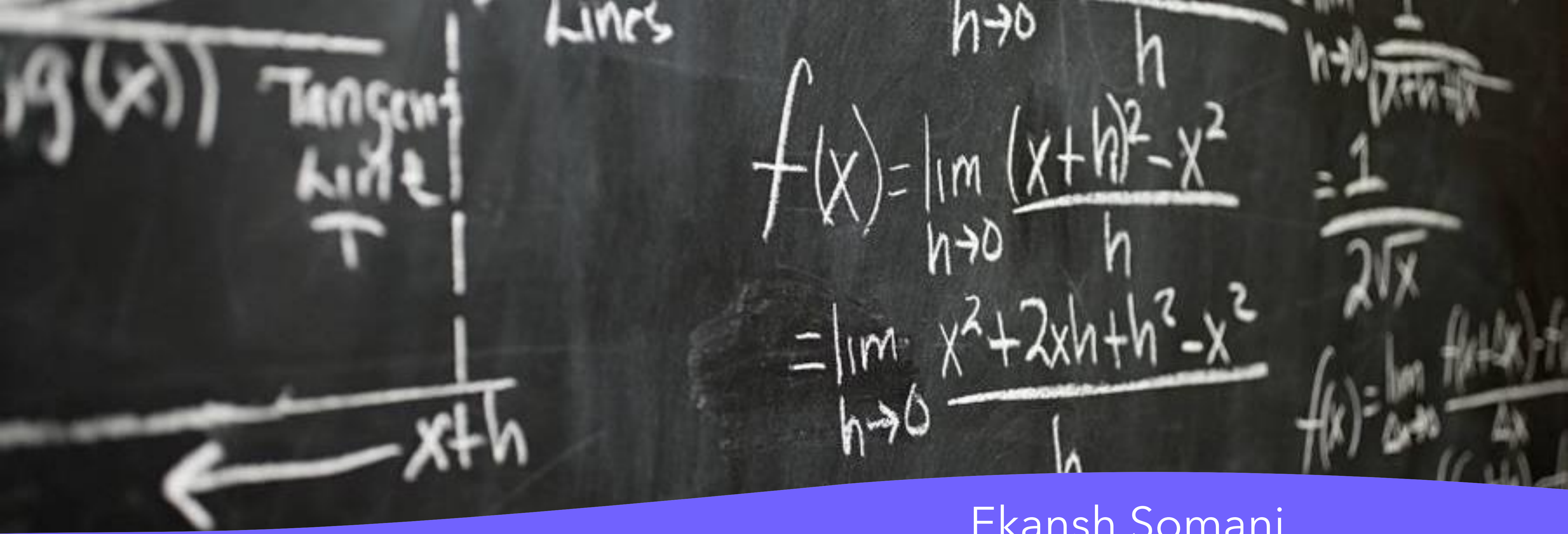
# If a Butterfly’s flapping of wings can create a tornado, it can also prevent one.

Edward Lorenz



# Lorenz Equations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# Content

| Introduction | Equation and Chaos. | Numerical Methods | Observation | Summary |
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| Introducing Lorentz equation, and some of its applications. | More details about the equation. Its uses in various environments. And the predictability of Weather. And Chaos. | Brief details about the used methods and code. | The graphs, their intricacies, and the conclusions drawn from them. | The overall learnings, and conclusions drawn from the project. |

# Introduction

- Lorenz Equations are a set of ordinary differential equations which gives chaotic solutions for certain parameter values and initial conditions.
- The equations have three variable parameters along with initial conditions.
- They were first developed and studied by Edward Lorenz in 1963.

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y$$

$$\frac{dz}{dt} = xy - \beta z,$$

# Equations, Predictions, And Chaos

# The Equations

1. Lorentz used the equations to describe the properties of a two-dimensional fluid layer which is:

uniformly warmed from below and

uniformly cooled from above where,

x → Rate of Convection

y → Horizontal Temperature Variation

z → Vertical Temperature Variation

sigma, pho, and beta → System parameter proportional to Prandtl number, Rayleigh Number, and physical dimensions of layer

He took the values, $\sigma = 10, \beta = \frac{8}{3}, \rho = 28$, famously known as Lorentz Attractor Values.

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy - \beta z,$$

2. They also arise in simplified models for lasers, dynamos, thermosyphons, brushless DC motors, Electric Circuits, Chemical Reactions and Forward Osmosis

3. They are non-linear, aperiodic, three-dimensional, and deterministic in nature.

# Prediction And Chaos

1. Lorentz equations diverge away over time for very small changes in initial values.
2. You need to measure the initial value to a high degree of precision and accuracy to be able to predict the weather for a few days.
3. This diversion is called Chaos.

# Numerical Methods

Methods of Iterative Guesses

# Euler Method

Extrapolates linearly over an arbitrary step size h using the slope (first derivative of y).

$$\frac{dy}{dx} = f(x, y)$$

$$y_{i+1} = y_i + f(x_i, y_i)h$$

# Runge-Kutta Method

1. Runge Kutta Methods try to achieve the accuracy of the Taylor series approach without solving the higher derivatives.
2. The Runge Kutta Method is derived for n-orders by comparing two equations obtained (1) from Taylor expansion of $y_{i+1}$ in terms of $y_i$ from Taylor method and (2) the straight replacing of k's in $\phi$, then replacement of $\phi$ in $y_{i+1}$, and doing Taylor expansion of f.
3. Classical Runge-Kutta Method is a fourth order method.

$$\frac{dy}{dx} = f(x, y)$$

$$y_{i+1} = y_i + \phi(x_i, y_i, h)h$$

$$\phi = \sum_{j=0}^{n} a_j k_j$$

$$k_j = f(x_i + p_{j-1}h, y_i + q_{j-1,1}k_1 h + q_{j-2,2}k_2 h + \ldots + q_{1,j-1}k_{j-1}h)$$

$$y_{n+1} = y_n + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4)$$

$$t_{n+1} = t_n + h$$

$$k_1 = f(t_n, y_n)$$

for n = 1,2,3.., using equation

$$k_2 = f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right),$$

$$k_3 = f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right),$$

$$k_4 = f(t_n + h, y_n + hk_3)$$

# Runge-Kutta Gil Method

1. The Runge-Kutta-Gill method is a variant of the classical Runge-Kutta method being used for approximating the solution of the differential equation given by, Y ′ (t) = f(t, Y )

$$y_{n+1} = y_n + \frac{1}{6}[k_1 + (2 - (\sqrt{2})k_2) + ((2 + \sqrt{2})k_3 + k_4] + O(h^5)$$

where

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1)$$

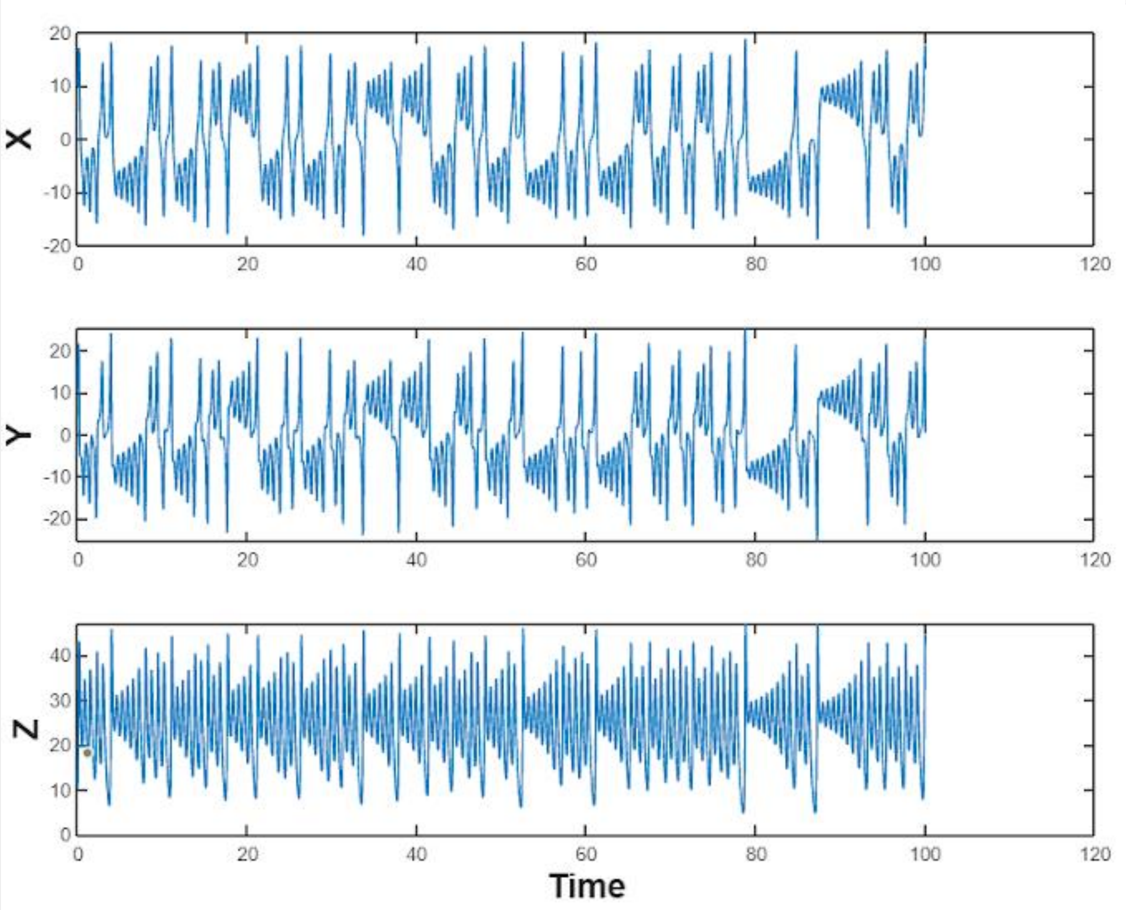
$$k_3 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}(-1 + \sqrt{2})k_1 + (1 - \frac{1}{2}\sqrt{2})k_2]$$

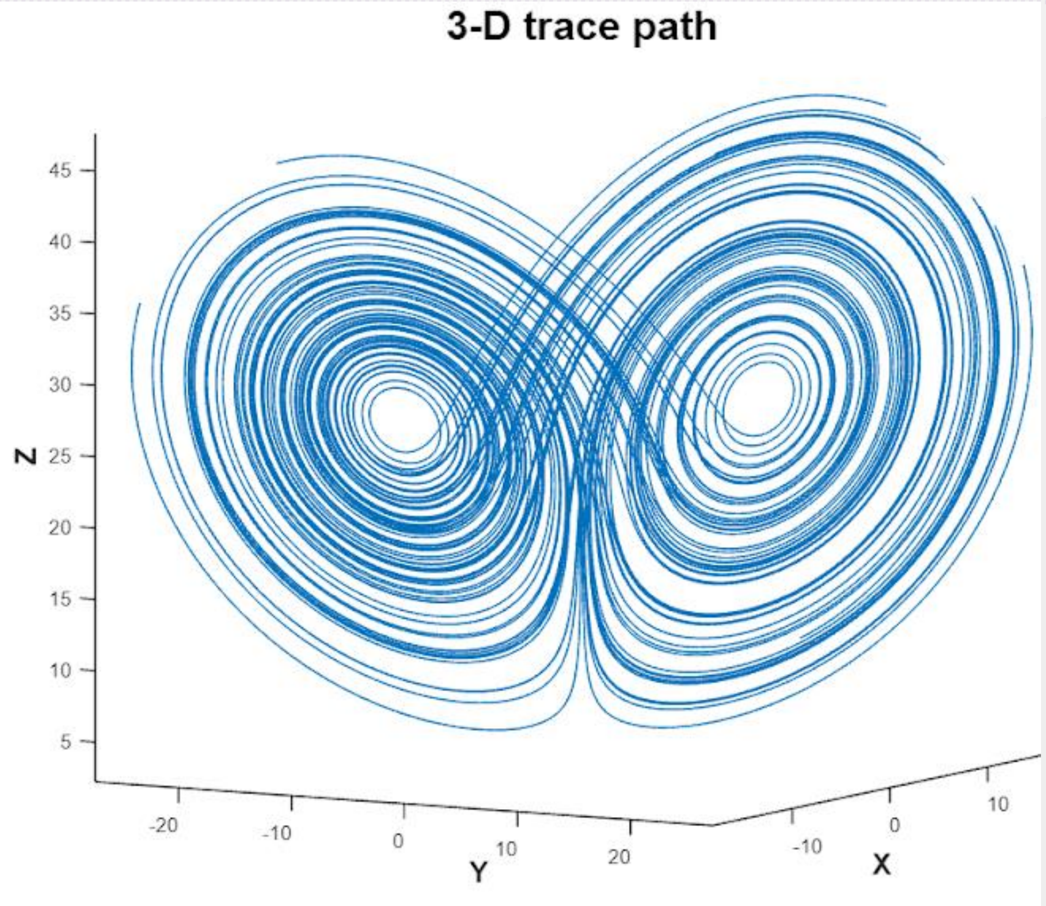
$$k_4 = hf[x_n + h, y_n - \frac{1}{2}\sqrt{2}k_2 + (1 + \frac{1}{2}\sqrt{2}k_3]$$

# Observations

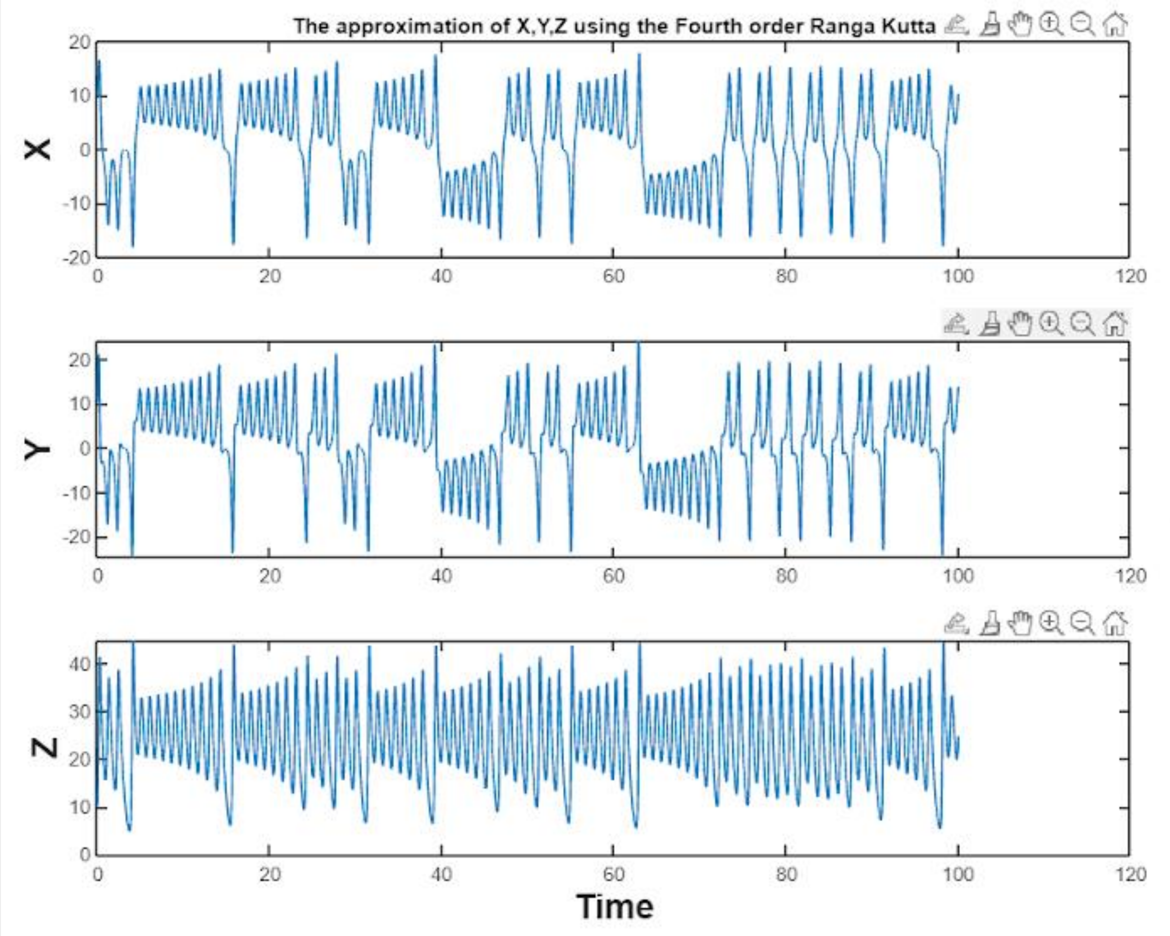
Subtitle

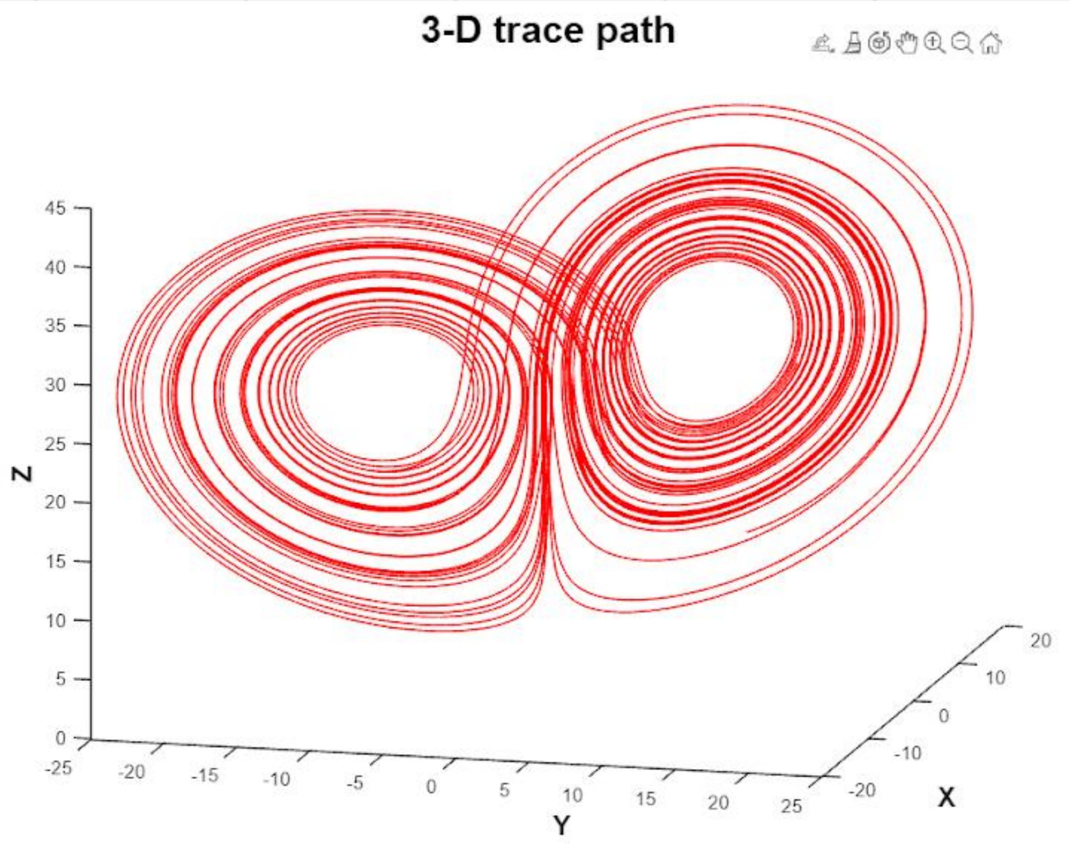
# Euler Method



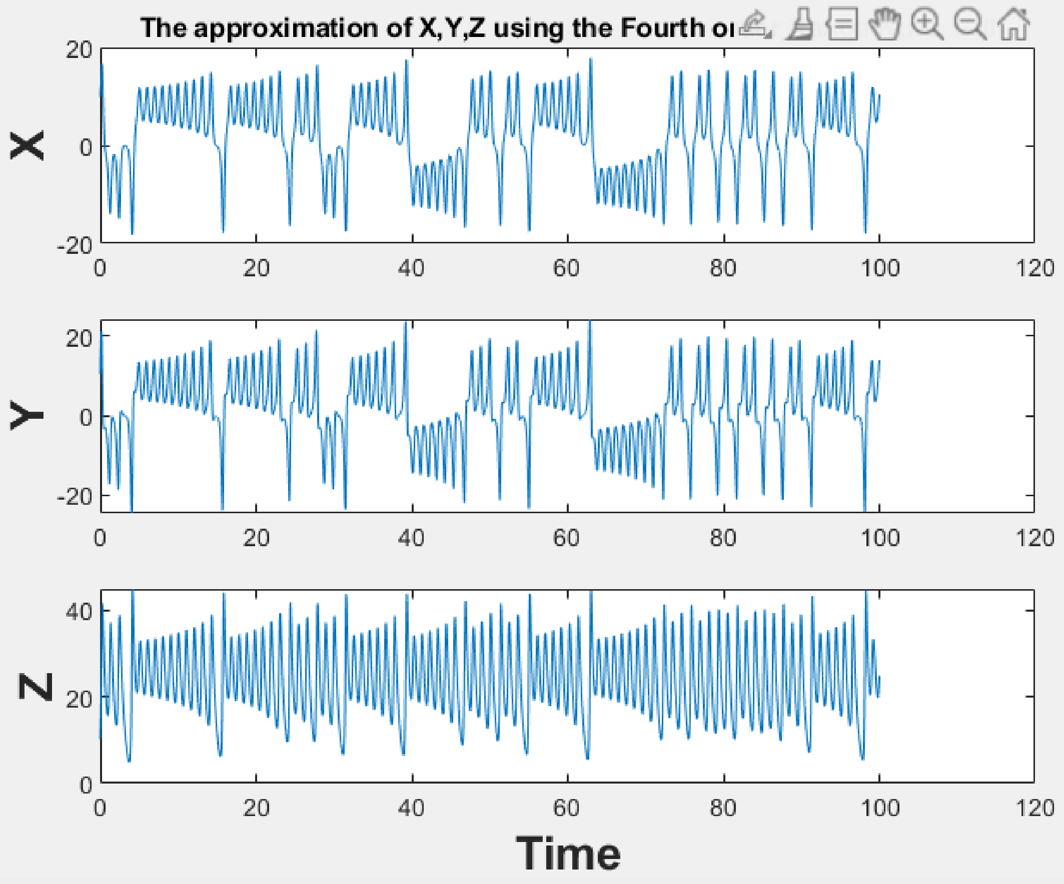



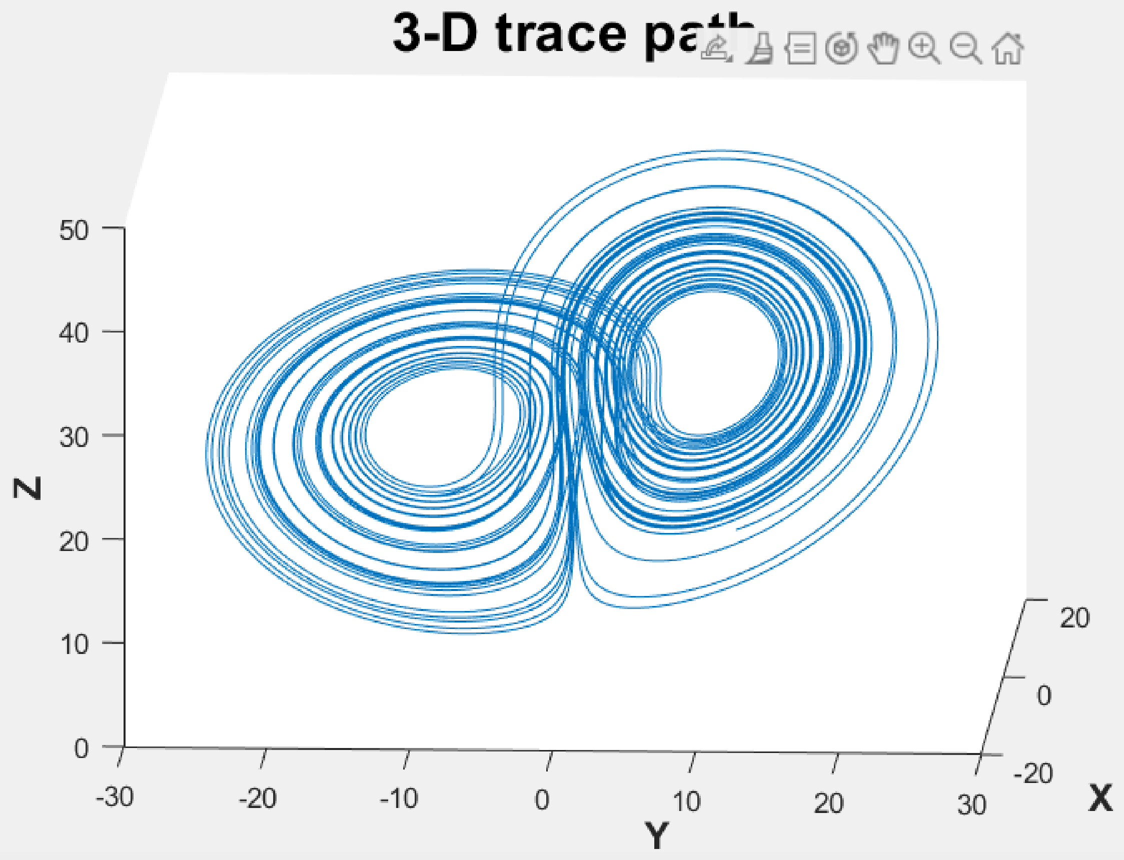
# Results from Runge Kutta Method



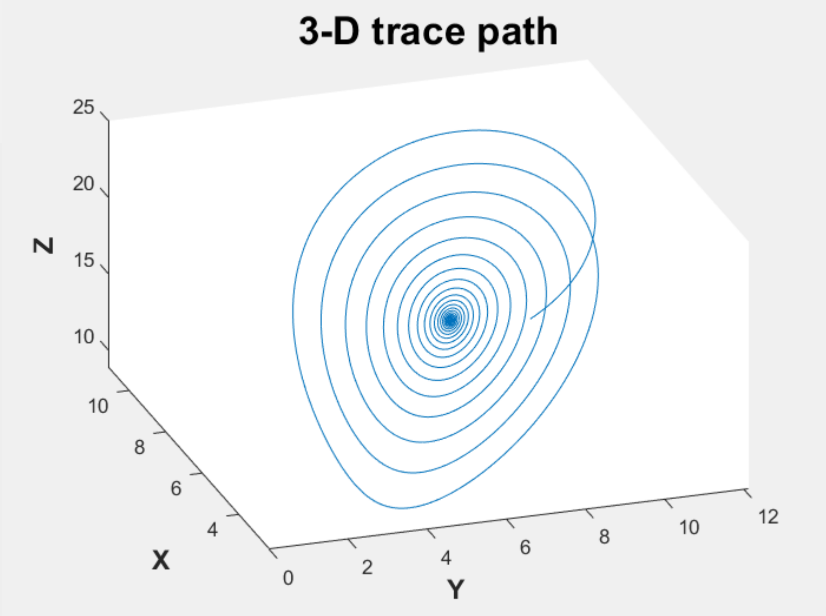



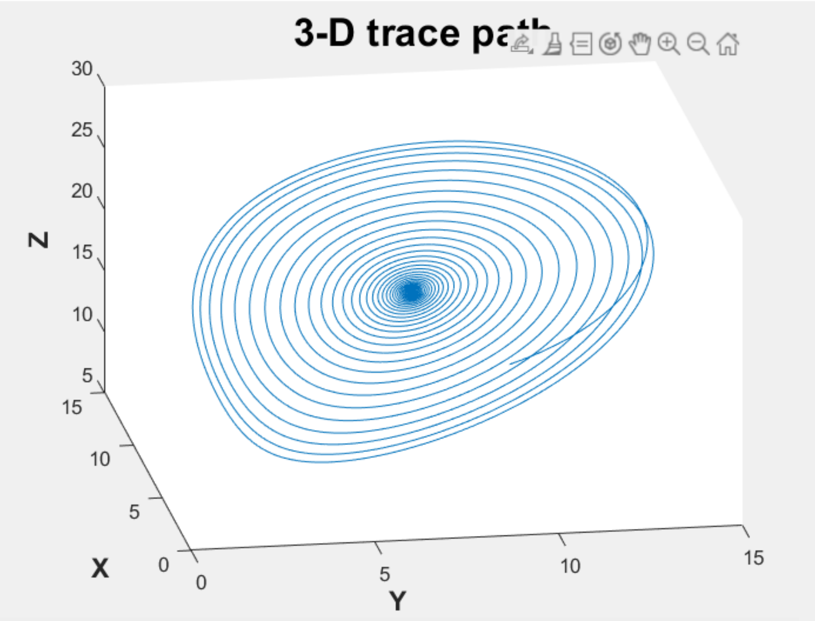
# Runge-Kutta Gill Method

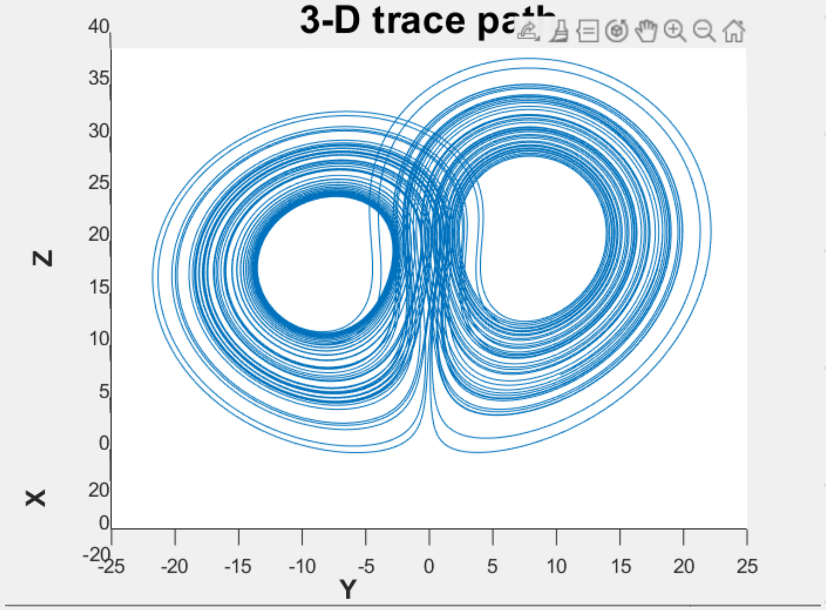


# Lorenz System for distinct values of $\rho$



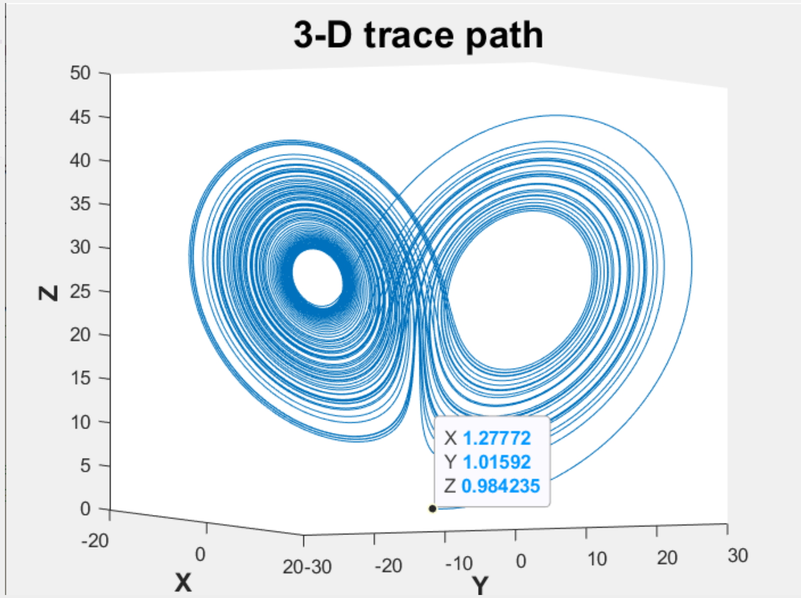



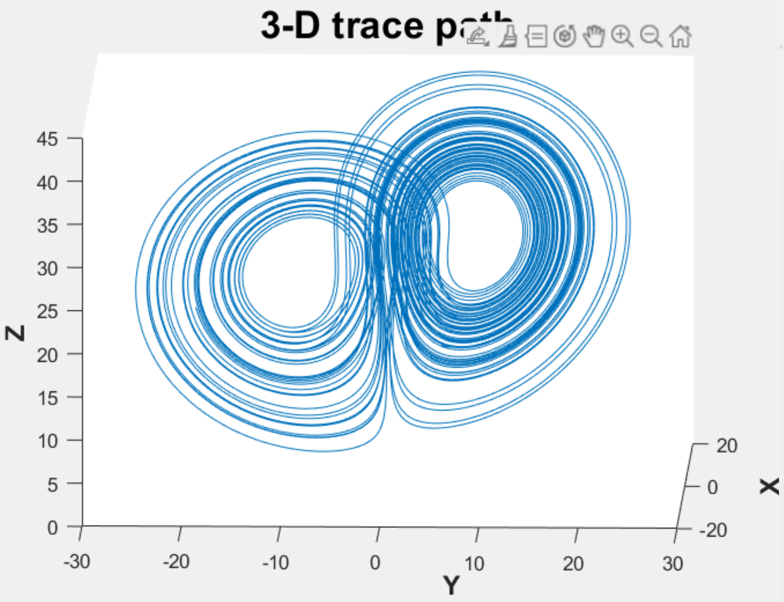

$\sigma = 10$
$\rho = 17$
$\beta = 8/3$

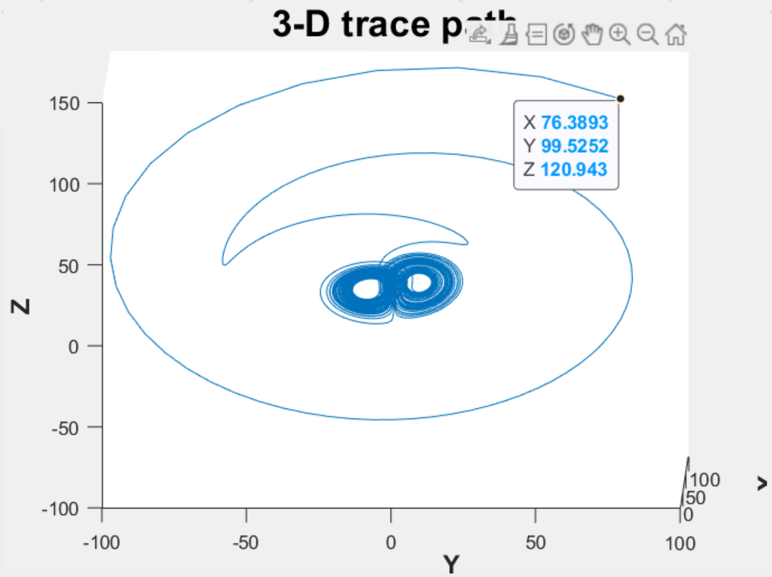
$\sigma = 10$
$\rho = 20$
$\beta = 8/3$

$\sigma = 10$
$\rho = 25$
$\beta = 8/3$

# Different Initial Conditions



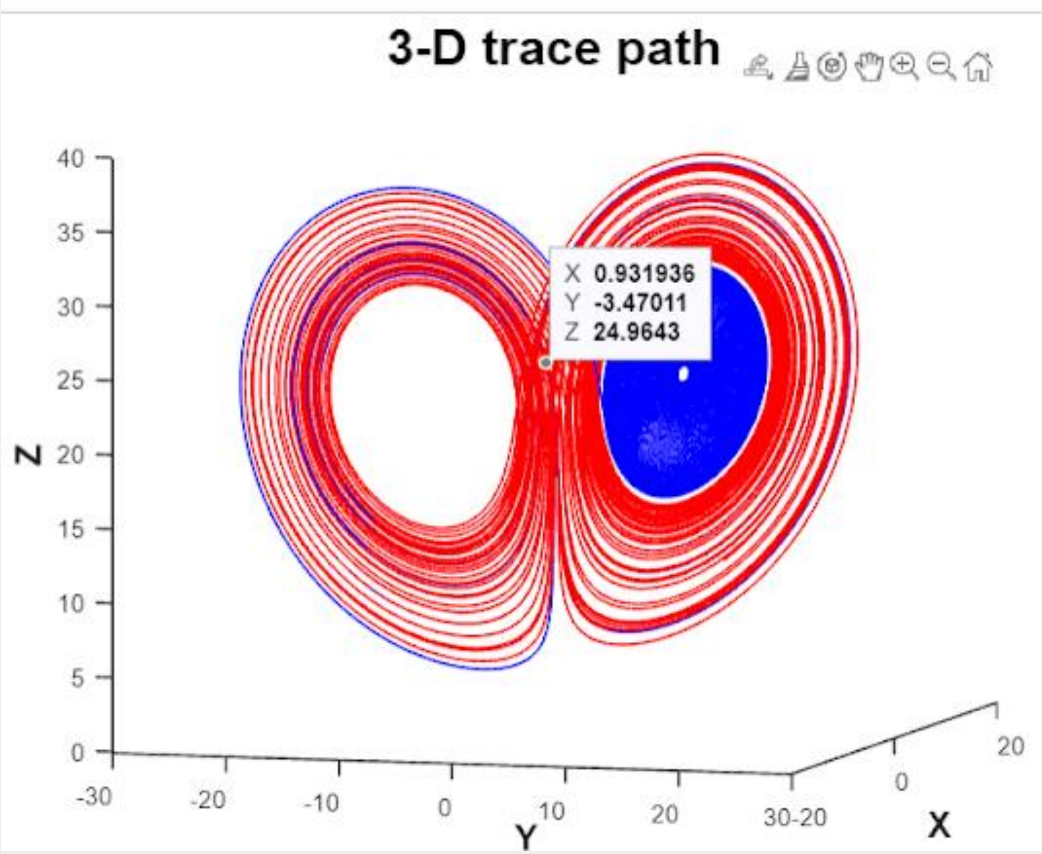




$X_0 = 1$
$Y_0 = 1$
$Z_0 = 1$

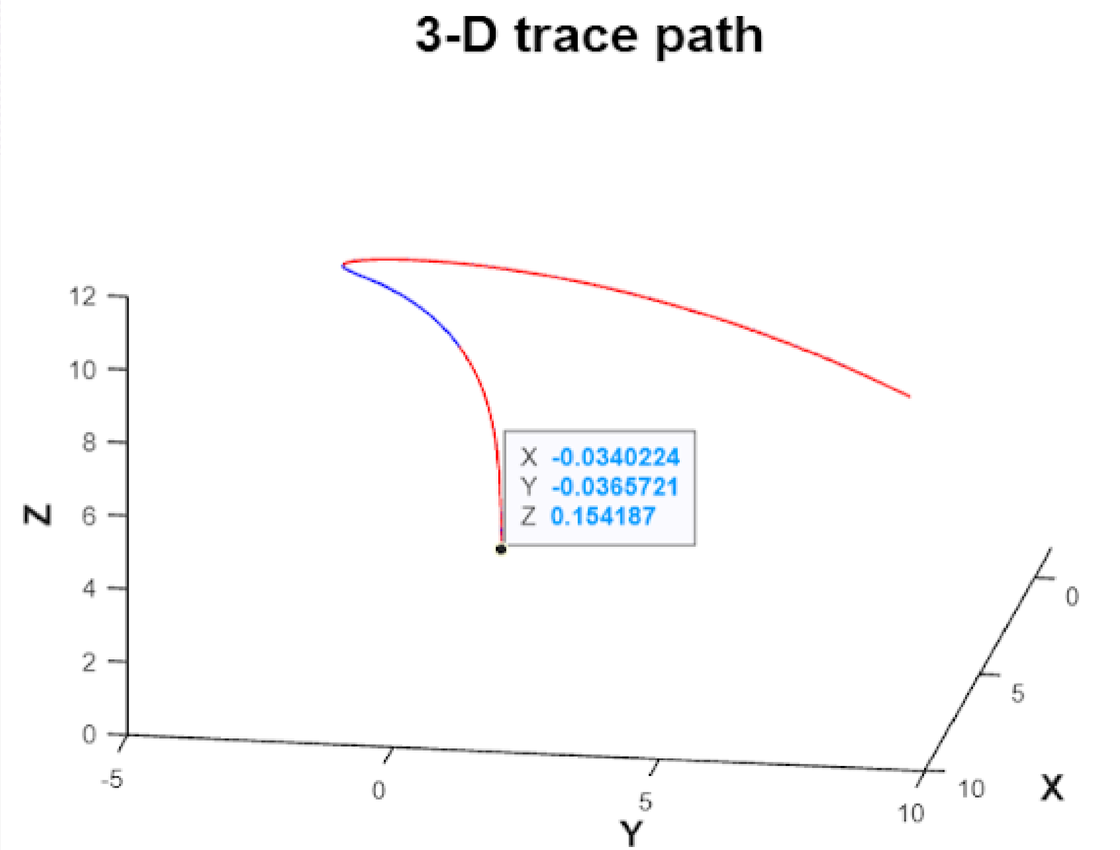
$X_0 = 5$
$Y_0 = 5$
$Z_0 = 15$

$X_0 = 15$
$Y_0 = 25$
$Z_0 = 50$

# Sensitivity

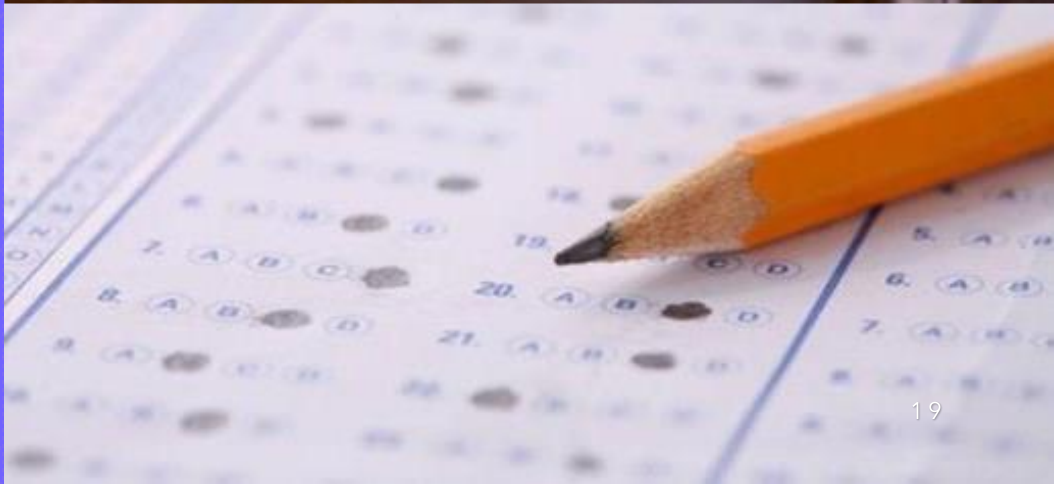



# Convergence



# Summary

- Lorenz equations hide within themselves a butterfly like structure, which reveals itself when the $\rho$ is high enough for the graph to not collapse to the two critical points.
- For small changes in the initial values, the resultant graphs trace out completely different paths eventually.
- In fine, the numerical methods can be used to solve the Lorenz Equation.
- The accuracy of the solution remains in doubt without verification from real world data.

# Thank you

Zeeshan, Ganesh, Sunny, Karthikeya, and Ekansh.