:

$$x\% = \frac{x}{100}$$

$$\text{Eg, } 23.5\% = \frac{23.5}{100} = \frac{235}{1000} = \frac{47}{200}$$

$$\text{Eg, } 401\% = \frac{401}{100} = 4.01$$

- To express any number as a percentage, multiply it by 100%.

$$\text{Eg, } 0.165 = 0.165 \times 100\% = 16.5\%.$$

- Expressing a quantity  $A$  as a percentage of a quantity  $B$ :

$$\frac{A}{B} \times 100\%$$

Eg, Express 63.7 as a percentage of 98.

$$\text{Answer: } \frac{63.7}{98} \times 100\% = 65\%$$

In words, we say that 63.7 is 65% of 98.

- Increase or decrease a quantity by a given percentage:

Eg, Increase 45 by 2.4%:

$$\text{Answer: } 45 \times \left(1 + \frac{2.4}{100}\right) = 45 \times 1.024 = 46.08$$

Eg, Decrease 45 by 90%:

$$\text{Answer: } 45 \times \left(1 - \frac{90}{100}\right) = 45 \times 0.1 = 4.5$$

- Percentage Increase and Percentage Decrease:

When a quantity increases, the percentage increase is

$$\frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100\%$$

When a quantity decreases, the percentage decrease is

$$\frac{\text{initial (bigger) value} - \text{final (smaller) value}}{\text{initial value}} \times 100\%$$

Percentage increase will always be  $> 0$  if the quantity has increased.

Percentage decrease will always be  $> 0$  if the quantity has decreased.

Percentage change is

$$\frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100\%$$

regardless of whether the quantity has increased or decreased. Percentage change can be either

positive or negative depending on whether the quantity has increased or decreased.

- When writing ratios such as  $a : b$ ,  $a, b$  are positive integers. Always reduce ratios to the simplest form, eg,  $10 : 6$  is to be reduced to  $5 : 3$ . The ratio  $a : b$  expressed in fraction form is  $\frac{a}{b}$ .

Eg, If 7 times of  $x$  is equal to 5 times of  $y$ , then  $x : y = 5 : 7$  (note the switching of the order)

- We can use ratios to increase and decrease quantities. For example, if we increase a quantity  $x$  in the ratio  $6 : 5$ , the new quantity is  $\frac{6}{5}x$ ; if we decrease a quantity  $x$  in the ratio  $5 : 6$ , the new quantity is  $\frac{5}{6}x$ .

- Various units of measurement:

Mass:

$$1 \text{ kg} = 1000 \text{ g}$$

$$1 \text{ g} = 1000 \text{ mg}$$

Length:

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ cm} = 10 \text{ mm}$$

Area:

$$1 \text{ km}^2 = 10^6 \text{ m}^2$$

$$1 \text{ m}^2 = 10000 \text{ cm}^2$$

Volume:

$$1 \text{ l} = 1000 \text{ ml}$$

$$1 \text{ cm}^3 = 1 \text{ ml}$$

$$1 \text{ m}^3 = 10^6 \text{ cm}^3 = 1000 \text{ l}$$

Time:

$$1 \text{ hr} = 60 \text{ min}$$

$$1 \text{ min} = 60 \text{ sec}$$

- Distance = Speed  $\times$  Time

- Average speed = (Total Distance) / (Total Time Taken)

- Conversion of units for speed:

$$26\text{km/h} = 26000\text{m/h} = \frac{26000}{3600}\text{m/s} = \frac{65}{9}\text{m/s}$$

$$35\text{m/s} = 0.035\text{km/s} = (0.035 \times 3600)\text{km/h} = 126\text{km/h}$$

- Density = Mass / Volume.

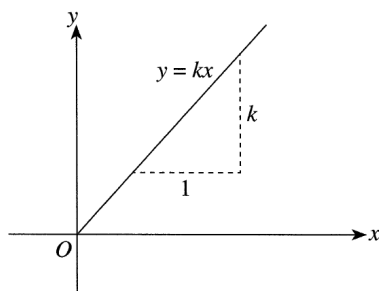
Units are usually  $\text{g/cm}^3$  or  $\text{kg/m}^3$ .

$$1\text{g/cm}^3 = 1000\text{kg/m}^3.$$

$$\text{Eg, } 0.235\text{g/cm}^3 = 235\text{kg/m}^3.$$

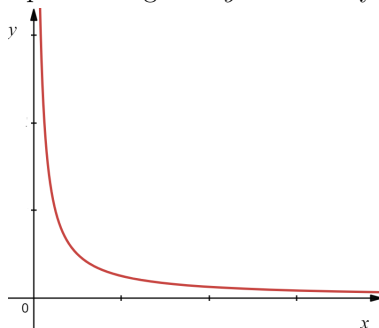
## Direct and Inverse Proportion

If  $y$  is directly proportional to  $x$ , then  $y = kx$ , where  $k$  is a constant and  $k \neq 0$ . The ratios  $\frac{x}{y}$  and  $\frac{y}{x}$  are constant. Furthermore, the graph on  $y$  against  $x$  (or of  $x$  against  $y$ ) is a straight line through the origin. Graph showing that  $y$  is directly proportional to  $x$ :



If  $y$  is inversely proportional to  $x$ , then  $y = \frac{k}{x}$ , where  $k$  is a constant and  $k \neq 0$ . The product  $xy$  is constant.

Graph showing that  $y$  is inversely proportional to  $x$ :



## Map Scales

- Linear scale:

$1 : n$  means 1 unit length on map represents  $n$  units length on ground.

Eg.  $1 : 5000$  means

1 cm represents 5000 cm

which implies 1 cm represents 50 m

which implies 1 cm represents 0.05 km

- Representative Fraction (RF):

If the linear scale is  $1 : n$ , the RF is expressed as  $\frac{1}{n}$ .

Eg, if 3 cm represents 6 m, then RF is  $\frac{1}{200}$ .

- Area Scale:

If linear scale is  $1 : 20000$ , then it means

1 cm represents 20000 cm

which implies 1 cm represents 0.2 km

which implies  $1^2 \text{ cm}^2$  represents  $(0.2)^2 \text{ km}^2$

which implies 1  $\text{cm}^2$  represents 0.04  $\text{km}^2$

## Number Patterns

Common number patterns:

- Constant difference

Eg,  $-5, -2, 1, 4, 7, 10, \dots$

The  $n^{\text{th}}$  term, denoted  $T_n$ , is given by

$$T_n = a + d(n - 1)$$

where  $a$  is the first term and  $d$  is the common difference.

For the sequence  $-5, -2, 1, 4, 7, 10, \dots$ ,

$$T_n = -5 + (n - 1)(3) = 3n - 8.$$

Alternatively,

$$T_n = b + dn$$

where  $b$  is the term that would have come before the first term (ie, the “zeroth” term).

- Constant multiple (or common ratio)

Eg, 3, 15, 75, 375, ...

$$T_n = a \times r^{n-1}$$

where  $a$  is the first term,  $r$  is the common ratio, that is,  $r$  is the number that when multiplied a term gives the next term.

For the sequence 3, 15, 75, 375, ...

$$T_n = 3 \times 5^{n-1}$$

- Perfect squares and perfect cubes

$$1, 4, 9, 16, 25, \dots : T_n = n^2$$

$$1, 8, 27, 64, 125, \dots : T_n = n^3$$

$$2, 8, 18, 32, 50, \dots : T_n = 2n^2$$

$$3, 10, 29, 66, 127, \dots : T_n = n^3 + 2$$

## Algebra

### Simultaneous Linear Equations

Method 1: Elimination

$$5x - 2y = 21 \quad \text{---(1)}$$

$$2x - y = 8 \quad \text{---(2)}$$

$$(1) \times 2 : 10x - 4y = 42 \quad \text{---(3)}$$

$$(2) \times 5 : 10x - 5y = 40 \quad \text{---(4)}$$

$$(3) - (4) : y = 2$$

$$\text{Sub into (1): } 5x - 2(2) = 21$$

$$5x - 4 = 21$$

$$5x = 25$$

$$x = 5$$

Method 2: Substitution

$$5x - 2y = 21 \quad \text{---(1)}$$

$$2x - y = 8 \quad \text{---(2)}$$

From (2):

$$y = 2x - 8 \quad \text{---(3)}$$

$$\text{Sub (3) into (1): } 5x - 2(2x - 8) = 21$$

$$5x - 4x + 16 = 21$$

$$x - 16 = 21$$

$$x = 5$$

Sub into (2):

$$2(5) - y = 8$$

$$10 - y = 8$$

$$y = 2$$

### Expansion

Eg.

$$2p - 3(p + 1)$$

$$= 2p - 3p - 3$$

$$= -p - 3$$

Eg.

$$5x - (x + 1)(2x - 3)$$

$$= 5x - (2x^2 - 3x + 2x - 3)$$

$$= 5x - 2x^2 + 3x - 2x + 3$$

$$= -2x^2 + 6x + 3$$

### Factorization and Identities

Factorization of Quadratic Expressions:

$$5x^2 + 9x - 2$$

$\times$	$5x$	$-1$
$x$	$5x^2$	$-x$
$2$	$10x$	$-2$

$$\therefore 5x^2 + 9x - 2 = (5x - 1)(x + 2)$$

Identities:

$$1. (a + b)^2 = a^2 + 2ab + b^2$$

$$2. (a - b)^2 = a^2 - 2ab + b^2$$

$$3. (a + b)(a - b) = a^2 - b^2$$

Common Factorisation Techniques:

- Common Factors

$$\text{Eg. } 6a^3b - 2a^2b = 2a^2b(3a - 1)$$

- Grouping

Eg.

$$6p^2 - 3pq - 10ap + 5aq$$

$$= 3p(2p - q) - 5a(2p - q)$$

$$= (3p - 5a)(2p - q)$$

- Using Difference of Two Squares

$$\text{Eg. } 9a^2 - 1$$

$$= (3a)^2 - (1)^2$$

$$= (3a + 1)(3a - 1)$$

Eg.  $16a^4 - 81$

$$= (4a^2 + 9)(4a^2 - 9)$$

$$= (4a^2 + 9)(2a + 3)(2a - 3)$$

- Combination of methods:

Eg.  $3x^3 - 12xy^2 = 3x(x^2 - 4y^2)$  common factor  
 $= 3x(x + 2y)(x - 2y)$  then diff. of 2 squares

Always try common factor first

Eg.

$$4 - p^2 + 6pq - 9q^2$$

$$= 4 - (p^2 - 6pq + 9q^2)$$

$$= (2)^2 - (p - 3q)^2$$

$$= (2 + (p - 3q))(2 - (p - 3q))$$

$$= (2 + p - 3q)(2 - p + 3q)$$

## Algebraic Fractions

Eg.

$$\begin{aligned} & \frac{x+2}{3} - \frac{x-5}{2} \\ &= \frac{2(x+2) - 3(x-5)}{6} \\ &= \frac{2x+4-3x+15}{6} \\ &= \frac{19-x}{6} \end{aligned}$$

Eg.

$$\begin{aligned} & \frac{5}{x+1} - \frac{2}{x-3} \\ &= \frac{5(x-3) - 2(x+1)}{(x+1)(x-3)} \\ &= \frac{5x-15-2x-2}{(x+1)(x-3)} \\ &= \frac{3x-17}{(x+2)(x-5)} \end{aligned}$$

Eg.

$$\begin{aligned} & \frac{5}{3x} + \frac{2}{x} \\ &= \frac{5}{3x} + \frac{6}{3x} \\ &= \frac{5+6}{3x} \\ &= \frac{11}{3x} \end{aligned}$$

Eg.

$$\begin{aligned} & \frac{3}{(x+2)^2} - \frac{4}{x+2} \\ &= \frac{3}{(x+2)^2} - \frac{4(x+2)}{(x+2)^2} \\ &= \frac{3-4(x+2)}{(x+2)^2} \\ &= \frac{3-4x-8}{(x+2)^2} \\ &= \frac{-4x-5}{(x+2)^2} \end{aligned}$$

Eg.

$$\begin{aligned} & \frac{7}{x^2-9} - \frac{1}{x-3} \\ &= \frac{7}{(x+3)(x-3)} - \frac{1}{x-3} \\ &= \frac{7}{(x+3)(x-3)} - \frac{x-3}{(x-3)^2} \\ &= \frac{7-(x+3)}{(x+3)(x-3)} \\ &= \frac{7-x-3}{(x+3)(x-3)} \\ &= \frac{4-x}{(x+3)(x-3)} \end{aligned}$$

Eg.

$$\begin{aligned} & \frac{9}{x-5} + \frac{3}{5-x} \\ &= \frac{9}{x-5} - \frac{3}{x-5} \\ &= \frac{9-3}{x-5} \\ &= \frac{6}{x-5} \end{aligned}$$

Eg.

$$\begin{aligned} & \frac{6p^3}{7q} \div \frac{2p}{21q^2} \\ &= \frac{6p^3}{7q} \times \frac{21q^2}{2p} \quad [\text{do cancelling}] \\ &= 9p^2q \end{aligned}$$

## Inequalities

Inequality sign is reversed when both sides are multiplied or divided by a negative number.

Eg.  $3(x - 1) < 4x + 1 \leq 7 + 2x$

$$\begin{array}{l|l} 3(x - 1) < 4x + 1 & 4x + 1 \leq 7 + 2x \\ 3x - 3 < 4x + 1 & 4x - 2x \leq 7 - 1 \\ 3x - 4x < 1 + 3 & 2x \leq 6 \\ -x < 4 & x \leq 3 \\ x > -4 & \end{array}$$

Ans:  $-4 < x \leq 3$