Problem definitions and evaluation criteria for the CEC 2006 special session on constrained real-parameter optimization

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Jing Liang
Zhengzhou University

Thomas Philip Runarsson
University of Iceland

Efrén Mezura-Montes
Universidad Veracruzana

Ponnuthurai N. Suganthan
Nanyang Technological University

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J. J. Liang[1], Thomas Philip Runarsson[2], Efrén Mezura-Montes[3]
Maurice Clerc[4], P. N. Suganthan[1], Carlos A. Coello Coello[3], K. Deb[5]

[1]School of EEE, Nanyang Technological University, Singapore, 639798

[2]Science Institute, University of Iceland

[3]Evolutionary Computation Group (EVOCINV),CINVESTAV-IPN,
Computer Science Section,México, D.F., 07300, MEXICO

[4]France Télécom,France

[5]Kanpur Genetic Algorithms Laboratory (KanGAL),
Indian Institute of Technology, Kanpur, PIN 208 016, India

liangjing@pmail.ntu.edu.sg, tpr@hi.is, emezura@computacion.cs.cinvestav.mx,
Maurice.Clerc@WriteMe.com, epnsugan@ntu.edu.sg, ccoello@delta.cs.cinvestav.mx, deb@iitk.ac.in

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Most optimization problems have constraints of different types (e.g., physical, time, geometric, etc.) which modify the shape of the search space. During the last few years, a wide variety of metaheuristics have been designed and applied to solve constrained optimization problems. Evolutionary algorithms and most other metaheuristics, when used for optimization, naturally operate as unconstrained search techniques. Therefore, they require an additional mechanism to incorporate constraints into their fitness function.

Historically, the most common approach to incorporate constraints (both in evolutionary algorithms and in mathematical programming) is the penalty functions, which were originally proposed in the 1940s and later expanded by many researchers. Penalty functions have, in general, several limitations. Particularly, they are not a very good choice when trying to solve problem in which the optimum lies in the boundary between the feasible and the infeasible regions or when the feasible region is disjoint. Additionally, penalty functions require a careful fine-tuning to determine the most appropriate penalty factors to be used with our metaheuristics.

In order to overcome the limitations of penalty functions approach, researchers have proposed a number of diverse approaches to handle constraints such as fitness approximation in constrained optimization, incorporation of knowledge such as cultural approaches in constrained optimization and so on. Additionally, the analysis of the role of the search engine has also become an interesting research topic in the last few years. For example, evolution strategies (ES), evolutionary programming (EP), differential evolution (DE) and particle swarm optimization (PSO) have been found advantageous by some researchers over other metaheuristics such as the binary genetic algorithms (GA).

In this report, 24 benchmark functions are described and guidelines for conducting experiments with performance evaluation criteria are given. The code which could be employed by C/C++/C#, Matlab, Java for them could be found at http://www.ntu.edu.sg/home/EPNSugan/. The mathematical formulas and properties of these functions are described in Section 1. In Section 2, the evaluation criteria are given. And a suggested results format is given in Section 3.

1. Definitions of the Function Suite

In this section, 24 optimization problems with constraints are described. They are all transformed into the following format:

$$Minimize \quad f(\vec{x}), \vec{x} = [x_1, x_2, \ldots, x_n] \tag{1}$$

subject to:

$$\begin{aligned} &\mathrm{g}_i(\vec{x}) \le 0, i = 1, \ldots, q \\ &\mathrm{h}_j(\vec{x}) = 0, j = q+1, \ldots, m \end{aligned} \tag{2}$$

Usually equality constraints are transformed into inequalities of the form

$$|\mathrm{h}_j(\vec{x})| - \epsilon \le 0, for\ j = q+1, \ldots, m \tag{3}$$

A solution $\vec{x}$ is regarded as **feasible** if $\mathrm{g}_i(\vec{x}) \le 0, for\ j = 1, \ldots, q\ and\ |\mathrm{h}_j(\vec{x})| - \epsilon \le 0, for\ j = q+1, \ldots, m$.In this special session ϵ is set to 0.0001.

g01

Minimize [1]:

$$f(\vec{x}) = 5\sum_{i=1}^{4} x_i - 5\sum_{i=1}^{4} x_i^2 - \sum_{i=5}^{13} x_i \tag{4}$$

subject to:

$$\begin{aligned}
g_1(\vec{x}) &= 2x_1 + 2x_2 + x_{10} + x_{11} - 10 \le 0\\
g_2(\vec{x}) &= 2x_1 + 2x_3 + x_{10} + x_{12} - 10 \le 0\\
g_3(\vec{x}) &= 2x_2 + 2x_3 + x_{11} + x_{12} - 10 \le 0\\
g_4(\vec{x}) &= -8x_1 + x_{10} \le 0\\
g_5(\vec{x}) &= -8x_2 + x_{11} \le 0\\
g_6(\vec{x}) &= -8x_3 + x_{12} \le 0\\
g_7(\vec{x}) &= -2x_4 - x_5 + x_{10} \le 0\\
g_8(\vec{x}) &= -2x_6 - x_7 + x_{11} \le 0\\
g_9(\vec{x}) &= -2x_8 - x_9 + x_{12} \le 0
\end{aligned} \tag{5}$$

where the bounds are $0 \le x_i \le 1$ $(i = 1, \ldots, 9)$, $0 \le x_i \le 100$ $(i = 10, 11, 12)$ and $0 \le x_{13} \le 1$. The global minimum is at $\vec{x}^* = (1, 1, 1, 1, 1, 1, 1, 1, 1, 3, 3, 3, 1)$ where six constraints are active (g_1, g_2, g_3, g_7, g_8 and g_9) and $f(\vec{x}^*) = -15$.

g02

Minimize [4]:

$$f(\vec{x}) = -\left| \frac{\sum_{i=1}^{n} \cos^4(x_i) - 2\prod_{i=1}^{n} \cos^2(x_i)}{\sqrt{\sum_{i=1}^{n} i x_i^2}} \right| \tag{6}$$

subject to:

$$\begin{aligned}
g_1(\vec{x}) &= 0.75 - \prod_{i=1}^{n} x_i \le 0\\
g_2(\vec{x}) &= \sum_{i=1}^{n} x_i - 7.5n \le 0
\end{aligned} \tag{7}$$

where $n = 20$ and $0 < x_i \le 10$ $(i = 1, \ldots, n)$. The global minimum $\vec{x}^* = (3.16246061572185,$ $3.12833142812967, 3.09479212988791, 3.06145059523469, 3.02792915885555, 2.99382606701730,$ $2.95866871765285, 2.92184227312450, 0.49482511456933, 0.48835711005490, 0.48231642711865,$ $0.47664475092742, 0.47129550835493, 0.46623099264167, 0.46142004984199, 0.45683664767217,$ $0.45245876903267, 0.44826762241853, 0.44424700958760, 0.44038285956317)$, the best we found is $f(\vec{x}^*) = -0.80361910412559$ (which, to the best of our knowledge, is better than any reported value), constraint g_1 is close to being active.

g03

Minimize [5]:

$$f(\vec{x}) = -(\sqrt{n})^n \prod_{i=1}^{n} x_i \tag{8}$$

subject to:

$$h_1(\vec{x}) = \sum_{i=1}^{n} x_i^2 - 1 = 0 \tag{9}$$

where $n = 10$ and $0 \leq x_i \leq 1$ $(i = 1, \ldots, n)$. The global minimum is at $\vec{x}^* = (0.31624357647283069, 0.316243577414338339, 0.316243578012345927, 0.316243575664017895, 0.316243578205526066, 0.31624357738855069, 0.316243575472949512, 0.316243577164883938, 0.316243578155920302, 0.316243576147374916)$ where $f(\vec{x}^*) = -1.00050010001000$.

g04

Minimize [2]:

$$f(\vec{x}) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141 \tag{10}$$

subject to:

$$\begin{aligned}
g_1(\vec{x}) &= 85.334407 + 0.0056858x_2x_5 + 0.0006262x_1x_4 - 0.0022053x_3x_5 - 92 \leq 0 \\
g_2(\vec{x}) &= -85.334407 - 0.0056858x_2x_5 - 0.0006262x_1x_4 + 0.0022053x_3x_5 \leq 0 \\
g_3(\vec{x}) &= 80.51249 + 0.0071317x_2x_5 + 0.0029955x_1x_2 + 0.0021813x_3^2 - 110 \leq 0 \\
g_4(\vec{x}) &= -80.51249 - 0.0071317x_2x_5 - 0.0029955x_1x_2 - 0.0021813x_3^2 + 90 \leq 0 \\
g_5(\vec{x}) &= 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_3x_4 - 25 \leq 0 \\
g_6(\vec{x}) &= -9.300961 - 0.0047026x_3x_5 - 0.0012547x_1x_3 - 0.0019085x_3x_4 + 20 \leq 0
\end{aligned} \tag{11}$$

where $78 \leq x_1 \leq 102$, $33 \leq x_2 \leq 45$ and $27 \leq x_i \leq 45$ $(i = 3, 4, 5)$. The optimum solution is $\vec{x}^* = 78, 33, 29.9952560256815985, 45, 36.7758129057882073)$ where $f(\vec{x}^*) = -3.0665538671783 32e+004$. Two constraints are active (g_1 and g_6).

g05

Minimize [3]:

$$f(\vec{x}) = 3x_1 + 0.000001x_1^3 + 2x_2 + (0.000002/3)x_2^3 \tag{12}$$

subject to:

$$\begin{aligned}
g_1(\vec{x}) &= -x_4 + x_3 - 0.55 \leq 0 \\
g_2(\vec{x}) &= -x_3 + x_4 - 0.55 \leq 0 \\
h_3(\vec{x}) &= 1000 \sin(-x_3 - 0.25) + 1000 \sin(-x_4 - 0.25) + 894.8 - x_1 = 0 \\
h_4(\vec{x}) &= 1000 \sin(x_3 - 0.25) + 1000 \sin(x_3 - x_4 - 0.25) + 894.8 - x_2 = 0 \\
h_5(\vec{x}) &= 1000 \sin(x_4 - 0.25) + 1000 \sin(x_4 - x_3 - 0.25) + 1294.8 = 0
\end{aligned} \tag{13}$$

where $0 \leq x_1 \leq 1200$, $0 \leq x_2 \leq 1200$, $-0.55 \leq x_3 \leq 0.55$ and $-0.55 \leq x_4 \leq 0.55$. The best known solution [4] $\vec{x}^* = (679.945148297028709, 1026.06697600004691, 0.118876369094410433, -0.39623348521517826)$ where $f(\vec{x}^*) = 5126.4967140071$.

g06

Minimize [1]:

$$f(\vec{x}) = (x_1 - 10)^3 + (x_2 - 20)^3 \tag{14}$$

subject to:

$$\begin{aligned}
g_1(\vec{x}) &= -(x_1 - 5)^2 - (x_2 - 5)^2 + 100 \leq 0 \\
g_2(\vec{x}) &= (x_1 - 6)^2 + (x_2 - 5)^2 - 82.81 \leq 0
\end{aligned} \tag{15}$$

where $13 \leq x_1 \leq 100$ and $0 \leq x_2 \leq 100$. The optimum solution is $\vec{x}^* = (14.09500000000000064, 0.8429607892154795668)$ where $f(\vec{x}^*) = -6961.81387558015$. Both constraints are active.

g07

Minimize [3]:

$$
\begin{aligned}
f(\vec{x}) = {} & x_1^2 + x_2^2 + x_1 x_2 - 14x_1 - 16x_2 + (x_3 - 10)^2 + 4(x_4 - 5)^2 + (x_5 - 3)^2 \\
& +2(x_6 - 1)^2 + 5x_7^2 + 7(x_8 - 11)^2 + 2(x_9 - 10)^2 + (x_{10} - 7)^2 + 45
\end{aligned} \tag{16}
$$

subject to:

$$
\begin{aligned}
\mathrm{g}_1(\vec{x}) &= -105 + 4x_1 + 5x_2 - 3x_7 + 9x_8 \leq 0 \\
\mathrm{g}_2(\vec{x}) &= 10x_1 - 8x_2 - 17x_7 + 2x_8 \leq 0 \\
\mathrm{g}_3(\vec{x}) &= -8x_1 + 2x_2 + 5x_9 - 2x_{10} - 12 \leq 0 \\
\mathrm{g}_4(\vec{x}) &= 3(x_1 - 2)^2 + 4(x_2 - 3)^2 + 2x_3^2 - 7x_4 - 120 \leq 0 \\
\mathrm{g}_5(\vec{x}) &= 5x_1^2 + 8x_2 + (x_3 - 6)^2 - 2x_4 - 40 \leq 0 \\
\mathrm{g}_6(\vec{x}) &= x_1^2 + 2(x_2 - 2)^2 - 2x_1 x_2 + 14x_5 - 6x_6 \leq 0 \\
\mathrm{g}_7(\vec{x}) &= 0.5(x_1 - 8)^2 + 2(x_2 - 4)^2 + 3x_5^2 - x_6 - 30 \leq 0 \\
\mathrm{g}_8(\vec{x}) &= -3x_1 + 6x_2 + 12(x_9 - 8)^2 - 7x_{10} \leq 0
\end{aligned} \tag{17}
$$

where $-10 \leq x_i \leq 10$ $(i = 1, \ldots, 10)$. The optimum solution is $\vec{x}^* = (2.17199634142692, 2.3636830416034,$ $8.77392573913157, 5.09598443745173, 0.990654756560493, 1.43057392853463, 1.32164415364306,$ $9.82872576524495, 8.2800915887356, 8.3759266477347)$ where $\texttt{g07}(\vec{x}^*) = 24.30620906818$ (The recorded results may suffer from rounding errors which may cause slight infeasibility sometimes in the best given solutions). Six constraints are active ($\mathrm{g}_1, \mathrm{g}_2, \mathrm{g}_3, \mathrm{g}_4, \mathrm{g}_5$ and g_6).

g08

Minimize [4]:

$$
f(\vec{x}) = -\frac{\sin^3(2\pi x_1)\sin(2\pi x_2)}{x_1^3(x_1 + x_2)} \tag{18}
$$

subject to:

$$
\begin{aligned}
\mathrm{g}_1(\vec{x}) &= x_1^2 - x_2 + 1 \leq 0 \\
\mathrm{g}_2(\vec{x}) &= 1 - x_1 + (x_2 - 4)^2 \leq 0
\end{aligned} \tag{19}
$$

where $0 \leq x_1 \leq 10$ and $0 \leq x_2 \leq 10$. The optimum is located at $\vec{x}^* = (1.22797135260752599, 4.24537336612274885)$ where $f(\vec{x}^*) = -0.0958250414180359$.

g09

Minimize [3]:

$$
\begin{aligned}
f(\vec{x}) = {} & (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2 \\
& +10x_5^6 + 7x_6^2 + x_7^4 - 4x_6 x_7 - 10x_6 - 8x_7
\end{aligned} \tag{20}
$$

subject to:

$$
\begin{aligned}
\mathrm{g}_1(\vec{x}) &= -127 + 2x_1^2 + 3x_2^4 + x_3 + 4x_4^2 + 5x_5 \leq 0 \\
\mathrm{g}_2(\vec{x}) &= -282 + 7x_1 + 3x_2 + 10x_3^2 + x_4 - x_5 \leq 0 \\
\mathrm{g}_3(\vec{x}) &= -196 + 23x_1 + x_2^2 + 6x_6^2 - 8x_7 \leq 0 \\
\mathrm{g}_4(\vec{x}) &= 4x_1^2 + x_2^2 - 3x_1 x_2 + 2x_3^2 + 5x_6 - 11x_7 \leq 0
\end{aligned} \tag{21}
$$

where $-10 \leq x_i \leq 10$ for $(i = 1, \ldots, 7)$. The optimum solution is $\vec{x}^* = (2.33049935147405174,$ $1.95137236847114592, -0.477541399510615805, 4.36572624923625874, -0.624486959100388983,$

1.03813099410962173, 1.5942266780671519) where $f(\vec{x}^*) = 680.630057374402$. Two constraints are active (g_1 and g_4).

g10

Minimize [3]:

$$f(\vec{x}) = x_1 + x_2 + x_3 \tag{22}$$

subject to:

$$\begin{aligned}
g_1(\vec{x}) &= -1 + 0.0025(x_4 + x_6) \leq 0 \\
g_2(\vec{x}) &= -1 + 0.0025(x_5 + x_7 - x_4) \leq 0 \\
g_3(\vec{x}) &= -1 + 0.01(x_8 - x_5) \leq 0 \\
g_4(\vec{x}) &= -x_1 x_6 + 833.33252 x_4 + 100 x_1 - 83333.333 \leq 0 \\
g_5(\vec{x}) &= -x_2 x_7 + 1250 x_5 + x_2 x_4 - 1250 x_4 \leq 0 \\
g_6(\vec{x}) &= -x_3 x_8 + 1250000 + x_3 x_5 - 2500 x_5 \leq 0
\end{aligned}$$

where $100 \leq x_1 \leq 10000$, $1000 \leq x_i \leq 10000$ $(i = 2, 3)$ and $10 \leq x_i \leq 1000$ $(i = 4, \ldots, 8)$. The optimum solution is $\vec{x}^* = (579.306685017979589, 1359.97067807935605, 5109.97065743133317, 182.01769963061534, 295.601173702746792, 217.982300369384632, 286.41652592786852, 395.601173702746735)$, where $f(\vec{x}^*) = 7049.24802052867$. All constraints are active (g_1, g_2 and g_3).

g11

Minimize [4]:

$$f(\vec{x}) = x_1^2 + (x_2 - 1)^2 \tag{23}$$

subject to:

$$h(\vec{x}) = x_2 - x_1^2 = 0 \tag{24}$$

where $-1 \leq x_1 \leq 1$ and $-1 \leq x_2 \leq 1$. The optimum solution is $\vec{x}^* = (-0.707036070037170616, 0.500000004333606807)$ where $f(\vec{x}^*) = 0.7499$.

g12

Minimize [4]:

$$f(\vec{x}) = -(100 - (x_1 - 5)^2 - (x_2 - 5)^2 - (x_3 - 5)^2)/100 \tag{25}$$

subject to:

$$g(\vec{x}) = (x_1 - p)^2 + (x_2 - q)^2 + (x_3 - r)^2 - 0.0625 \leq 0$$

where $0 \leq x_i \leq 10$ $(i = 1, 2, 3)$ and $p, q, r = 1, 2, \ldots, 9$. The feasible region of the search space consists of 9^3 disjointed spheres. A point (x_1, x_2, x_3) is feasible if and only if there exist p, q, r such that the above inequality holds. The optimum is located at $\vec{x}^* = (5, 5, 5)$ where $f(\vec{x}^*) = -1$. The solution lies within the feasible region.

g13

Minimize [3]:

$$f(\vec{x}) = e^{x_1 x_2 x_3 x_4 x_5} \tag{26}$$

subject to:

$$\begin{aligned} \mathrm{h}_1(\vec{x}) &= x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0 \\ \mathrm{h}_2(\vec{x}) &= x_2 x_3 - 5 x_4 x_5 = 0 \\ \mathrm{h}_3(\vec{x}) &= x_1^3 + x_2^3 + 1 = 0 \end{aligned} \tag{27}$$

where $-2.3 \leq x_i \leq 2.3$ $(i = 1, 2)$ and $-3.2 \leq x_i \leq 3.2$ $(i = 3, 4, 5)$. The optimum solution is $\vec{x}^* = (-1.71714224003, 1.59572124049468, 1.8272502406271, -0.763659881912867, -0.76365986736498)$ where $f(\vec{x}^*) = 0.053941514041898$.

g14

Minimize [8]:

$$f(\vec{x}) = \sum_{i=1}^{10} x_i \left(c_i + \ln \frac{x_i}{\sum_{j=1}^{10} x_j} \right) \tag{28}$$

subject to:

$$\begin{aligned} \mathrm{h}_1(\vec{x}) &= x_1 + 2x_2 + +2x_3 + x_6 + x_{10} - 2 = 0 \\ \mathrm{h}_2(\vec{x}) &= x_4 + 2x_5 + x_6 + x_7 - 1 = 0 \\ \mathrm{h}_3(\vec{x}) &= x_3 + x_7 + x_8 + 2x_9 + x_{10} - 1 = 0 \end{aligned} \tag{29}$$

where the bounds are $0 < x_i \leq 10$ $(i = 1, \ldots, 10)$, and $c_1 = -6.089$, $c_2 = -17.164$, $c_3 = -34.054$, $c_4 = -5.914$, $c_5 = -24.721$, $c_6 = -14.986$, $c_7 = -24.1$, $c_8 = -10.708$, $c_9 = -26.662$, $c_{10} = -22.179$,. The best known solution is at $x^* = (0.0406684113216282, 0.147721240492452, 0.783205732104114, 0.00141433931889084, 0.485293636780388, 0.000693183051556082, 0.0274052040687766, 0.0179509660214818, 0.0373268186859717, 0.0968844604336845)$ where $f(x^*) = -47.7648884594915$.

g15

Minimize [8]:

$$f(\vec{x}) = 1000 - x_1^2 - 2x_2^2 - x_3^2 - x_1 x_2 - x_1 x_3 \tag{30}$$

subject to:

$$\begin{aligned} \mathrm{h}_1(\vec{x}) &= x_1^2 + x_2^2 + +x_3^2 - 25 = 0 \\ \mathrm{h}_2(\vec{x}) &= 8x_1 + 14x_2 + 7x_3 - 56 = 0 \end{aligned} \tag{31}$$

where the bounds are $0 \leq x_i \leq 10$ $(i = 1, 2, 3)$. The best known solution is at $x^* = (3.51212812611795133, 0.216987510429556135, 3.55217854929179921)$ where $f(x^*) = 961.715022289961$.

g16

Minimize [8]:

$$\begin{aligned} f(\vec{x}) =& 0.000117 y_{14} + 0.1365 + 0.00002358 y_{13} + 0.000001502 y_{16} + 0.0321 y_{12} \\ &+ 0.004324 y_5 + 0.0001 \frac{c_{15}}{c_{16}} + 37.48 \frac{y_2}{c_{12}} - 0.0000005843 y_{17} \end{aligned} \tag{32}$$

subject to:

$$
\begin{aligned}
g_1(\vec{x}) &= \frac{0.28}{0.72} y_5 - y_4 \leq 0 \\
g_2(\vec{x}) &= x_3 - 1.5x_2 \leq 0 \\
g_3(\vec{x}) &= 3496 \frac{y_2}{c_{12}} - 21 \leq 0 \\
g_4(\vec{x}) &= 110.6 + y_1 - \frac{62212}{c_{17}} \leq 0 \\
g_5(\vec{x}) &= 213.1 - y_1 \leq 0 \\
g_6(\vec{x}) &= y_1 - 405.23 \leq 0 \\
g_7(\vec{x}) &= 17.505 - y_2 \leq 0 \\
g_8(\vec{x}) &= y_2 - 1053.6667 \leq 0 \\
g_9(\vec{x}) &= 11.275 - y_3 \leq 0 \\
g_{10}(\vec{x}) &= y_3 - 35.03 \leq 0 \\
g_{11}(\vec{x}) &= 214.228 - y_4 \leq 0 \\
g_{12}(\vec{x}) &= y_4 - 665.585 \leq 0 \\
g_{13}(\vec{x}) &= 7.458 - y_5 \leq 0 \\
g_{14}(\vec{x}) &= y_5 - 584.463 \leq 0 \\
g_{15}(\vec{x}) &= 0.961 - y_6 \leq 0 \\
g_{16}(\vec{x}) &= y_6 - 265.916 \leq 0 \\
g_{17}(\vec{x}) &= 1.612 - y_7 \leq 0 \\
g_{18}(\vec{x}) &= y_7 - 7.046 \leq 0 \\
g_{19}(\vec{x}) &= 0.146 - y_8 \leq 0 \\
g_{20}(\vec{x}) &= y_8 - 0.222 \leq 0 \\
g_{21}(\vec{x}) &= 107.99 - y_9 \leq 0 \\
g_{22}(\vec{x}) &= y_9 - 273.366 \leq 0 \\
g_{23}(\vec{x}) &= 922.693 - y_{10} \leq 0 \\
g_{24}(\vec{x}) &= y_{10} - 1286.105 \leq 0 \\
g_{25}(\vec{x}) &= 926.832 - y_{11} \leq 0 \\
g_{26}(\vec{x}) &= y_{11} - 1444.046 \leq 0 \\
g_{27}(\vec{x}) &= 18.766 - y_{12} \leq 0 \\
g_{28}(\vec{x}) &= y_{12} - 537.141 \leq 0 \\
g_{29}(\vec{x}) &= 1072.163 - y_{13} \leq 0 \\
g_{30}(\vec{x}) &= y_{13} - 3247.039 \leq 0 \\
g_{31}(\vec{x}) &= 8961.448 - y_{14} \leq 0 \\
g_{32}(\vec{x}) &= y_{14} - 26844.086 \leq 0 \\
g_{33}(\vec{x}) &= 0.063 - y_{15} \leq 0 \\
g_{34}(\vec{x}) &= y_{15} - 0.386 \leq 0 \\
g_{35}(\vec{x}) &= 71084.33 - y_{16} \leq 0 \\
g_{36}(\vec{x}) &= -140000 + y_{16} \leq 0 \\
g_{37}(\vec{x}) &= 2802713 - y_{17} \leq 0 \\
g_{38}(\vec{x}) &= y_{17} - 12146108 \leq 0
\end{aligned}
\tag{33}
$$

where:

$$
\begin{aligned}
y_1 &= x_2 + x_3 + 41.6 \\
c_1 &= 0.024x_4 - 4.62 \\
y_2 &= \frac{12.5}{c_1} + 12 \\
c_2 &= 0.0003535x_1^2 + 0.5311x_1 + 0.08705y_2x_1 \\
c_3 &= 0.052x_1 + 78 + 0.002377y_2x_1 \\
y_3 &= \frac{c_2}{c_3} \\
y_4 &= 19y_3 \\
c_4 &= 0.04782(x_1 - y_3) + \frac{0.1956(x_1 - y_3)^2}{x_2} + 0.6376y_4 + 1.594y_3 \\
c_5 &= 100x_2 \\
c_6 &= x_1 - y_3 - y_4 \\
c_7 &= 0.950 - \frac{c_4}{c_5} \\
y_5 &= c_6c_7 \\
y_6 &= x_1 - y_5 - y_4 - y_3 \\
c_8 &= (y_5 + y_4)0.995 \\
y_7 &= \frac{c_8}{y_1} \\
y_8 &= \frac{c_8}{3798} \\
c_9 &= y_7 - \frac{0.0663y_7}{y_8} - 0.3153 \\
y_9 &= \frac{96.82}{c_9} + 0.321y_1 \\
y_{10} &= 1.29y_5 + 1.258y_4 + 2.29y_3 + 1.71y_6 \\
y_{11} &= 1.71x_1 - 0.452y_4 + 0.580y_3 \\
c_{10} &= \frac{12.3}{752.3} \\
c_{11} &= (1.75y_2)(0.995x_1) \\
c_{12} &= 0.995y_{10} + 1998 \\
y_{12} &= c_{10}x_1 + \frac{c_{11}}{c_{12}} \\
y_{13} &= c_{12} - 1.75y_2 \\
y_{14} &= 3623 + 64.4x_2 + 58.4x_3 + \frac{146312}{y_9 + x_5} \\
c_{13} &= 0.995y_{10} + 60.8x_2 + 48x_4 - 0.1121y_{14} - 5095 \\
y_{15} &= \frac{y_{13}}{c_{13}} \\
y_{16} &= 148000 - 331000y_{15} + 40y_{13} - 61y_{15}y_{13} \\
c_{14} &= 2324y_{10} - 28740000y_2
\end{aligned}
\tag{34}
$$

$$y_{17} = 14130000 - 1328y_{10} - 531y_{11} + \frac{c_{14}}{c_{12}}$$
$$c_{15} = \frac{y_{13}}{y_{15}} - \frac{y_{13}}{0.52}$$
$$c_{16} = 1.104 - 0.72y_{15}$$
$$c_{17} = y_9 + x_5$$

and where the bounds are $704.4148 \leq x_1 \leq 906.3855$, $68.6 \leq x_2 \leq 288.88$, $0 \leq x_3 \leq 134.75$, $193 \leq x_4 \leq 287.0966$ and $25 \leq x_5 \leq 84.1988$. The best known solution is at $x^* = (705.174537070090537, 68.5999999999999943, 102.899999999999991, 282.324931593660324, 37.5841164258054832)$ where $f(x^*) = -1.90515525853479$.

g17

Minimize [8]:

$$f(\vec{x}) = f(x_1) + f(x_2) \tag{35}$$

where

$$f_1(x_1) = \begin{cases} 30x_1 & 0 \leq x_1 < 300 \\ 31x_1 & 300 \leq x_1 < 400 \end{cases}$$

$$f_2(x_2) = \begin{cases} 28x_2 & 0 \leq x_2 < 100 \\ 29x_2 & 100 \leq x_2 < 200 \\ 30x_2 & 200 \leq x_2 < 1000 \end{cases}$$

subject to:

$$\begin{aligned}
\mathrm{h}_1(\vec{x}) &= -x_1 + 300 - \frac{x_3x_4}{131.078}\cos(1.48477 - x_6) + \frac{0.90798x_3^2}{131.078}\cos(1.47588) \\
\mathrm{h}_2(\vec{x}) &= -x_2 - \frac{x_3x_4}{131.078}\cos((1.48477 + x_6) + \frac{0.90798x_4^2}{131.078}\cos(1.47588) \\
\mathrm{h}_3(\vec{x}) &= -x_5 - \frac{x_3x_4}{131.078}\sin((1.48477 + x_6) + \frac{0.90798x_4^2}{131.078}\sin(1.47588) \\
\mathrm{h}_4(\vec{x}) &= 200 - \frac{x_3x_4}{131.078}\sin((1.48477 - x_6) + \frac{0.90798x_3^2}{131.078}\sin(1.47588)
\end{aligned} \tag{36}$$

where the bounds are $0 \leq x_1 \leq 400$, $0 \leq x_2 \leq 1000$, $340 \leq x_3 \leq 420$, $340 \leq x_4 \leq 420$, $-1000 \leq x_5 \leq 1000$ and $0 \leq x_6 \leq 0.5236$. The best known solution is at $x^* = (201.784467214523659, 99.9999999999999005, 383.071034852773266, 420, -10.9076584514292652, 0.0731482312084287128)$ where $f(x^*) = 8853.53967480648$.

g18

Minimize [8]:

$$f(\vec{x}) = -0.5(x_1x_4 - x_2x_3 + x_3x_9 - x_5x_9 + x_5x_8 - x_6x_7) \tag{37}$$

subject to:

$$\begin{aligned}
\mathrm{g}g_1(\vec{x}) &= x_3^2 + x_4^2 - 1 \le 0\\
\mathrm{g}g_2(\vec{x}) &= x_9^2 - 1 \le 0\\
\mathrm{g}g_3(\vec{x}) &= x_5^2 + x_6^2 - 1 \le 0\\
\mathrm{g}g_4(\vec{x}) &= x_1^2 + (x_2 - x_9)^2 - 1 \le 0\\
\mathrm{g}g_5(\vec{x}) &= (x_1 - x_5)^2 + (x_2 - x_6)^2 - 1 \le 0\\
\mathrm{g}g_6(\vec{x}) &= (x_1 - x_7)^2 + (x_2 - x_8)^2 - 1 \le 0\\
\mathrm{g}g_7(\vec{x}) &= (x_3 - x_5)^2 + (x_4 - x_6)^2 - 1 \le 0\\
\mathrm{g}g_8(\vec{x}) &= (x_3 - x_7)^2 + (x_4 - x_8)^2 - 1 \le 0\\
\mathrm{g}g_9(\vec{x}) &= x_7^2 + (x_8 - x_9)^2 - 1 \le 0\\
\mathrm{g}g_{10}(\vec{x}) &= x_2x_3 - x_1x_4 \le 0\\
\mathrm{g}g_{11}(\vec{x}) &= -x_3x_9 \le 0\\
\mathrm{g}g_{12}(\vec{x}) &= x_5x_9 \le 0\\
\mathrm{g}g_{13}(\vec{x}) &= x_6x_7 - x_5x_8 \le 0
\end{aligned} \tag{38}$$

where the bounds are $-10 \le x_i \le 10$ ($i = 1, \ldots, 8$) and $0 \le x_9 \le 20$. The best known solution is at $x^* = (-0.657776192427943163, -0.153418773482438542, 0.323413871675240938, -0.946257611651304398, -0.657776194376798906, -0.753213434632691414, 0.323413874123576972, -0.346462947962331735, 0.59979466285217542)$ where $f(x^*) = -0.866025403784439$.

g19

Minimize [8]:

$$f(\vec{x}) = \sum_{j=1}^{5}\sum_{i=1}^{5} c_{ij}x_{(10+i)}x_{(10+j)} + 2\sum_{j=1}^{5} d_jx_{(10+j)}^3 - \sum_{i=1}^{10} b_ix_i \tag{39}$$

subject to:

$$\mathrm{g}_j(\vec{x}) = -2\sum_{i=1}^{5} c_{ij}x_{(10+i)} - 3d_jx_{(10+j)}^2 - e_j + \sum_{i=1}^{10} a_{ij}x_i \le 0 \qquad j = 1, \ldots, 5 \tag{40}$$

where $\vec{b} = [-40, -2, -0.25, -4, -4, -1, -40, -60, 5, 1]$ and the remaining data is on Table 1. The bounds are $0 \le x_i \le 10$ ($i = 1, \ldots, 15$). The best known solution is at $x^* = (1.66991341326291344e-17, 3.95378229282456509e-16, 3.94599045143233784, 1.06036597479721211e-16, 3.2831773458454161, 9.99999999999999822, 1.12829414671605333e-17, 1.2026194599794709e-17, 2.50706276000769697e-15, 2.24624122987970677e-15, 0.370764847417013987, 0.278456024942955571, 0.523838487672241171, 0.388620152510322781, 0.298156764974678579)$ where $f(x^*) = 32.6555929502463$.

j	1	2	3	4	5
e_j	-15	-27	-36	-18	-12
c_{1j}	30	-20	-10	32	-10
c_{2j}	-20	39	-6	-31	32
c_{3j}	-10	-6	10	-6	-10
c_{4j}	32	-31	-6	39	-20
c_{5j}	-10	32	-10	-20	30
d_j	4	8	10	6	2
a_{1j}	-16	2	0	1	0
a_{2j}	0	-2	0	0.4	2
a_{3j}	-3.5	0	2	0	0
a_{4j}	0	-2	0	-4	-1
a_{5j}	0	-9	-2	1	-2.8
a_{6j}	2	0	-4	0	0
a_{7j}	-1	-1	-1	-1	-1
a_{8j}	-1	-2	-3	-2	-1
a_{9j}	1	2	3	4	5
a_{10j}	1	1	1	1	1

Table 1: Data set for test problem g19

g20

Minimize [8]:

$$f(\vec{x}) = \sum_{i=1}^{24} a_i x_i \tag{41}$$

subject to:

$$\begin{aligned}
\mathrm{g}_i(\vec{x}) &= \frac{(x_i + x_{(i+12)})}{\sum_{j=1}^{24} x_j + e_i} \leq 0 \qquad i = 1,2,3 \\
\mathrm{g}_i(\vec{x}) &= \frac{(x_{(i+3)} + x_{(i+15)})}{\sum_{j=1}^{24} x_j + e_i} \leq 0 \qquad i = 4,5,6 \\
\mathrm{h}_i(\vec{x}) &= \frac{x_{(i+12)}}{b_{(i+12)} \sum_{j=13}^{24} \frac{x_j}{b_j}} - \frac{c_i x_i}{40 b_i \sum_{j=1}^{12} \frac{x_j}{b_j}} = 0 \qquad i = 1,\ldots,12 \\
\mathrm{h}_{13}(\vec{x}) &= \sum_{i=1}^{24} x_i - 1 = 0 \\
\mathrm{h}_{14}(\vec{x}) &= \sum_{i=1}^{12} \frac{x_i}{d_i} + k \sum_{i=13}^{24} \frac{x_i}{b_i} - 1.671 = 0
\end{aligned} \tag{42}$$

where $k = (0.7302)(530)(\frac{14.7}{40})$ and the data set is detailed on Table 2. The bounds are $0 \leq x_i \leq 10$ ($i = 1,\ldots,24$). The best known solution is at $x^* = (1.28582343498528086e-18, 4.83460302526130664e-34, 0, 0, 6.30459929660781851e-18, 7.57192526201145068e-34, 5.03350698372840437e-34, 9.28268079616618064e-34, 0, 1.76723384525547359e-17, 3.55686101822965701e-34, 2.99413850083471346e-34, 0.158143376337580827, 2.29601774161699833e-19, 1.06106938611042947e-18, 1.31968344319506391e-18, 0.530902525044209539, 0, 2.89148310257773535e-18, 3.34892126180666159e-18, 0, 0.310999974151577319, 5.41244666317833561e-05, 4.84993165246959553e-16)$ .This solution is a little infeasible and no feasible solution is found so far.

i	a_i	b_i	c_i	d_i	e_i
1	0.0693	44.094	123.7	31.244	0.1
2	0.0577	58.12	31.7	36.12	0.3
3	0.05	58.12	45.7	34.784	0.4
4	0.2	137.4	14.7	92.7	0.3
5	0.26	120.9	84.7	82.7	0.6
6	0.55	170.9	27.7	91.6	0.3
7	0.06	62.501	49.7	56.708	
8	0.1	84.94	7.1	82.7	
9	0.12	133.425	2.1	80.8	
10	0.18	82.507	17.7	64.517	
11	0.1	46.07	0.85	49.4	
12	0.09	60.097	0.64	49.1	
13	0.0693	44.094			
14	0.0577	58.12			
15	0.05	58.12			
16	0.2	137.4			
17	0.26	120.9			
18	0.55	170.9			
19	0.06	62.501			
20	0.1	84.94			
21	0.12	133.425			
22	0.18	82.507			
23	0.1	46.07			
24	0.09	60.097			

Table 2: Data set for test problem g20

g21

Minimize [6]:

$$f(\vec{x}) = x_1 \tag{43}$$

subject to:

$$\begin{aligned}
\mathrm{g}_1(\vec{x}) &= -x_1 + 35x_2^{0.6} + 35x_3^{0.6} \leq 0 \\
\mathrm{h}_1(\vec{x}) &= -300x_3 + 7500x_5 - 7500x_6 - 25x_4x_5 + 25x_4x_6 + x_3x_4 = 0 \\
\mathrm{h}_2(\vec{x}) &= 100x_2 + 155.365x_4 + 2500x_7 - x_2x_4 - 25x_4x_7 - 15536.5 = 0 \\
\mathrm{h}_3(\vec{x}) &= -x_5 + \ln(-x_4 + 900) = 0 \\
\mathrm{h}_4(\vec{x}) &= -x_6 + \ln(x_4 + 300) = 0 \\
\mathrm{h}_5(\vec{x}) &= -x_7 + \ln(-2x_4 + 700) = 0
\end{aligned} \tag{44}$$

where the bounds are $0 \leq x_1 \leq 1000$, $0 \leq x_2, x_3 \leq 40$, $100 \leq x_4 \leq 300$, $6.3 \leq x_5 \leq 6.7$, $5.9 \leq x_6 \leq 6.4$ and $4.5 \leq x_7 \leq 6.25$. The best known solution is at $x^* = (193.724510070034967, 5.56944131553368433e-27, 17.3191887294084914, 100.047897801386839, 6.68445185362377892, 5.99168428444264833, 6.21451648886070451)$ where $f(x^*) = 193.724510070035$.

g22

Minimize [6]:

$$f(\vec{x}) = x_1 \tag{45}$$

subject to:

$$
\begin{aligned}
&\mathrm{g}_1(\vec{x}) = -x_1 + x_2^{0.6} + x_3^{0.6} + x_4^{0.6} \le 0\\
&\mathrm{h}_1(\vec{x}) = x_5 - 100000x_8 + 1\times 10^7 = 0\\
&\mathrm{h}_2(\vec{x}) = x_6 + 100000x_8 - 100000x_9 = 0\\
&\mathrm{h}_3(\vec{x}) = x_7 + 100000x_9 - 5\times 10^7 = 0\\
&\mathrm{h}_4(\vec{x}) = x_5 + 100000x_{10} - 3.3\times 10^7 = 0\\
&\mathrm{h}_5(\vec{x}) = x_6 + 100000x_{11} - 4.4\times 10^7 = 0\\
&\mathrm{h}_6(\vec{x}) = x_7 + 100000x_{12} - 6.6\times 10^7 = 0\\
&\mathrm{h}_7(\vec{x}) = x_5 - 120x_2x_{13} = 0\\
&\mathrm{h}_8(\vec{x}) = x_6 - 80x_3x_{14} = 0\\
&\mathrm{h}_9(\vec{x}) = x_7 - 40x_4x_{15} = 0\\
&\mathrm{h}_{10}(\vec{x}) = x_8 - x_{11} + x_{16} = 0\\
&\mathrm{h}_{11}(\vec{x}) = x_9 - x_{12} + x_{17} = 0\\
&\mathrm{h}_{12}(\vec{x}) = -x_{18} + \ln(x_{10} - 100) = 0\\
&\mathrm{h}_{13}(\vec{x}) = -x_{19} + \ln(-x_8 + 300) = 0\\
&\mathrm{h}_{14}(\vec{x}) = -x_{20} + \ln(x_{16}) = 0\\
&\mathrm{h}_{15}(\vec{x}) = -x_{21} + \ln(-x_9 + 400) = 0\\
&\mathrm{h}_{16}(\vec{x}) = -x_{22} + \ln(x_{17}) = 0\\
&\mathrm{h}_{17}(\vec{x}) = -x_8 - x_{10} + x_{13}x_{18} - x_{13}x_{19} + 400 = 0\\
&\mathrm{h}_{18}(\vec{x}) = x_8 - x_9 - x_{11} + x_{14}x_{20} - x_{14}x_{21} + 400 = 0\\
&\mathrm{h}_{19}(\vec{x}) = x_9 - x_{12} - 4.60517x_{15} + x_{15}x_{22} + 100 = 0
\end{aligned}
\tag{46}
$$

where the bounds are $0 \le x_1 \le 20000$, $0 \le x_2, x_3, x_4 \le 1\times 10^6$, $0 \le x_5, x_6, x_7 \le 4\times 10^7$, $100 \le x_8 \le 299.99$, $100 \le x_9 \le 399.99$, $100.01 \le x_{10} \le 300$, $100 \le x_{11} \le 400$, $100 \le x_{12} \le 600$, $0 \le x_{13}, x_{14}, x_{15} \le 500$, $0.01 \le x_{16} \le 300$, $0.01 \le x_{17} \le 400$, $-4.7 \le x_{18}, x_{19}, x_{20}, x_{21}, x_{22} \le 6.25$. The best known solution is at $x^* = (236.430975504001054, 135.82847151732463, 204.818152544824585, 6446.54654059436416,$ $3007540.83940215595, 4074188.65771341929, 32918270.5028952882, 130.075408394314167,$ $170.817294970528621, 299.924591605478554, 399.258113423595205, 330.817294971142758,$ $184.51831230897065, 248.64670239647424, 127.658546694545862, 269.182627528746707,$ $160.000016724090955, 5.29788288102680571, 5.13529735903945728, 5.59531526444068827,$ $5.434444793144534 99, 5.07517453535834395)$ where $f(x^*) = 236.430975504001$.

g23

Minimize [10]:

$$f(\vec{x}) = -9x_5 - 15x_8 + 6x_1 + 16x_2 + 10(x_6 + x_7) \tag{47}$$

subject to:

$$
\begin{aligned}
&\mathrm{g}_1(\vec{x}) = x_9x_3 + 0.02x_6 - 0.025x_5 \le 0\\
&\mathrm{g}_2(\vec{x}) = x_9x_4 + 0.02x_7 - 0.015x_8 \le 0\\
&\mathrm{h}_1(\vec{x}) = x_1 + x_2 - x_3 - x_4 = 0\\
&\mathrm{h}_2(\vec{x}) = 0.03x_1 + 0.01x_2 - x_9(x_3 + x_4) = 0\\
&\mathrm{h}_3(\vec{x}) = x_3 + x_6 - x_5 = 0\\
&\mathrm{h}_4(\vec{x}) = x_4 + x_7 - x_8 = 0
\end{aligned}
\tag{48}
$$

where the bounds are $0 \leq x_1, x_2, x_6 \leq 300$, $0 \leq x_3, x_5, x_7 \leq 100$, $0 \leq x_4, x_8 \leq 200$ and $0.01 \leq x_9 \leq 0.03$. The best known solution is at $x^* = (0.00510000000000259465, 99.9947000000000514, 9.01920162996045897e-18, 99.9999000000000535, 0.000100000000027086086, 2.75700683389584542e-14, 99.9999999999999574, \cancel{2000.0100000100000100008})$ where $f(x^*) = -400.055099999999584$.

g24

Minimize [7]

$$f(\vec{x}) = -x_1 - x_2 \tag{49}$$

subject to:

$$\begin{aligned} g_1(\vec{x}) &= -2x_1^4 + 8x_1^3 - 8x_1^2 + x_2 - 2 \leq 0 \\ g_2(\vec{x}) &= -4x_1^4 + 32x_1^3 - 88x_1^2 + 96x_1 + x_2 - 36 \leq 0 \end{aligned} \tag{50}$$

where the bounds are $0 \leq x_1 \leq 3$ and $0 \leq x_2 \leq 4$. The feasible global minimum is at $x^* = (2.329520197477623.17849307411774)$ where $f(x^*) = -5.50801327159536$. This problem has a feasible region consisting on two disconnected sub-regions.

Prob.	n	Type of function	ρ	LI	NI	LE	NE	a
g01	13	quadratic	0.0111%	9	0	0	0	6
g02	20	nonlinear	99.9971%	0	2	0	0	1
g03	10	polynomial	0.0000%	0	0	0	1	1
g04	5	quadratic	52.1230%	0	6	0	0	2
g05	4	cubic	0.0000%	2	0	0	3	3
g06	2	cubic	0.0066%	0	2	0	0	2
g07	10	quadratic	0.0003%	3	5	0	0	6
g08	2	nonlinear	0.8560%	0	2	0	0	0
g09	7	polynomial	0.5121%	0	4	0	0	2
g10	8	linear	0.0010%	3	3	0	0	6
g11	2	quadratic	0.0000%	0	0	0	1	1
g12	3	quadratic	4.7713%	0	1	0	0	0
g13	5	nonlinear	0.0000%	0	0	0	3	3
g14	10	nonlinear	0.0000%	0	0	3	0	3
g15	3	quadratic	0.0000%	0	0	1	1	2
g16	5	nonlinear	0.0204%	4	34	0	0	4
g17	6	nonlinear	0.0000%	0	0	0	4	4
g18	9	quadratic	0.0000%	0	13	0	0	6
g19	15	nonlinear	33.4761%	0	5	0	0	0
g20	24	linear	0.0000%	0	6	2	12	16
g21	7	linear	0.0000%	0	1	0	5	6
g22	22	linear	0.0000%	0	1	8	11	19
g23	9	linear	0.0000%	0	2	3	1	6
g24	2	linear	79.6556%	0	2	0	0	2

Table 3: Details of the 24 test problems. n is the number of decision variables, $\rho = |F|/|S|$ is the estimated ratio between the feasible region and the search space, LI is the number of linear inequality constraints, NI the number of nonlinear inequality constraints, LE is the number of linear equality constraints and NE is the number of nonlinear equality constraints. a is the number of active constraints at $\vec{x}$.

2. Performance Evaluation Criteria

Global optima: The fitness value of the best known solutions are listed in Table 4.
Runs/ problem: 25
Max_FES: $500,000$
Population Size: You are free to have an appropriate population size to suit your algorithm while not exceeding the Max_FES.

Prob.	n	$\mathbf{f}(\vec{x}^*)$	Bounds
g01	13	-15.0000000000	$0 \le x_i \le 1$ $(i = 1, \ldots, 9)$,$0 \le x_i \le 100$ $(i = 10, 11, 12)$ and $0 \le x_{13} \le 1$
g02	20	-0.8036191042	$0 < x_i \le 10$ $(i = 1, \ldots, n)$
g03	10	-1.0005001000	$0 \le x_i \le 1$ $(i = 1, \ldots, n)$
g04	5	-30665.5386717834	$78 \le x_1 \le 102$, $33 \le x_2 \le 45$ and $27 \le x_i \le 45$ $(i = 3, 4, 5)$
g05	4	5126.4967140071	$0 \le x_1 \le 1200$, $0 \le x_2 \le 1200$, $-0.55 \le x_3 \le 0.55$ and $-0.55 \le x_4 \le 0.55$
g06	2	-6961.8138755802	$13 \le x_1 \le 100$ and $0 \le x_2 \le 100$
g07	10	24.3062090681	$-10 \le x_i \le 10$ $(i = 1, \ldots, 10)$
g08	2	-0.0958250415	$0 \le x_1 \le 10$ and $0 \le x_2 \le 10$
g09	7	680.6300573745	$-10 \le x_i \le 10$ for $(i = 1, \ldots, 7)$
g10	8	7049.2480205286	$100 \le x_1 \le 10000$, $1000 \le x_i \le 10000(i = 2, 3)$ and $10 \le x_i \le 1000$ $(i = 4, \ldots, 8)$
g11	2	0.7499000000	$-1 \le x_1 \le 1$ and $-1 \le x_2 \le 1$
g12	3	-1.0000000000	$0 \le x_i \le 10$ $(i = 1, 2, 3)$and $p, q, r = 1, 2, \ldots, 9$
g13	5	0.0539415140	$-2.3 \le x_i \le 2.3$ $(i = 1, 2)$ and $-3.2 \le x_i \le 3.2$ $(i = 3, 4, 5)$
g14	10	-47.7648884595	$0 < x_i \le 10$ $(i = 1, \ldots, 10)$
g15	3	961.7150222899	$0 \le x_i \le 10$ $(i = 1, 2, 3)$
g16	5	-1.9051552586	$704.4148 \le x_1 \le 906.3855$,$68.6 \le x_2 \le 288.88$, $0 \le x_3 \le 134.75$, $193 \le x_4 \le 287.0966$, $25 \le x_5 \le 84.1988$
g17	6	8853.5396748064	$0 \le x_1 \le 400$, $0 \le x_2 \le 1000$, $340 \le x_3 \le 420$, $340 \le x_4 \le 420$, $-1000 \le x_5 \le 1000$ and $0 \le x_6 \le 0.5236$
g18	9	-0.8660254038	$-10 \le x_i \le 10$ $(i = 1, \ldots, 8)$ and $0 \le x_9 \le 20$
g19	15	32.6555929502	$0 \le x_i \le 10$ $(i = 1, \ldots, 15)$
g20	24	0.2049794002	$0 \le x_i \le 10$ $(i = 1, \ldots, 24)$
g21	7	193.7245100700	$0 \le x_1 \le 1000$,$0 \le x_2, x_3 \le 40$, $100 \le x_4 \le 300$, $6.3 \le x_5 \le 6.7$, $5.9 \le x_6 \le 6.4$ and $4.5 \le x_7 \le 6.25$
g22	22	236.4309755040	$0 \le x_1 \le 20000$, $0 \le x_2, x_3, x_4 \le 1 \times 10^6$, $0 \le x_5, x_6, x_7 \le 4 \times 10^7$, $100 \le x_8 \le 299.99$,$100 \le x_9 \le 399.99$, $100.01 \le x_{10} \le 300$, $100 \le x_{11} \le 400$, $100 \le x_{12} \le 600$, $0 \le x_{13}, x_{14}, x_{15} \le 500$,$0.01 \le x_{16} \le 300$, $0.01 \le x_{17} \le 400$, $-4.7 \le x_{18}, x_{19}, x_{20}, x_{21}, x_{22} \le 6.25$
g23	9	-400.0551000000	$0 \le x_1, x_2, x_6 \le 300$, $0 \le x_3, x_5, x_7 \le 100$, $0 \le x_4, x_8 \le 200$ and $0.01 \le x_9 \le 0.03$
g24	2	-5.5080132716	$0 \le x_1 \le 3$ and $0 \le x_2 \le 4$

Table 4: $f(\vec{x})$ and the bounds for the 24 problems.

*The best known solutions for g20 is slightly infeasible.

1) Record the function error value ($f(\vec{x}) - f(\vec{x}^*)$) for the achieved best solution $\vec{x}$ after $\mathbf{5 \times 10^3, 5 \times 10^4, 5 \times 10^5}$ FES for each run.
Equality constraints are transformed into inequalities of the form

$$|\mathrm{h}_j(\vec{x})| - \epsilon \le 0, for\ j = q + 1, \ldots, m \tag{51}$$

A solution $\vec{x}$ is regarded as **feasible** if $\mathrm{g}_i(\vec{x}) \le 0, for\ j = 1, \ldots, q\ and\ |\mathrm{h}_j(\vec{x})| - \epsilon \le 0, for\ j = q + 1, \ldots, m$.In this special session ϵ is set to 0.0001.
For each function, present the following: best, median, worst result, mean value and standard deviation for the 25 runs. Please indicate the number of violated constraints (including the number of violations by more than 1, 0.01, 0.0001) and the mean violations $\overline{v}$at the median solution. $\overline{v} = (\sum_{i=1}^{q} G_i(\vec{x}) + \sum_{j=q+1}^{m} H_j(\vec{x}))/m$, where $G_i(\vec{x}) = \begin{cases} g_i(\vec{x}) & \text{if } g_i(\vec{x}) > 0 \\ 0 & \text{if } g_i(\vec{x}) \le 0 \end{cases}$ $H_j(\vec{x}) = \begin{cases} |h_j(\vec{x})| & \text{if } |h_j(\vec{x})| - \epsilon > 0 \\ 0 & \text{if } |h_j(\vec{x})| - \epsilon \le 0 \end{cases}$.

If feasible solutions better than the provided best-known solutions are found, please send email to epnsugant@ntu.edu.sg about the details.

* **If the participant uses method of penalties, please notice that $f(\vec{x})$ here is the function value of the problem without penalties.**

2) Record the FES needed in each run for finding a solution satisfying the following condition: $f(\vec{x}) - f(\vec{x}^*) \leq 0.0001$ and $\vec{x}$ is feasible.

For each function, present the following: best, median, worst result, mean value and standard deviation for the 25 runs.

3) Feasible Rate, Success Rate & Success Performance for Each Problem

Feasible Run: A run during which at least one feasible solution is found in Max_FES.

Successful Run: A run during which the algorithm finds a feasible solution $\vec{x}$ satisfying $f(\vec{x}) - f(\vec{x}^*) \leq 0.0001$.

Feasible Rate = (# of feasible runs) / total runs

Success Rate = (# of successful runs) / total runs

Success Performance = mean (FEs for successful runs)×(# of total runs) / (# of successful runs)

The above three quantities are computed for each problem separately.

4) Convergence Graphs (or Run-length distribution graphs) for Each Problem

The graph would show the median run of the total runs with termination by the Max_FES. The semi-log graphs should show $log10(f(\vec{x}) - f(\vec{x}^*))$ vs FES and $log10(\overline{v})$ vs FES for each problem. $\vec{x}$ here is the best solution till now.

*Needn't plot the points which satisfy $(f(\vec{x}) - f(\vec{x}^*))leq0$

5) Algorithm Complexity

a) $T1 = (\sum_{i=1}^{24} t1_i)/24$. $t1_i$ = the computing time of 10000 evaluations for problem i.

b) $T2 = (\sum_{i=1}^{24} t2_i)/24$. $t2_i$ = the complete computing time for the algorithm with 10000 evaluations for problem i.

The complexity of the algorithm is reflected by: $T1, T2, and (T2 - T1)/T1$

6) Parameters

We discourage participants searching for a distinct set of parameters for each problem/dimension/etc. Please provide details on the following whenever applicable:

a) All parameters to be adjusted.

b) Corresponding dynamic ranges.

c) Guidelines on how to adjust the parameters.

d) Estimated cost of parameter tuning in terms of number of FEs.

e) Actual parameter values used.

7) Encoding

If the algorithm requires encoding, then the encoding scheme should be independent of the specific problems and governed by generic factors such as the search ranges.

3. Results Format

Participants are suggested to present their results in the following format:

PC Configure:
System: CPU:
RAM: Language:
Algorithm:

Parameters Setting
a) All parameters to be adjusted.
b) Corresponding dynamic ranges.
c) Guidelines on how to adjust the parameters.
d) Estimated cost of parameter tuning in terms of number of FEs.
e) Actual parameter values used.

Results Achieved

FES		g01	g02	g03	g04	g05	g06
5×10^3	Best	5.4871(0)					
	Median	5.5622(0)					
	Worst	5.5772(0)					
	c	0,0,0					
	$\overline{v}$	0					
	Mean	5.5422					
	Std	0.0482					
5×10^4	Best	5.5712e-005(0)					
	Median	7.5782e-005(0)					
	Worst	9.1048e-005(0)					
	c	0,0,0					
	$\overline{v}$	0					
	Mean	7.4180e-005					
	Std	1.7722e-005					
5×10^5	Best	0(0)					
	Median	0(0)					
	Worst	0(0)					
	c	0,0,0					
	$\overline{v}$	0					
	Mean	0					
	Std	0					

Table 5: Error Values Achieved When FES= 5×10^3 , FES= 5×10^4, FES= 5×10^5 for Problems 1-6. (Please keep 4 digits after the decimal point as the example data in the table)

c is the number of violated constraints at the median solution: the sequence of three numbers indicate the number of violations (including inequality and equalities) by more than 1.0, more than 0.01 and more than 0.0001 respectively. $\overline{v}$ is the mean value of the violations of all constraints at the median solution.The numbers in the parenthesis after the fitness value of the best, median, worst solution are the number of constraints which can not satisfy feasible condition at the best, median and worst solutions respectively.

***Sorting method for the final results :**

1. Sort feasible solutions in front of infeasible solutions;
2. Sort feasible solutions according to their function errors f(x)-f(x*)
3. Sort infeasible solutions according to their mean value of the violations of all constraints.

FES		g07	g08	g09	g10	g11	g12
	Best						
	Median						
	Worst						
$\mathbf{5 \times 10^3}$	c						
	$\overline{v}$						
	Mean						
	Std						
	Best						
	Median						
	Worst						
$\mathbf{5 \times 10^4}$	c						
	$\overline{v}$						
	Mean						
	Std						
	Best						
	Median						
	Worst						
$\mathbf{5 \times 10^5}$	c						
	$\overline{v}$						
	Mean						
	Std						

Table 6: Error Values Achieved When FES= 5×10^3 , FES= 5×10^4, FES= 5×10^5 for Problems 7-12.

FES		g13	g14	g15	g16	g17	g18
	Best						
	Median						
	Worst						
$\mathbf{5 \times 10^3}$	c						
	$\overline{v}$						
	Mean						
	Std						
	Best						
	Median						
	Worst						
$\mathbf{5 \times 10^4}$	c						
	$\overline{v}$						
	Mean						
	Std						
	Best						
	Median						
	Worst						
$\mathbf{5 \times 10^5}$	c						
	$\overline{v}$						
	Mean						
	Std						

Table 7: Error Values Achieved When FES= 5×10^3 , FES= 5×10^4, FES= 5×10^5 for Problems 13-18.

FES		g19	g20	g21	g22	g23	g24
	Best						
	Median						
	Worst						
$\mathbf{5 \times 10^3}$	c						
	$\overline{v}$						
	Mean						
	Std						
	Best						
	Median						
	Worst						
$\mathbf{5 \times 10^4}$	c						
	$\overline{v}$						
	Mean						
	Std						
	Best						
	Median						
	Worst						
$\mathbf{5 \times 10^5}$	c						
	$\overline{v}$						
	Mean						
	Std						

Table 8: Error Values Achieved When FES= 5×10^3 , FES= 5×10^4, FES= 5×10^5 for Problems 19-24.

Prob.	**Best**	**Median**	**Worst**	**Mean**	**Std**	**Feasible Rate**	**Success Rate**	**Success Performance**
g01	48444	49182	49999	49208	777.8344	100%	100%	49208
g02								
g03								
g04								
g05								
g06								
g07								
g08								
g09								
g10								
g11								
g12								
g13								
g14								
g15								
g16								
g17								
g18								
g19								
g20								
g21								
g22								
g23								
g24								

Table 9: Number of FES to achieve the fixed accuracy level $((f(\vec{x}) - f(\vec{x}^*)) \leq 0.0001)$, Success Rate, Feasible Rate and Success Performance.

Convergence Map

The semi-log graphs should show $log10(f(\vec{x}) - f(\vec{x}^*))$ vs FES and $log10(\overline{v})$ vs FES for each problem. Please use +, x, o, etc. to differentiate graphs. FEs should go to 500,000.

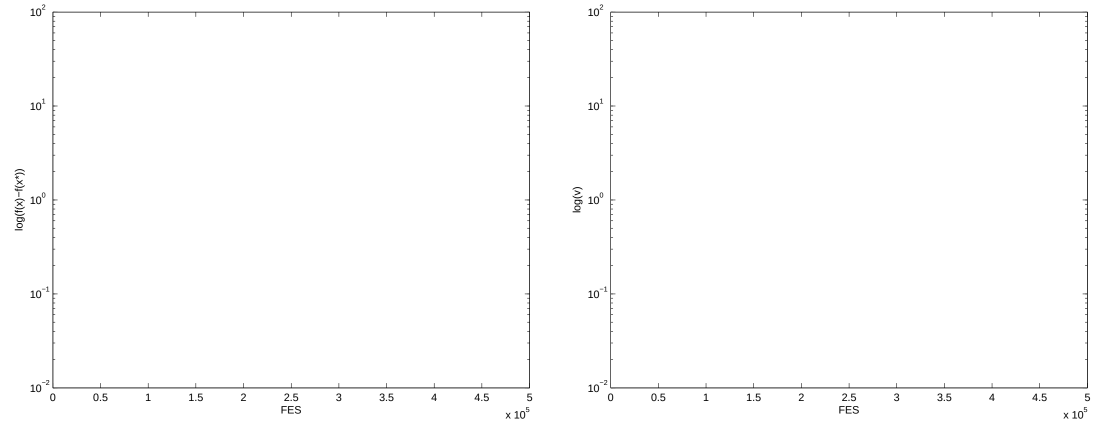

Figure 1: Convergence Graph for Problems 1-6

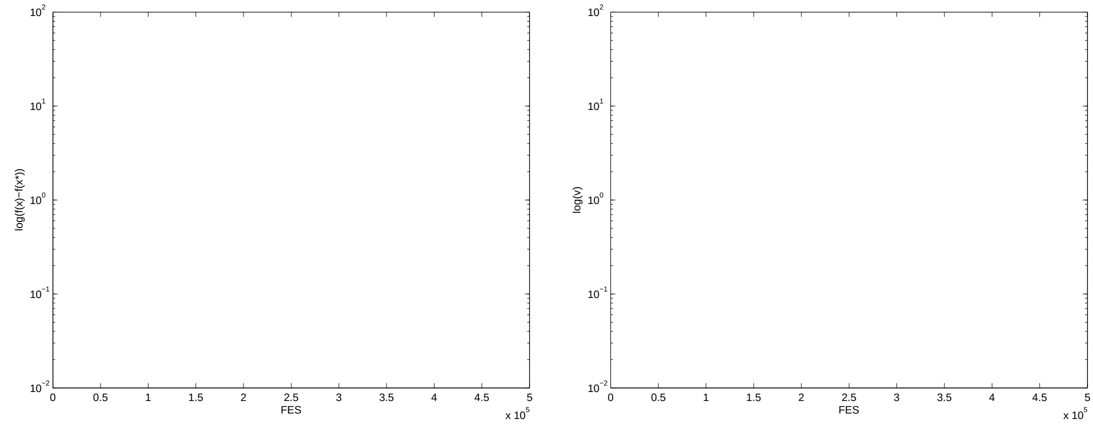

Figure 2: Convergence Graph for Problems 7-12

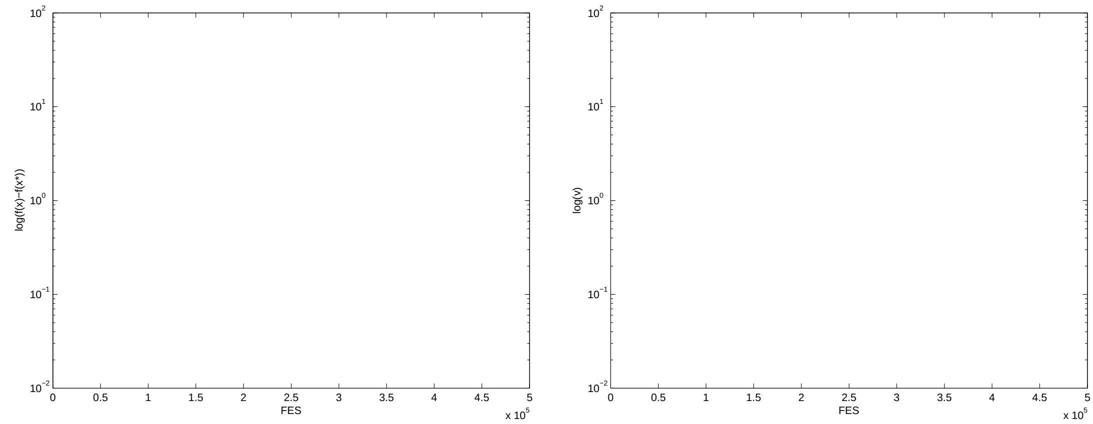

Figure 3: Convergence Graph for Problems 13-18

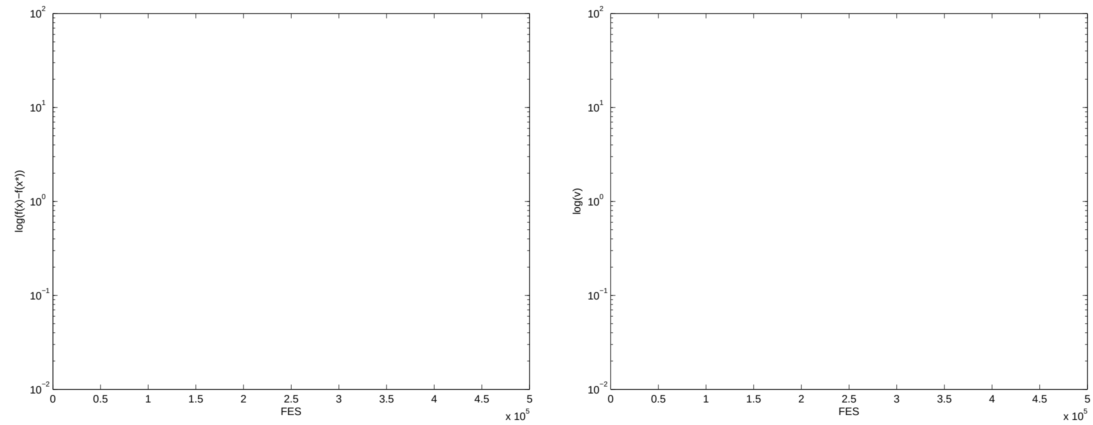

Figure 4: Convergence Graph for Problems 19-24

Algorithm Complexity

$T1$	$T2$	$(T2-T1)/T1$

Table 10: Computational Complexity

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