Defn:

Let $a_0, a_1, \ldots, a_n$ be non-negative integers and let x be an integer such that $x \geq 2$ and $x > \max\{a_0, \ldots, a_n\}$. We say $a_n a_{n-1} \cdots a_1 a_0$ is a number written in base x if this is "shorthand" for writing $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 x^0$.

e.g.

1011 in base 10 means $\boxed{1} \cdot 10^3 + \boxed{0} \cdot 10^2 + \boxed{1} \cdot 10^1 + \boxed{1} \cdot 10^0$

1011 in base 2 means $\boxed{1} \cdot 2^3 + \boxed{0} \cdot 2^2 + \boxed{1} \cdot 2^1 + \boxed{1} \cdot 2^0$

1011 in base 7 means $\underbrace{1 \cdot 7^3 + 0 \cdot 7^2 + 1 \cdot 7^1 + 1 \cdot 7^0}_{\text{polynomial in powers of the base}}$

$\frac{1}{3} = \boxed{1} \cdot 3^{-1}$ 0.1

$a_n 1^n + a_{n-1} \cdot 1^{n-1} + \cdots + a_0 1^0 = 0$

$0.111 = \boxed{1}\, 3^{-1} + \boxed{1} \cdot 3^{-2} + 1 \cdot 3^{-3}$

$2 \quad 10 \; 5 \; 2 \quad 6 \quad 9$

Some noteworthy observations;

1) In base x, the largest value for a single digit must be $x-1$; e.g. 7

2) The smallest value for a single digit must be 0;

3) There are x possible values for each digit of a number in base x; $\quad 2 \quad 0, 1$

4) * The highest power in a polynomial representation of a number in base x is "1 less" than the number of digits representing the number.

$$5201_6 = 5 \times 6^3 + 2 \times 6^2 + 0 \cdot 6^1 + 1 \times 6^0$$

What is $x+1$ in base x?

$x+1 = a_n x^n + \cdots + a_0 x^0$

$= \boxed{1} \cdot x^1 + \boxed{1} \cdot x^0$

0.53

11 x

0.1 $\boxed{\frac{5}{10} + \frac{3}{100}}$

x in base $x+1$?

$\frac{1}{10}$

$x = x \cdot (x+1)^0$

3

a_3 a_2

$x^3 = 1 \cdot x^3 + 0 \cdot x^2 + \cdots + 0 \cdot x^0$

1000 a_1

$\boxed{\frac{1}{3} \text{ written as } 0.\boxed{1}?}$

$\frac{1}{3} = \frac{\boxed{1}}{x}$

x

$1/3 = \frac{1}{x}$

3

0.1

0.5 $\frac{5}{10}$

0.712 $\frac{7}{10} + \frac{1}{100} + \frac{2}{1000}$

$\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$

$x^3 = x \cdot (x^2)^1 + 0 \cdot (x^2)^0$

$x0$