Additional Maths Quadratic Chapter 1

Instructions. Answer all questions. Where a sketch is requested, sketch the graph clearly and label all key features (intercepts, turning point, and line of symmetry where appropriate).

Section 1: Completing the Square

Q1. Express $x^2 + \frac{5}{2}x - 7$ in the form $(x + a)^2 + b$.

Q2. Express $5x^2 - 20x + 7$ in the form $a(x + b)^2 + c$.

Q3. Express $\frac{1}{2}x^2 - 3x + 4$ in the form $a(x + b)^2 + c$.

Q4. Express $-\frac{2}{3}x^2 + 4x - 5$ in the form $a(x + b)^2 + c$.

Section 2: Sketching Quadratic Graphs

Q1. (i) Express $y = 3x^2 - 12x + 7$ in the form $a(x - h)^2 + k$.

(ii) Hence, sketch the graph and state the turning point, the x-intercepts (if any), the y-intercept, and the line of symmetry.

Q2. (i) Express $y = -2x^2 + 8x + 1$ in the form $a(x - h)^2 + k$.

(ii) Hence, sketch the graph and state the turning point, the x-intercepts (if any), the y-intercept, and the line of symmetry.

Q3. Sketch the graph of $y = 3(2x - 1)(x + 2)$ and state the x-intercepts, the y-intercept, the turning point, and the line of symmetry.

Q4. Sketch the graph of $y = (x + 3)(5 - 2x)$ and state the x-intercepts, the y-intercept, the turning point, and the line of symmetry.

Section 3: Maximum/Minimum Values

Q1. Given $f(x) = 2x^2 - 8x + 11$, find the smallest possible value of $f(x)$ and state the value of x at which it occurs.

Q2. Explain why $x^2 + 6x + 10$ is always positive for all real x, and state its minimum value.

Q3. Explain why $-4x^2 - 4x - 2$ is always negative for all real x, and state its maximum value.

Q4. Find the maximum value of $y = -2x^2 + 7x - 3$ and the value of x at which it occurs.

Section 4: Applications of Quadratic Functions

Q1. A cable between two towers has height above the roadway

$$y = \frac{1}{900}(x - 150)^2 + 9, \qquad 0 \le x \le 300,$$

where x (m) is the horizontal distance from the left tower.

(i) Find the height of each tower above the roadway.

(ii) Find the distance between the towers.

(iii) Find the possible distances from the left tower where the cable is 20 m above the roadway.

Q2. A bridge cable is supported by vertical wires. The two end wires are 35 m long and are 180 m apart. The shortest wire, at the midpoint, is 8 m long. Let x be the horizontal distance (m) from the left end wire and y the wire length (m).

(i) Find y in the form $y = a(x - h)^2 + k$.

(ii) Find the wire length when $x = 50$.

Q3. A flare is launched from a platform 2 m above ground. Its height (m) is modelled by

$$y = -\frac{1}{25}x^2 + \frac{8}{5}x + 2,$$

where x is the horizontal distance (m) from the launch point.

(i) Find the maximum height and the horizontal distance at which it occurs.

(ii) Find the horizontal distance from the launch point when the flare hits the ground.

(iii) A mast at $x = 30$ m is 15 m tall. Determine whether the flare clears the mast.

Q4. Two shells are fired from ground level at the origin. Their paths are modelled (in metres) by

$$\text{Shell 1: } y = -\frac{1}{200}x^2 + 3x, \qquad \text{Shell 2: } y = -\frac{1}{500}(x - 350)^2 + 245.$$

(i) Assuming Shell 1 hits an enemy frigate at sea level, how far from the origin is the frigate?

(ii) Find the maximum height of Shell 1.

(iii) Determine whether Shell 2 will hit the same frigate. Justify your answer.

(iv) What is the horizontal distance from the origin to where Shell 2 lands?

Solutions

Section 1

Q1. $\left(x+\frac{5}{4}\right)^2-\frac{137}{16}$.

Q2. $5(x-2)^2-13$.

Q3. $\frac{1}{2}(x-3)^2-\frac{1}{2}$.

Q4. $-\frac{2}{3}(x-3)^2+1$.

Section 2

Q1. $y=3(x-2)^2-5$. Turning point $(2,-5)$, line of symmetry $x=2$, y-intercept $(0,7)$, x-intercepts $\left(2\pm\frac{\sqrt{15}}{3},0\right)$.

Q2. $y=-2(x-2)^2+9$. Turning point $(2,9)$, line of symmetry $x=2$, y-intercept $(0,1)$, x-intercepts $\left(2\pm\frac{3\sqrt{2}}{2},0\right)$.

Q3. x-intercepts $(-2,0),\left(\frac{1}{2},0\right)$; y-intercept $(0,-6)$; line of symmetry $x=-\frac{3}{4}$; turning point $\left(-\frac{3}{4},-\frac{75}{8}\right)$ (minimum).

Q4. x-intercepts $(-3,0),\left(\frac{5}{2},0\right)$; y-intercept $(0,15)$; line of symmetry $x=-\frac{1}{4}$; turning point $\left(-\frac{1}{4},\frac{121}{8}\right)$ (maximum).

Section 3

Q1. $f(x)=2(x-2)^2+3$, so minimum value is 3 at $x=2$.

Q2. $(x+3)^2+1\geq 1>0$, so it is always positive. Minimum value is 1.

Q3. $-4\left(x+\frac{1}{2}\right)^2-1\leq -1<0$, so it is always negative. Maximum value is -1 at $x=-\frac{1}{2}$.

Q4. $y=-2\left(x-\frac{7}{4}\right)^2+\frac{25}{8}$, so maximum value is $\frac{25}{8}$ at $x=\frac{7}{4}$.

Section 4

Q1. (i) $y(0)=34$ and $y(300)=34$, so each tower is 34 m high.

(ii) Distance between towers: 300 m.

(iii) $20=\frac{1}{900}(x-150)^2+9\Rightarrow(x-150)^2=9900\Rightarrow x=150\pm 30\sqrt{11}\approx$ 50.5 m or 249.5 m.

Q2. (i) Vertex $(90,8)$: $y=a(x-90)^2+8$. Using $(0,35)$ gives $35=8100a+8\Rightarrow a=\frac{1}{300}$, so $y=\frac{1}{300}(x-90)^2+8$.

(ii) $y(50)=\frac{1}{300}(50-90)^2+8=\frac{40}{3}$ m.

Q3. (i) Vertex at $x=20$, maximum height $y(20)=18$ m.

(ii) $0=-\frac{1}{25}x^2+\frac{8}{5}x+2\Rightarrow x=20\pm 15\sqrt{2}$; physical root $x=20+15\sqrt{2}\approx 41.2$ m.

(iii) $y(30)=14<15$, so it does not clear the mast.

Q4. (i) $0=-\frac{1}{200}x^2+3x\Rightarrow x=600$ m (non-zero root).

(ii) Maximum at $x=300$: $y(300)=450$ m.

(iii) At $x=600$, Shell 2 has $y=-\frac{1}{500}(250)^2+245=120>0$, so it is still airborne and does not hit the frigate.

(iv) Shell 2 lands when $0=-\frac{1}{500}(x-350)^2+245\Rightarrow(x-350)^2=350^2\Rightarrow x=700$ m.