"WARM UP" QUESTIONS:

How many numbers can be represented by:

(a) a 1 digit number in base 2? $(2^1) = 2$

(b) a 3 digit number in base 5? $(5^3) = 125$

(c) a 5 digit number in base 3? $(3^5) = 243$

(d) a 2 digit number in base 14? $(14^2) = 196$

(e) a 9 digit number in base 1? $(1^9) = 1$

↳ :-) this is a cheeky question. Can you think about problems with this?

What are the largest numbers for:

(a) a 2 digit number in base 6? (55)

(b) a 2 digit number in base 8? (77)

(c) a 3 digit number in base 4? (333)

(d) a 4 digit number in base 3? (2222)

(e) a 6 digit number in base 20? (JJJJJJ)

Convert the following numbers between bases:

(a) 758 in base 10 to base 6 (decimal to senary)

(b) a831 in base 11 to base 13 (undecimal to trixadecimal)

(c) 11011 in base 2 to base 8 (binary to octal)

(d) 29 in hexadecimal to base 2 (hexadecimal to binary)

Convert numbers between bases:

(a) 758 in base 10 to base 6

1a

Solution: each digit in a base 6 number can take 6 possible values (i.e. 0 to 5).

We need to find $x_0, x_1, \ldots$ and so on, in the following relationship.

$$758 = x_0 + x_1 6 + x_2 6^2 + x_3 6^3 + x_4 6^4 + \cdots \quad \text{——— (1)}$$

First, we divide both sides of (1) by 6, to get

$$\frac{758}{6} = \frac{x_0}{6} + x_1 + x_2 6 + x_3 6^2 + x_4 6^3 + \cdots$$

$$\boxed{126} + \frac{\boxed{2}}{6} = \frac{\boxed{x_0}}{6} + \boxed{x_1 + x_2 6 + x_3 6^2 + x_4 6^3 + \cdots}$$

, so $\boxed{x_0 = 2}$. We equate the remaining terms and continue solving.

$$126 = x_1 + x_2 6 + x_3 6^2 + x_4 6^3 + \cdots \quad \text{——— (11)}$$

divide both sides of (11) by 6, to get

Convert numbers between bases:

758 in base 10 to base 6

2a

solution:

$$126 = x_1 + x_2 6 + x_3 6^2 + x_4 6^3 + \cdots \quad \text{———} \quad (11)$$

divide both sides of (11) by 6, to get

$$\frac{126}{6} = \frac{x_1}{6} + x_2 + x_3 6 + x_4 6^2 + \cdots$$

$$\boxed{21} + \frac{\boxed{0}}{6} = \frac{\boxed{x_1}}{6} + \boxed{x_2 + x_3 6 + x_4 6^2 + \cdots}.$$ So $\boxed{x_1 = 0}$. We equate the remaining terms and continue solving.

$$21 = x_2 + x_3 6 + x_4 6^2 + x_5 6^3 + \cdots \quad \text{———} \quad (111)$$

divide both sides of (111) by 6, to get

$$\frac{21}{6} = \frac{x_2}{6} + x_3 + x_4 6 + x_5 6^2 + \cdots$$

$$\boxed{3} + \frac{\boxed{3}}{6} = \frac{\boxed{x_2}}{6} + \boxed{x_3 + x_4 6 + x_5 6^2 + \cdots}.$$ So $\boxed{x_2 = 3}$. We equate the remaining terms and continue solving.

Convert numbers between bases:

758 in base 10 to base 6

(3a)

solution:

$$\frac{21}{6} = \frac{x_2}{6} + x_3 + x_4 6 + x_5 6^2 + \cdots$$

$$\boxed{3} + \frac{\boxed{3}}{6} = \frac{\boxed{x_2}}{6} + \boxed{x_3 + x_4 6 + x_5 6^2 + \cdots}.$$ So $\boxed{x_2 = 3}$. We equate the remaining terms and continue solving.

$$3 = x_3 + x_4 6 + x_5 6^2 + \cdots \qquad \text{(IV)}$$

divide both sides of (IV) by 6, to get

$$\boxed{0} + \frac{\boxed{3}}{6} = \frac{\boxed{x_3}}{6} + \boxed{x_4 + x_5 6 + x_6 6^2 + \cdots}.$$ So $\boxed{x_3 = 3}$. We equate the remaining terms and continue solving.

$$0 = x_4 + x_5 6 + x_6 6^2 + \cdots.$$ Since the l.h.s. is 0, all of the "x"s on the r.h.s. must be 0 too.

So, 758 in base 10 is $x_3 x_2 x_1 x_0 = 3302$ in base 6.

check: $2 \cdot 6^0 + 0 \cdot 6^1 + 3 \cdot 6^2 + 3 \cdot 6^3$
$= 2 + 108 + 648 = 758$ ✓

Convert numbers between bases:

(b) a831 in base 11 to base 13

Solution: 7b

Each digit in a base 11 number can take on 11 values.
That is, 0, 1, 2, ..., 9, a.
Similarly, each digit in a base 13 number can take on 13 values.
That is, 0, 1, 2, ..., 9, a, b, c

So, use our successive calculations. To simplify the calculations, first convert a831 to base 10,

so $a831 = \boxed{a} \times 11^3 + 8 \times 11^2 + 3 \times 11 + 1 \times 11^0 = \boxed{10} \times 11^3 + 8 \times 11^2 + 3 \times 11 + 1 \times 11^0 = 14312$

Then,

$$a831 = 14312 = x_0 + x_1 13 + x_2 13^2 + x_3 13^3 + \cdots \quad (1)$$

divide by 13,

$$\frac{14312}{13} = \frac{x_0}{13} + x_1 + x_2 13 + x_3 13^2 + \cdots$$

"c" in base 13

$$\boxed{1100} + \frac{\boxed{12}}{13} = \frac{\boxed{x_0}}{13} + \boxed{x_1 + x_2 13 + x_3 13^2 + \cdots}$$

So $\boxed{x_0 = c}$. Equate the remaining terms and continue solving.

Convert numbers between bases:

a831 in base 11 to base 13

2b

$$1100 = x_1 + x_2 13 + x_3 13^2 + \cdots \quad \text{———} \quad (II)$$

divide by 13, to get

$$\frac{1100}{13} = \frac{x_1}{13} + x_2 + x_3 13 + \cdots$$

$$84 + \frac{\boxed{8}}{13} = \frac{\boxed{x_1}}{13} + \boxed{x_2 + x_3 13 + x_4 13^2 + \cdots}$$ So $\boxed{x_1 = 8}$. Equate the remaining terms and continue solving.

$$84 = x_2 + x_3 13 + x_4 13^2 + \cdots \quad \text{———} \quad (III)$$

divide by 13, to get

$$6 + \frac{\boxed{6}}{13} = \frac{\boxed{x_2}}{13} + \boxed{x_3 + x_4 13 + \cdots}.$$ So $\boxed{x_2 = 6}$. Equate the remaining terms and continue solving

$$6 = x_3 + x_4 13 + x_5 13^2 + \cdots \quad \text{———} \quad (IV)$$

So $\boxed{x_3 = 6}$.

divide by 13, to get $\boxed{0} + \frac{\boxed{6}}{13} = \frac{\boxed{x_3}}{13} + \boxed{x_4 + x_5 13 + x_6 13^2 + \cdots}$

Equate the remaining and continue solving.

Convert numbers between bases:

a831 in base 11 to base 13

(3b)

$0 = x_4 + x_5 13 + \cdots$ Since the l.h.s. is 0, all of the remaining "x"s must be 0 too!

So a831 in base 11 is (14312 in base 10 is) $x_3 x_2 x_1 x_0 = 668c$ in base 13

$$\text{Check: } 668c = 6 \times 13^3 + 6 \times 13^2 + 8 \times 13 + c$$
$$= 13182 + 1014 + 104 + 12$$
$$= 14312 \checkmark$$

Convert numbers between bases:

(c) 11011 in base 2 to base 8

Check: $11011_2 = 1+2+2^3+2^4$
$= 27 = 3+3\times 8$
$= 33_8$ ✓

Solution: let's solve it quickly. For practice, use the longer method to get the same answer.

Notice that each digit in a "base 8" number can take on 8 values (i.e. 0, 1, 2, --, 7). This means each digit is "3 bits", because 3 bits can be used to represent 8 numbers too. So, let's break up 11011 into groups of 3 bits. We have to include a 0 on the left to make things look balanced. So, $11011 = \boxed{011}\boxed{011}$

each of these in base 10 equals
$0\cdot 2^2 + 1\cdot 2 + 1\cdot 2^0 = 0+2+1$
$=3$

And, see that $11011 = \underbrace{\boxed{3}\boxed{3}}_{\text{base } 10} = \underbrace{\boxed{3}\boxed{3}}_{\text{base } 8} = 33$ in base 8

This equality is true because $3<8$!!!

This equality is true because each digit in a base 8 number lies between 0 and 7!!!

So, 11011 in base 2
=
33 in base 8

Convert numbers between bases:

(d) 29 in base 16 to base 2

solution: a quick solution. For practice, use the longer method to get the same answer.

Each digit in a base 16 number has 16 possible values. That is, 0, 1, 2, ..., 9 a, b, c, d, e, f.

So, each digit is "4 bits", because 4 bits can also be used to represent 16 possibilities.

Look at each digit of 29. The first, "2", is 0010 in 4 bits. The second, "9", is 1001.

$$\text{So, } 29 = \underbrace{\boxed{2}\,\boxed{9}}_{\text{base } 10} = \underbrace{\boxed{0010}\,\boxed{1001}}_{\text{base } 2} = \underbrace{\boxed{10}\,\boxed{1001}}_{\text{base } 2} = 101001 \text{ in base } 2$$

We drop the "00" in the first yellow box, by convention!!

Check:

$$101001_2 = 1 + 2^3 + 2^5$$
$$= 41$$
$$= 9 + 2 \cdot 16$$
$$= 29_{16}$$

So, 29 in base 16

=

101001 in base 2