Student Handout
Chapter 1: Simple Linear Equations and Inequalities
(For practice sets like Exercises 1A, 1B, 1C)

Section 1 (Exercise 1A): Simplifying and Expanding Expressions

Quick checklist (use this every time)

- Combine only terms with the **same letter part** (e.g. x with x, y with y).

- When adding/subtracting fractions, make the **bottom numbers the same** first.

- When expanding, multiply the number outside the bracket by **every term** inside.

- A minus sign in front of a bracket changes **every sign** inside the bracket.

- If asked for **one fraction**, make one bottom number and combine into a single fraction.

A1. Combining like terms

You can combine terms only if they match in the letter part.

Example:
$$\frac{5}{6}x + \frac{1}{3}x$$

Make bottom numbers the same:
$$\frac{1}{3} = \frac{2}{6} \quad \Rightarrow \quad \frac{5}{6}x + \frac{2}{6}x = \frac{7}{6}x$$

Example (mixing x and y):
$$\frac{3}{5}x + \frac{1}{4}y - \frac{1}{10}x - \frac{3}{8}y$$

Group same letters:
$$\left(\frac{3}{5} - \frac{1}{10}\right)x + \left(\frac{1}{4} - \frac{3}{8}\right)y$$

Work each part:
$$\frac{3}{5} = \frac{6}{10} \Rightarrow \frac{6}{10} - \frac{1}{10} = \frac{1}{2}, \qquad \frac{1}{4} = \frac{2}{8} \Rightarrow \frac{2}{8} - \frac{3}{8} = -\frac{1}{8}$$

So:
$$\frac{1}{2}x - \frac{1}{8}y$$

A2. Expanding brackets (including fractions outside)

Rule: multiply the outside number by **every** term inside.

Example:

$$\frac{3}{4}(8x - 12) = \frac{3}{4} \cdot 8x - \frac{3}{4} \cdot 12 = 6x - 9$$

Example (bracket inside bracket):

$$\frac{1}{4}\left[12a - 5(2a - 4b)\right]$$

Do the inner bracket first:

$$5(2a - 4b) = 10a - 20b$$

Now subtract it (watch the minus sign!):

$$12a - (10a - 20b) = 12a - 10a + 20b = 2a + 20b$$

Multiply by $\frac{1}{4}$:

$$\frac{1}{4}(2a + 20b) = \frac{1}{2}a + 5b$$

A3. Writing as a single fraction

Goal: one fraction only.

Example:

$$\frac{5x - 1}{6} - \frac{x + 4}{9}$$

Use a common bottom number (here 18):

$$\frac{5x - 1}{6} = \frac{3(5x - 1)}{18}, \qquad \frac{x + 4}{9} = \frac{2(x + 4)}{18}$$

Combine:
$$\frac{3(5x - 1) - 2(x + 4)}{18} = \frac{15x - 3 - 2x - 8}{18} = \frac{13x - 11}{18}$$

A4. Common mistake to avoid

Minus sign in front of a bracket changes every sign inside.

Example:

$$12a - 5(2a - 4b)$$

The -5 affects both terms:

$$12a - (10a - 20b) = 12a - 10a + 20b$$

Section 2 (Exercise 1B): Solving Equations

Quick checklist (use this every time)

- Do the same thing to **both sides**.

- Undo steps in reverse order: **add/subtract first**, then **multiply/divide**.

- If there are fractions, multiply both sides by a good number to **clear the bottoms**.

- If the letter is on both sides, move all letter terms to **one side**.

- After you find the answer, do a quick check by substituting back (optional but helpful).

B1. One-step equations

Example:

$$\frac{1}{4}x = 9$$

Undo "divide by 4" by multiplying by 4:

$$x = 9 \times 4 = 36$$

Example:

$$\frac{5}{6}y = -10$$

Undo "times $\frac{5}{6}$" by multiplying by $\frac{6}{5}$:

$$y = -10 \times \frac{6}{5} = -12$$

B2. Two-step equations

Example:

$$\frac{1}{5}x + 7 = 3$$

Subtract 7 first:

$$\frac{1}{5}x = -4$$

Then multiply by 5:

$$x = -20$$

B3. The letter appears on both sides

Example:

$$x = 18 - \frac{1}{2}x$$

Add $\frac{1}{2}x$ to both sides:

$$x + \frac{1}{2}x = 18 \Rightarrow \frac{3}{2}x = 18 \Rightarrow x = 12$$

B4. Fractions with brackets: clear the bottoms

Example:
$$\frac{4x - 1}{3} = 5$$

Multiply both sides by 3:
$$4x - 1 = 15 \Rightarrow 4x = 16 \Rightarrow x = 4$$

Example:
$$\frac{2y + 3}{5} = \frac{y - 1}{2}$$

Multiply both sides by 10 (works for both 5 and 2):
$$2(2y + 3) = 5(y - 1) \Rightarrow 4y + 6 = 5y - 5 \Rightarrow y = 11$$

B5. The letter is in the bottom

Example:
$$\frac{3}{a} = \frac{1}{4}$$

Multiply both sides by a:
$$3 = \frac{a}{4} \Rightarrow a = 12$$

Example:
$$\frac{15}{b - 2} = -5$$

Multiply both sides by $(b - 2)$:
$$15 = -5(b - 2) \Rightarrow 15 = -5b + 10 \Rightarrow b = -1$$

(Also remember: $b \neq 2$ because dividing by 0 is not allowed.)

B6. Word problem style (two numbers)

Example: A smaller number is $\frac{3}{5}$ of a larger number. Their sum is 64.
   Let larger $= L$, smaller $= S$.

$$S = \frac{3}{5}L, \qquad S + L = 64$$

Substitute:
$$\frac{3}{5}L + L = 64 \Rightarrow \frac{8}{5}L = 64 \Rightarrow L = 40 \Rightarrow S = \frac{3}{5} \cdot 40 = 24$$

B7. "Given a solution, find k"

Example: Given $x = \frac{3}{2}$ is a solution of $5x - 2 = kx + 1$, find k.
   Substitute $x = \frac{3}{2}$:

$$5\left(\frac{3}{2}\right) - 2 = k\left(\frac{3}{2}\right) + 1 \Rightarrow \frac{15}{2} - 2 = \frac{3k}{2} + 1 \Rightarrow \frac{9}{2} = \frac{3k}{2} \Rightarrow k = 3$$

Section 3 (Exercise 1C): Inequalities

Quick checklist (use this every time)

- Solve like an equation: do the same thing to both sides.

- If you multiply or divide by a **negative number**, flip the sign.

- "At most" means $\leq$, "at least" means $\geq$.

- If asked for an integer answer, solve first, then choose the correct whole number.

- For packing problems: **maximum** $\Rightarrow$ round down, **minimum** $\Rightarrow$ round up.

C1. What the signs mean

$$x < 5 \text{ means } x \text{ is smaller than 5}, \quad x \leq 5 \text{ means } x \text{ is 5 or smaller.}$$
$$x > 5 \text{ means } x \text{ is bigger than 5}, \quad x \geq 5 \text{ means } x \text{ is 5 or bigger.}$$

C2. Solving inequalities (same steps as equations)

Example:
$$7x - 3 \leq 18 \Rightarrow 7x \leq 21 \Rightarrow x \leq 3$$

C3. The most important rule: negative numbers flip the sign

Example:
$$-5y > 20$$

Divide by -5 (negative), so flip $>$ to $<$:

$$y < -4$$

Example:

$$3 - 2t < 11 \Rightarrow -2t < 8 \Rightarrow t > -4 \quad \text{(sign flips when dividing by } -2)$$

C4. Smallest / greatest integer

Example: Find the smallest integer n such that $4n + 1 > 30$.

$$4n > 29 \Rightarrow n > \frac{29}{4} = 7.25$$

Smallest integer bigger than 7.25 is $\boxed{8}$.

Example: Find the greatest even integer x such that $4x \leq 50$.

$$x \leq 12.5$$

Greatest even integer ≤ 12.5 is $\boxed{12}$.

C5. Word problems: "at most" and "at least"

Example: Mei has at most \$25. Each notebook costs \$3.20. Number of notebooks is n.

$$3.2n \leq 25 \Rightarrow n \leq \frac{25}{3.2} = 7.8125$$

So the maximum whole number is $\boxed{7}$ notebooks.

C6. Packing / filling: rounding down vs rounding up

Example (maximum, round down): A tank holds at most 45 L. Each bottle holds 1.5 L.

$$\text{bottles} \leq \frac{45}{1.5} = 30 \Rightarrow \boxed{30}$$

Example (minimum, round up): A van carries at most 12 boxes. Need to carry 95 boxes.

$$\text{vans} \geq \frac{95}{12} \approx 7.92 \Rightarrow \boxed{8}$$

C7. Reverse-style question (find the number that makes the inequality work)

Example: The solution of $kd > 30$ is $d < -5$. Find k.
    To end up with $d <$, the sign must have flipped, so k is negative.

$$kd > 30 \Rightarrow d < \frac{30}{k} \quad \text{(flip because } k < 0\text{)}$$

We want $d < -5$, so
$$\frac{30}{k} = -5 \Rightarrow k = -6$$

Chapter 1: Simple Linear Equations and Inequalities
Practice Sets (Exercises 1A, 1B, 1C)

Section 1: Exercise 1A

1. Simplify each of the following expressions.

   (a) $\dfrac{5}{6}x + \dfrac{1}{3}x$

   (b) $\dfrac{7}{8}y - \dfrac{1}{4}y$

   (c) $\dfrac{3m}{5} + \dfrac{m}{10}$

   (d) $\dfrac{9p}{4} - \dfrac{p}{8}$

2. Simplify each of the following expressions.

   (a) $\dfrac{3}{5}x + \dfrac{1}{4}y - \dfrac{1}{10}x - \dfrac{3}{8}y$

   (b) $\dfrac{1}{2}a - \dfrac{2}{7}b + \dfrac{3}{4}a + \dfrac{5}{14}b$

   (c) $\dfrac{4}{9}c - \dfrac{1}{6}d + \dfrac{1}{3}c + \dfrac{5}{12}d$

   (d) $3f - \dfrac{7}{4}h + \dfrac{5}{2}h - \dfrac{3}{5}f$

3. Expand and simplify each of the following expressions.

   (a) $\dfrac{3}{4}(8x - 12)$

   (b) $\dfrac{2}{3}(9p + 6)$

   (c) $\dfrac{1}{4}\big[12a - 5(2a - 4b)\big]$

   (d) $\dfrac{1}{2}\big[6x - 4 - 2(3 - 5x)\big]$

4. Express each of the following as a fraction in its simplest form.

   (a) $\dfrac{3t}{7} + \dfrac{2}{5}(t - 1)$

   (b) $\dfrac{5x - 1}{6} - \dfrac{x + 4}{9}$

   (c) $\dfrac{2y + 5}{8} + \dfrac{3y - 1}{12}$

   (d) $\dfrac{1}{3}(a + 2) - \dfrac{1}{4}(2a - 5)$

5. Expand and simplify each of the following expressions.

(a) $x - \dfrac{3}{5}(10x - 5y)$

(b) $-\dfrac{1}{4}\big[8(m - n) - 2(3m + n)\big]$

6. Express each of the following as a fraction in its simplest form.

(a) $\dfrac{5(x - 2)}{3} + \dfrac{7(2x + 1)}{4}$

(b) $\dfrac{a + 1}{2} + \dfrac{a - 3}{5} - \dfrac{3a + 4}{10}$

Section 2: Exercise 1B

1. Solve each of the following equations.

(a) $\dfrac{1}{4}x = 9$

(b) $\dfrac{5}{6}y = -10$

(c) $\dfrac{1}{5}x + 7 = 3$

(d) $\dfrac{z}{3} - 5 = 1$

(e) $4p - \dfrac{7}{2} = 5$

(f) $6 - \dfrac{3}{5}q = 1.8$

2. Solve each of the following equations.

(a) $x = 18 - \dfrac{1}{2}x$

(b) $\dfrac{3}{4}y = \dfrac{1}{2}y + 5$

(c) $\dfrac{z}{3} + 2 = \dfrac{z}{6} + 5$

(d) $\dfrac{3}{5}p - 1 = \dfrac{1}{5}p + 3$

3. Solve each of the following equations.

(a) $\dfrac{3}{a} = \dfrac{1}{4}$

(b) $\dfrac{15}{b - 2} = -5$

4. A smaller number is $\dfrac{3}{5}$ of a larger number. The sum of the two numbers is 64.

(i) Find the larger number.

(ii) Hence find the smaller number.

5. Solve each of the following equations.

   (a) $\dfrac{4x-1}{3} = 5$

   (b) $\dfrac{2y+3}{5} = \dfrac{y-1}{2}$

   (c) $\dfrac{1}{3}(6p-9) = \dfrac{1}{2}(p+3)$

   (d) $\dfrac{3q+6}{4} - \dfrac{q+2}{2} = 1$

6. (a) Given that $x = \dfrac{3}{2}$ is a solution of $5x - 2 = kx + 1$, find k.

   (b) If $3x + 2y = 5y$, find $\dfrac{x}{y}$.

Section 3: Exercise 1C

1. Fill in each box with $<, >, \leq$ or $\geq$.

   (a) If $a > b$, then $2a \ \square\ 2b$.

   (b) If $m < n$, then $-m \ \square\ -n$.

   (c) If $x \leq y$, then $3 - x \ \square\ 3 - y$.

   (d) If $p \geq q$, then $\dfrac{p}{-4} \ \square\ \dfrac{q}{-4}$.

2. Solve each inequality.

   (a) $7x - 3 \leq 18$

   (b) $-5y > 20$

   (c) $3 - 2t < 11$

   (d) $\dfrac{1}{3}z \geq -2$

3. (a) Find the smallest integer n such that $4n + 1 > 30$.

   (b) Given that p satisfies $15p \geq 100$, find the smallest prime number p.

   (c) Find the greatest even integer x that satisfies $4x \leq 50$.

4. Mei has at most \$25. Each notebook costs \$3.20.

   (i) Write an inequality for the number of notebooks n she can buy.

   (ii) Find the maximum number of notebooks she can buy.

5. A tank holds at most 45 L. Each bottle holds 1.5 L. Find the maximum number of bottles that can be filled.

6. A van can carry at most 12 boxes. What is the minimum number of vans needed to carry 95 boxes?

7. A baker uses 180 g of flour per cake. She has 3.6 kg of flour. Find the maximum number of cakes she can bake.

8. The solution of the inequality $kd > 30$ is $d < -5$. Find k.

9. Given that x is an integer such that $2x \geq 11$ and $x < 8$, write down a possible value of x.

10. The perimeter of a square is at most 75 m.

   (i) Find the greatest possible side length.
   (ii) Hence find the greatest possible area, correct to 4 significant figures.

Answers

Section 1: Exercise 1A

1. (a) $\dfrac{7}{6}x$

   (b) $\dfrac{5}{8}y$

   (c) $\dfrac{7}{10}m$

   (d) $\dfrac{17}{8}p$

2. (a) $\dfrac{1}{2}x - \dfrac{1}{8}y$

   (b) $\dfrac{5}{4}a + \dfrac{1}{14}b$

   (c) $\dfrac{7}{9}c + \dfrac{1}{4}d$

   (d) $\dfrac{12}{5}f + \dfrac{3}{4}h$

3. (a) $6x - 9$

   (b) $6p + 4$

   (c) $\dfrac{1}{2}a + 5b$

   (d) $8x - 5$

4. (a) $\dfrac{29t - 14}{35}$

   (b) $\dfrac{13x - 11}{18}$

   (c) $\dfrac{12y + 13}{24}$

   (d) $\dfrac{23 - 2a}{12}$

5. (a) $3y - 5x$

   (b) $\dfrac{5n - m}{2}$

6. (a) $\dfrac{62x - 19}{12}$

   (b) $\dfrac{4a - 5}{10}$

Section 2: Exercise 1B

1. (a) $x = 36$

   (b) $y = -12$

   (c) $x = -20$

   (d) $z = 18$

   (e) $p = \dfrac{17}{8}$

(f) $q = 7$

2. (a) $x = 12$

  (b) $y = 20$

  (c) $z = 18$

  (d) $p = 10$

3. (a) $a = 12$

  (b) $b = -1$

4. (i) 40

  (ii) 24

5. (a) $x = 4$

  (b) $y = 11$

  (c) $p = 3$

  (d) $q = 2$

6. (a) $k = 3$

  (b) $\dfrac{x}{y} = 1$

Section 3: Exercise 1C

1. (a) $>$

  (b) $>$

  (c) $\geq$

  (d) $\leq$

2. (a) $x \leq 3$

  (b) $y < -4$

  (c) $t > -4$

  (d) $z \geq -6$

3. (a) 8

  (b) 7

  (c) 12

4. (i) $3.2n \leq 25$

  (ii) 7

5. 30

6. 8

7. 20

8. $k = -6$

9. $x = 6$ (also 7 works)

10. (i) $s \leq 18.75$, so greatest $s = 18.75$ m

  (ii) $A_{\max} = 18.75^2 = 351.6$ m^2 (4 s.f.)