A Maths Kinematics Questions - Version 1

Questions

Q1. A particle moves in a straight line so that, t seconds after passing O,

$$s = t^3 - 12t^2 + 36t.$$

(i) Find v and a.

(ii) Find the times when the particle is at rest and the corresponding positions.

(iii) Find the total distance travelled from $t = 0$ until it first returns to O.

(iv) Find the average speed over that time.

Q2. A particle moves so that

$$s = \frac{1}{4}t^4 - \frac{11}{3}t^3 + 17t^2 - 24t, \quad t \geq 0.$$

(i) Find v and factorise it fully.

(ii) Hence find all t when the particle is at rest.

(iii) Find all $t > 0$ when the particle is at O, and state whether it *crosses* O or only *touches* O at each such time.

(iv) Find the total distance travelled in the first 6 seconds.

Q3. A particle passes O at $t = 0$ and has velocity

$$v = t^2 - 6t + 5.$$

(i) Find s in terms of t.

(ii) Find the times when it is at rest.

(iii) Find the first two times $t > 0$ when it is at O.

(iv) Find the total distance travelled in the first 8 seconds.

Q4. A particle has acceleration

$$a = 6 - 8e^{-t}, \quad v(0) = 2, \quad s(0) = 0.$$

(i) Find $v(t)$ and $s(t)$.

(ii) Find when $a = 0$.

(iii) Hence find the minimum value of v and the displacement then.

(iv) Does the particle ever come to rest?

Q5. A particle satisfies

$$a = -4\sin t + 3\cos t, \quad v(0) = 1, \quad s(0) = 0.$$

(i) Find v and s.

(ii) Show the first time it is at rest is $t = \frac{\pi}{2}$.

(iii) Find s and a at $t = \frac{\pi}{2}$.

(iv) Find the total distance travelled for $0 \leq t \leq 2\pi$.

Q6. A particle passes O with velocity 5 m/s and has

$$a = 6t - k.$$

Given that $v(2) = -7$ m/s:

(i) Find k.

(ii) Find all times when the particle is at rest.

(iii) Find the distance between the two rest positions.

Q7. A particle travels in a straight line so that, t seconds after passing a fixed point O, its velocity v m/s is

$$v = -(t-1)(t-5)(t-8), \qquad s(0) = 0.$$

(i) Find the acceleration a in terms of t.

(ii) Find the displacement s in terms of t.

(iii) Find the values of t when the particle is instantaneously at rest, and state the intervals of t for which the particle moves in the positive direction.

(iv) Find the total distance travelled in the first 8 seconds.

Q8. A particle passes O with velocity 18 m/s and has

$$a = -9e^{-\frac{3}{10}t}, \quad s(0) = 0.$$

It comes to rest at a point P.

(i) Find the time taken to reach P.

(ii) Find OP.

(iii) Find the average speed during the first 6 seconds.

Q9. Two particles P and Q move on a straight line with fixed points A and B 30 m apart.
Take the direction $A \to B$ as positive.
At $t = 0$, P passes A with velocity $v_P = 10 - 2t$.
At $t = 0$, Q passes B with velocity $v_Q = t - 6$.

(i) Find the two times when P and Q meet.

(ii) Find the distances of the meeting points from A.

(iii) Find the relative speed at the first meeting.

Q10. Two particles move from O with displacements

$$s_P = t^3 - 6t^2 + 9t, \qquad s_Q = 4t.$$

(i) Find all times $t \geq 0$ when they are at the same position.

(ii) Find v_P at those times and state when P is instantaneously at rest at the meeting.

(iii) Find the total distance travelled by P in the first 5 seconds.

Q11. A particle moves along a straight line with displacement s metres measured from a fixed point O (positive away from O). Its acceleration is constant

$$a = -6 \text{ m/s}^2.$$

At $t = 0$, the particle is at point A where $s = 30$, and it is moving away from O with speed u m/s. It passes through O at $t = 5$.

(i) Find u.

(ii) Find the time when the particle is furthest from O, and that maximum value of s.

(iii) Find the speed when it passes through O.

(iv) Find the time when it next passes through A (i.e. when $s = 30$ again for $t > 0$).

Q12. A particle moves in a straight line so that its displacement x metres at time t seconds is

$$x = 6\cos 2t + 8\sin 2t.$$

(i) Using the R-formula, write x in the form $x = R\sin(2t + \alpha)$ where $R > 0$ and α is a positive acute angle (in radians).

(ii) Hence state the maximum and minimum values of x.

(iii) Find the period of the motion.

(iv) Find the first time $t > 0$ when $x = 0$ and $\dfrac{dx}{dt} > 0$.

(v) Find the maximum speed, and the acceleration when the particle first comes to rest for $t > 0$.

Q13. A particle passes O at $t = 0$ with velocity

$$v = t^2 - 5t + 5.$$

(i) Find when the particle is at rest.

(ii) Find the distance travelled in the fourth second (i.e. from $t = 3$ to $t = 4$).

Q14. A particle passes O at $t = 0$ with velocity

$$v = 2 - \frac{3}{t+1}, \quad t \geq 0, \qquad s(0) = 0.$$

(i) Find $a(t)$.

(ii) Find $s(t)$.

(iii) Find the time when the particle is at rest.

(iv) Find the total distance travelled in the first 1 second.

Q15. A train leaves station P and later stops at station Q. Its speed v (km/h) after t hours is

$$v = 240t - 60t^2, \quad t \geq 0.$$

(i) Find the time taken to travel from P to Q.

(ii) Find the distance PQ.

(iii) Find the maximum speed and when it occurs.

(iv) Find the distance travelled while $v > 180$ km/h.

Solutions

Q1. (i) $v = 3t^2 - 24t + 36 = 3(t-2)(t-6)$, $a = 6t - 24$.

(ii) Rest at $t = 2, 6$; $s(2) = 32$, $s(6) = 0$.

(iii) First return to O at $t = 6$. Distance $= 32 + 32 = 64$ m.

(iv) Average speed $= 64/6 = 32/3$ m/s.

Q2. (i) $v = s' = (t-1)(t-4)(t-6)$.

(ii) Rest at $t = 1, 4, 6$.

(iii) $s = \dfrac{t(t-6)^2(3t-8)}{12}$, so $s = 0$ at $t = \dfrac{8}{3}$ and $t = 6$. At $t = \dfrac{8}{3}$ it crosses O; at $t = 6$ it touches O.

(iv) Distance $0 \to 6$: $\dfrac{125}{12} + \dfrac{63}{4} + \dfrac{16}{3} = \dfrac{63}{2}$ m.

Q3. (i) $s = \int v\,dt = \dfrac{1}{3}t^3 - 3t^2 + 5t$.

(ii) Rest: $v = (t-1)(t-5) = 0 \Rightarrow t = 1, 5$.

(iii) $s = 0 \Rightarrow t = 0,\ \dfrac{9 \pm \sqrt{21}}{2}$.

(iv) Distance $0 \to 8$: $|s(1)-s(0)|+|s(5)-s(1)|+|s(8)-s(5)| = 40$ m.

Q4. (i) $v = 6t + 8e^{-t} - 6$, $\quad s = 3t^2 - 6t - 8e^{-t} + 8$.

(ii) $a = 0 \Rightarrow e^{-t} = \dfrac{3}{4} \Rightarrow t = \ln\dfrac{4}{3}$.

(iii) Minimum $v = 6\ln\dfrac{4}{3}$; at this time $s = 3(\ln\frac{4}{3})^2 - 6\ln\frac{4}{3} + 2$.

(iv) No; $v_{\min} > 0$.

Q5. (i) $v = 4\cos t + 3\sin t - 3$, $\quad s = 4\sin t - 3\cos t - 3t + 3$.

(ii) First rest at $t = \dfrac{\pi}{2}$.

(iii) $s(\pi/2) = 7 - \dfrac{3\pi}{2}$, $\quad a(\pi/2) = -4$.

(iv) Total distance $= 16 + 12\tan^{-1}\left(\dfrac{3}{4}\right)$ m ≈ 23.72 m.

Q6. (i) $v = 3t^2 - kt + 5$. Use $v(2) = -7$: $12 - 2k + 5 = -7 \Rightarrow k = 12$.

(ii) Rest: $3t^2 - 12t + 5 = 0 \Rightarrow t = \dfrac{6 \pm \sqrt{21}}{3}$.

(iii) $s = t^3 - 6t^2 + 5t$. Distance between rest positions $= \dfrac{28}{9}\sqrt{21}$ m.

Q7. (i) $a = \dfrac{dv}{dt} = -3t^2 + 28t - 53$.

(ii) $s = \int v\,dt = -\dfrac{1}{4}t^4 + \dfrac{14}{3}t^3 - \dfrac{53}{2}t^2 + 40t$.

(iii) Rest at $t = 1, 5, 8$. $v > 0$ on $(0,1) \cup (5,8)$ and $v < 0$ on $(1,5)$.

(iv) $s(1) = \dfrac{215}{12}$, $s(5) = -\dfrac{425}{12}$, $s(8) = -\dfrac{32}{3}$. Distance $= 96$ m.

Q8. (i) $v = 30e^{-\frac{3}{10}t} - 12$. Rest: $30e^{-\frac{3}{10}t} = 12 \Rightarrow t = \dfrac{10}{3}\ln\dfrac{5}{2}$.

(ii) $s = 100(1 - e^{-\frac{3}{10}t}) - 12t$, so $OP = 60 - 40\ln\dfrac{5}{2}$ m.

(iii) $s(6) = 28 - 100e^{-1.8}$. Distance in first 6 s $= 2OP - s(6)$, so average speed $= \dfrac{2OP - s(6)}{6} \approx 5.87$ m/s.

Q9. (i) $x_P = 10t - t^2$, $x_Q = 30 + \dfrac{1}{2}t^2 - 6t$. Meet: $3t^2 - 32t + 60 = 0$, so $t = \dfrac{16 \pm 2\sqrt{19}}{3}$.

(ii) Positions from A: $x = 10t - t^2 = \dfrac{148 \pm 4\sqrt{19}}{9}$ m.

(iii) Relative speed at first meeting: $v_P - v_Q = (10 - 2t) - (t - 6) = 16 - 3t = 2\sqrt{19}$ m/s.

Q10. (i) $s_P = s_Q \Rightarrow t^3 - 6t^2 + 9t = 4t \Rightarrow t = 0, 1, 5$.

(ii) $v_P = 3t^2 - 12t + 9 = 3(t-1)(t-3)$; at $t = 1$, $v_P = 0$.

(iii) $s_P(0) = 0$, $s_P(1) = 4$, $s_P(3) = 0$, $s_P(5) = 20$. Distance $= 4 + 4 + 20 = 28$ m.

Q11. (i) $v = u - 6t$, $s = 30 + ut - 3t^2$. Given $s(5) = 0$: $30 + 5u - 75 = 0 \Rightarrow u = 9$.

(ii) Furthest when $v = 0$: $9 - 6t = 0 \Rightarrow t = \dfrac{3}{2}$; $s_{\max} = \dfrac{147}{4}$ m.

(iii) $v(5) = 9 - 30 = -21$, speed $= 21$ m/s.

(iv) $s = 30 \Rightarrow 30 + 9t - 3t^2 = 30 \Rightarrow t = 3$ s.

Q12. (i) Compare $R\sin(2t + \alpha) = R(\sin 2t\cos\alpha + \cos 2t\sin\alpha)$ with $8\sin 2t + 6\cos 2t$. Then $R\cos\alpha = 8$, $R\sin\alpha = 6$, so $R = 10$ and $\alpha = \tan^{-1}\left(\dfrac{3}{4}\right)$.

(ii) $\max x = 10$, $\min x = -10$.

(iii) Period $T = \pi$.

(iv) $x = 0 \Rightarrow 2t + \alpha = n\pi$ and $x' = 20\cos(2t + \alpha) > 0 \Rightarrow n$ even. Smallest $t > 0$: $t = \pi - \dfrac{\alpha}{2}$.

(v) Max speed $= 20$. First rest: $x' = 0 \Rightarrow 2t + \alpha = \dfrac{\pi}{2} \Rightarrow t = \dfrac{\pi}{4} - \dfrac{\alpha}{2}$. Then $x'' = -40\sin(2t + \alpha) = -40$.

Q13. (i) Rest: $t = \dfrac{5 \pm \sqrt{5}}{2}$.

(ii) Turning time in $(3,4)$ is $t_r = \dfrac{5 + \sqrt{5}}{2}$. With $s = \dfrac{1}{3}t^3 - \dfrac{5}{2}t^2 + 5t$, distance in fourth second

$$(s(3) - s(t_r)) + (s(4) - s(t_r)) = \frac{5\sqrt{5} - 8}{6} \text{ m.}$$

Q14. (i) $a = v' = \dfrac{3}{(t+1)^2}$.

(ii) $s = \int v\,dt = 2t - 3\ln(t+1)$.

(iii) $v = 0 \Rightarrow 2 - \dfrac{3}{t+1} = 0 \Rightarrow t = \dfrac{1}{2}$.

(iv) Distance $0 \to 1$: turning at $t = \dfrac{1}{2}$, so $D = \ln\left(\dfrac{729}{512}\right) = 6\ln 3 - 9\ln 2$ m.

Q15. (i) Stop when $v = 0 \Rightarrow t = 4$ h.

(ii) $PQ = \int_0^4 (240t - 60t^2)\, dt = [120t^2 - 20t^3]_0^4 =$ 640 km.

(iii) Max at $t = 2$: $v_{\max} = 240$ km/h.

(iv) $v > 180 \Rightarrow t \in (1, 3)$, distance $= \int_1^3 (240t - 60t^2)\, dt = 440$ km.