Crash game

v1.0 of May 23, 2020

Algorithm

Set RTP $= 0.99$ or another number less than 1.

0. Player selects any multiplier $x > 1$.
1. The program generates random number r uniformly distributed between 0 and 1.
   In practical applications standard random function can rarely be equal to 0 but cannot be equal to 1. We will need a nonzero r below. Therefore we assign $p = 1 - r$ (which also is uniformly distributed between 0 and 1).
2. Calculate
$$\xi = \frac{\mathrm{RTP}}{p}$$
3. Set
$$\mathrm{Win} = \begin{cases} x, & \text{if } \xi > x \\ 0, & \text{if } \xi \leq x \end{cases}$$
4. Pay the player Win $\times$ Bet.

Average RTP

Mathematical expectation of payout ratio is equal to

$$\mathbb{E}(\mathrm{Win}) = 0 \cdot \mathbb{P}(\mathrm{Win} = 0) + x \cdot \mathbb{P}(\mathrm{Win} = x) =$$
$$x \cdot \mathbb{P}(\xi > x) = x \cdot \mathbb{P}\left(\frac{\mathrm{RTP}}{p} > x\right) = x \cdot \mathbb{P}\left(p < \frac{\mathrm{RTP}}{x}\right)$$

In the last expression note that $\frac{\mathrm{RTP}}{x} < 1$. Since p is uniformly distributed for any $a < 1$ we have $\mathbb{P}(p < a) = a$ and

$$\mathbb{E}(\mathrm{Win}) = x \cdot \mathbb{P}\left(p < \frac{\mathrm{RTP}}{x}\right) = x\frac{\mathrm{RTP}}{x} = \mathrm{RTP}.$$

That proves that the described algorithm provides average RTP equal to the value assigned to the algorithm parameter for any x.

Standard deviation of RTP

Variance of RTP is the following:

$$\mathbb{D}(\mathrm{Win}) = \mathbb{E}(\mathrm{Win}^2) - (\mathbb{E}\mathrm{Win})^2 =$$

$$x^2 \cdot \mathbb{P}(\mathrm{Win} = x) - (x \cdot \mathbb{P}(\mathrm{Win} = x))^2 = x\mathrm{RTP} - \mathrm{RTP}^2 = (x - \mathrm{RTP}) \cdot \mathrm{RTP}.$$

Standard deviation

$$\mathrm{Stddev} = \sqrt{\mathbb{D}(\mathrm{Win})} = \sqrt{(x - \mathrm{RTP}) \cdot \mathrm{RTP}}.$$