CS2102 Database Systems

Semester 1 2019/2020

Tutorial 08 (*Selected Answers*)

Quiz

1. Given schema $R(A, B, C, D)$ with FDs $F = \{A \rightarrow BCD,\ C \rightarrow D\}$, find all the completely non-trivial FDs in the following FD projections:
   a) F_{ABC}
   b) F_{CD}
   c) F_{AC}
   d) F_{ABD}
   e) F_{BCD}
   f) F_{AB}
2. Given schema $R(A, B, C, D)$ with FDs $F = \{A \rightarrow BCD,\ C \rightarrow D\}$, determine whether or not the following decompositions are lossless-join decomposition.
   a) Decomposition $\{R_1(A, B, C),\ R_2(C, D)\}$
   b) Decomposition $\{R_1(A, C),\ R_2(A, B, D)\}$
   c) Decomposition $\{R_1(B, C, D),\ R_2(A, B)\}$
3. Given schema $R(A, B, C, D)$ with FDs $F = \{A \rightarrow BCD,\ C \rightarrow D\}$, determine whether or not the following decompositions are dependency-preserving decomposition.
   a) Decomposition $\{R_1(A, B, C),\ R_2(C, D)\}$
   b) Decomposition $\{R_1(A, C),\ R_2(A, B, D)\}$
   c) Decomposition $\{R_1(B, C, D),\ R_2(A, B)\}$
4. Is there a dependency-preserving decomposition that is not a lossless-join decomposition? If yes, give an example. If no, explain.

Tutorial Questions *[Discussion: 5(ab), 5(cd), 5(ef), 6(ab), 6(cd), 6(ef), 7(a), 7(b)]*

5. Given schema $R(A, B, C, D, E)$ with FDs $F = \{AB \rightarrow C,\ AC \rightarrow D,\ E \rightarrow ABCD\}$ and decomposition $\delta = \{R_1(A, B, C),\ R_2(A, B, E),\ R_3(A, C, D)\}$.
   a) Is δ a lossless-join decomposition? Explain.
   b) Is δ a dependency-preserving decomposition? Explain.
   c) Is R in BCNF? Explain.
   d) Is δ in BCNF? Explain.
   e) Is R in 3NF? Explain.
   f) Is δ in 3NF? Explain.

Solution:

a) Consider $R_I(A, B, C, E)$.
   The decomposition of R into $\{R_3(A, C, D), R_I(A, B, C, E)\}$ is a lossless-join decomposition because $(R_3 \cap R_I) \rightarrow R_3$[1].
   The decomposition of R_I into $\{R_1(A, B, C), R_2(A, B, E)\}$ is a lossless-join decomposition because $(R_1 \cap R_2) \rightarrow R_1$[2].
   Therefore, the decomposition of R into δ is a lossless-join decomposition by Theorem 2.
b) In this question, we are only interested in the *union minimal cover of the projection*.
   Compute $F_{R_1} = \{AB \rightarrow C\}$. Compute $F_{R_2} = \{E \rightarrow AB\}$. Compute $F_{R_3} = \{AC \rightarrow D\}$.
   Let $G = F_{R_1} \cup F_{R_2} \cup F_{R_3} = \{AB \rightarrow C,\ E \rightarrow AB,\ AC \rightarrow D\}$. We need to proof $G \vDash E \rightarrow CD$.
   Compute E^+ w.r.t. G and we have $E^+ = ABCDE$. Therefore $G \vDash E \rightarrow CD$ and thus, $G \vDash F$.
   This is a dependency-preserving decomposition.

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[1] $R_3 \cap R_I = AC$ and $AC^+ = ACD$ w.r.t. R_3 so $AC \rightarrow ACD$ and AC is the superkey of R_3
[2] $R_1 \cap R_2 = AB$ and $AB^+ = ABC$ w.r.t. R_1 so $AB \rightarrow ABC$ and AB is the superkey of R_1

c) Consider $ABC \rightarrow D$. (1) $ABC \rightarrow D$ is non-trivial, (2) ABC is not a superkey of R[3]. Therefore, R is not in BCNF.

d) Consider the union minimal cover of projection computed in (b).
Consider R_1 with F_{R_1}. Consider $AB \rightarrow C$, we have AB as superkey of R_1. Thus, R_1 is in BCNF.
Consider R_2 with F_{R_2}. Consider $E \rightarrow AB$, we have E as superkey of R_2. Thus, R_2 is in BCNF.
Consider R_3 with F_{R_3}. Consider $AC \rightarrow D$, we have AC as superkey of R_3. Thus, R_3 is in BCNF.
Therefore, δ is in BCNF.

e) Consider $ABC \rightarrow D$. (1) $ABC \rightarrow D$ is non-trivial, (2) ABC is not a superkey of R, (3) D is not a prime attribute of R[4]. Therefore, R is not in 3NF.

f) Consider the union minimal cover of projection computed in (b).
Consider R_1 with F_{R_1}. Consider $AB \rightarrow C$, we have AB as superkey of R_1. Thus, R_1 is in 3NF.
Consider R_2 with F_{R_2}. Consider $E \rightarrow AB$, we have E as superkey of R_2. Thus, R_2 is in 3NF.
Consider R_3 with F_{R_3}. Consider $AC \rightarrow D$, we have AC as superkey of R_3. Thus, R_3 is in 3NF.
Therefore, δ is in 3NF[5].

6. Given schema $R(A, B, C, D, E)$ with FDs $F = \{A \rightarrow E,\ AB \rightarrow D,\ CD \rightarrow AE,\ E \rightarrow B,\ E \rightarrow D\}$ and decomposition $\delta = \{R_1(B, D, E),\ R_2(A, C, E)\}$.
   a) Is δ a lossless-join decomposition? Explain.
   b) Is δ a dependency-preserving decomposition? Explain.
   c) Is R in BCNF? Explain.
   d) Is δ in BCNF? Explain.
   e) Is R in 3NF? Explain.
   f) Is δ in 3NF? Explain.

Solution:

a) The decomposition of R into δ is a lossless-join decomposition because $(R_1 \cap R_2) \rightarrow R_1$[6].

b) In this question, we are only interested in the *union minimal cover of the projection*.
Compute $F_{R_1} = \{E \rightarrow BD\}$. Compute $F_{R_2} = \{A \rightarrow E,\ CE \rightarrow A\}$.
Let $G = F_{R_1} \cup F_{R_2} = \{E \rightarrow BD,\ A \rightarrow E,\ CE \rightarrow A\}$. We consider $G \vDash CD \rightarrow AE$.
Compute CD^+ w.r.t. G and we have $CD^+ = CD$. Therefore $G \nvDash CD \rightarrow AE$ and thus, $G \nvDash F$.
This is not a dependency-preserving decomposition

c) Consider $A \rightarrow E$. (1) $A \rightarrow E$ is non-trivial, (2) A is not a superkey of R[7]. Therefore, R is not in BCNF.

d) Consider the union minimal cover of projection computed in (b).
Consider R_2 with F_{R_2}. Consider $A \rightarrow E$, we have $A^+ = AE$ w.r.t. F_{R_2}. Therefore, A is not the superkey of R_2. Thus, R_2 is not in BCNF.
Therefore, δ is not in BCNF.

e) Consider $E \rightarrow B$. (1) $E \rightarrow B$ is non-trivial, (2) E is not a superkey of R[8], (3) B is not a prime attribute of R[9]. Therefore, R is not in 3NF.

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[3] Since $ABC^+ = ABCD \subset R$
[4] The key of R is only E
[5] Or simply, since δ is in BCNF, therefore δ is also in 3NF
[6] $R_1 \cap R_2 = E$ and $E^+ = ABE$ w.r.t. R_1 so $E \rightarrow ABE$ and E is the superkey of R_1
[7] Since $A^+ = ABDE \subset R$
[8] Since $E^+ = BDE \subset R$
[9] The key of R are $\{\{A, C\}, \{C, D\}, \{C, E\}\}$

f) Consider the union minimal cover of projection computed in (b).
Consider R_1 with F_{R_1}. Consider $E \rightarrow BD$, we have E as superkey of R_1. Thus, R_1 is in 3NF.
Consider R_2 with F_{R_2}. Consider $CE \rightarrow A$, we have CE as superkey of R_2.
Consider R_2 with F_{R_2}. Consider $A \rightarrow E$, we have E as a prime attribute of R_2[10].
Therefore, δ is in 3NF.

7. Given schema $R(A, B, C, D, E)$ with FDs $F = \{AB \rightarrow CDE,\ AC \rightarrow BDE,\ B \rightarrow D,\ C \rightarrow B,\ C \rightarrow D,\ B \rightarrow E\}$.
   a) Find a lossless-join BCNF decomposition of R. Is your BCNF decomposition dependency-preserving?
   b) Find a lossless-join and dependency-preserving 3NF decomposition of R.

Solution:

a) There are many ways to decompose R into a lossless-join BCNF decomposition. The following is the step that is used step-by-step by Algorithm 6.

Let $\theta = \{R(A, B, C, D, E)\}$ and $\delta = \{\ \}$
$B \rightarrow DE$ violates BCNF of R (proof omitted)
  Decompose R into $R_1(B, D, E)$ and $R_2(A, B, C)$
  Then $\theta = \{R_1(B, D, E), R_2(A, B, D)\}$ and $\delta = \{\ \}$
R_1 is in BCNF (proof omitted)
  Then $\theta = \{R_2(A, B, D)\}$ and $\delta = \{R_1(B, D, E)\}$
$C \rightarrow B$ violates BCNF of R_2 (proof omitted)
  Decompose into $R_3(B, C)$ and $R_4(A, C)$
  Then $\theta = \{R_3(B, C), R_4(A, C)\}$ and $\delta = \{R_1(B, D, E)\}$
R_3 and R_4 are in BCNF by Lemma 3
  Then $\theta = \{\ \}$ and $\delta = \{R_1(B, D, E), R_3(B, C), R_4(A, C)\}$
Therefore, one lossless-join BCNF decomposition of R is $\delta = \{R_1(B, D, E),\ R_3(B, C),\ R_4(A, C)\}$.

The decomposition δ is not a dependency-preserving decomposition because
$G = F_{R_1} \cup F_{R_3} \cup F_{R_4} = \{B \rightarrow DE,\ C \rightarrow B\}$ and $G \not\models AB \rightarrow C$ (to name one).

b) Using minimal cover $G = \{B \rightarrow D,\ AB \rightarrow C,\ C \rightarrow B,\ B \rightarrow E\}$ (step omitted, use Algorithm 2). The following is the step that used step-by-step by Algorithm 7.

Union of minimal cover $G = \{B \rightarrow DE,\ AB \rightarrow C,\ C \rightarrow B\}$
With $B \rightarrow DE$, create $R_1(B, D, E)$
With $AB \rightarrow C$, create $R_2(A, B, C)$
With $C \rightarrow B$, create $R_3(B, C)$
With key $\{A, B\}$, create $R_4(A, B)$
Remove R_3 as it is redundant because $R_3 \subset R_2$.
Remove R_4 as it is redundant because $R_4 \subset R_2$.
Therefore, one lossless-join and dependency-preserving 3NF decomposition of R is $\delta = \{R_1(B, D, E),\ R_2(A, B, C)\}$

[10] The key of R_2 are $\{\{A, C\}, \{C, E\}\}$