Journal of Machine Learning Research 1 (2000) 1-48 Submitted 4/00; Published 10/00

Extending the Multi-Object Tracking Problem to the Case of Exchangeable Targets

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Abstract

The multi-object tracking problem involves associating objects with tracks by storing the data collected in an efficient manner. The general form of this problem can be applied to fields as diverse as signal processing and traffic control. An algorithm that is developed in order to solve this problem should also overcome various limitations imposed by signal noise, incomplete data, and object interference. In this paper, we examine a method to improve observational updates of data within first- and second-order Markov models. We examine and compare variations on these models to help better examine our data, and we seek to extend the problem to the case of exchangeable targets.

Keywords: Object Tracking, Markov Models, Identity Problem

1. Introduction

The application of abstract algebraic methods to machine learning was our primary motivation for choosing this topic. The history and advances in harmonic analysis and group theory comprise an endless source of fascination among mathematicians, and natural questions arise about the relevance and applicability of such rich and abstract theories. We sought to combine purely stochastic models with theoretical models that can be applied to computational methods, and to determine the space in which such models have a clear advantage over other common models in machine learning.

Multi-object tracking systems associate tracks $r_1, r_2, ..., r_n$ with real world objects $o_1, o_2, ..., o_n$ called targets. When the objects are well separated and good quality observations are available, it is relatively easy to determine the correspondence between each track and target. However, when two objects come very close, are occluded, or observations are not available, the association between tracks and objects becomes uncertain (Kondor, 2008). The ideal solution would involve keeping a probability distribution over all $n!$ permutations and updating it appropriately according to observations. However, due to the exponential growth of the factorial function, such an approach quickly becomes infeasible for values of $n \geq 10$ or 12. Hence different methods are needed in order to store these probabilities efficiently while reducing the amount of information lost.

One conventional solution that would come to mind might be to store $n \times n$ matrices to efficiently map each track and object that would update given new information. Unfortunately, such a method would completely ignore the high-level interactions between each object, and would not update well given new information. A more robust approach is needed that would keep track of all possible permutations of the model, with little loss of information upon projection of the probability distribution.

To strike a balance between information loss and computational cost, we consider a second-order model which keeps track of some higher order information which is most relevant to the tracking problem, while adding only $\mathcal{O}(n^4)$ computational cost. This makes the method feasible till $n \leq 30$ (Kondor et al., 2007). The focus of our paper relied on a variation of this method using hidden Markov models to examine its strengths and weaknesses.

2. Higher Order Stochastic Models

The primary goal of the identity management problem is to efficiently store the probability of each permutation. Letting $O := \{o_1, \ldots, o_n\}$ represent the set of objects and $R := \{r_1, \ldots, r_n\}$ represent tracks associated with each object, we hope to recover the permutation $\dot{\sigma}$ that properly associates each object with its track. We update the model in two ways, mixing events where tracks r_i and r_j may switch with a probability depending on their geographical locations (see section 4.1 for more details), and observations which inform the model of object o_i is on track r_j with a certain probability p.

Given the symmetric group S_n, define $p : S_n \to \mathbb{R}$ as the probability distribution over each possible permutation $\sigma : O \to R$.

We define a representation ρ_λ of S_n, where λ is an integer partition on n, as a group homomorphism from S_n to $\mathbb{R}^{d_{\rho_\lambda}}$, where d_{ρ_λ} is the degree of the representation ρ_λ. Then the Fourier transform of $p : S_n \to \mathbb{R}$ is given by

$$\hat{p}(\rho_\lambda) = \sum_{\sigma \in S_n} p(\sigma)\rho_\lambda(\sigma)$$

and the inverse Fourier transform is be given by:

$$p(\sigma) = \frac{1}{n!} \sum_{\lambda} d_{\rho_\lambda} \mathrm{tr}\left[\rho_\lambda(\sigma^{-1})\hat{p}(\lambda)\right]$$

Instead of storing the probability distribution on S_n, we can store the Fourier transform of the probability distribution on a decomposition of S_n called the product of irreducible representations. This decomposition allows us to maintain and update the probability distribution in a hierarchical manner. This enables us to throw away the higher order components and only store up to second or third order components which provide the most useful information, in a very efficient manner rather than resorting to $\mathcal{O}(n!)$ calculations.

We implemented the second-order version of this model and experimentally compared it to the naive model. We hope to extend it to a higher order general implementation of this model which we could not do due to time constraints.

The first order model is very simple; it only keeps track of the $n \times n$ probabilistic matrix storing the probability of object o_i being on track r_j, and updates this internal representation based on updates (Shin et al., 2003). It is very efficient to implement; however it is not expressive enough to store the higher order interactions between objects. Therefore in a setting where there is a lot of overlap between tracks and the probability of mixing is high, it is unable to take advantage of all the information available.

The second order model keeps track of more complex probabilistic variables, for example the probability that object o_i is on track r_j while simultaneously object $o_{i'}$ is on track $r_{j'}$. Hence it is able to track more nuanced relationships between the objects. We implement mixing updates by matrix multiplication with the appropriate mixing matrix, and observation updates by Bayesian normalization.

3. Improving Observational Updates of Data

In addition to determining an optimal set of partitions of a given number of objects n, our primary goal was to use the categorization of the objects in order to improve the predictability of our algorithm. One possible vector of attack included using partial observations on the data at certain intervals in order to determine the likelihood of transposing the association of two objects with their respective tracks. (Jagabathula and Shah, 2009)

If σ' is a similar probability distribution with the tracks i and j swapped, we use the transposition $\begin{pmatrix} i & j \end{pmatrix}$ to transition from $\sigma \mapsto \sigma'$ by $\sigma' = \begin{pmatrix} i & j \end{pmatrix} \sigma$. If p_t represents the probability distribution at time t, then the probability of switching tracks using partial observations takes on the form of a Bayesian update as follows (Kondor, 2011):

$$p_{t+1}(\sigma') = \frac{p(\mathcal{O} \mid \sigma') \sum_{\sigma \in S_n} p(\sigma' \mid \sigma) p_t(\sigma)}{\sum_{\sigma' \in S_n} p(\mathcal{O} \mid \sigma') \sum_{\sigma \in S_n} p(\sigma' \mid \sigma) p_t(\sigma)} \tag{1}$$

Let m represent the number of subclasses or ‘teams’ of the objects, with the team of each track $r_k k$ denoted by m_k. Let the probability that we observe object o_i at track r_j be a small constant parameter β, and given that $\sigma \neq \sigma'$, from (1) it can be derived that

$$p(\sigma' \mid \sigma) = \begin{cases} \dfrac{\beta}{2m} & \text{if } m_i = m_j \\ \\ \beta \left(1 - \binom{m}{2} \dfrac{1}{2m}\right) & \text{if } m_i \neq m_j \end{cases}$$

whereas if $\sigma = \sigma'$, we have that $p(\sigma' \mid \sigma) = 1 - \binom{n}{2}\beta$.

4. Experiments

One motivating example was the relationship between the tracking of migratory birds. We decided to apply this algorithm on a dataset that used GPS to record the migration of 22 lesser black-backed gulls and 5 herring gulls primarily across Western Europe and the west coast of Africa over the course of several months (Stienen et al., 2014). We sought to compare this data with positions measured by geolocation trackers, that are also much cheaper to install and use than GPS devices. These trackers use metrics such as sunlight to give approximate locations and are error-prone, as they tend to be off by hundreds of

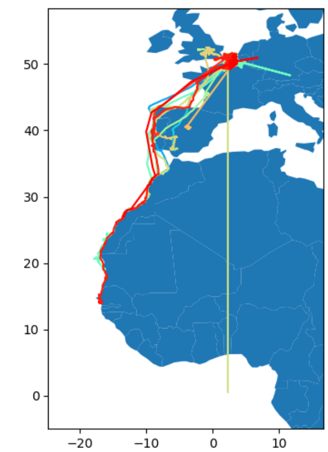


Figure 1: Map of paths of seagulls across several months

kilometers. Using this information, we applied an error function on the dataset to simulate the probability of track-switching when using geolocation devices.

The main goal of our project was using partial observations in the data to measure the robustness of this algorithm given the frequency of overlap between the trajectories of the birds. We used these algebraic results with the traditional Markov model. We also examined whether the categorization of these objects (in this case, two distinct varieties of gulls) would enable us to predict tracks more accurately.

In addition to location, the dataset stored information about the relative heights, temperature, barometric pressure, and ground speed—all metrics useful for determining additional partial observations about the data. Each track represents the location of each of the sensors, and each object represents the bird associated with that track. We used an error function to simulate a certain probability of track-switching to model noisy data. We decided to model this function based on the error posed by geolocation trackers on top of the GPS data. In a study that directly compared locations tracked by GPS devices with those tracked by geolocation devices over a similar geographic area, one particularly volatile method demonstrated that the geolocation data deviated from GPS data by 495.5 ± 1031 km (Rakhimberdiev et al., 2016).

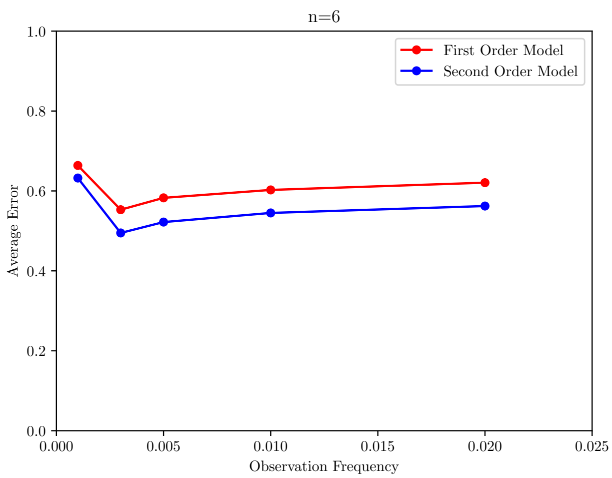
4.1 Error distribution

The error distribution we chose was a skew normal distribution, calculated using F-divergences (Lazo and Rathie, 1978), where the k-th moment of a distribution X is given by:

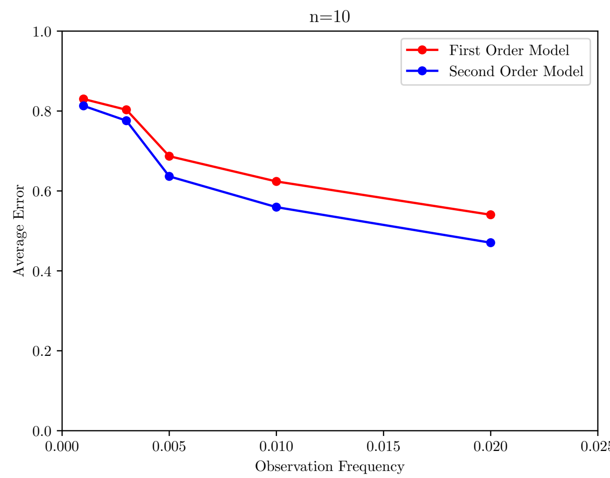
$$\mu_X(k) = \left(\frac{d_2}{d_1}\right)^k \cdot \frac{\Gamma\left(\frac{d_1}{2}+k\right)}{\Gamma\left(\frac{d_1}{2}\right)} \cdot \frac{\Gamma\left(\frac{d_2}{2}-k\right)}{\Gamma\left(\frac{d_2}{2}\right)}$$

where d_1 and d_2 refer to the degrees of freedom. Given the desired mean and variance, we obtained a method (Poe, 2019) that used some initial assumptions derived experimentally from simulations, with an equation to find an optimal d_1 and d_2. This allowed us to recover a distribution with the desired location and skew of the data.

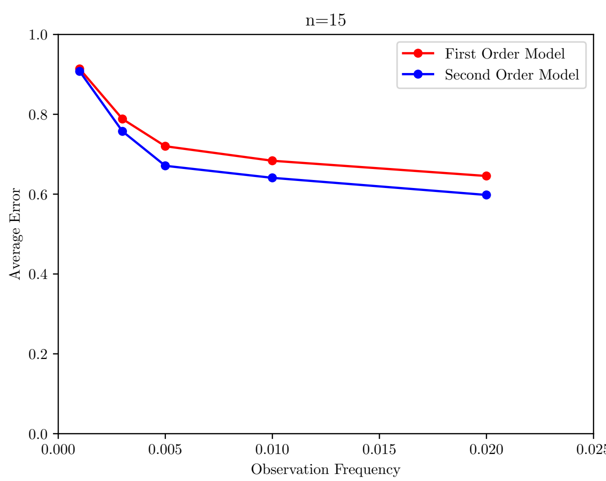
4.2 Initial first- and second-order results



(a) 6 tracks



(b) 10 tracks



(c) 15 tracks

Figure 2: Graphs plotting the average error in the first and second order model against the frequency of observation in the dataset (lower is better).

We ran the models with a multitude of parameters for varying subsets of tracks and for different frequencies of observations to see the particular strengths of each approach. The results can be observed in Figure 2. Regardless of the number of objects, the second order model performs considerably better than the first order model, specially in situations where there are frequent observations that can utilize the higher-order correlations being tracked by the second order model. We do however note that the second order model is considerably slower than the first order model, especially on large datasets including $\geq 30,000$ points.

The average error pertains to the average difference between 1 and the predicted probability of object o_i being on track r_i. The default guess which would assign probability $\frac{1}{n}$ for every object and every track would have a baseline error of $\frac{n-1}{n}$.

All the code used to run the above experiments is available on GitHub. A copy of the dataset is available on Google Drive (could not be uploaded to GitHub due to file size limitations).

5. Conclusion

We attempted to solve the multi-object tracking problem which involves associating objects with unlabeled tracks. We built models incorporating ideas from the representation theoretic decomposition of S_n to prevent reasoning about probability distributions on all $n!$ permutations. We ran our models on a dataset of 22 lesser black-backed gulls and 5 herring gulls and created a probability distribution to switch those tracks in order to simulate an error function. We implemented both a first-order and second-order Markov model based on the general algorithm and ran it on the dataset, and compared both models.

We found that the second order model performed considerably better than the first order model, specially when the frequency of incoming observations was higher. This can be adequately explained by the fact that since the second order model stores higher order correlations, it was able to make use of the observations better than the first order model. The second order model was considerably slower than the first order model which makes us question the relative computation vs precision trade-off, and the conditions where it would be worthwhile. If the tracks are highly entangled, and the number of tracks relatively small, say $10 \leq n \leq 30$, then the higher order models would be better suited.

The implementation of our algorithm was not very precise, likely due to the sparseness of the data and the sensitivity of the noise function. Moreover, we were not able to extend this model to include higher order Fourier components, due to time constraints. Nevertheless, we are confident that the principles examined in our implementation pose some unique advantages in cases where observations are not readily available, and would likely benefit other models with smooth distributions.

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