CS2102

Tutorial 07

Question 7(a)

- Question: using Armstrong's axiom, proof
  - Union if $a \rightarrow b$ and $a \rightarrow c$, then $a \rightarrow bc$

Question 7(a)

- Question: using Armstrong’s axiom, proof
  - Union if $a \to b$ and $a \to c$, then $a \to bc$
- What we know?
  - $a \to b$
  - $a \to c$

Question 7(a)

- Question: using Armstrong's axiom, proof
  - Union if $a \rightarrow b$ and $a \rightarrow c$, then $a \rightarrow bc$
- What we know?
  - $a \rightarrow b$
  - $a \rightarrow c$
- Aim: $a \rightarrow bc$ (Make RHS bc)
  - Two candidates:
    - $a \rightarrow b$ augmentation $ac \rightarrow bc$
    - $a \rightarrow c$ augmentation $ab \rightarrow bc$

Question 7(a)

- Question: using Armstrong's axiom, proof
  - Union if $a \rightarrow b$ and $a \rightarrow c$, then $a \rightarrow bc$
- What we know?
  - $a \rightarrow b$
  - $a \rightarrow c$
- Aim: $a \rightarrow bc$ (Make RHS bc)
  - Two candidates:
    - $a \rightarrow b$ augmentation $ac \rightarrow bc$
      - Then need to find $a \rightarrow ac$ to complete with transitivity [$a \rightarrow c$ augmentation with a]
    - $a \rightarrow c$ augmentation $ab \rightarrow bc$
      - Then need to find $a \rightarrow ab$ to complete with transitivity [$a \rightarrow b$ augmentation with b]

Question 7(b)

- Question: using Armstrong's axiom, proof
  - Decomposition if $a \rightarrow bc$, then $a \rightarrow b$

Question 7(b)

- Question: using Armstrong's axiom, proof
  - Decomposition   if $a \rightarrow bc$, then $a \rightarrow b$
- What we know?
  - $a \rightarrow bc$

Question 7(b)

- Question: using Armstrong's axiom, proof
  - Decomposition if $a \rightarrow bc$, then $a \rightarrow b$
- What we know?
  - $a \rightarrow bc$
- Aim: $a \rightarrow b$
  - Can be solved immediately if we have $bc \rightarrow b$
    - Then $a \rightarrow bc$; $bc \rightarrow b$ can be combined into $a \rightarrow bc \rightarrow b$

Question 7(b)

- Question: using Armstrong's axiom, proof
  - Decomposition if $a \to bc$, then $a \to b$
- What we know?
  - $a \to bc$
- Aim: $a \to b$
  - Can be solved immediately if we have $bc \to b$
    - Then $a \to bc$; $bc \to b$ can be combined into $a \to bc \to b$
  - But $bc \to b$ is always implied
    - due to Reflexivity

Question 8(a)

- Question: using Armstrong’s axiom, proof
  - Pseudo-transitivity   if $a \rightarrow b$ and $bc \rightarrow d$, then $ac \rightarrow d$

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Question 8(a)

- Question: using Armstrong’s axiom, proof
  - Pseudo-transitivity if $a \rightarrow b$ and $bc \rightarrow d$, then $ac \rightarrow d$
- What we know?
  - $a \rightarrow b$
  - $bc \rightarrow d$
- Aim: $ac \rightarrow d$
  - If we have $ac \rightarrow bc$ we can solve by transitivity: $ac \rightarrow bc \rightarrow d$

Question 8(a)

- Question: using Armstrong's axiom, proof
  - Pseudo-transitivity   if $a \to b$ and $bc \to d$, then $ac \to d$
- What we know?
  - $a \to b$
  - $bc \to d$
- Aim: $ac \to d$
  - If we have $ac \to bc$ we can solve by transitivity: $ac \to bc \to d$
  - But $ac \to bc$ can be formed from $a \to b$
    - Augmentation with c

Question 8(b)

- Question: using Armstrong's axiom, proof
  - Composition if $a \to b$ and $c \to d$, then $ac \to bd$
- What we know?
  - $a \to b$
  - $c \to d$
- Aim: $ac \to bd$
  - Make RHS bd, we have two candidates
    - $a \to b$ augmentation $ad \to bd$
    - $c \to d$ augmentation $bc \to bd$
  - Make LHS ac, we have two candidates
    - $a \to b$ augmentation $ac \to bc$
    - $c \to d$ augmentation $ac \to ad$

Question 8(b)

- Question: using Armstrong's axiom, proof
  - Composition if $a \rightarrow b$ and $c \rightarrow d$, then $ac \rightarrow bd$

Question 8(b)

- Question: using Armstrong's axiom, proof
  - Composition if $a \rightarrow b$ and $c \rightarrow d$, then $ac \rightarrow bd$
- What we know?
  - $a \rightarrow b$
  - $c \rightarrow d$

Question 8(b)

- Question: using Armstrong's axiom, proof
  - Composition if $a \to b$ and $c \to d$, then $ac \to bd$
- What we know?
  - $a \to b$
  - $c \to d$
- Aim: $ac \to bd$
  - Make RHS bd, we have two candidates
  - Make LHS ac, we have two candidates

Question 8(b)

- Question: using Armstrong’s axiom, proof
  - Composition if $a \rightarrow b$ and $c \rightarrow d$, then $ac \rightarrow bd$
- What we know?
  - $a \rightarrow b$
  - $c \rightarrow d$
- Aim: $ac \rightarrow bd$
  - Make RHS bd, we have two candidates
    - $a \rightarrow b$ augmentation $ad \rightarrow bd$
    - $c \rightarrow d$ augmentation $bc \rightarrow bd$
  - Make LHS ac, we have two candidates
    - $c \rightarrow d$ augmentation $ac \rightarrow ad$
    - $a \rightarrow b$ augmentation $ac \rightarrow bc$

Question 8(b)

- Question: using Armstrong's axiom, proof
  - Composition   if $a \to b$ and $c \to d$, then $ac \to bd$
- What we know?
  - $a \to b$
  - $c \to d$
- Aim: $ac \to bd$
  - Make RHS bd, we have two candidates
    - $a \to b$ augmentation $ad \to bd$
    - $c \to d$ augmentation $bc \to bd$
  - Make LHS ac, we have two candidates
    - $c \to d$ augmentation $ac \to ad$
    - $a \to b$ augmentation $ac \to bc$

$ac \to ad \to bd$

$ac \to bc \to bd$

Question 9(a)

- Question: Use extended Armstrong's axioms to show $F \vDash CDG \rightarrow E$
- What we know?
  - $ABC \rightarrow E;\ \ BD \rightarrow A;\ \ CG \rightarrow B$
- Aim: $CDG \rightarrow E$
  - Find $CDG \rightarrow ABC$
  - ➢ Then by transitivity: $CDG \rightarrow ABC \rightarrow E$

Question 9(a)

- Question: Use extended Armstrong's axioms to show $F \vDash CDG \rightarrow E$
- What we know?
  - $ABC \rightarrow E; \quad BD \rightarrow A; \quad CG \rightarrow B$
- Aim: $CDG \rightarrow ABC$
  - Find $CDG \rightarrow A$
  - Find $CDG \rightarrow B$
    - Find $CDG \rightarrow CG$ (Reflexivity)
    - Then by transitivity: $CDG \rightarrow CG \rightarrow B$
  - Find $CDG \rightarrow C$
    - Reflexivity
  - ➢ Then by union: $CDG \rightarrow ABC$

Question 9(a)

- Question: Use extended Armstrong's axioms to show $F \vDash CDG \rightarrow E$
- What we know?
  - $ABC \rightarrow E;\;\; BD \rightarrow A;\;\; CG \rightarrow B$
- Aim: $CDG \rightarrow A$
  - Find $CDG \rightarrow BD$
  - ➢ Then by transitivity: $CDG \rightarrow BD \rightarrow A$

Question 9(a)

- Question: Use extended Armstrong's axioms to show $F \vDash CDG \rightarrow E$
- What we know?
  - $ABC \rightarrow E; \;\; BD \rightarrow A; \;\; CG \rightarrow B$
- Aim: $CDG \rightarrow BD$
  - Find $CDG \rightarrow B$
    - <done before>
  - Find $CDG \rightarrow D$
    - Reflexivity
  - ➢ Then by union: $CDG \rightarrow BD$

Question 9(a)

- Question: Use extended Armstrong's axioms to show $F \vDash CDG \rightarrow E$
- What we know?
  - $ABC \rightarrow E;\ \ BD \rightarrow A;\ \ CG \rightarrow B$
- Aim: $CDG \rightarrow E$
  - <omitted>
    - ➢ Then by union $CDG \rightarrow BD$
    - ➢ Then by transitivity $CDG \rightarrow BD \rightarrow A$
    - ➢ Then by union $CDG \rightarrow ABC$
    - ➢ Then by transitivity $CDG \rightarrow ABC \rightarrow E$

Question 9(b)

- Question: Compute CDG^+
- Steps: Using algorithm #1
  - $\theta = CDG$

Question 9(b)

- Question: Compute CDG^+
- Steps: Using algorithm #1
  - $\theta = CDG$
  - With $CG \to B$ then $\theta = \theta \cup \{B\} = BCDG$
    - $CG \subseteq CDG = \theta$ and $B \notin \theta$

Question 9(b)

- Question: Compute CDG^+
- Steps: Using algorithm #1
  - $\theta = CDG$
  - With $CG \to B$ then $\theta = \theta \cup \{B\} = BCDG$
    - $CG \subseteq CDG = \theta$ and $B \notin \theta$
  - With $BD \to A$ then $\theta = \theta \cup \{A\} = ABCDG$
    - $BD \subseteq BCDG = \theta$ and $A \notin \theta$

Question 9(b)

- Question: Compute CDG^+
- Steps: Using algorithm #1
  - $\theta = CDG$
  - With $CG \rightarrow B$ then $\theta = \theta \cup \{B\} = BCDG$
    - $CG \subseteq CDG = \theta$ and $B \notin \theta$
  - With $BD \rightarrow A$ then $\theta = \theta \cup \{A\} = ABCDG$
    - $BD \subseteq BCDG = \theta$ and $A \notin \theta$
  - With $ABC \rightarrow E$ then $\theta = \theta \cup \{E\} = ABCDEG$
    - $ABC \subseteq ABCDG = \theta$ and $E \notin \theta$

Question 9(b)

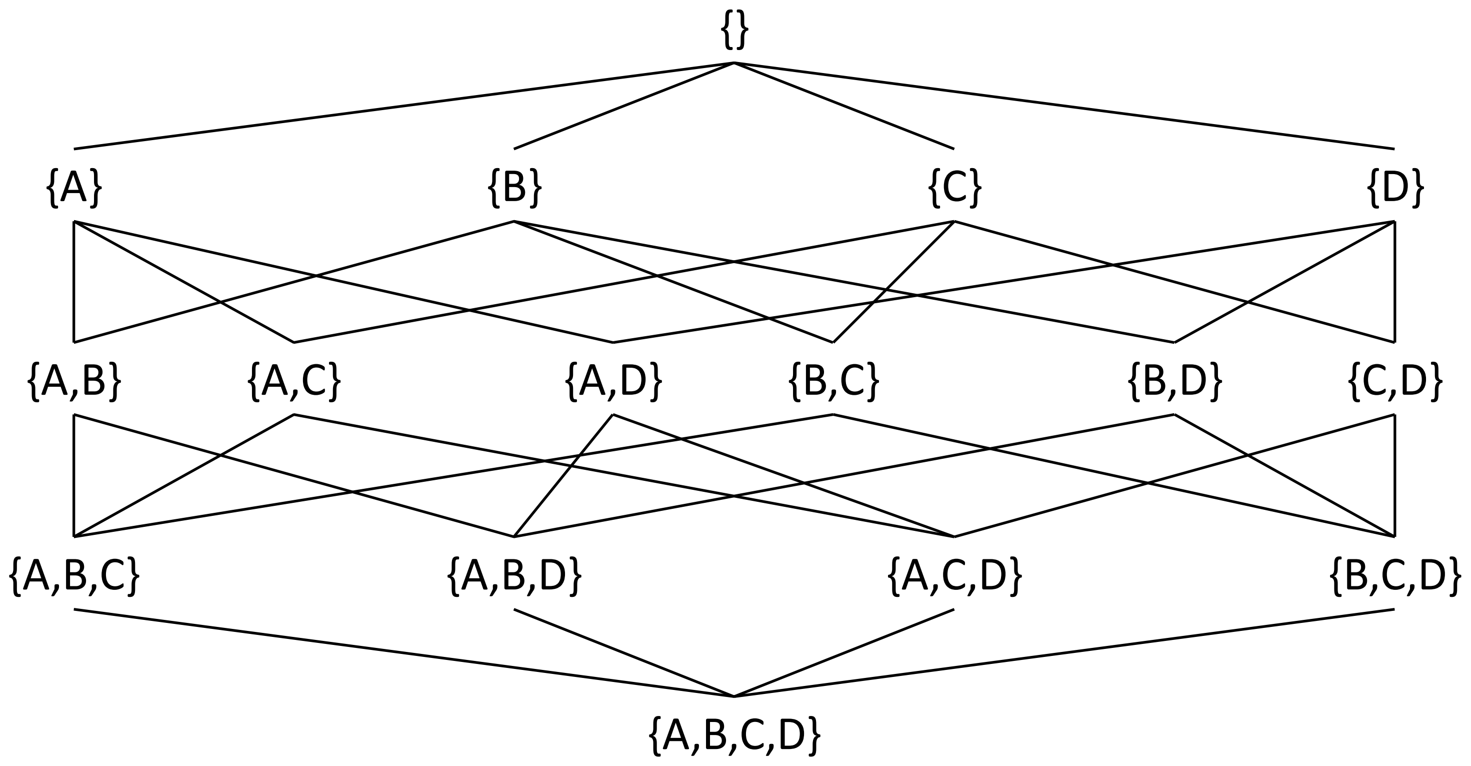
- Question: Compute CDG^+
- Steps: Using algorithm #1
  - $\theta = CDG$
  - With $CG \rightarrow B$ then $\theta = \theta \cup \{B\} = BCDG$
    - $CG \subseteq CDG = \theta$ and $B \notin \theta$
  - With $BD \rightarrow A$ then $\theta = \theta \cup \{A\} = ABCDG$
    - $BD \subseteq BCDG = \theta$ and $A \notin \theta$
  - With $ABC \rightarrow E$ then $\theta = \theta \cup \{E\} = ABCDEG$
    - $ABC \subseteq ABCDG = \theta$ and $E \notin \theta$
  - ➢ $CDG^+ = ABCDEG = R$

Question 9(b)

- Question: Compute CDG^+
- Steps: Using question 9(a)
  - Final step: $CDG \rightarrow ABC \rightarrow E$
  - We have
    - $CDG \rightarrow CDG$
    - $CDG \rightarrow ABC$
    - $CDG \rightarrow E$
  - By union: $CDG \rightarrow ABCDEG$

Question 9(c)

- Question: find all keys of R
- How?
  - Brute force method
    - Similar to T01 Q12A
    - But with 5 attributes
    - Find from smallest subset



Question 9(c)

- Question: find all keys of R
- How?
  - By reasoning
    - Since $CDG^+ = ABCDEG = R$
      - CDG is a superkey
    - Check if CDG is a key
      - Check CD^+; CG^+; DG^+ then determine that CDG is a key
    - Consider that CDG does not appear on the RHS of any FD
      - $(R - C)^+$ cannot contain C
      - $(R - D)^+$ cannot contain D
      - $(R - G)^+$ cannot contain G
    - CDG is the only key since every superkey must contain CDG

Question 10

- Question: find prime attributes of R
- How?
  - Brute force method
    - Find all keys of R
      - $AB \rightarrow R$
      - $AC \rightarrow R$
    - Union all attributes in keys of R
      - A, B, C

Question 10

- Question: find prime attributes of R
- How?
  - Smarter method
    - Note: A does not appear on any RHS $\Rightarrow$ A must be part of any superkey
    - This limits the search space to include A
      - One attribute
        - $A^+ = A$ $\Rightarrow$ A is not superkey
      - Two attributes
        - $AB^+ = R$ $\Rightarrow$ AB is a key
        - $AC^+ = R$ $\Rightarrow$ AC is a key
        - $AD^+ = AD$ $\Rightarrow$ AD is not a key
        - $AE^+ = AE$ $\Rightarrow$ AE is not a key

Question 10

- Question: find prime attributes of R
- How?
  - Smarter method
    - Note: A does not appear on any RHS $\Rightarrow A$ must be part of any superkey
    - This limits the search space to include A
      - Known keys: AB, AC
      - Three attributes: must include A but must exclude AB and AC
        - $ADE^+ = ADE \quad \Rightarrow ADE$ is not a key
        - No more 3 attributes that satisfies the condition
      - Four attributes: must include A but must exclude AB and AC
        - No 4 attributes that satisfies the condition
        - We can stop the search here

Question 11

- Question: find one minimal cover of F
- How?
  - Algorithm #2
  - Start with $G = \{AB \rightarrow CDE,\ \ AC \rightarrow BDE,\ B \rightarrow C,\ C \rightarrow B,\ C \rightarrow D,\ B \rightarrow E\}$
    - Consider $AB \rightarrow CDE$
      - Consider A
        - $A^+ = A$ $\quad\Rightarrow$ B is NOT redundant
      - Consider B
        - $B^+ = BCDE$ $\quad\Rightarrow B \rightarrow CDE$ is implied, A is redundant
  - Let $G = \{B \rightarrow CDE,\ \ AC \rightarrow BDE,\ B \rightarrow C,\ C \rightarrow B,\ C \rightarrow D,\ B \rightarrow E\}$

Question 11

- Question: find one minimal cover of F
- How?
  - Algorithm #2
  - Start with $G = \{B \to CDE,\ \ AC \to BDE,\ B \to C,\ C \to B,\ C \to D,\ B \to E\}$
    - Consider $AC \to BDE$
      - Consider A
        - $A^+ = A$ $\Rightarrow$ C is NOT redundant
      - Consider C
        - $C^+ = BCDE$ $\Rightarrow C \to CDE$ is implied, A is redundant
  - Let $G = \{B \to CDE,\ \ C \to BDE,\ B \to C,\ C \to B,\ C \to D,\ B \to E\}$

Question 11

- Question: find one minimal cover of F
- How?
  - Algorithm #2
  - Start with $G = \{B \to CDE,\ \ C \to BDE,\ B \to C,\ C \to B,\ C \to D,\ B \to E\}$
    - The rest of the FDs have single attribute for LHS
    - None of these can be redundant
  - $G = \{B \to CDE,\ \ C \to BDE,\ B \to C,\ C \to B,\ C \to D,\ B \to E\}$
    - Contains no redundant attribute in FD
    - Apply decomposition and remove duplicates
  - $G = \{B \to C,\ B \to D,\ B \to E,\ \ C \to B,\ C \to D,\ C \to E\}$

Question 11

- Question: find one minimal cover of F
- How?
  - Algorithm #2
  - Start with $G = \{B \to C,\ B \to D,\ B \to E,\ \ C \to B,\ C \to D,\ C \to E\}$
    - Consider $B \to C$
      - $B^+ = BDE$ w.r.t. $\{B \to D,\ B \to E,\ \ C \to B,\ C \to D,\ C \to E\}$
      - Not redundant
    - Consider $B \to D$
      - $B^+ = BCDE$ w.r.t. $\{B \to C,\ B \to E,\ \ C \to B,\ C \to D,\ C \to E\}$
      - Redundant
      - $G = \{B \to C,\ B \to E,\ \ C \to B,\ C \to D,\ C \to E\}$

Question 11

- Question: find one minimal cover of F
- How?
  - Algorithm #2
  - Start with $G = \{B \to C,\ B \to E,\ \ C \to B,\ C \to D,\ C \to E\}$
    - Consider $B \to E$
      - $B^+ = BCDE$ w.r.t. $\{B \to D,\ \ C \to B,\ C \to D,\ C \to E\}$
      - Redundant
      - $G = \{B \to C,\ \ C \to B,\ C \to D,\ C \to E\}$
    - Consider $C \to B$
      - $C^+ = CDE$ w.r.t. $\{B \to C,\ C \to D,\ C \to E\}$
      - Not redundant
      - $G = \{B \to C,\ \ C \to B,\ C \to D,\ C \to E\}$

Question 11

- Question: find one minimal cover of F
- How?
  - Algorithm #2
  - Start with $G = \{B \to C,\ \ C \to B,\ C \to D,\ C \to E\}$
    - Consider $C \to D$
      - $B^+ = BCE$ w.r.t. $\{B \to C,\ \ C \to B,\ C \to E\}$
      - Not redundant
      - $G = \{B \to C,\ \ C \to B,\ C \to D,\ C \to E\}$
    - Consider $C \to E$
      - $C^+ = BCD$ w.r.t. $\{B \to C,\ C \to B,\ C \to D\}$
      - Not redundant
      - $G = \{B \to C,\ \ C \to B,\ C \to D,\ C \to E\}$

Question 11

- Question: find one minimal cover of F
- How?
  - Algorithm #2
  - Ends with $G = \{B \rightarrow C,\ \ C \rightarrow B,\ C \rightarrow D,\ C \rightarrow E\}$
    - This is one possible minimal cover