Some solutions for problem III

1. Pseudo-transitivity
   a. The proof can be done using the definition of an FD or using the Armstrong axioms.

Armstrong:
Assume that X -> Y (1), Z -> V (2), and Z (belongs) Y (3)
Since Z (belongs) Y (3) then Y -> Z (4), by reflexivity
Since X -> Y (1) and Y -> Z (4) then X -> Z (5), by transitivity
Since X -> Z (5) and Z -> V (2) then X -> V (QED), by transitivity
   b. Transitivity can be deduced from pseudo transitivity alone, therefore the Armstrong axioms in which transitivity is replaced by pseudo-transitivity are still complete.

2. The rule is not correct. It can be shown by showing an example instance of a table that verifies X -> Y but such that Y -> X is false.
The simplest is to use X={A} and Y={B} fro R(A,B).
In the example below {A}-> {B} but, of course {B} is not a subset of {A}.

A	B
1	2
2	2
3	3

3. F={ {A}->{B},{C}->{D}, {B,D}->{E}, {D}->{A,D}, {A,C}->{E,B} }

g. C+ (0) = {C}
   C+ (1) = {C, D} by using {C}->{D}
   C+ (2) = {C, D, A} by using {D}->{A,D}
   C+ (3) = {C, D, A, B} by using {A}->{B}
   C+ (4) = {C, D, A, B, E} by using {B,D}->{E}

C+ = {C, D, A, B, E} , we can stop, we have every attribute.
{C} is a superkey
There is no proper subset which is a superkey (only one proper subset -> and it is not a superkey), therefore {C} is a candidate key.
It is the only one.
{C} is a primary key.

h. Minimal cover
1. Simplify the right-hand side
F'={ {A}->{B},{C}->{D}, {B,D}->{E}, {D}->{A}, {D}->{D}, {A,C}->{E} , {A,C}->{B} }

2. Simplify the left-hand side

F"={ {A}->{B},{C}->{D}, {D}->{E}, {D}->{A}, {D} ->{D}, {C}->{E} }

{A,C}->{B} can be removed because {A}->{B} is there (and {A} -> {A,B})

{B,D}->{E}, can be replaced by {D}->{E}, (because {D}->{A} and {A}->{B})

{A,C}->{E} can be replaced by {C}->{E}, (because {C}->{D} and {D}->{E})

3. Eliminate redundant rules

Min(F)={ {A}->{B},{C}->{D}, {D}->{E}, {D}->{A} }

{D} ->{D}, can be removed because it is trivial

{C}->{E} can be removed because it can obtained from {C}->{D}, {D}->{E},