# Part 1

1. **Cards (Sum, without replacement)**
   A box contains 5 identical cards numbered $1, 2, 4, 7, 8$. Two cards are drawn at random, one after another, **without replacement**. The **sum** is recorded.

   (a) Copy and complete the possibility diagram (write **X** on the diagonal).

| 1st \ 2nd | 1 | 2 | 4 | 7 | 8 |
|---|---|---|---|---|---|
| 1 | X | | | | |
| 2 | | X | | | |
| 4 | | | X | | |
| 7 | | | | X | |
| 8 | | | | | X |

   (b) Find $P(\text{sum} = 9)$.

   (c) Find $P(\text{sum is prime})$.

   (d) Find $P(\text{sum} \geq 10)$.

   (e) Find $P(\text{sum is even})$.

2. **Dice (Union of events)**
   Two unbiased six-sided dice are thrown.
   Let $A$: "sum is a **prime** number".
   Let $B$: "difference (larger−smaller) is 2".
   Find $P(A)$, $P(B)$, $P(\text{both } A \text{ and } B \text{ occur})$, $P(A \text{ or } B \text{ or both occur})$.

3. **Balls (Product + "at least" via complement)**
   A bag contains 5 balls numbered $1, 2, 4, 5, 9$. Two balls are drawn, one after another, **without replacement**. The **product** is recorded.
   Find the probability that the product is

   (a) a perfect square,

   (b) a multiple of 10,

   (c) greater than 20,

   (d) **at least one** of the balls drawn is numbered 9.

4. **Two spinners (Sum table)**
   Spinner $P$ has outcomes $2, 4, 6, 8$ (equally likely).
   Spinner $Q$ has outcomes $1, 3, 5$ (equally likely).
   Each spinner is spun once. The **sum** is recorded.

   (a) Complete the possibility diagram.

| $P \backslash Q$ | 1 | 3 | 5 |
|---|---|---|---|
| 2 | | | |
| 4 | | | |
| 6 | | | |
| 8 | | | |

   (b) Find $P(\text{sum is prime})$.

(c) Find $P(\text{sum is a multiple of 3})$.

(d) Find $P(\text{sum} > 10)$.

(e) Find $P(\text{sum is odd})$.

5. **Cards (Difference table, without replacement)**
A box contains 4 cards numbered $1, 3, 4, 6$. Two cards are drawn, one after another, **without replacement**. The **difference** (larger−smaller) is recorded.

(a) Complete the possibility diagram (write **X** on the diagonal).

| 1st \ 2nd | 1 | 3 | 4 | 6 |
|---|---|---|---|---|
| 1 | X | | | |
| 3 | | X | | |
| 4 | | | X | |
| 6 | | | | X |

(b) Find $P(\text{difference} = 1)$.

(c) Find $P(\text{difference is prime})$.

(d) Find $P(\text{difference} \leq 2)$.

6. **Coloured balls (Mixed colours, without replacement)**
A bag contains 2 red, 3 blue, and 1 green ball. Two balls are drawn, one after another, **without replacement**.
Find the probability that

(a) both balls are blue,

(b) the two balls are different colours,

(c) at most one blue ball is drawn,

(d) at least one green ball is drawn.

7. **With vs without replacement (compare probabilities)**
A box contains 4 cards numbered $1, 2, 4, 7$. Two cards are drawn.

(a) Without replacement, find $P(\text{sum} > 8)$.

(b) With replacement, find $P(\text{sum} > 8)$.

(c) A student says (b) must be larger because "there are more favourable outcomes". Is the student correct?

8. **Dice (Product properties)**
Two unbiased six-sided dice are thrown.
Find the probability that the **product** is

(a) 12,

(b) a multiple of 5,

(c) a perfect square.

9. **Two-digit number from digits (without replacement)**
Cards numbered 1 to 9 are placed in a bag. Two cards are drawn, one after another, **without replacement**, and a **two-digit number** is formed (the first card drawn forms the tens digit).
Find the probability that the number is

(a) divisible by 4,

(b) divisible by 3 **or** 5.

10. **Three dice (counting)**
Three unbiased six-sided dice are thrown.
Find the probability that

(a) exactly two dice show the same number,

(b) all three numbers are different, and these three numbers form an arithmetic progression when suitably ordered.

## Part 2

1. **With replacement (2 draws)**
A bag contains the following balls:

| Colour | Red | Yellow |
|---|---|---|
| Number of balls | 5 | 3 |

A ball is drawn at random, **replaced**, then a second ball is drawn.

(a) Draw the **full** probability tree diagram.

(b) Find $P$(two red).

(c) Find $P$(exactly one red).

(d) Find $P$(at least one yellow).

2. **Two bags (independent)**
Two bags contain marbles as follows:

| Bag | Black | Red |
|---|---|---|
| A | 4 | 6 |
| B | 5 | 3 |

Peter takes one marble from Bag A, then one marble from Bag B.

(a) Draw the **full** probability tree diagram.

(b) Find $P$(A is red AND B is black).

(c) Find $P$(one red and one black).

(d) Find $P$(both marbles are the same colour).

(e) Find $P$(marble from Bag B is red).

3. **3 colours, without replacement (2 draws)**
A bag contains:

| Colour | Green | Blue | Red |
|---|---|---|---|
| Number of balls | 3 | 4 | 5 |

Two balls are drawn one after another, **without replacement**.

(a) Draw the **full** probability tree diagram.

(b) Find $P$(both balls are the same colour).

(c) Find $P$(exactly one green is drawn).

(d) Find $P$(at least one blue is drawn).

(e) Find $P$(second ball is red).

4. **Without replacement (2 draws)**
A bag contains 5 white and 7 black counters. Two counters are taken one after another, **without replacement**.

(a) Draw the **full** probability tree diagram.

(b) Find $P$(white then black).

(c) Find $P$(one of each colour).

(d) Find $P$(two black).

(e) Find $P$(at most one black).

5. **Mixed replacement rule (2 draws)**
A box contains 4 red and 8 blue balls. One ball is drawn at random.

- If the first ball is **red**, it is **not replaced**.
- If the first ball is **blue**, it **is replaced**.

A second ball is then drawn.

(a) Draw the **full** probability tree diagram.

(b) Find $P$(both balls are red).

(c) Find $P$(balls are of different colours).

(d) Find $P$(second ball is red).

(e) Find $P$(at least one blue is drawn).

6. **Mixed replacement rule with 3 colours (2 draws)**
A bag contains 3 pink, 5 yellow, 4 black balls. One ball is drawn at random.

- If the first ball is **black**, it is **not replaced**.
- Otherwise (pink or yellow), it **is replaced**.

A second ball is then drawn.

(a) Draw the **full** probability tree diagram.

(b) Find $P$(second ball is black).

(c) Find $P$(at least one black ball is drawn).

(d) Find $P$(both balls are the same colour).

7. **3 draws, without replacement**
A bag contains 5 green, 4 yellow, 3 red balls. Three balls are drawn one after another, **without replacement**.

(a) Draw the **full** probability tree diagram (3 stages).

(b) Find $P$(at least two green are drawn).

(c) Find $P$(exactly one yellow is drawn).

(d) Find $P$(all three balls are different colours).

(e) Find $P$(third ball is red).

8. **3 draws, with replacement**
A bag contains 2 red and 3 blue balls. Three balls are drawn one after another, **with replacement**.

(a) Draw the **full** probability tree diagram (3 stages).

(b) Find $P$(exactly two red are drawn).

(c) Find $P$(at least one red is drawn).

(d) Find $P$(all three are the same colour).

9. **All-different vs two-same (3 draws, without replacement)**
A bag contains 2 red, 2 blue, 2 green balls. Three balls are drawn one after another, **without replacement**.

(a) Draw the **full** probability tree diagram (3 stages).

(b) Find $P$(all three are different colours).

(c) Find $P$(exactly two are the same colour).

(d) Find $P$(at least one green is drawn).

10. **Find an unknown number (2 draws, without replacement)**
A bag contains 10 balls, of which $r$ are red and the rest are blue. Two balls are drawn one after another, **without replacement**.

(a) Draw the **full** probability tree diagram in terms of $r$.

(b) Given that $P(\text{both balls are red}) = \dfrac{1}{15}$, find $r$.

11. **Find an unknown number (mixed replacement rule)**
A bag contains $x$ red balls and 3 blue balls. One ball is drawn at random.

- If the first ball is **red**, it is **replaced**.
- If the first ball is **blue**, it is **not replaced**.

A second ball is then drawn.

(a) Draw the **full** probability tree diagram in terms of $x$.

(b) Given that $P(\text{second ball is red}) = \dfrac{11}{20}$, find $x$.

12. **Third draw extension (without replacement)**
A bag contains 5 blue and 3 yellow balls. Three balls are drawn one after another, **without replacement**.

(a) Draw the **full** probability tree diagram (3 stages).

(b) Find $P$(first two balls are blue).

(c) Find $P$(at most one blue in the first two draws).

(d) Find $P$(at least two blue balls in three draws).

## Part 3

1. **Travel choice + late / not late (tree + repeated days)**
   Each morning, Zara travels by Taxi, Bus or MRT.

| Mode | Taxi | Bus | MRT |
|---|---|---|---|
| Probability she uses this mode | $\frac{3}{10}$ | $\frac{2}{5}$ | $\frac{3}{10}$ |
| Probability she is late (if she uses this mode) | $\frac{1}{20}$ | $\frac{1}{5}$ | $\frac{1}{10}$ |

   Assume each day behaves the same way and is unaffected by previous days.

   (a) Draw the **full** probability tree diagram for one morning.

   (b) Find the probability Zara is late on a particular morning.

   (c) Over two consecutive mornings, find the probability she is late on **exactly one** day.

   (d) Over three consecutive mornings, find the probability she is on time on **at least one** day.

   (e) From Monday to Friday, find the probability she takes the Bus on all 5 days.

2. **Three boxes: good vs bad (no tree needed)**
   One fruit is selected from each box:

| Box | Total | Bad | Good |
|---|---|---|---|
| A | 10 | 2 | 8 |
| B | 8 | 2 | 6 |
| C | 6 | 1 | 5 |

   (a) Find $P$(all three are good).

   (b) Find $P$(exactly one is bad).

   (c) Find $P$(at least one is bad).

   (d) Find $P$(either all good OR all bad).

   (e) Find $P$(at least two are good).

3. **Algebra with two draws (quadratic)**
   A box contains 12 beads. $n$ are white and the rest are black. Two beads are chosen at random, **without replacement**.
   You are told:

   - The probability that the two beads are of **different colours** is $\frac{9}{22}$.
   - There are **more black beads than white beads**.

   (a) Find $n$.

   (b) Find $P$(both beads are black).

   (c) Find $P$(at least one bead is white).

4. **Three draws from 3 colours (tree)**
   A bag contains:

| Colour | Red | Blue | Green |
|---|---|---|---|
| Number of balls | 5 | 4 | 3 |

Three balls are drawn one after another, **without replacement**.

(a) Draw the **full** probability tree diagram (3 stages).

(b) Find $P$(at least two red balls are drawn).

(c) Find $P$(exactly one green ball is drawn).

(d) Find $P$(all three balls are different colours).

(e) Find $P$(the first two balls are the same colour, and the third is different).

5. **Maze in words (tree + algebra)**
Allyson starts at junction A. Let $p$ be a fixed integer from 1 to 6. At **every** junction, she either:

- goes straight with probability $\frac{p}{7}$, or
- changes direction with probability $\frac{7-p}{7}$.

Once she passes a junction, she cannot turn back. The game ends when she reaches a checkpoint or a dead end.

The maze behaves like this:

- At A: straight $\to$ B; change direction $\to$ C
- At B: straight $\to$ checkpoint Y (no prize); change direction $\to$ D
- At C: straight $\to$ D; change direction $\to$ checkpoint Z (no prize)
- At D: straight $\to$ checkpoint X (prize); change direction $\to$ Dead End

(a) Draw the **full** probability tree diagram.

(b) Write $P$(wins prize at X) in terms of $p$.

(c) Given that $P(\text{wins prize at X}) = \frac{72}{343}$, find $p$.

(d) Hence find $P$(hits the Dead End).

(e) Hence find $P$(ends at Y) and $P$(ends at Z).

6. **Target by area + total score**
A target consists of 3 concentric circles with radii 1 cm, 3 cm, 5 cm.

- Region A: inside radius 1 cm, score 60
- Region B: ring between 1 cm and 3 cm, score 20
- Region C: ring between 3 cm and 5 cm, score 5

A shooter hits the target every time and is equally likely to hit any part of the target. Shots are independent.

(a) After 3 shots, find $P(\text{total score } = 85)$.

(b) After 3 shots, find $P(\text{total score } \geq 120)$.

(c) After 2 shots, find $P(\text{total score } \geq 80)$.

7. **Product table + die (higher-order reasoning)**
There are two bags:

| Bag P cards | 1 | 2 | 3 | 4 |
|---|---|---|---|---|

| Bag Q cards | 1 | 2 | 3 | 5 |
|---|---|---|---|---|

A card is drawn from each bag. Let $a$ be the **product**.

(a) Copy and complete the probability diagram for $a$.

| $P\backslash Q$ | 1 | 2 | 3 | 5 |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |

An unbiased die has faces: $2, 4, 7, 9, 11, 14$. It is thrown once. Let the outcome be $b$.

(b) Find $P(a \text{ is prime})$.

(c) Find $P(a + b \text{ is odd})$.

(d) Find $P(a + b > 15)$.

(e) Find $P(a + b \text{ is a multiple of } 3)$.

8. **City blood types (large population)**
A city has blood types as follows:

| Type | O | A | B | AB |
|---|---|---|---|---|
| Percentage | 45% | 40% | 11% | 4% |

Three people are selected at random. Assume the city is so large that these percentages remain effectively unchanged from one selection to the next.

(a) Find $P(\text{at least one person is type AB})$.

(b) Find $P(\text{exactly one person is type O})$.

(c) Find $P(\text{at least two people are type O})$.

(d) Find $P(\text{all three have different blood types})$.

# Answers

### Part 1

**Q1.** (a)

| 1st \ 2nd | 1 | 2 | 4 | 7 | 8 |
|---|---|---|---|---|---|
| 1 | X | 3 | 5 | 8 | 9 |
| 2 | 3 | X | 6 | 9 | 10 |
| 4 | 5 | 6 | X | 11 | 12 |
| 7 | 8 | 9 | 11 | X | 15 |
| 8 | 9 | 10 | 12 | 15 | X |

(b) $\frac{1}{5}$; (c) $\frac{3}{10}$; (d) $\frac{2}{5}$; (e) $\frac{2}{5}$.

**Q2.** $P(A) = \frac{5}{12}$, $P(B) = \frac{2}{9}$, $P(\text{both } A \text{ and } B) = 0$, $P(A \text{ or } B \text{ or both}) = \frac{23}{36}$.

**Q3.** (a) $\frac{3}{10}$; (b) $\frac{1}{5}$; (c) $\frac{1}{5}$; (d) $\frac{2}{5}$.

**Q4.** (a)

| $P\backslash Q$ | 1 | 3 | 5 |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 4 | 5 | 7 | 9 |
| 6 | 7 | 9 | 11 |
| 8 | 9 | 11 | 13 |

(b) $\frac{3}{4}$; (c) $\frac{1}{3}$; (d) $\frac{1}{4}$; (e) 1.

**Q5.** (a)

| 1st \ 2nd | 1 | 3 | 4 | 6 |
|---|---|---|---|---|
| 1 | X | 2 | 3 | 5 |
| 3 | 2 | X | 1 | 3 |
| 4 | 3 | 1 | X | 2 |
| 6 | 5 | 3 | 2 | X |

(b) $\frac{1}{6}$; (c) $\frac{5}{6}$; (d) $\frac{1}{2}$.

**Q6.** (a) $\frac{1}{5}$; (b) $\frac{11}{15}$; (c) $\frac{4}{5}$; (d) $\frac{1}{3}$.

**Q7.** (a) $\frac{1}{3}$; (b) $\frac{5}{16}$; (c) No (since $\frac{5}{16} < \frac{1}{3}$).

**Q8.** (a) $\frac{1}{9}$; (b) $\frac{11}{36}$; (c) $\frac{2}{9}$.

**Q9.** (a) $\frac{2}{9}$; (b) $\frac{29}{72}$.

**Q10.** (a) $\frac{5}{12}$; (b) $\frac{1}{6}$.

### Part 2

**Q1.** (b) $\frac{25}{64}$; (c) $\frac{15}{32}$; (d) $\frac{39}{64}$.

**Q2.** (b) $\frac{3}{8}$; (c) $\frac{21}{40}$; (d) $\frac{19}{40}$; (e) $\frac{3}{8}$.

**Q3.** (b) $\frac{19}{66}$; (c) $\frac{9}{22}$; (d) $\frac{19}{33}$; (e) $\frac{5}{12}$.

**Q4.** (b) $\frac{35}{132}$; (c) $\frac{35}{66}$; (d) $\frac{7}{22}$; (e) $\frac{15}{22}$.

**Q5.** (b) $\frac{1}{11}$; (c) $\frac{46}{99}$; (d) $\frac{31}{99}$; (e) $\frac{10}{11}$.

**Q6.** (b) $\frac{31}{99}$; (c) $\frac{5}{9}$; (d) $\frac{259}{792}$.

**Q7.** (b) $\frac{4}{11}$; (c) $\frac{28}{55}$; (d) $\frac{3}{11}$; (e) $\frac{1}{4}$.

**Q8.** (b) $\frac{36}{125}$; (c) $\frac{98}{125}$; (d) $\frac{7}{25}$.

**Q9.** (b) $\frac{2}{5}$; (c) $\frac{3}{5}$; (d) $\frac{4}{5}$.

**Q10.** (b) $r = 3$.

**Q11.** (b) $x = 3$.

**Q12.** (b) $\frac{5}{14}$; (c) $\frac{9}{14}$; (d) $\frac{5}{7}$.

### Part 3

**Q1.** (b) $\frac{1}{8}$; (c) $\frac{7}{32}$; (d) $\frac{511}{512}$; (e) $\frac{32}{3125}$.

**Q2.** (a) $\frac{1}{2}$; (b) $\frac{47}{120}$; (c) $\frac{1}{2}$; (d) $\frac{61}{120}$; (e) $\frac{107}{120}$.

**Q3.** (a) $n = 3$; (b) $\frac{6}{11}$; (c) $\frac{5}{11}$.

**Q4.** (b) $\frac{4}{11}$; (c) $\frac{27}{55}$; (d) $\frac{3}{11}$; (e) $\frac{29}{132}$.

**Q5.** (b) $\dfrac{2p^2(7-p)}{343}$; (c) $p = 3$; (d) $\frac{96}{343}$; (e) $P(Y) = \frac{9}{49}$, $P(Z) = \frac{16}{49}$.

**Q6.** (a) $\frac{768}{15625}$; (b) $\frac{73}{15625}$; (c) $\frac{17}{625}$.

**Q7.** (a)

| $P\backslash Q$ | 1 | 2 | 3 | 5 |
|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 5 |
| 2 | 2 | 4 | 6 | 10 |
| 3 | 3 | 6 | 9 | 15 |
| 4 | 4 | 8 | 12 | 20 |

(b) $\frac{5}{16}$; (c) $\frac{1}{2}$; (d) $\frac{5}{12}$; (e) $\frac{29}{96}$.

**Q8.** (a) $\frac{1801}{15625}$; (b) $\frac{3267}{8000}$; (c) $\frac{1701}{4000}$; (d) $\frac{4611}{25000}$.