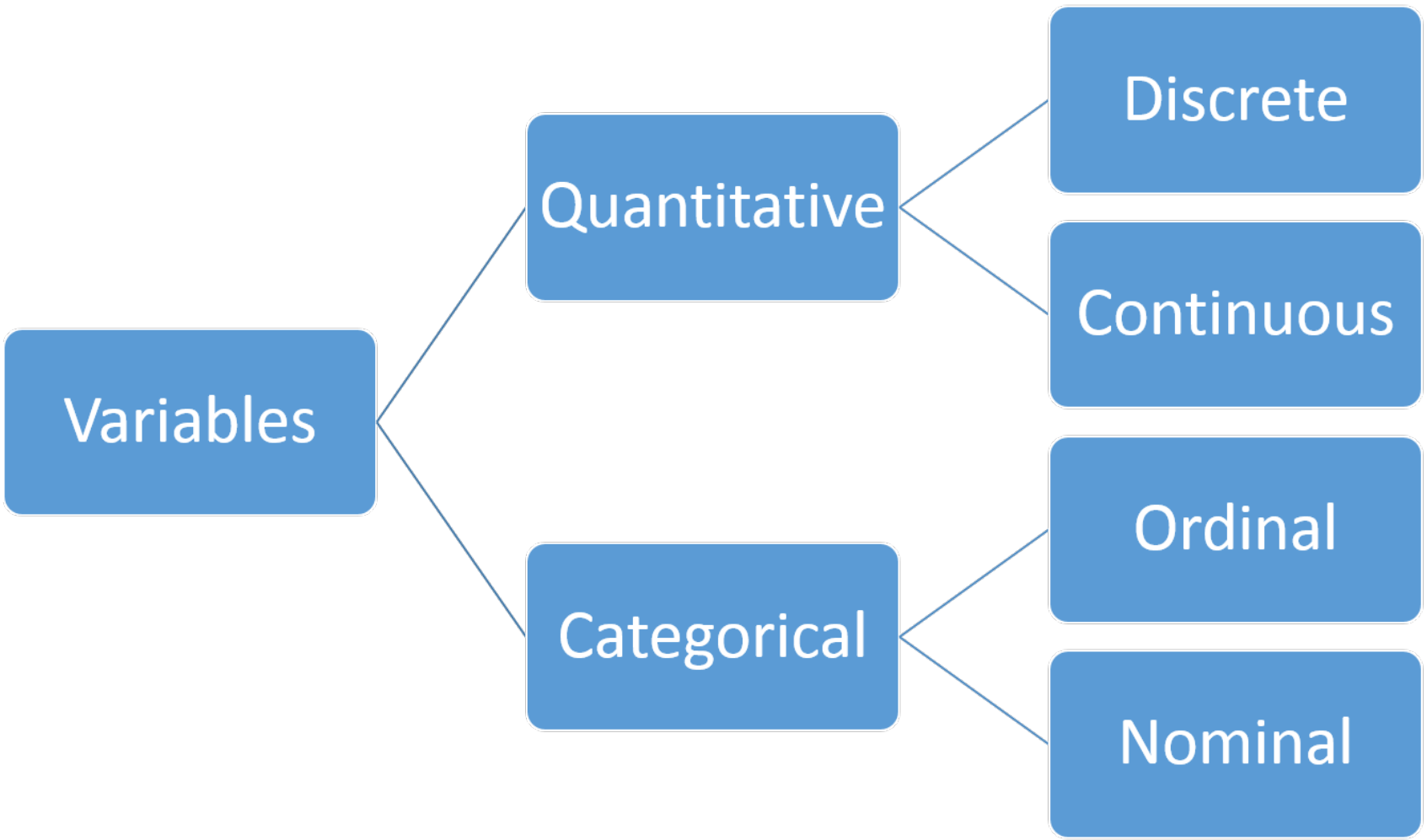
Basic Probability and Statistics

Types of Data



Descriptive Statistics

- There are two major ways of describing data descriptively: numerical and graphical summaries.
- One variable: the numerical and graphical summaries will be covered.
- For two variables: association between two variables will be covered.

Numerical and Graphical Summaries

- There are two major ways of describing numerical data:

  - **Numerical summaries**/descriptive measures: number of observations (sample size), location, variability and other measures.

  - **Graphical summaries**: histogram, boxplot, QQ plot (for checking normality of a dataset), scatter plot for bivariate data.

An Example: Yearly Sales

- `> sales <- read.csv("C:/Data/yearly_sales.csv")`

- The function head() displays the first few records in the data set

```
> head(sales)
  cust_id sales_total num_of_orders gender
1  100001      800.64             3      F
2  100002      217.53             3      F
3  100003       74.58             2      M
4  100004      498.60             3      M
5  100005      723.11             4      F
6  100006       69.43             2      F
> total = sales$sales_total
```

Summary of the Center

- Center of data should include the information on: mean, median and mode.

- About the total sales, we roughly can have

```
> n = length(total); n
[1] 10000
> summary(total)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  30.02   80.29  151.65  249.46  295.50 7606.09
```

Summary of the Variability

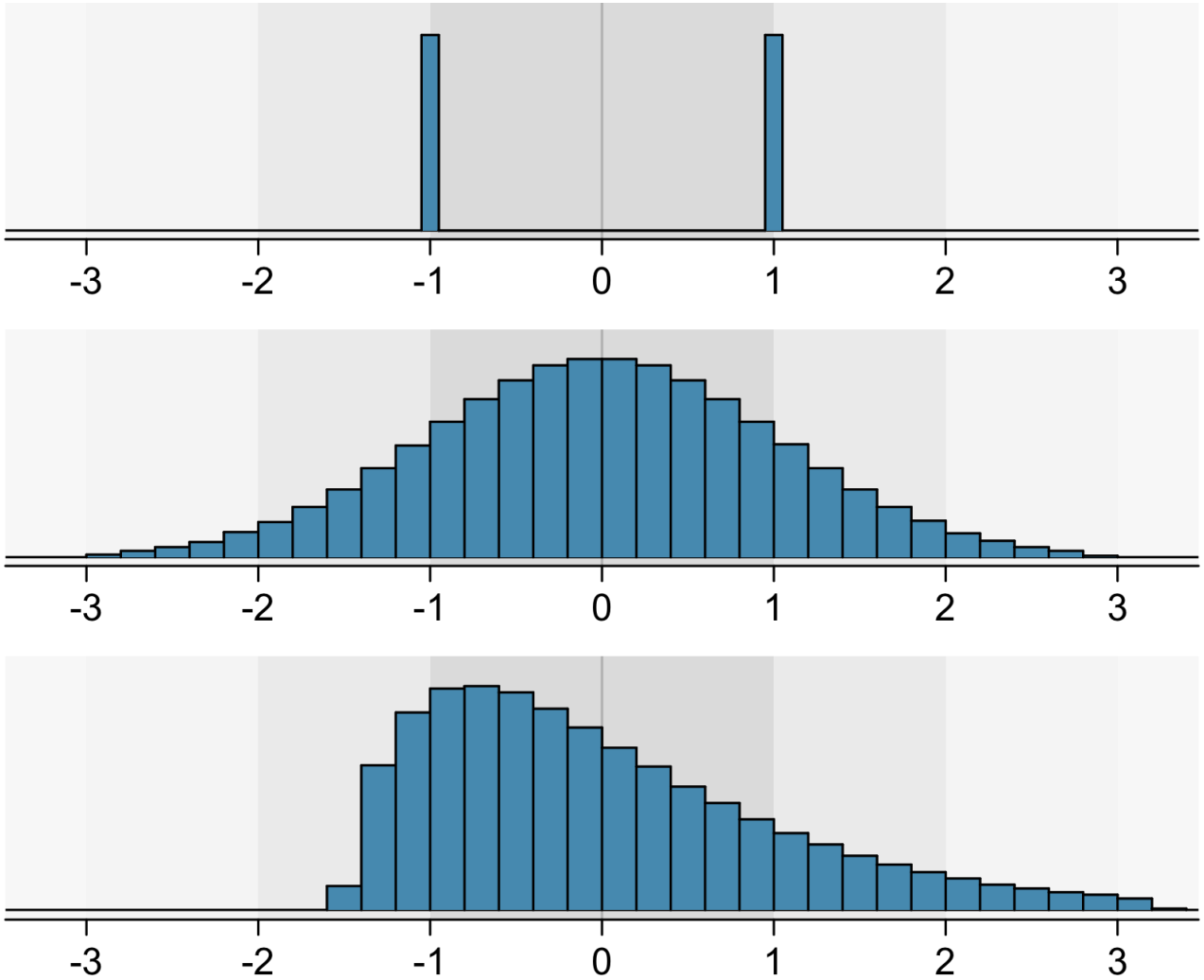
```
> range(total)
[1]   30.02 7606.09
> var(total)
[1] 101793.4
> sd(total)
[1] 319.0508
> IQR(total)
[1] 215.21
> total[order(total)[1:5]] # The 5 smallest observations
[1] 30.02 30.03 30.04 30.05 30.06
> total[order(total)[(n-4):n]] #The 5 largest observations
[1] 3873.24 4046.90 6091.15 6428.06 7606.09
```

A Note on Numerical Summaries

- For a sample, when the mean is the same or approximately the same as median, then the sample is close to symmetric.
- Mean is sensitive to the outlier while median is not.
- When mean is much larger than median, sample is right skewed; while when mean is much smaller than median then sample is left skewed.

Numerical Summaries Are Not Enough

- No matter how many of the summary measures we report, nothing beats a picture.
- All 3 samples below had a sample mean of 0 and a sample variance of 1.



Histogram and Density Plot

- A histogram is a graph that uses bars to portray the frequencies or relative frequencies of the possible outcomes for a quantitative variable.

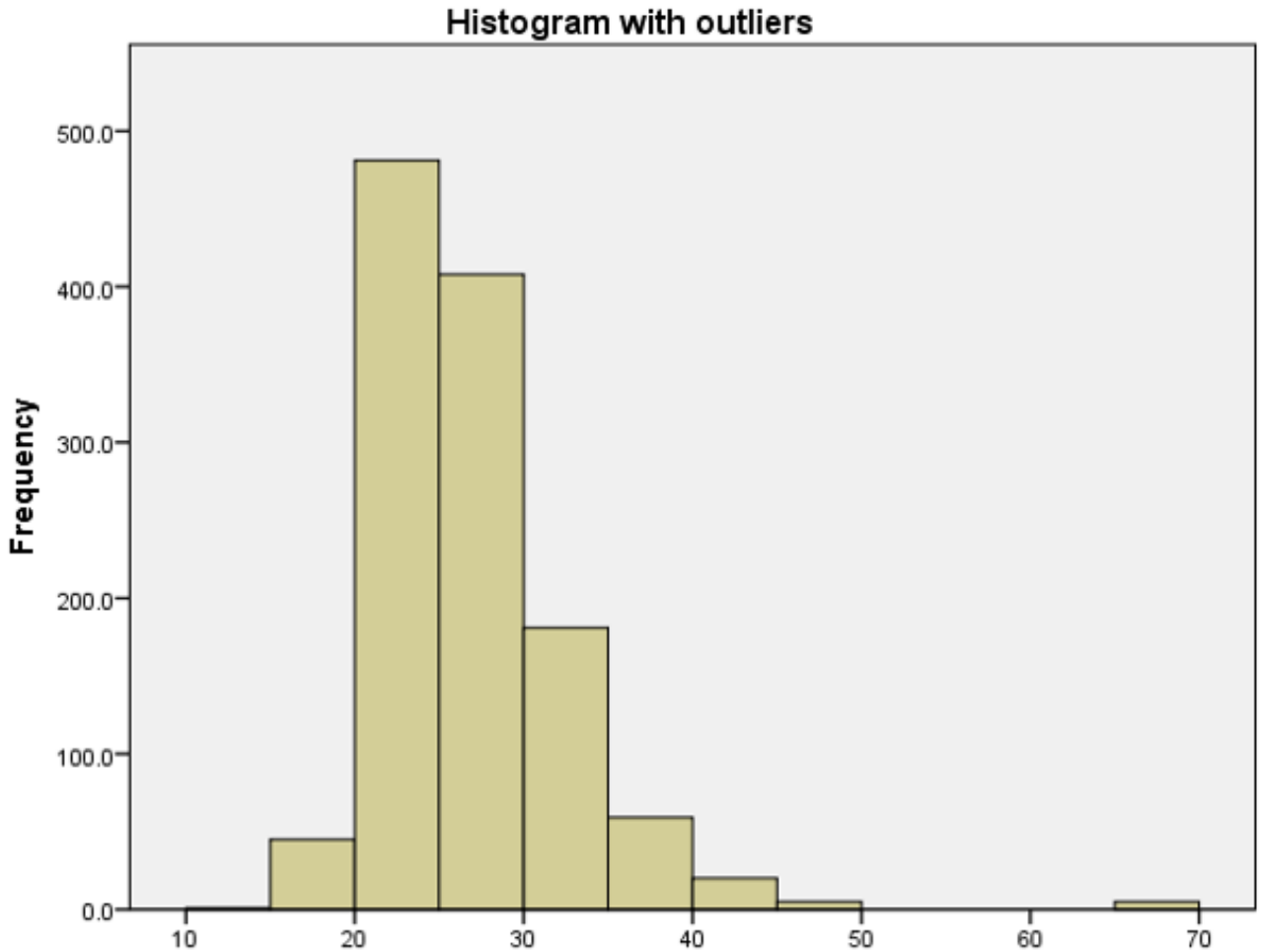
- Density plots can be thought of as plots of smoothed histograms.

- What do we look for in a histogram?
  - The overall pattern. Do the data cluster together, or is there a gap such that one or more observations deviate from the rest?

  - Do the data have a single mound? This is known as a unimodal distribution. Data with two mound are known as bimodal, and data with many mounds are referred to as multimodal.

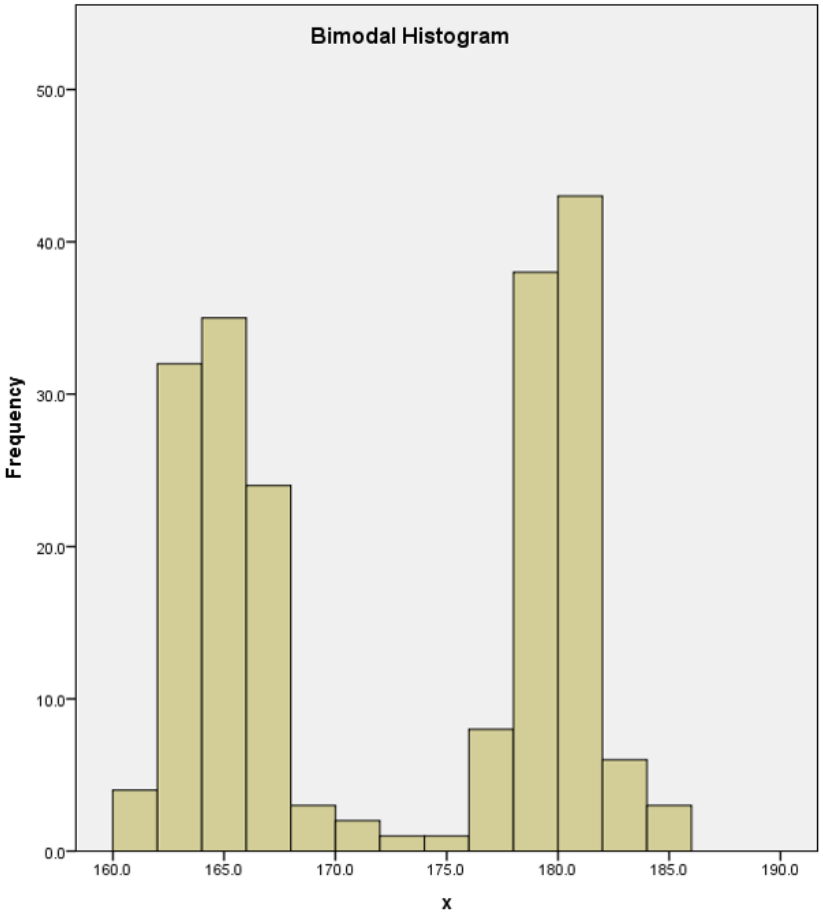
  - Is the distribution symmetric or skewed? Any suspected outliers?

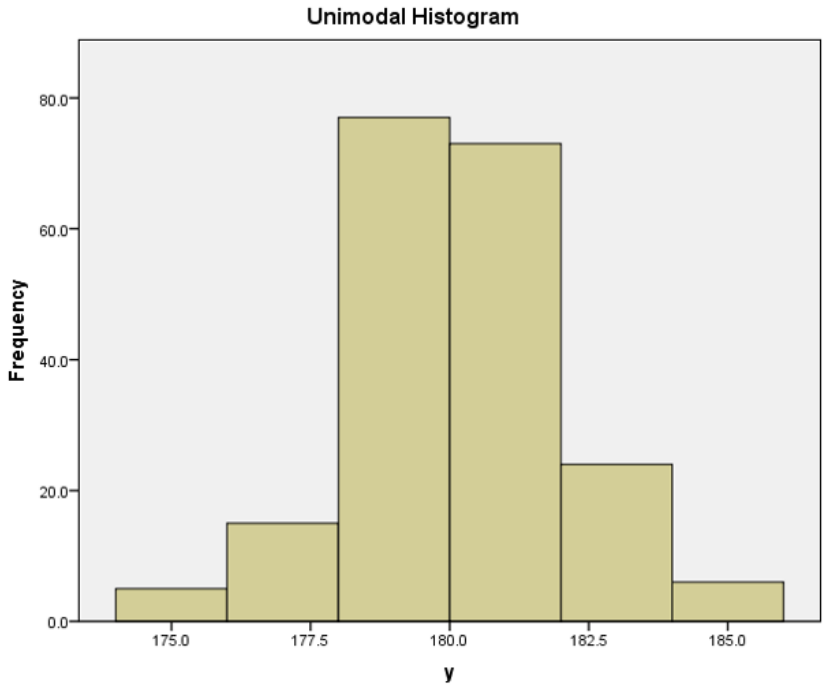
A Histogram With Suspected Outliers



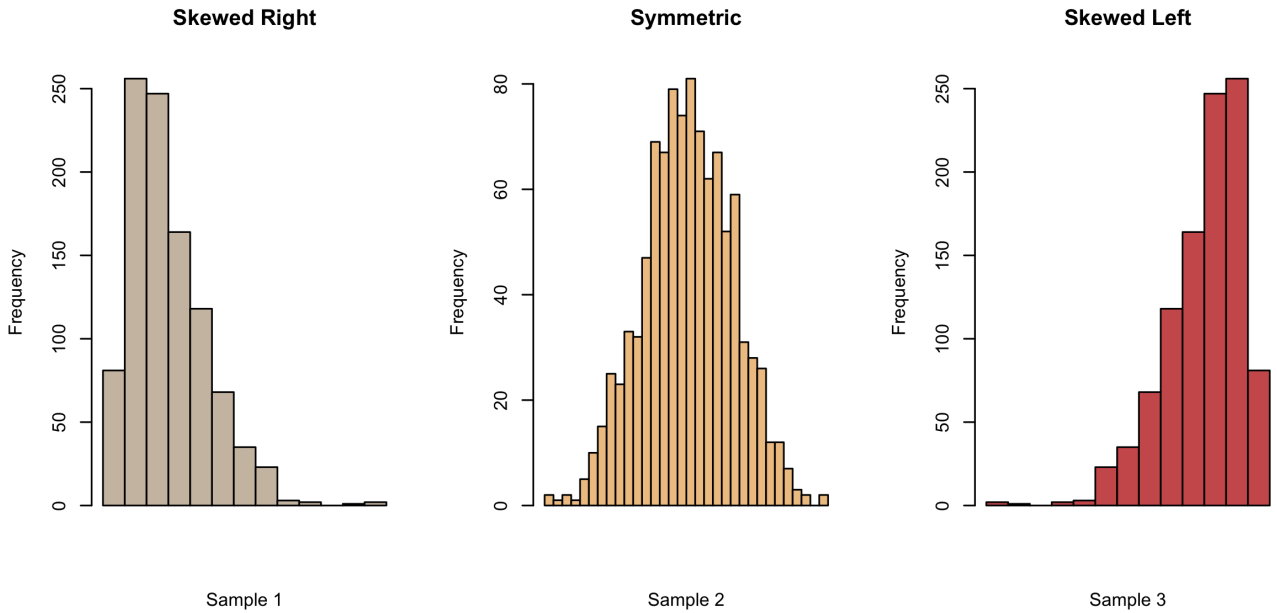
- This histogram is unimodal, but it has suspected outliers on the right.

Unimodal and Bimodal Histograms





Skewness of Histograms



- Income is typically right-skewed.
- IQ is typically symmetric.
- Life-span is typically left-skewed.

Histogram and Density Plot in R

There are many ways to plot histograms in R:

- The `hist` function in the base `graphics` package;
- `truehist` in package `MASS`;
- `histogram` in package `lattice`;
- `geom_histogram` in package `ggplot2`.

```
## Default S3 method:
hist(x, breaks = "Sturges",
     freq = NULL, probability = !freq,
     include.lowest = TRUE, right = TRUE,
     density = NULL, angle = 45, col = "lightgray", border = NULL,
     main = paste("Histogram of" , xname),
     xlim = range(breaks), ylim = NULL,
     xlab = xname, ylab,
     axes = TRUE, plot = TRUE, labels = FALSE,
     nclass = NULL, warn.unused = TRUE, ...)
```

Histogram and Normal Density Plot in R

```
> hist(total, freq=FALSE, main = paste("Histogram of Total Sales"),
+      xlab = "total sales", ylab="Probability",
+      axes = TRUE, col = "grey")
```

The histogram is highly right skewed.

Boxplots

- Boxplots provide a skeletal representation of a distribution, and they are very well suited for showing distributions for multiple variables.

- A boxplot helps us to identify median, lower and upper quantiles, IRQ, and outlier(s).

Boxplots in R

The code should be

```
> boxplot(total, xlab = "Total Sales")
```

The median is very low, close to 200. Box plot shows many outliers and extreme outliers.

If the sample is unimodal then the distribution is highly right skewed.

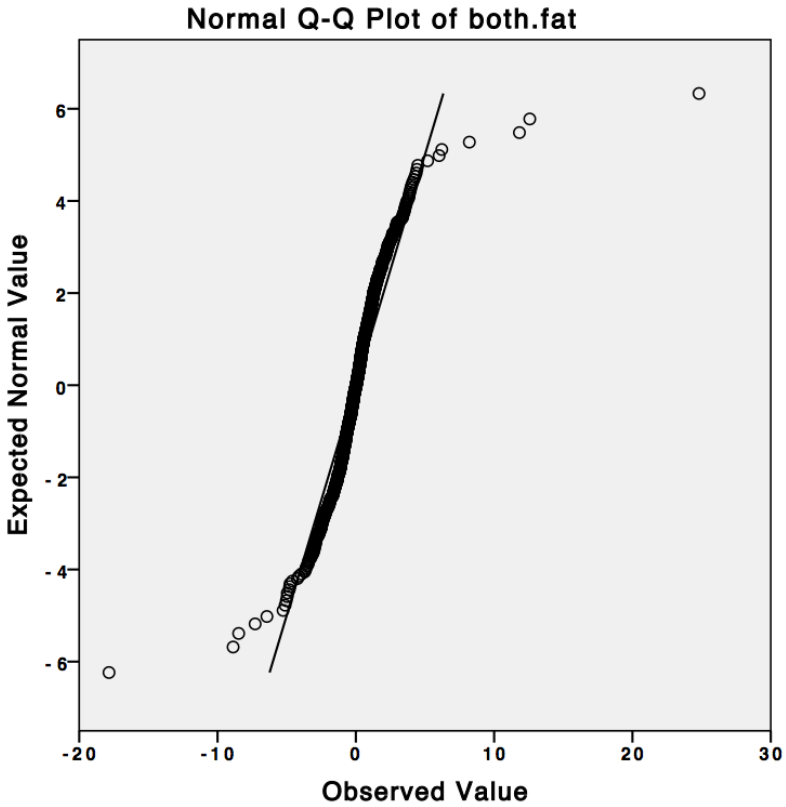
QQ Plots

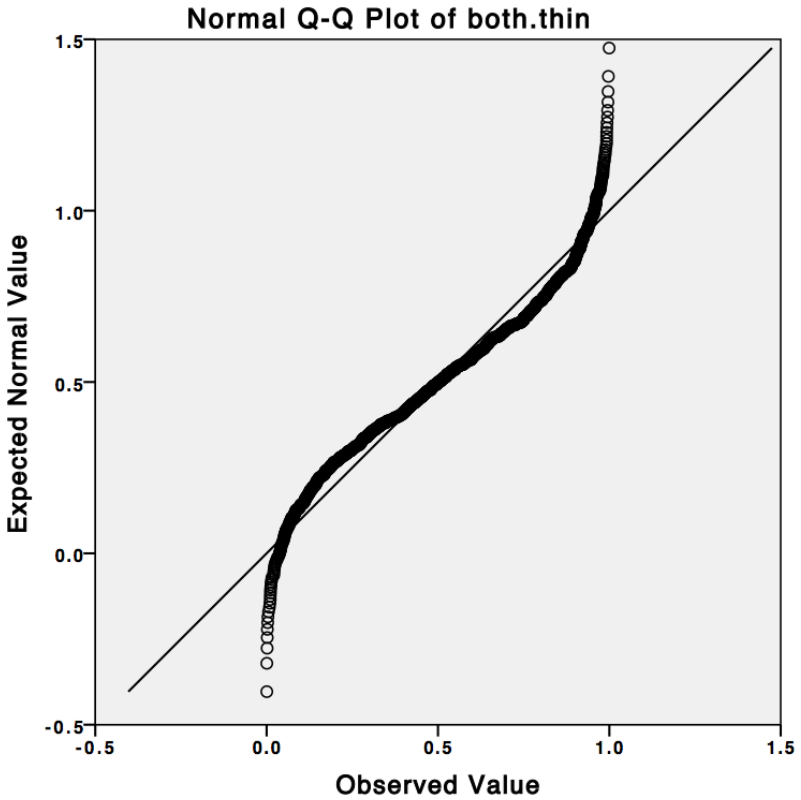
- **The purpose of plotting a QQ plot of a sample is to see if the sample follows (approximately) a normal distribution or not.**

- A QQ-plot matches the standardized sample quantiles against the theoretical quantiles of a N(0; 1) distribution. If they fall on a straight line, then we would say that there is evidence that the sample came from a normal distribution.

- From the points on the plot, we can usually tell whether our sample has longer or shorter tail than normal.

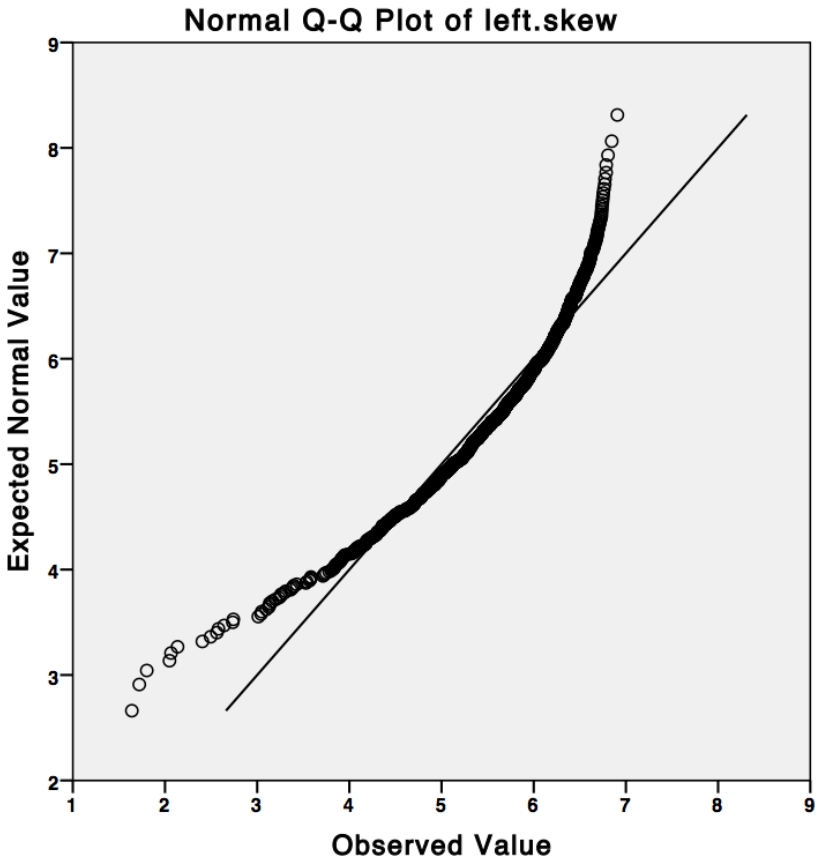
QQ plots (1)

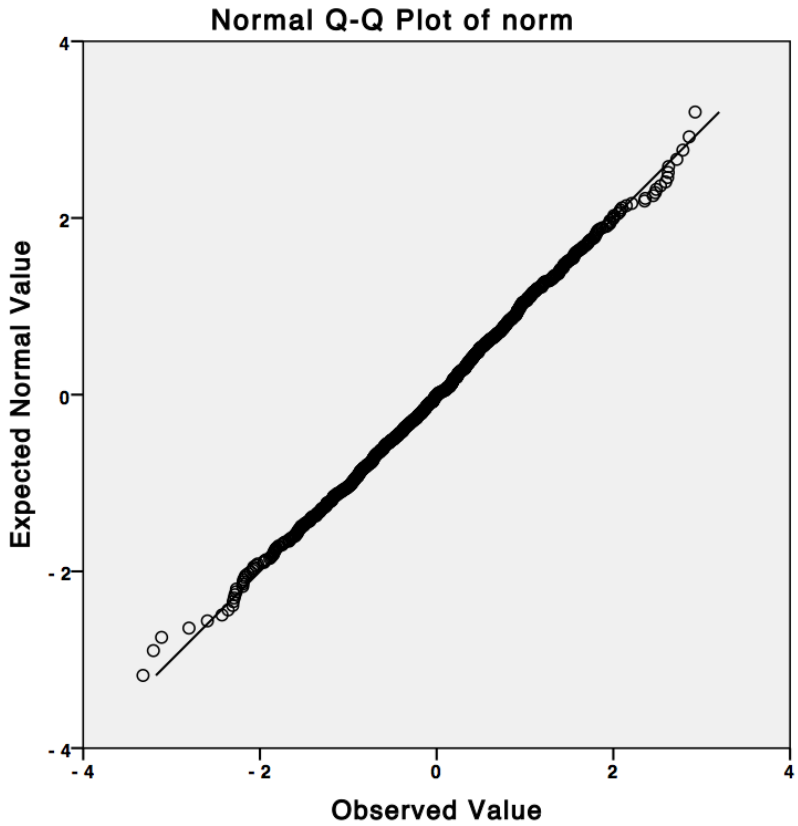




- Figure in the left is a data with both longer tails than normal.
- Figure in the right is a data with both shorter tails than normal.

QQ plots (2)





- Figure in the left is a data with left tail longer than normal but right tail is shorter than normal.
- Figure in the right is a data with both tails are normal.

QQ Plots in R

The code should be

```
> qqnorm(total, main = "QQ Plot", pch = 20)
> qqline(total, col = "red")
```

The QQ plot is extremely right skewed.

Quantifying the Association: Correlation Value

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- r is always between -1 and 1.
- A positive value for r indicates a positive association and a negative value for r indicates a negative association.

```
> order = sales$num_of_orders
> cor(total, order)
[1] 0.7508015
```

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- If there is association, is it linear or non-linear type?
- Are some observations unusual, departing from the overall trend?
- If y is the response then check if the **variance** of y is stable when the value of the x-feature changes?

Scatterplots in R

```
> plot(order,total, pch = 20, col = "darkblue")
```

Boxplots of Multiple Groups in R

```
> attach(sales)
> boxplot(total ~ gender)
```

There is no obvious difference in the total sales of the customer's gender. The median of two groups are similar, and the IRQ are about the same.

Association of 3 Variables

- Can you figure out a way to visualize the association of the three features: total sales, number of orders and the gender of the customers?

Summary of a Categorical Variable

- For a single categorical variable, we can use **frequency table** (which also can produce the proportion or percentage) as numerical summaries. The category with the highest frequency is the **modal category**.

- Common graphical to display a categorical variable is **bar plot** or pie chart.

Barplot and Pie Chart

```
> count = table(gender)
> count # frequency table
gender
   F    M
5035 4965
> barplot(count)
> pie(count)
```

Two Categorical Variables

- Contingency table is often used to summarize the two categorical variables.

- Odd ratio is useful too.

Two Categorical Variables

- Categorizing the number of orders into two categories: small and large size.

```
> order.size = ifelse(order<=5, "small", "large")
> table(order.size)
order.size
large small
  324  9676
```

- Contingency table of frequency

```
> table = table(gender,order.size);table
      order.size
gender large small
     F   142  4893
     M   182  4783
```

Contingency Tables

- Contingency table of joint proportion

```
> prop.table(table)
        order.size
gender   large   small
     F 0.0142 0.4893
     M 0.0182 0.4783
```

- Contingency table of proportion by gender

```
> tab = prop.table(table, "gender") # proportion by gender
> tab
        order.size
gender       large       small
     F 0.02820258 0.97179742
     M 0.03665660 0.96334340
```

Among orders by females, 2.82% are large orders while 3.67% of orders by males are large.

Odds Ratio

- For a probability of success π, the **odds of success** is defined as $odds = \pi/(1-\pi)$.

- If we consider having a large order is a success, then for the female groups, the odds of success, or the odds of large order, is 0.029.

```
> tab[1]/(1-tab[1])
[1] 0.02902105
```

- For the male group, the odds of having large order is 0.038.

```
> tab[2]/(1-tab[2])
[1] 0.03805143
```

- The odds ratio is the ratio of two odds, 0.76. What does this value mean?

```
> OR = tab[1]/(1-tab[1])/(tab[2]/(1-tab[2])); OR
[1] 0.7626796
```

User Defined Functions

- Characteristics of a function:
  -has a name
  -has parameters
  -has a body
  -returns something

- How to write/define:
  `name <-` function`(parameters) { function body }`

- At the end of the `function body`, some command to ask for evaluation and return normally be included.

Function to calculate OR

- We would want to form a function that helps us to calculate OR for a 2×2 matrix or table.

```
> OR<-function(x){
+   if(any(x==0)) {x<-x+0.5}
+   odds.ratio<-x[1,1]*x[2,2]/(x[2,1]*x[1,2])
+
+   return(odds.ratio) }
> #OR(table)
>
```

- Function is named as "OR".

- It has one argument/parameter, `x`, which is the 2×2 matrix/table.
- Question: write a function to find the median of a given vector.