FM Lab Experiment -1: The Bernoulli’s Apparatus

Group-2

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1. Answers to the questions posed in the essential background

Q. Read up on Bernoulli’s equation in Fluid Mechanics. Fundamentally, what is Bernoulli’s equation about? What are the assumptions under which this equation is valid?

The Bernoulli theorem is an approximate relation between pressure, velocity and elevation, and is valid in regions of steady, incompressible flow where along with zero frictional
forces.

Bernoulli’s equation states that the “sum of the kinetic energy (the velocity head), the pressure energy (static head) and Potential energy (elevation head) per unit weight of
the fluid at any point remains constant,” provided the assumptions.

The assumptions under which the equation is valid are-

- The fluid is ideal/perfect, i.e., viscosity is zero.
- The flow is steady. There is no energy loss while flowing.
- The flow is incompressible.

Q. To circumvent the assumptions, several modifications to Bernoulli’s equation have been proposed. Read up on these modifications from your Fluid Mechanics textbook and report them in your submission.

Every functioning fluid is viscous and provides flow resistance. Therefore, there are some fluid flow losses between the two portions. Since the original form of Bernoulli's equation was based on the erroneous assumption that fluids were frictionless and non-viscous, it has been changed to consider losses. Equations (1), (2), and (3) do not take into account any potential energy exchange brought on by heat or work.

1. Modified Bernoulli's equation for a real fluid is equal to:

$$\frac{P_1}{\gamma} + \frac{{V_1}^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{{V_2}^2}{2g} + z_2 + h_L$$

Where h_L is equal to the loss of energy between points 1 and 2

2. The modified Bernoulli’s equation for incompressible flow with one inlet and one outlet:

$$\frac{P_{in}}{\gamma} + \frac{{V_{in}}^2}{2g} + z_{in} = \frac{P_{out}}{\gamma} + \frac{{V_{out}}^2}{2g} + z_{out} + h_{friction} - h_{pump} + h_{turbine}$$

3. Modified Bernoulli’s equation when the kinetic energy correction factor is considered.

$$\frac{P_{in}}{\gamma} + \alpha\frac{{V_{in}}^2}{2g} + z_{in} = \frac{P_{out}}{\gamma} + \alpha\frac{{V_{out}}^2}{2g} + z_{out} + h_{friction} - h_{pump} + h_{turbine}$$

Q. What is a head loss in a pipe flow? Why is it important? Can you quantify head loss?

The head loss (or the pressure loss) represents the reduction in the total head or pressure (sum of elevation head, velocity head, and pressure head) of the fluid as it flows through a hydraulic system. **The head loss** also represents the energy used in overcoming friction caused by the pipe walls and other technological equipment. Head loss is unavoidable in real moving fluids. It is present because of the friction between adjacent fluid particles moving relative to one another (especially in turbulent flow).

The head loss that occurs in pipes depends on the **flow velocity, pipe diameter,** and **length**, and a **friction factor** based on the roughness of the pipe and the **Reynolds number** of the flow. Although the **head loss represents a loss of energy**, it **does not represent a loss of total energy** of the fluid. The total energy of the fluid is conserved as a consequence of the **law of conservation of energy**. In reality, the head loss due to friction results in an equivalent **increase in the fluid's internal energy** (temperature increases).

head loss is formed **key characteristic** of any hydraulic system. In systems in which some certain flowrate must be maintained (e.g.,, to provide sufficient cooling or heat transfer from a reactor core
), **the equilibrium** of the **head loss** and the **head added** by a pump determine the flow rate through the system.

Q. What is the friction factor, and what does it depend on?

The friction factor is an empirical equation that relates the head loss,
or pressure loss, due to friction along a given pipe length to the average fluid flow velocity for an incompressible fluid.

(f_d= 64/Re)

It depends on the following factors-

- Reynolds number
- Relative toughness (ε/d) where (ε) is the roughness of pipe walls and d is the diameter of the pipe.
- Temperature

Q. How do you evaluate the friction factor for a given flow? Give examples.

In physical terms, friction factor is defined as,

$$f = \frac{\Delta p}{\rho {u_m}^2} * \frac{H}{L}$$

where,

Δp → Pressure Drop Across the pipe

u_m → Mean Velocity

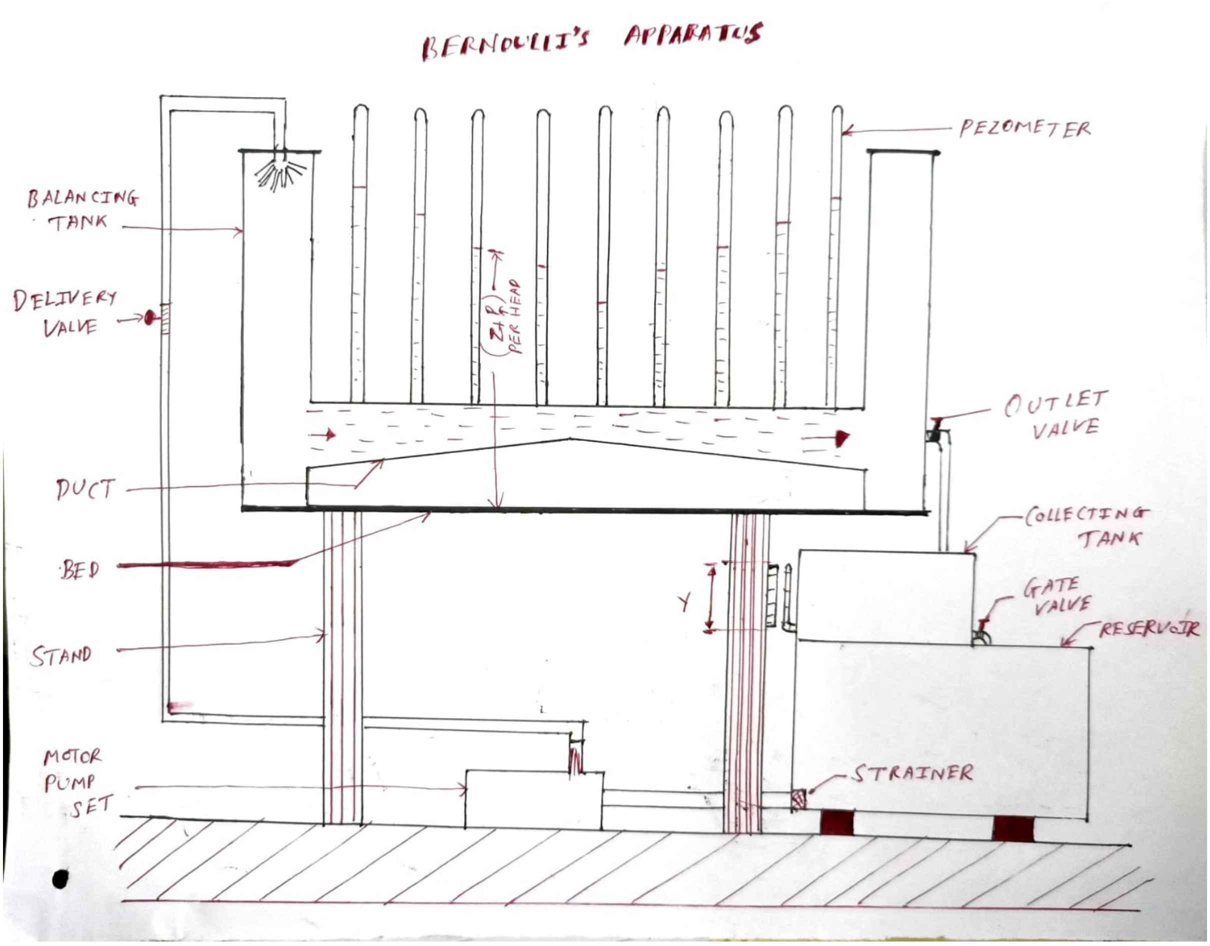
H → Channel Height

L → Channel Length

We can use this equation to calculate friction factor from experiments. Empirically, relation between reynold's number and friction factor can be equated as,

$$\begin{cases} f = \frac{64}{R_e} & \text{If Laminar Flow} \\ \frac{1}{\sqrt{f}} = -2.0log(\frac{e/D}{3.7} + \frac{2.51}{R_e\sqrt{f}}) & \text{If Turbulent flow} \end{cases}$$

2. Schematic of the experimental set up along with a photograph



3. Outline of experimental procedures

1. The first basic step is to make sure that the switches given on the panel are OFF.
2. Make sure to close all the given valves.
3. Fill the reservoir tank with water.
4. Open the by pass delivery valve.
5. The next step is to turn ON the main power supply and the pump, respectively.
6. Partially close by delivery valve, so as to fill balancing tank and wait until overflow occurs in balancing Tank.
7. Control the flow of water through test section with the help of outlet valve provided at the end of test section. The water will start flowing through the flow channel.
8. The level in the piezometer tubes will start rising.
9. Make sure that the water level is maintained in balancing tank i.e. overflow is still occurring. If not, partially close the delivery valve.
10. Measure flow rate with the help of measuring/collecting tank and a stop watch to record time.
11. Measure pressure head (i.e. height of water level in tubes) by piezometer tubes.
12. Repeat steps 7 to 11 for different flow rates.
13. Once the Experiment is completed turn off the Pump carefully.
14. Turn OFF the power supply and drain the water from all the tanks with the help of drain valves.

4. Experimental Observations

Time taken = 150s

Q = (400 mm * 250 mm * 100 mm) / 150 s

= 6.67 * 10^-5 m^3/s

Distance fromreference point(m)	Cross-section area (*10^(-4)) m^2	Velocity(m/s)	Pressure head(m)	Velocity head(m)	Total head(*(10^-2) m)
0.03	6.1575	0.10832318	0.26	0.000598669	26.0598669
0.059	4.9088	0.13587842	0.258	0.000941987	25.8941987
0.088	3.4636	0.1925742	0.256	0.001892083	25.78920828
0.117	2.4053	0.27730429	0.25	0.00392335	25.39233504
0.146	1.5	0.44466667	0.241	0.010088186	25.10881859
0.175	2.4053	0.27730429	0.238	0.00392335	24.19233504
0.204	3.4636	0.1925742	0.239	0.001892083	24.08920828
0.233	4.9088	0.13587842	0.241	0.000941987	24.1941987
0.262	6.1575	0.10832318	0.244	0.000598669	24.4598669

Time taken= 167s

Q = (400 mm * 250 mm * 100mm) / 167 s

= 5.99 * 10^-5 m^3/s

Distance fromreference point(m)	Cross-section area (*10^(-4)) m^2	Velocity(m/s)	Pressure head(m)	Velocity head(m)	Total head(*(10^-2) m)
0.03	6.1575	0.09727974	0.235	0.000482824	23.54828239
0.059	4.9088	0.12202575	0.233	0.000759708	23.37597083
0.088	3.4636	0.17294145	0.23	0.001525956	23.15259564
0.117	2.4053	0.24903338	0.229	0.003164165	23.21641646
0.146	1.5	0.39933333	0.219	0.008136077	22.71360771
0.175	2.4053	0.24903338	0.216	0.003164165	21.91641646
0.204	3.4636	0.17294145	0.217	0.001525956	21.85259564
0.233	4.9088	0.12202575	0.219	0.000759708	21.97597083
0.262	6.1575	0.09727974	0.221	0.000482824	22.14828239

Time taken = 193s

Q = (400 mm * 250 mm * 100 mm) / 193 s

= 5.18 * 10^-5 m^3/s

Distance fromreference point(m)	Cross-section area (*10^(-4)) m^2	Velocity(m/s)	Pressure head(m)	Velocity head(m)	Total head(*(10^-2) m)
0.03	6.1575	0.08412505	0.225	0.000361073	22.53610727
0.059	4.9088	0.10552477	0.223	0.000568137	22.35681366
0.088	3.4636	0.14955538	0.22	0.001141164	22.11411638
0.117	2.4053	0.21535775	0.219	0.002366274	22.13662735
0.146	1.5	0.34533333	0.213	0.006084444	21.90844444
0.175	2.4053	0.21535775	0.21	0.002366274	21.23662735
0.204	3.4636	0.14955538	0.211	0.001141164	21.21411638
0.233	4.9088	0.10552477	0.214	0.000568137	21.45681366
0.262	6.1575	0.08412505	0.216	0.000361073	21.63610727

Error Inclusion

$$\text{Pressure Head Error: } \Delta h_P = \pm 1mm$$

$$\text{Time Error: } \Delta t = \pm 0.1s$$

$$\text{Diameter Error: } \Delta D = \pm 1mm$$

$$\text{Volume Flow Rate Error: } \frac{dQ}{Q} = \frac{dt}{t} + \frac{dh}{h}$$
$$\implies \frac{dQ}{6*10^{-5}} = \frac{0.1}{170} + \frac{1}{100}$$
$$\implies \Delta Q = \pm 6.35 * 10^{-7} m^3/s$$

$$\text{Velocity Error: } \frac{dV}{V} = \frac{dQ}{Q} + 2\frac{dD}{D}$$
$$\implies \frac{dV}{0.15} = \frac{6.35}{600} + 2 * \frac{1}{11}$$
$$\implies \Delta V = \pm 0.028 m/s$$

$$\text{Velocity Head Error: } \Delta h_v = \pm 5.6 * 10^{-4} = \pm 0.5 mm$$

$$\text{Total Head Error:} \Delta h_t = \Delta h_P + \Delta h_v = \pm 1.5 mm$$

Reynold’s Number Calculation

From the Diameter vs distance curve, we can see that it is almost linear in the first half with equation, $D = (2.8 - 12.24x) \times 10^{-2}$

↳ to find, $Re = \frac{\rho V D}{\mu} = \frac{\rho Q \cancel{D} \times 4}{\mu \pi D^2} = \frac{4 \rho Q}{\mu D \pi}$

↳ $$Re_{avg} = \frac{\int_0^{0.116} \frac{4 \rho Q \times 100 \; dx}{\pi \mu (2.8 - 12.24x)}}{\int_0^{0.116} dx}$$

$$Re_{avg} = \frac{\frac{4 \cancel{\rho}^{\,1000} Q \times 100}{\pi \cancel{\mu}_{\,10^{-3}}} \left[0.0816993\,209 \left(\frac{2.8}{1.38} \right) \right]}{0.116}$$

$$\left[Re_{avg} = \frac{0.07359989 \times Q \times 1000 \times 100}{10^{-3} \times 0.116} \right]$$

For $Q = 6.67 \times 10^{-5}$

$(Re_{avg})_1 = 4231.9939$

For $Q = 5.99 \times 10^{-5}$

$(Re_{avg})_2 = 3800.55$

For $Q = 5.18 \times 10^{-5}$

$(Re_{avg})_3 = 3373.87$

For laminar flow $Re < 2000$

So, flow in all reading is turbulant.

Friction Factor Calculation

to Find ~~Fre~~ Friction Factor f we assume that head loss due to minor losses is zero or there are no minor losses.

$$h_e = f\frac{L}{D}\frac{v^2}{2g}$$

$$\int dh_e = \frac{f}{2g}\int \frac{Q^2 (4)^2\, dx}{\pi^2 D^5}$$

→ For the limit we consider ~~to~~ head loss between first two tubes.

$$h_{e(2\to1)} = \frac{f Q^2(16)}{2g\pi^2}\int_0^{0.029}\frac{dx}{(2.8-12.24x)^5 \times (10^{-2})^5}$$

$$h_{e(2\to1)} = \frac{f Q^2\,(16)}{2g\pi^2}\left[(5.72\times10^{-4})\times 10^{10}\right]$$

$$\left[h_{e(2\to1)} = 0.47269 \times f \times Q^2 \times 10^{10} \times 10^{-4}\right] \quad \text{— (i)}$$

→ For $Q = 6.67\times10^{-5}$

$$h_{e(2\to1)} = \underbrace{(\text{Total head})_1 - (\text{velocity head})_2}_{\text{of 2nd reading}} - (\text{Total head})_2$$

ideal head where $f = 0$.

$$h_{e(2\to1)} = (26.0598669 - 0.0941987 - 25.8941987)\times10^{-2}$$

$$h_{e(2\to1)} = 7.14695\times10^{-4}$$

→ Puting Q, $h_{e(2\to1)}$ in (i),

$$\left[f_1 = 0.33985\right]$$

similarly for $Q = 5.99 \times 10^{-5}$

$h_{l(2 \to 1)} = 9.63 \times 10^{-4}$

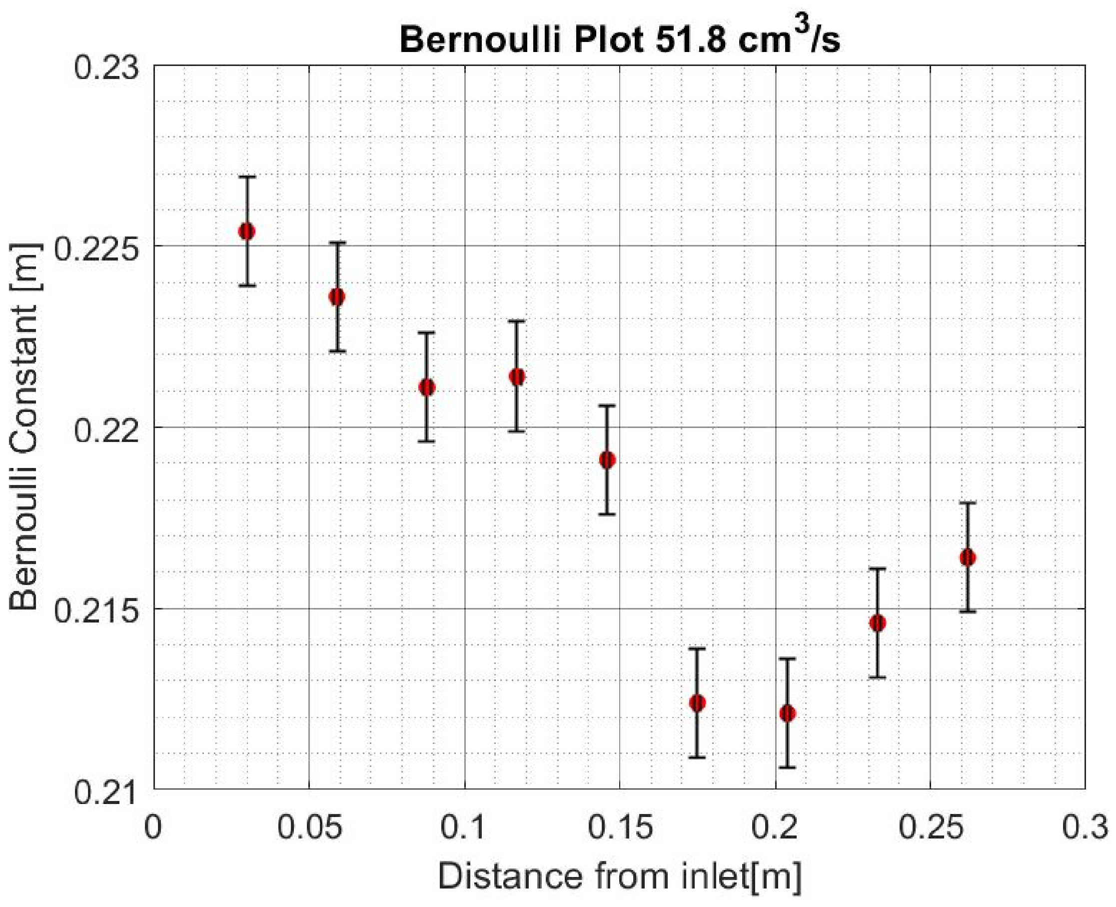
$\to \boxed{F_2 = 0.5678}$

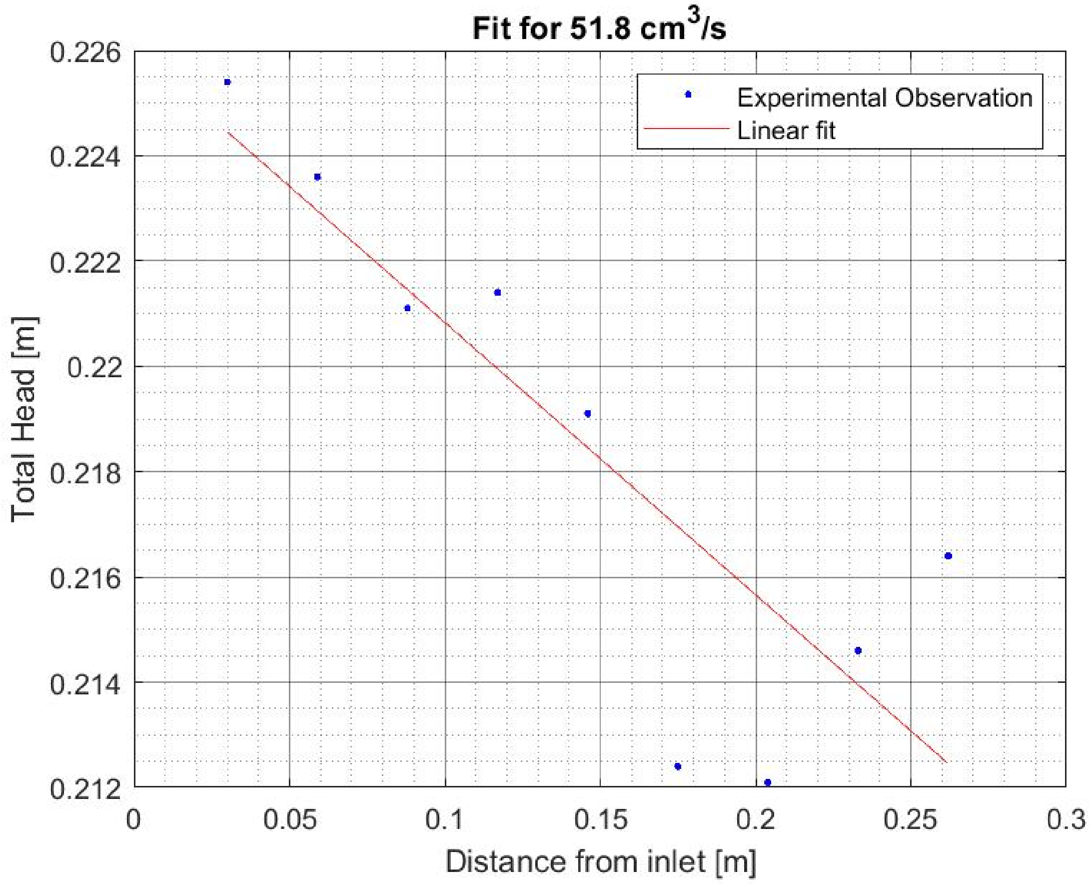
similarly for $Q = 5.18 \times 10^{-5}$

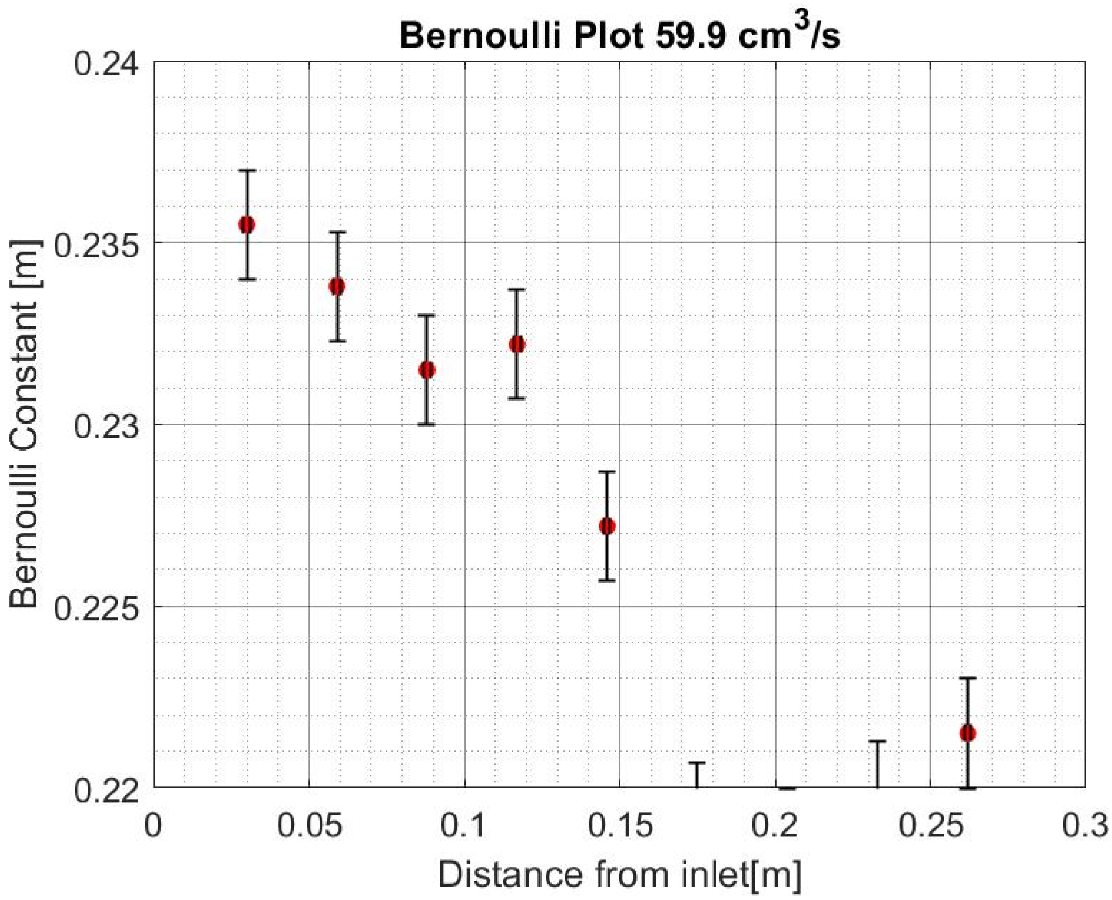
$h_{l(2 \to 1)} = 12.25 \times 10^{-4}$

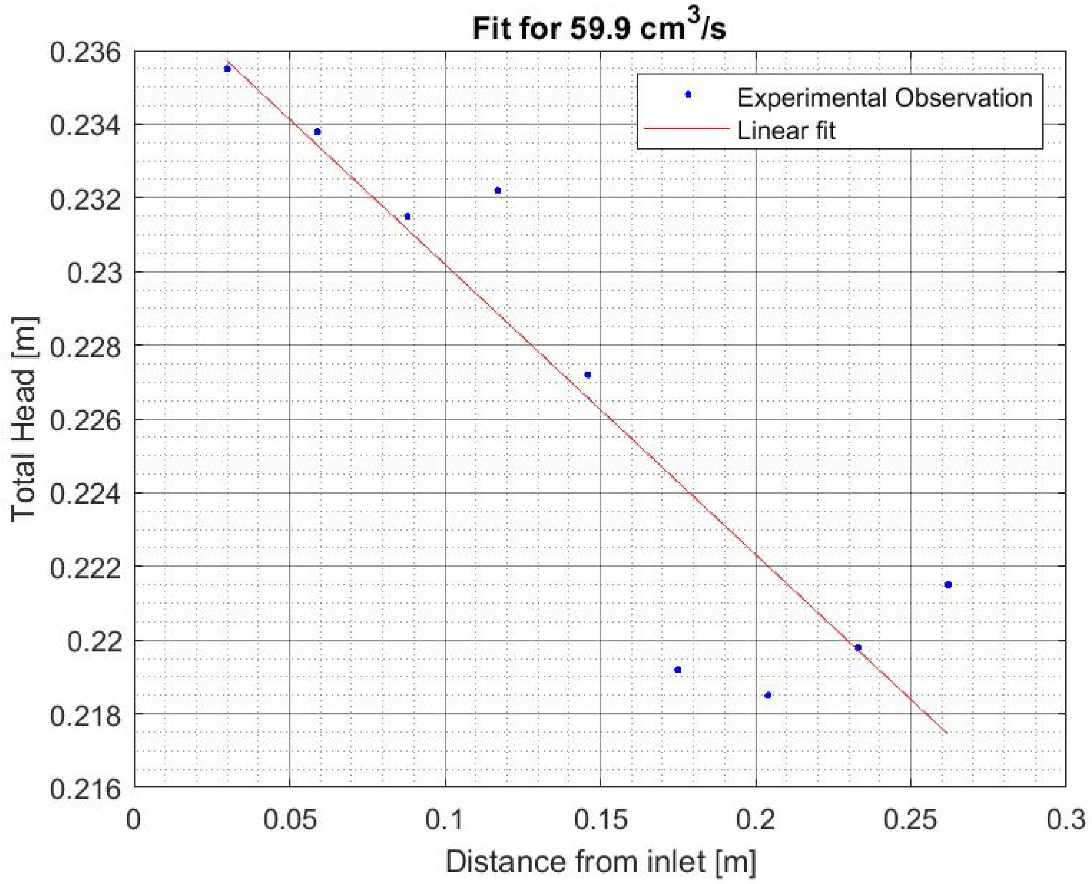
$\to \boxed{F_3 = 0.9658}$

Bernoulli Constant vs Ref Distance Plot

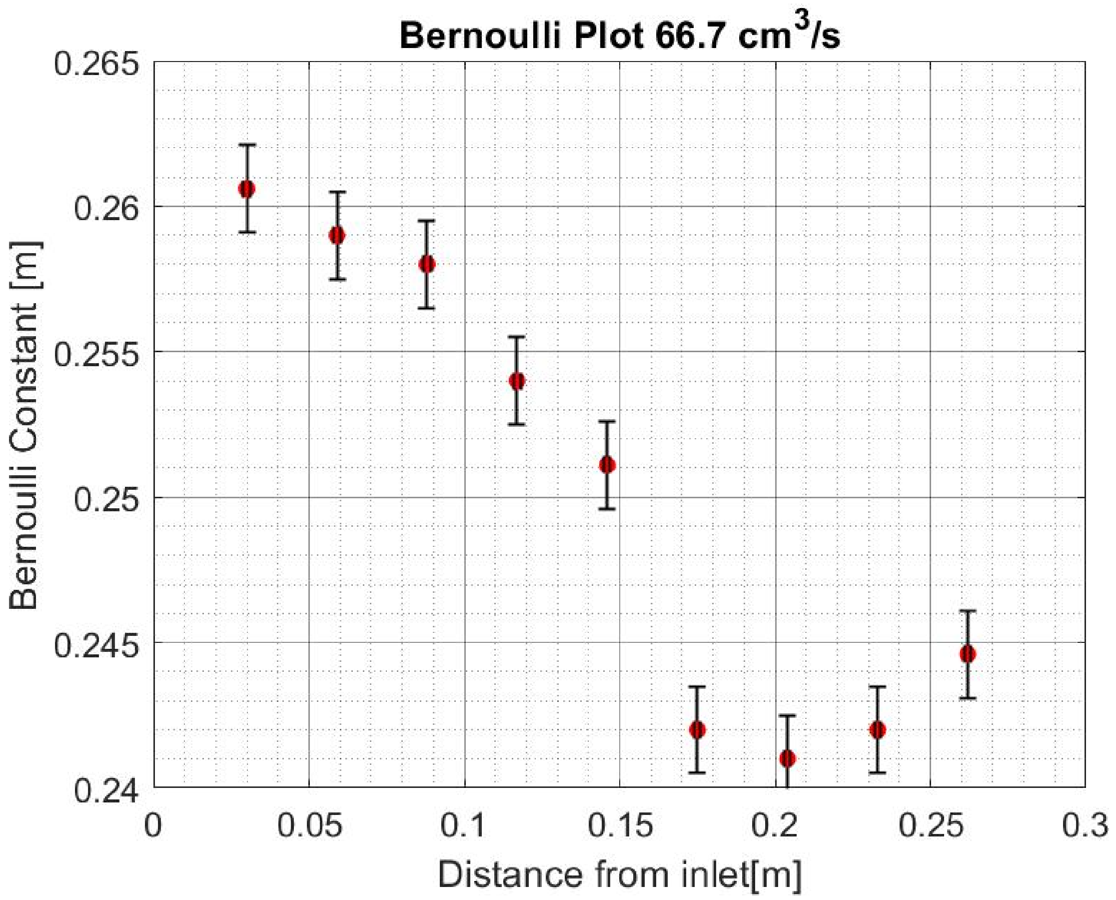



Fit for 51.8 cm3/s
Experimental Observation
Linear fit
Total Head [m]
Distance from inlet [m]
0.226
0.224
0.222
0.22
0.218
0.216
0.214
0.212
0
0.05
0.1
0.15
0.2
0.25
0.3

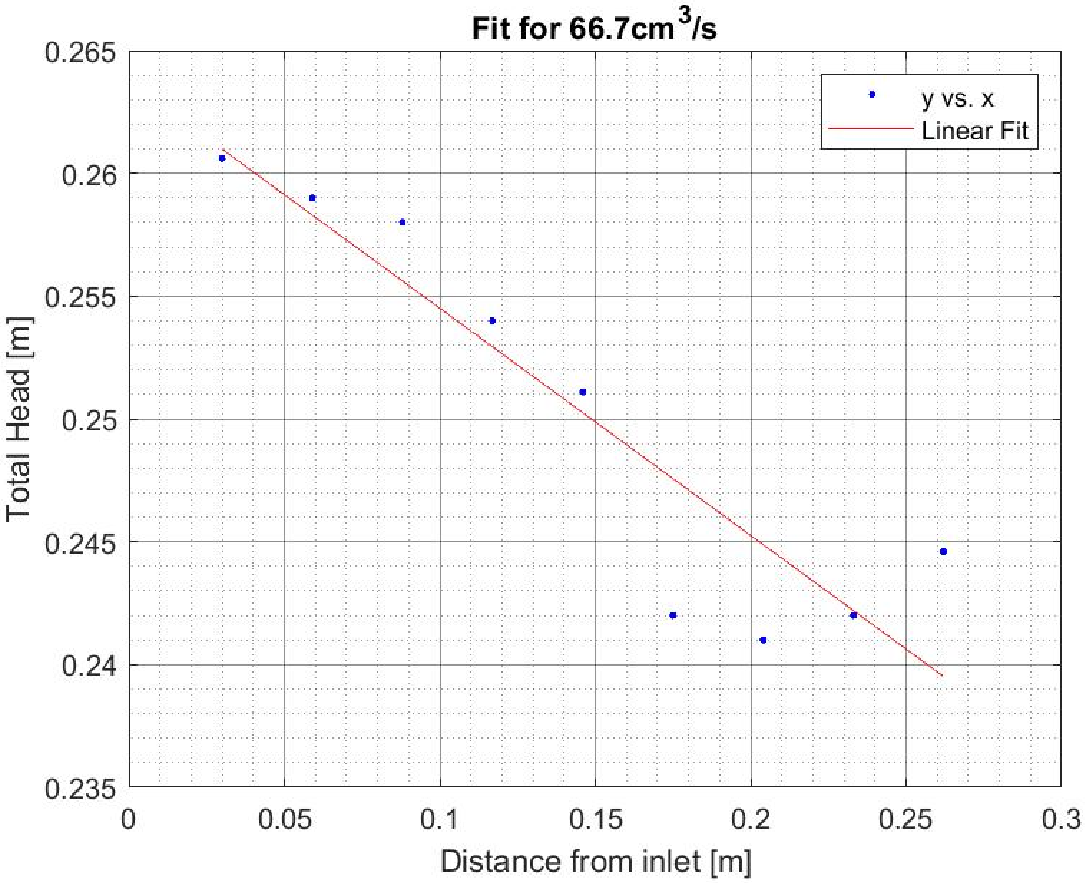

Bernoulli Plot 59.9 cm3/s
Bernoulli Constant [m]
Distance from inlet[m]
0.24
0.235
0.23
0.225
0.22
0
0.05
0.1
0.15
0.2
0.25
0.3



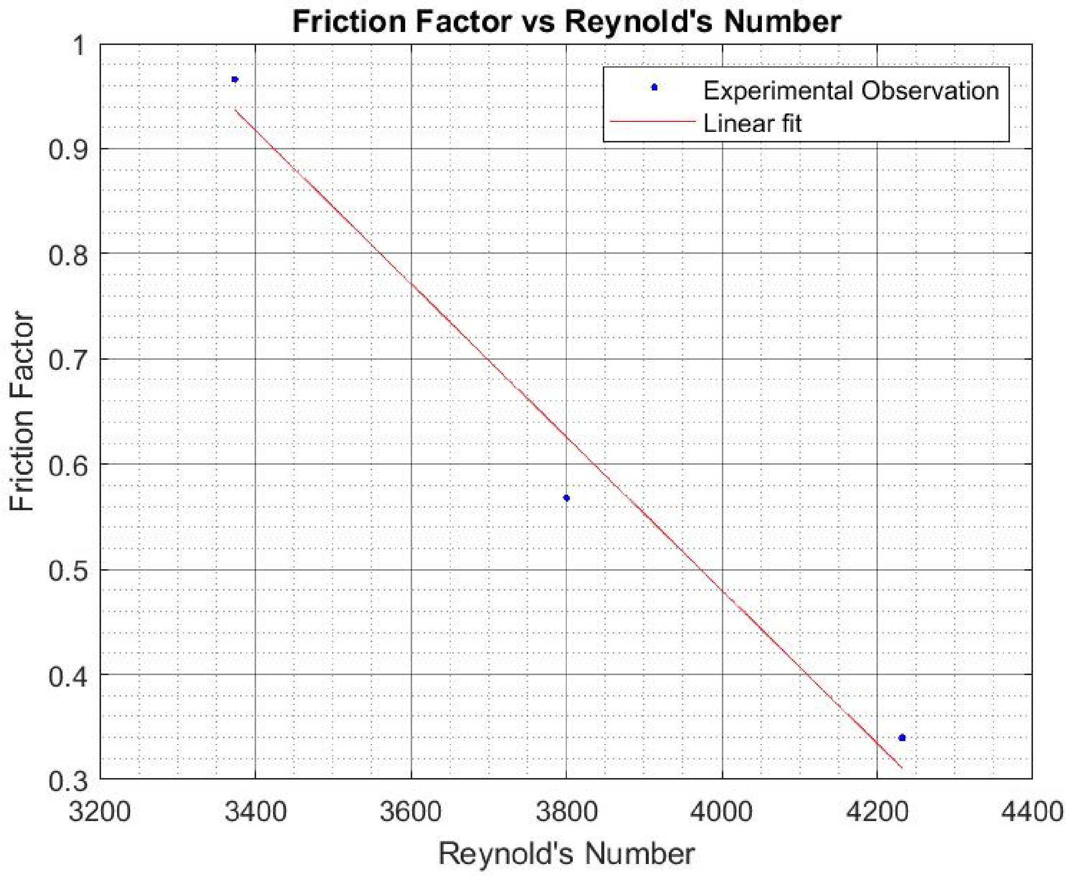
Fit for 59.9 cm³/s
Experimental Observation
Linear fit
Total Head [m]
Distance from inlet [m]
0.236
0.234
0.232
0.23
0.228
0.226
0.224
0.222
0.22
0.218
0.216
0
0.05
0.1
0.15
0.2
0.25
0.3



Bernoulli Plot 66.7 cm³/s
Bernoulli Constant [m]
Distance from inlet[m]
0.265
0.26
0.255
0.25
0.245
0.24
0
0.05
0.1
0.15
0.2
0.25
0.3


Fit for 66.7cm^3/s
y vs. x
Linear Fit
Total Head [m]
Distance from inlet [m]
0.265
0.26
0.255
0.25
0.245
0.24
0.235
0
0.05
0.1
0.15
0.2
0.25
0.3

f vs Re Plot



5. Interpretations or Tasks 1.2

Q. From the data that has been recorded by you, check whether the same satisfies the Bernoulli's equation. Also include the observed results in your report.

If we carefully look at the bernoulli equation, than we can predict that on plotting the total head versus distance from inlet of the pipe, we'll see a (slightly) decreasing graph due to the presence of losses.

$$\frac{P_1}{\gamma} + \frac{{V_1}^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{{V_2}^2}{2g} + z_2 + h_L$$

But here are what the graphs look like,

Q. If not, state what factors might cause a mismatch between theory and experimental data.

According to the experimental findings, the total head value first decreases and then increases. But after increase does not reach up to the initial head, which indicates that there are frictional losses in the pipe.

1. Theoretically, the total energy should be conserved during the flow inside the cross-sectional pipes' convergent and divergent regions. Friction between the fluid and the inside surface of the pipe still results in energy loss while moving from one end to the other. Therefore, the total head will vary depending on the distance from the reference.
2. There are also minor small head losses, which impact the outcomes of our experiments.
3. Readings should be taken as soon as possible since the water level in the pitot tube's pipes fluctuates over time. If we take the first pipe reading immediately, we should take the other readings afterward. Therefore, subsequent pitot tube measurements may vary by 1 or 2 mm throughout that time interval.
4. Since the pipes should be straight while taking the readings and the reading least count is in millimeters, it should be extremely accurate, but there may be some inaccuracy in the pitot tube readings.

Q. Based on the “Essential Background” questions given at the beginning of this document, try to quantify as many of these “factors” as possible and subsequently incorporate them in the modified Bernoulli’s equation (see Q2 in the Essential Background) in order to get a closer agreement between the theoretical prediction and the observations.

We have to account for the losses in the modified Bernoulli's equation. This is because in real life the are always losses due to friction which we have to account in equation. Theses calculation and equations are shown above. The energy expended to overcome friction induced by the pipe walls is represented by the head loss owing to fluid friction (Hf). Even though it represents a loss of energy from the perspective of fluid flow, it typically does not represent a substantial loss of the fluid's overall energy. Additionally, it does not go against the law of conservation of energy because the head loss brought on by friction causes an equivalent rise in the fluid's internal energy (u). The amount of these losses increases as the fluid passes through pumps, valves, fittings, entrances, exits, and other pipe features with rough inner surfaces.

So the final modified equation turns out as below. Where α is the Kinetic energy co-efficient and

*h*_lt is the head loss.

The values of α & h_{lT} can be determined by the below formulas:

$$\left(\frac{P_1}{\rho g} + \alpha_1 \frac{v_1^2}{2g} + z_1\right) - \left(\frac{P_2}{\rho g} + \alpha_2 \frac{v_2^2}{2g} + z_2\right) = \frac{h_{lT}}{g} \quad \ldots\ldots (1)$$

here, α = Kinetic Energy Coefficient
h_{lT} = Head Loss

$$\alpha = \left(\frac{U}{\bar{V}}\right)^3 \frac{2n^2}{(3+n)(3+2n)} \ \&$$

$h_{lT} = h_l + h_{lm}$, h_l = Major Losses & h_{lm} = Minor Losses

$$h_l = \frac{P_1 - P_2}{\rho} - g(z_2 - z_1), \text{ for } z_1 = z_2,\ h_l = \frac{P_1 - P_2}{\rho} \quad \ldots.(2)$$

$$h_{lm} = K\frac{\bar{V}^2}{2},\ K = \text{Loss Coefficient} \ldots..\ (3)$$

Now, lets quantity the head loss. From equation 2 and 3. we could determine the generalized total head loss. Now, consider the pipe flow:

$$\Longrightarrow\ h_l\ =\ f\,\frac{L}{D}\,\frac{\bar{V}^2}{2},\ \text{where,}$$

$$f\ =\ \text{friction factor}$$

$$f\ =\ \frac{64}{R_e},\qquad \text{for Laminar Flow}\}$$

$$\frac{1}{\sqrt{f}}\ =\ -\,2.0\ \log\left(\frac{e/D}{3.7}\ +\ \frac{2.51}{R_e\sqrt{f}}\right),\quad \text{for Turbulant Flow}\}\ \ \ldots.(4)$$

where,

$L\ =$ Length of Pipe
$D\ =$ Diameter of the Pipe
$e\ =$ Roughness of the Pipe
$R_e\ =$ Reynold's Number

$$\Longrightarrow\ h_{lm}\ =\ K\frac{\bar{V}^2}{2},\ K\ =\ \text{Loss Coefficient}$$

Hence,

$$\Longrightarrow\ h_{lT}\ =\ f\,\frac{L}{D}\,\frac{\bar{V}^2}{2}\ +\ K\frac{\bar{V}^2}{2}\quad \ldots\ldots..(5)$$

The friction factor is a dimensionless quantity useful to calculate the friction losses in the flow in the pipe, It can be calculated using the equation below: