Tutorial 6 (Week 9): Functional dependencies.

1. Is the following rule correct? Prove your answer.
   $\forall$ X$\in$R $\forall$Y$\in$R (if X$\rightarrow$Y, then Y$\subseteq$X)

   The rule is not correct. For instance, {id} -> {name}, but {name} is not a subset of {id}. For a formal proof, build an instance table that verifies {id} -> {name} (the empty table {name, id} does that.

2. The following rule is called Pseudo-transitivity.
   $\forall$ X$\in$R $\forall$Y$\in$R $\forall$Z$\in$R $\forall$V$\in$R (if X$\rightarrow$Y and Z$\rightarrow$V and Z$\subseteq$Y, then X$\rightarrow$V)
   a. Prove it using the Armstrong axioms.

   1. We know that X -> Y
   2. We know that Z -> V
   3. We know that Z $\subseteq$ Y
   4. Therefore Y -> Z by reflexivity with (3)
   5. Therefore X -> Z, by transitivity of (1) and (3)
   6. Therefore X -> V, by transitivity of (5) and (2)

   Q.E.D.

   b. Argue that if we replace transitivity with pseudo-transitivity in the Armstrong's axioms we still have a set of axioms that is complete.

   Transitivity can be deduced from pseudo-transitivity alone; therefore the Armstrong axioms in which transitivity is replaced by pseudo-transitivity are still complete.

3. Consider the set of functional dependencies F={ {A}$\rightarrow${B},{C}$\rightarrow${D}, {B,D}$\rightarrow${E}, {D}$\rightarrow${A,D}, {A,C}$\rightarrow${E,B} } on the relation scheme R= {A,B,C,D,E}.
   a. Give an example instance of R that complies with the functional dependencies.

   Empty instance or an instance with only one tuple.

   b. Give an example instance of R that violates the functional dependencies.

   (1,1,1,1,1) and (1,2,2,2,2).

   c. Compute F+ the closure of F.

   …

   d. Give an example of a trivial functional dependency in F+

   {A,B}->{A}

   Give an example of a trivial functional dependency NOT in F+: impossible.

   e. Give an example of a completely non trivial functional dependency in F+

   {A}->{B}

   Give an example of a non-completely non-trivial and non-trivial functional dependency in F+

   {A,C}->{B,C}

   f. Compute {C}+ the closure of the set of attributes {C}.

C+ (0) = {C}

C+ (1) = {C, D} by using {C}->{D}

C+ (2) = {C, D, A} by using {D}->{A,D}

C+ (3) = {C, D, A, B} by using {A}->{B}

C+ (4) = {C, D, A, B, E} by using {B,D}->{E}

C+ = {C, D, A, B, E}, we can stop, we have every attribute. {C} is a superkey

There is no proper subset which is a superkey (only one proper subset -> and it is not a superkey), therefore {C} is a candidate key.

It is the only one. {C} is a primary key.

g. Compute a minimal cover of F

1. Simplify the right-hand side

F'={ {A}->{B},{C}->{D}, {B,D}->{E}, {D}->{A}, {D}->{D}, {A,C}->{E} , {A,C}->{B} }

2. Simplify the left-hand side

F"={ {A}->{B},{C}->{D}, {D}->{E}, {D}->{A}, {D} ->{D}, {C}->{E} }

{A,C}->{B} can be removed because {A}->{B} is there (and {A} -> {A,B})

{B,D}->{E}, can be replaced by {D}->{E}, (because {D}->{A} and {A}->{B})

{A,C}->{E} can be replaced by {C}->{E}, (because {C}->{D} and {D}->{E})

3. Eliminate redundant rules

Min(F)={ {A}->{B},{C}->{D}, {D}->{E}, {D}->{A} }

{D} ->{D}, can be removed because it is trivial

{C}->{E} can be removed because it can obtained from {C}->{D}, {D}->{E}.