CS2102 Database Systems

2013/2014 Semester I

Tutorial #5 Functional Dependencies

1. Is the following rule correct?

   $\forall X \in R\ \forall Y \in R\ (\text{if } X \rightarrow Y \text{ then } Y \subseteq X)$

The rule is not correct.
For instance, {id} -> {name}, but {name} is not a subset of {id}.

2. The following rule is called pseudo-transitivity. Use Armstrong axioms to prove it.

   $\forall X \in R\ \forall Y \in R\ \forall Z \in R\ \forall W \in R\ (\text{if } X \rightarrow Y \text{ and } Z \rightarrow W \text{ and } Z \subseteq Y, \text{ then } X \rightarrow W)$

Assume that $X \rightarrow Y$ (1), $Z \rightarrow W$ (2), and $Z \subseteq Y$ (3)
Since $Z \subseteq Y$ (3), then $Y \rightarrow Z$ (4) (reflexivity)
Since $X \rightarrow Y$ (1) and $Y \rightarrow Z$ (4), then $X \rightarrow Z$ (5) (transitivity)
Since $X \rightarrow Z$ (5) and $Z \rightarrow W$ (2) then $X \rightarrow W$ (transitivity)

3. Consider the set of functional dependencies:

$F = \{ \{A\} \rightarrow \{B\}, \{C\} \rightarrow \{D\}, \{B, D\} \rightarrow \{E\}, \{D\} \rightarrow \{A, D\}, \{A, C\} \rightarrow \{E, B\} \}$ on the relation scheme $R = \{A, B, C, D, E\}$.

   a. Give an example instance of R that complies with the functional dependencies.

Empty instance or an instance with only one tuple.

   b. Give an example instance of R that violates the functional dependencies.

(1,1,1,1,1) and (1,2,2,2,2).

   c. Compute F^+, the closure of F.

   d. Give an example of a trivial functional dependency in F^+.

AB->A

   e. Give an example of a non-trivial functional dependency in F^+.

A->B

f. Compute $\{C\}^+$, the closure of the set of attributes {C}.

C+ (0) = {C}
C+ (1) = {C, D} by using {C}->{D}
C+ (2) = {C, D, A} by using {D}->{A,D}
C+ (3) = {C, D, A, B} by using {A}->{B}
C+ (4) = {C, D, A, B, E} by using {B,D}->{E}
C+ = {C, D, A, B, E}, we can stop, we have every attribute. {C} is a superkey
There is no proper subset which is a superkey (only one proper subset -> and it is not a superkey), therefore {C} is a candidate key.
It is the only one. {C} is a primary key.

g. Compute a minimal cover of F.

1. Simplify the right-hand side
F'={ {A}->{B},{C}->{D}, {B,D}->{E}, {D}->{A}, {D}->{D}, {A,C}->{E} , {A,C}->{B} }

2. Simplify the left-hand side
F''={ {A}->{B},{C}->{D}, {D}->{E}, {D}->{A}, {D} ->{D}, {C}->{E} }
{A,C}->{B} can be removed because {A}->{B} is there (and {A} -> {A,B})
{B,D}->{E}, can be replaced by {D}->{E}, (because {D}->{A} and {A}->{B})
{A,C}->{E} can be replaced by {C}->{E}, (because {C}->{D} and {D}->{E})

3. Eliminate redundant rules
Min(F)={ {A}->{B},{C}->{D}, {D}->{E}, {D}->{A} }
{D} ->{D}, can be removed because it is trivial
{C}->{E} can be removed because it can obtained from {C}->{D}, {D}->{E}.