LAB REPORT-1

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Experiment-III: Visualization and quantification of free and forced vortices

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The Essential Background:

1. *What are free and forced vortices? Briefly discuss a few applications/occurrences of such vortices.*

- **Free Vortex:** A free vortex is formed when water flows out of a vessel through a hole at the center of the bottom of the container. No external force is required to rotate the water.

  Example: Whirlpools in rivers and tornadoes are natural free vortices.

- **Forced Vortex:** External force is required in this case. It can be created by rotating the water container or paddling the fluid.

  Example: Rotational flow created by impellers of a pump in turbomachinery.

Applications:

- Study of the vortices is critical to examine the behavior of natural events such as hurricanes, tornadoes, and whirlpools (free vortices).

- Engineers and designers need to study the vortices so that they can characterize forced vortices generated in machinery such as centrifugal pumps or turbines.

- Vortices often have adverse effects, as have been seen during hurricanes, tornadoes, or scour holes created downstream of a dam outlet; however, understanding vortex behavior has enabled engineers to design turbomachinery and hydraulic structures that take advantage of these phenomena. For example, hydrodynamic separators have been developed, based on vortex behavior (swirling flow), to separate solid materials from liquids. This type of separator is used in water treatment plants.

Objective:

The objective of this lab experiment is to study and compare the water surface profiles of free and forced vortices.

Equipment:

Transparent cylinders have been used.

Fig: Cylinder 1: Forced Vortex and Cylinder 2: Free vortex

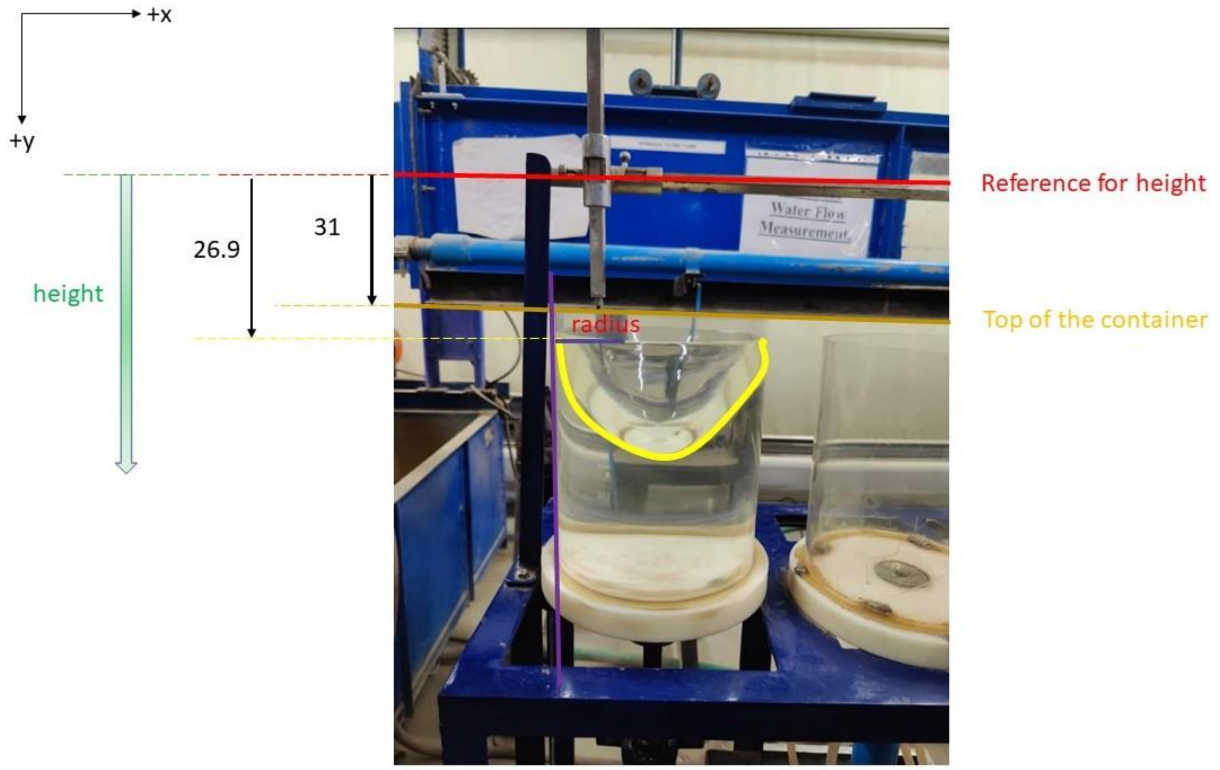
Cylinder 1:

1. It is kept on rotating machinery which, when powered, rotates, and in turn, the cylindrical vessel rotates to develop a forced vortex.

Cylinder 2:

1. contains two pairs of diametrically opposite inlet tubes in order to create a swirling motion of the water entering the vessel during the free vortex experiment.
2. and an outlet is a hole at the center of the bottom.

FORCED VORTEX

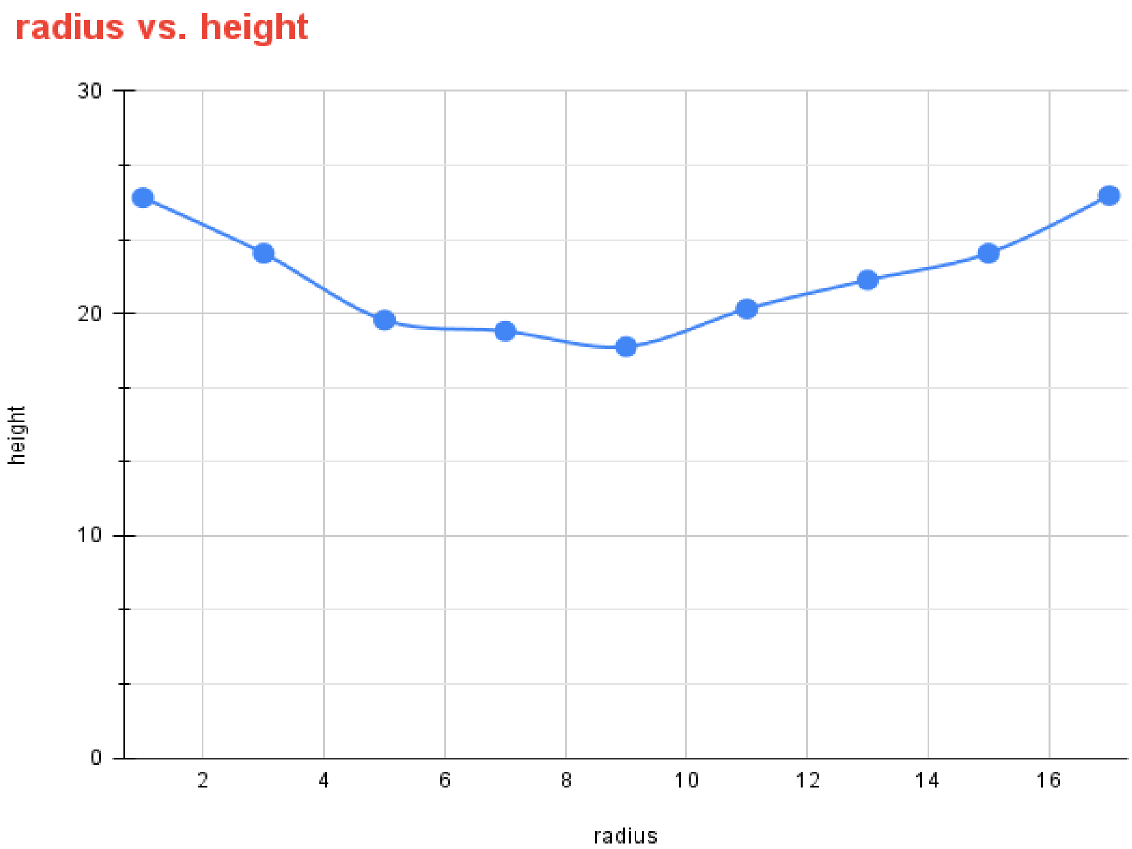


Experimental Data:

Case1: The speed of the motor = 130 rpm

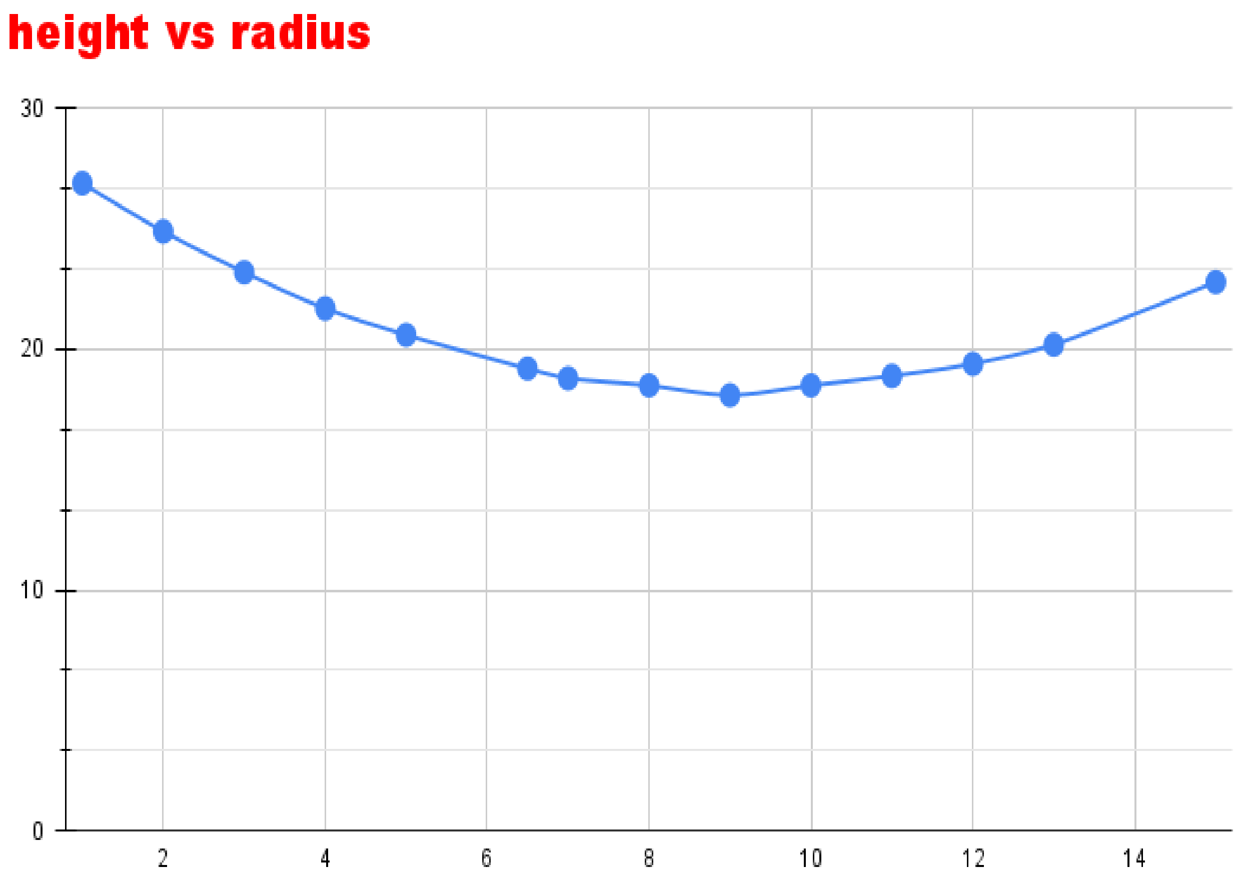
Radius	Height
1	25.2
3	22.7
5	19.7
7	19.2
9	18.5
11	20.2
13	21.5
15	22.7
17	25.3

Note: The heights in the above table are with respect to the reference shown in the figure.



Case2: Speed of the motor = 155rpm

radius	height (h)
1	26.9
2	24.9
3	23.2
4	21.7
5	20.6
6.5	19.2
7	18.8
8	18.5
9	18.1
10	18.5
11	18.9
12	19.4
13	20.2
15	22.8



Theoretical computation:

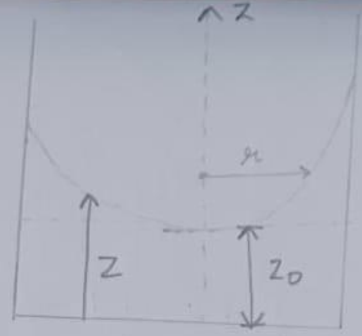


Fig: Typical Surface Profile of a forced vortex

Fluid rotates with uniform angular velocity Ω about z-axis.

The fluid is essentially taken to be rotating as a solid body.

Tangential velocity at a radius r $= \Omega r$

Assumption: inviscid flow [Viscosity = 0]

compressible [density of water is constant]

Navier Stoke's Eq^n:

$$\rho \frac{D\vec{V}}{Dt} = -\nabla P + \rho \vec{g} + \mu \nabla^2 \vec{V}$$

Total derivative

$\rho \left[\frac{\partial V}{\partial t} + (V \cdot \nabla) V \right]$

Change of velocity with time

Pressure gradient

Body force term

μ: viscosity of fluid

Applying assump^n of inviscid flow:

Navier Stokes reduces to Euler's eq^n

$$\rho \frac{Du}{Dt} = -\nabla P + F$$

acc^n of fluid element at radius r, must be $\frac{v_T^2}{r}$

v_T = tangential velocity

For radial eq^m:

$$\frac{\partial P}{\partial r} = \rho \frac{v_T^2}{r} \quad ; \quad \underline{v_T = \Omega r}$$

$$\underline{\frac{\partial P}{\partial r} = \rho \Omega^2 r} \quad \text{—— ①}$$

For vertical eq^m:

$$\frac{\partial P}{\partial z} = -\rho g \quad \text{—— ②}$$

Static pressure, P is varying in both r & z dir^{n}s,

So $P = f(r, z)$.

Integrating ① & ②:

$$\frac{dP}{dr} = \frac{\partial P}{\partial r} + \frac{\partial P}{\partial z}\frac{dz}{dr}$$

$$dP = \frac{\partial P}{\partial r}dr + \frac{\partial P}{\partial z}dz$$

Substituting for $\partial P/\partial r$ & $\partial P/\partial z$

$$dP = \rho \Omega^2 r\,dr - \rho g\,dz$$

Integrating :

$$p - p_0 = \frac{1}{2}\rho \Omega^2 r^2 - \rho g (z - z_0) \quad \text{——} (3)$$

Pressure at free surface = Atmospheric pressure
= p_0

At $z = z_0$, $r = 0$.

$$\Rightarrow \quad z - z_0 = \frac{\Omega^2 r^2}{2g} \quad \Rightarrow \quad \boxed{z = z_0 + \frac{\Omega^2 r^2}{2g}}$$

Hence the free surface has a parabolic distribuⁿ.
Eqⁿ (3) can be re-arranged in a form analogous to Bernouli's eqⁿ.

$$p + \frac{1}{2}\rho r^2 \Omega^2 + \rho g z = p_0 + \rho r^2 \Omega^2 + \rho g z_0 \quad (4)$$

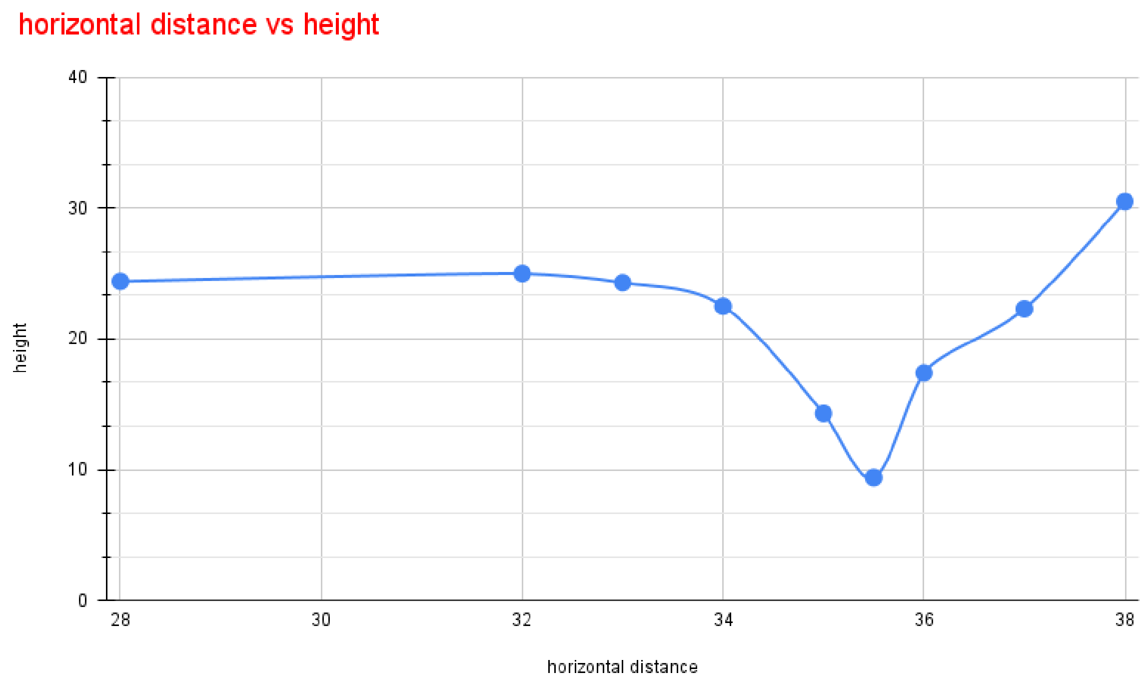
Eqⁿ (4) clearly shows that the energy associated with the streamlines in a forced vortex ↑ses with ↑sing radius.

The energy to sustain the flow comes from the motor driving the vessel.

FREE VORTEX

Experimental Data:

horizontal distance	height
28	24.4
32	25
33	24.3
34	22.5
35	14.3
35.5	9.4
36	17.4
37	22.3
38	30.5



Theoretical computation:

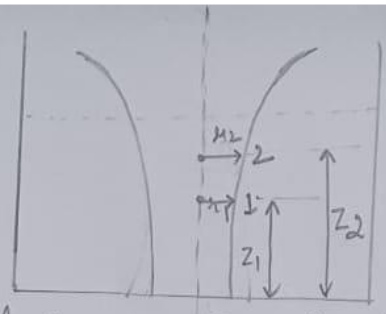


Fig: Typical Surface Profile of a free vortex.

Obtaining relationship b/w velocity & radius:

External Torque on the water as a system must be zero, i.e. Time rate change of angular momentum must be zero.

Consider a particle of mass m at a radius r from the axis of rot^n, having a tangential velocity v.

Angular Momentum : mvr.

Time rate of change of momentum $= \frac{\partial}{\partial t}(mvr) = 0$

$mvr = \text{constant} = C$

$vr = C$ [∵ m is constant]

$V = \frac{C}{r}$ (C is known as strength of vortex)

$\boxed{V \propto \frac{1}{r}}$ ⇒ tangential velocity is inversely proportional to distance r.

As derived in the case of FORCED VORTEX, we have

$$dP = \frac{\rho v^2}{r} dr - \rho g dz$$

Putting $V = \frac{C}{r}$

$$dP = \rho \times \frac{c^2}{r^2 \times r} dr - \rho g \, dz$$

$$= \frac{\rho c^2}{r^3} dr - \rho g \, dz$$

Consider point 1 & 2 in the fluid having radii r_1 & r_2 from the central axis, & at heights z_1 & z_2 from the bottom of the vessel.

Integrating above eq^n for points 1 & 2:

$$\int_1^2 dP = \int_1^2 \frac{\rho c^2}{r^3} dr - \int_1^2 \rho g \, dz$$

$$P_2 - P_1 = \rho c^2 \int_1^2 \frac{dr}{r^3} - \rho g \int_1^2 dz$$

$$= \rho c^2 \left[\frac{r^{-3+1}}{-3+1} \right]_1^2 - \rho g (z_2 - z_1)$$

$$P_2 - P_1 = -\frac{\rho c^2}{2} \left[\frac{1}{r_2^2} - \frac{1}{r_1^2} \right] - \rho g (z_2 - z_1)$$

$$\because V_2 = c/r_2 \; ; \; V_1 = c/r_1$$

$$P_2 - P_1 = \rho/2 \left(V_1^2 - V_2^2 \right) - \rho g (z_2 - z_1)$$

Dividing both sides by ρg:

$$\frac{P_2 - P_1}{\rho g} = \frac{V_1^2 - V_2^2}{2g} - (z_2 - z_1)$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$