Sec 1 G2 Numbers — Additional Practice (Sections 2.1–2.5)

Instructions. Answer all questions. Use a calculator only where stated.

2.1 Concept of negative numbers and the number line

What negative numbers mean

- **Positive numbers** are above 0 (often used for *above*, *gain*, *profit*).
- **Negative numbers** are below 0 (often used for *below*, *loss*, *debt*).

Examples

- Temperature: $-3°$C means 3 degrees below 0°C.
- Money: $-\$20$ can mean owing \$20 or losing \$20.
- Height: -50 m (below sea level), $+50$ m (above sea level).

Reading a number line

- Numbers increase as you move **to the right**.
- Numbers decrease as you move **to the left**.
- So -7 is **less than** -2 because -7 is further left.

Key comparisons

- Any negative number < 0.
- Any positive number > 0.
- Among negatives: the one with the **larger** magnitude is actually **smaller**.
  - Example: $-9 < -2$.

"Which is closer to 0?"

Think "distance from 0".

- Example: -1 is closer to 0 than -5, so -1 is closer.

Practice Questions

Q1. A scuba diver is 6 m below sea level.

(a) Represent his position as an integer.

(b) He rises 4 m. Represent the change as an integer.

(c) Find his new position as an integer.

Q2. Consider -3.5, -1, 0.6, 2.4, $\frac{5}{2}$.

(a) Arrange in ascending order.

(b) Fill in: -1 ___ -3.5 ($>$, $<$, $=$).

(c) Which number is closest to 0?

Q3. Points $A = -4$, $B = 1.5$, $C = -0.5$ on a number line.

(a) Which point is leftmost?

(b) Fill in: -0.5 ___ 1.5 ($>$, $<$, $=$).

(c) Give one integer between C and B.

Q4. Insert $<$, $>$, or $=$.

(a) -7 ___ -2

(b) -3 ___ 0

(c) 4 ___ -4

(d) -1.2 ___ -1.20

Q5. Average monthly temperature is 28°C. Differences (from 28) for months A, B, C are -3, $+1$, -5 respectively.

(a) What does -5°C mean?

(b) Find each actual temperature.

(c) Order months from coldest to hottest.

Q6. A shop's profit/loss over 4 weeks is $+120$, -45, $+80$, -150 dollars.

(a) Which week has the greatest profit?

(b) What does -45 mean?

(c) Find the total over 4 weeks.

Q7. A hot-air balloon is 300 m above ground. Changes in height: $+50$, -120, $+30$, -80 (in order).

(a) Height after each change.

(b) Greatest height reached?

(c) Is it ever below ground?

Q8. List all integers n with $-12 \leq n \leq 6$ that are:

(a) negative multiples of 4

(b) positive multiples of 3

Q9. List all **positive and negative composite integers** n such that $-15 < n < 15$.

Q10. A game score starts at 0. Events: $+7$, -3, $+4$, -10.

(a) Score after each event.

(b) Final score.

(c) When does the score become negative for the first time?

2.2 Addition and subtraction of integers

Adding integers

- If the signs are the **same**, add the sizes and keep the sign.
  - Example: $(-6) + (-4) = -10$.
- If the signs are **different**, subtract the smaller size from the larger size, and keep the sign of the larger size.
  - Example: $9 + (-14) = -(14 - 9) = -5$.

Subtracting integers

A very important rule:

- **Subtracting a negative becomes adding.**
- **Subtracting a positive stays subtracting.**

Examples

- $12 - (-7) = 12 + 7 = 19$
- $-3 - 5 = -8$
- $-3 - (-5) = -3 + 5 = 2$

Using context (profit/loss, rise/fall)

Treat each change as a signed number, then add them.

- Example: Start at -250 m (below sea level), rise $+310$ m $\Rightarrow -250 + 310 = 60$ m (now above sea level).

Practice Questions

Q1. Evaluate **without a calculator**.

(a) $-18 + 25$

(b) $14 - (-9)$

(c) $-7 - 13$

(d) $-30 - (-12)$

Q2. Evaluate **without a calculator**.

(a) $8 + (-3) + (-11)$

(b) $-5 + 14 - 9$

(c) $-12 - (-7) + (-10)$

(d) $25 - (-8) - 17$

Q3. Find the missing integer.

(a) $x + (-7) = -15$

(b) $12 - y = -3$

(c) $-4 - z = 9$

(d) $a - (-8) = -2$

Q4. A submarine is 250 m below sea level. It dives 120 m, then rises 310 m.

(a) Write the starting position and each change using integers.

(b) Final position (integer).

(c) How far from sea level is it finally?

Q5. Morning temperature is -4°C. It increases by 9°C, then decreases by 12°C.

(a) Final temperature.

(b) How many degrees below 0°C is it?

Q6. Heights relative to sea level:

- Place A: 430 m below sea level
- Place B: 4095 m above sea level
- Place C: 1800 m above sea level

(a) Represent each height as an integer.

(b) Difference in height between B and A.

(c) Difference in height between C and A.

Q7. A stall's monthly profit/loss: Jan +350, Feb −120, Mar +80, Apr −400.

(a) Which month has the greatest profit?

(b) What does −400 mean?

(c) Total profit/loss for the 4 months.

Q8. City A is GMT +8, City B is GMT −5.

(a) How many hours ahead is A compared to B?

(b) If it is 3:00 pm in B, what time is it in A?

(c) If it is 9:30 am in A, what time is it in B?

Q9. Find the sum and average of $-3, 7, -10, 2$.

Q10. Points $P = -6$, $R = 5$ on a number line. Point $Q = x$ is exactly halfway between them. Find x.

2.3 Multiplication, division and combined operations of integers

Sign rules for multiplication/division

- Same signs ⇒ **positive**
  - Example: $(-6) \times (-2) = +12$
- Different signs ⇒ **negative**
  - Example: $(-6) \times 2 = -12$

Same rules for division:

- $(-48) \div (-6) = +8$
- $45 \div (-9) = -5$

Powers (indices) with negatives

Be careful with brackets.

- $(-3)^2 = 9$ (because the negative is inside the square)
- $-3^2 = -(3^2) = -9$ (square happens first, then the minus)

Useful idea: brackets tell you what is being powered.

Roots

- $\sqrt{81} = 9$
- $\sqrt[3]{-125} = -5$ (cube root keeps the sign)

Order of operations (BODMAS)

This is the order you calculate in:

1. **B**rackets
2. **O**rders (powers, roots)
3. **D**ivision and **M**ultiplication (left to right)
4. **A**ddition and **S**ubtraction (left to right)

Example

$$3 + 5 \times (-2) - 18 \div (-3)$$

Do $\times$ and $\div$ first:

- $5 \times (-2) = -10$
- $18 \div (-3) = -6$

So:

$$3 + (-10) - (-6) = 3 - 10 + 6 = -1$$

Brackets can change the value

- $5 + 4 \times 3 = 5 + 12 = 17$
- $(5 + 4) \times 3 = 9 \times 3 = 27$

Practice Questions

Q1. Evaluate **without a calculator**.

(a) $(-6) \times 7$

(b) $(-48) \div (-6)$

(c) $45 \div (-9)$

(d) $(-3) \times (-5) \times 2$

Q2. Evaluate **without a calculator**.

(a) -3^2

(b) $(-3)^2$

(c) $\sqrt[3]{-125}$

(d) $-\sqrt{81}$

(e) $(-2)^3 \times (-2)^2$

Q3. Evaluate **without a calculator**: $3 + 5 \times (-2) - 18 \div (-3)$.

Q4. Evaluate **without a calculator**:

$$\left[(-4)^2 - \sqrt{64}\right] \div (-2) + 3.$$

Q5. A drone descends at 40 cm per second.

(a) Represent the change in height in 2 minutes by a negative number.

(b) If its height was 5000 cm, find its height after 2 minutes.

Q6. Water leaks from a tank at 250 ml per minute.

(a) Represent the amount leaked in 12 minutes by a negative number (in ml).

(b) The tank started with 5 L. How much remains after 12 minutes?

Q7. In a 12-question test: $+5$ marks per correct answer, -2 per wrong answer, -1 per unanswered.

(a) Lowest possible score.

(b) Score if a student gets 7 correct, 3 wrong, 2 unanswered.

Q8. Insert **one pair of brackets** to make each statement correct.

(a) $5 + 4 \times 3 - 2 = 25$

(b) $16 - 4 \div 2^2 = 3$

Q9. A student writes: $(-3)^2 - 3^2 = -9 - 9 = -18$.

(a) State the mistake.

(b) Give the correct answer.

Q10. Let $x = -\frac{7}{3}$.

(a) Write the reciprocal of x.

(b) Evaluate $x \times$ (its reciprocal).

(c) Evaluate $x \div \frac{14}{9}$.

2.4 Rational numbers, irrational numbers and real numbers

Rational numbers (numbers you can write as a fraction)

A **rational number** can be written as $\dfrac{a}{b}$ where a, b are integers and $b \neq 0$.
These include:

- Integers: $-4, 0, 7$
- Fractions: $\frac{2}{5}$
- Terminating decimals: 0.72
- Recurring decimals: $0.\overline{3}$

Examples

- $0.72 = \frac{72}{100} = \frac{18}{25}$
- $0.\overline{3} = \frac{1}{3}$

Terminating vs recurring decimals

- **Terminating**: ends (like 0.125)
- **Recurring**: repeats a pattern forever (like $0.41\overline{6}$)

Irrational numbers (cannot be written as a fraction)

These cannot be written exactly as a fraction, and their decimals go on forever **without repeating**.
Common examples:

- π
- $\sqrt{2}$, $\sqrt{5}$ (when the number under the square root is *not* a perfect square)

Examples

- $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$ is irrational because $\sqrt{2}$ is irrational.
- $\sqrt{16} = 4$ is rational (perfect square).

Real numbers

Real numbers include *all* rational and irrational numbers.
So:

- rational numbers $\subset$ real numbers
- irrational numbers $\subset$ real numbers

Comparing surds (like $\sqrt{5}$)

You can compare by knowing nearby squares:

- $2^2 = 4$, $3^2 = 9$ so $\sqrt{5}$ is between 2 and 3.
- Roughly, $\sqrt{5} \approx 2.24$ (good enough for ordering).

Practice Questions

Q1. Express as a fraction in simplest form (or mixed number where stated).

(a) 0.72

(b) -2.5

(c) 3.04

(d) $5\frac{3}{8}$ as an improper fraction

Q2. Using a calculator, evaluate and state whether the decimal is **terminating** or **recurring**.

(a) $\frac{7}{16}$

(b) $\frac{5}{12}$

(c) $-\frac{11}{40}$

(d) $\frac{13}{6}$

Q3. From $\sqrt{50}$, 3.1416, $0.\overline{12}$, $\frac{\pi}{3}$, $-\sqrt{81}$, $\frac{2}{7}$, $\sqrt{\frac{1}{4}}$:

(a) List the rational numbers.

(b) List the irrational numbers.

Q4. Arrange in ascending order: $-\sqrt{5},\ -2.1,\ -2,\ 1.7,\ \sqrt{3}$.

Q5. Which of the following are **integers**?

$$-4,\ 0,\ \frac{5}{2},\ \sqrt{16},\ -\sqrt{7},\ 3.0,\ -\frac{8}{4}$$

Q6. Express each as a fraction in simplest form.

(a) $0.\overline{3}$

(b) $0.\overline{27}$

(c) $1.\overline{2}$

Q7. (a) Express $0.\overline{18}$ as a fraction in simplest form.

(b) Hence, find $0.\overline{18} + 0.\overline{09}$ as a fraction in simplest form.

Q8. State whether each number is **rational** or **irrational**.

(a) $\dfrac{2\pi}{\pi}$

(b) $\dfrac{\sqrt{12}}{\sqrt{3}}$

(c) $\sqrt{2} + \sqrt{8}$

(d) $\dfrac{\sqrt{50}}{5}$

Q9. (a) Between which two consecutive integers does $\sqrt{2}$ lie? Give $\sqrt{2}$ correct to 2 d.p.

(b) Between which two consecutive integers does $\sqrt{7}$ lie? Give $\sqrt{7}$ correct to 2 d.p.

Q10. Two students claim:

- A: "Every irrational number is a real number."
- B: "Every real number is rational."

Who is correct? Give a brief reason.

2.5 Operations on real numbers

This section is mainly about doing calculations correctly with:

- fractions
- mixed numbers
- decimals
- negatives
- and sometimes roots

Converting between mixed and improper fractions

- Mixed number → improper:
$$2\frac{5}{6} = \frac{2 \times 6 + 5}{6} = \frac{17}{6}$$
- Improper → mixed: divide top by bottom.
$$\frac{17}{12} = 1\frac{5}{12}$$

Simplifying fractions

Divide top and bottom by the same number.

- $\frac{18}{24}$ divide by 6 $\Rightarrow \frac{3}{4}$

Adding/subtracting fractions

You need a **common denominator**.

- $\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20}$

Multiplying fractions

Multiply tops, multiply bottoms, then simplify.

- $\frac{3}{8} \times \frac{5}{6} = \frac{15}{48} = \frac{5}{16}$

Dividing fractions

Flip the second fraction (take the reciprocal), then multiply.

- $\frac{3}{8} \div \left(\frac{1}{4}\right) = \frac{3}{8} \times 4 = \frac{3}{2}$

Decimals with negatives

Treat them like signed numbers and combine carefully.

- $0.008 - (-1.02) = 0.008 + 1.02 = 1.028$

"Halfway between two numbers"

Halfway means "average of the two numbers":

$$\text{halfway} = \frac{\text{first} + \text{second}}{2}$$

Example:

$$\text{halfway between } -6 \text{ and } 5 = \frac{-6+5}{2} = -\frac{1}{2}$$

Practice Questions

Q1. (a) Simplify $\frac{18}{24}$.

(b) Convert $2\frac{5}{6}$ to an improper fraction.

(c) Convert $\frac{17}{12}$ to a mixed number.

Q2. Evaluate **without a calculator**.

(a) $\frac{3}{4} + \frac{2}{5}$

(b) $\frac{5}{6} - \frac{1}{4}$

(c) $1\frac{1}{3} + 2\frac{1}{6}$

(d) $-\frac{3}{5} + \frac{7}{10}$

Q3. Evaluate **without a calculator**.

(a) $\frac{3}{8} \div \left(-\frac{1}{4}\right)$

(b) $-2\frac{1}{2} \times \frac{3}{5}$

(c) $\frac{5}{12} \div \frac{8}{9}$

Q4. Evaluate **without a calculator**:

$$\frac{3}{5} - \left[\frac{1}{2} \div \left(-\frac{3}{4}\right)\right].$$

Q5. Evaluate **without a calculator**.

(a) $1.25 + 0.81 - 5.4$

(b) $0.008 - (-1.02)$

(c) $(-3.18) - 5.4 + 0.02$

(d) $6.7 + (-10.3) + 4$

Q6. Find the number exactly halfway between -0.625 and $-\frac{1}{3}$. Express your answer as a fraction in simplest form.

Q7. In a class, $\frac{1}{3}$ walk to school, $\frac{2}{5}$ take the bus, and the rest take MRT. Find the fraction who take MRT.

Q8. Joanne spent $\frac{1}{4}$ of her money on food, $\frac{3}{10}$ on transport, and donated $\frac{2}{5}$ of the remainder. She saved the rest.

(a) Find the fraction of her money that was donated.

(b) If she saved \$216, find her total money and the amount spent on food.

Q9. An empty file has mass 250 g. When filled with papers, the mass is 0.73 kg. If it can hold 240 sheets, find the mass of 1 sheet (in grams).

Q10. Using a calculator, evaluate (give answers correct to **5 significant figures**).

(a) $\sqrt[3]{19} \times \pi^2$

(b) $\sqrt[5]{3.75 + 0.006}$

Solutions

2.1 Concept of negative numbers and the number line

Q1. (a) -6 (b) $+4$ (c) -2 (below is negative; rising is positive)

Q2. (a) $-3.5 < -1 < 0.6 < 2.4 < \frac{5}{2}$ (b) $>$ (c) 0.6

Q3. (a) A (b) $<$ (c) 0 or 1

Q4. (a) $<$ (b) $<$ (c) $>$ (d) $=$

Q5. (a) 5°C below average (b) A 25°C, B 29°C, C 23°C (c) C, A, B

Q6. (a) $+120$ (b) loss \$45 (c) $120-45+80-150 = +5$

Q7. (a) $350, 230, 260, 180$ m (b) 350 m (c) No

Q8. (a) $-12, -8, -4$ (b) $3, 6$

Q9. $\pm4, \pm6, \pm8, \pm9, \pm10, \pm12, \pm14$

Q10. (a) $7, 4, 8, -2$ (b) -2 (c) after the -10 event

2.2 Addition and subtraction of integers

Q1. (a) 7 (b) 23 (c) -20 (d) -18

Q2. (a) -6 (b) 0 (c) -15 (d) 16

Q3. (a) $x = -8$ (b) $y = 15$ (c) $z = -13$ (d) $a = -10$

Q4. (a) start -250, dive -120, rise $+310$ (b) -60 (c) 60 m below sea level

Q5. (a) $-4 + 9 - 12 = -7$°C (b) 7°C below 0

Q6. (a) A -430, B $+4095$, C $+1800$ (b) 4525 m (c) 2230 m

Q7. (a) Jan (b) loss \$400 (c) $350-120+80-400 = -90$ (overall loss \$90)

Q8. (a) $8-(-5) = 13$ h (b) $15{:}00+13 = 04{:}00$ (next day) (c) $09{:}30 - 13 = 20{:}30$ (previous day)

Q9. Sum $= -4$, average $= -1$

Q10. $x = \dfrac{-6+5}{2} = -\dfrac{1}{2}$

2.3 Multiplication, division and combined operations of integers

Q1. (a) -42 (b) 8 (c) -5 (d) 30

Q2. (a) -9 (b) 9 (c) -5 (d) -9 (e) -32

Q3. $3 + 5(-2) - 18/(-3) = 3 - 10 + 6 = -1$

Q4. $(16 - 8)/(-2) + 3 = -4 + 3 = -1$

Q5. (a) $-40\times120 = -4800$ cm (b) $5000-4800 = 200$ cm

Q6. (a) $-250 \times 12 = -3000$ ml (b) $5000 - 3000 = 2000$ ml $= 2$ L

Q7. (a) $12(-2) = -24$ (b) $7(5) - 3(2) - 2(1) = 27$

Q8. (a) $(5 + 4) \times 3 - 2 = 25$ (b) $(16 - 4) \div 2^2 = 3$

Q9. (a) $(-3)^2$ is $+9$, not -9 (b) $9 - 9 = 0$

Q10. (a) $-\frac{3}{7}$ (b) 1 (c) $-\frac{7}{3} \times \frac{9}{14} = -\frac{3}{2}$

2.4 Rational numbers, irrational numbers and real numbers

Q1. (a) $\frac{18}{25}$ (b) $-\frac{5}{2}$ (c) $\frac{76}{25}$ (d) $\frac{43}{8}$

Q2. (a) 0.4375 terminating (b) $0.41\overline{6}$ recurring (c) -0.275 terminating (d) $2.1\overline{6}$ recurring

Q3. (a) $3.1416,\ 0.\overline{12},\ -9,\ \frac{2}{7},\ \frac{1}{2}$ (b) $\sqrt{50},\ \frac{\pi}{3}$

Q4. $-\sqrt{5} < -2.1 < -2 < 1.7 < \sqrt{3}$ ($\sqrt{5} \approx 2.236$, $\sqrt{3} \approx 1.732$)

Q5. Integers: $-4,\ 0,\ 4,\ 3,\ -2$

Q6. (a) $\frac{1}{3}$ (b) $\frac{3}{11}$ (c) $\frac{11}{9}$

Q7. (a) $\frac{2}{11}$ (b) $\frac{2}{11} + \frac{1}{11} = \frac{3}{11}$

Q8. (a) rational (=2) (b) rational (=2) (c) irrational ($= 3\sqrt{2}$) (d) irrational ($= \sqrt{2}$)

Q9. (a) between 1 and 2; $\sqrt{2} \approx 1.41$ (b) between 2 and 3; $\sqrt{7} \approx 2.65$

Q10. A correct; B false (e.g. $\sqrt{2}$ is real but irrational)

2.5 Operations on real numbers

Q1. (a) $\frac{3}{4}$ (b) $\frac{17}{6}$ (c) $1\frac{5}{12}$

Q2. (a) $\frac{23}{20} = 1\frac{3}{20}$ (b) $\frac{7}{12}$ (c) $\frac{7}{2} = 3\frac{1}{2}$ (d) $\frac{1}{10}$

Q3. (a) $-\frac{3}{2}$ (b) $-\frac{3}{2}$ (c) $\frac{15}{32}$

Q4. $\frac{1}{2} \div (-\frac{3}{4}) = -\frac{2}{3}$ so $\frac{3}{5} - (-\frac{2}{3}) = \frac{19}{15} = 1\frac{4}{15}$

Q5. (a) -3.34 (b) 1.028 (c) -8.56 (d) 0.4

Q6. $-\frac{23}{48}$

Q7. $1 - (\frac{1}{3} + \frac{2}{5}) = \frac{4}{15}$

Q8. (a) remainder $= \frac{9}{20}$, donated $= \frac{2}{5} \cdot \frac{9}{20} = \frac{9}{50}$ (b) saved fraction $= \frac{27}{100}$ so total $= 216 \div \frac{27}{100} = 800$, food $= \frac{1}{4} \cdot 800 = 200$

Q9. 0.73 kg $= 730$ g, papers $= 730 - 250 = 480$ g, per sheet $= 480/240 = 2$ g

Q10. (a) ≈ 26.336 (b) ≈ 1.3030