

Problem-2

Gauss Elimination

Forward Elimination

- n-1 Loops (kth)
 - n-(k+1) Loops (ith)
 - Division $a_{i,k} / a_{k,k}$
 - n-(k+1) Loops (jth)
 - 2 Operations (1 subtraction and 1 multiplication) $a_{i,j} - factor \cdot a_{k,j}$
 - 2 Operations Again

$$\begin{aligned} \text{Total} &= \sum_{k=1}^{n-1} (n-k-1) * (1 + 2(n-k-1) + 2) \\ &= \sum_{k=1}^{n-1} (n-k-1)(2n-2k+1) \\ &= \sum_{k=1}^{n-1} (2n^2 + 2k^2 - 4kn + k - n - 1) \\ &= 2n^2(n-1) + \frac{(n-1)n(2n-1)}{3} - 4n \frac{n(n+1)}{2} + \frac{n(n+1)}{2} - (n+1)(n-1) \\ &= \frac{2n^3}{3} - \frac{11n^2}{2} - \frac{n}{6} + 1 \end{aligned}$$

Back Substitution

- 1 Operation ($b_n / a_{n,n}$)
- n-1 Loops (ith)
 - n-i-1 loop (jth)
 - 2 Operations
 - 1 Operations

$$\begin{aligned}
\text{Total} &= 1 + \sum_{i=1}^{n-1} \left(\sum_{j=i+1}^n 2 + 1 \right) \\
&= 1 + \sum_{i=1}^{n-1} (2n - 2i - 1) \\
&= 1 + 2n(n-1) - (n-1)n - n + 1 \\
&= n^2 - 2n + 2
\end{aligned}$$

Thus, Overall

$$\text{Complexity} = O(n^3)$$

Gauss Seidel Iteration

- For each iteration (Max 20-30 iterations, Not related with n)
 - n Equations

$$x_j = \frac{b_j - a_{j1}x_1 - a_{j2}x_2 - \dots - a_{jn}x_n}{a_{jj}}$$

- $n-1$ Multiplications
- n subtractions
- 1 division

$$Total < 30 * n * (n - 1 + n + 1) < 60n^2$$

Usually 7-8 iterations

$$\Rightarrow Total \approx 16n^2$$

$$\text{Thus, Complexity} = O(n^2)$$