

Introduction
Relation: Set of tuples; Relational database scheme: set of schemas; Relational database: collection of tables
Integrity Constraints
Domain constraints, key constraints, foreign key constraints, other general constraints
Key Constraints
Superkey: subset of attributes in a relation that uniquely identifies its tuples
Key: a superkey that satisfies the additional property \rightarrow *not null & no proper subset of a key is a superkey*
 \rightarrow Minimal subset of attributes that uniquely identifies its tuples
 \rightarrow Can have multiple in a relation (candidate keys) but one is selected as the **primary key**
Foreign key: refers to the primary key of a second relation
 \rightarrow Each foreign key value in referencing relation must either *appear as primary key value* in referenced relation or be a null value
* Referencing and referenced relations could be the same relation

Relational Algebra
Unary operators (input: one relation)
Closure of relation: unary operator takes in a relation as input and gives a relation as output
Selection σ
Selects tuples from relation R that satisfies condition C
* won't affect columns
* may remove rows
* won't add rows
* won't reorder/rename columns
Projection π
Projects attributes given by a list L of attributes from relation R
* may remove columns, rows
* won't add columns, rows
* may reorder columns
* won't rename columns
* o/p is a set (no duplicates)
Renaming ρ
 $\rho(B_1, B_2 \dots B_n)(R)$ renames R($A_1 \dots A_n$) to S($B_1, B_2 \dots B_n$)
* won't add/remove rows, columns
* won't reorder columns
* may rename columns
Binary operators (input: two relations)
Closure of relation: binary operators takes in two relations as inputs and gives a relation as output
Cross-product \times
Returns a relation with schema (A, B, C, X, Y)
Union \cup
Returns a relation containing all tuples that occur in R, S or both
Intersection \cap
Returns a relation containing all tuples that occur in R & S
Set-difference $-$
Returns a relation containing all tuples that occur in R but not S

SQL

Null values
 \rightarrow Result of comparison operations involving NULL is UNKNOWN (eg. $\leq \geq =$)
 \rightarrow Result of arithmetic operations involving NULL is NULL (eg. $+$ / $-$ *)

x	y	x AND y	x OR y	NOT x
FALSE	FALSE	FALSE	FALSE	TRUE
FALSE	UNKNOWN	FALSE	UNKNOWN	TRUE
FALSE	TRUE	FALSE	TRUE	TRUE
UNKNOWN	FALSE	FALSE	UNKNOWN	UNKNOWN
UNKNOWN	UNKNOWN	UNKNOWN	UNKNOWN	UNKNOWN
UNKNOWN	TRUE	FALSE	TRUE	UNKNOWN
TRUE	FALSE	FALSE	TRUE	FALSE
TRUE	UNKNOWN	UNKNOWN	TRUE	FALSE
TRUE	TRUE	TRUE	TRUE	FALSE

x	y	x IS
NULL	NULL	DISTINCT
non-null	non-null	FROM y
non-null	non-null	TRUE
non-null	non-null	x <> y

* use IS NULL to check for NULL

$\pi_{\text{select-list}}(\sigma_{\text{condition}}(r_1 \times r_2 \times \dots \times r_n))$

\downarrow projection \downarrow selection \downarrow cross product

SELECT WHERE FROM

SQL Syntax
Create table
CREATE TABLE [IF NOT EXISTS] table_name ([
 column_name data_type
 column_constraints[...]
 table_constraints]
[...]
);

Drop table
DROP TABLE [IF EXISTS] table_name

Table modification
INSERT INTO table_name [[column]] VALUES ()
DELETE FROM table_name [WHERE ...]
UPDATE table_name SET ... [WHERE ...]

Alter table
ALTER TABLE table_name ALTER COLUMN column_name DROP DEFAULT;
ALTER TABLE table_name DROP COLUMN column_name;
ALTER TABLE table_name ADD COLUMN column_name column_type;

Constraints
 \rightarrow PRIMARY KEY
 \rightarrow REFERENCES ... [ON DELETE action] [ON UPDATE action]
 action \rightarrow NO ACTION / RESTRICT / CASCADE / SET DEFAULT / SET NULL
 \rightarrow NOT NULL
 \rightarrow UNIQUE
 \rightarrow CHECK
 \rightarrow DEFAULT

Query
SELECT [DISTINCT] select_list
FROM from_list
[WHERE condition]

Renaming Column
SELECT 'Price of ' || pizza || ' is ' || round(price/1.3) || ' USD' AS menu \rightarrow Price of Diavola is 18 USD

Pattern Matching
attr LIKE pattern
 \rightarrow underscore (_) : match any single character
 \rightarrow percent (%) : match sequence of 0 or more characters

Conditional Expressions
 \rightarrow CASE [expression]
 WHEN condition THEN result
 [WHEN ...]
 [ELSE result]
END
 \rightarrow NULLIF (result, 'absent')

\rightarrow COALESCE (arg1, arg2, ... argn)
Returns the first non-null value in its argument, & returns null if all null

Multi-Relation Queries
 \rightarrow Set operations ['ALL' preserves duplicate records]

- $Q_1 \cup Q_2$ Q_1 UNION Q_2 Q_1 UNION ALL Q_2
- $Q_1 \cap Q_2$ Q_1 INTERSECT Q_2 Q_1 INTERSECT ALL Q_2
- $Q_1 - Q_2$ Q_1 EXCEPT Q_2 Q_1 EXCEPT ALL Q_2

 \rightarrow Join

- Inner Join aka join *eliminates all with no match (null)
- Left Join *all rows/values from left table preserved
- Right Join *all rows/values from right table preserved
- Outer Join aka full join *gets all dangling tuples
- Natural Join can be put with left/right

Views (a virtual relation that can be used for querying)
CREATE VIEW view_name [(column1, column2, ...)] AS
Aggregate Functions (computes a single value from a set of tuples)
 \rightarrow MIN(_), MAX(_), AVG(_), SUM(_), COUNT(_)
* take note that COUNT(_) counts null values too, COUNT(*) is to count number of rows
 \rightarrow ORDER BY column1 [ASC | DESC]
 [, column2 [ASC | DESC] [...]]
 \rightarrow LIMIT[number | ALL] \rightarrow top n rows
 \rightarrow OFFSET number \rightarrow removes top n rows
 \rightarrow GROUP BY column1 [, column2 [...]]
Divides the rows into groups such that aggregate functions can be applied to each group
* In a query, two tuples belong to the same group if the values are NOT DISTINCT
Remember that 2 null values are non-distinct!
* For each column A in relation R that appears in SELECT, one of the following conditions must hold:
1. Column A appears in the GROUP BY clause
2. Column A appears in aggregated expression in SELECT
3. The primary/candidate key of R appears in the GROUP BY clause
 \rightarrow HAVING
Replaces "WHERE" for aggregated functions
Condition is same as GROUP BY but SELECT is replaced by HAVING

Subqueries (inner/nested queries)
* a tuple variable declared in a subquery/query Q can be used only in Q & any subquery nested in Q
* if a tuple variable is declared both locally as well as in an outer query, the local declaration applies
 \rightarrow Scalar subqueries return at most one tuple with one column
 \rightarrow Common Table Expressions (a temporary named result set that can be queried)
WITH
 cte1 AS (subquery1) [,
 cte2 AS (subquery2) [...]]
query
 \rightarrow Types of subqueries

- EXISTS Returns true if result subquery is non-empty
- IN Subquery must return exactly one column, else if empty, false
- ANY / SOME
- ALL

Universal Quantification
 $\rightarrow \forall f \Rightarrow \sim(\forall f) \Rightarrow \sim(\exists \sim f)$
SELECT _ FROM _
WHERE NOT EXISTS (SELECT _ FROM _
WHERE _
AND NOT EXISTS (_insert subquery_));

Entity Relationship Data Model
 \rightarrow Entity, Attribute, Entity Set, Relationship, Relationship Sets
Keys
 \rightarrow Each entity set has a key, attributes that form a primary key are underlined
Key Constraints

- Many-to-many ≥ 0
- One-to-many Each $S \leq 1$, R , each $R \geq 0$ S
- One-to-one Each $S \leq 1$, R , each $R \leq 1$ S
- N-ary

Participation Constraints

- Partial participation constraints
- Total participation constraints

* if \Rightarrow means = 1

Roles
 \rightarrow Used when one entity set appears ≥ 2 times in a relationship set

Weak Entity Sets
 \rightarrow An entity set that does not have its own key (i.e. its existence is dependent on owner entity's existence)
 \rightarrow Can only be uniquely identified by considering the primary key of another entity (i.e. identifying owner)

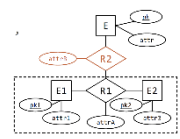
- Must be many-to-one relationship from WES to owner ES
- WES must have total participation in identifying relationship
- Partial key of a WES is a set of attributes of weak entity sets that uniquely identifies a weak entity for owner

IS-A Hierarchies
 \rightarrow Subclass-superclass relationship

- Overlap constraints: satisfied if entity in superclass could belong to multiple subclasses
- Covering constraints: satisfied if every entity in a superclass has to belong to some subclass

* typically not reflected in ER diagram

Aggregation
 \rightarrow When a relationship with corresponding entities is aggregated into a higher level entity
CREATE TABLE R2 (pk type REFERENCES E, pk1 type, pk2 type, attr type,
PRIMARY KEY (pk, pk1, pk2)
FOREIGN KEY (pk1, pk2) REFERENCES R1(pk1, pk2)
);



Stored Procedure, Functions
Transaction \rightarrow consists of one or more update/retrieval operations
BEGIN code { COMMIT | ROLLBACK } (must end with either)
 \rightarrow COMMIT success \rightarrow update database
 \rightarrow ROLLBACK failure \rightarrow restore database to state before BEGIN

ACID Properties

- Atomicity \rightarrow either all or none of the effects of the transactions are reflected in the DB
- Consistency \rightarrow user-defined property should be preserved (constraints)
- Isolation \rightarrow isolated other concurrent transaction executions, can run concurrently
- Durability \rightarrow commit is permanent

Constraint Check
By default, constraints are checked at the end of each SQL statement execution
 \rightarrow A violation will cause the statement to be rolledback which can be deferred to the end of the transaction
 \rightarrow Default: NOT DEFERRABLE / DEFERRABLE INITIALLY IMMEDIATE
 \rightarrow Wait till transaction completes before checking constraint: DEFERRABLE INITIALLY DEFERRED

Stored Procedures and Function
A procedure/function/subroutine that is available to applications that access a DBMS and is stored in the DB
Create Function
 \rightarrow Stored functions may have return types
CREATE [OR REPLACE] FUNCTION
func_name ([arg1 type1 [, arg2 type2 [, ...]]])
RETURNS ret_type AS func_def
LANGUAGE lang;
 \rightarrow Eg. CREATE OR REPLACE FUNCTION hello_world()
RETURNS CHAR(11) AS
\$\$ BEGIN RETURN 'Hello World'; END; \$\$
LANGUAGE plpgsql;

Create Procedure
 \rightarrow Stored procedures may not have return type (mainly deals with side-effects)
CREATE [OR REPLACE] PROCEDURE
proc_name ([arg1 type1 [, arg2 type2 [, ...]]])
AS proc_def
LANGUAGE lang;

Remove Function/Procedure
DROP { FUNCTION | PROCEDURE } [IF EXISTS] name;

Declaration (before BEGIN)
DECLARE var type;

(below are for PLPGSQL)
Assignment
var := expr;

Selection (IF statement)
IF cond THEN stmt;
[ELSIF cond THEN stmt [...]]
[ELSE stmt];

Iteration
 \rightarrow WHILE cond LOOP stmt; END LOOP;
 \rightarrow FOR var in (expr ...) LOOP
 { stmt; | EXIT; | EXIT WHEN cond; } [...]
END LOOP;
 \rightarrow LOOP { stmt; | EXIT; | EXIT WHEN cond; } [...]
END LOOP;

Operations
Arithmetic and bitwise
 \rightarrow Simple +, -, *, /, %, ^
 \rightarrow Bitwise &, |, # (xor), ~(not), <<, >>
 \rightarrow Others | / (square root), | / (cube root), @ (absolute value), ! (factorial postfix), !! (factorial prefix)
Comparison
 \rightarrow Simple <, >, <=, >=, =, <>

Functional Dependencies
Constraints on schemas that specify that the values for certain set of attributes determine unique values for another set of attributes (i.e. uniquely identifies)
Notations
We use R($A_1, A_2, \dots A_n$) to denote relation schema with n attributes
We use lowercase letter a, b, ... except r to denote subsets of attributes in R

- Let a, b \subseteq R and $A, A_1 \in R$
 - We use ab to denote a \cup b union
 - We use $A(A_1)$ to denote {A, A_1 } set
 - We use A.b to denote {A} \cup b union
 - We use b - A to denote b - {A} set diff

Definition
Let r be a relation instance of relation schema R
 \rightarrow r satisfies FD $a \rightarrow b$ if for every pair of tuples t_1 and t_2 in r such that $\pi_a(t_1) = \pi_a(t_2)$, it is also true that $\pi_b(t_1) = \pi_b(t_2)$
 \rightarrow an FD f holds on R if and only if for any relation instance r of R, r satisfies R
r is a legal instance of R if r satisfies all FDs that holds on R

Trivial vs Non-Trivial
For FD $a \rightarrow b$,
Trivial: b is a subset of a
Non-Trivial: b is not a subset of a

Completely Non-Trivial: a and b have completely different attributes

* completely non-trivial implies non-trivial

* an empty set is a subset of everything → trivial

Closure

- $F \models G$ if $F \models g$ for all $g \in G$
- The closure of F (F^+) is the set of all FDs implied by F

F is equivalent to G if $F^+ = G^+$, i.e. $F \models G, G \models F$ and $G \models F \rightarrow$ closures are the same

* Anything trivial is always true/is implied

Armstrong's Axioms

- Reflexivity if $b \subseteq a$ then $a \rightarrow b$
- Augmentation if $a \rightarrow b$ then $ac \rightarrow bc$
- Transitivity if $a \rightarrow b$ then $b \rightarrow c$ then $a \rightarrow c$
- Extension
 - Union if $a \rightarrow b$ then $a \rightarrow c$ then $a \rightarrow bc$
 - Decomposition if $a \rightarrow b$ then $a \rightarrow b'$ where $b' \subseteq b$
 - Specific case if $a \rightarrow bc$ then $a \rightarrow b$ and $a \rightarrow c$

Superkeys, keys and prime attributes

Superkey: A set of attributes α is a superkey of schema R (with FDs F) if $F \models \alpha \rightarrow R$

Prime Attributes: An attribute $A \in R$ is a prime attribute if A is contained in some key of R

Attribute Closure

Given a set of attribute α , other attributes that we can know is called *attribute closure* of α

- The closure of α (wrt F) is $\alpha^+ = \{A \in R \mid F \models \alpha \rightarrow A\}$

Algorithm 1 (get attribute closure)

Input A set of attributes $\alpha \subseteq R$ and a set of FDs F on R Example: let $F = \{A \rightarrow C, B \rightarrow C, CD \rightarrow E\}$
Output α^+ (wrt F) • Show that $F \models AD \rightarrow E$

1. initialize $\theta = \alpha$ 1. initialize $\Rightarrow \theta = AD$
2. while (there exists some FD $b \rightarrow c \in F$ such that $b \subseteq \theta$ and $c \not\subseteq \theta$)
 - 2. with $A \rightarrow C \Rightarrow \theta = ACD$
 - 3. with $CD \rightarrow E \Rightarrow \theta = ACDE$
 - 4. therefore $AD^+ = ACDE$
 - thus $F \models AD \rightarrow E$
3. $\theta = \theta \cup c$
4. return θ

Minimal Covers

Some FDs are *redundant* → can be removed.

Smallest set of FDs is called minimal cover (may have ≥ 1 unique minimal covers)

A minimal cover:

- Every FD is of the form $a \rightarrow A$ (single attribute on the right)
- For each FD $a \rightarrow A$ in G , a has no redundant attributes
- There are no redundant FDs in G
- G and F are equivalent

* Each set of FDs has at least one minimal cover (trivially, it is itself)

1. Given an FD $a \rightarrow b$, an attribute $A \in a$ is a redundant attribute in FD if:

- $(F - \{a \rightarrow b\}) \cup \{(a - A) \rightarrow b\}$ is equivalent to F
- i.e. having $(a - A) \rightarrow b$ instead of $a \rightarrow b$ does not change F^+

2. Given an FD $f \in F$, f is a redundant FD if

- $F - f$ is equivalent to F

Algorithm 2 (get a minimal cover)

Input A set of FDs F
Output A minimal cover for F

1. initialize $G = \emptyset$
2. for each $(FD \ a \rightarrow B_1 \dots B_n \text{ in } F)$
3. $G = G \cup \{a \rightarrow B_i \mid i \in [1, n]\}$
4. for each $(FD \ a \rightarrow B \text{ in } G)$
5. initialize $a' = a$
6. for each $(A \in a)$ do
7. if $(B \text{ in } (a' - A)^+ \text{ w.r.t. } G)$ then
8. replace $a' \rightarrow B$ in G by $(a' - A) \rightarrow B$
9. $a' = a' - A$
10. for each $(FD \ a \rightarrow B \text{ in } G)$
11. if $(B \text{ in } a^+ \text{ w.r.t. } G - \{a \rightarrow B\})$ then
12. remove $a \rightarrow B$ from G
13. return G

- Example: $F = \{ABCD \rightarrow E, E \rightarrow D, A \rightarrow B, AC \rightarrow D\}$

- Find a minimal cover of F

- Steps

➢ Decompose FDs already decomposed

➢ Remove redundant attributes

start with $G = \{ABCD \rightarrow E, E \rightarrow D, A \rightarrow B, AC \rightarrow D\}$

1. A in $ABCD \rightarrow E$ is non-redundant $BCD^+ = BCD \text{ w.t. } G$

2. B in $ABCD \rightarrow E$ is redundant $ACD^+ = ABCDE \text{ w.t. } G$

* $G = \{ACD \rightarrow E, E \rightarrow D, A \rightarrow B, AC \rightarrow D\}$

3. C in $ABCD \rightarrow E$ is non-redundant $AD^+ = ABD \text{ w.t. } G$

4. D in $ABCD \rightarrow E$ is redundant $AC^+ = ABCDE \text{ w.t. } G$

* $G = \{AC \rightarrow E, E \rightarrow D, A \rightarrow B, AC \rightarrow D\}$

5. A in $AC \rightarrow D$ is non-redundant $C^+ = C \text{ w.t. } G$

6. C in $AC \rightarrow D$ is non-redundant $A^+ = AB \text{ w.t. } G$

➢ Remove redundant FDs:

start with $G = \{AC \rightarrow E, E \rightarrow D, A \rightarrow B, AC \rightarrow D\}$

1. $AC \rightarrow E$ is non-redundant $AC^+ = ABCD \text{ w.t. } G - \{AC \rightarrow E\}$

2. $E \rightarrow D$ is non-redundant $E^+ = E \text{ w.t. } G - \{E \rightarrow D\}$

3. $A \rightarrow B$ is non-redundant $A^+ = A \text{ w.t. } G - \{A \rightarrow B\}$

4. $AC \rightarrow D$ is redundant $AC^+ = ABCDE \text{ w.t. } G - \{AC \rightarrow D\}$

➢ Minimal cover is $G = \{AC \rightarrow E, E \rightarrow D, A \rightarrow B\}$

Decomposition

The decomposition of schema R is a set of schemas $\{R_1, R_2, \dots, R_n\}$ (called fragments) such that

- $R_i \subseteq R$ for each R_i (each fragment is simpler than the original schema)
- * need not be a proper subset

- $R = R_1 \cup R_2 \cup \dots \cup R_n$ (no attributes are missing)

Lossless-join Decomposition

$\{R_1, R_2, \dots, R_n\}$ is a lossless-join decomposition wrt F if no information is lost by performing a join

Lemma 1

If $\{R_1, R_2, \dots, R_n\}$ is a decomposition of R , then for any relation r or R

A natural join of the decomposition of R will lead to a superset of R

Lossy-join decomposition will produce more tuples! → gain data, lose information

Theorem 1

The decomposition of R with FDs F into $\{R_1, R_2\}$ is a lossless-join decomposition wrt F if

- $F \models R_1 \cap R_2 \rightarrow R_1$
- OR
- $F \models R_1 \cap R_2 \rightarrow R_2$

i.e. when $R_1 \cap R_2$ is a superkey for either R_1 or R_2

Corollary 1

If $a \rightarrow b$ is a completely non-trivial FD that holds on R , then the decomposition of R into $\{R - b, ab\}$ is a lossless-join decomposition

- Since $(R - b) \cap ab = a$ and $a \rightarrow b$ so $a \rightarrow ab$

Theorem 2

If $\{R_1, R_2, \dots, R_n\}$ is a lossless-join decomposition of R , and $\{R_{1,1}, R_{1,2}\}$ is a lossless-join decomposition of R_1 , then

$\{R_{1,1}, R_{1,2}, R_2, \dots, R_n\}$ is a lossless-join decomposition of R

* only works on splits into 2

To check for lossless-join, must check for lossless-join of all the combinations of the decompositions. If any of them are not lossless-join, can conclude it is not lossless-join.

Dependency-preserving Decomposition

Only use attributes that appear in the decomposition

Decomposition $\{R_1, R_2, \dots, R_n\}$ of R is dependency-preserving if

- $(F_{R1} \cup F_{R2} \cup \dots \cup F_{Rn})$ is equivalent to F
 - $(F_{R1} \cup F_{R2} \cup \dots \cup F_{Rn}) \models F$
 - $(F_{R1} \cup F_{R2} \cup \dots \cup F_{Rn})^+ \models F^+$
 - $(F_{R1} \cup F_{R2} \cup \dots \cup F_{Rn}) \models F^+ \models F$
 - $(F_{R1} \cup F_{R2} \cup \dots \cup F_{Rn}) \models F^+ \models F \models (F_{R1} \cup F_{R2} \cup \dots \cup F_{Rn})$
- Guarantees that for each update to a decomposed relation, FD violations can be detected w/o computing joins

Algorithm 3 (find FDs of a decomposition)

Input A set of attributes $\alpha \subseteq R$ and a set of FDs F on R
Output F_α Example: Let $R(A, B, C)$ with FDs $F = \{A \rightarrow B, B \rightarrow C, C \rightarrow B\}$

- Compute F_{AC}
- initialize $\Rightarrow \theta = \emptyset$
- let $b = B, b^+ = BC \Rightarrow \theta = \{B \rightarrow BC\}$
- let $b = C, C^+ = BC \Rightarrow \theta = \{B \rightarrow BC, C \rightarrow BC\}$
- let $b = BC, BC^+ = BC \Rightarrow \theta = \{B \rightarrow BC, C \rightarrow BC, BC \rightarrow BC\}$

1. Initialize $\theta = \alpha$
2. for each $(b \subseteq \alpha$ such that $b \neq \emptyset$)
3. $\theta = \theta \cup \{b \rightarrow (b^+ \cap \alpha)\}$ // w.r.t F
4. return θ

Lemma 2

For every decomposition $\{R_1, R_2, \dots, R_n\}$ of R ,

- $F \models (F_{R1} \cup F_{R2} \cup \dots \cup F_{Rn})$
- By definition, $F_\alpha = \{b \rightarrow c \in F^+ \mid b \subseteq \alpha \subseteq R\}$
- For all $b \rightarrow c$ in F_α , we also have $b \rightarrow c$ in F^+

Hence, we only need to check if $(F_{R1} \cup F_{R2} \cup \dots \cup F_{Rn}) \models F^+$

Algorithm 4 (check if decomposition is dependency-preserving)

Input A decomposition $\{R_1, \dots, R_n\}$ of R with FDs F

Output YES (if dependency-preserving) or NO (otherwise)

1. for each $(R_i \in \{R_1, \dots, R_n\})$
2. compute F_{R_i}
3. let $G = F_{R_1} \cup \dots \cup F_{R_n}$
4. for each $(FD \ a \rightarrow b \in F)$
5. compute a^+ w.r.t. G
6. if $(b \not\subseteq a^+)$ then return NO
7. return YES

Normal Forms

Restricts the set of data dependencies that are allowed to hold on a schema to avoid certain undesirable

redundancy and update problems in database

Boyce-Codd normal form (BCNF)

R is in BCNF if for every FD $a \rightarrow A$ in F either

1. $a \rightarrow A$ is trivial ($a \in A$) OR
2. a is a superkey of R

R violates BCNF if both are not satisfied

A decomposition of R is in BCNF if all of its decompositions is in BCNF

- if every decomposed element is a single element, i.e. fulfills BCNF, then it fulfills BCNF by default (everything is trivial). BUT it might not be lossless-join/dependency-preserving

Checking if a relation schema R_i is in BCNF, check if there exists some non-trivial FD f which holds on R_i that violates BCNF

1. If F is the set of FDs that hold on R_i (i.e. R_i is not from a decomposition)
- check for any violating non-trivial FD in F
- * if there are no non-trivial FD, R_i is in BCNF
2. If F is the set of FDs that hold on R , and R_i is a decomposed relation schema of R
- check for any violating non-trivial FD in F_α (check if any FD in the **projection** of R_i violates BCNF too)

Lemma 3

For any relation schema R with exactly two attributes, R is in BCNF

Algorithm 5 (checks if a schema violates BCNF)

Input F is a set of FDs that hold on schema R

and R_i is either R or a decomposed schema of R

Output A completely non-trivial FD that violates BCNF

if R_i is not in BCNF; otherwise *null*

- if $(R_i$ has exactly 2 attributes)
- return null
- for each $(a \subseteq R$ such that $a \neq \emptyset)$
- let $X = a^+ \cap R_i$ w.r.t. F // compute $a \rightarrow X$
- if $(a \subset X \subset R_i)$
- return $a \rightarrow (X - a)$
- return null

$a \subset X$ implies non-trivial (since trivial FD means $a = X$)

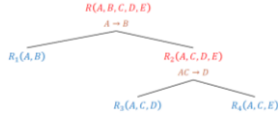
$X \subset R_i$ implies non-superkey (superkey means $X = R_i$)

Algorithm 6 (lossless-join, non dependency-preserving BCNF decomposition)

- Input Schema R with FDs F

- Output A lossless BCNF decomposition of R

1. initialize $\delta = \emptyset; i = 1; \theta = \{R\}$
2. while $(\theta \neq \emptyset)$
3. remove some R' from θ
4. let $f = \text{Algorithm5}(F, R')$
5. if $(f = \text{null})$ then $\delta = \delta \cup \{R'\}$ // in BCNF
6. else
 - 7. let f be $a \rightarrow b$ // completely non-trivial
 - 8. let $c = R' - b$
 - 9. $\theta = \theta \cup \{R_i(ab), R_{i+1}(c)\}$ // decompose
 - 10. $i = i + 2$
11. return δ



At the worst-case, each fragment will have exactly 2 attributes

3rd normal form (3NF)

R is in 3NF if for every FD $a \rightarrow A$ in F either

1. $a \rightarrow A$ is trivial ($a \in A$) OR
2. a is a superkey of R OR
3. A is a prime attribute

R violates 3NF if all 3 are not satisfied

A decomposition of R is in 3NF if all of its decompositions is in 3NF

Every decomposition in BCNF is definitely in 3NF

Algorithm 7 (lossless-join, dependency-preserving decompositions in 3NF)

Minimal cover input as it guarantees all FD to have no redundancy → all FDs are completely non-trivial

- Input Schema R with FDs F which is a *minimal cover*

- Output A lossless and dependency-preserving 3NF decomposition of R

1. initialize $\delta = \emptyset$
2. apply union rule to combine FDs in F
3. let $G = \{f_1, f_2, \dots, f_n\}$ be the resultant set of FDs
4. for each $(FD \ f_i \text{ of the form } a_i \rightarrow b_i \text{ in } G)$
5. create a relation schema $R_i(a_i, b_i)$ for FD f_i
6. insert $R_i(a_i, b_i)$ into δ
7. choose a key K of R and insert $R_{n+1}(K)$ into δ
8. remove redundant relation schema from δ
9. \Rightarrow delete R_i from δ if $\exists R_j \in \delta : i \neq j \wedge R_i \subseteq R_j$
10. return δ

Since minimal cover is non-deterministic, 3NF is non-deterministic as well

* but always try to start with BCNF first and if can't, then use 3NF