Recursion and Tail Recursion
Functional Programming

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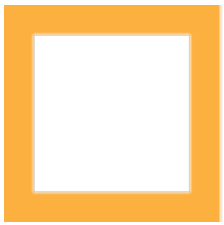

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Spring 2018

Outline

Stacks in CPUs

Recursion

Hand-in Tail recursion

Frame content

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What do we need to know? What does the frame contain?

Frame content

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Frame content

What do we need to know? What does the frame contain?

- Space for return value and arguments

Frame content

What do we need to know? What does the frame contain?

- Space for return value and arguments
- Return address

Frame content

What do we need to know? What does the frame contain?

- Space for return value and arguments
- Return address: Where do we go after we're done?

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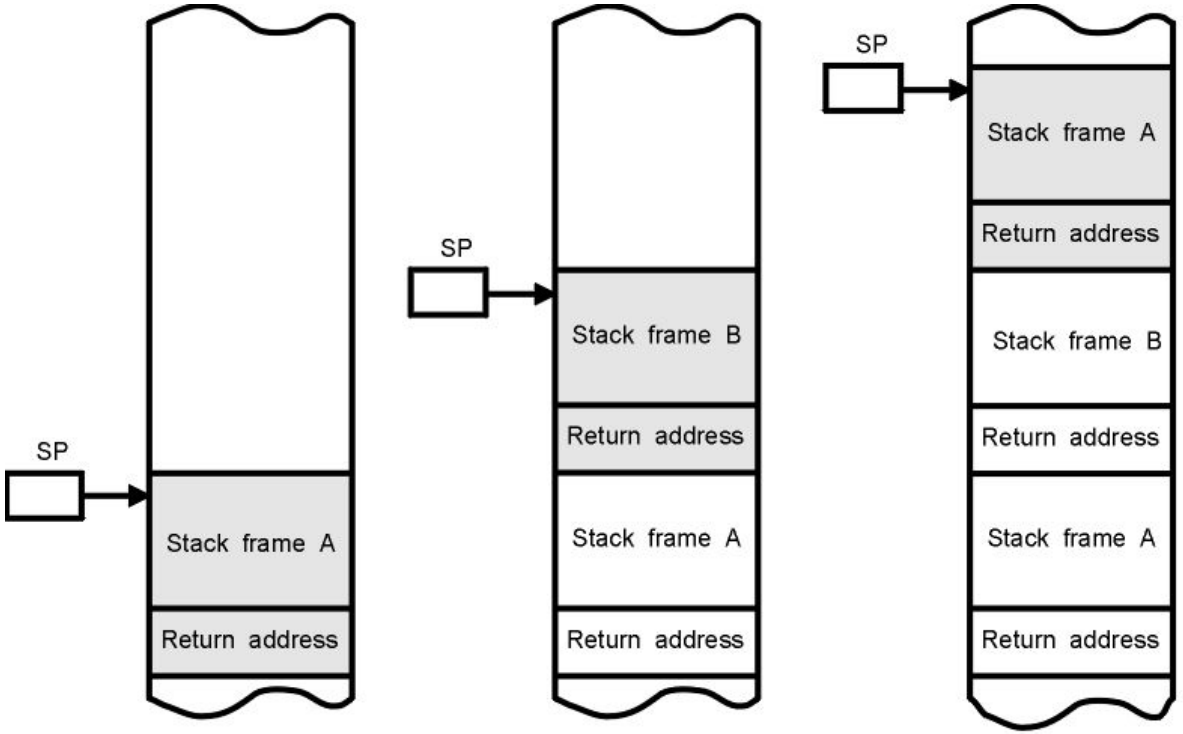
Disclaimer: Implementation specific

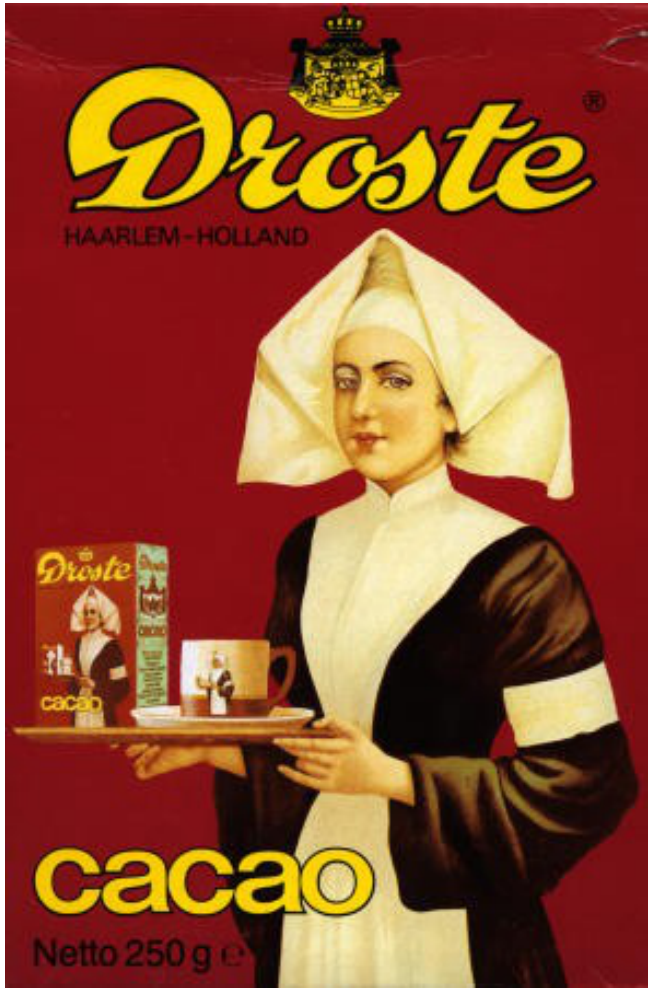
Stack frames

What happens every time we call a function?

We create a new stack frame!



Recursion



Recursion on the stack

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```
public void fact(int n) {
  if (n == 0) return 1;
  return n*fact(n - 1);
  }
```

```
fact(5)
5*fact(4)
5*4*fact(3)
5*4*3*fact(2)
5*4*3*2*fact(1)
5*4*3*2*1*fact(0)
5*4*3*2*1*1
5*4*3*2*1
5*4*3*2
5*4*6
5*24
120
```

Tail recursive version on the stack

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```
public void fact(int n) { return fact(1, n); }

private void fact(int acc, int n) {
  if (n == 0) return acc;
  return fact(n*acc, n - 1);
  }
```

```
fact(5)
fact(1, 5)
fact(5, 4)
fact(20, 3)
fact(60, 2)
fact(120, 1)
fact(120, 0)
120
```

Exercise 1 - Stack frames in fibonacci

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```
public long fibonacci(int n) {
  if (n <= 1) return n;
  else return fibonacci(n-1) + fibonacci(n-2);
  }
```

1. How many stack frames do we create with `fibonacci(2)`?
2. How many stack frames do we create with `fibonacci(5)`?
3. How many stack frames do we create with `fibonacci(10)`?
4. What is the general formula for how many stack frames the `fibonacci` function creates?

Exercise 2 - Factorial in Java

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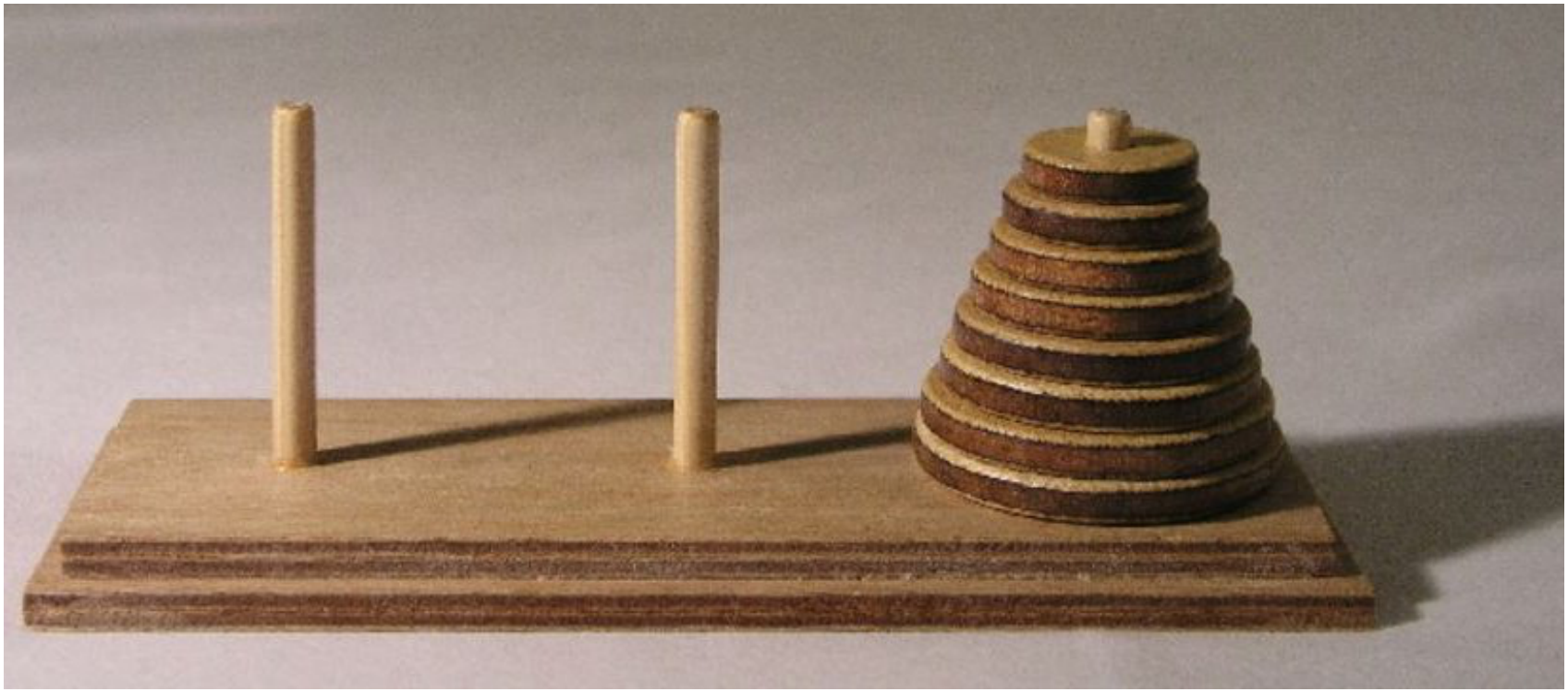
In a new Java project:

1. Implement a function for factorial using `BigInteger`
   `public static BigInteger factorial(BigInteger i);`
2. Run factorial with `100_000` as input. What happens?
3. Try to run it with `10`. Better now?
4. In `Run -> Set Project Configuration -> Customize... -> VM Options` write '`-Xss20m`'
5. Run it with `100_000` as input. What happened?

1. Only one disk can be moved at a time.
2. Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack i.e. a disk can only be moved if it is the uppermost disk on a stack.
3. No disk may be placed on top of a smaller disk.



Exercise 4 - Reverse list

With a list of the `Path` type we made last Tuesday:

Make a recursive fundtion that reverses the order of list elements.

Hand-in

1. Finish the virtual CPU

Hand-in

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2. Program a tail recursive version of factorial

Hand-in

1. Finish the virtual CPU
2. Program a tail recursive version of factorial

Hint: Use the factorial implementation from Anders on GitHub under **general/ 02 Assignment**