ST2132 HW3
Ethan Keck Jun Wei (March 7, 2024)

Problem 1.

Answer: a)

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = 26$$
$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = 27.14286 \approx 27.14$$

b) $H_0 : \mu = \mu_0, H_1 = \mu > \mu_0$
c) Since $\sigma^2 = 4^2$ is known,

$$t = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{26 - 26}{4\sqrt{15}} = 0$$
$$z_{0.05} = 1.645$$

d) Since $t < z_{0.05}$ Charles should not reject H_0.
e) For Charles to reject the H_0,

$$t > z_{0.05}$$
$$\frac{\bar{x} - 26}{4/\sqrt{15}} > 1.645$$
$$\bar{x} > 27.698 \approx 27.70$$

f) Since σ^2 is unknown,

$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{26 - 26}{s/\sqrt{15}} = 0$$
$$t_{0.05}(15 - 1) = 1.761$$

Since $T < t_{0.05}(15 - 1)$, Charles does not reject H_0. □

Problem 2.

Answer: a) Let $H_0 : p_0 = 1/2, H_1 : p_0 \neq 1/2$

$$t = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$
$$= \frac{35/50 - 1/2}{\sqrt{1/2(1 - 1/2)/50}} = 2.8284$$
$$z_{0.025} = 1.96$$

Since $|t| > z_{0.025}$, Micheal rejects H_0.
b) To reject H_0

$$|t| \geq z_{0.025}$$
$$\left| \frac{h/50 - 1/2}{\sqrt{(1/2)(1-1/2)/50}} \right| \geq 1.96$$
$$|h - 25| \geq 1.96 \cdot 50 \cdot \sqrt{0.005}$$
$$h - 25 \leq -1.96 \cdot 50 \cdot \sqrt{0.005}, h - 25 \geq 1.96 \cdot 50 \cdot \sqrt{0.005}$$
$$h < 18.070, h > 31.929$$
$$\Rightarrow h \leq 18, h \geq 32$$

$\square$

Problem 3.

Answer: a)

$$\begin{aligned} \beta(\mu') &= P\left(\bar{X} < \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}; H_1\right) \\ &= P\left(Z < \frac{\mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}} - \mu'}{\sigma/\sqrt{n}}\right) \\ &= P\left(Z < z_\alpha + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) \\ &= \Phi\left(z_\alpha + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) \end{aligned}$$

b)

$$z_\alpha + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}} = z_{1-\beta} = -z_\beta$$
$$\sqrt{n} = -\frac{\sigma(z_\alpha + z_\beta)}{\mu_0 - \mu'}$$
$$n = \left(\frac{\sigma(z_\alpha + z_\beta)}{\mu_0 - \mu'}\right)^2$$

c)

$$n = \left(\frac{\sigma(z_\alpha + z_\beta)}{\mu_0 - \mu'}\right)^2$$
$$= \left(\frac{1300(1.645 + 1.96)}{25000 - 27000}\right)^2 = 5.4908 \approx 5$$

$\square$

Problem 4.

Answer: Using R:
a) $H_0 : p_1 = p_2, H_1 : p_1 \neq p_2$
b) $\hat{p}_A = 0.1204795, \hat{p}_B = 0.1188495$,
$(\hat{p}_A)_{mle} = 0.1204795, (\hat{p}_B)_{mle} = 0.1188495$
c) Let $\hat{p} = \frac{y_a + y_b}{n+m}$

$$t = \frac{y_a/n - y_b/m}{\sqrt{\hat{p}(1-\hat{p})(1/n + 1/m)}} = 1.362603$$

d) Given $z_{0.025} = 1.96$, then $|t| < z_{0.025}$, we fail to reject H_0 at the 5% significance. Hence, the two versions of the website are not likely to give different CTR at 5% significance. $\square$

Problem 5.

Answer: Using R:
a) $H_0 : \mu_x = \mu_y, H_1 : \mu_x > \mu_y$,
b) Since $\sigma_x^2 = \sigma_y^2$, we compute:

$$t = \frac{\bar{x} - \bar{y}}{s_p\sqrt{\frac{1}{n} + \frac{1}{m}}} = -8.328589$$

We compare this to $t_{0.05}(n + m - 2)$. From R, $n + m - 2 = 19061$,
thus we take $t_{0.05}(n + m - 2) \approx 1.645$.
Since $t < t_{0.05}(n + m - 2)$, we not reject H_0 at 5% significance.
c) Since $\sigma_x^2 \neq \sigma_y^2$, we compute:

$$T = \frac{\bar{x} - \bar{y}}{s_p\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}} = -8.336874,$$

$$r = \left\lfloor \frac{(\frac{s_x^2}{n} + \frac{s_y^2}{m})^2}{\frac{1}{n-1}(\frac{s_x^2}{n})^2 + \frac{1}{m-1}(\frac{s_y^2}{m})^2} \right\rfloor$$

We compare this to $t_{0.05}(r)$. From R, $r = 10704$,
thus we take $t_{0.05}(r) \approx 1.645$.
Since $t < t_{0.05}(r)$, we not reject H_0 at 5% significance.
d) Using R, we compute:

$$t = \frac{s_x^2}{s_y^2} = 1.004922,$$

$$F_{0.025}(13390, 5671) \approx 1.00,$$

$$F_{1-0.025}(13390, 5671) = \frac{1}{F_{0.025}(5671, 13390)} \approx 1.00$$

Since $t \geq F_{0.025}(13390, 5671)$, we reject H_0 at 5% significance level. $\square$