1.

According to the law of mass action, for elementary reaction like

$$aA + bB \rightarrow mG + nD$$

At a given temperature, the reaction rate is proportional to the product of c_A and c_B powers of the reactant concentration and the mathematical expression is

$$v = kc_A^a c_B^b$$

For the reaction

$$E + S \underset{k_2}{\overset{k_1}{\rightleftharpoons}} ES \xrightarrow{k_3} E + P$$

The equations for the rate of changes of the four species, E, S, ES, and P are

$$v_E = k_2[ES] + k_3[ES] - k_1[E][S]$$
$$v_S = k_2[ES] - k_1[E][S]$$
$$v_{ES} = k_1[E][S] - k_2[ES] - k_3[ES]$$
$$v_P = k_3[ES]$$

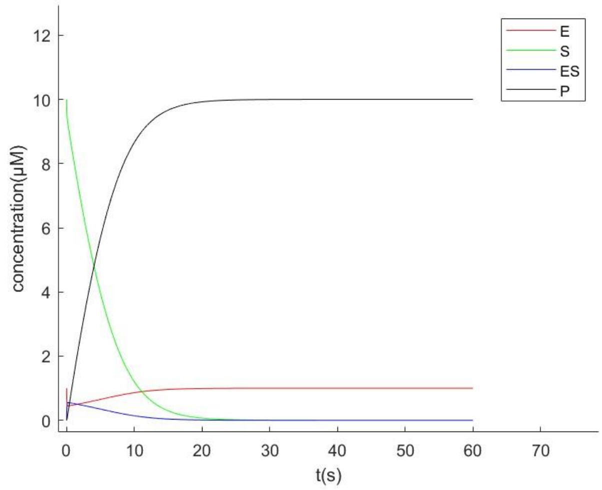
2.

In order solve the equations for the rate of changes of the four species, E, S, ES, and P, I write and run a code on MATLAB

```
clear;
clc;
close all;

h=1e-3;        %step length
t=0:h:60;      %generate a vector of arguments

N=length(t);
x_0 = 1;
x = x_0;
y_0 = 10;
y = y_0;
z_0 = 0;
z = z_0;
w_0 = 0;
w = w_0;

%%fourth-order Runge-Kutta iteration
for i=2:N
    t_n=t(i-1);
    x_n=x(i-1);
    y_n=y(i-1);
    z_n=z(i-1);
    w_n=w(i-1);
```

```
    kx1=10*z_n+2.5*z_n-5/3*x_n*y_n;
    ky1=10*z_n-5/3*x_n*y_n;
    kz1=5/3*x_n*y_n-10*z_n-2.5*z_n;
    kw1=2.5*z_n;

    kx2=10*(z_n+kz1*h/2)+2.5*(z_n+kz1*h/2)-5/3*(x_n+kx1*h/2)*(y_n+ky1*h/2);
    ky2=10*(z_n+kz1*h/2)-5/3*(x_n+kx1*h/2)*(y_n+ky1*h/2);
    kz2=5/3*(x_n+kx1*h/2)*(y_n+ky1*h/2)-10*(z_n+kz1*h/2)-2.5*(z_n+kz1*h/2);
    kw2=2.5*(z_n+kz1*h/2);

    kx3=10*(z_n+kz2*h/2)+2.5*(z_n+kz2*h/2)-5/3*(x_n+kx2*h/2)*(y_n+ky2*h/2);
    ky3=10*(z_n+kz2*h/2)-5/3*(x_n+kx2*h/2)*(y_n+ky2*h/2);
    kz3=5/3*(x_n+kx2*h/2)*(y_n+ky2*h/2)-10*(z_n+kz2*h/2)-2.5*(z_n+kz2*h/2);
    kw3=2.5*(z_n+kz2*h/2);

    kx4=10*(z_n+kz3*h/2)+2.5*(z_n+kz3*h/2)-5/3*(x_n+kx3*h/2)*(y_n+ky3*h/2);
    ky4=10*(z_n+kz3*h/2)-5/3*(x_n+kx3*h/2)*(y_n+ky3*h/2);
    kz4=5/3*(x_n+kx3*h/2)*(y_n+ky3*h/2)-10*(z_n+kz3*h/2)-2.5*(z_n+kz3*h/2);
    kw4=2.5*(z_n+kz3*h/2);

    x(i)=x_n+h/6*(kx1+2*kx2+2*kx3+kx4);
    y(i)=y_n+h/6*(ky1+2*ky2+2*ky3+ky4);
    z(i)=z_n+h/6*(kz1+2*kz2+2*kz3+kz4);
    w(i)=w_n+h/6*(kw1+2*kw2+2*kw3+kw4);
end

figure();
hold on;
plot(t,x,'red');
plot(t,y,'green');
plot(t,z,'blue');
plot(t,w,'black');
legend('E','S','ES','P');
xlabel('t(s)');
ylabel('concentration(µM)');
hold off;
```

And here is the result



The figure of the concentration of four components in 60s

3.

For the reaction

$$E + S \underset{k_2}{\overset{k_1}{\rightleftharpoons}} ES \overset{k_3}{\rightarrow} E + P$$

ES continues to generate and break down, and when the reaction is in a constant state there is

$$k_1[E][S] = k_2[ES] + k_3[ES]$$

If we represent the total amount of enzymes with Et there is

$$k_1([Et] - [ES])[S] = k_2[ES] + k_3[ES]$$

With calculation we can get

$$\frac{[Et][S] - [ES][S]}{[ES]} = \frac{k_2 + k_3}{k_1}$$

If represent $\frac{k_2 + k_3}{k_1}$ with k_m there is

$$[ES] = \frac{[Et][S]}{k_m + [S]}$$

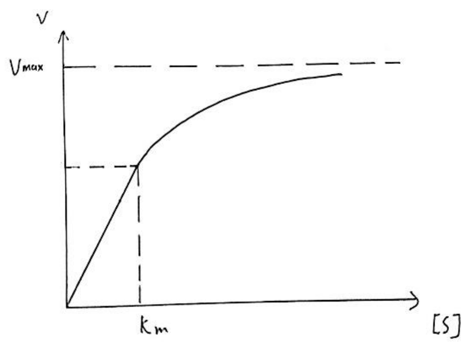
And as the velocity, V, of the enzymatic reaction has been defined as the rate of change of the product P there is

$$v = k_3[ES] = \frac{k_3[Et][S]}{k_m + [S]}$$

When the concentration of S are small, we can ignore the $[S]$ in the denominator and $\frac{k_3[Et]}{k_m}$ is constant, so the velocity V increases approximately linearly. When the concentration of S is large, we can then ignore the k_m in the denominator and there is

$$v_{max} = k_3[ES] = k_3[Et]$$

Here in the picture, we can find the v_{max}



V
Vmax
Km
[S]