Assignment - 2

Course: Signals, Systems, and Random Processes

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Q1

Given following $x(t)$ with period $T_0 = 2$:

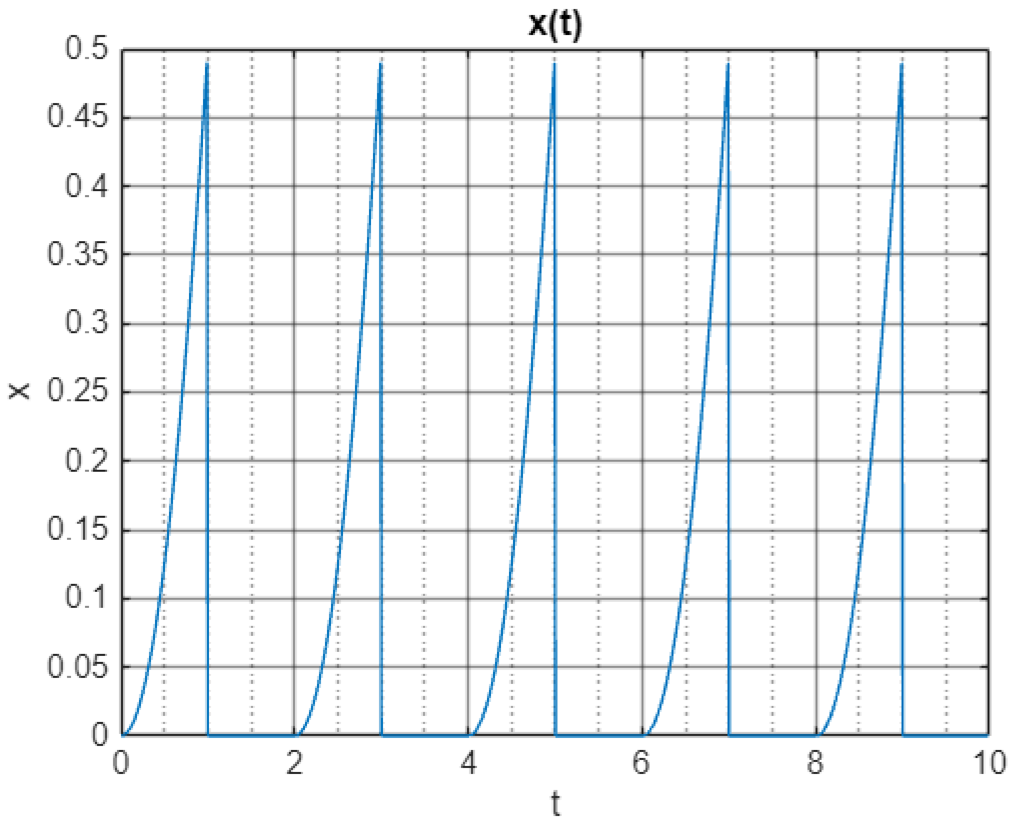
$$x(t) = \begin{cases} t^2/2 & 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$

a

Plot the function and it's magnitude line spectra using MATLAB.

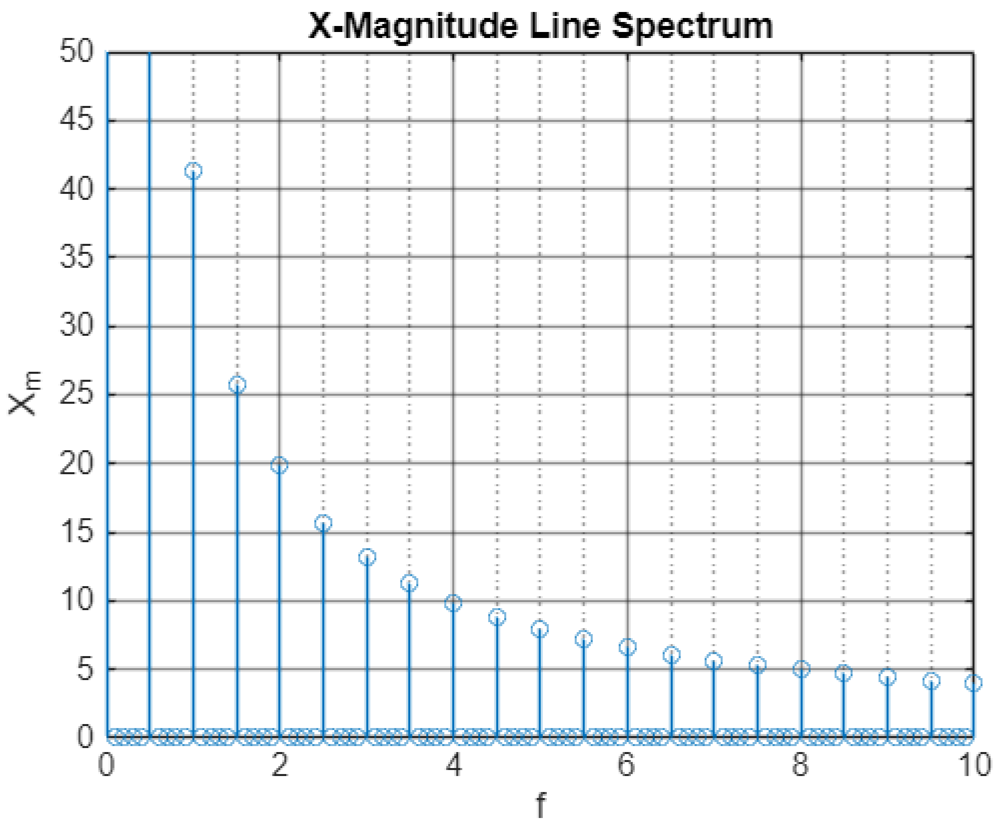
```
fs = 100; % sampling frequency = 100 Hz
t = 0 : 1/fs : 10 - 1/fs;
x = repmat([ (t(1:fs).^2)./2, zeros(1, fs)], [1 5]);

figure1 = figure;
axes1 = axes('Parent',figure1);
hold(axes1,'on');
plot(t, x);

title('x(t)')
xlim([0,10]);
ylim([0, 0.5]);   % optional to resize axis limits
xlabel('t');
ylabel('x');

box(axes1,'on');
```

```
grid(axes1,'on');
hold(axes1,'off');
% Set the remaining axes properties
set(axes1,'GridAlpha',0.5,'MinorGridAlpha',0.4,'XMinorGrid','on')

set(gca,'FontSize',12)
```




```
X = fft(x);
X_magnitude = abs(X);
f = (0: length(X)-1) * fs/length(X);

figure2 = figure;
axes2 = axes('Parent',figure2);
hold(axes2,'on');
stem(f, X_magnitude);

title('X-Magnitude Line Spectrum');
xlim([0,10]);
ylim([0, 50]);   % optional to resize axis limits
xlabel('f');
ylabel('X_m');

box(axes2,'on');
grid(axes2,'on');
hold(axes2,'off');

% Set the remaining axes properties
```

```
set(axes2,'GridAlpha',0.5,'MinorGridAlpha',0.4,'XMinorGrid','on')

set(gca,'FontSize',12)
```



b

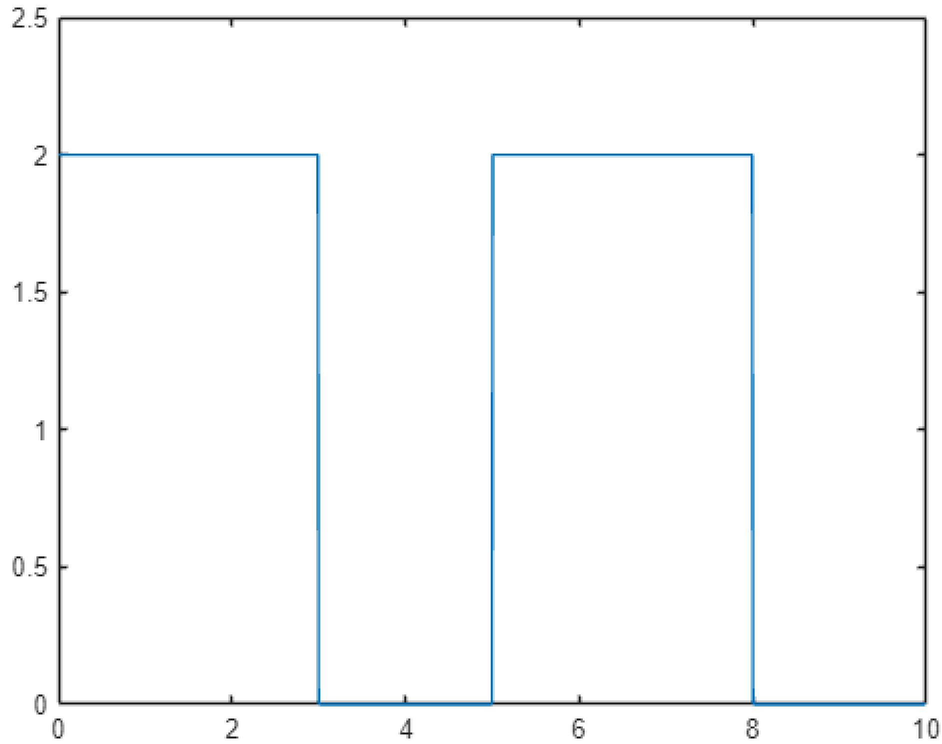
List out the magnitudes of 3f, 5f, and 7f (where f is the fundamental frequency of the given signal) components.

$f_0 = \frac{1}{T_0} = 0.5Hz$

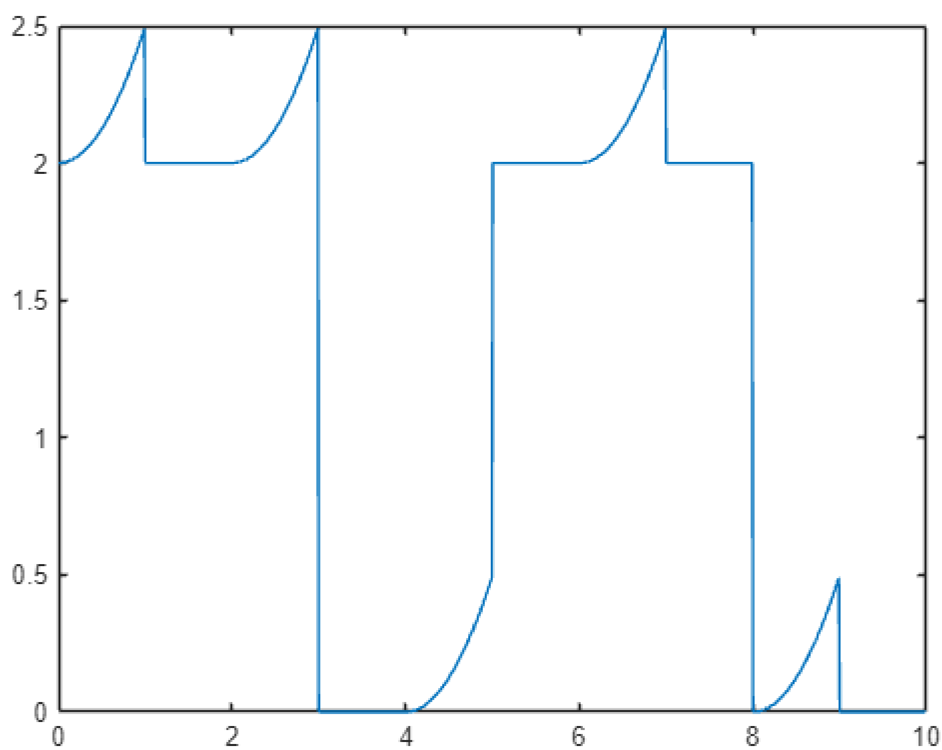
frequency	$3f_0$	$5f_0$	$7f_0$
magnitude	25.6884	15.6442	11.2303

c

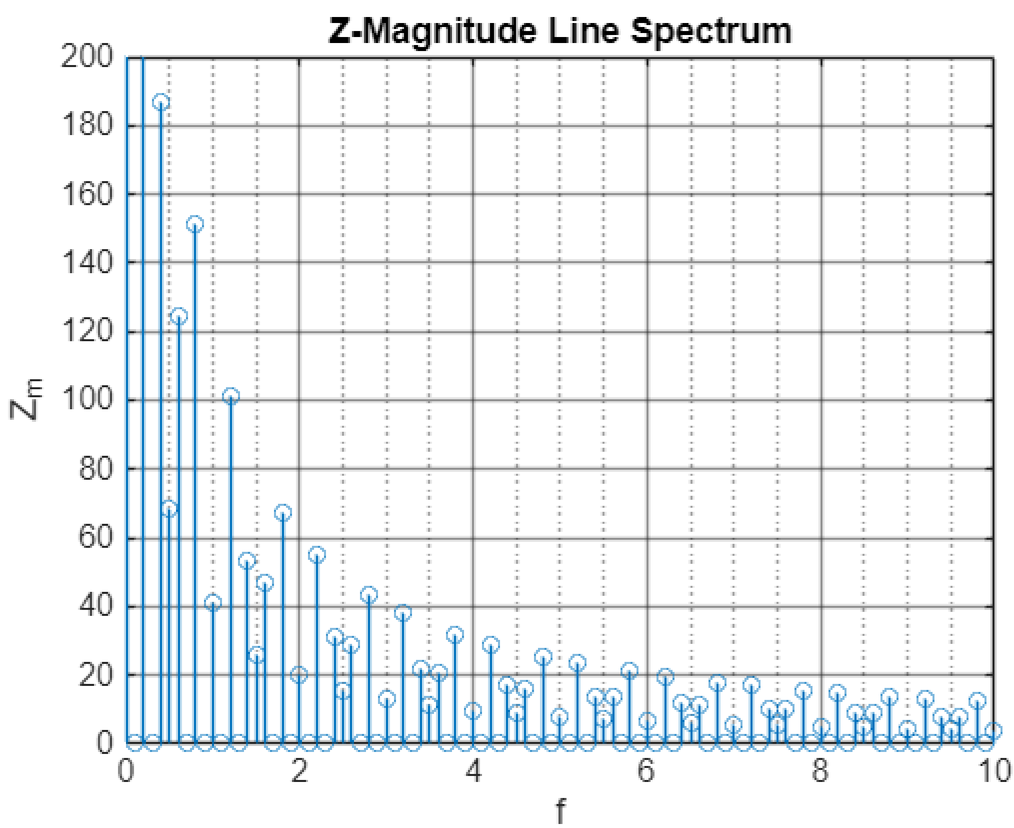
```
y = repmat([zeros(1, 3*fs) + 2, zeros(1, 2*fs)], [1 2]);
plot(t, y);
ylim([0, 2.5]);
```




```
z = y + x;
plot(t, z)
```

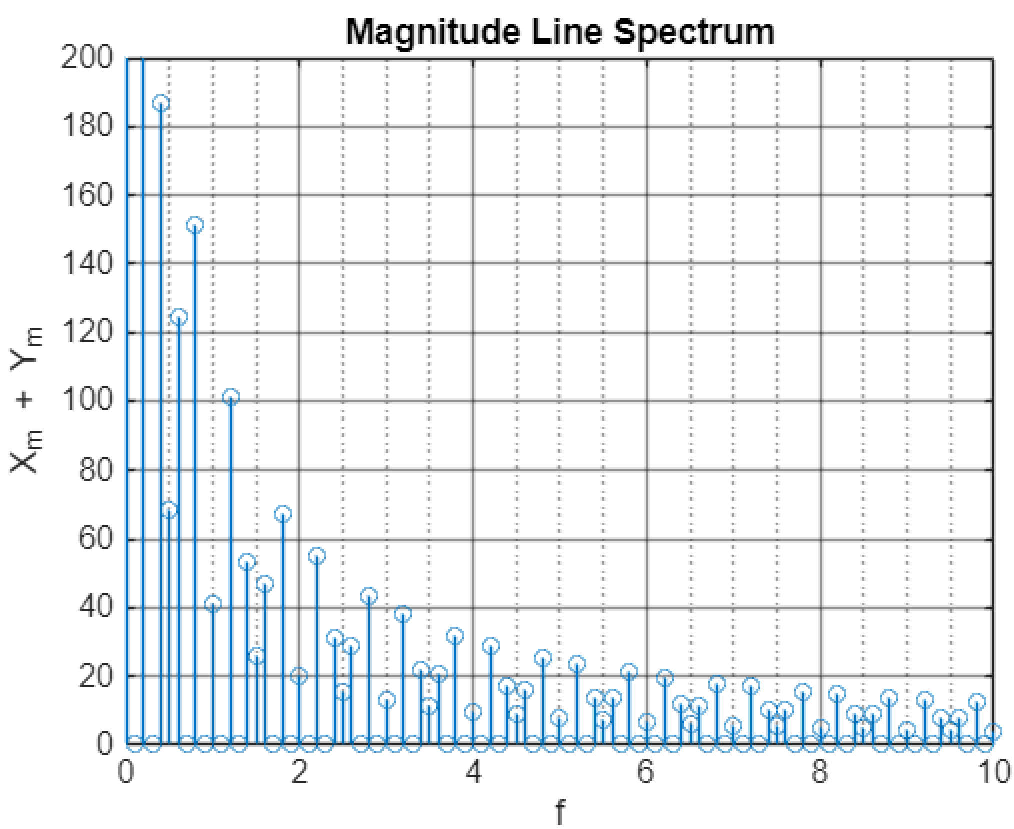


```
Y_magnitude = abs(fft(y));
Z_magnitude = abs(fft(z));

figure3= figure;
axes3 = axes('Parent',figure3);
hold(axes3,'on');
stem(f, Z_magnitude);

title('Z-Magnitude Line Spectrum');
xlim([0,10]);
ylim([0, 200]);   % optional to resize axis limits
xlabel('f');
ylabel('Z_m');

box(axes3,'on');
grid(axes3,'on');
hold(axes3,'off');

% Set the remaining axes properties
set(axes3,'GridAlpha',0.5,'MinorGridAlpha',0.4,'XMinorGrid','on')

set(gca,'FontSize',12)
```




```
figure4= figure;
axes4 = axes('Parent',figure4);
hold(axes4,'on');
```

```
stem(f, X_magnitude + Y_magnitude);

title('Magnitude Line Spectrum');
xlim([0,10]);
ylim([0, 200]);   % optional to resize axis limits
xlabel('f');
ylabel('X_m + Y_m');

box(axes4,'on');
grid(axes4,'on');
hold(axes4,'off');

% Set the remaining axes properties
set(axes4,'GridAlpha',0.5,'MinorGridAlpha',0.4,'XMinorGrid','on')

set(gca,'FontSize',12)
```



Q2

Say $x(t)$ is a signal with period $T_o = 2\,\mathrm{s}$, and each period has the form of $u(t) - 2u(t-1) + u(t-2)$. Approximate $x(t)$ to $x'(t)$ with 30 harmonics and plot $x'(t)$ using MATLAB. Also plot the magnitude line spectrum for $x'(t)$.

$$x'(t) = a_0 + \sum_{k=1}^{30} \left(a_k \cos k\omega_0 t + b_k \sin k\omega_0 t \right)$$

where,

$$\omega_0 = \frac{2\pi}{T_0} = \pi$$

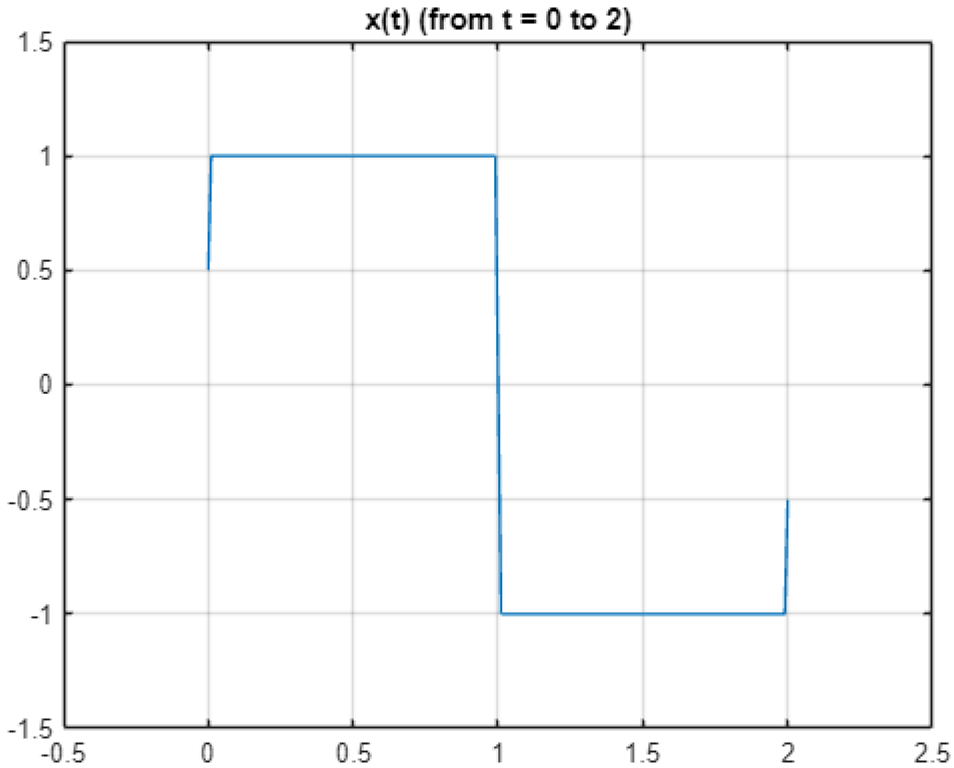
$$a_0 = \frac{1}{T_0} \int_0^{T_0} x(t) dt$$

$$a_k = \frac{2}{T_0} \int_0^{T_0} x(t) \cos kw_0 t \, dt$$

$$b_k = \frac{2}{T_0} \int_0^{T_0} x(t) \sin kw_0 t \, dt$$

Let's first try to plot x(t)

```
t = 0:0.01:2; % Define the time vector
u = @(t) heaviside(t); % Define the unit step function

% Define and plot the signal
x = u(t) - 2 * u(t - 1) + u(t - 2);
plot(t, x)
title('x(t) (from t = 0 to 2)');
xlim([-0.5,2.5]);
ylim([-1.5, 1.5]);
grid on;
```



Based on the above plot, we can say that $a_0 = 0$. Now it's a lengthy task to compute all the a_k and b_k by hand. So let's have matlab do it for us.

$$x'(t) = \sum_{k=1}^{30} \left(a_k \cos k\pi t + b_k \sin k\pi t \right)$$

where,

$$a_k = \int_0^2 x(t) \cos k\pi t \, dt$$

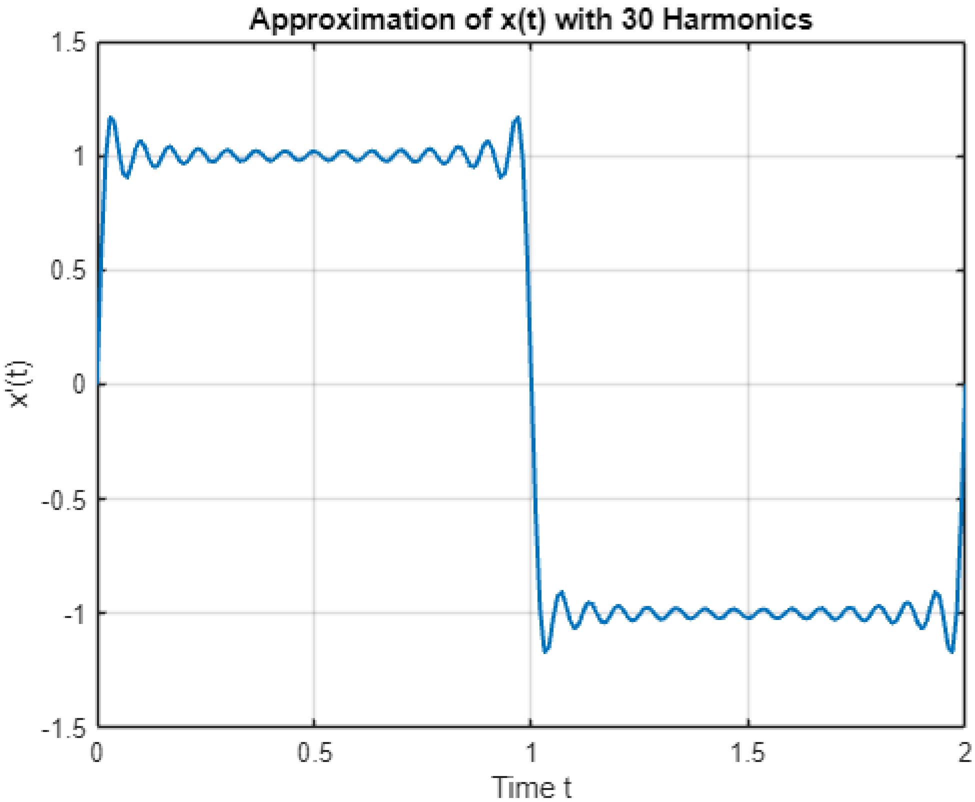
$$b_k = \int_0^2 x(t) \sin k\pi t \, dt$$

```
x = @(t) u(t) - 2 * u(t - 1) + u(t - 2);

a_k = @(k) integral(@(t) x(t).*cos(k*pi*t), 0, 2);
b_k = @(k) integral(@(t) x(t).*sin(k*pi*t), 0, 2);

a = zeros(1, 30);
b = zeros(1, 30);
for k = 1:30
    a(k) = a_k(k);
    b(k) = b_k(k);
end
```

```
% Create a table for a_k and b_k
disp(table((1:30)', a', b', 'VariableNames', {'k', 'a_k', 'b_k'}));
```

k	a_k	b_k
1	1.9429e-16	1.2732
2	2.0817e-17	1.0235e-16
3	2.2204e-16	0.42441
4	6.9389e-18	-2.8796e-16
5	-4.2934e-17	0.25465
6	-1.7174e-16	-2.2204e-16
7	4.1286e-16	0.18189
8	1.1796e-16	-4.1633e-16
9	3.0531e-16	0.14147
10	1.1796e-16	4.5797e-16
11	6.6093e-16	0.11575
12	-3.1919e-16	3.1398e-16
13	-8.3267e-17	0.097942
14	-1.2143e-17	-8.3267e-17
15	7.7369e-16	0.084883
16	2.7582e-16	7.3726e-17
17	1.4225e-16	0.074896
18	-1.0061e-16	-8.4134e-17
19	2.8623e-16	0.067013
20	1.1276e-17	-4.1633e-17
21	-3.4044e-17	0.06063
22	6.6353e-16	-1.0061e-16
23	2.8449e-16	0.055358
24	6.3578e-16	-6.9389e-18
25	3.1832e-16	0.05093
26	-1.0408e-17	-2.5327e-16
27	-7.867e-16	0.047157
28	-3.5735e-16	1.5959e-16
29	-1.0408e-16	0.043905
30	3.5041e-16	-5.3256e-16

```
x_t_prime = zeros(size(t)); % Start with the a0 term
for k = 1:30
    x_t_prime = x_t_prime + a(k) * cos(k * pi * t) + b(k) * sin(k * pi * t);
end

figure;
plot(t, x_t_prime, 'LineWidth', 1.5);
xlabel('Time t');
ylabel('x''(t)');
title('Approximation of x(t) with 30 Harmonics');
grid on;
```



Q3

Convert the following continuous-time signals into discrete-time signals using the sampling theorem. Make sure that there is no aliasing.

a

$x(t) = 1 + \text{sinc}(300\pi t)$

$x[n] = 1 + \text{sinc}(300\pi T_s n)$

$$f_m = \frac{\omega_m}{2\pi} = 150$$

$$T_s = \frac{1}{f_s} = \frac{1}{2f_m} = \frac{1}{2 * 150} = 1/300$$

$x[n] = 1 + \text{sinc}(\pi n) \qquad \text{where, } n \in Z$

b

$x(t) = 1 + \cos(2000\pi t) + sin(4000\pi t)$

$$f_m = \frac{\omega_m}{2\pi} = 2000$$

$$T_s = \frac{1}{2f_m} = \frac{1}{4000}$$

$$x[n] = x(nT_s) = 1 + \cos\left(\pi \frac{n}{2}\right) + \sin(\pi n) \qquad \text{where, } n \in Z$$

c

$x(t) = 10\sin 40\pi t \cos 300\pi t = 5\sin 340\pi t - 5\sin 260\pi t$

$f_m = \dfrac{340\pi}{2\pi} = 170$

$T_s = \dfrac{1}{2 * 170} = \dfrac{1}{340}$

$x[n] = x(nT_s) = 10\sin\dfrac{2}{17}\pi n \cos\dfrac{15}{17}\pi n = 5\sin\pi n - 5\sin\dfrac{13}{17}\pi n$

where, $n \in Z$

Q4

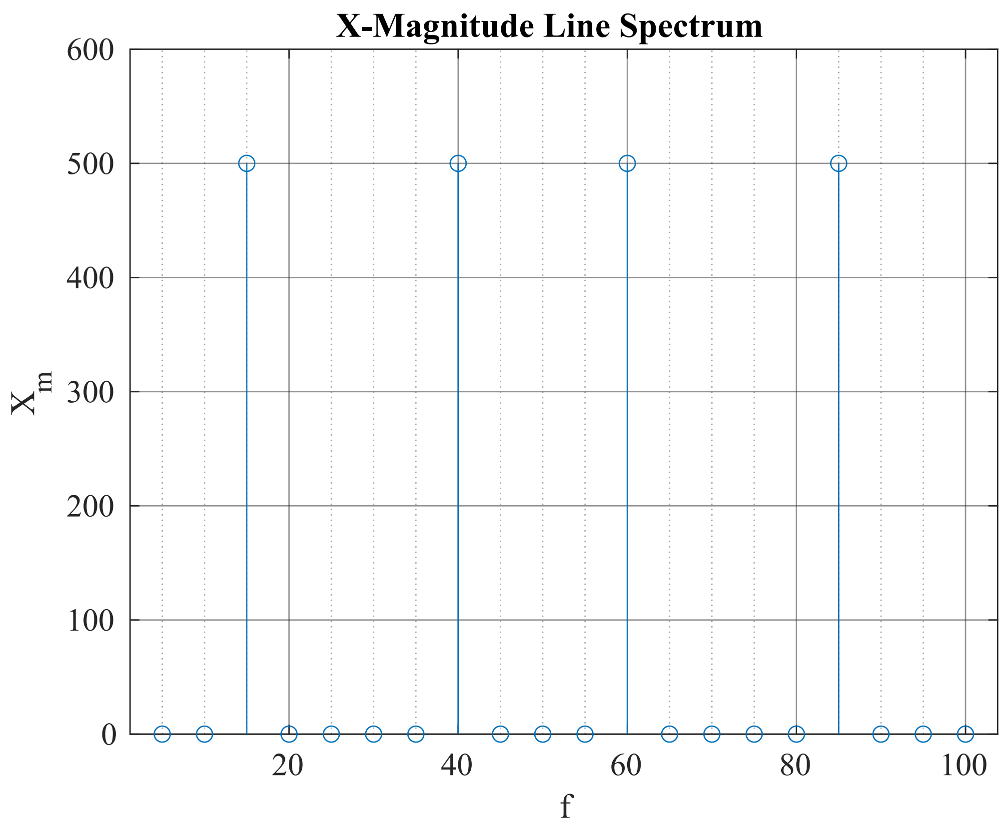
Find out the FT of the given signal in MATLAB without using the FFT built-in function and plot the phase and magnitude graph for the given signal. Compare your results with the built-in function FFT.

$x = \sin 30\pi t + \sin 80\pi t$

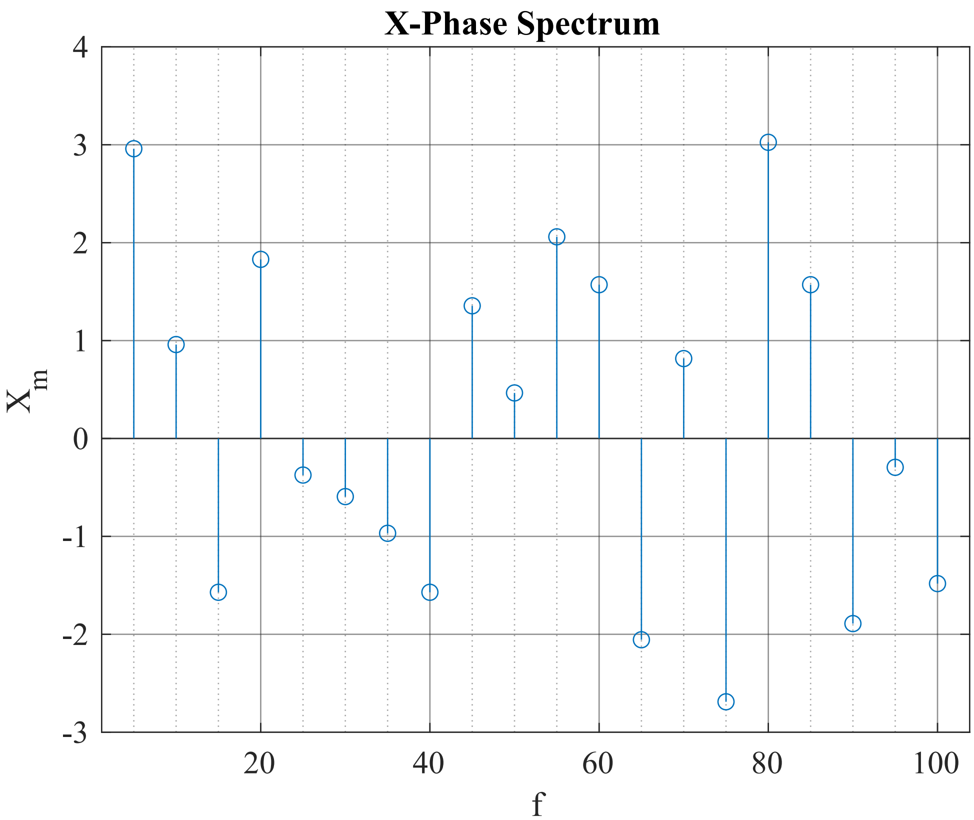
$X(\omega) = \sum_{0}^{T} x(t)e^{j\omega t}$

```
t = 0:1/100:10-1/100; % Time vector
x = sin(2*pi*15*t) + sin(2*pi*40*t); % Signal

X = zeros([1 20]);
omega_0 = 10*pi;
for k = 1:20
    Angle = k * omega_0 * t;
    X(k) = x * (cos(Angle) + 1i*sin(Angle))';
end

f = 5:5:100;

figure1 = figure;
axes1 = axes('Parent',figure1);
hold(axes1,'on');
stem(f, abs(X))
title('X-Magnitude Line Spectrum');
% xlim([0,10]);
% ylim([0, 50]);   % optional to resize axis limits
xlabel('f');
ylabel('X_m');

box(axes1,'on');
grid(axes1,'on');
hold(axes1,'off');

% Set the remaining axes properties
set(axes1,'GridAlpha',0.5,'MinorGridAlpha',0.4,'XMinorGrid','on')
```

```
set(gca,'FontSize',12)
```




```
figure2 = figure;
axes2 = axes('Parent',figure2);
hold(axes2,'on');
stem(f, angle(X))
title('X-Phase Spectrum');
% xlim([0,10]);
% ylim([0, 50]);   % optional to resize axis limits
xlabel('f');
ylabel('X_m');

box(axes2,'on');
grid(axes2,'on');
hold(axes2,'off');

% Set the remaining axes properties
set(axes2,'GridAlpha',0.5,'MinorGridAlpha',0.4,'XMinorGrid','on')

set(gca,'FontSize',12)
```



Q5

Consider the following signal:

$x(t) = r(t+1) - 2r(t) + r(t-1)$

$y(t) = \frac{dx}{dt}$

a

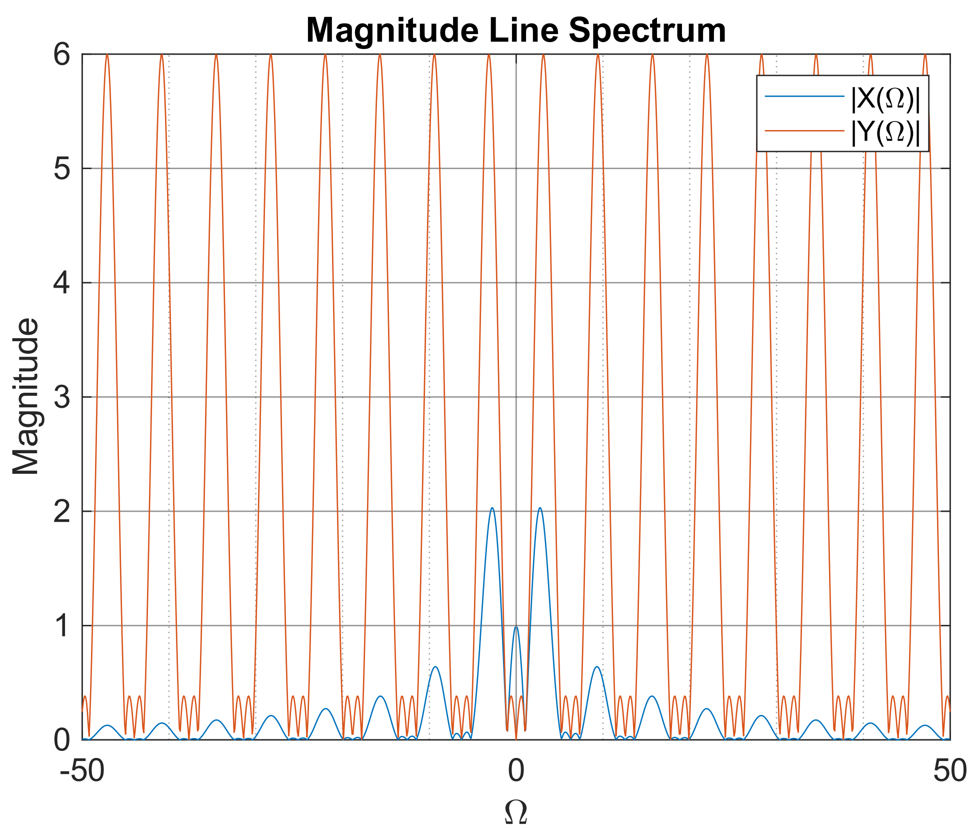
Obtain $X(\Omega)$ and $Y(\Omega)$, plot their magnitude and phase spectrums, and comment whether $X(\Omega)$ and $Y(\Omega)$ are real or imaginary.

$$\begin{aligned} \mathrm{X}(\Omega) &= \int_{-\infty}^{+\infty} x(t) e^{j\Omega t} dt \\ &= \int_{-\frac{3}{2}}^{-\frac{1}{2}} e^{j\Omega t} dt + \int_{\frac{1}{2}}^{\frac{3}{2}} e^{j\Omega t} dt - \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{j\Omega t} dt \\ &= \frac{1}{j\Omega}\left(e^{3/2 j\Omega} + 2e^{-1/2 j\Omega} - 2e^{1/2 j\Omega} - e^{-3/2 j\Omega}\right) \end{aligned}$$

$$\begin{aligned} \mathrm{Y}(\Omega) &= \int_{-\infty}^{+\infty} \frac{dx}{dt} e^{j\Omega t} dt \\ &= e^{j\Omega t} x(t)|_{-\infty}^{+\infty} - j\Omega \int_{-\infty}^{+\infty} x(t) e^{j\Omega t} dt \\ &= -j\Omega * \mathrm{X}(\Omega) \end{aligned}$$

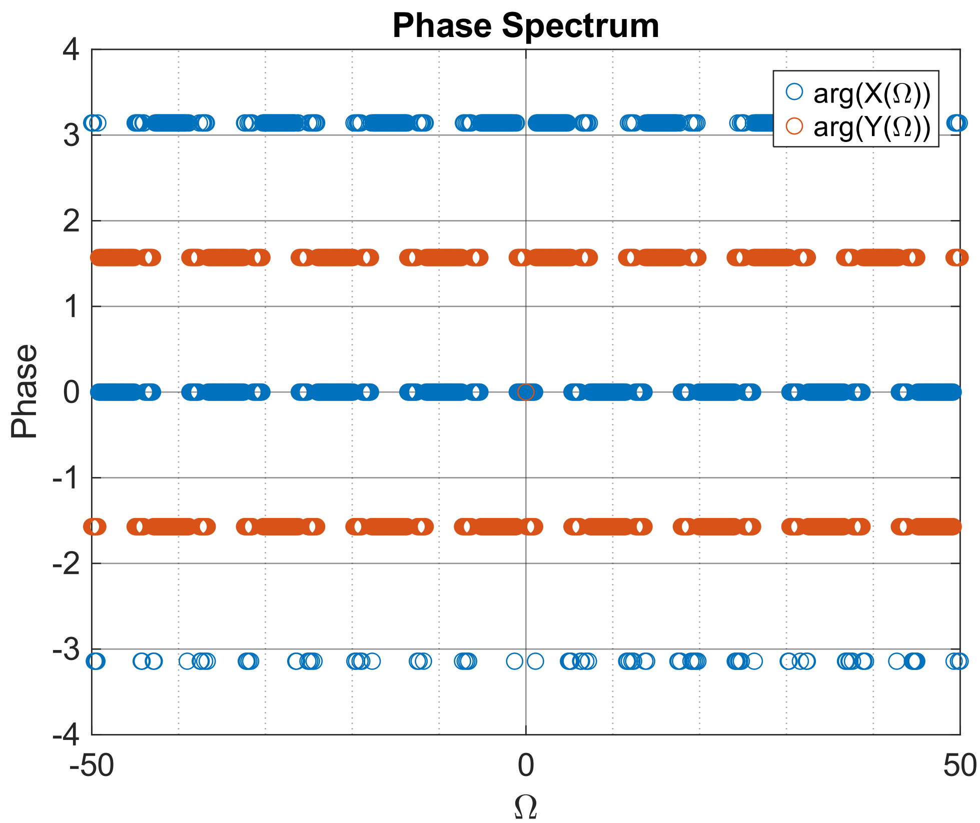
```
omega = -50:0.1:50;
Y = - exp(1.5i .* omega) - 2 * exp(-0.5i .* omega) + 2 * exp(0.5i .* omega) +
exp(-1.5i .* omega);
X = Y./ (-1i * omega);
```

```
figure1 = figure;
axes1 = axes('Parent',figure1);
hold(axes1,'on');
plot(omega, abs(X), 'DisplayName', '|X(\Omega)|')
plot(omega, abs(Y), 'DisplayName', '|Y(\Omega)|')
title('Magnitude Line Spectrum');
% xlim([0,10]);
% ylim([0, 50]);   % optional to resize axis limits
xlabel('\Omega');
ylabel('Magnitude');
legend

box(axes1,'on');
grid(axes1,'on');
hold(axes1,'off');

% Set the remaining axes properties
set(axes1,'GridAlpha',0.5,'MinorGridAlpha',0.4,'XMinorGrid','on')

set(gca,'FontSize',12)
```




```
figure2 = figure;
axes2 = axes('Parent',figure2);
hold(axes2,'on');
scatter(omega, angle(X), 'DisplayName', 'arg(X(\Omega))')
```

```
scatter(omega, angle(Y), 'DisplayName', 'arg(Y(\Omega))')
title('Phase Spectrum');
% xlim([0,10]);
% ylim([0, 50]);   % optional to resize axis limits
xlabel('\Omega');
ylabel('Phase');
legend

box(axes2,'on');
grid(axes2,'on');
hold(axes2,'off');

% Set the remaining axes properties
set(axes2,'GridAlpha',0.5,'MinorGridAlpha',0.4,'XMinorGrid','on')

set(gca,'FontSize',12)
```



We could already tell from the derived equations that $X(\Omega)$ and $Y(\Omega)$ were 90^o out of phase. Based on the above phase spectrum now, it is clear that the phase of $X(\Omega)$ is either $0, +\pi, \text{or } -\pi$. Thus $X(\Omega)$ is real, and $Y(\Omega)$ is imaginary.

b

Determine from the above spectra which of these two signals are smoother. Use MATLAB integration function int to find the fourier transforms. Plot $20\log_{10}|Y(\Omega)|$ and $20\log_{10}|X(\Omega)|$ and decide. Would

you say in general that computing the derivative of a signal generates high frequencies or possible discontinuities.

We can easily conclude based on the magnitude line spectra that $x(t)$ is much smoother than $y(t)$. The reason being that magnitude of high frequencies in it's spectra is significantly lower due to $X(\Omega)$ being a decaying function of Ω. It implies components of $x(t)$ are mainly low frequency sine and cosine waves while $y(t)$ has a lot of high frequency components. Thus, $y(t)$ would fluctuate a lot more than $x(t)$ and would seem noisy/less smooth in comparison to x(t).

```
figure1 = figure;
axes1 = axes('Parent',figure1);
hold(axes1,'on');
plot(omega, 20.*log10(abs(X)), 'DisplayName', '20 log_{10}|X(\Omega)|')
plot(omega, 20.*log10(abs(Y)), 'DisplayName', '20 log_{10}|Y(\Omega)|')
title('Magnitude Line Spectrum');
% xlim([0,10]);
% ylim([0, 50]);   % optional to resize axis limits
xlabel('\Omega');
ylabel('log(Magnitude)');
legend
```