# MA2104 - Multivariable Calculus Suggested Solutions

(Semester 1 : AY2018/19)

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## Q1

a. False

b. True

c. True

d. True

e. True

f. False

## Q2

a.
$$\int_0^1 \int_0^{2-x^2} f(x,y) \ dy \ dx$$

b. 3

c. 4

d. 7

## Q3

a. Let the density, in circular coordinates, be $f(r,\theta,z) = kz$

$$
\begin{aligned}
\int_0^{2\pi} \int_0^2 \int_0^{\sqrt{4-r^2}} kzr \ dz \ dr \ d\theta &= 2\pi \int_0^2 \frac{1}{2} kr(4-r^2) \ dr \\
&= \pi k \int_0^2 4r - r^3 \ dr \\
&= \pi k \left[ 2r^2 - \frac{1}{4}r^4 \right]_0^2 \\
&= 4\pi k \\
&= 1
\end{aligned}
$$

$$\therefore k = \frac{1}{4\pi}$$
$$\therefore f(r,\theta,z) = \frac{z}{4\pi}$$

b.

$$\int_0^{2\pi} \int_0^1 \int_0^{\sqrt{4-r^2}} 2zr \; dz \; dr \; d\theta = \frac{7}{2}\pi$$

c. Take volume of half a sphere minus a spherical cap, and divide that by 2

$$\frac{1}{2}\left(\frac{1}{2}\frac{4}{3}\pi(2)^3 - \frac{\pi(2-\sqrt{2})^2}{3}(6-(2-\sqrt{2}))\right) = \frac{8\pi}{3} - \frac{\pi(2-\sqrt{2})^2(4+\sqrt{2})}{3}$$

$$= \frac{8\pi}{3} - \frac{2\pi(8-5\sqrt{2})}{3}$$

# Q4

a. Yes. The potential function

$$\frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2$$

works

b. Let $E$ be the solid bounded by $S$ and $D$. Note that the flux across $D$ is 0, as $z = 0$ in $D$. Hence, the flux across $S$ is the same as the flux across the surface of $E$, and we can apply Divergence Theorem.

$$\nabla \cdot F = 3$$

$$\iint_S F \cdot dA = \iiint_E 3 \; dV = \pi$$

c. 20

# Q5

a. Change coordinates to $u, v, z$, where $u = x + y, v = x - 2y$, then the volume has $-1 \le v \le 1, 0 \le u \le 2, \frac{2}{3}(2u+v) - 3 \le z \le \frac{2}{3}(2u+v) - 1$

$$\frac{\partial(u,v)}{\partial(x,y)} = \left|\begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}\right| = -3$$

$$\int_{-1}^1 \int_0^2 \int_{\frac{2}{3}(2u+v)-3}^{\frac{2}{3}(2u+v)-1} \frac{1}{3} \; dz \; du \; dv = \frac{8}{3}$$

b.

$$-1 \le y \le 1$$
$$0 \le z \le 1 - y^2$$
$$0 \le x \le 1 - z$$

Hence

$$0 \le x \le 1 - z \le 1$$
$$0 \le z \le 1 - x$$
$$y^2 \le 1 - z \implies -\sqrt{1-z} \le y \le \sqrt{1-z}$$

Hence

$$\int_{-1}^1 \int_0^{1-y^2} \int_0^{1-z} f(x,y,z) \; dx \; dz \; dy = \int_0^1 \int_0^{1-x} \int_{-\sqrt{1-z}}^{\sqrt{1-z}} f(x,y,z) \; dy \; dz \; dx$$

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