Problem-4

$$E(x) = \frac{e^{-x}}{x}$$
$$S(z) = \frac{sin(z)}{z}$$
$$E'(x) = -\frac{1+x}{x^2}e^{-x}$$
$$S'(z) = \frac{zcos(z) - sin(z)}{z^2}$$
$$E''(x) = e^{-x}\frac{x^2+2x+2}{x^3}$$
$$S''(z) = -\frac{(z^2-2)sin(z) + 2zcos(z)}{z^3}$$
$$E'''(x) = -e^{-x}\frac{x^3+3x^2+6x+6}{x^4}$$
$$S'''(z) = \frac{3(z^2-20sin(z) - z(z^2-6)cos(z))}{z^4}$$
$$E''''(x) = e^{-x}\frac{x^4+4x^3+12x^2+24x+24}{x^5}$$
$$S''''(z) = \frac{4z(z^2-6)cos(z) + (z^4-12z^2+24)sin(z)}{z^5}$$

Computing Sections

Trapezoid Method

$$E_a = -\frac{(b-a)^3}{12n^2}\bar{f}''$$
$$\text{Given, } E_a < tol$$
$$\Rightarrow n^2 > -\frac{(b-a)^3}{12 * tol}\bar{f}''$$

Here, we can keep the max value of double derivative instead of average to get the max possible error.

Simpson's Rule

$$E_a = -\frac{(b-a)^5}{180n^4}\bar{f}^4 \Rightarrow n^4 > -\frac{(b-a)^5}{12 * tol}\bar{f}^4$$

Now we apply the integral formulae as given in the book.

Approximations

$$S(z) = \int_0^z \frac{sin(z)}{z} \approx \int_{0.1}^z \frac{sin(z)}{z}$$

Code Notation

Eit → Trapezoidal answer for E(x)
Eis → Simpson's Rule over E(x)
Sit → Trapezoidal Answer for S(z)
Sis → Simpson's Rule over S(z)

TrapezoidE → Trapezoidal Function Applied over E(x)
Funcx(x) → E(x)
DoubleDerx → Double derivative of E(x)
Maxix → Finds maximum of E(x)
SimpsonE → Simpson Applied over E(x)

Output

```
>> [Eit, Eis, Sit, Sis] = T4_20110065(5, 10, 10e-3)

Eit =

   0.001278390340290


Eis =

   0.001159687491527


Sit =

   1.558220820129614


Sis =

   1.558417737396075
```