

Indices

1. Simplify each of the following, leaving your answer in **positive index form**.

(a) $(ab^2)^3 \times (a^3b^{-1})^2$

(b) $(6a^2b^{-3})^2 \div (3a^{-1}b^2)$

(c) $(x^{-2}y^3)^4 \div (x^5y^{-1})^2$

2. Simplify each of the following, leaving your answer in **positive index form**.

(a) $(2p^{-3}q^2)^3(4pq^{-1})^{-2}$

(b) $\frac{(-3m^2n^{-1})^2}{9m^{-3}n^4}$

(c) $(5x^0y^{-2})^{-2} \div (25y^3)$

3. Simplify each of the following, leaving your answer in **positive index form**.

(a) $6a \div a^{-2} + a^2 \times a - \frac{4a^{-1}}{2a^{-3}}$

(b) $(4k)^3 - 12k(2k^2) + \frac{8}{k^{-2}}$

4. (a) Express $3^{2n+1} \times 9^{n-2} \div 27^n$ in the form 3^{an+b} .

(b) Hence find n if the expression equals 27.

(c) Express $\frac{(3^n)^4 \cdot 81}{9^{n-1}}$ as a single power of 3, and hence find n if it equals 3^{10} .

5. Evaluate each of the following **without using a calculator**.

(a) $4^{-2} + \left(\frac{1}{4}\right)^{-1} - (-2)^0$

(b) $\left(\frac{3}{4}\right)^{-2} - 2^{-3}$

(c) $\left(\frac{1000}{27}\right)^{-5/3}$ (give your answer as a decimal)

6. Simplify each of the following, leaving your answer in **positive index form**.

(a) $\sqrt[5]{p^7} \times \sqrt[3]{8p^2}$

(b) $(p^{-3/4}q^{2/5})^4 \times (p^{1/2}q^{-1/5})^5$

(c) $\frac{p^{2/3}q^{-4/5}}{(p^{-1/3}q^{1/5})^{-3}}$

7. Simplify each of the following, leaving your answer in **positive index form**.

(a) $\sqrt[3]{a^2} \times \sqrt[4]{a^3} \div \sqrt[6]{a}$

(b) $\frac{\sqrt[5]{243c^{10}}}{\sqrt[3]{27c^4}}$

(c) $\left(\sqrt[4]{16d^6}\right)^2 \div \sqrt{d^3}$

8. Evaluate each of the following **without using a calculator**.

(a) $216^{2/3} - 16^{3/4}$

(b) $81^{3/4} + 8^{-2/3}$

(c) $(-125)^{1/3} \times (-8)^{-2/3}$

9. Simplify each of the following, leaving your answer in **positive index form**.

(a) $\frac{(3x^{-2}y^5)^2(-2x^3y^{-1})^3}{9x^4y^2}$

(b) $\left(\frac{x^{-4}y^2z^{-3}}{x^3y^{-1}z}\right)^{-2}$

(c) $\frac{ab^{n-1}}{c^2} \times \frac{c^nd^2}{a^2b} \div \frac{b^{n+3}}{c^{n-2}d}$

(d) $\frac{(a+b)^m}{ab^2} \div \frac{(a+b)^{m+3}}{a^2b}$

10. Simplify each of the following, leaving your answer in **positive index form**.

(a) $\frac{(2x^{-1}y^3)^4(-3x^2y^{-5})^2}{6x^3y^{-1}}$

(b) $\frac{(5p^{1/2}q^{-1/3})^6}{(25p^{-1}q^{2/3})^2}$

11. Solve each of the following equations.

(a) $16^x = 8^5$

(b) $9^{n-2} = 27^{n+1}$

(c) $\frac{10^p}{10^3} = 0.001$

(d) $4^{2k-1} = \frac{1}{64}$

12. Given that $x^{12}(x^3)^{2n-1} \div (x^{-2})^n = x^{33}$:

- (a) Write the left-hand side as $x^{(\text{something})}$.
- (b) Hence find n .
13. Given that
- $$\frac{(-6p^2q^{-1})^3}{-216pq^2} \div \frac{p^2}{q^4} = \frac{p^{a+b}}{q^{a-b}},$$
- (a) simplify the left-hand side to the form $\frac{p^M}{q^N}$,
- (b) form a pair of simultaneous equations in a and b ,
- (c) hence find a and b .
14. Given that $3^u \times 9^v = 3^{20}$ and $27^u \div 3^v = 3^4$:
- (a) form two linear equations in u and v ,
- (b) find u and v ,
- (c) hence evaluate 3^{u-2v} .
15. (a) Express 25 and 125 as powers of 5.
- (b) Solve $5^{2x+1} \times 25 = 125^{x-2}$.
16. (a) Simplify $\frac{(2x^{-1}y^2)^3}{(4x^2y^{-1})^2}$ in **positive index form**.
- (b) Hence evaluate it when $x = 2$ and $y = 4$.
17. A student tried to simplify $\left(\frac{3x^2}{2y}\right)^2$.
- Student A wrote $\frac{9x^4}{2y^2}$. Student B wrote $\frac{3x^4}{4y^2}$.
- (i) State the misconception made by each student.
- (ii) Give the correct simplified expression.
18. Compound interest: \$18000 is invested at 3.9% per annum **compounded monthly**.
- (a) Find the amount after 4 years (nearest cent).
- (b) Find the total interest earned (nearest cent).
19. A loan charges 4.8% per annum **compounded quarterly**. The interest added at the end of the first year is \$293.23.

- (a) Find the principal (nearest dollar).
- (b) Find the total amount owed after 3 years (nearest cent).
- 20.** Cheryl invested \$5000 and received \$6200 after 6 years, compounded yearly.
- (a) Find the annual interest rate, correct to 2 decimal places.
- (b) Using this nominal annual rate, find the amount after 6 years if compounded monthly (nearest cent).
- 21.** Calculate each, giving your answer in **standard form**, correct to **3 s.f.**
- (a) $(7.2 \times 10^8)(3.5 \times 10^{-4})$
- (b) $(9.6 \times 10^{-5}) \div (3.2 \times 10^4)$
- (c) $(4.05 \times 10^7) - (7.8 \times 10^6)$
- (d) $\frac{(5.6 \times 10^9) + (2.4 \times 10^8)}{(2.0 \times 10^3)^2}$
- (e) $\frac{(3.7 \times 10^{-4}) + (5.1 \times 10^{-5})}{6.0 \times 10^{-3}}$
- 22.** Given $P = 6.8 \times 10^3$ and $Q = 3.25 \times 10^4$, express each in **standard form** (3 s.f.).
- (a) $3P \times 2Q$
- (b) $Q - P$
- (c) $\frac{Q}{P}$
- (d) $\left(\frac{P}{Q}\right)^2$
- 23.** Given $x = 4 \times 10^{-3}$ and $y = 7 \times 10^{-4}$, evaluate $x + 12y$, giving your answer in **standard form**.
- 24.** (Use $1 \text{ MHz} = 10^6 \text{ Hz}$, $1 \text{ GHz} = 10^9 \text{ Hz}$.)
- (a) Convert 480 000 000 Hz to MHz (standard form).
- (b) Convert 72 GHz to Hz (standard form).
- (c) Convert 3.6×10^{14} Hz to GHz (standard form).
- 25.** Atomic radii: $1 \text{ pm} = 10^{-12} \text{ m}$, $1 \text{ nm} = 10^{-9} \text{ m}$.
- (a) A lithium atom has radius 160 pm. Express this in metres in standard form.

- (b) A chlorine atom has radius 0.099 nm. Express this in metres in standard form.
- (c) Express lithium : chlorine as a ratio of two integers in simplest form.

26. A probe travels at 2.4×10^4 m/s and the distance is 7.2×10^9 m.

- (a) Find the travel time in seconds in standard form (3 s.f.).
- (b) Convert your answer into hours (3 s.f.) and days (3 s.f.).

27. The table shows approximate world population:

- 1600: 5.50×10^8
- 1700: 6.20×10^8
- 1800: 9.90×10^8
- 1900: 1.65×10^9

Find:

- (a) the increase from 1600 to 1700 (standard form),
- (b) the number of times the population in 1900 is as large as that in 1700 (3 s.f.),
- (c) the percentage of the 1900 world population represented by 1.28×10^9 (3 s.f.).

28. The diameter of a circular microorganism is 12 nanometres (nm).

(Use $\pi = 3.142$ and $1 \text{ nm} = 10^{-9} \text{ m}$.)

- (a) Find the circumference in metres, in standard form (3 s.f.).
- (b) Find the area in square metres, in standard form (3 s.f.).

29. The mass of a hydrogen atom is 1.67×10^{-24} g and the mass of an oxygen atom is 2.66×10^{-23} g.

- (a) Find the mass of a water molecule (2 H and 1 O), in standard form (3 s.f.).
- (b) Find the approximate number of water molecules in 250 g of water, in standard form (3 s.f.).

30. Calculate

$$N = \frac{(2.5 \times 10^6)(4 \times 10^{-3})^2}{5 \times 10^4}$$

and give your answer in **standard form** (3 s.f.).

Concise Solutions (Final Answers Only)

1. (a) a^9b^4 (b) $\frac{12a^5}{b^8}$ (c) $\frac{y^{14}}{x^{18}}$
2. (a) $\frac{q^8}{2p^{11}}$ (b) $\frac{m^7}{n^6}$ (c) $\frac{y}{625}$
3. (a) $7a^3 - 2a^2$ (b) $40k^3 + 8k^2$
4. (a) 3^{n-3} (b) $n = 6$ (c) 3^{2n+6} , so $n = 2$
5. (a) $\frac{49}{16}$ (b) $\frac{119}{72}$ (c) 0.00243
6. (a) $2p^{31/15}$ (b) $\frac{q^{3/5}}{p^{1/2}}$ (c) $\frac{1}{p^{1/3}q^{1/5}}$
7. (a) $a^{5/4}$ (b) $c^{2/3}$ (c) $4d^{3/2}$
8. (a) 28 (b) $\frac{109}{4}$ (c) $-\frac{5}{4}$
9. (a) $-8xy^5$ (b) $\frac{x^{14}z^8}{y^6}$ (c) $\frac{c^{2n-4}d^3}{ab^5}$ (d) $\frac{a}{b(a+b)^3}$
10. (a) $\frac{24y^3}{x^3}$ (b) $\frac{25p^5}{q^{10/3}}$
11. (a) $x = \frac{15}{4}$ (b) $n = -7$ (c) $p = 0$ (d) $k = -1$
12. (a) x^{8n+9} (b) $n = 3$
13. (a) $\frac{p^3}{q}$ (b) $a + b = 3, a - b = 1$ (c) $a = 2, b = 1$
14. (a) $u + 2v = 20, 3u - v = 4$ (b) $u = 4, v = 8$ (c) $3^{u-2v} = 3^{-12} = \frac{1}{531441}$
15. (a) $25 = 5^2, 125 = 5^3$ (b) $x = 9$
16. (a) $\frac{y^8}{2x^7}$ (b) 256
17. A: didn't square the 2. B: didn't square the 3. Correct: $\frac{9x^4}{4y^2}$

18. (a) \$21033.55 (b) \$3033.55
19. (a) \$6000 (b) \$6923.37
20. (a) 3.65% (b) \$6222.17
21. (a) 2.52×10^5 (b) 3.00×10^{-9} (c) 3.27×10^7 (d) 1.46×10^3 (e) 7.02×10^{-2}
22. (a) 1.33×10^9 (b) 2.57×10^4 (c) 4.78×10^0 (d) 4.38×10^{-2}
23. 1.24×10^{-2}
24. (a) 4.80×10^2 MHz (b) 7.20×10^{10} Hz (c) 3.60×10^5 GHz
25. (a) 1.60×10^{-10} m (b) 9.90×10^{-11} m (c) 160 : 99
26. (a) 3.00×10^5 s (b) 8.33×10^1 h, 3.47×10^0 days
27. (a) 7.00×10^7 (b) 2.66 (c) 77.6%
28. (a) 3.77×10^{-8} m (b) 1.13×10^{-16} m²
29. (a) 2.99×10^{-23} g (b) 8.35×10^{24}
30. 8.00×10^{-4}