

## MODULE - 1

### MODERN PHYSICS

#### Dual nature of matter

The photoelectric effect and the Compton scattering conclusively established the particle behavior of light. The phenomena of interference, diffraction and polarization give exclusive evidence for the wave behavior of light. On one hand, light resembles a collection of particles having energy  $E$  and momentum  $p$  and on the other hand, it is regarded as a continuous wave of frequency ( $\gamma$ ). Either of these separate pictures is not in a position to explain all the experimental results and hence, we have to conclude that light behaves as an advancing wave in some phenomenon and it behaves as a flux of particles in some other phenomena. Therefore, we say that light exhibits *wave-particle duality*.

#### de Broglie's hypothesis

We know that the phenomenon such as interference, diffraction, polarization etc. can be explained only with the help of wave theory of light. While phenomenon such as photoelectric effect, Compton effect, spectrum of blackbody radiation can be explained only with the help of Quantum theory of radiation. Thus radiation is assumed to exhibit dual nature. i.e. both the particle and wave nature. In 1924, **Louis de-Broglie** made a bold hypothesis, which can be stated as follows

**“If radiation which is basically a wave can exhibit particle nature under certain circumstances, and since nature likes symmetry, then entities which exhibit particle nature ordinarily, should also exhibit wave nature under suitable circumstances.”**

Thus according to De-Broglie's hypothesis, there is wave associated with the moving particle. Such waves are called **Matter waves** and wavelength of the wave associated with the particle is called **De-Broglie wavelength**.

#### Expression for de Broglie wavelength:

As a photon travels with the velocity  $c$ , we can express its momentum as,

$$\begin{aligned}
 p &= \frac{E}{c} \\
 p &= \frac{h\gamma}{c} \\
 p &= \frac{h}{\lambda}
 \end{aligned}
 \tag{10}$$

Thus, the wavelength  $\lambda$  and momentum  $p$  of a photon are related to each other through the expression

$$\lambda = \frac{h}{p} \tag{11}$$

De Broglie proposed that the relation (11) between the momentum and the wavelength of a photon is a universal one and must be applicable to photons and material particles as well.

Now, let us consider a moving particle. A particle of mass  $m$  moving with a velocity  $v$  carries a momentum  $p = mv$  and it must be associated with a wave of wavelength

$$\lambda = \frac{h}{mv} \tag{12}$$

Relation (12) is known as ***de Broglie equation*** and the wavelength  $\lambda$  is called the ***de Broglie wavelength***.

### **De-Broglie wavelength associated with an accelerated charged particle:**

If a charged particle, say an electron is accelerated by a potential difference of  $V$  volts, then its kinetic energy is given by

$$\begin{aligned}
 E_K &= eV. \\
 \text{or } \frac{1}{2}mv^2 &= eV \\
 v &= \sqrt{\frac{2eV}{m}}
 \end{aligned}$$

Then the electron wavelength is given by,

$$\lambda = \frac{h}{mv} = \frac{h}{m} \sqrt{\frac{m}{2eV}}$$

$$\text{Thus, } \lambda = \frac{h}{\sqrt{2meV}} \quad (13)$$

By substituting the values of constants  $h$ ,  $m$  and  $e$  in eq. (13), we get,

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times (9.11 \times 10^{-31}) \times (1.602 \times 10^{-19} V)}} = \frac{1.226 \times 10^{-9}}{\sqrt{V}} m$$

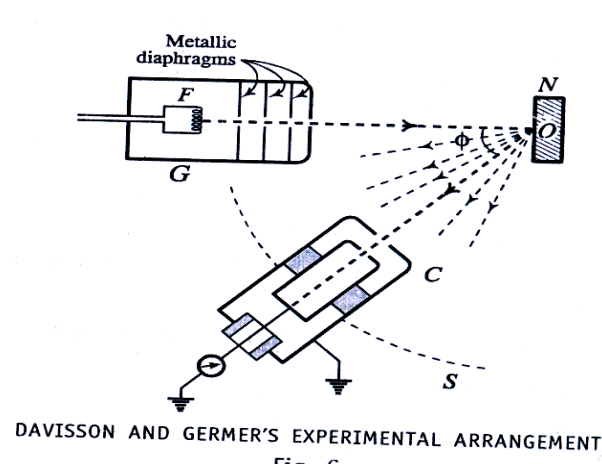
$$\text{or } \lambda = \frac{1.226}{\sqrt{V}} nm$$

### Davisson-Germer's experiment:

Davisson and Germer were studying the phenomenon of scattering of electrons from material targets and they observed *diffraction of electrons* in a crystal of nickel, similar to *X-ray waves undergoing diffraction* in crystals, thus proving the wave behavior of electrons.

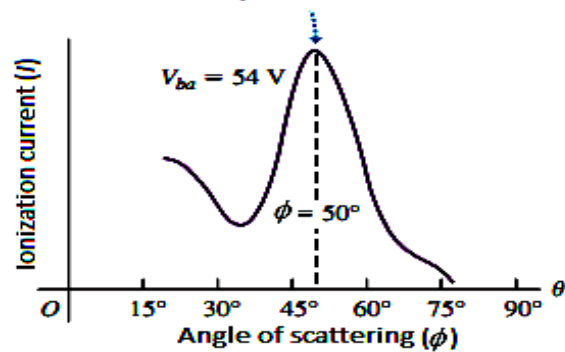
The experimental apparatus is as shown in the above fig. consists of an electron gun G, which produces a narrow and collimated beam of electrons accelerated to known potential  $V$ . a solid nickel crystal used as a target mounted on a rotatable stand and an ionization chamber (detector) C which is connected to a galvanometer to collect and measure the current due to the scattered electrons.

Electron beam is made to strike the nickel crystal C. Electrons scattered from the crystal is collected by the ionization chamber at various scattering angles  $\phi$  and the corresponding value of ionization current  $I$  is noted. Experiment is repeated for various accelerating potentials  $V$ . A graph representing  $\phi$  and  $I$  for different values of  $V$  is plotted. The graph obtained is shown in



the

fig.

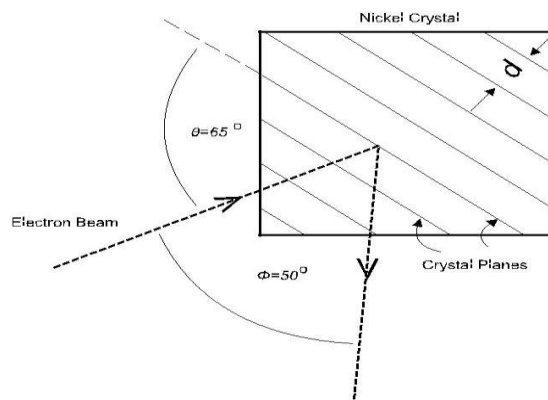


Initially, when accelerating potential was 40 V a smooth curve was obtained. When potential was increased to 44 V ionization current started to increase and reaches maximum for the accelerating potential of 54 V. The scattering angle corresponding to the accelerating Potential 54 V when Ionization current became maximum was found to be  $\phi = 50^\circ$ .

Davisson and Germer interpreted the result as follows:

Electrons in the incident beam behave like waves. Thus when electrons strike the crystal they undergo Bragg's diffraction from the different planes of the crystal. The bump in the curve corresponds to constructive interference caused by the scattered electrons. According to Bragg's law the condition for constructive interference is given by

$$2d \sin \theta = n\lambda$$



where  $d$  = Interplanar spacing for the crystal,  $\theta$  = glancing angle made by incident beam with the crystal plane,  $n$ =order and  $\lambda$  is wavelength of the wave. Thus, when bump in the curve is maximum,  $\theta = 65^\circ$  (see fig of crystal),  $n=1$  and for nickel crystal,  $d = 0.91 \text{ \AA}$ . Thus, wavelength of the waves associated with the electrons is given by

$$\lambda = 2 \times 0.91 \times 10^{-10} \times \sin(65^\circ) = 1.65 \times 10^{-10} \text{ m}$$

According to de Broglie's hypothesis for an electron accelerated by potential difference of 54 V the de Broglie wavelength is given by

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ \AA} = \frac{1.226 \times 10^{-9}}{\sqrt{54}} = 1.66 \times 10^{-10} \text{ m}$$

The experimentally determined value is in good agreement with the value calculated according to de Broglie's hypothesis. Thus Davisson and Germer experiment not only confirms the wave associated with moving particle it also verifies the de Broglie's hypothesis.

**Phase velocity and Group velocity****Phase velocity ( $v_p$ )**

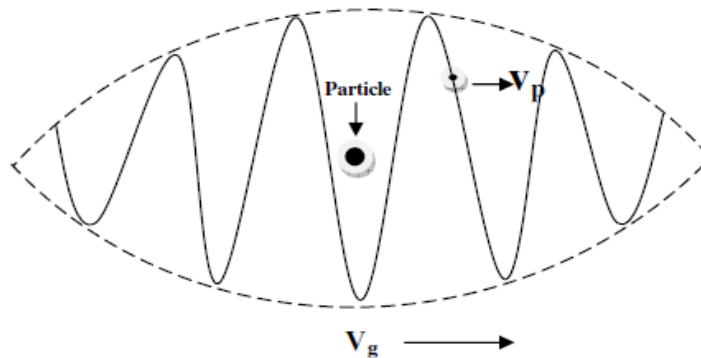
The velocity with which a wave travels is called phase velocity and is also called wave velocity. If a point is marked on the wave representing the phase of the particle then the velocity with which the phase propagates from one point to another is called phase velocity. It is given by

$v_p = \frac{\omega}{k}$  Where ' $\omega$ ' is the angular frequency and ' $k$ ' is wave number. Substituting for  $\omega = 2\pi\nu$  and  $k = \frac{2\pi}{\lambda}$ . We get  $v_p = \nu\lambda$ ,

Therefore  $v_p = \frac{h\nu}{\frac{h}{\lambda}} = \frac{E}{p}$  Where ' $E$ ' is the energy and ' $p$ ' is momentum

$v_p = \frac{mc^2}{mv} = \frac{c^2}{v}$  Where ' $c$ ' is the velocity of light and ' $v$ ' is the velocity of the particle.

From the above expression it is evident that the phase velocity is not only greater than the particle velocity it is also greater than the velocity of light. Hence there is no physical meaning for phase velocity of matter waves.

**Group velocity ( $v_g$ )**

Since the velocity of matter waves must be equal to that of the particle velocity and since no physical can be associated with phase velocity the concept of group velocity is introduced. Matter wave can be considered as a resultant wave due to the superposition of many component waves whose velocities differ slightly. Thus a wave group or wave packet is formed. The velocity with which the wave group travels is called group velocity which is same as particle velocity. It is denoted by  $v_g$ .

**Properties of Matter waves**

The following are the properties associated with the matter waves

- 1) Matter waves are associated only with particles in motion
- 2) They are not electromagnetic in nature
- 3) Group velocity is associated with matter waves
- 4) The phase velocity has no physical meaning for matter waves
- 5) The amplitude of the matter wave at a given point is associated with the probability of finding the particle at that point.
- 6) The wavelength of matter waves is given by  $\lambda = \frac{h}{p}$

**WAVE MECHANICS****Heisenberg's Uncertainty Principle**

Statement: "The simultaneous determination of the exact position and momentum of a moving particle is impossible"

Explanation: According to this principle if  $\Delta x$  is the error involved in the measurement of position and  $\Delta p$  is the error involved in the measurement of momentum during their simultaneous measurement, then the product of the corresponding uncertainties is given by

$$\Delta x \times \Delta p \geq \frac{h}{4\pi} \quad \text{similarly} \quad \Delta E \times \Delta t \geq \frac{h}{4\pi}$$

The product of the errors is of the order of Planck's constant. If one quantity is measured with high accuracy then the simultaneous measurement of the other quantity becomes less accurate.

Physical significance: According to Newtonian physics the simultaneous measurement of position and momentum are "exact". But the existence of matter waves induces serious problems due to the limit to accuracy associated with the simultaneous measurement. Hence the "Exactness" in Newtonian physics is replaced by "Probability" in quantum mechanics.

Suggested reading: Principle of complementarity and Schrodinger's cat.

**Application of Heisenberg's uncertainty principle**

**Non-existence of the electron in the nucleus:** Beta rays are emitted by the nucleus. When it was first observed it was believed that electrons exist inside the nucleus and are emitted at certain instant. If the electron can exist inside the atomic nucleus then uncertainty in its position must not exceed the diameter of the nucleus. The diameter of the nucleus is of the order of  $10^{-14}$ .