

# CS2102

# Database Systems

*Slides adapted from Prof. Chan Chee Yong*

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LECTURE 02

INTRODUCTION

# Relational data model

## History

- Introduced by **Edgar Codd** of IBM Research Lab in 1970
- Data is modeled using **relations**
- Relations are simply tables with rows & columns

<i>studentID</i>	<i>name</i>	<i>birthDate</i>	<i>cap</i>
3118	Alice	1999-12-25	3.8
1423	Bob	2000-05-27	4.0
5609	Carol	1999-06-11	4.3

(3118, 'Alice', 1998-12-25, 3.8) is NOT a student  
but contains all information  
needed to model the student

- **Definitions**
  - **Degree/Arity** : *number of columns*
  - **Cardinality** : *number of rows*

# Relational data model

## Relation schema

- Each relation has a definition called a **relation schema**
- Schema specifies **attributes** and **data constraints**
- Data constraints include **domain constraints**
  - Students (*studentID*: integer, *name*: string, *birthDate*: date, *cap* : numeric)
- Each row in a relation is called a **tuple/record**
- It has one **component** for each attribute of relation
  - Example: (1423, “Bob”, 2000-05-07, 4.0)

<i>studentID</i>	<i>name</i>	<i>birthDate</i>	<i>cap</i>
3118	Alice	1999-12-25	3.8
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Tuple is like set, but position matters.  
Set of {A,B,C} is equivalent to set of {B,C,A}, but tuple (A,B,C) is not equivalent to tuple (B,C,A)

- Relational data model
  - Integrity constraints
    - Key constraints
    - Foreign key constraints
  - Relational algebra
    - Unary operators
    - Binary operators
    - Closure properties
- 

## Overview

- Relational data model
  - Integrity constraints
    - Key constraints
    - Foreign key constraints
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    - Unary operators
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## Relational data model

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# Relational data model

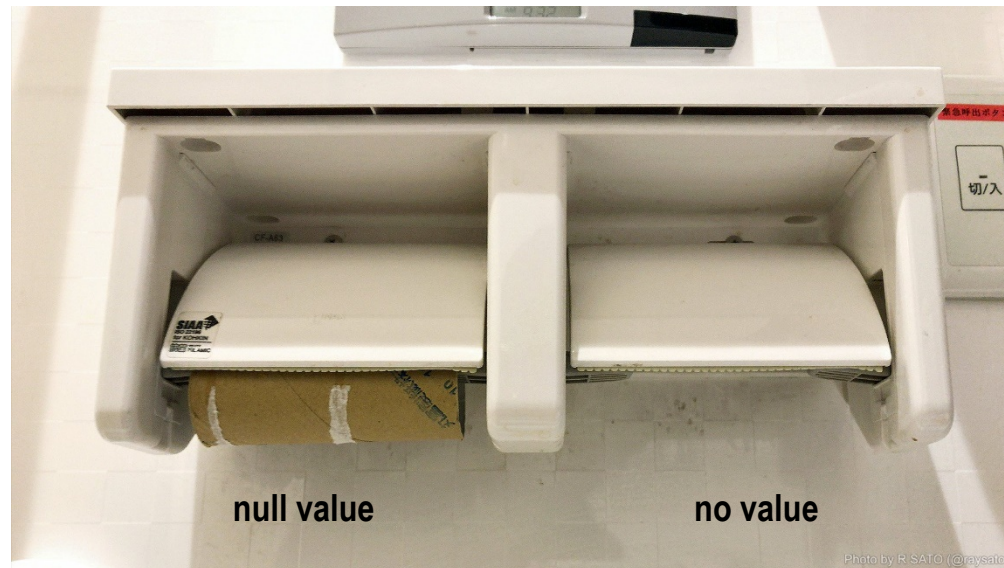
## Domain

- **Domain** is defined as a set of atomic values
  - Examples: integer, numeric, string
  - The special value **null** is a member of each domain
  - A **null** value means value is either not applicable or unknown

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(image: reddit)

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**HOW A 'NULL' LICENSE PLATE  
LANDED ONE HACKER IN TICKET  
HELL**



(source: <https://www.wired.com/story/null-license-plate-landed-one-hacker-ticket-hell/>)



# Relational data model

## Relations

- A **relation** is defined as a set of tuples
- Consider a relation schema  $R(A_1, A_2, \dots, A_n)$  with  $n$  attributes  $A_1, A_2, \dots, A_n$
- Example
  - Students (*studentID*: integer, *name*: string, *birthDate*: date, *cap* : numeric)
  - $R$
  - $A_1$
  - $A_2$
  - $A_3$
  - $A_4$

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  - $R$     Students
  - $A_1$    studentID
  - $A_2$    name
  - $A_3$    birthDate
  - $A_4$    cap

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  - $D_1$
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  - $D_2$  string
  - $D_3$  date
  - $D_4$  numeric

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a set

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a set

consisting of tuples



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of  $n$  elements



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where each element

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a set consisting of tuples of  $n$  elements where each element is within domain



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  - Example
    - Students (***studentID***: integer, ***name***: string, ***birthDate***: date, ***cap*** : numeric)
    - $\{(1423, \text{"Bob"}, 2000-05-27, 4.0), (3118, \text{"Alice"}, 1999-12-25, 3.8)\}$

# Relational data model

## Relational database schema

- A **relational database schema** consists of a set of schemas
- Example:
  - Students (*studentID*: integer, *name*: string, *birthDate*: date, *cap* : numeric)
  - Courses (*courseID*: integer, *name*: string, *credits* : integer)
  - Enrolls (*sID*: integer, *grade*: numeric, *cID*: integer)
- ❖ *Relational database schema*  
= relational schemas + data constraints

# Relational data model

## Relational database

- A **relational database** is a collection of tables

- Example:

- Students

<i>studentID</i>	<i>name</i>	<i>birthDate</i>	<i>cap</i>
3118	Alice	1999-12-25	3.8
1423	Bob	2000-05-27	4.0
5609	Carol	1999-06-11	4.3

- Courses

<i>courseID</i>	<i>name</i>	<i>credits</i>
101	Programming in C	3.8
112	Discrete Mathematics	4.0
311	Database Systems	4.3

- Enrolls

<i>sID</i>	<i>cID</i>	<i>grade</i>
3118	101	3.0
3118	112	4.0
1423	311	4.5

- Relational data model
  - Integrity constraints
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- 

## Integrity constraints

# Integrity constraints (ICs)

## Definitions

- **Integrity constraint**
    - A condition that restricts the data that can be stored in database instance
      - ICs are specified when schema is defined
      - ICs are checked when relations are updated
  - **Legal relation instance**
    - A relation that satisfies all specified ICs
- ❖ A DBMS enforces ICs
- ❖ Allow only legal instances to be stored



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# Integrity constraints (ICs)

## Types

- Domain constraints
  - Restrict attribute values of relations
- Key constraints
- Foreign key constraints
- Other general constraints

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# Key constraints

## Superkey

- A **superkey** is a subset of attributes in a relation that uniquely identifies its tuples
- No two distinct tuples of relation have the same values in all attributes of superkey

# Key constraints

## Superkey

- A **superkey** is a subset of attributes in a relation that uniquely identifies its tuples
  - No two distinct tuples of relation have the same values in all attributes of superkey
- Example
  - Which of the following could be a superkey of the table on the right?

<i>sID</i>	<i>cID</i>	<i>grade</i>
3118	101	3.0
3118	112	4.0
5609	112	1.0
1423	311	4.5

1. sID
2. cID
3. grade
4. (sID, cID)
5. (sID, grade)
6. (cID, grade)
7. (sID, cID, grade)

# Key constraints

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# Key constraints

## Key

- A **key** is a superkey that satisfies the additional property
  - Not null & no proper subset of a key is a superkey
  - Minimal subset of attributes that uniquely identifies its tuples

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# Key constraints

## Notation

- We indicate a key with an arrow
- Example
  - Students (*studentID*: integer, *name*: string, *birthDate*: date, *cap* : numeric)
  - If studentID is a key, then we write
    - studentID → (studentID, name, birthDate, cap)
    - OR
    - studentID -> (studentID, name, birthDate, cap)
    - if there's no arrow symbol



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    - if there's no arrow symbol
- More generally
  - LHS → RHS
  - If every unique value of LHS is associated with exactly one value of RHS
    - *Example:* the same studentID cannot belong to different name
    - *Therefore:* studentID → name

# Key constraints

## Properties

- A relation can have multiple keys
  - These are called **candidate keys**
  - One of the candidate keys is then selected as the **primary keys**
- We denote primary key with underline

## Example

- Students (studentID, name, birthDate, cap)
  - studentID is the primary key, hence it is underlined
- Enrolls (sID, cID, grade)
  - sID and cID are the primary keys, hence they are underlined

# Foreign key constraints

## Foreign key

- A subset of attributes in a relation is a **foreign key** if it refers to the primary key of a second relation

- Example

<i>studentID</i>	<i>name</i>	<i>birthDate</i>	<i>cap</i>
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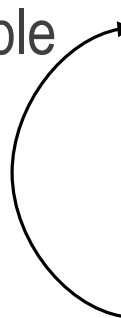
Courses

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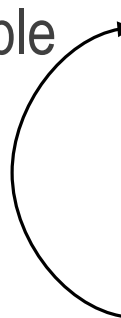
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**referenced relation**

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Courses

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# Foreign key constraints

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- A subset of attributes in a relation is a **foreign key** if it refers to the primary key of a second relation
- **Foreign key constraints**
  - Each foreign key value in **referencing relation** must either
    - Appear as primary key value in **referenced relation**, or
    - Be a **null** value
  - Referencing & referenced relations could be the same relation
  - Also called **referential integrity constraints**



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  - Referencing & referenced relations could be the same relation
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- Relational data model
  - Integrity constraints
    - Key constraints
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  - Relational algebra
    - Unary operators
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## Integrity constraints

# Relational algebra

## What is it?

- A **formal language** for asking queries on relations

## Basic

- A query is composed of a collection of operators
  - **Relational operators**
- Each operator takes one/two relations as input and computes an output relation
- Basic relational algebra operators
  - **Unary operators** (*input: one relation*)
    - Selection  $\sigma$ ; Projection  $\pi$ ; Renaming  $\rho$
  - **Binary operators** (*input: two relations*)
    - Cross-product  $\times$ ; Union  $\cup$ ; Intersection  $\cap$ ; Difference  $-$

# Relational algebra

## Example database

- Consider a database consisting of the following 6 relations
  - Pizzas (pizza)
  - Contains (pizza, ingredient)
  - Restaurants (rname, area)
  - Sells (rname, pizza, price)
  - Customers (cname, area)
  - Likes (cname, pizza)
- Foreign key constraints:
  - Contains.pizza is referencing Pizzas.pizza
  - Sells.rname is referencing Restaurants.rname
  - Sells.pizza is referencing Pizzas.pizza
  - Likes.cname is referencing Customers.cname
  - Likes.pizza is referencing Pizzas.pizza

# Relational algebra

## Example database

Pizzas

<u>pizza</u>
Diavola
Funghi
Hawaiian
Margherita
Marinara
Siciliana

Customers

<u>cname</u>	<u>area</u>
Homer	West
Lisa	South
Maggie	East
Moe	Central
Ralph	Central
Willie	North

Restaurants

<u>rname</u>	<u>area</u>
Corleone Corner	North
Gambino Oven	Central
Lorenzo Tavern	Central
Mamma's Place	South
Pizza King	East

Contains

<u>pizza</u>	<u>ingredient</u>
Diavola	Cheese
Diavola	Chilli
Diavola	Salami
Funghi	Ham
Funghi	Mushroom
Hawaiian	Ham
Hawaiian	Pineapple
Margherita	Cheese
Margherita	Tomato
Marinara	Seafood
Siciliana	Anchovies
Siciliana	Capers
Siciliana	Cheese

Likes

<u>cname</u>	<u>pizza</u>
Homer	Hawaiian
Homer	Margherita
Lisa	Funghi
Maggie	Funghi
Moe	Funghi
Moe	Siciliana
Ralph	Diavola

Sells

<u>rname</u>	<u>pizza</u>	<u>price</u>
Corleone Corner	Diavola	24
Corleone Corner	Hawaiian	25
Corleone Corner	Margherita	19
Gambino Oven	Siciliana	16
Lorenzo Tavern	Funghi	23
Mamma's Place	Marinara	22
Pizza King	Diavola	17
Pizza King	Hawaiian	21

# Unary operators

## Selection: $\sigma_c$

- $\sigma_c(R)$  selects tuples from relation  $R$  that satisfies selection condition  $c$
- **Selection condition** is a boolean combination of terms
- A **term** is one of the following forms:
  1. attribute **op** constant  $op \in \{=, \neq, <, \leq, >, \geq\}$
  2. attribute<sub>1</sub> **op** attribute<sub>2</sub>
  3. term<sub>1</sub>  $\wedge$  term<sub>2</sub> Conjunction (and)
  4. term<sub>1</sub>  $\vee$  term<sub>2</sub> Disjunction (or)
  5.  $\neg$  term Negation (not)
  6. (term)
- ❖ *Operator precedence:  $()$ , **op**,  $\neg$ ,  $\wedge$ ,  $\vee$*

# Unary operators

## Selection: $\sigma_c$

- $\sigma_c(R)$  selects tuples from relation  $R$  that satisfies selection condition  $c$
- Selection removes rows
- Better known as filter

Table

<i>attr1</i>	<i>attr2</i>	<i>Attr3</i>
Val1_1	Val1_2	Val1_3
Val2_1	Val2_2	Val2_3
Val3_1	Val3_2	Val3_3
Val4_1	Val4_2	Val4_3
Val5_1	Val5_2	Val5_3
Val6_1	Val6_2	Val6_3
:	:	:

# Unary operators

## Selection: $\sigma_c$

- $\sigma_c(R)$  selects tuples from relation  $R$  that satisfies selection condition  $c$

- Selection removes rows
- Better known as filter

Table				Table		
attr1	attr2	Attr3	satisfy c?	attr1	attr2	Attr3
Val1_1	Val1_2	Val1_3	→	Val1_1	Val1_2	Val1_3
Val2_1	Val2_2	Val2_3	→ ✗	Val2_1	Val2_2	Val2_3
Val3_1	Val3_2	Val3_3	→ ✗	Val3_1	Val3_2	Val3_3
Val4_1	Val4_2	Val4_3	→	Val4_1	Val4_2	Val4_3
Val5_1	Val5_2	Val5_3	→	Val5_1	Val5_2	Val5_3
Val6_1	Val6_2	Val6_3	→ ✗	Val6_1	Val6_2	Val6_3
:	:	:		:	:	:

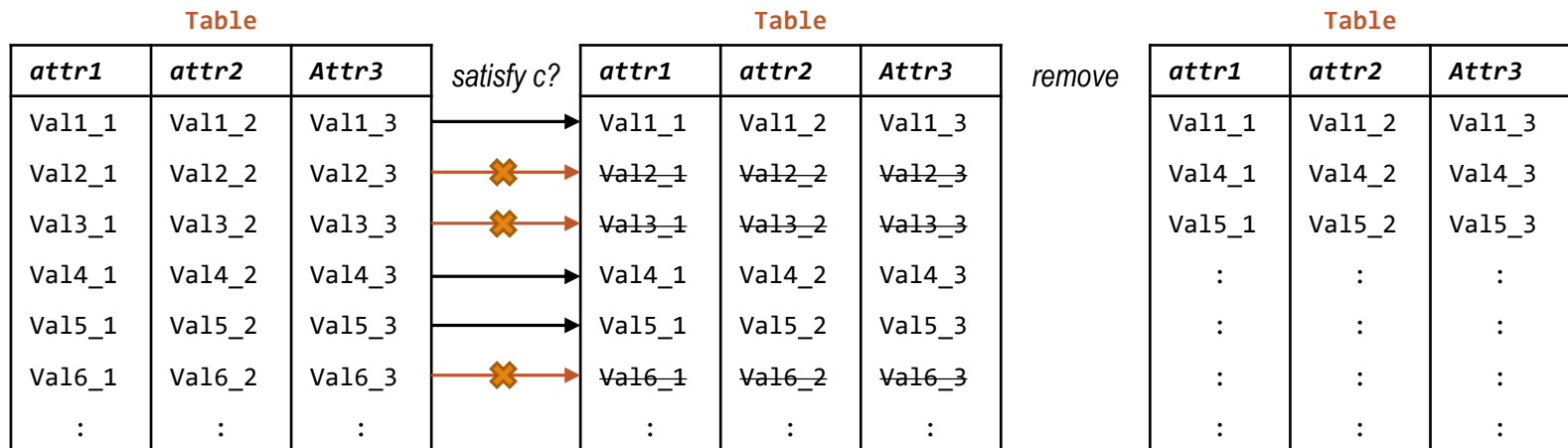


# Unary operators

## Selection: $\sigma_c$

- $\sigma_c(R)$  selects tuples from relation  $R$  that satisfies selection condition  $c$

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- Better known as filter



# Unary operators

**Selection:**  $\sigma_c$

- $\sigma_c(R)$  selects tuples from relation  $R$  that satisfies selection condition  $c$

**Example:** Find all restaurants, the pizzas that they sell, and their prices, where the price is under \$20

# Unary operators

## Selection: $\sigma_c$

- $\sigma_c(R)$  selects tuples from relation  $R$  that satisfies selection condition  $c$

**Example:** Find all restaurants, the pizzas that they sell, and their prices, where the price is under \$20

- $\sigma_{\text{price} < 20}(\text{Sells})$

Sells

<u>rname</u>	<u>pizza</u>	<u>price</u>
Corleone Corner	Diavola	24
Corleone Corner	Hawaiian	25
Corleone Corner	Margherita	19
Gambino Oven	Siciliana	16
Lorenzo Tavern	Funghi	23
Mamma's Place	Marinara	22
Pizza King	Diavola	17
Pizza King	Hawaiian	21




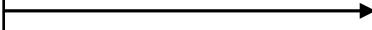




# Unary operators

## Selection: $\sigma_c$

- $\sigma_c(R)$  selects tuples from relation  $R$  that satisfies selection condition  $c$

**Example:** Find all restaurants, the pizzas that they sell, and their prices, where the price is under \$20

- $\sigma_{\text{price} < 20}(\text{Sells})$

Sells				$\sigma_{\text{price} < 20}(\text{Sells})$		
<u>rname</u>	<u>pizza</u>	<u>price</u>	price under \$20?	<u>rname</u>	<u>pizza</u>	<u>price</u>
Corleone Corner	Diavola	24		<del>Corleone Corner</del>	<del>Diavola</del>	<del>24</del>
Corleone Corner	Hawaiian	25		<del>Corleone Corner</del>	<del>Hawaiian</del>	<del>25</del>
Corleone Corner	Margherita	19		Corleone Corner	Margherita	19
Gambino Oven	Siciliana	16		Gambino Oven	Siciliana	16
Lorenzo Tavern	Funghi	23		<del>Lorenzo Tavern</del>	<del>Funghi</del>	<del>23</del>
Mamma's Place	Marinara	22		<del>Mamma's Place</del>	<del>Marinara</del>	<del>22</del>
Pizza King	Diavola	17		Pizza King	Diavola	17
Pizza King	Hawaiian	21		<del>Pizza King</del>	<del>Hawaiian</del>	<del>21</del>

# Unary operators

## Selection: $\sigma_c$

- $\sigma_c(R)$  selects tuples from relation  $R$  that satisfies selection condition  $c$

**Example:** Find all restaurants, the pizzas that they sell, and their prices, where the price is under \$20

- $\sigma_{\text{price} < 20}(\text{Sells})$

Sells

<u>rname</u>	<u>pizza</u>	<u>price</u>
Corleone Corner	Diavola	24
Corleone Corner	Hawaiian	25
Corleone Corner	Margherita	19
Gambino Oven	Siciliana	16
Lorenzo Tavern	Funghi	23
Mamma's Place	Marinara	22
Pizza King	Diavola	17
Pizza King	Hawaiian	21

$\sigma_{\text{price} < 20}(\text{Sells})$

<u>rname</u>	<u>pizza</u>	<u>price</u>
Corleone Corner	Margherita	19
Gambino Oven	Siciliana	16
Pizza King	Diavola	17

# Unary operators

## Selection: $\sigma_c$

- $\sigma_c(R)$  selects tuples from relation  $R$  that satisfies selection condition  $c$

**Example:** Find all restaurants, the pizzas that they sell, and their prices, where (1) either the price is under \$20 or the pizza is “Marinara”, and (2) the pizza is not “Diavola”

Sells

<u>rname</u>	<u>pizza</u>	<u>price</u>
Corleone Corner	Diavola	24
Corleone Corner	Hawaiian	25
Corleone Corner	Margherita	19
Gambino Oven	Siciliana	16
Lorenzo Tavern	Funghi	23
Mamma's Place	Marinara	22
Pizza King	Diavola	17
Pizza King	Hawaiian	21

<u>rname</u>	<u>pizza</u>	<u>price</u>
Corleone Corner	Margherita	19
Gambino Oven	Siciliana	16
Mamma's Place	Marinara	22

# Unary operators

## Selection: $\sigma_c$

- $\sigma_c(R)$  selects tuples from relation  $R$  that satisfies selection condition  $c$

**Example:** Find all restaurants, the pizzas that they sell, and their prices, where (1) either the price is under \$20 or the pizza is “Marinara”, and (2) the pizza is not “Diavola”

- $\sigma_{(\text{price} < 20 \vee \text{pizza} = \text{"Marinara"}) \vee (\text{pizza} \neq \text{"Diavola"})}(\text{Sells})$

Sells

<u>rname</u>	<u>pizza</u>	<u>price</u>
Corleone Corner	Diavola	24
Corleone Corner	Hawaiian	25
Corleone Corner	Margherita	19
Gambino Oven	Siciliana	16
Lorenzo Tavern	Funghi	23
Mamma's Place	Marinara	22
Pizza King	Diavola	17
Pizza King	Hawaiian	21

$\sigma_{(\text{price} < 20 \vee \text{pizza} = \text{"Marinara"}) \vee (\text{pizza} \neq \text{"Diavola"})}(\text{Sells})$

<u>rname</u>	<u>pizza</u>	<u>price</u>
Corleone Corner	Margherita	19
Gambino Oven	Siciliana	16
Mamma's Place	Marinara	22

# Unary operators

## Projection: $\pi_{\ell}$

- $\pi_{\ell}(R)$  projects attributes given by a list  $\ell$  of attributes from relation  $R$
- Projection removes columns
- Duplicate records are removed in the output relation

**Example:** Find all restaurants and the pizzas that they sell

Sells

<u>rname</u>	<u>pizza</u>	<u>price</u>
Corleone Corner	Diavola	24
Corleone Corner	Hawaiian	25
Corleone Corner	Margherita	19
Gambino Oven	Siciliana	16
Lorenzo Tavern	Funghi	23
Mamma's Place	Marinara	22
Pizza King	Diavola	17
Pizza King	Hawaiian	21

<u>rname</u>	<u>pizza</u>
Corleone Corner	Diavola
Corleone Corner	Hawaiian
Corleone Corner	Margherita
Gambino Oven	Siciliana
Lorenzo Tavern	Funghi
Mamma's Place	Marinara
Pizza King	Diavola
Pizza King	Hawaiian



# Unary operators

## Projection: $\pi_{\ell}$

- $\pi_{\ell}(R)$  projects attributes given by a list  $\ell$  of attributes from relation  $R$
- Projection removes columns
- Duplicate records are removed in the output relation

**Example:** Find all restaurants and the pizzas that they sell

- $\pi_{\text{rname,pizza}}(\text{Sells})$

Sells

<u>rname</u>	<u>pizza</u>	<u>price</u>
Corleone Corner	Diavola	24
Corleone Corner	Hawaiian	25
Corleone Corner	Margherita	19
Gambino Oven	Siciliana	16
Lorenzo Tavern	Funghi	23
Mamma's Place	Marinara	22
Pizza King	Diavola	17
Pizza King	Hawaiian	21

$\pi_{\text{rname,pizza}}(\text{Sells})$

<u>rname</u>	<u>pizza</u>
Corleone Corner	Diavola
Corleone Corner	Hawaiian
Corleone Corner	Margherita
Gambino Oven	Siciliana
Lorenzo Tavern	Funghi
Mamma's Place	Marinara
Pizza King	Diavola
Pizza King	Hawaiian

# Unary operators

## Projection: $\pi_{\ell}$

- $\pi_{\ell}(R)$  projects attributes given by a list  $\ell$  of attributes from relation  $R$
- Projection removes/rearranges columns
- Duplicate records are removed in the output relation

**Example:** Find all restaurants area

Restaurants

<u>rname</u>	<u>area</u>
Corleone Corner	North
Gambino Oven	Central
Lorenzo Tavern	Central
Mamma's Place	South
Pizza King	East

<u>area</u>
North
Central
South
East

# Unary operators

## Projection: $\pi_{\ell}$

- $\pi_{\ell}(R)$  projects attributes given by a list  $\ell$  of attributes from relation  $R$
- Projection removes/rearranges columns
- Duplicate records are removed in the output relation

**Example:** Find all restaurants area

- $\pi_{\text{area}}(\text{Restaurants})$

Restaurants

<u>rname</u>	<u>area</u>
Corleone Corner	North
Gambino Oven	Central
Lorenzo Tavern	Central
Mamma's Place	South
Pizza King	East

$\pi_{\text{area}}(\text{Restaurants})$

<u>area</u>
North
Central
South
East

# Unary operators

**Renaming:**  $\rho_{S(B_1, B_2, \dots, B_n)}(R)$

- $\rho_{S(B_1, B_2, \dots, B_n)}(R)$  renames  $R(A_1, A_2, \dots, A_n)$  to  $S(B_1, B_2, \dots, B_n)$ 
  - When the attributes are not renamed  $\rho_S(R)$
  - When the table is not renamed  $\rho_{(B_1, B_2, \dots, B_n)}(R)$

**Example:** Rename Restaurants(rname, area) to Shops(sname, region)

Restaurants

<u>rname</u>	area
Corleone Corner	North
Gambino Oven	Central
Lorenzo Tavern	Central
Mamma's Place	South
Pizza King	East

<u>sname</u>	region
Corleone Corner	North
Gambino Oven	Central
Lorenzo Tavern	Central
Mamma's Place	South
Pizza King	East

# Unary operators

**Renaming:**  $\rho_{S(B_1, B_2, \dots, B_n)}(R)$

- $\rho_{S(B_1, B_2, \dots, B_n)}(R)$  renames  $R(A_1, A_2, \dots, A_n)$  to  $S(B_1, B_2, \dots, B_n)$ 
  - When the attributes are not renamed  $\rho_S(R)$
  - When the table is not renamed  $\rho_{(B_1, B_2, \dots, B_n)}(R)$

**Example:** Rename Restaurants(rname, area) to Shops(sname, region)

- $\rho_{\text{Shops}(\text{sname}, \text{region})}(\text{Restaurants})$

Restaurants

<u>rname</u>	area
Corleone Corner	North
Gambino Oven	Central
Lorenzo Tavern	Central
Mamma's Place	South
Pizza King	East

$\rho_{\text{Shops}(\text{sname}, \text{region})}(\text{Restaurants})$

<u>sname</u>	region
Corleone Corner	North
Gambino Oven	Central
Lorenzo Tavern	Central
Mamma's Place	South
Pizza King	East

# Union compatibility

**Definition:** Two relations  $R_1$  and  $R_2$  are **union compatible** if

1. They have the same number of attributes, and
  2. The corresponding attributes have the same domains
- ❖ The schema of the result of  $R_1 \oplus R_2$  where  $\oplus$  are binary operator requiring union compatibility is identical to the schema of  $R_1$  and  $R_2$  respectively

# Union compatibility

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- ❖ The schema of the result of  $R_1 \oplus R_2$  where  $\oplus$  are binary operator requiring union compatibility is identical to the schema of  $R_1$  and  $R_2$  respectively

**Example:** Consider the following database

- Student (*sid*: integer, *dob*: date, *name*: string)
- GradStudent (*sid*: integer, *name*: string)
- Report (*reportID*: integer, *title*: string, *pubdate*: date)

**Questions:**

- Which of the following are valid binary operations?
1. Student  $\cup$  GradStudent
  2.  $\pi_{sid, name}(\text{Student}) \cup \text{GradStudent}$
  3. Student  $\cap$  Report
  4.  $\pi_{sid, name, dob}(\text{Student}) - \text{Report}$

# Union compatibility

**Definition:** Two relations  $R_1$  and  $R_2$  are **union compatible** if

1. They have the same number of attributes, and
  2. The corresponding attributes have the same domains
- ❖ The schema of the result of  $R_1 \oplus R_2$  where  $\oplus$  are binary operator requiring union compatibility is identical to the schema of  $R_1$  and  $R_2$  respectively

**Example:** Consider the following database

- Student (*sid*: integer, *dob*: date, *name*: string)
- GradStudent (*sid*: integer, *name*: string)
- Report (*reportID*: integer, *title*: string, *pubdate*: date)

**Questions:**

- Which of the following are valid binary operations?
- ~~1. Student  $\cup$  GradStudent~~
  2.  $\pi_{sid, name}(\text{Student}) \cup \text{GradStudent}$
  - ~~3. Student  $\cap$  Report~~
  4.  $\pi_{sid, name, dob}(\text{Student}) - \text{Report}$



# Binary operators

## Union: $R \cup S$

- Returns a relation containing all tuples that occur in  $R$ ,  $S$ , or both

## Intersection: $R \cap S$

- Returns a relation containing all tuples that occur in both  $R$  and  $S$

## Set-difference: $R - S$

- Returns a relation containing all tuples that occur in  $R$  but not in  $S$

❖ Union ( $\cup$ ), intersection ( $\cap$ ), and set-difference ( $-$ ) operators require input relations to be **union compatible**

# Binary operators

## Union: $R \cup S$

- Returns a relation containing all tuples that occur in  $R$ ,  $S$ , or both

### Example:

- Find all customer/restaurant names
- **Solution:**

Restaurants

<u>rname</u>	area
Corleone Corner	North
Gambino Oven	Central
Lorenzo Tavern	Central
Mamma's Place	South
Pizza King	East

Customers

<u>cname</u>	area
Homer	West
Lisa	South
Maggie	East
Moe	Central
Ralph	Central
Willie	North

rname

Corleone Corner
Gambino Oven
Lorenzo Tavern
Mamma's Place
Pizza King
Homer
Lisa
Maggie
Moe
Ralph
Willie

# Binary operators

## Union: $R \cup S$

- Returns a relation containing all tuples that occur in  $R$ ,  $S$ , or both

### Example:

- Find all customer/restaurant names
- **Solution:**  $\pi_{\text{rname}}(\text{Restaurants}) \cup \pi_{\text{cname}}(\text{Customers})$

Restaurants

<u>rname</u>	area
Corleone Corner	North
Gambino Oven	Central
Lorenzo Tavern	Central
Mamma's Place	South
Pizza King	East

Customers

<u>cname</u>	area
Homer	West
Lisa	South
Maggie	East
Moe	Central
Ralph	Central
Willie	North

$\pi_{\text{rname}}(\text{Restaurants})$   
 $\cup \pi_{\text{cname}}(\text{Customers})$

<u>rname</u>
Corleone Corner
Gambino Oven
Lorenzo Tavern
Mamma's Place
Pizza King
Homer
Lisa
Maggie
Moe
Ralph
Willie

# Binary operators

## Intersection: $R \cap S$

- Returns a relation containing all tuples that occur in both  $R$  and  $S$

### Example:

- Find all pizzas that contain both cheese and chilli
- **Solution:**

Contains

<u>pizza</u>	<u>ingredient</u>
Diavola	Cheese
Diavola	Chilli
Diavola	Salami
Funghi	Ham
Funghi	Mushroom
Hawaiian	Ham
Hawaiian	Pineapple
Margherita	Cheese
Margherita	Tomato
Marinara	Seafood
Siciliana	Anchovies
Siciliana	Capers
Siciliana	Cheese

<u>pizza</u>
Diavola
Margherita
Siciliana

<u>pizza</u>
Diavola

<u>pizza</u>
Diavola

# Binary operators

## Intersection: $R \cap S$

- Returns a relation containing all tuples that occur in both  $R$  and  $S$

### Example:

- Find all pizzas that contain both cheese and chilli
- **Solution:**  $\pi_{\text{pizza}}(\sigma_{\text{ingredient}='cheese'}(\text{Contains})) \cap \pi_{\text{pizza}}(\sigma_{\text{ingredient}='chilli'}(\text{Contains}))$

**Contains**

<u>pizza</u>	<u>ingredient</u>
Diavola	Cheese
Diavola	Chilli
Diavola	Salami
Funghi	Ham
Funghi	Mushroom
Hawaiian	Ham
Hawaiian	Pineapple
Margherita	Cheese
Margherita	Tomato
Marinara	Seafood
Siciliana	Anchovies
Siciliana	Capers
Siciliana	Cheese

$\pi_{\text{pizza}}(\sigma_{\text{ingredient}='cheese'}(\text{Contains}))$

<u>pizza</u>
Diavola
Margherita
Siciliana

$\pi_{\text{pizza}}(\sigma_{\text{ingredient}='chilli'}(\text{Contains}))$

<u>pizza</u>
Diavola

$\pi_{\text{pizza}}(\sigma_{\text{ingredient}='cheese'}(\text{Contains}))$   
 $\cap \pi_{\text{pizza}}(\sigma_{\text{ingredient}='chilli'}(\text{Contains}))$

<u>pizza</u>
Diavola

# Binary operators

## Set-difference: $R - S$

- Returns a relation containing all tuples that occur in  $R$  but not in  $S$

### Example:

- Find all pizzas that contain cheese but not chilli
- **Solution:**

Contains

<u>pizza</u>	<u>ingredient</u>
Diavola	Cheese
Diavola	Chilli
Diavola	Salami
Funghi	Ham
Funghi	Mushroom
Hawaiian	Ham
Hawaiian	Pineapple
Margherita	Cheese
Margherita	Tomato
Marinara	Seafood
Siciliana	Anchovies
Siciliana	Capers
Siciliana	Cheese

<u>pizza</u>
Diavola
Margherita
Siciliana

<u>pizza</u>
Margherita
Siciliana

<u>pizza</u>
Diavola

# Binary operators

## Set-difference: $R - S$

- Returns a relation containing all tuples that occur in  $R$  but not in  $S$

### Example:

- Find all pizzas that contain cheese but not chilli
- **Solution:**  $\pi_{\text{pizza}}(\sigma_{\text{ingredient}='cheese'}(\text{Contains})) - \pi_{\text{pizza}}(\sigma_{\text{ingredient}='chilli'}(\text{Contains}))$

**Contains**

<u>pizza</u>	<u>ingredient</u>
Diavola	Cheese
Diavola	Chilli
Diavola	Salami
Funghi	Ham
Funghi	Mushroom
Hawaiian	Ham
Hawaiian	Pineapple
Margherita	Cheese
Margherita	Tomato
Marinara	Seafood
Siciliana	Anchovies
Siciliana	Capers
Siciliana	Cheese

$\pi_{\text{pizza}}(\sigma_{\text{ingredient}='cheese'}(\text{Contains}))$

<u>pizza</u>
Diavola
Margherita
Siciliana

$\pi_{\text{pizza}}(\sigma_{\text{ingredient}='cheese'}(\text{Contains})) - \pi_{\text{pizza}}(\sigma_{\text{ingredient}='chilli'}(\text{Contains}))$

<u>pizza</u>
Margherita
Siciliana

$\pi_{\text{pizza}}(\sigma_{\text{ingredient}='chilli'}(\text{Contains}))$

<u>pizza</u>
Diavola

# Binary operator

## Cross-product: $\times$

- Consider a relation  $R_1(A, B, C)$  and  $R_2(X, Y)$
- $R_1 \times R_2$  returns a relation with schema  $(A, B, C, X, Y)$  defined as follows:
  - $R_1 \times R_2 = \{(a, b, c, x, y) \mid (a, b, c) \in R_1, (x, y) \in R_2\}$
- Also known as cartesian product

## Example

- Find all customer-restaurant pairs that are located in the central area
- Idea
  - Find all customers in central
  - Find all restaurants in central
  - Cross-product



# Binary operator

## Cross-product: $\times$

- Consider a relation  $R_1(A, B, C)$  and  $R_2(X, Y)$
- $R_1 \times R_2$  returns a relation with schema  $(A, B, C, X, Y)$  defined as follows:
  - $R_1 \times R_2 = \{(a, b, c, x, y) \mid (a, b, c) \in R_1, (x, y) \in R_2\}$
- Also known as cartesian product

## Example

- Find all customer-restaurant pairs that are located in the central area
- Idea
  - Find all customers in central  $R_1 = \pi_{\text{cname}}(\sigma_{\text{area}='Central'}(\text{Customers}))$
  - Find all restaurants in central  $R_2 = \pi_{\text{rname}}(\sigma_{\text{area}='Central'}(\text{Restaurants}))$
  - Cross-product  $R_1 \times R_2$

# Binary operator

## Cross-product: $\times$

### Example

- Find all customer-restaurant pairs that are located in the central area
- Idea
  - Find all customers in central
  - Find all restaurants in central
  - Cross-product

$$R_1 = \pi_{\text{cname}}(\sigma_{\text{area}='Central'}(\text{Customers}))$$

$$R_2 = \pi_{\text{rname}}(\sigma_{\text{area}='Central'}(\text{Restaurants}))$$

$$R_1 \times R_2$$

Customers

<u>cname</u>	area
Homer	West
Lisa	South
Maggie	East
Moe	Central
Ralph	Central
Willie	North

Restaurants

<u>rname</u>	area
Corleone Corner	North
Gambino Oven	Central
Lorenzo Tavern	Central
Mamma's Place	South
Pizza King	East

$R_1$

<u>cname</u>
Moe
Ralph

$R_2$

<u>rname</u>
Gambino Oven
Lorenzo Tavern

$R_1 \times R_2$

cname	rname
Moe	Gambino Oven
Moe	Lorenzo Tavern
Ralph	Gambino Oven
Ralph	Lorenzo Tavern

# Closure properties

## Definition

- A set  $S$  is **closed** under an operation  $\oplus$  if for any two members of the set  $x_1 \in S$  and  $x_2 \in S$ , the result  $x_1 \oplus x_2 \in S$  (i.e., the result is the member of the set  $S$ )
- Quick examples
  - Positive integer is closed under addition (*but not subtraction*)
  - Integer is closed under addition, subtraction, and multiplication (*but not division*)

## Closure of relation under unary operators

- Unary operator takes in a relation as input and gives a relation as output

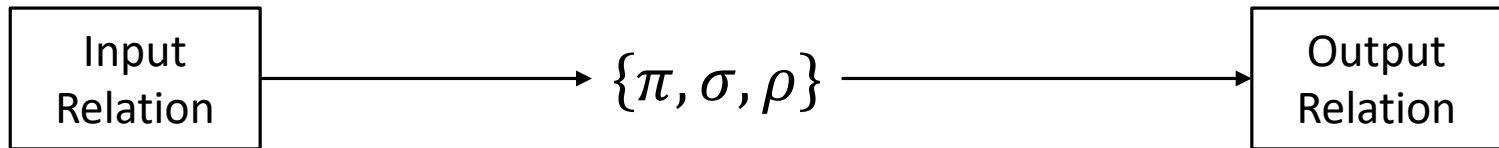
## Closure of relation under binary operators

- Binary operator takes in two relations as inputs and gives a relation as output

# Closure properties

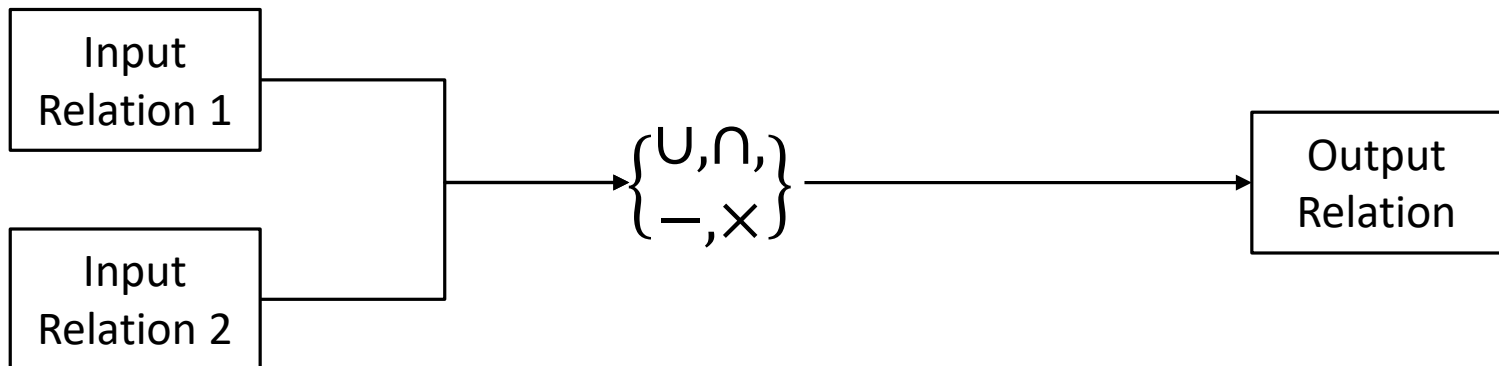
## Closure of relation under unary operators (as diagrams)

- Unary operator takes in a relation as input and gives a relation as output



## Closure of relation under binary operators (as diagrams)

- Binary operator takes in two relations as inputs and gives a relation as output



# Closure properties

## Composition

- Operators can be composed to form relational algebra expressions
- Examples:
  - $\pi_{\text{cname}}(\sigma_{\text{area}='Central'}(\text{Customers}))$
  - $\pi_{\text{pizza}}(\sigma_{\text{ingredient}='cheese'}(\text{Contains}))$   
 $\cap$   
 $\pi_{\text{pizza}}(\sigma_{\text{ingredient}='chilli'}(\text{Contains}))$
  - $\pi_{\text{cname}}(\sigma_{\text{area}='Central'}(\text{Customers}))$   
 $\times$   
 $\pi_{\text{rname}}(\sigma_{\text{area}='Central'}(\text{Restaurants}))$

# Closure properties

## Diagrams

- $\pi_{\text{cname}}\left(\sigma_{\text{area}='Central'}(\text{Customers})\right)$   
 $\times$   
 $\pi_{\text{rname}}\left(\sigma_{\text{area}='Central'}(\text{Restaurants})\right)$

---

# Closure properties

## Diagrams

- $\pi_{\text{cname}}\left(\sigma_{\text{area}='Central'}(\text{Customers})\right)$   
 $\times$   
 $\pi_{\text{rname}}\left(\sigma_{\text{area}='Central'}(\text{Restaurants})\right)$

# Closure properties

## Diagrams

- $\pi_{\text{cname}}\left(\sigma_{\text{area}='Central'}(\text{Customers})\right)$   
 $\times$   
 $\pi_{\text{rname}}\left(\sigma_{\text{area}='Central'}(\text{Restaurants})\right)$

Customers



# Closure properties

## Diagrams

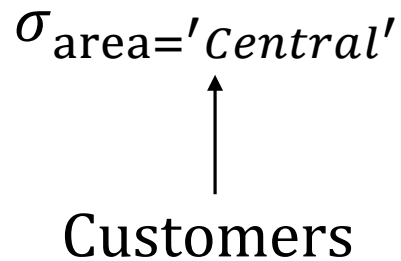
- $\pi_{\text{cname}}\left(\sigma_{\text{area}='Central'}(\text{Customers})\right)$   
 $\times$   
 $\pi_{\text{rname}}\left(\sigma_{\text{area}='Central'}(\text{Restaurants})\right)$

Customers

# Closure properties

## Diagrams

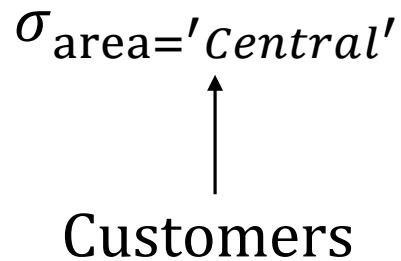
- $\pi_{\text{cname}}\left(\sigma_{\text{area}='Central'}(\text{Customers})\right)$   
 $\times$   
 $\pi_{\text{rname}}\left(\sigma_{\text{area}='Central'}(\text{Restaurants})\right)$



# Closure properties

## Diagrams

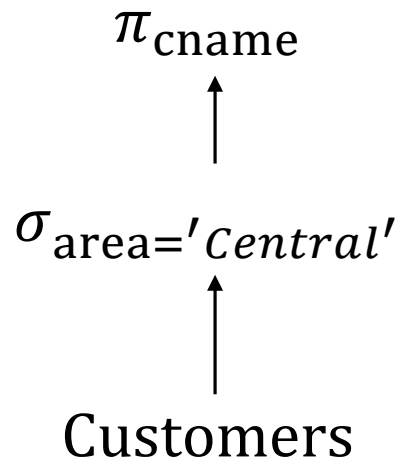
- $\pi_{\text{cname}}(\sigma_{\text{area}='Central'}(\text{Customers}))$   
×  
 $\pi_{\text{rname}}(\sigma_{\text{area}='Central'}(\text{Restaurants}))$



# Closure properties

## Diagrams

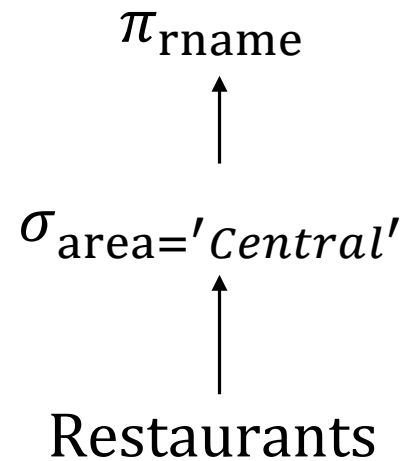
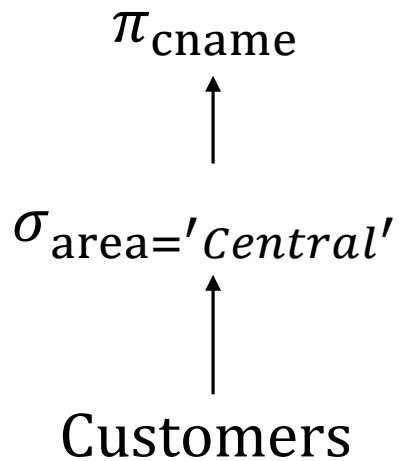
- $\pi_{\text{cname}}(\sigma_{\text{area}='Central'}(\text{Customers}))$   
×  
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# Closure properties

## Diagrams

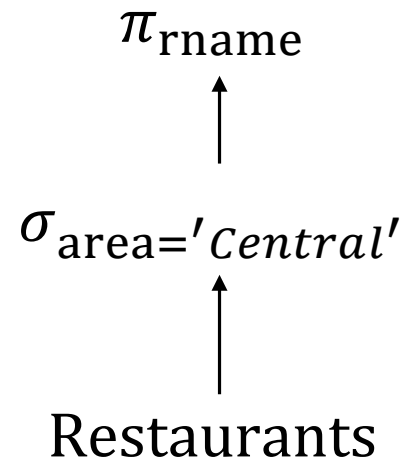
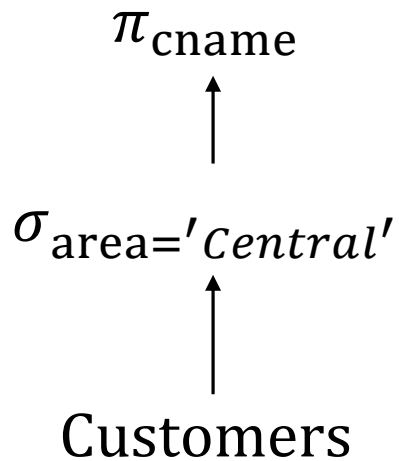
- $\pi_{\text{cname}}(\sigma_{\text{area}='Central'}(\text{Customers}))$   
×  
 $\pi_{\text{rname}}(\sigma_{\text{area}='Central'}(\text{Restaurants}))$



# Closure properties

## Diagrams

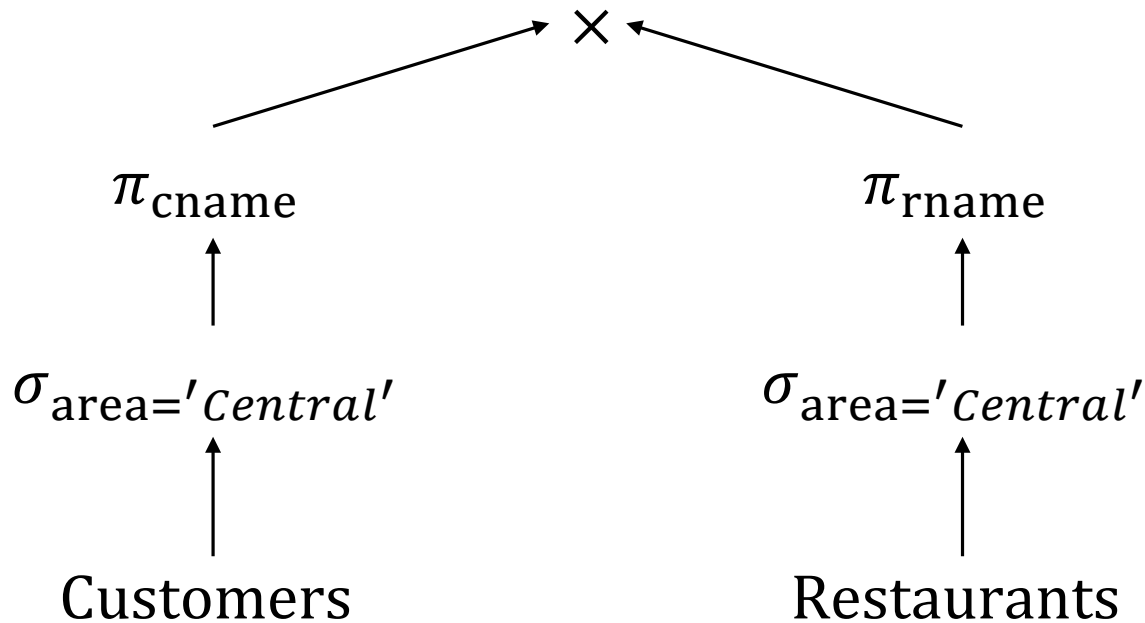
- $\pi_{\text{cname}}(\sigma_{\text{area}='Central'}(\text{Customers}))$   
 $\times$   
 $\pi_{\text{rname}}(\sigma_{\text{area}='Central'}(\text{Restaurants}))$



# Closure properties

## Diagrams

- $\pi_{\text{cname}}(\sigma_{\text{area}='Central'}(\text{Customers}))$   
 $\times$   
 $\pi_{\text{rname}}(\sigma_{\text{area}='Central'}(\text{Restaurants}))$



# Relational algebra problems

## Question

- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

## Visualization

Likes

<u>cname</u>	<u>pizza</u>
Homer	Hawaiian
Homer	Margherita
Lisa	Funghi
Maggie	Funghi
Moe	Funghi
Moe	Siciliana
Ralph	Diavola

R

<u>cname</u>	<u>cname2</u>
Lisa	Maggie
Lisa	Moe
Maggie	Moe

- What is R?



# Relational algebra problems

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- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

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Homer	Hawaiian
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Maggie	Funghi
Moe	Funghi
Moe	Siciliana
Ralph	Diavola

R

<u>cname</u>	<u>cname2</u>
Lisa	Maggie
Lisa	Moe
Maggie	Moe

- What is R?

- $R = \pi_{\text{cname}, \text{cname2}} \left( \sigma_{(\text{pizza} = \text{pizza2}) \wedge (\text{cname} < \text{cname2})} \left( \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes}) \right) \right)$
- Too complicated!**

# Relational algebra problems

## Question

- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

## Visualization

- What is  $R$ ?
  - $R = \pi_{\text{cname}, \text{cname2}} \left( \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})} \left( \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes}) \right) \right)$
  - Simplify:

# Relational algebra problems

## Question

- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

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- What is R?
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- Simplify:
  - **Method 1:** draw diagram
    - *My drawing is not so good, any other method?*

# Relational algebra problems

## Question

- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

## Visualization

- What is R?
  - $R = \pi_{\text{cname}, \text{cname2}} \left( \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})} \left( \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes}) \right) \right)$
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  - **Method 1:** draw diagram
    - *My drawing is not so good, any other method?*
  - **Method 2:** sequence of steps

# Relational algebra problems

## Question

- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

## Visualization

- What is R?
  - $R = \pi_{\text{cname}, \text{cname2}} \left( \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})} \left( \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes}) \right) \right)$
  - Simplify:
    - Method 1:** draw diagram
      - My drawing is not so good, any other method?*
    - Method 2:** sequence of steps
      - $R_1 = \pi_{\text{cname}} \left( \sigma_{\text{area}='Central'}(\text{Customers}) \right)$
      - $R_2 = \pi_{\text{rname}} \left( \sigma_{\text{area}='Central'}(\text{Restaurants}) \right)$
      - $R_{\text{answer}} = R_1 \times R_2$

# Relational algebra problems

## Question

- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

## Visualization

- What is R?
  - $R = \pi_{\text{cname}, \text{cname2}} \left( \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})} \left( \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes}) \right) \right)$
- Simplification using method 2

# Relational algebra problems

## Question

- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

## Visualization

- What is R?
  - $R = \pi_{\text{cname}, \text{cname2}} \left( \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})} \left( \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes}) \right) \right)$
- Simplification using method 2
  - $R_1 = \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes})$

# Relational algebra problems

## Question

- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

## Visualization

- What is  $R$ ?
  - $R = \pi_{\text{cname}, \text{cname2}} \left( \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})} \left( \begin{array}{c} R_1 \end{array} \right) \right)$
  - Simplification using method 2
    - $R_1 = \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes})$



# Relational algebra problems

## Question

- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

## Visualization

- What is R?
  - $R = \pi_{\text{cname}, \text{cname2}} \left( \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})} \left( \begin{array}{c} \text{Likes} \\ R_1 \end{array} \right) \right)$
  - Simplification using method 2
    - $R_1 = \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes})$
    - $R_2 = \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})}(R_1)$

# Relational algebra problems

## Question

- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

## Visualization

- What is  $R$ ?
  - $R = \pi_{\text{cname}, \text{cname2}} \left( \text{Join} \left( \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes}), \text{Likes} \right) \right)$
  - Simplification using method 2
    - $R_1 = \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes})$
    - $R_2 = \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})}(R_1)$

# Relational algebra problems

## Question

- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

## Visualization

- What is R?
  - $R = \pi_{\text{cname}, \text{cname2}} \left( \begin{array}{c} \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes}) \\ \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})}(R_1) \end{array} R_2 \right)$
- Simplification using method 2
  - $R_1 = \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes})$
  - $R_2 = \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})}(R_1)$
  - $R = \pi_{\text{cname}, \text{cname2}}(R_2)$

# Relational algebra problems

## Question

- Find customer pairs  $(C_1, C_2)$  such that they like some common pizza and  $C_1 < C_2$  (i.e., *lexicographical order*)

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- What is R?
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- Simplification using method 2
  - $R_1 = \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes})$
  - $R_2 = \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})}(R_1)$
  - $R = \pi_{\text{cname}, \text{cname2}}(R_2)$

# Relational algebra problems

## Computation

- $R_1(\text{cname}, \text{pizza}, \text{cname2}, \text{pizza2}) = \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes})$
- $R_2(\text{cname}, \text{pizza}, \text{cname2}, \text{pizza2}) = \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})}(R_1)$
- $R(\text{cname}, \text{cname2}) = \pi_{\text{cname}, \text{cname2}}(R_2)$

Likes

<u>cname</u>	<u>pizza</u>
Homer	Hawaiian
Homer	Margherita
Lisa	Funghi
Maggie	Funghi
Moe	Funghi
Moe	Siciliana
Ralph	Diavola

$R_1$

cname	pizza	cname2	pizza2
Homer	Hawaiian	Homer	Hawaiian
Homer	Hawaiian	Homer	Margherita
...	...	...	...
Lisa	Funghi	Maggie	Funghi
Lisa	Funghi	Moe	Funghi
...	...	...	...
Maggie	Funghi	Moe	Funghi
...	...	...	...
Ralph	Diavola	Moe	Siciliana
Ralph	Diavola	Ralph	Diavola

R

cname	cname2
Lisa	Maggie
Lisa	Moe
Maggie	Moe

# Relational algebra problems

## Computation

- $R_1(\text{cname}, \text{pizza}, \text{cname2}, \text{pizza2}) = \text{Likes} \times \rho_{\text{Likes2}(\text{cname2}, \text{pizza2})}(\text{Likes})$
- $R_2(\text{cname}, \text{pizza}, \text{cname2}, \text{pizza2}) = \sigma_{(\text{pizza}=\text{pizza2}) \wedge (\text{cname} < \text{cname2})}(R_1)$
- $R(\text{cname}, \text{cname2}) = \pi_{\text{cname}, \text{cname2}}(R_2)$

Likes		$R_1$				}	49 rows!	$R$	
<u>cname</u>	<u>pizza</u>	<u>cname</u>	<u>pizza</u>	<u>cname2</u>	<u>pizza2</u>			<u>cname</u>	<u>cname2</u>
Homer	Hawaiian	Homer	Hawaiian	Homer	Hawaiian			Lisa	Maggie
Homer	Margherita	Homer	Hawaiian	Homer	Margherita			Lisa	Moe
Lisa	Funghi	...	...	...	...			Maggie	Moe
Maggie	Funghi	Lisa	Funghi	Maggie	Funghi				
Moe	Funghi	Lisa	Funghi	Moe	Funghi				
Moe	Siciliana	...	...	...	...				
Ralph	Diavola	Maggie	Funghi	Moe	Funghi				
		...	...	...	...				
		Ralph	Diavola	Moe	Siciliana				
		Ralph	Diavola	Ralph	Diavola				

# Summary

- ❑ DBMS used to store, update, and query data
- ❑ Relational data model
  - ❑ Tabular representation of data
  - ❑ Integrity constraints specify restrictions on data based on application semantics
  - ❑ Relational algebra provides formal language for querying relations
    - ❑ Selection  $\sigma$
    - ❑ Projection  $\pi$
    - ❑ Renaming  $\rho$
    - ❑ Union  $\cup$
    - ❑ Intersection  $\cap$
    - ❑ Set-difference  $-$
    - ❑ Cross-product  $\times$