

Numbers

- Natural Numbers: $\mathbb{N} = \{1, 2, 3, \dots\}$
- Whole Numbers: $\mathbb{N} \cup \{0\} = \{0, 1, 2, 3, \dots\}$
- Integers: $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$
- Rational Numbers: $\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$. Rational Numbers comprise of fractions (includes all proper and improper fractions and mixed numbers). All terminating decimals (eg, 10.87) and recurring decimals (eg, $0.3\overline{71} = 0.3717171\dots$) are rational numbers because these can all be expressed as fractions. All integers are rational numbers.
- Irrational Numbers: numbers that cannot be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Eg. π , $\sqrt{2}$, $\sqrt{5}$, $-4\sqrt{7}$, e , $2.75e$, etc. Irrational numbers are non-recurring decimals. Any non-zero rational number multiplied to an irrational number results in an irrational number. For example, $-\frac{3}{4}\pi$ is irrational.
- Perfect Squares: $\{1, 4, 9, 16, 25, \dots\}$
- Perfect Cubes: $\{1, 8, 27, 64, \dots\}$
- Prime Numbers: Positive integers at least 2 whose only positive divisors are 1 and itself. $\{2, 3, 5, 7, 11, 13, 17, 19, 23, \dots\}$
- Composite Numbers: Positive integers ≥ 4 that are not prime, in other words, having factors apart from 1 and itself.

Equality and Inequality Symbols

Symbol	Meaning	Example
=	is equal to	$0.1 = \frac{1}{10}$
\neq	is not equal to	$0.11 \neq \frac{1}{10}$
>	is greater than	$0.1 > 0.01$
\geq	is greater than or equal to	$a \geq 5$
<	is less than	$0.05 < 5$
\leq	is less than or equal to	$b \leq 5$

Prime Factorization, HCF, LCM

- Example of prime factorization:

2	4356
2	2178
3	1089
3	363
11	121
	11

Hence $4356 = 2^2 \times 3^2 \times 11^2$ (in index notation).

- Example of HCF and LCM using prime factorization:

$$4800 = 2^6 \times 3 \times 5^2$$

$$5544 = 2^3 \times 3^2 \times 7 \times 11$$

$$\text{HCF} = 2^3 \times 3$$

[take common prime factors and lowest power of each]

$$\text{LCM} = 2^6 \times 3^2 \times 5^2 \times 7 \times 11$$

[take all prime factors and highest power of each]

- Examples of square roots and cube roots using prime factorization:

$$54756 = 2^2 \times 3^4 \times 13^2$$

$$\sqrt{54756} = 2 \times 3^2 \times 13 = 234$$

$$1728 = 2^6 \times 3^3$$

$$\sqrt[3]{1728} = 2^2 \times 3 = 12$$

Approximation

Significant Figures

Rules of identifying number of significant digits:

1. All non-zero digits are significant.
2. Zeros between non-zero digits are significant.
Eg. 302 (3 sf)
Eg. 10.2301 (6 sf)
3. In a whole number, zeros after the last nonzero digit may or may not be significant. It depends on the estimation being made.

Eg.

$$7436000 = 7000000 \text{ (1 sf)}$$

$$7436000 = 7400000 \text{ (2 sf)}$$

$$7436000 = 7440000 \text{ (3 sf)}$$

$$7436000 = 7436000 \text{ (4 sf)}$$

$$7436000 = 7436000 \text{ (5 sf)}$$

$$7436000 = 7436000 \text{ (6 sf)}$$

4. In a decimal number, zeros before the 1st non-zero digit are not significant.

Eg. 0.004 (1 sf)

Eg. 0.07008 (4 sf)

5. In a decimal number, zeros after the last non-zero digit are significant.

Eg. 6.40 (3 sf)

Eg. 12.000 (5 sf)

Eg. 20300.000 (8 sf)

Eg. 0.0700800 (6 sf)

Decimal Place Rounding

Examples:

$$0.7374 = 0.74 \text{ (2 dp)}$$

$$58.301 = 58.30 \text{ (2 dp)}$$

$$207.6296 = 207.630 \text{ (3 dp)}$$

$$207.6296 = 207.63 \text{ (2 dp)}$$

$$207.977 = 208.0 \text{ (1 dp)}$$

$$207.977 = 207.98 \text{ (2 dp)}$$

$$18.997 = 19.00 \text{ (2 dp)}$$

Standard Form

$\pm A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

Examples:

$$1350000 = 1.35 \times 10^6$$

$$0.000375 = 3.75 \times 10^{-4}$$

Common Prefixes

10^{12}	trillion	tera	T
10^9	billion	giga	G
10^6	million	mega	M
10^3	thousand	kilo	k
10^{-3}	thousandth	milli	m
10^{-6}	millionth	micro	μ
10^{-9}	billionth	nano	n
10^{-12}	trillionth	pico	p

Indices

Rules of Indices:

Assume that a, b, m, n are non-zero.

$$a^0 = 1$$

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(a^n)^m = a^{nm}$$

$$a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

Caution:

1. For indices (powers) that are not integers, the above rules of indices hold only for $a, b > 0$.

2. Likewise, if some of the indices are negative or there is division by either a or b , then the above rules hold only for $a, b > 0$.

3. $\sqrt{(-8)^2} = \sqrt{64} = 8$, NOT -8 .

4. $\sqrt[3]{-27} = -3$.

Equalities of Indices:

1. If $a^m = a^n$, then $m = n$.

2. If $a^m = b^m$, then $a = b$.

Percentage, Ratio, Rate

- To express a percentage as a fraction or decimal, divide by 100:

$$x\% = \frac{x}{100}$$

$$\text{Eg, } 23.5\% = \frac{23.5}{100} = \frac{235}{1000} = \frac{47}{200}$$

$$\text{Eg, } 401\% = \frac{401}{100} = 4.01$$

- To express any number as a percentage, multiply it by 100%.

$$\text{Eg, } 0.165 = 0.165 \times 100\% = 16.5\%.$$

- Expressing a quantity A as a percentage of a quantity B :

$$\frac{A}{B} \times 100\%$$

Eg, Express 63.7 as a percentage of 98.

$$\text{Answer: } \frac{63.7}{98} \times 100\% = 65\%$$

In words, we say that 63.7 is 65% of 98.

- Increase or decrease a quantity by a given percentage:

Eg, Increase 45 by 2.4%:

$$\text{Answer: } 45 \times \left(1 + \frac{2.4}{100}\right) = 45 \times 1.024 = 46.08$$

Eg, Decrease 45 by 90%:

$$\text{Answer: } 45 \times \left(1 - \frac{90}{100}\right) = 45 \times 0.1 = 4.5$$

- Percentage Increase and Percentage Decrease:

When a quantity increases, the percentage increase is

$$\frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100\%$$

When a quantity decreases, the percentage decrease is

$$\frac{\text{initial (bigger) value} - \text{final (smaller) value}}{\text{initial value}} \times 100\%$$

Percentage increase will always be > 0 if the quantity has increased.

Percentage decrease will always be > 0 if the quantity has decreased.

Percentage change is

$$\frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100\%$$

regardless of whether the quantity has increased or decreased. Percentage change can be either

positive or negative depending on whether the quantity has increased or decreased.

- When writing ratios such as $a : b$, a, b are positive integers. Always reduce ratios to the simplest form, eg, $10 : 6$ is to be reduced to $5 : 3$. The ratio $a : b$ expressed in fraction form is $\frac{a}{b}$.

Eg, If 7 times of x is equal to 5 times of y , then $x : y = 5 : 7$ (note the switching of the order)

- We can use ratios to increase and decrease quantities. For example, if we increase a quantity x in the ratio $6 : 5$, the new quantity is $\frac{6}{5}x$; if we decrease a quantity x in the ratio $5 : 6$, the new quantity is $\frac{5}{6}x$.

- Various units of measurement:

Mass:

$$1 \text{ kg} = 1000 \text{ g}$$

$$1 \text{ g} = 1000 \text{ mg}$$

Length:

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ cm} = 10 \text{ mm}$$

Area:

$$1 \text{ km}^2 = 10^6 \text{ m}^2$$

$$1 \text{ m}^2 = 10000 \text{ cm}^2$$

Volume:

$$1 \text{ l} = 1000 \text{ ml}$$

$$1 \text{ cm}^3 = 1 \text{ ml}$$

$$1 \text{ m}^3 = 10^6 \text{ cm}^3 = 1000 \text{ l}$$

Time:

$$1 \text{ hr} = 60 \text{ min}$$

$$1 \text{ min} = 60 \text{ sec}$$

- Distance = Speed \times Time

- Average speed = (Total Distance) / (Total Time Taken)

- Conversion of units for speed:

$$26\text{km/h} = 26000\text{m/h} = \frac{26000}{3600}\text{m/s} = \frac{65}{9}\text{m/s}$$

$$35\text{m/s} = 0.035\text{km/s} = (0.035 \times 3600)\text{km/h} = 126\text{km/h}$$

- Density = Mass / Volume.

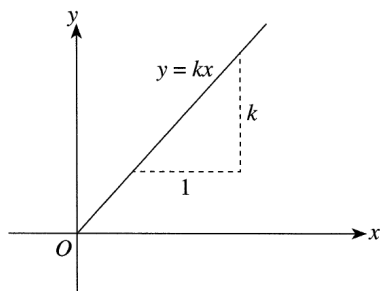
Units are usually g/cm^3 or kg/m^3 .

$$1\text{g/cm}^3 = 1000\text{kg/m}^3.$$

$$\text{Eg, } 0.235\text{g/cm}^3 = 235\text{kg/m}^3.$$

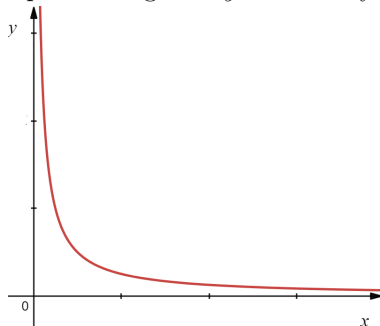
Direct and Inverse Proportion

If y is directly proportional to x , then $y = kx$, where k is a constant and $k \neq 0$. The ratios $\frac{x}{y}$ and $\frac{y}{x}$ are constant. Furthermore, the graph on y against x (or of x against y) is a straight line through the origin. Graph showing that y is directly proportional to x :



If y is inversely proportional to x , then $y = \frac{k}{x}$, where k is a constant and $k \neq 0$. The product xy is constant.

Graph showing that y is inversely proportional to x :



Map Scales

- Linear scale:

$1 : n$ means 1 unit length on map represents n units length on ground.

Eg. $1 : 5000$ means

1 cm represents 5000 cm

which implies 1 cm represents 50 m

which implies 1 cm represents 0.05 km

- Representative Fraction (RF):

If the linear scale is $1 : n$, the RF is expressed as $\frac{1}{n}$.

Eg, if 3 cm represents 6 m, then RF is $\frac{1}{200}$.

- Area Scale:

If linear scale is $1 : 20000$, then it means

1 cm represents 20000 cm

which implies 1 cm represents 0.2 km

which implies 1^2 cm^2 represents $(0.2)^2 \text{ km}^2$

which implies 1 cm^2 represents 0.04 km^2

Number Patterns

Common number patterns:

- Constant difference

Eg, $-5, -2, 1, 4, 7, 10, \dots$

The n^{th} term, denoted T_n , is given by

$$T_n = a + d(n - 1)$$

where a is the first term and d is the common difference.

For the sequence $-5, -2, 1, 4, 7, 10, \dots$,

$$T_n = -5 + (n - 1)(3) = 3n - 8.$$

Alternatively,

$$T_n = b + dn$$

where b is the term that would have come before the first term (ie, the “zeroth” term).

- Constant multiple (or common ratio)

Eg, 3, 15, 75, 375, ...

$$T_n = a \times r^{n-1}$$

where a is the first term, r is the common ratio, that is, r is the number that when multiplied a term gives the next term.

For the sequence 3, 15, 75, 375, ...

$$T_n = 3 \times 5^{n-1}.$$

- Perfect squares and perfect cubes

$$1, 4, 9, 16, 25, \dots : T_n = n^2$$

$$1, 8, 27, 64, 125, \dots : T_n = n^3$$

$$2, 8, 18, 32, 50, \dots : T_n = 2n^2$$

$$3, 10, 29, 66, 127, \dots : T_n = n^3 + 2$$

Simultaneous Linear Equations

Method 1: Elimination

$$5x - 2y = 21 \quad \text{---(1)}$$

$$2x - y = 8 \quad \text{---(2)}$$

$$(1) \times 2 : 10x - 4y = 42 \quad \text{---(3)}$$

$$(2) \times 5 : 10x - 5y = 40 \quad \text{---(4)}$$

$$(3) - (4) : y = 2$$

Sub into (1): $5x - 2(2) = 21$

$$5x - 4 = 21$$

$$5x = 25$$

$$x = 5$$

Method 2: Substitution

$$5x - 2y = 21 \quad \text{---(1)}$$

$$2x - y = 8 \quad \text{---(2)}$$

From (2):

$$y = 2x - 8 \quad \text{---(3)}$$

Sub (3) into (1): $5x - 2(2x - 8) = 21$

$$5x - 4x + 16 = 21$$

$$x - 16 = 21$$

$$x = 5$$

Sub into (2):

$$2(5) - y = 8$$

$$10 - y = 8$$

$$y = 2$$

Inequalities

Inequality sign is reversed when both sides are multiplied or divided by a negative number.

$$\text{Eg. } -3x + 4 \geq 12$$

$$-3x \geq 12 - 4$$

$$-3x \geq 8$$

$$x \leq -\frac{8}{3}$$

$$\text{Eg. } 3(x - 1) < 4x + 1 \leq 7 + 2x$$

$$3(x - 1) < 4x + 1 \quad \left| \quad 4x + 1 \leq 7 + 2x \right.$$

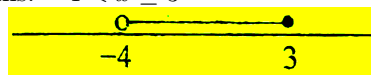
$$3x - 3 < 4x + 1 \quad \left| \quad 4x - 2x \leq 7 - 1 \right.$$

$$3x - 4x < 1 + 3 \quad \left| \quad 2x \leq 6 \right.$$

$$-x < 4 \quad \left| \quad x \leq 3 \right.$$

$$x > -4$$

$$\text{Ans: } -4 < x \leq 3$$

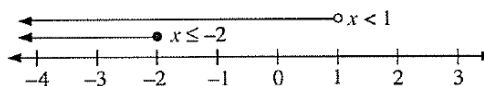


$$\text{Eg. } 5x + 4 \leq 3x < 6 - 3x$$

$$5x + 4 \leq 3x \quad \left| \quad 3x < 6 - 3x \right.$$

$$2x \leq -4 \quad \left| \quad 6x < 6 \right.$$

$$x \leq -2 \quad \left| \quad x < 1 \right.$$



$$\text{Ans: } x \leq -2.$$

Expansion

Eg.

$$2p - 3(p + 1)$$

$$= 2p - 3p - 3$$

$$= -p - 3$$

Eg.

$$5x - (x + 1)(2x - 3)$$

$$= 5x - (2x^2 - 3x + 2x - 3)$$

$$= 5x - 2x^2 + 3x - 2x + 3$$

$$= -2x^2 + 6x + 3$$

Factorization and Identities

Factorization of Quadratic Expressions:

$$5x^2 + 9x - 2$$

×	$5x$	-1
x	$5x^2$	$-x$
2	$10x$	-2

$$\therefore 5x^2 + 9x - 2 = (5x - 1)(x + 2)$$

Identities:

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)(a - b) = a^2 - b^2$

Common Factorisation Techniques:

- Common Factors

Eg. $6a^3b - 2a^2b = 2a^2b(3a - 1)$

- Grouping

Eg.

$$\begin{aligned} 6p^2 - 3pq - 10ap + 5aq \\ = 3p(2p - q) - 5a(2p - q) \\ = (3p - 5a)(2p - q) \end{aligned}$$

- Using Difference of Two Squares

Eg. $9a^2 - 1$

$$\begin{aligned} &= (3a)^2 - (1)^2 \\ &= (3a + 1)(3a - 1) \end{aligned}$$

Eg. $16a^4 - 81$

$$\begin{aligned} &= (4a^2 + 9)(4a^2 - 9) \\ &= (4a^2 + 9)(2a + 3)(2a - 3) \end{aligned}$$

- Combination of methods:

Eg. $3x^3 - 12xy^2 = 3x(x^2 - 4y^2)$ common factor
 $= 3x(x + 2y)(x - 2y)$ then diff. of 2 squares

Always try common factor first

Eg.

$$\begin{aligned} 4 - p^2 + 6pq - 9q^2 \\ = 4 - (p^2 - 6pq + 9q^2) \\ = (2)^2 - (p - 3q)^2 \\ = (2 + (p - 3q))(2 - (p - 3q)) \\ = (2 + p - 3q)(2 - p + 3q) \end{aligned}$$

Algebraic Fractions

Eg.

$$\begin{aligned} \frac{x+2}{3} - \frac{x-5}{2} &= \frac{2(x+2)}{6} - \frac{3(x-5)}{6} \\ &= \frac{2(x+2) - 3(x-5)}{6} \\ &= \frac{2x+4-3x+15}{6} \\ &= \frac{19-x}{6} \end{aligned}$$

Eg.

$$\begin{aligned} \frac{5}{x+1} - \frac{2}{x-3} \\ &= \frac{5(x-3)}{(x+1)(x-3)} - \frac{2(x+1)}{(x-3)(x+1)} \\ &= \frac{5(x-3) - 2(x+1)}{(x+1)(x-3)} \\ &= \frac{5x-15-2x-2}{(x+1)(x-3)} \\ &= \frac{3x-17}{(x+2)(x-5)} \end{aligned}$$

Eg.

$$\begin{aligned} \frac{5}{3x} + \frac{2}{x} \\ &= \frac{5}{3x} + \frac{6}{3x} \\ &= \frac{5+6}{3x} \\ &= \frac{11}{3x} \end{aligned}$$

Eg.

$$\begin{aligned} \frac{3}{(x+2)^2} - \frac{4}{x+2} \\ &= \frac{3}{(x+2)^2} - \frac{4(x+2)}{(x+2)^2} \\ &= \frac{3-4(x+2)}{(x+2)^2} \\ &= \frac{3-4x-8}{(x+2)^2} \\ &= \frac{-4x-5}{(x+2)^2} \end{aligned}$$

Eg.

$$\begin{aligned}
& \frac{7}{x^2-9} - \frac{1}{x-3} \\
&= \frac{7}{(x+3)(x-3)} - \frac{1}{x-3} \\
&= \frac{7}{(x+3)(x-3)} - \frac{x-3}{(x-3)^2} \\
&= \frac{7-(x+3)}{(x+3)(x-3)} \\
&= \frac{7-x-3}{(x+3)(x-3)} \\
&= \frac{4-x}{(x+3)(x-3)}
\end{aligned}$$

Eg.

$$\begin{aligned}
& \frac{3x+2}{x^2-4} + \frac{1}{x+2} - \frac{2}{x-2} \\
&= \frac{3x+2}{(x+2)(x-2)} + \frac{1}{x+2} - \frac{2}{x-2} \\
&= \frac{3x+2+(x-2)-2(x+2)}{(x+2)(x-2)} \\
&= \frac{3x+2+x-2-2x-4}{(x+2)(x-2)} \\
&= \frac{2x-4}{(x+2)(x-2)} \\
&= \frac{2(x-2)}{(x+2)(x-2)} \\
&= \frac{2}{x+2}
\end{aligned}$$

Eg.

$$\begin{aligned}
& \frac{9}{x-5} + \frac{3}{5-x} \\
&= \frac{9}{x-5} - \frac{3}{x-5} \\
&= \frac{9-3}{x-5} \\
&= \frac{6}{x-5}
\end{aligned}$$

Eg.

$$\begin{aligned}
& \frac{2x}{3y-8x} + \frac{11x}{80x-30y} \\
&= \frac{2x}{3y-8x} + \frac{11x}{-10(3y-8x)} \\
&= \frac{2x}{3y-8x} - \frac{11x}{10(3y-8x)} \\
&= \frac{20x-11x}{10(3y-8x)} \\
&= \frac{9x}{10(3y-8x)}
\end{aligned}$$

Eg.

$$\begin{aligned}
& \frac{4}{x^2-4} + \frac{1}{2-x} \\
&= \frac{4}{(x+2)(x-2)} - \frac{1}{x-2} \\
&= \frac{4-(x+2)}{(x+2)(x-2)} \\
&= \frac{4-x-2}{(x+2)(x-2)} \\
&= \frac{2-x}{(x+2)(x-2)} \\
&= \frac{-x-2}{(x+2)(x-2)} \\
&= -\frac{1}{x+2}
\end{aligned}$$

Eg.

$$\begin{aligned}
& \frac{6p^3}{7q} \div \frac{2p}{21q^2} \\
&= \frac{6p^3}{7q} \times \frac{21q^2}{2p} \quad [\text{do cancelling}] \\
&= 9p^2q
\end{aligned}$$

Eg.

$$\begin{aligned}
& \frac{4pq^2+4pqr}{9pqr^2+9pq^2r} = \frac{4pq(q+r)}{9pqr(r+q)} \\
&= \frac{4}{9r}
\end{aligned}$$

Eg.

$$\begin{aligned}\frac{5k^2 - 17k - 12}{5k^2 - 10k - 40} &= \frac{(5k + 3)(k - 4)}{5(k^2 - 2k - 8)} \\ &= \frac{(5k + 3)(k - 4)}{5(k - 4)(k + 2)} \\ &= \frac{5k + 3}{5(k + 2)}\end{aligned}$$

Eg.

$$\begin{aligned}\frac{xy - z^2 - xz + yz}{y^2 - 2yz + z^2} &\div \frac{11}{2xz + x^2 + z^2} \\ &= \frac{xy - xz + yz - z^2}{y^2 - 2yz + z^2} \times \frac{2xz + x^2 + z^2}{11} \\ &= \frac{x(y - z) + z(y - z)}{(y - z)^2} \times \frac{(x + z)^2}{11} \\ &= \frac{(x + z)(y - z)}{(y - z)^2} \times \frac{(x + z)^2}{11} \\ &= \frac{(x + z)^3}{11(y - z)}\end{aligned}$$

Eg.

$$\begin{aligned}\frac{1}{x} + 1 \\ &= \left(\frac{1}{x} + 1\right) \div (x + 1) \\ &= \frac{1 + x}{x} \times \frac{1}{x + 1} \\ &= \frac{1}{x}\end{aligned}$$

Eg.

$$\begin{aligned}\frac{1 - \frac{1}{x}}{1 - \frac{1}{x^2}} \\ &= \left(1 - \frac{1}{x}\right) \div \left(1 - \frac{1}{x^2}\right) \\ &= \left(\frac{x - 1}{x}\right) \div \left(\frac{x^2 - 1}{x^2}\right) \\ &= \frac{x - 1}{x} \times \frac{x^2}{x^2 - 1^2} \\ &= \frac{x - 1}{x} \times \frac{x^2}{(x + 1)(x - 1)} \\ &= \frac{x}{x + 1}\end{aligned}$$

Making Subject of Formula

Eg: Make a the subject

$$y = m(x - a) + b$$

$$y - b = m(x - a)$$

$$m(x - a) = y - b$$

$$x - a = \frac{y - b}{m}$$

$$a = x - \frac{y - b}{m}$$

Eg: Make x the subject

$$ax - by = 3 - 2x$$

$$ax + 2x = 3 + by$$

$$x(a + 2) = by + 3$$

$$x = \frac{by + 3}{a + 2}$$

Eg: Make d the subject

$$T = 0.25\pi d^2$$

$$\pi d^2 = 4T$$

$$d^2 = \frac{4T}{\pi}$$

$$d = \pm \sqrt{\frac{4T}{\pi}}$$

Note the \pm when taking square-roots in this type of question.

Eg: Make c the subject

$$d = \frac{8 - c}{c + 7}$$

$$d(c + 7) = 8 - c$$

$$cd + 7d = 8 - c$$

$$cd + c = 8 - 7d$$

$$c(d + 1) = 8 - 7d$$

$$c = \frac{8 - 7d}{d + 1}$$

Eg: Make q the subject

$$\begin{aligned}
 5a &= \sqrt{\frac{b^2}{q} - \frac{3c}{4}} \\
 25a^2 &= \frac{b^2}{q} - \frac{3c}{4} \\
 \frac{b^2}{q} &= 25a^2 + \frac{3c}{4} \\
 \frac{b^2}{q} &= \frac{100a^2 + 3c}{4} \\
 \frac{q}{b^2} &= \frac{4}{100a^2 + 3c} \\
 q &= \frac{4b^2}{100a^2 + 3c}
 \end{aligned}$$

Eg: Make y the subject

$$\begin{aligned}
 \frac{x(yz - w^2)}{2} - \frac{y}{3} &= 6y \\
 3x(yz - w^2) - 2y &= 36y \\
 3xyz - 3w^2x - 2y &= 36y \\
 3xyz - 38y &= 3w^2x \\
 y(3xz - 38) &= 3w^2x \\
 y &= \frac{3w^2x}{3xz - 38}
 \end{aligned}$$

Eg: Make x the subject

$$\begin{aligned}
 p\sqrt{x} + q &= r + s\sqrt{x} \\
 p\sqrt{x} - s\sqrt{x} &= r - q \\
 \sqrt{x}(p - s) &= r - q \\
 \sqrt{x} &= \frac{r - q}{p - s} \\
 x &= \left(\frac{r - q}{p - s}\right)^2
 \end{aligned}$$

Solving Quadratic Equations and Equations Involving Algebraic Fractions

Three methods of solving quadratic equations:

1. Factorisation

$$\begin{aligned}
 3x^2 - 2x - 8 &= 0 \\
 (3x + 4)(x - 2) &= 0 \\
 3x + 4 = 0 \text{ or } x - 2 &= 0 \\
 x = -\frac{4}{3} \text{ or } x &= 2
 \end{aligned}$$

2. Using General Formula

$$\begin{aligned}
 ax^2 + bx + c &= 0 \\
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
 \end{aligned}$$

Eg. $12x^2 - x - 25 = 0$

$$\begin{aligned}
 a = 12, b = -1, c &= -25 \\
 x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(12)(-25)}}{2(12)} \\
 x &= \frac{1 \pm \sqrt{201}}{24} \\
 x &= \frac{1 + \sqrt{201}}{24} \text{ or } \frac{1 - \sqrt{201}}{24} \\
 x &= 3.79(2\text{dp}) \text{ or } -3.29(2\text{dp})
 \end{aligned}$$

3. Complete the Square

$$\begin{aligned}
 x^2 - 8x + 13 &= (x + p)^2 + q \\
 x^2 - 8x + 13 &= x^2 - 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 + 13 \\
 &= x^2 - 8x + 16 - 16 + 13 \\
 &= (x - 4)^2 - 3 \\
 \text{To Solve } x^2 - 8x + 13 &= 0 \\
 \text{First Change To } (x - 4)^2 - 3 &= 0 \\
 (x - 4)^2 &= 3 \\
 x - 4 &= \pm\sqrt{3} \\
 x - 4 &= \sqrt{3} \text{ or } x - 4 = -\sqrt{3} \\
 x &= \sqrt{3} + 4 \text{ or } x = -\sqrt{3} + 4 \\
 x &= 5.73(2\text{dp}) \text{ or } 2.27(2\text{dp})
 \end{aligned}$$

Equations involving algebraic fractions:

Eg:

$$\begin{aligned}
 3x &= \frac{1}{x} - 4 \\
 3x^2 &= 1 - 4x \\
 3x^2 + 4x - 1 &= 0 \\
 x &= \frac{-4 \pm \sqrt{4^2 - 4(3)(-1)}}{2(3)} \\
 &= \frac{-4 \pm \sqrt{28}}{6} \\
 x &= \frac{-4 + \sqrt{28}}{6} \quad \text{or} \quad x = \frac{-4 - \sqrt{28}}{6} \\
 x &= 0.22 \quad \text{or} \quad x = -1.55(2 \text{ d.p.})
 \end{aligned}$$

Eg:

$$\begin{aligned}
 x + 1 &= \frac{20}{x + 2} \\
 (x + 1)(x + 2) &= 20 \\
 x^2 + 3x + 2 &= 20 \\
 x^2 + 3x - 18 &= 0 \\
 (x + 6)(x - 3) &= 0 \\
 x + 6 = 0 \quad \text{or} \quad x - 3 = 0 \\
 x = -6 \quad \text{or} \quad x = 3
 \end{aligned}$$

Eg:

$$\begin{aligned}
 \frac{2-x}{x+1} + \frac{1}{x-3} &= \frac{3}{5} \\
 5(2-x)(x-3) + 5(x+1) &= 3(x+1)(x-3) \\
 5(2x-6-x^2+3x) + 5x+5 &= 3(x^2-3x+x-3) \\
 5(-x^2+5x-6) + 5x+5 &= 3(x^2-2x-3) \\
 -5x^2+25x-30+5x+5 &= 3x^2-6x-9 \\
 8x^2-36x+16 &= 0 \\
 2x^2-9x+4 &= 0 \\
 (2x-1)(x-4) &= 0 \\
 2x-1=0 \quad \text{or} \quad x-4=0 \\
 x = \frac{1}{2} \quad \text{or} \quad x = 4
 \end{aligned}$$

Eg:

$$\begin{aligned}
 \frac{2x+5}{x^2+4x+3} + \frac{2}{x+3} &= 1 \\
 \frac{2x+5}{(x+1)(x+3)} + \frac{2}{x+3} &= 1 \\
 2x+5+2(x+1) &= (x+1)(x+3) \\
 2x+5+2x+2 &= x^2+4x+3 \\
 4x+7 &= x^2+4x+3 \\
 x^2-4 &= 0 \\
 x^2 &= 4 \\
 x &= \pm\sqrt{4} \\
 x &= 2 \quad \text{or} \quad x = -2
 \end{aligned}$$

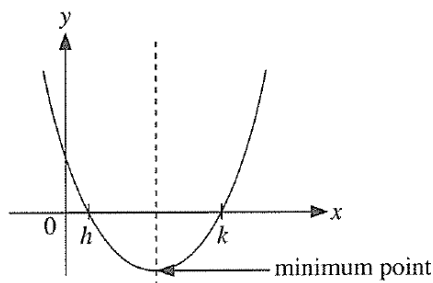
Eg:

$$\begin{aligned}
 \frac{x^2+2}{(5x-4)(2x-1)} &= \frac{1}{3} \\
 3(x^2+2) &= (5x-4)(2x-1) \\
 3x^2+6 &= 10x^2-13x+4 \\
 7x^2-13x-2 &= 0 \\
 (7x+1)(x-2) &= 0 \\
 \therefore 7x+1=0 \quad \text{or} \quad x-2=0 \\
 x = -\frac{1}{7} \quad \text{or} \quad x = 2
 \end{aligned}$$

Quadratic Graphs

- Suppose that a quadratic function $y = ax^2 + bx + c$ ($a \neq 0$) can be factorized into $a(x-h)(x-k)$.

Consider the case where $a > 0$. The graph is concave upwards (smiley face) and there is a minimum point.



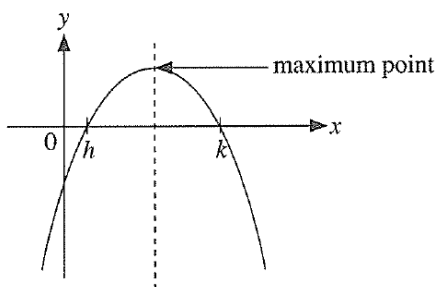
The graph intersects the x -axis at $(h, 0)$ and $(k, 0)$.

The line of symmetry is $x = \frac{h+k}{2}$.

The minimum point also occurs at $x = \frac{h+k}{2}$. To find the y -coordinate of the minimum point, substitute $x = \frac{h+k}{2}$ into the expression $y = ax^2 + bx + c$.

2. Suppose that a quadratic function $y = ax^2 + bx + c$ ($a \neq 0$) can be factorized into $a(x - h)(x - k)$.

Consider the case where $a < 0$. The graph is concave downwards (sad face) and there is a maximum point.



The graph intersects the x -axis at $(h, 0)$ and $(k, 0)$.

The line of symmetry is $x = \frac{h+k}{2}$.

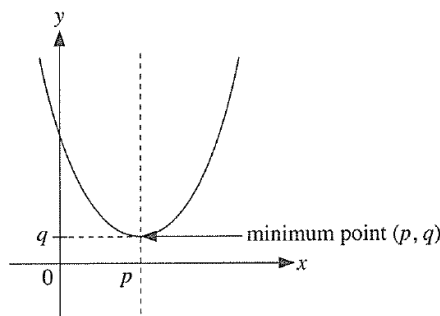
The maximum point also occurs at $x = \frac{h+k}{2}$. To find the y -coordinate of the maximum point, substitute $x = \frac{h+k}{2}$ into the expression $y = ax^2 + bx + c$.

3. Suppose we complete the square quadratic function $y = ax^2 + bx + c$ ($a \neq 0$) to obtain $y = a(x - p)^2 + q$.

If $a > 0$, then the graph is smiley face, and has a minimum point at (p, q) . The minimum value of y is q .

The equation $ax^2 + bx + c = h$ has two solutions if $h > q$, one solution if $h = q$, and no solutions if $h < q$.

The line of symmetry $x = p$.

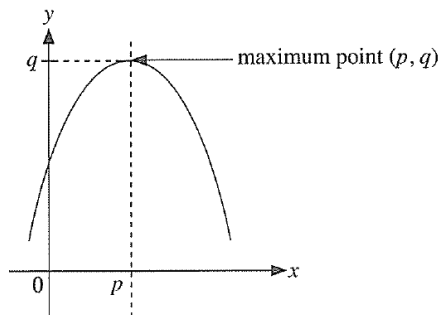


4. Suppose we complete the square quadratic function $y = ax^2 + bx + c$ ($a \neq 0$) to obtain $y = a(x - p)^2 + q$.

If $a < 0$, then the graph is sad face, and has a maximum point at (p, q) . The maximum value of y is q .

The equation $ax^2 + bx + c = h$ has two solutions if $h < q$, one solution if $h = q$, and no solutions if $h > q$.

The line of symmetry $x = p$.



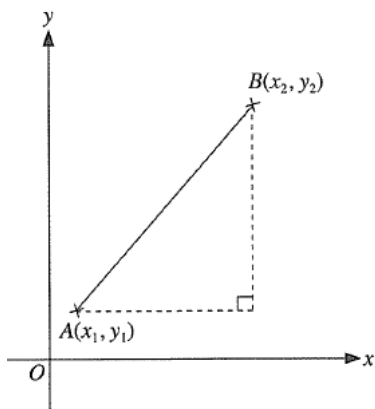
Coordinate Geometry

- The length of a line segment with end points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

or equivalently

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



- The gradient of a line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$\frac{y_2 - y_1}{x_2 - x_1}$$

or equivalently

$$\frac{y_1 - y_2}{x_1 - x_2}.$$

- Collinear Points:**

If gradient of AB = gradient of BC = gradient of AC , we say that A, B, C are collinear. This means that A, B, C lie on the same straight line (not necessarily in the order $A - B - C$; it could also be in any order such as $B - C - A$.)

Note: For collinear points, we only need to check the equality of one pair of gradients, for example, gradient of AB = gradient of BC . Then automatically, gradient of AB = gradient of AC and gradient of AC = gradient of BC as well.

When checking that gradient of AB = gradient of BC , there must be a common point (in this case, B) in order to conclude that the points are collinear.

If gradient of AB = gradient of CD (ie, no common points), then we cannot conclude any of the points are collinear.

- A horizontal line is parallel to the x -axis. If all the points on the line has y -coordinate

equal to b , then the equation of the line is

$$y = b.$$

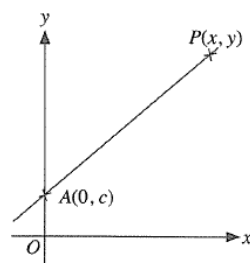
A vertical line is parallel to the y -axis. If all the points on the line has x -coordinate equal to a , then the equation of the line is

$$x = a.$$

If a line has a gradient m and y -intercept c , then its equation is

$$y = mx + c.$$

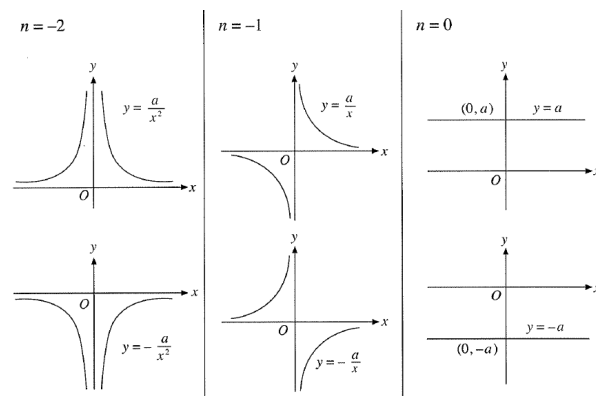
The equation $y = mx + c$ is known as the gradient-intercept form.

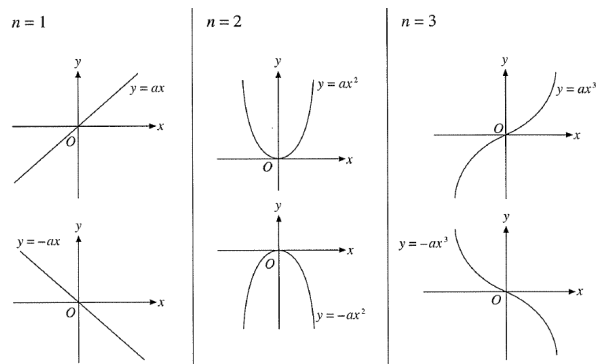


Graphs and Functions

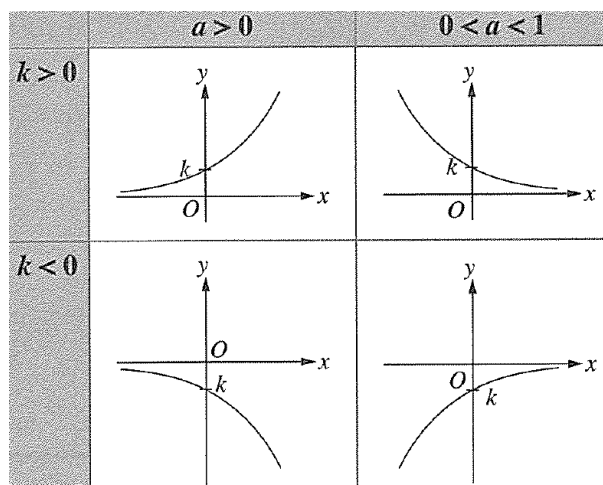
Below is a summary of the graphs of the form $y = ax^n$, where $n = -2, -1, 0, 1, 2$ and 3 .

In the below sketches, we have assumed $a > 0$.

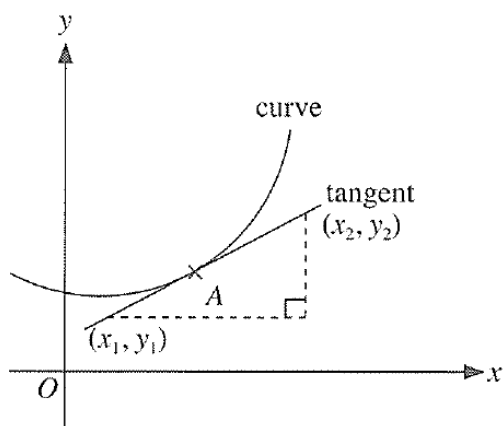




These are graphs of the form $y = ka^x$.



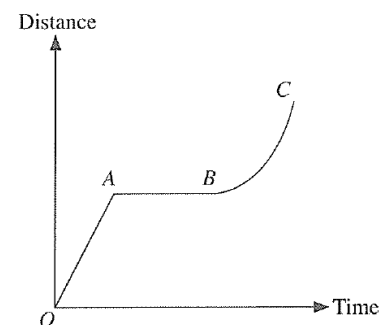
Tangent to graphs:



A straight line which touches a curve at a single point A is called a tangent to the curve at the point A .

The gradient of the curve at the point A is equal to the gradient of the tangent to the curve at A . In general, the gradient of the curve at any point is equal to the gradient of the tangent to the curve at that point.

Distance-Time Graphs:



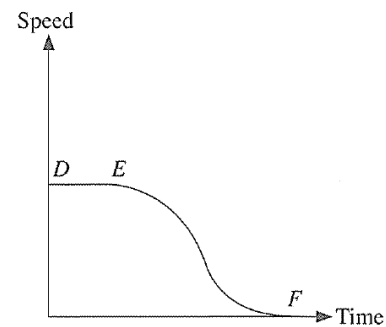
The gradient of a distance-time graph gives the speed of the object.

OA is a straight line, i.e. speed of object = gradient of OA .

AB is a horizontal line, i.e. speed of object = zero.

BC is a curve, i.e. instantaneous speed of object = gradient of the tangent to the curve at a point.

Speed-Time Graphs:



The gradient of a speed-time graph gives the acceleration of the object.

The area under a speed-time graph gives the distance travelled by the object.

DE is a horizontal line, i.e. acceleration of object = zero.

EF is a curve, i.e. instantaneous acceleration of object = gradient of tangent to the curve at a point.

Total distance travelled = area under graph from D to F