Statistical Analysis Of 273 years of Sunspot Counts

Abstract

💡 Sunspots are regions of reduced temperature that appear on sun's surface due to blocking of convection. This block is usually caused by high concentration of magnetic flux, and appears as dark spots that are darker than the rest of the sun. Sunspots decay as magnetic pressure causes the magnetic field to disperse and thus increases the convection once again. They can last from anywhere between a few days to many weeks. They usually form in active regions ("temporary region in sun's atmosphere characterized by strong and complex magnetic field")[1] The statistical analysis of sunspots helps us in understanding the magnetic activity over sun's surface (also called solar magnetic field) and how it varies with time. Sunspots have been observed since 17^{th} century, and sunspot time series is the longest continuously observed time series of any natural phenomena. In this article we will try to analyze the sunspots and develop an intuitive understanding of how the sunspots vary with time. In the process, we will also get an understanding of the magnetic activity over sun's surface.

Keywords: Active Region, Solar Magnetic Field, and Solar Cycle

Introduction

What are sunspots?

Sunspots are solar phenomena that appear as dark areas on the surface of the Sun. They are where intense magnetic fields poke through the outer layers of material, and they can be up to 8 times larger than Earth. Sunspots can help us predict the amount of light that will reach Earth during an eclipse. They can also help us in predicting solar flares and coronal mass ejections, and send an advance warning signal to satellites and astronauts at risk.

Sunspots were first noticed by Galileo Galilei in 1610 when he pointed a telescope at the Sun. They were named in 1848 by Samuel Heinrich Schwabe, a German astronomer who noticed they seemed to come and go on an 11-year cycle. The number of sunspots goes up and down every 11 years because that's how long it takes for the Sun's magnetic field to refresh itself.

About The Datasheet

- [ ] Column 1 in the given datasheet represents the year
- [ ] Column 2 represents the month.
- [ ] Column 3 represents the date in a fraction of the year for the middle of the corresponding month. For instance 1794.042(1794+1/24) this signifies the January month and 1794.123(1794+3/24) signifies the February month and so on.
- [ ] Column 4 represents the Monthly smoothed total sunspot number.

How are sunspots identified ?

- Sunspots are black regions that appear on the Sun's surface. Because they are cooler than other parts of the Sun's surface, they seem dark. Solar flares are a burst of energy created by magnetic field lines twisting, crossing, or reforming near sunspots.
- Scientist **Fabricius** monitored sunspot clusters for months with a pinhole camera, revealing that they vanished over the Sun's western border, then reappeared two weeks later on the other side. **After this proved, Fabricius** became the **first solar scientist** to prove they were a part of the Sun's spinning surface.
- Land-based and space-based solar telescopes are used to observe sunspots. Sunspots and sunspot areas are studied with specialized tools such as spectroscopes and spectroheliоscopes.

For this analysis, we have the sunspot data from 1749 to upto 2 Feb 2022.[2]

Methods Used

Circular Analysis

The circular analysis technique maps pseudo-periodic time data of maxima or minima on a circular graph in an attempt to try to find its time period.

Here's How it Works

- Let us say that we have n independent events occurring at time $t_1, t_2, t_3, ..., t_n$ respectively.
- Then we can plot these events on a circle on unit circumference using the formula,

$$\theta_i(\tau) = 2\pi\frac{t_i}{\tau}$$

where, $i = 1, 2, ..., n$ & τ is an arbitrary constant.

- We can also define the arithmetic mean of the coordinates of all of these points as,

$$\bar{x}_\tau = \frac{\sum_i cos(\theta_i(\tau))}{n}$$

$$\bar{y}_\tau = \frac{\sum_i sin(\theta_i(\tau))}{n}$$

- Now, maximizing the $\mathcal{R}$-statistic with respect to τ will give us our time period.

$$\mathcal{R}(\tau) = \sqrt{\bar{x}^2 + \bar{y}^2}$$

$$\hat{T} = \arg_\tau \max \mathcal{R}(\tau)$$

In other words, value of τ for which the events are most closely spaced on the graph is our time period. Circular analysis is used for various applications like predicting seasonality and plant phenology. Figure 2 shows the circular analysis applied to a simple sinusoidal function, $f(x) = sin(2\pi t)$ (figure 1). As expected, all of the values converge to a single point for the periodic data. On the other hand, if we add a little bit of gaussian noise to this function (figure 3), then the events spread out a bit (figure 4), but the time period still remains the same. When we draw the graph of $\mathcal{R}$-statistic with τ(figure 5), we see that the $\mathcal{R}$-statistic has extremely high values for $\tau = T$. It's concept is kind analogous to what happens when you give resonant frequency to sound wave propagating through a glass tube in physics. The amplitude knows no bounds.

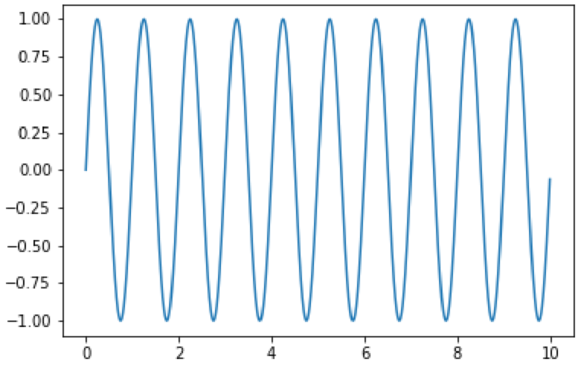

Fig-1: A Simple Sinusoidal Wave $f(x) = sin(2\pi t)$

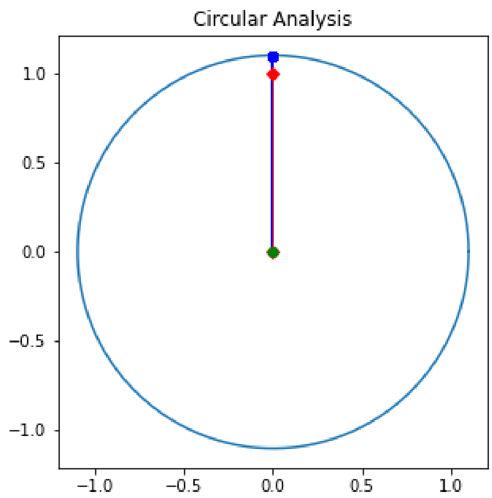

Fig-2: Circular Analysis Of The Sinusoidal Wave. Local maxima converge at one single point.

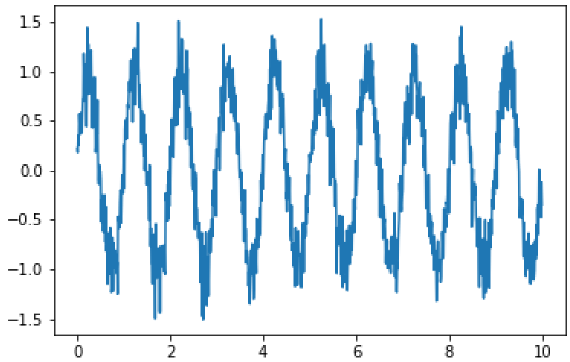

Fig-3: $f(x) = sin(2\pi t) + X$, where X is a Gaussian Random variable.

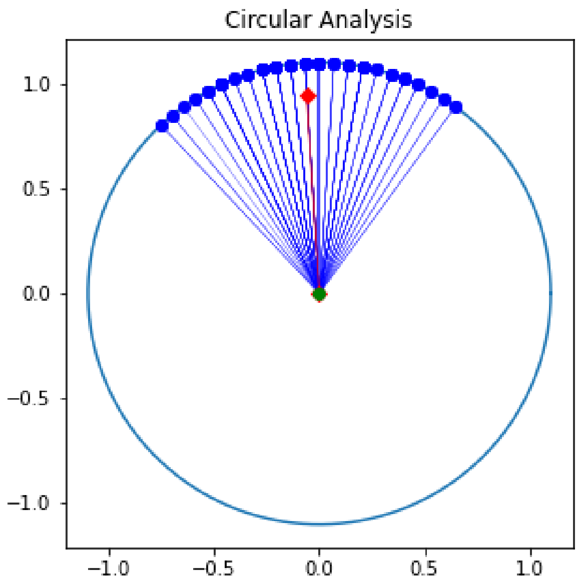

Fig-4: Circular Analysis with noise. The random variable makes the maxima diverge away from a point.

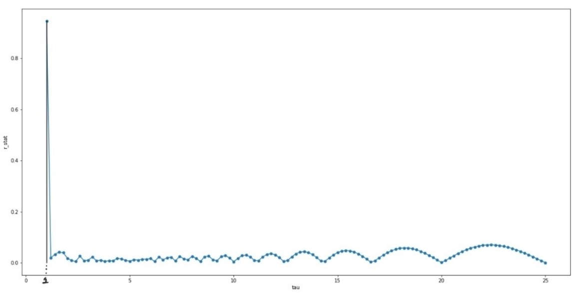

Fig-5: Plot of the $\mathcal{R}$-Statistic with the arbitrary constant τ. We get the global maxima at 1(Maddeningly Higher than the rest of tau values)

Spiral Analysis

Marc Weber C. and Wolfgang Muller recently a developed a new approach for visualizing statistical data.[3] Spiral graphs can visualize data-sets and are ideally suited to complement human's ability to detect structures. In other words, Spiral graph or spiral analysis is a form of visualizing periodic events or quasi periodic events in order to better understand its trends or variations in its plot filled with local maxima (or minima) and to observe how those maxima cluster at a certain angle or sector, through the showing of the shades of color, with a certain color being assigned a certain value of the function of the event under consideration.

The spiral graph is considered to be more advantageous than a traditional rectangular graph. This is because in the Cartesian plot, the space is governed by the horizontal and vertical axes and a data point has to represented by at least one pixel, and also the coordinate point not under the function is left as empty space, which is considered as inefficient. Whereas, in the spiral graph, the data is shown much more densely using only a few pixels, while there is almost no empty space. The second reason is that, just like a circular graph, a spiral graph shows the corresponding points in consecutive periods are shown near to each other, thereby allowing a better visual comparison. In a Cartesian graph, the cycles would not be overlapping, unlike the Spiral graphs.

Here's How it Works

In polar coordinates, spiral can be described as

$$r = f(\phi), \qquad \frac{df}{d\phi} > 0, \qquad \phi \in \mathbb{R}^+$$

Several different f lead to different kinds of spirals. For example, $r = a\phi^k$ are called Archimedean spirals, and $r = ae^{k\phi}$ lead to logarithmic spirals. Logarithmic spirals have a special property that all arcs cut a ray emanating from origin at the same angle. Whereas for $k = 1$, a ray emanating from origin would cross two consecutive arc at a constant distance of $2\pi a$ in the Archimedes' spiral. For visualization of data, Archimedes' spiral are most appropriate.

To plot the x-axis of time onto the spiral, we use the following formula

$$\phi = 2\pi\frac{t}{T}$$
$$r_i = r_0 + \frac{i}{L - r_0}$$

Here,

> r_i → Starting radius after each successive revolution for the spiral
>
> i → Number of local maximum that have occurred before a particular time t
>
> T → Time Period

Thus, for plotting the linear time-axis onto the spiral, we will also need information regarding the occurring of maxima.

With this method, we have plotted and assigned a particular section to each week onto the spiral. Now, we need to somehow show the number of sunspots observed in each week on the spiral. In more general terms, we have folded the time axis onto the spiral, now we need to show the y-axis. We do that with the help of intensity of a color. As an example, we can think of the y-axis in greyscale, i.e. we adjust the ratio of white and black in each section based on the output that we have from y-axis over there (figure 6).

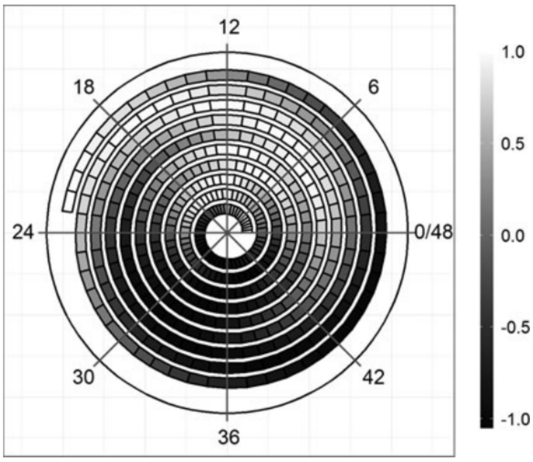

Fig-6: An example of how the visual analysis is done with spiral graphs.[4] Here, we have taken pure white to be 1 and pure black to be -1.

Let us try to get an intuitive understanding of the approach with the help of a few examples.

Simple Sinusoidal Wave

Recall the simple sinusoidal wave from circular analysis example (figure 1)

The number of years is represented on the x-axis, while the amplitude is on the y-axis. Although the above graph is continuous, we would consider a very short time interval to obtain discrete amplitude values.

In the spiral graph, the highest amplitude will be indicated by the brightest color, while the minimum amplitude is indicated by the darkest color.

Now, we will divide each cycle of the sin(ωt) graph into four equal parts. For the first part, the value of amplitude increases from minimum to maximum; it is evident from the above spiral graph that the intensity changes from dark to bright in the first quadrant.

The amplitude decreases from maximum to minimum for the second part of the cycle. It can be visualized from the spiral graph that the intensity changes from bright to dark in the second quadrant.

Since the third and fourth sections have the lowest amplitudes, the spiral graph's third and fourth quadrants are the darkest.

On the sin(ωt) graph, draw a line parallel to the x-axis at 1 to plot the local maximum. The discontinuous graph segments are obtained, and the maxima of each segment are shown on the spiral graph as blue points.

When there is no noise, all local maximum of all segments will have the same amplitude. As a result, all blue spots were seen on the vertical axis. (figure 7)

Here the successive revolution of the spiral are so closely spaced, that they give a feeling as if this spiral was a disc.

Combining the circular analysis with spiral gives us a better understanding of the data. (figure 8)

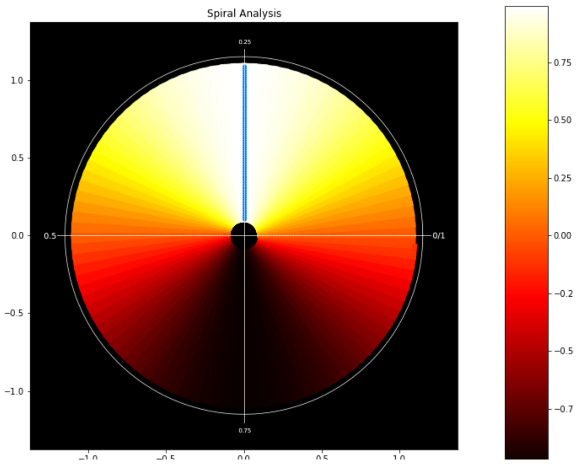

Fig-7: Spiral analysis of simple sinusoidal wave

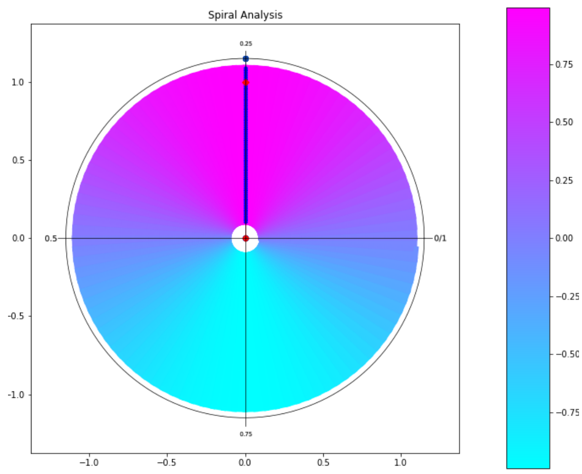

Fig-8: Combination of Circular analysis with Spiral Analysis for a simple sinusoidal wave.

Adding Noise to the Sinusoidal Wave

If we add a little bit of Gaussian Noise to the sinusoidal wave (figure

3), then we obtain figure 9 and 10.

The linearly graph clearly becomes a bit noisy and hard to understand. But the spiral graph on the other hand gives a better visual understanding of the data. If we try to find a physical analogy, then between fig-8 and 10 it looks like fig-8 represents laminar flow, and by adding noise to the linear graph, we add a little bit of turbulence to the graph, but the key properties of the graph remains same. But with physical analogy or not, it is clear that circular analysis helps us in finding the time period of data, and spiral analysis makes it easy to visualize the data with noise. Even for the pseudo-periodic data, we will be able to predict how the cycle of amplitude goes, and how it has evolved over 200 years, which although is likely to not give us a lot of information considering that sun has been around for a billion years, and is going to be around for another billion years or so. But it will however provide a good understanding of the solar cycle of the sun.

Fourier Analysis

Fourier transform is used to convert the time domain signals into frequency domain because analysis of the signals in the frequency domain is easy. The Discrete Fourier transform is a useful technique in digital signal processing that allows us to determine the spectrum of a signal within a specified period.

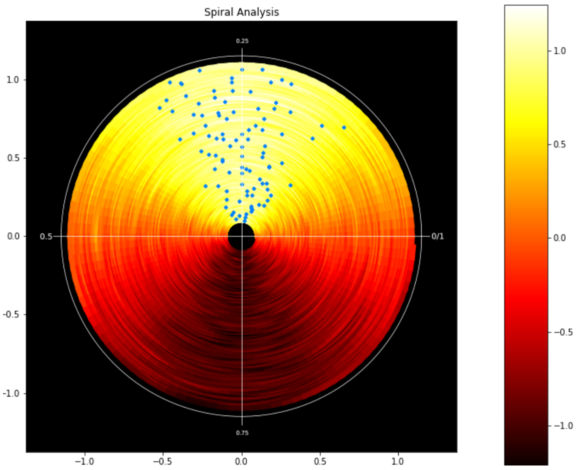

Fig-9: Spiral analysis of Gaussian Sinusoidal Wave

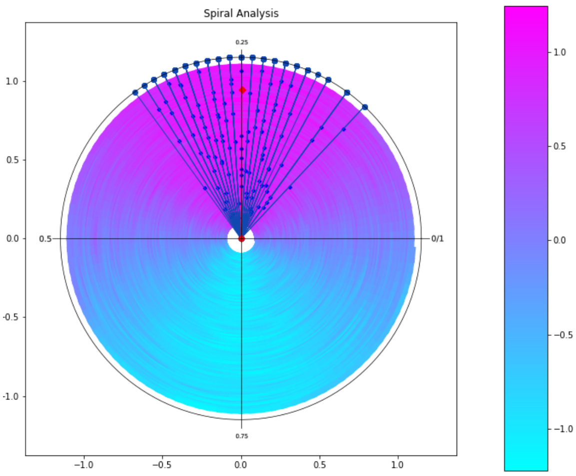

Fig-10: Combination of Spiral and Circular Analysis for gaussian sinusoidal wave.

To understand how fourier transform works, let us take four signals with added noise and different frequencies, i.e., 10 Hz, 5 Hz, 15 Hz, 20 Hz. (Figure 11), We can observe from the figures that these signals are noisy and unclear. When these signals are added together, we obtain the signal shown in Figure 12. We can't tell the constituting frequencies of the signal from the time domain. On the other hand, a Fourier transform can do it for us; it converts signals from the time domain to the frequency domain, allowing us to see the frequencies that resulted in that signal. At the involved individual signal frequencies, we can see a sharp peak indicating the presence of a signal at that frequency.

We can see in figure 13 that there are sharp peaks at the involved frequencies, i.e., at 5, 10, 15, 20 Hz. Hence Fourier transform helps us identify individual signal frequencies from a group of frequencies mixture.

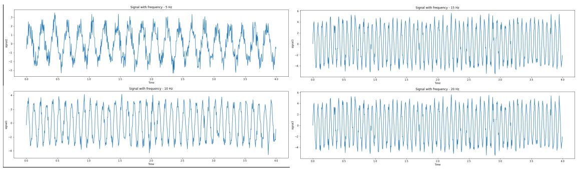

Fig-11: Sinusoidal waves of frequency 5, 10, 15 & 20 Hz with some noise.

Fig-12: Combination of signals of frequency 5, 10, 15 and 20 Hz along with Noise.

Fig-13: The Fourier Transform Of the Signal in Fig-12

Here the frequencies are already clearly visible. But generally, we can fold the signal to reduce the noise. Performing folding five times for this signal gives figure 7.

Fig-14: Graph after 5-times folding of the signal

Sunspot Analysis and Visualization

Circular Analysis

The activity of the sun varies periodically, and it is difficult to predict or get the time period of sunspot counts. In our project, we use circular analysis to find the time period of appearance of sunspot counts using monthly averaged data for the presence of sunspot counts. The python code written to plot the circular analysis used sunspot counts data from 1749 to 2022. The monthly averaged data are used for this particular analysis.

Figure 15 shows the plot of number of sunspot counts for 273 years. The plot shows that it is irregular and has many local maxima from which it is difficult to get the time period. Instead of plotting each point of local maxima on a circle, we are finding those points which are more relevant by drawing a horizontal line at count equals 20, which disconnects the continuous graph into individual peaks by which all the minor local maxima can be avoided. All the global maxima of these individual peaks are collected, which were the local maxima of the continuous graph.

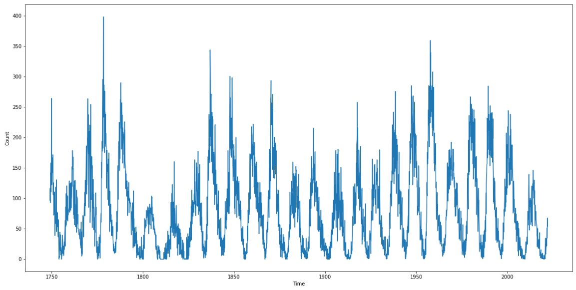

Fig-15: Weekly plot of the sunspot counts over 273 years.

On doing the circular analysis and plotting the $\mathcal{R}$-statistic, we get figure 16. and any other $\mathcal{R}$-statistic will have a lesser length than this.

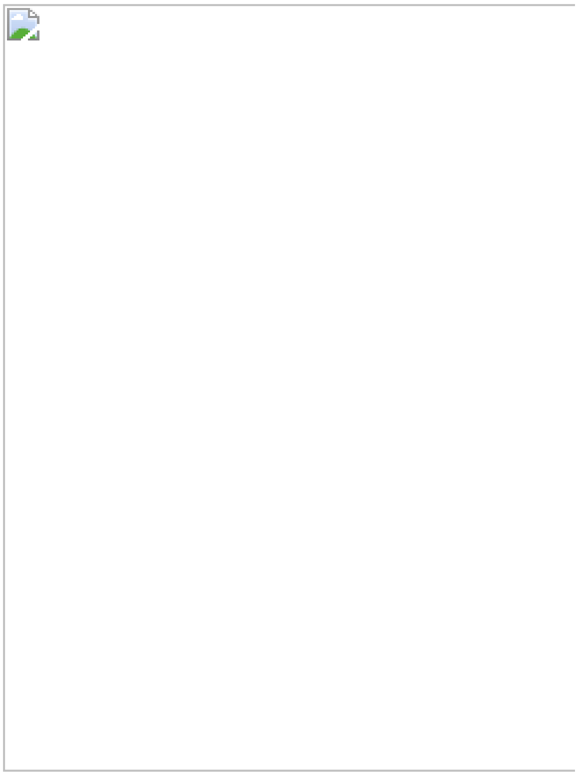

Fig-16: Circular analysis and $\mathcal{R}$-statistic for the sunspot counts.

The $\mathcal{R}$-statistic vs 'τ' graph is then plotted to get a global maximum value, which comes at τ equals 11, as shown in figure 17.

Spiral Analysis

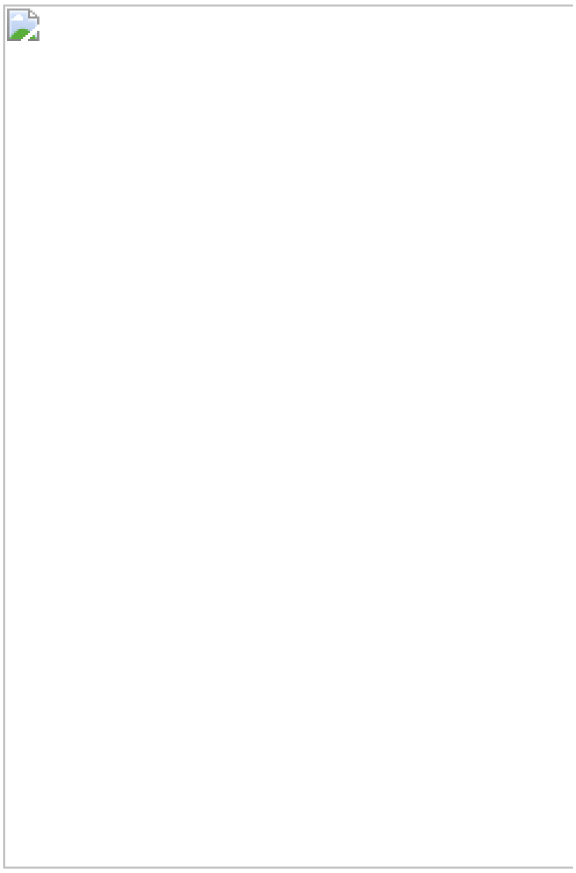

Fig-17: $\mathcal{R}$-statistic vs τ graph for sunspots

the number of sunspots, the color changes from white to yellow, red and eventually black. Along with it, the local maxima are shown with blue dots, which allows us to observe how the maxima are clustering at the time period of11 years. (figure 18 and 19)

As shown in the above diagram, the highest number of sunspots is assigned the color white, and as we decrease

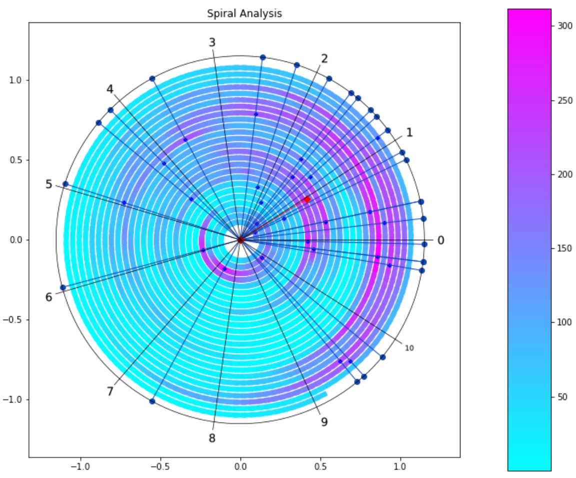

Fig-19: The combination of spiral and circular analysis

Figure 20 shows the graph that we have obtained by applying FFT to our data.

Fig-20: Fourier Transform Without any folding.

Using folding technique, adding together the fourier transform of n-folds give us figure 21 and 22.

Fig-21: Fourier Transform folded two times. T = 10.21223

Fig-22: Fourier Transform folded six times. T=11.0565

frequencies present other than the noise, the one with higher amplitude corresponds to a time period of 11 years, the other one is only a thirtieth or fortieth of that corresponding to a time

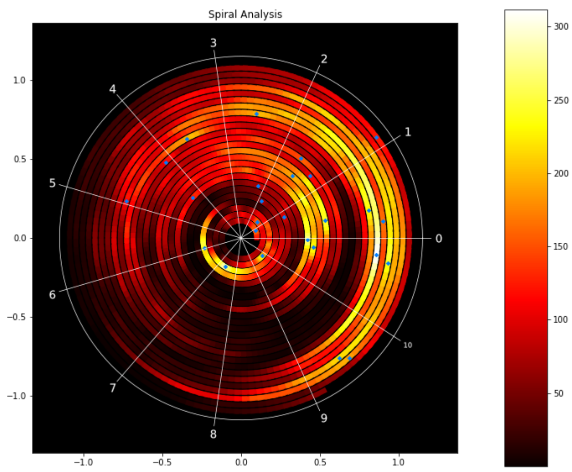

Fig-18: Spiral analysis of sunspots.

Fourier Analysis

The sunspot data that we have contains noises of various frequencies. So, if we apply fourier transform to our data, we can get all the individual constituent frequencies, and differentiate the required sunspots' data from the other noises owing to the high amplitude of frequency corresponding to the time period.

We can assume that the dominant frequency corresponds to the data of the sunspots, while the others represent noises. Using the result, we get the time period as 10.62072 years, which is approximately equal to the result that we obtained in circular analysis.

Results And Discussion

Both circular and fourier analysis reveal the time period of the sunspots to be 11 years. Which makes it appear as if the solar cycle of the magnetic field on the sun's surface changes every 11 years. From the spiral graph (figure 19), and period of 300-400 years. In other words, our signal wave consist of two frequencies instead of one. The higher time-period can probably be explained by the delay caused due to slow dying of sunspots. My hypothesis is that the thermal convection and activity on the sun's surface must somehow be affecting the magnetic activity whose delay lead to further delay of sun-spot appearance in the next cycle and the loop repeats. It will also obtain why we don't obtain the amplitudes for just a few particular values of ϕ in figure 19 as in the case of sinusoidal waves (figure 10). The amplitudes are instead initially in the first quadrant, then they slowly rotate to 3rd quadrant, then 4th, and then 1st, and now they seem to be in the process of subtly translating to the 2nd quadrant (its more clear in figure 18).

even the linear graph (figure 15), it is clear that the sunspots rise rapidly with the increase of magnetic concentration on particular regions on the sun's surface, however as we can expect due to inhibition of convection, they don't seem to disperse so rapidly. The reason behind that being that magnetic flux disperses quickly due to the flux pressure, but convection (rate of heat flow) takes time to increase, and even as it increases, it can take time for temperatures to rise for the complete region of sunspot depending on how big it really is. This delay can also be explained by the umbra and penumbra structure of sunspots, where umbra are depression on the sun's surface. The depression will explain the extra time require for levelling of the sun's surface.

From the fourier transform (figure 20), we observe that there are two key

Conclusion

Circular and spiral analysis turn out to be a great way for visualizing the sunspot data to better comprehend the understanding of the solar activity on sun's surface. Along with fourier analysis, they help us in reverifying the time period of 11 years first found by the German astronomer Samuel Heinrich Schwabe in 1848.

At the same time, our analysis also yields us new results, such as the presence of one more time period in the data of around 300-400 years which helps us in reverifying our physical understanding of sunspots. We however can also see a major drawback of spiral analysis. That while it can yield a better qualitative understanding of the data, we still need to rely on other traditional techniques for quantitative understanding. Moreover, the qualitative understanding of the data from spiral graph highly depends on choosing the right color-amplitude scale.

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