837 in base 10:

$$837 = 7 + 3 \times 10 + 8 \times 100 = 7 * 1 + 3 * 10 + 8 * 10^2$$

2459 in base 10:

$$2459 = 9 + 5 * 10 + 4 * 10^2 + 2 * 10^3$$

837 in base 10:

$$\boxed{837} = \boxed{7} + \boxed{3} \times 10 + \boxed{8} \times 100 = 7*1 + 3*10 + 8*10^2$$

2459 in base 10:

$$\boxed{2459} = \boxed{9} + \boxed{5} * 10 + \boxed{4} * 10^2 + \boxed{2} * 10^3$$

digits that can take non-negative values less than 10. So, values from 0 to 9

837 in base 10:

$$837 = 7 + 3 \times 10 + 8 \times 100 = 7 * 1 + 3 * 10 + 8 * 10^2$$

2459 in base 10:

$$2459 = 9 + 5 * 10 + 4 * 10^2 + 2 * 10^3$$

837 in base 10:

$837 = 7 + 3 \times 10 + 8 \times 100 = 7 * 1 + 3 * 10 + 8 * 10^{\boxed{2}}$

The highest power is 1 less than the number of digits

2459 in base 10:

$2459 = 9 + 5 * 10 + 4 * 10^2 + 2 * 10^{\boxed{3}}$

837 in base 10:

$$837 = 7 + 3 \times 10 + 8 \times 100 = 7 * 1 + 3 * 10 + 8 * 10^2$$

2459 in base 10:

$$2459 = 9 + 5 * 10 + 4 * 10^2 + 2 * 10^3$$

837 in base 10:

$$837 = 7 + 3\times 10 + 8\times 100 = 7*1 + 3*10 + 8*10^2 \quad \text{———} \quad (1)$$

divide both sides of (1) by 10

$$83 + \frac{7}{10} = \frac{837}{10} = \frac{7}{10} + 3 + 8*10$$

837 in base 10:

$$837 = 7 + 3\times 10 + 8\times 100 = 7*1 + 3*10 + 8*10^2 \quad \text{—— (I)}$$

divide both sides of (I) by 10

$$\boxed{83} + \boxed{\frac{7}{10}} = \frac{837}{10} = \boxed{\frac{7}{10}} + \boxed{3 + 8*10}$$

Whenever we divide the equations "like" (I) by 10, we can equate corresponding terms on both sides

$$83 = 3 + 8*10 \quad \text{—— (II)}$$

An equation like (I), but of 1-degree less in the highest power of 10

837 in base 10:

$$837 = 7 + 3\times 10 + 8\times 100 = 7*1 + 3*10 + 8*10^2 \text{ ——— (I)} \qquad \boxed{\frac{7}{10}}$$

$$83 = 3 + 8*10 \text{ ——— (II)} \qquad \boxed{\frac{3}{10}}$$

what happens when we divide (II) by 10?

$$8 + \frac{3}{10} = \frac{83}{10} = \frac{3}{10} + 8$$

$$8 = 8 \text{ ——— (III)}$$

837 in base 10:

$$837 = 7 + 3\times 10 + 8\times 100 = 7*1 + 3*10 + 8*10^2 \quad \text{———} \quad (\mathrm{I}) \qquad \boxed{\frac{7}{10}}$$

$$83 = 3 + 8*10 \quad \text{———} \quad (\mathrm{II}) \qquad \boxed{\frac{3}{10}}$$

$$8 = 8 \quad \text{———} \quad (\mathrm{III}) \qquad \boxed{\frac{8}{10}}$$

Divide by 10 one more time

$$\frac{8}{10} = \frac{8}{10}$$

837 in base 10:

$$837 = 7 + 3\times 10 + 8\times 100 = 7*1 + 3*10 + 8*10^2 \quad \text{———} \quad (I)$$

$$83 = 3 + 8*10 \quad \text{———} \quad (II)$$

$$8 = 8 \quad \text{———} \quad (III)$$

$$\boxed{\frac{7}{10}}$$

$$\boxed{\frac{3}{10}}$$

$$\boxed{\frac{8}{10}}$$

837 in base 10!

$\boxed{\frac{7}{10}}$

$\boxed{\frac{3}{10}}$

$\boxed{\frac{8}{10}}$

The "numerators" give the digits of the number in base 10

837 in base 10:

What happens when we expand 837 in a different way?

In powers of 2, say, instead of powers of 10?

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots$$

837 in base 10:

What happens when we expand 837 in a different way? In powers of 2, say, instead of powers of 10?

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots$$

We don't know $x_0, x_1, \ldots$! But, whatever their values, we require them to be smaller than 2. So, only "1"s and "0"s.

837 in base 10:

What happens when we expand 837 in a different way?

In powers of 2, say, instead of powers of 10?

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots$$

837 in base 10:

What happens when we expand 837 in a different way? In powers of 2, say, instead of powers of 10?

$$837 = \boxed{x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots}$$

Sums like these are called "series expansions".
This is a series expansion in powers of 2.
We've already seen expansions in powers of 10

837 in base 10:

What happens when we expand 837 in a different way?
In powers of 2, say, instead of powers of 10?

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \quad \text{———} \quad (1)$$

What happens when we divide (1) by 2?

$$418 + \frac{1}{2} = \frac{837}{2} = \frac{x_0}{2} + x_1 + x_2 2 + x_3 2^2 + \cdots$$

837 in base 10:

What happens when we expand 837 in a different way?
In powers of 2, say, instead of powers of 10?

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \quad \text{———} \quad (1)$$

What happens when we divide (1) by 2?

$$\boxed{418} + \boxed{\frac{1}{2}} = \frac{837}{2} = \boxed{\frac{x_0}{2}} + \boxed{x_1 + x_2 2 + x_3 2^2 + \cdots} \qquad \text{So, } \boxed{x_0 = 1}$$

$$418 = x_1 + x_2 2 + x_3 2^2 + x_4 2^3 + \cdots \quad \text{———} \quad (11)$$

837 in base 10:

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \quad \text{———} \quad (I)$$

$$\boxed{x_0 = 1}$$

$$418 = x_1 + x_2 2 + x_3 2^2 + x_4 2^3 + \cdots \quad \text{———} \quad (II)$$

Divide (II) by 2

$$209 = \frac{418}{2} = \frac{x_1}{2} + x_2 + x_3 2 + x_4 2^2 + \cdots$$

837 in base 10:

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \text{ ——— (I)}$$

$$\boxed{x_0 = 1}$$

$$418 = x_1 + x_2 2 + x_3 2^2 + x_4 2^3 + \ldots \text{ ——— (II)}$$

Divide (II) by 2

$$\boxed{209} + \boxed{0} = \frac{418}{2} = \boxed{\frac{x_1}{2}} + \boxed{x_2 + x_3 2 + x_4 2^2 + \ldots}$$ So, $\boxed{x_1 = 0}$

$$209 = x_2 + x_3 2 + x_4 2^2 + x_5 2^3 + \ldots \text{ ——— (III)}$$

837 in base 10:

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \quad \text{———} \quad (\mathrm{I})$$

$$418 = x_1 + x_2 2 + x_3 2^2 + x_4 2^3 + \cdots \quad \text{———} \quad (\mathrm{II})$$

$$209 = x_2 + x_3 2 + x_4 2^2 + x_5 2^3 + \cdots \quad \text{———} \quad (\mathrm{III})$$

$$\boxed{x_0 = 1}$$

$$\boxed{x_1 = 0}$$

Divide (III) by 2 and ... you get the idea ☺

837 in base 10:

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \qquad \text{(I)} \qquad \boxed{x_0 = 1}$$

$$418 = x_1 + x_2 2 + x_3 2^2 + x_4 2^3 + \cdots \qquad \text{(II)} \qquad \boxed{x_1 = 0}$$

$$209 = x_2 + x_3 2 + x_4 2^2 + x_5 2^3 + \cdots \qquad \text{(III)} \qquad \boxed{x_2 = 1}$$

$$104 = x_3 + x_4 2 + x_5 2^2 + x_6 2^3 + \cdots \qquad \text{(IV)} \qquad \boxed{x_3 = 0}$$

$$52 = x_4 + x_5 2 + x_6 2^2 + x_7 2^3 + \cdots \qquad \text{(V)} \qquad \boxed{x_4 = 0}$$

$$26 = x_5 + x_6 2 + x_7 2^2 + \cdots \qquad \text{(VI)} \qquad \boxed{x_5 = 0}$$

$$13 = x_6 + x_7 2 + x_8 2^2 + \cdots \qquad \text{(VII)} \qquad \boxed{x_6 = 1}$$

$$6 = x_7 + x_8 2 + x_9 2^2 + \cdots \qquad \text{(VIII)} \qquad \boxed{x_7 = 0}$$

$$3 = x_8 + x_9 2 + x_{10} 2^2 + \cdots \qquad \text{(IX)} \qquad \boxed{x_8 = 1}$$

$$1 = x_9 + x_{10} 2 + x_{11} 2^2 + \cdots \qquad \text{(X)} \qquad \boxed{x_9 = 1}$$

837 in base 10!

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \qquad (1)$$

$$837 = 1 + 0*2 + 1*2^2 + 0*2^3 + 0*2^4 + 0*2^5 + 1*2^6 + 0*2^7 + 1*2^8 + 1*2^9$$

$$= 1 + 2^2 + 2^6 + 2^8 + 2^9$$

$\boxed{x_0 = 1}$

$\boxed{x_1 = 0}$

$\boxed{x_2 = 1}$

$\boxed{x_3 = 0}$

$\boxed{x_4 = 0}$

$\boxed{x_5 = 0}$

$\boxed{x_6 = 1}$

$\boxed{x_7 = 0}$

$\boxed{x_8 = 1}$

$\boxed{x_9 = 1}$

837 in base 10:

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \quad \text{———} \quad (1)$$

$$837 = 1 + 0*2 + 1*2^2 + 0*2^3 + 0*2^4 + 0*2^5 + 1*2^6$$
$$+ 0*2^7 + 1*2^8 + 1*2^9$$

$$= 1 + 2^2 + 2^6 + 2^8 + 2^9$$

837 in base 10:

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \qquad (1)$$

$$837 = 1 + 0*2 + 1*2^2 + 0*2^3 + 0*2^4 + 0*2^5 + 1*2^6 + 0*2^7 + 1*2^8 + 1*2^9$$

$$= 1 + 2^2 + 2^6 + 2^8 + 2^9$$

Check: $\boxed{1 + 4 + 64 + 256 + 512 = 837}$ ✓

837 in base 10:

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \tag{1}$$

$$837 = 1 + 0*2 + 1*2^2 + 0*2^3 + 0*2^4 + 0*2^5 + 1*2^6 + 0*2^7 + 1*2^8 + 1*2^9$$

$$= 1 + 2^2 + 2^6 + 2^8 + 2^9$$

837 in base 10:

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \qquad \text{———} \quad (1)$$

$$837 = 1 + 0*2 + 1*2^2 + 0*2^3 + 0*2^4 + 0*2^5 + 1*2^6$$
$$+ 0*2^7 + 1*2^8 + 1*2^9$$

$$= 1 + 2^2 + 2^6 + 2^8 + 2^9$$

We say 837 in base 10 is 1101000101 in base 2

837 in base 10:

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \quad \text{———} \quad (1)$$

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$$= 1 + 2^2 + 2^6 + 2^8 + 2^9$$

We say 837 in base 10 is 1101000101 in base 2

Because the series is in powers of 2!

837 in base 10:

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \qquad \text{———} \qquad (1)$$

$$837 = 1 + 0*2 + 1*2^2 + 0*2^3 + 0*2^4 + 0*2^5 + 1*2^6$$
$$+ 0*2^7 + 1*2^8 + 1*2^9$$

$$= 1 + 2^2 + 2^6 + 2^8 + 2^9$$

We say 837 in base 10 is 1101000101 in base 2

837 in base 10:

$$837 = x_0 + x_1 2 + x_2 2^2 + x_3 2^3 + \cdots \quad \text{———} \quad (1)$$

$$837 = 1 + 0*2^{1} + 1*2^{2} + 0*2^{3} + 0*2^{4} + 0*2^{5} + 1*2^{6} + 0*2^{7} + 1*2^{8} + 1*2^{9}$$

$$= 1 + 2^2 + 2^6 + 2^8 + 2^9$$

We say 837 in base 10 is 1101000101 in base 2

9 8 7 6 5 4 3 2 1 0

The digits are written from the highest degree term (i.e. $1*2^9$) to the lowest degree term (i.e. $1*2^0$)

We say 837 in base 10 is 1101000101 in base 2

We say 837 in base 10 is 1101000101 in base 2

<u>none</u> of the digits in base 2 are equal to or larger than 2

We say 837 in base 10 is 1101000101 in base 2

We could have guessed that 837 in base 10 would have a representation in base 2 with 10 digits, because $2^9 < 837 < 2^{10}$!!

We say 837 in base 10 is $\underbrace{1101000101}_{\text{10 digits}}$ in base 2

We say 837 in base 10 is 1101000101 in base 2

837 in base 10 is ? ? ? in base 3 ?

837 in base 10 is ? ? ? in base 7 ?

837 in base 10 is ? ? ? in base 16 ???

837 in base 10 is ??? in base 16 ???

For bases greater than base 10, the digits can take on values greater than 10

For base 16, a.k.a. "Hexadecimal", each digit can take values
0, 1, 2, . . . , 8, 9, a, b, c, d, e, f

837 in base 10 is ? ? ? in base 16 ???

For base 16, a.k.a. "Hexadecimal", each digit can take values
0, 1, 2, . . . , 8, 9, a, b, c, d, e, f

10 11 12 13 14 15

base 16 ???

For base 16, a.k.a. "Hexadecimal", each digit can take values
0, 1, 2, . . . , 8, 9, a, b, c, d, e, f

837 in base 10 is ??? in base 16 ???

837 in base 10 is ? ? ? in base 16 ???