In the Lecture Series Introduction to Database Systems

Functional Dependencies

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Anomalies: Example

Assume that the position determines the salary:
position $\rightarrow$ salary

company

eNumber	firstName	lastName	address	depart-ment	position	salary
1XU3	Dewi	Srijaya	12a Jln Lempeng	Toys	Clerk	2000
4W3E	Izabel	Leong	10 Outram Park	Sports	Trainee	1200
3XXE	John	Smith	107 Clementi Rd	Toys	Clerk	2000
5SD2	Axel	Bayer	55 Cuscaden Rd	Sports	Trainee	1200
6RG5	Winnie	Lee	10 West Coast Rd	Sports	Manager	2500
755Y	Sylvia	Tok	22 East Coast Ln	Toys	Manager	2600
2SD3	Eric	Wei	100 Jurong drive	Toys	Assistant manager	2200
?	?	?	?	?	Security guard	1500

key

key

Redundant storage

Update anomaly

Insertion anomaly

Potential deletion anomaly

Normalization: Example

employee

eNumber	firstName	lastName	address	depart-ment	position
1XU3	Dewi	Srijaya	12a Jln Lempeng	Toys	Clerk
4W3E	Izabel	Leong	10 Outram Park	Sports	Trainee
3XXE	John	Smith	107 Clementi Rd	Toys	Clerk
5SD2	Axel	Bayer	55 Cuscaden Rd	Sports	Trainee
6RG5	Winnie	Lee	10 West Coast Rd	Sports	Manager
755Y	Sylvia	Tok	22 East Coast Ln	Toys	Manager
2SD3	Eric	Wei	100 Jurong drive	Toys	Assistant manager

key key

- Redundant storage?
  - NO
- Update anomaly?
  - NO
- Deletion anomaly?
  - NO
- Insertion anomaly?
  - NO

salary

Position	salary
Clerk	2000
Trainee	1200
Manager	2500
Assistant manager	2200
Security guard	1500

key

Learning Objectives

- Definitions
- Reasoning (Armstrong's axioms)
- Closure and Equivalence
- Minimal Cover

Functional Dependencies

For a relation scheme R, a functional dependency from a set S of attribute of R to a set T of attribute of R exists if and only if:

For every instance of |R| of R, if two tuples in |R| agree on the values of the attributes in S, then they agree on the values of the attributes in T.

We write: $S \rightarrow T$

Functional Dependencies

company(eNumber, firstName, lastName, address, department, position, salary)

{position} $\rightarrow$ {salary}

If two tuples in the relation employee have the same value for the attribute position then they must have the same value for the salary attribute.

Functional Dependencies

company(eNumber, firstName, lastName, address, department, position, salary)

$\{position\} \rightarrow \{salary\}$

$\forall X1\ \forall X2\ \forall X3\ \forall X4\ \forall X5\ \forall X6\ \forall X7\ \forall X8\ \forall X9\ \forall X10\ \forall P\ \forall S1\ \forall S2$

$((company(X1, X2, X3, X4, X5, \mathbf{P}, S1)$

$\wedge\ company(X6, X7, X8, X9, X10, \mathbf{P}, S2))$

$\Rightarrow\ (\mathbf{S1 = S2}))$

$\forall E1\ \forall E2$

$(E1 \in company \wedge E2 \in company \wedge \mathbf{E1.position = E2.position}$

$\Rightarrow\ (\mathbf{E1.salary = E2.salary})$

Functional Dependencies

company(eNumber, firstName, lastName, address, department, position, salary)

{position} $\rightarrow$ {salary}

```
CHECK ( NOT EXISTS (
SELECT *
FROM company c1, company c2
WHERE c1.position=c2.position AND c1.salary <> c2.salary))
```

Functional Dependencies

employee(eNumber, firstName, lastName, address, department, position)

salary(position, salary)

$\{position\} \rightarrow \{salary\}$

In employee: FOREIGN KEY (position) REFERENCES salary(position)

In salary: PRIMARY KEY (position)

Trivial FDs

$\boldsymbol{X} \rightarrow Y$

$\boldsymbol{Y} \subset X$

{firstName, address} $\rightarrow$ {firstName}

Non-Trivial FDs

$\boldsymbol{X} \rightarrow \mathrm{Y}$

$\boldsymbol{Y} \not\subset \mathrm{X}$

{eNumber} $\rightarrow$ {address}

{firstName, lastName} $\rightarrow$ {firstName, address}

Completely Non-Trivial FDs

$\boldsymbol{X} \rightarrow \mathrm{Y}$

$\boldsymbol{Y} \cap \mathrm{X} = \varnothing$

{firstName, lastName} $\rightarrow$ {address}

Functional Dependencies

company(eNumber, firstName, lastName, address, department, position, salary)

{eNumber} $\rightarrow$ {firstName, lastName, address, department, position, salary}

{firstName, lastName} $\rightarrow$ {eNumber, address, department, position, salary}

If two tuples in the relation employee relation have the same first name and last name then they must be the same tuple (no duplicate)

Superkeys

A set of attributes whose knowledge determines the value of the entire tuple is a **superkey**

company(eNumber, firstName, lastName, address, department, position, salary)

{firstName, lastName}

{eNumber}

{firstName, lastName, address}

{eNumber, address}

Candidate Keys

A minimal (for inclusion) set of attributes whose knowledge determines the value of the entire tuple is a **candidate key**

company(eNumber, firstName, lastName, address, department, position, salary)

{firstName, lastName}

{eNumber}

Primary Keys

*The designer chooses one candidate key to be the **primary key***

Reasoning about Functional Dependencies

It is sometimes possible to infer new functional dependencies from a set of given functional dependencies

(independently from any particular instance of the relation scheme or of any additional knowledge)

Reasoning about Functional Dependencies

For example:

From

$\{eNumber\} \rightarrow \{firstName\}$

and

$\{eNumber\} \rightarrow \{lastName\}$

We can infer

$\{eNumber\} \rightarrow \{firstName, lastName\}$

Armstrong's Axioms

- Be X, Y, Z be subsets of the relation scheme of a relation R
- **Reflexivity**:
  If $Y \subset X$, then $X \to Y$
- **Augmentation**:
  If $X \to Y$, then $X \cup Z \to Y \cup Z$
- **Transitivity**:
  If $X \to Y$ and $Y \to Z$, then $X \to Z$

Armstrong's Axioms

employee(eNumber, firstName, lastName, address, department, position, salary)

Reflexivity: If $Y \subset X$, then $X \rightarrow Y$

If {firstName} $\subset$ {firstName, lastName},

Then {firstName, lastName} $\rightarrow$ {firstName}

Armstrong's Axioms

employee(eNumber, firstName, lastName, address, department, position, salary)

Augmentation: If $X \rightarrow Y$, then $X \cup Z \rightarrow Y \cup Z$

If {position} $\rightarrow$ {salary},

then {position, eNumber} $\rightarrow$ {salary, eNumber}

Armstrong's Axioms

employee(eNumber, firstName, lastName, address, department, position, salary)

Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

If {eNumber} $\rightarrow$ {position}
and {position} $\rightarrow$ {salary},
Then {eNumber} $\rightarrow$ {salary}

Armstrong's Axioms

Consider the scheme {name, room, tel} with the set of functional dependencies:

$$\{\{room\} \rightarrow \{tel\}, \{tel\} \rightarrow \{name\}\}$$

We can deduce that the following functional dependency holds:

$$\{room\} \rightarrow \{name\}$$

Proof:

1. Let R= {name, room, tel}
2. Let {room} → {tel} be a functional dependency on R
3. Let {tel} → {name} be a functional dependency on R
4. Therefore {room} → {name} holds on R by Transitivity of (2) and (3)

Q.E.D.

Armstrong's Axioms

Weak-Augmentation

Let X, Y, Z be subsets of the relation R

$$\text{If } X \rightarrow Y \text{ , then } X \cup Z \rightarrow Y$$

Proof

1. Let R be a relation scheme
2. Let $X \rightarrow Y$ be a functional dependency on R
3. Therefore $X \cup Z \rightarrow Y \cup Z$ by Augmentation of (2) with Z
4. We know that $Y \cup Z \rightarrow Y$ by Reflexivity because $Y \subset Y \cup Z$
5. Therefore $X \cup Z \rightarrow Y$ by Transitivity of (3) and (4)

Q.E.D.

Armstrong's Axioms

Armstrong's axioms are sound

Armstrong's axioms are complete

Closure of a Set of Functional Dependencies

For a set F of functional dependencies, we call the closure of F, noted F+, the set of all the functional dependencies that F entails

Armstrong's Axioms

Armstrong's axioms are complete

F+ can be computed by applying the Armstrong Axioms in all possible ways

Closure of a Set of Functional Dependencies

Consider the relation scheme R(A,B,C,D)

- F = {{A} →{B},{B,C} →{D}}
- F+ = {{A} →{A}, {B}→{B}, {C}→{C}, {D}→{D}, […], {A}→{B}, {A,B}→{B}, {A,D}→{B,D}, {A,C}→{B,C}, {A,C,D}→{B,C,D}, {{A} →{A,B}, {A,B}→{A,B}, {A,D}→{A,B,D}, {A,C}→{A,B,C}, {A,C,D}→{A,B,C,D}, {B,C} →{D}, […], {A,C} →{D}, […]}

Equivalence of Sets of Functional Dependencies

Two sets of functional dependencies F and G are equivalent if and only if

$$F^+ = G^+$$

Finding Keys: Example

Example: Consider the relation scheme R(A,B,C,D)

with functional dependencies:

$$\{A\} \rightarrow \{C\} \text{ and } \{B\} \rightarrow \{D\}.$$

Is {A,B} a candidate key?

Finding Keys: Example

Example: {A,B} is a superkey.

Proof

1. We know that $\{A\} \rightarrow \{C\}$
2. Therefore $\{A,B\} \rightarrow \{A,B,C\}$, by augmentation of (1) with{A,B}
3. We know that $\{B\} \rightarrow \{D\}$
4. Therefore $\{A,B,C\} \rightarrow \{A,B,C,D\}$, by augmentation of (3) with {A, B, C}
5. Therefore $\{A,B\} \rightarrow \{A,B,C,D\}$ by transitivity of (2) and (4)

Q.E.D

Finding Keys: Example

Example: {A,B} is a candidate key (minimal)

We must show that neither {A} nor {B} alone are candidate keys

This can be done by producing counter example relation instance verifying the functional dependencies given but neither $\{A\} \rightarrow \{A,B,C,D\}$ nor $\{B\} \rightarrow \{A,B,C,D\}$

We will however learn an algorithm to do otherwise

Attribute Closure

For a set A of attributes, we call the **closure** of A *(with respect to a set of functional dependencies F)*, noted A+, the maximum set of attributes such that $A \rightarrow A+$ *(as a consequence of F)*

Closure of a Set of Attributes: Example

Consider the relation scheme R(A,B,C,D) with functional dependencies

$$\{A\}\rightarrow\{C\} \text{ and } \{B\}\rightarrow\{D\}.$$

- $\{A\}+ = \{A,C\}$
- $\{B\}+ = \{B,D\}$
- $\{A,B\}+ = \{A,B,C,D\}$

Closure of a Set of Attributes: Algorithm

- Input:
  - R a relation scheme
  - F a set of functional dependencies
  - $X \subset R$
- Output:
  - X+ the closure of X w.r.t. F

Closure of a Set of Attributes: Algorithm

- $X^{(0)} := X$
- Repeat
  - $X^{(i+1)} := X^{(i)} \cup A$, where A is the union of the sets Z of attributes such that there exist $Y \rightarrow Z$ in F, and $Y \subset X^{(i)}$
- Until $X^{(i+1)} := X^{(i)}$
- Return $X^{(i+1)}$

Closure of a Set of Attributes: Example

$R = \{A,B,C,D,E,G\}$

$F = \{ \{A,B\}\rightarrow\{C\}, \{C\}\rightarrow\{A\}, \{B,C\}\rightarrow\{D\},$
$\{A,C,D\}\rightarrow\{B\}, \{D\}\rightarrow\{E,G\}, \{B,E\}\rightarrow\{C\},$
$\{C,G\}\rightarrow\{B,D\}, \{C,E\}\rightarrow\{A,G\}\}$

$X = \{B,D\}$

Closure of a Set of Attributes: Example

$R = \{A,B,C,D,E,G\}$

$F = \{ \{A,B\}\rightarrow\{C\}, \{C\}\rightarrow\{A\}, \{B,C\}\rightarrow\{D\}, \{A,C,D\}\rightarrow\{B\}, \{D\}\rightarrow\{E,G\},$
$\{B,E\}\rightarrow\{C\}, \{C,G\}\rightarrow\{B,D\}, \{C,E\}\rightarrow\{A,G\}\}$

$X = \{B,D\}$

- $X^{(0)} = \{B,D\}$
- $\{D\}\rightarrow\{E,G\}$
- $X^{(1)} = \{B,D,E,G\}$
- $\{B,E\}\rightarrow\{C\}$,
- $X^{(2)} = \{B,C,D,E,G\}$
- $\{C,E\}\rightarrow\{A,G\}$
- $X^{(3)} = X^{(4)} = X+ = \{A,B,C,D,E,G\}$

Testing Equivalence Based on Attribute Closures

- Let R be a relational scheme
- Let F and G be two sets of functional dependencies on R

- for each (X→Y) in F
  - compute $X^{+(G)}$
  - if $Y \not\subset X^{+(G)}$ return false

- for each (X→Y) in G
  - Compute $X^{+(F)}$
  - if $Y \not\subset X^{+(F)}$ return false

- return true

Example:
If F contains {A} → {B,C}
but $\{A\}^{+(G)}$ → {A,B},

then {A}→{C} is entailed by F
but not by G,

therefore F and G are not equivalent

Equivalence of Sets of Functional Dependencies

Every set F of functional dependencies is equivalent to a set of functional dependencies $Y \rightarrow Z$ such that Z is a singleton, i.e. every right-hand side has a single attribute

Example: $\{D\} \rightarrow \{E,G\}$ is equivalent to $\{D\} \rightarrow \{E\}$ and $\{D\} \rightarrow \{G\}$

Minimal Set of Dependencies

A set of dependencies F is minimal if and only if:

1. Every right-hand side is a single attribute
2. For no functional dependency $X \rightarrow A$ in F and proper subset Z of X is $F - \{X \rightarrow A\} \cup \{Z \rightarrow A\}$ equivalent to F
3. For no functional dependency $X \rightarrow A$ in F is the set $F - \{X \rightarrow A\}$ equivalent to F

Minimal Cover

A set of functional dependencies F is a **minimal cover** of a set of functional dependencies G if and only if

- F is minimal
- F is equivalent to G

- (an **extended minimal cover** is obtained by undoing step 1 on a minimal cover)

Minimal Cover

- Every set of functional dependencies has a minimal cover
- There might be several different minimal cover of the same set

Minimal Cover: Example

$F = \{ \{A,B\}\rightarrow\{C\}, \{C\}\rightarrow\{A\}, \{B,C\}\rightarrow\{D\}, \{A,C,D\}\rightarrow\{B\}, \{D\}\rightarrow\{E,G\}, \{B,E\}\rightarrow\{C\}, \{C,G\}\rightarrow\{B,D\}, \{C,E\}\rightarrow\{A,G\}\}$

A set of dependencies F is minimal if and only if:

1. Every right-hand side is a single attribute
2. For no functional dependency $X\rightarrow A$ in F and proper subset Z of X is $F - \{X\rightarrow A\} \cup \{Z\rightarrow A\}$ equivalent to F
3. For no functional dependency $X \rightarrow A$ in F is the set $F - \{X\rightarrow A\}$ equivalent to F

Minimal Cover: Example, Step (1)

F = { {A,B}→{C}, {C}→{A}, {B,C}→{D}, {A,C,D}→{B}, <u>{D}→{E,G}</u>, {B,E}→{C}, <u>{C,G}→{B,D}</u>, <u>{C,E}→{A,G}</u>}

F' = { {A,B}→{C}, {C}→{A}, {B,C}→{D}, {A,C,D}→{B}, <u>{D}→{G}</u>, <u>{D}→{E}</u>, {B,E}→{C}, <u>{C,G}→{B}</u>, <u>{C,G}→{D}</u>, <u>{C,E}→{A}</u>, <u>{C,E}→{G}</u>}

A set of dependencies F is minimal if and only if:

1. Every right-hand side is a single attribute.
   (Therefore, we should break down every dependency with more than one attribute on the right-hand side.)

Minimal Cover: Example, Step (2)

F' = {{C}→{A}, {C,E}→{A}, {A,C,D}→{B},
{C,G}→{B}, {A,B}→{C}, {B,E}→{C}, {B,C}→{D},
{C,G}→{D}, {D}→{E}, {C,E}→{G}, {D}→{G}}

Since {D}→{G}, we have {C,D}→{C,G}
Since {C,G}→{B}, we have {C,D}→{B}
Therefore, A in {A,C,D}→{B} is redundant

F'' = {{C}→{A}, {C,D}→{B}, {C,G}→{B}, {A,B}→{C},
{B,E}→{C}, {B,C}→{D}, {C,G}→{D}, {D}→{E},
{C,E}→{G}, {D}→{G}}

2. For no functional dependency X→A in F and proper subset Z of X is F – {X→A} ∪ {Z→A} equivalent to F.

(Therefore, we should remove redundant attributes from the left-hand side of the dependencies.)

Minimal Cover: Example, Step (3)

F'' = {{C}→{A}, {C,D}→{B}, <u>{C,G}→{B}</u>, {A,B}→{C}, {B,E}→{C}, {B,C}→{D}, {C,G}→{D}, {D}→{E}, {C,E}→{G}, {D}→{G}}

Since {C,G}→{D}, we have {C,G}→{C,D}
Since {C,D}→{B}, we have {C,G}→{B}
Therefore, {C,G}→{B} is redundant

F''' = {{C}→{A}, {C,D}→{B}, {A,B}→{C}, {B,E}→{C}, {B,C}→{D}, {C,G}→{D}, {D}→{E}, {C,E}→{G}, {D}→{G}}

3. For no functional dependency X →A in F is the set F – {X→A} equivalent to F.
   (Therefore, we should remove redundant dependencies.)

Extended Minimal Cover: Example

F''' = {{C}→{A}, {C,D}→{B}, {A,B}→{C}, {B,E}→{C}, {B,C}→{D}, {C,G}→{D}, {D}→{E}, {C,E}→{G}, {D}→{G}}

F'''' = {{C}→{A}, {C,D}→{B}, {A,B}→{C}, {B,E}→{C}, {B,C}→{D}, {C,G}→{D}, {D}→{E,G}, {C,E}→{G}}

(an **extended minimal cover** is obtained by undoing step 1 on a minimal cover)

Minimal Cover: Algorithm

We can apply steps (1), (2), (3) iteratively in various orders

However only (1) + (2) + (3) is guaranteed to lead to a minimal cover!:

- Put functional dependencies in single attribute rhs form
- Minimize left side of each functional dependency
- Delete redundant functional dependencies

Credits

The content of this lecture is based on chapter 8 of the book "Introduction to database Systems"
By
S. Bressan and B. Catania,
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