A number is represented as 14 (in base x) and 110 (in base y). You are told $x - y = 5$. What is x ?

$$14_x = 1 \cdot x^1 + 4 \cdot x^0 = 1 \cdot y^2 + 1 \cdot y^1 + 0 \cdot y^0 = 110_y .$$

This is the equation $\quad x + 4 = y^2 + y$ — (I)

Also, $\quad x - y = 5$ — (II)

(I) − (II) gives $\quad 4 + y = y^2 + y - 5$

$$4 + 5 = y^2$$

$$9 = y^2$$

$$3 = y \quad \text{(since } y \text{ must be positive)}$$

so, $x - y = x - 3 = 5$, so $x = 8$.

You are told 219 (in base 10) is 432 in some other base. What base is 432 written in?

Hint: the quadratic equation $ax^2+bx+c=0$ *has roots* $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$219_{10} = 2\times10^2 + 1\times10^1 + 9\times10^0 = 4x^2 + 3x^1 + 2x^0 = 432_x$$

Solve this quadratic equation.

$$4x^2 + 3x + 2 = 219$$

Using the hint,

$$x = \left(-3 \pm \left(9 - 4\times4\times(-217)\right)^{1/2}\right)/8$$

$$= \left(-3 \pm \sqrt{3481}\right)/8$$

$$= \left(-3 \pm 59\right)/8$$

$$= 56/8 = 7$$

since x must be positive

An instruction for a hypothetical CPU is 1940 in hexadecimal. The "19" is the opcode. How many bits are used to represent the opcode in this architecture? Give your answer in **bytes**.

1940 is a 4-digit number written in base 16.
A single digit can represent 16 numbers.
2 digits can represent $16 \times 16 = 256$ numbers.
So, the opcode has 256 possibilities; the same number of possibilities as 2^8, i.e. an 8-digit binary number. So, the opcode can be represented by 8 bits (1 byte).

A hypothetical CPU executes instructions formatted in binary. The opcode in these instructions can be represented by 2 octal digits (i.e., each digit is in base 8). Which of the following options is a valid opcode format in binary that the CPU expects?

- a. 10
- b. 00000010
- c. 0010
- d. 000010 ✔

An octal digit can represent 8 numbers. This is 2^3, so these numbers can be represented by 3 bits. So, 2 octal digits need 6 bits.