## The answer to Problem III

1. The rule is not correct. For instance, {id} -> {name}, but {name} is not a subset of {id}.

2. Pseudo-transitivity

    a. Armstrong:

Assume that X -> Y (1), Z -> V (2), and Z (belongs) Y (3)
Since Z (belongs) Y (3) then Y -> Z (4), by reflexivity
Since X -> Y (1) and Y -> Z (4) then X -> Z (5), by transitivity
Since X -> Z (5) and Z -> V (2) then X -> V (QED), by transitivity

    b. Transitivity can be deduced from pseudo transitivity alone; therefore the Armstrong axioms in which transitivity is replaced by pseudo-transitivity are still complete.

3. F={ {A}->{B},{C}->{D}, {B,D}->{E}, {D}->{A,D}, {A,C}->{E,B} }

a. Empty instance or an instance with only one tuple.

b. (1,1,1,1,1) and (1,2,2,2,2).

c. ……

d. AB->A

e. A->B

f. AC->BC

g. C+ (0) = {C}
C+ (1) = {C, D} by using {C}->{D}
C+ (2) = {C, D, A} by using {D}->{A,D}
C+ (3) = {C, D, A, B} by using {A}->{B}
C+ (4) = {C, D, A, B, E} by using {B,D}->{E}

C+ = {C, D, A, B, E}, we can stop, we have every attribute. {C} is a superkey
There is no proper subset which is a superkey (only one proper subset -> and it is not a superkey), therefore {C} is a candidate key.
It is the only one. {C} is a primary key.

h. Minimal cover
1. Simplify the right-hand side
F'={ {A}->{B},{C}->{D}, {B,D}->{E}, {D}->{A}, {D}->{D}, {A,C}->{E} , {A,C}->{B} }
2. Simplify the left-hand side
F"={ {A}->{B},{C}->{D}, {D}->{E}, {D}->{A}, {D} ->{D}, {C}->{E} }
{A,C}->{B} can be removed because {A}->{B} is there (and {A} -> {A,B})
{B,D}->{E}, can be replaced by {D}->{E}, (because {D}->{A} and {A}->{B})
{A,C}->{E} can be replaced by {C}->{E}, (because {C}->{D} and {D}->{E})
3. Eliminate redundant rules

Min(F)={ {A}->{B},{C}->{D}, {D}->{E}, {D}->{A} }
{D} ->{D}, can be removed because it is trivial
{C}->{E} can be removed because it can obtained from {C}->{D}, {D}->{E}.