HCF and LCM: Second Revision Set

Section 1: Highest Common Factor (HCF) and Lowest Common Multiple (LCM)

1.1. Find the HCF of 693 and 840.

1.2. Find the LCM of 84 and 132.

1.3. Find the HCF of 150, 420 and 630.

1.4. Find the LCM of 90, 120 and 168.

1.5. Find the HCF and LCM of the following, **leaving your answers in index form**:

$$2^3 \times 3^4 \times 5, \qquad 2^5 \times 3^2 \times 7, \qquad 2^2 \times 3^3 \times 5^2 \times 7.$$

1.6. Given that $756 = 2^2 \times 3^3 \times 7$.

(a) Express 420 as a product of prime factors.

(b) Hence, find the HCF and LCM of 756 and 420.

1.7. The numbers 450 and 525 are given.

(a) Express each as a product of prime factors (index form).

(b) Hence, find their HCF and LCM.

(c) Find the smallest integer k such that $450k$ is a perfect square.

1.8. (a) Express 540 as a product of prime factors (index form).

(b) Find the smallest integer k such that $540k$ is a perfect square.

(c) Find the smallest integer m such that $540m$ is a perfect cube.

1.9. Given that $960 = 2^6 \times 3 \times 5$, find the smallest positive integer q such that $\dfrac{960}{q}$ is a cube number.

1.10. (a) Express 252 as a product of prime factors.

(b) Find the smallest integer t such that $\sqrt{252t}$ is a whole number.

(c) Hence evaluate $\sqrt{252t}$.

1.11. Written as a product of prime factors, $2160 = 2^4 \times 3^3 \times 5$. Find the smallest positive integer k such that $\dfrac{2160}{k}$ is a cube number.

1.12. The numbers 2750 and 1980 are given.

(a) Find their HCF and LCM.

(b) Hence find the smallest positive integer n such that $2750n$ is a multiple of 1980.

1.13. Find the smallest integers x and y such that $2^x \times 3^2 \times 5^y \times 7$ is a multiple of 840.

1.14. (a) Express 882 as a product of prime factors.

(b) Find the smallest integer r such that $882r$ is a perfect cube.

(c) Hence find $\sqrt[3]{882r}$.

1.15. Let $N = 2^2 \times 3^5 \times 5 \times 7^2$. Find the smallest positive integer k such that Nk is **both** a perfect square and a perfect cube. (Leave k in index form.)

Section 2: HCF–LCM Word Problems

2.1. 180 boys and 252 girls are to be divided into mixed groups such that the number of boys in each group is the same and the number of girls in each group is the same.

(a) Find the largest possible number of groups.

(b) Hence find the number of boys per group and girls per group.

2.2. A company packs 1170 boxes of snacks, 810 bottles of juice and 1350 sandwiches into as many goodie bags as possible. Each bag must have the same contents.

(a) Find the maximum number of goodie bags.

(b) State the contents of each bag.

2.3. Three traffic lights turn red at fixed intervals of 50 s, 65 s and 78 s. They turn red together at **07:48.30**. Find the next time they will turn red simultaneously again.

2.4. On a screen, three images light up once every 56 s, 90 s and 84 s respectively. Find the interval, in minutes, at which all three images light up together.

2.5. Three comets pass through our solar system every 72 years, 90 years and 150 years respectively. They were last seen together in the year 1920. In which year will they be seen together again?

2.6. The floor of a hall is 4.8 m by 6.3 m and is to be tiled with identical square tiles.

(a) Find the largest possible side length (in cm) of each tile.

(b) Find the number of tiles needed.

2.7. Three toy cars A, B, C start at the same point on a circular track. They take 72 s, 90 s and 105 s respectively to complete one round. The race starts at **13:18**.

(a) Find the time when they next meet at the start point again.

(b) How many rounds has car C completed then?

2.8. Mrs Tan baked some muffins. There are always 5 muffins left over whether the muffins are packed into packets of 7, 9 or 12. What is the minimum possible number of muffins she baked?

2.9. A ribbon is shared equally among 4 tailors. They cut their ribbons into 7-cm, 9-cm, 12-cm and 14-cm pieces respectively. To avoid wastage, there must be no leftover after cutting. Find the least possible total length of the original ribbon.

2.10. Cylindrical cans A, B, C have heights 9 cm, 12 cm and 20 cm respectively. They are stacked (using only one type per stack) to the same height.

(a) Find the minimum common height.

(b) Find the minimum number of B cans required.

2.11. Three sprinklers turn on every 40 s, 72 s and 90 s. They start together at **15 06 20**.

(a) When will they next start together?

(b) How many cycles will the 40 s sprinkler complete by then?

2.12. Three alarm clocks ring every 35 min, 50 min and 84 min. They ring together at **06:00 on Monday**. When will they next ring together?

2.13. Three drone cars race around a track. Car A takes 18 s per lap, B takes 30 s per lap, and C takes 45 s per lap.

(a) Find the time taken for them to next meet at the start point.

(b) By how many laps is A ahead of C at that time?

2.14. Poles of lengths 2.64 m, 3.30 m and 4.62 m are cut into equal pieces of greatest possible length (no leftover).

(a) Find the length of each piece (in cm).

(b) How many pieces come from each pole?

2.15. 210 pencils, 294 erasers and 378 rulers are packed into as many identical stationery sets as possible with none left over.

(a) Find the maximum number of sets.

(b) State the contents of each set.

Answers

Section 1

1.1. 21

1.2. 924

1.3. 30

1.4. 2520

1.5. HCF $= 2^2 \times 3^2$; LCM $= 2^5 \times 3^4 \times 5^2 \times 7$

1.6. (a) $420 = 2^2 \times 3 \times 5 \times 7$ (b) HCF $= 84$, LCM $= 3780$

1.7. (a) $450 = 2 \times 3^2 \times 5^2$, $525 = 3 \times 5^2 \times 7$ (b) HCF $= 75$, LCM $= 3150$ (c) $k = 2$

1.8. (a) $540 = 2^2 \times 3^3 \times 5$ (b) $k = 15$ (c) $m = 50$

1.9. $q = 15$ (since $960/15 = 64 = 4^3$)

1.10. (a) $252 = 2^2 \times 3^2 \times 7$ (b) $t = 7$ (c) $\sqrt{252t} = 42$

1.11. $k = 10$ (since $2160/10 = 216 = 6^3$)

1.12. (a) HCF $= 110$, LCM $= 49500$ (b) $n = 18$

1.13. $x = 3,\ y = 1$

1.14. (a) $882 = 2 \times 3^2 \times 7^2$ (b) $r = 84$ (c) $\sqrt[3]{882r} = 42$

1.15. $k = 2^4 \times 3 \times 5^5 \times 7^4$

Section 2

2.1. (a) 36 groups (b) 5 boys, 7 girls

2.2. (a) 90 bags (b) 13 snack boxes, 9 juice bottles, 15 sandwiches

2.3. 08 21 00

2.4. 42 min

2.5. 3720

2.6. (a) 30 cm (b) 336 tiles

2.7. (a) 14 00 (b) 24 rounds

2.8. 257

2.9. 1008 cm

2.10. (a) 180 cm (b) 15

2.11. (a) 15 12 20 (b) 9 cycles

2.12. Tuesday 17:00

2.13. (a) 90 s (b) 3 laps

2.14. (a) 66 cm (b) 4 pieces, 5 pieces, 7 pieces

2.15. (a) 42 sets (b) 5 pencils, 7 erasers, 9 rulers