# HCF and LCM: Practice Questions

## Section 1: Highest Common Factor and Lowest Common Multiple

1. Find the **highest common factor (HCF)** of 84 and 156.

2. Find the HCF of 144, 180 and 240.

3. Find the **lowest common multiple (LCM)** of 48 and 180.

4. Find the LCM of 72, 90 and 150.

5. Find the HCF and LCM of the following, **leaving your answers in index form**:

$$2^3 \times 3^2 \times 5, \qquad 2^2 \times 3^4 \times 7, \qquad 2^4 \times 3 \times 5^2.$$

6. Given that $720 = 2^4 \times 3^2 \times 5$.

   (a) Express 504 as a product of prime factors (index form).

   (b) Hence, find the HCF and LCM of 720 and 504.

7. The numbers 900 and 315, written as the product of prime factors, are

$$900 = 2^2 \times 3^2 \times 5^2 \quad \text{and} \quad 315 = 3^2 \times 5 \times 7.$$

   Hence, find

   (a) the smallest integer $k$ such that $900k$ is a perfect cube,

   (b) the smallest integer $m$ such that $315m$ is a perfect square,

   (c) the largest integer which is a factor of both 900 and 315,

   (d) the smallest integer which is divisible by both 900 and 315.

8. The numbers 450 and 672 are given.

   (a) Find their HCF and LCM.

   (b) Hence, find the smallest positive integer $n$ such that $450n$ is a multiple of 672.

9. (a) Express 756 as a product of prime factors (index form).

   (b) Find the smallest positive integer $t$ such that $756t$ is a perfect square.

   (c) Find the smallest positive integer $u$ such that $756u$ is a perfect cube.

10. Find the smallest positive integer $p$ such that $\sqrt{630p}$ is a whole number. Hence, find $\sqrt{630p}$.

11. Find the smallest positive integer $q$ such that $\dfrac{540}{q}$ is a perfect cube. (State $q$ and the cube.)

12. Find the smallest integers $x$ and $y$ such that $2^x \times 3^y \times 5^2$ is a multiple of 360.

13. Given that $588 = 2^2 \times 3 \times 7^2$ and $756 = 2^2 \times 3^3 \times 7$.

    (a) Find the HCF and LCM, leaving answers in index form.

    (b) Find the smallest positive integer $n$ such that $588n$ is a multiple of 756.

14. The numbers 1260 and 840 are given.

    (a) Find the HCF and LCM.

    (b) Find the smallest positive integer $k$ such that $\dfrac{1260}{k}$ is a perfect square.

    (c) Find the smallest positive integer $m$ such that $840m$ is a perfect cube. (Leave $m$ in prime factors.)

15. Let $N = 2^3 \times 3 \times 5^2 \times 7^3$. Find the smallest positive integer $k$ such that $Nk$ is **both** a perfect square and a perfect cube. (Leave $k$ in index form.)

## Section 2: HCF–LCM Word Problems

1. 132 boys and 210 girls signed up for a camp. They are to be divided into mixed groups such that the number of boys in each group is the same and the number of girls in each group is the same.

    (a) Find the largest possible number of groups.

    (b) Hence, find the number of boys in each group.

    (c) Hence, find the number of girls in each group.

2. A company packs 864 boxes of snacks, 1056 bottles of water and 720 sandwiches into as many goodie bags as possible. Each goodie bag must have the same contents.

    (a) What is the maximum number of goodie bags that can be prepared?

    (b) How many of each item are in each goodie bag?

3. Three traffic lights will turn red at fixed intervals of 50 s, 1 min 20 s, and 1 min 35 s respectively. They all turn red simultaneously at 09 15 00. Find the next time when they will turn red simultaneously again.

4. Three school bells ring at fixed intervals of 1 min 12 s, 1 min 30 s, and 1 min 48 s respectively. They ring together at 14 12. Find the next time they will ring together again.

5. Three comets pass through our solar system every 60 years, 84 years and 210 years respectively. They were last seen together in the year 1900. In which year will they be seen together again?

6. The floor of a hall is 4.8 m by 7.5 m and is to be laid with identical square tiles.

    (a) Find the largest possible side length (in cm) of each tile.

(b) Find the number of tiles needed.

7. A contractor has ropes of lengths 3.6 m, 4.2 m and 5.4 m. He wants to cut them into equal pieces of the greatest possible length, with no leftover.

   (a) Find the length of each piece in cm.

   (b) Find how many pieces can be cut from each rope.

8. Three toy cars $A, B, C$ start at the same point on a circular track. They take 96 s, 120 s, and 150 s respectively to complete one round. The race starts at 09 50.

   (a) At what time will they next meet at the start point again?

   (b) How many rounds has car $A$ completed by then?

9. Three buses arrive at a bus stop every 18 min, 24 min, and 45 min respectively. They arrive together at 08 20. Find the next time they will arrive together again.

10. A bakery has some tarts. There are always 5 tarts left over whether the tarts are packed into boxes of 6, 8 or 15. What is the minimum possible number of tarts?

11. A ribbon is shared equally among 5 tailors. One tailor cuts his share into 6 cm pieces, another into 9 cm pieces, another into 15 cm pieces, another into 18 cm pieces, and the last into 24 cm pieces. To avoid wastage, each tailor must have no leftover after cutting. Find the least possible total length of the original ribbon (in cm).

12. Cylindrical cans $A, B, C$ (same radii) have heights 12 cm, 18 cm, and 20 cm respectively. They are stacked (using only one type per stack) to reach the same height.

    (a) Find the minimum common height.

    (b) Find the minimum number of $B$ cans required.

13. Boxes $P, Q, R$ have heights 9 cm, 14 cm, and 21 cm respectively. They are stacked to form towers of equal height.

    (a) Find the minimum height of each tower.

    (b) Find the minimum number of $Q$ boxes needed.

14. Three robot arms repeat a cycle every 12 s, 18 s, and 30 s respectively. They begin a cycle together at 16 05 30.

    (a) When will they next begin a cycle together?

    (b) How many cycles will the 12 s arm complete by that time?

15. A charity wants to pack 168 bottles, 252 food packs and 420 masks into as many identical care packs as possible, with none left over.

    (a) Find the maximum number of care packs.

    (b) State the contents of each care pack.

# Answers

## Section 1

1. 12

2. 12

3. 720

4. 1800

5. HCF $= 2^2 \times 3$;  LCM $= 2^4 \times 3^4 \times 5^2 \times 7$

6. (a) $504 = 2^3 \times 3^2 \times 7$  (b) HCF $= 72$, LCM $= 5040$

7. (a) $k = 30$  (b) $m = 35$  (c) 45  (d) 6300

8. (a) HCF $= 6$, LCM $= 50400$  (b) $n = 112$

9. (a) $756 = 2^2 \times 3^3 \times 7$  (b) $t = 21$  (c) $u = 98$

10. $p = 70$;  $\sqrt{630p} = 210$

11. $q = 20$;  $\dfrac{540}{20} = 27$

12. $x = 3,\ y = 2$

13. (a) HCF $= 2^2 \times 3 \times 7$;  LCM $= 2^2 \times 3^3 \times 7^2$  (b) $n = 9$

14. (a) HCF $= 420$, LCM $= 2520$  (b) $k = 35$  (c) $m = 3^2 \times 5^2 \times 7^2$

15. $k = 2^3 \times 3^5 \times 5^4 \times 7^3$

## Section 2

1. (a) 6 groups  (b) 22 boys  (c) 35 girls

2. (a) 48 bags  (b) 18 snack boxes, 22 bottles, 15 sandwiches

3. 11 21 40

4. 14 30

5. 2320

6. (a) 30 cm  (b) 400 tiles

7. (a) 60 cm  (b) 6 pieces, 7 pieces, 9 pieces

8. (a) 10 30  (b) 25 rounds

9. 14 20

10. 125

11. 1800 cm

12. (a) 180 cm  (b) 10

13. (a) 126 cm  (b) 9

14. (a) 16 08 30  (b) 15 cycles

15. (a) 84 packs  (b) 2 bottles, 3 food packs, 5 masks