CS2102 Database Systems

Semester 1 2019/2020

Tutorial 07 (*Selected Answers*)

Quiz

Questions 1-6 uses the following schema $R(A, B, C, D)$ with set of FDs $F = \{AB \rightarrow C, B \rightarrow A, C \rightarrow D\}$

1. Which of the following FDs are *logically implied* by F?
   a) $A \rightarrow D$
   b) $AB \rightarrow D$
   c) $B \rightarrow C$
   d) $B \rightarrow D$
   e) $B \rightarrow CD$
   f) $C \rightarrow A$
2. Which of the following set of FDs are equivalent to F?
   a) $G_1 = \{B \rightarrow A, B \rightarrow D, C \rightarrow D\}$
   b) $G_2 = \{B \rightarrow A, C \rightarrow D, B \rightarrow D, B \rightarrow C\}$
   c) $G_3 = \{B \rightarrow AC, C \rightarrow D\}$
3. Compute the attribute closure of B w.r.t. F
4. What is/are the key(s) of R w.r.t. F?
5. What are the prime attributes of R w.r.t. F?
6. Compute one minimal cover of F

Tutorial Questions *[Discussion: 7(a), 7(b), 8(a), 8(b), 9(ab), 10, 11]*

7. The more *extended* set of Armstrong's axioms has union and decomposition. Proof them only using Armstrong's axioms.
   a) Union if $a \rightarrow b$ and $a \rightarrow c$, then $a \rightarrow bc$
   b) Decomposition if $a \rightarrow bc$, then $a \rightarrow b$

Solution: [*the proof-assisted version*]

a) Proof

```
A->B [Given]
A->C [Given]
A->AB [Augmentation (1) with A]
AB->BC [Augmentation (2) with B]
A->BC [Transitivity (3) and (4)]
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b) Proof

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A->BC [Given]
BC->B [Reflexivity]
A->B [Transitivity (1) and (2)]
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8. The more *extended* set of Armstrong's axioms has two more rules in addition to union and decomposition. For each of the rules below, proof them using only Armstrong's axiom.
   a) Pseudo-transitivity if $a \rightarrow b$ and $bc \rightarrow d$, then $ac \rightarrow d$
   b) Composition if $a \rightarrow b$ and $c \rightarrow d$, then $ac \rightarrow bd$

Solution: [*the proof-assisted version*]

a) Proof

```
A->B [Given]
BC->D [Given]
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AC->BC [Augmentation (1) with C]
AC->D [Transitivity (3) and (2)]
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b) Proof

```
A->B [Given]
C->D [Given]
AC->BC [Augmentation (1) with C]
BC->BD [Augmentation (2) with B]
AC->BD [Transitivity (3) and (4)]
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9. Consider $R(A,B,C,D,E,G)$ with FDs $F = \{ABC \rightarrow E, BD \rightarrow A, CG \rightarrow B\}$.
   a) Use extended Armstrong's axioms to show that $F \vDash CDG \rightarrow E$
   b) Compute CDG^+
   c) Find all the keys of R

Solution: [*the proof-assisted version is given in the appendix*]

a) Proof

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ABC->E [Given]
BD->A [Given]
CG->B [Given]
CDG->BD [Augmentation (3) with D]
CDG->A [Transitivity (4) and (2)]
CDG->CG [Reflexivity]
CDG->B [Transitivity (6) and (3)]
CDG->BC [Augmentation (7) with C]
CDG->ABC [Union (5,8)]
CDG->E [Transitivity (9) and (1)]
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b) $CDG^+ = ABCDEG = R$

c) From (b), we know that CDG is a superkey of R. Since $DG^+ = DG$; $CG^+ = CGB$; and $CD^+ = CD$, we can conclude that CDG is a key of R.
Since each of C, D, or G does not appear on the RHS of any FD in F, it follows that $C \notin (R - C)^+$; $D \notin (R - D)^+$; and $G \notin (R - G)^+$.
Therefore, any key of R must be a superset of CDG. Since CDG is already a key of R, CDG is the only key of R.

10. Consider the schema $R(A,B,C,D,E)$ with FDs $F = \{AB \rightarrow CDE, AC \rightarrow BDE, B \rightarrow C, C \rightarrow B, C \rightarrow D, B \rightarrow E\}$. Find all the prime attributes of R. Show your working.

Solution:

Observe that attribute A does not appear in any RHS of any FDs in F. Therefore, A must appear in every key. By process of enumeration of all subset of R containing A, starting from the simplest:

- $A^+ = A$ — A is NOT a superkey
- $AB^+ = ABCDE$ — AB is a superkey
  - $B^+ = BCDE$ — B is NOT a superkey $\Rightarrow$ AB is a key
- $AC^+ = ABCDE$ — AC is a superkey
  - $C^+ = BCDE$ — C is NOT a superkey $\Rightarrow$ AC is a key

- $AD^+ = AD$ — AD is NOT a superkey
- $AE^+ = AE$ — AE is NOT a superkey
- $\mathrm{ADE}^+ = \mathrm{ADE}$ — ADE is NOT a superkey

The two keys of R are AB and AC. Hence, the prime attributes are A, B, and C.

11. Consider the schema $R(A, B, C, D, E)$ with FDs $F = \{AB \to CDE, AC \to BDE, B \to C, C \to B, C \to D, B \to E\}$. Find one minimal cover of F. Show your working.

Solution:

Start with $G = F = \{AB \to CDE,\ AC \to BDE,\ B \to C,\ C \to B,\ C \to D,\ B \to E\}$

In this example, we don't start working from $a \to A$ to simplify the notation. $a \to A$ can always be obtained by decomposition rule of extended Armstrong's axioms.

Remove redundant attributes:

- Consider $AB \to CDE$
  - $A^+ = A$ — So B is not a redundant attribute in FD
  - $B^+ = BCDE$ — So A is a redundant attribute in FD
    - Let $G = \{B \to CDE,\ AC \to BDE,\ B \to C,\ C \to B,\ C \to D,\ B \to E\}$
- Consider $AC \to BDE$
  - $A^+ = A$ — So C is not a redundant attribute in FD
  - $C^+ = BCDE$ — So A is a redundant attribute in FD
    - Let $G = \{B \to CDE,\ C \to BDE,\ B \to C,\ C \to B,\ C \to D,\ B \to E\}$
- No other FD need to be considered since all LHS are a single attribute now

Remove redundant FDs

- Let $G = \{B \to C,\ B \to D,\ B \to E,\ C \to B,\ C \to D,\ C \to E\}$
  - after applying decomposition rule and removing duplicates
- Consider $B \to C$
  - $B^+ = BDE$ w.r.t. $\{B \to D,\ B \to E,\ C \to B,\ C \to D,\ C \to E\}$
    - $B \to C$ is not redundant
  - Thus, $G = \{B \to C,\ B \to D,\ B \to E,\ C \to B,\ C \to D,\ C \to E\}$
- Consider $B \to D$
  - $B^+ = BCDE$ w.r.t. $\{B \to C,\ B \to E,\ C \to B,\ C \to D,\ C \to E\}$
    - $B \to D$ is redundant
  - Thus, $G = \{B \to C,\ B \to E,\ C \to B,\ C \to D,\ C \to E\}$
- Consider $B \to E$
  - $B^+ = BCDE$ w.r.t. $\{B \to C,\ C \to B,\ C \to D,\ C \to E\}$
    - $B \to E$ is redundant
  - Thus, $G = \{B \to C,\ C \to B,\ C \to D,\ C \to E\}$
- Consider $C \to B$
  - $C^+ = CDE$ w.r.t. $\{B \to C,\ C \to D,\ C \to E\}$
    - $C \to B$ is not redundant
  - Thus, $G = \{B \to C,\ C \to B,\ C \to D,\ C \to E\}$
- Consider $C \to D$
  - $C^+ = BCE$ w.r.t. $\{B \to C,\ C \to B,\ C \to E\}$
    - $C \to D$ is not redundant
  - Thus, $G = \{B \to C,\ C \to B,\ C \to D,\ C \to E\}$
- Consider $C \to E$

- $C^+ = BCD$ w.r.t. $\{B \rightarrow C,\ C \rightarrow B,\ C \rightarrow D\}$
  - $C \rightarrow E$ is not redundant
- Thus, $G = \{B \rightarrow C,\ C \rightarrow B,\ C \rightarrow D,\ C \rightarrow E\}$

Thus, $G = \{B \rightarrow C,\ C \rightarrow B,\ C \rightarrow D,\ C \rightarrow E\}$ is a minimal cover of F