> $with(LinearAlgebra)$:

> $with(ArrayTools)$:

> $n := 10$:

> $A := RandomMatrix\left(n, shape = triangular_{lower}\right) + 100 \cdot n \cdot IdentityMatrix(n) + RandomMatrix(n, density = 0.1)$

$$A := \begin{bmatrix} 1010 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 67 \\ 42 & 1005 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 98 & -50 & 970 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -31 & 75 & -30 & 1015 & 0 & 0 & 0 & 0 & 0 & 0 \\ -21 & 58 & -17 & 33 & 1037 & 0 & 0 & 0 & 0 & 0 \\ -87 & 17 & -69 & -2 & -43 & 919 & 0 & 0 & 0 & 0 \\ -90 & -8 & 78 & 50 & 22 & 21 & 938 & 0 & 0 & 0 \\ -19 & -80 & 77 & 66 & -89 & -15 & -53 & 981 & 0 & 0 \\ 86 & 7 & -69 & 80 & -2 & 68 & 23 & -43 & 960 & 0 \\ 64 & -86 & -60 & -46 & -27 & -48 & -77 & -86 & 122 & 1027 \end{bmatrix} \quad (1)$$

> $b0 := RandomMatrix(n, 1)$

$$b0 := \begin{bmatrix} 29 \\ -50 \\ 91 \\ 71 \\ -64 \\ 36 \\ -1 \\ 57 \\ -96 \\ -41 \end{bmatrix} \quad (2)$$

> #Метод простой итерации

> $t := MatrixNorm(A)$:

> $B1 := IdentityMatrix(n) - \frac{A}{t}$:

> $NormMPI := evalf(MatrixNorm(B1, 1))$

$$NormMPI := 0.7127206330 \quad (3)$$

> $b1 := \frac{b0}{t}$:

> $x1[0] := Matrix(n, 1, [seq(0, i = 1..n)])$:

>

> $\varepsilon adm := 0.000001$

$$\varepsilon adm := 1.\ 10^{-6} \quad (4)$$

```
> ε1[0] := infinity :
> for i from 1 by 1 while ε1[i − 1] > εadm do
  x1[i] := evalf(B1 • x1[i − 1] + b1) :
  ε1[i] := NormMPI/(1 − NormMPI) .MatrixNorm(x1[i] - x1[ i − 1], 1) :
  end do:
> IMPI := i − 1
```

$$IMPI := 18 \tag{5}$$

```
> evalf(x1[i − 1]) :
>
> #Метод Якоби
  U := UpperTriangle(A, 1) :
> L := LowerTriangle(A, -1) :
> d := A − L − U:
> B2 := -d^(-1).(L + U) :
> b2 := d^(-1).b0 :
> NY := evalf(MatrixNorm(B2))
```

$$NY := 0.5998052580 \tag{6}$$

```
>  x2[0] := Matrix(n, 1, [seq(0, i = 1 ..n)]) :
>
> ε2[0] := infinity :
> for i from 1 by 1 while ε2[i − 1] > εadm do
  x2[i] := evalf(B2 • x2[i − 1] + b2) :
  ε2[i] := NY/(1 − NY) .MatrixNorm(x2[i] - x2[i − 1], 1) :
  end do:
> IMY := i − 1
```

$$IMY := 8 \tag{7}$$

```
>
> evalf(x2[i − 1]) :
> #Метод Зейделя
> B3 := -(L + d)^(-1).U:
> b3 := (L + d)^(-1).b0 :
> NZ := evalf(MatrixNorm(B3))
```

$$NZ := 0.06633663366 \tag{8}$$

```
>  x3[0] := Matrix(n, 1, [seq(0, i = 1 ..n)]) :
>
> ε3[0] := infinity :
> for i from 1 by 1 while ε3[i − 1] > εadm do
  x3[i] := evalf(B3 • x3[i − 1] + b3) :
  ε3[i] := NZ/(1 − NZ) .MatrixNorm(x3[i] - x3[i − 1], 1) :
  end do:
> IMZ := i − 1
```

(9)

$$IMZ := 3 \tag{9}$$

>

>

> $evalf(x3[i-1])$:

>

> #Точное решение

$x0 := evalf(A^{-1}.b0)$

$$x0 := \begin{bmatrix} 0.03017794640 \\ -0.05101241169 \\ 0.08813602131 \\ 0.07724682547 \\ -0.05926555075 \\ 0.04698602043 \\ -0.009714178428 \\ 0.03723030454 \\ -0.1039852379 \\ -0.02208546070 \end{bmatrix} \tag{10}$$

>

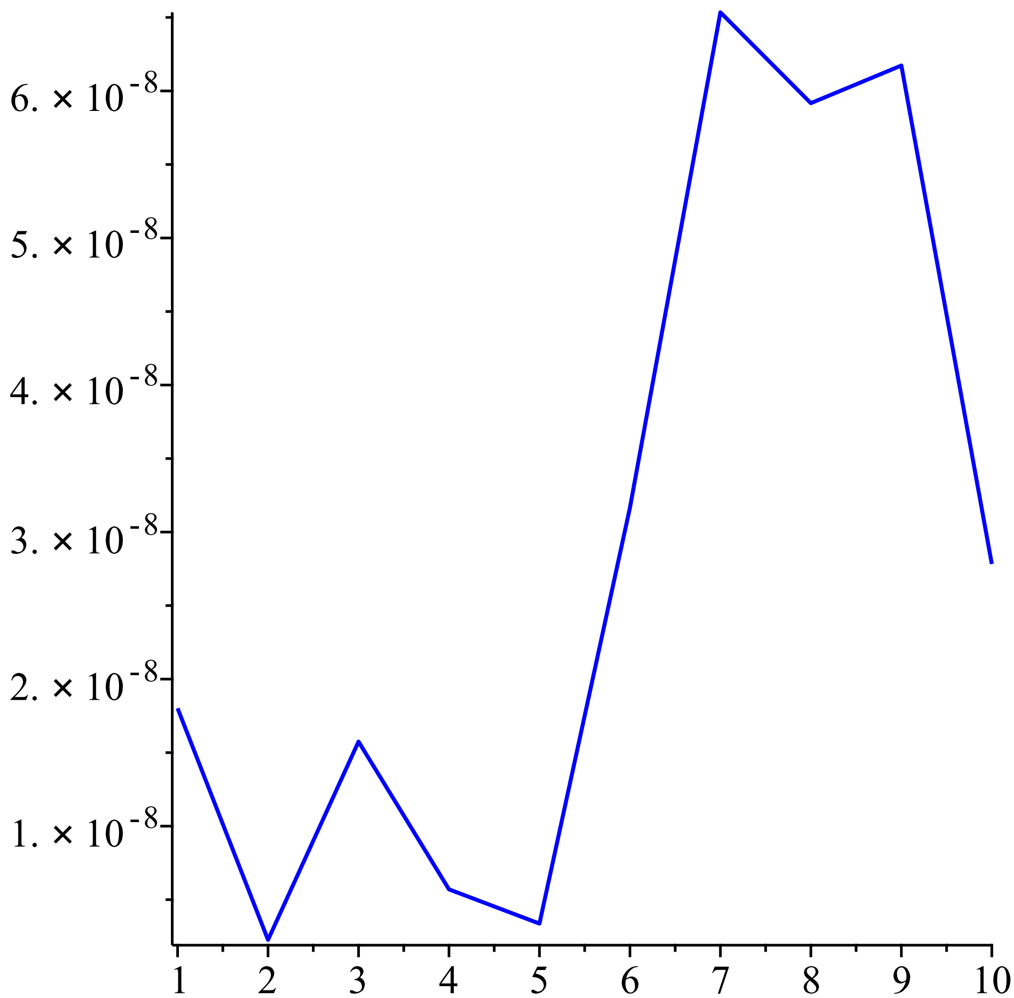
> $seq\_abc\_x := Matrix([seq([i], i=1..n)])$:

> $seq\_err\_x1 := Matrix([seq([\mathrm{abs}(x0[i]-x1[IMPI][i])], i=1..n)])$:

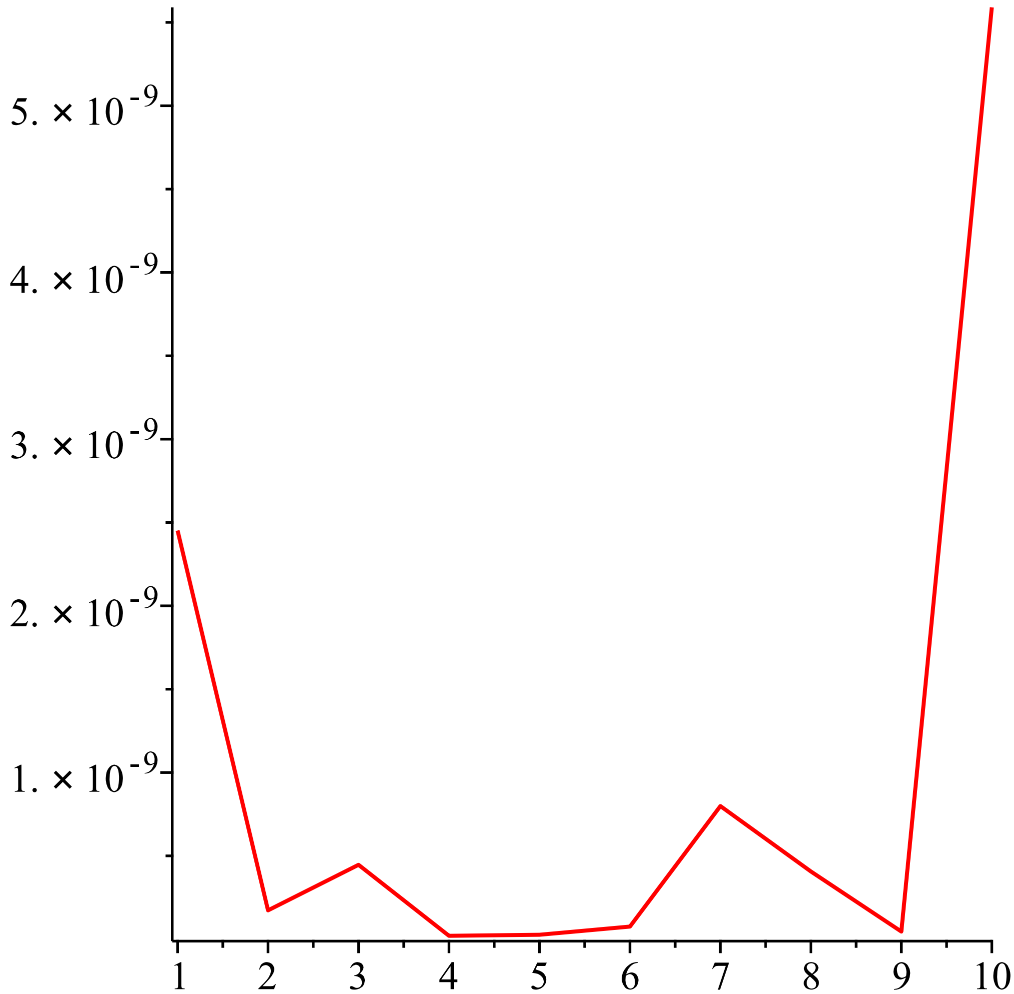
> $seq\_err\_x2 := Matrix([seq([\mathrm{abs}(x0[i]-x2[IMY][i])], i=1..n)])$:

> $seq\_err\_x3 := Matrix([seq([\mathrm{abs}(x0[i]-x3[IMZ][i])], i=1..n)])$:

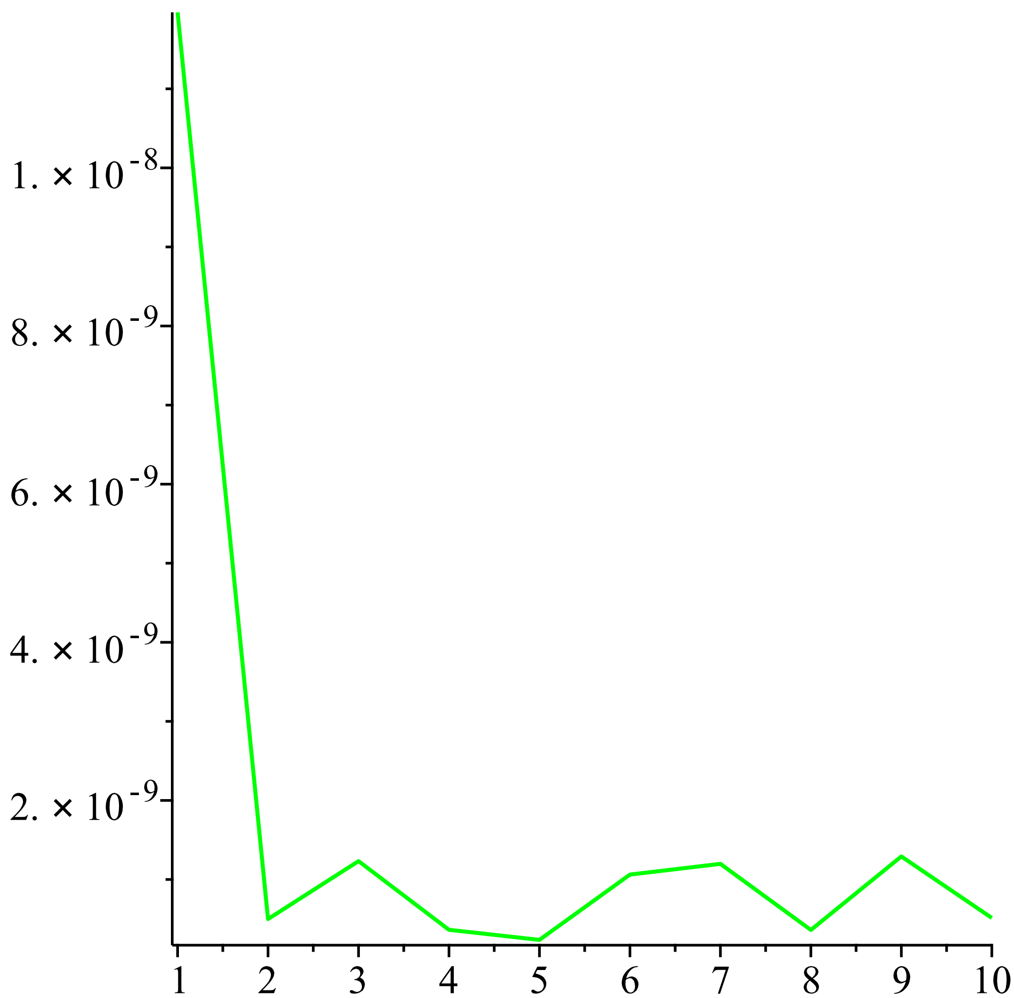
> $plot\_err\_x1 := plot(\langle seq\_abc\_x | seq\_err\_x1 \rangle, color=blue)$



> $plot\_err\_x2 := plot(\langle seq\_abc\_x | seq\_err\_x2 \rangle, color = red)$



> $plot\_err\_x3 := plot(\langle seq\_abc\_x | seq\_err\_x3 \rangle, color = green)$


1. × 10^-8
8. × 10^-9
6. × 10^-9
4. × 10^-9
2. × 10^-9
1
2
3
4
5
6
7
8
9
10

>

>

> $with(LinearAlgebra):$

>

> $with(ArrayTools):$

> $n := 10:$

>

> $A := RandomMatrix(n, generator = -100 ..100) + 500 \cdot n \cdot IdentityMatrix(n) + RandomMatrix(n, density = 0.1)$

$$A := \begin{bmatrix} 4942 & -89 & 63 & 97 & 78 & -94 & -60 & 35 & 94 & -53 \\ 28 & 5058 & 1 & -63 & -47 & 96 & -82 & -10 & -91 & 87 \\ 14 & -56 & 5033 & 64 & 81 & -99 & -62 & -74 & -58 & 6 \\ 58 & -93 & 16 & 4955 & 46 & 80 & 68 & 88 & -15 & 20 \\ -44 & -83 & 98 & 9 & 5008 & 98 & -100 & -53 & -34 & -61 \\ -90 & 68 & 27 & -68 & -93 & 4924 & 6 & -5 & 32 & 61 \\ 76 & -27 & 0 & 86 & 104 & 60 & 4963 & -41 & 80 & -180 \\ 60 & -68 & 22 & -25 & -17 & 68 & -90 & 4959 & 10 & 94 \\ -4 & 15 & 19 & -9 & 79 & -55 & 4 & -81 & 4915 & -88 \\ 36 & -8 & 23 & -57 & -74 & -17 & 58 & 59 & 38 & 5030 \end{bmatrix} \quad (1)$$

> $b0 := RandomMatrix(n, 1)$

$$b0 := \begin{bmatrix} 54 \\ 80 \\ 75 \\ 15 \\ 38 \\ 69 \\ 72 \\ -77 \\ -20 \\ 56 \end{bmatrix} \quad (2)$$

> *#Метод простых итераций*
$t := MatrixNorm(A):$

> $B1 := IdentityMatrix(n) - \frac{A}{t}:$

> $NormMPI := evalf(MatrixNorm(B1, 1))$

$$NormMPI := 0.2421221292 \quad (3)$$

> $b1 := \frac{b0}{t}:$

> $x1[0] := Matrix(n, 1, [seq(0, i = 1 ..n)]):$

>

```
> εadm := 0.000001
                                εadm := 1. 10^(−6)                                   (4)
> ε1[0] := infinity :
> for i from 1 by 1 while ε1[i − 1] > εadm do
  x1[i] := evalf(B1 · x1[i − 1] + b1) :
  ε1[i] := NormMPI/(1 − NormMPI) . MatrixNorm(x1[i] - x1[ i − 1], 1) :
  end do:
> IMPI := i − 1
                                IMPI := 6                                            (5)
> evalf(x1[i − 1]) :
>
> #Метод Якоби
  U := UpperTriangle(A, 1) :
> L := LowerTriangle(A, -1) :
> d := A − L − U :
> B2 := -d^(-1).(L + U) :
> b2 := d^(-1).b0 :
> NY := evalf(MatrixNorm(B2))
                                NY := 0.1341562121                                   (6)
>  x2[0] := Matrix(n, 1, [seq(0, i = 1 ..n)]) :
>
> ε2[0] := infinity :
> for i from 1 by 1 while ε2[i − 1] > εadm do
  x2[i] := evalf(B2 · x2[i − 1] + b2) :
  ε2[i] := NY/(1 − NY) . MatrixNorm(x2[i] - x2[i − 1], 1) :
  end do:
> IMY := i − 1
                                IMY := 4                                             (7)
>
> evalf(x2[i − 1]) :
> #Метод Зейделя
> B3 := -(L + d)^(-1).U :
> b3 := (L + d)^(-1).b0 :
> NZ := evalf(MatrixNorm(B3))
                                NZ := 0.1341562121                                   (8)
>  x3[0] := Matrix(n, 1, [seq(0, i = 1 ..n)]) :
>
> ε3[0] := infinity :
> for i from 1 by 1 while ε3[i − 1] > εadm do
  x3[i] := evalf(B3 · x3[i − 1] + b3) :
  ε3[i] := NZ/(1 − NZ) . MatrixNorm(x3[i] - x3[i − 1], 1) :
```

end do:

> $IMZ := i-1$

$$IMZ := 4 \tag{9}$$

>

> $evalf(x3[i-1])$

$$\begin{bmatrix} 0.0115896089257367 \\ 0.0155240253591480 \\ 0.0150439303526941 \\ 0.00286016188078976 \\ 0.00760460657544407 \\ 0.0139656251615052 \\ 0.0143830704193320 \\ -0.0156161243823562 \\ -0.00419368937158633 \\ 0.0112466471322716 \end{bmatrix} \tag{10}$$

>

> *#Точное решение*
> $x0 := evalf(A^{-1}.b0)$

$$x0 := \begin{bmatrix} 0.01158961391 \\ 0.01552402533 \\ 0.01504393154 \\ 0.002860162220 \\ 0.007604606942 \\ 0.01396562518 \\ 0.01438307031 \\ -0.01561612449 \\ -0.004193689348 \\ 0.01124664710 \end{bmatrix} \tag{11}$$

>

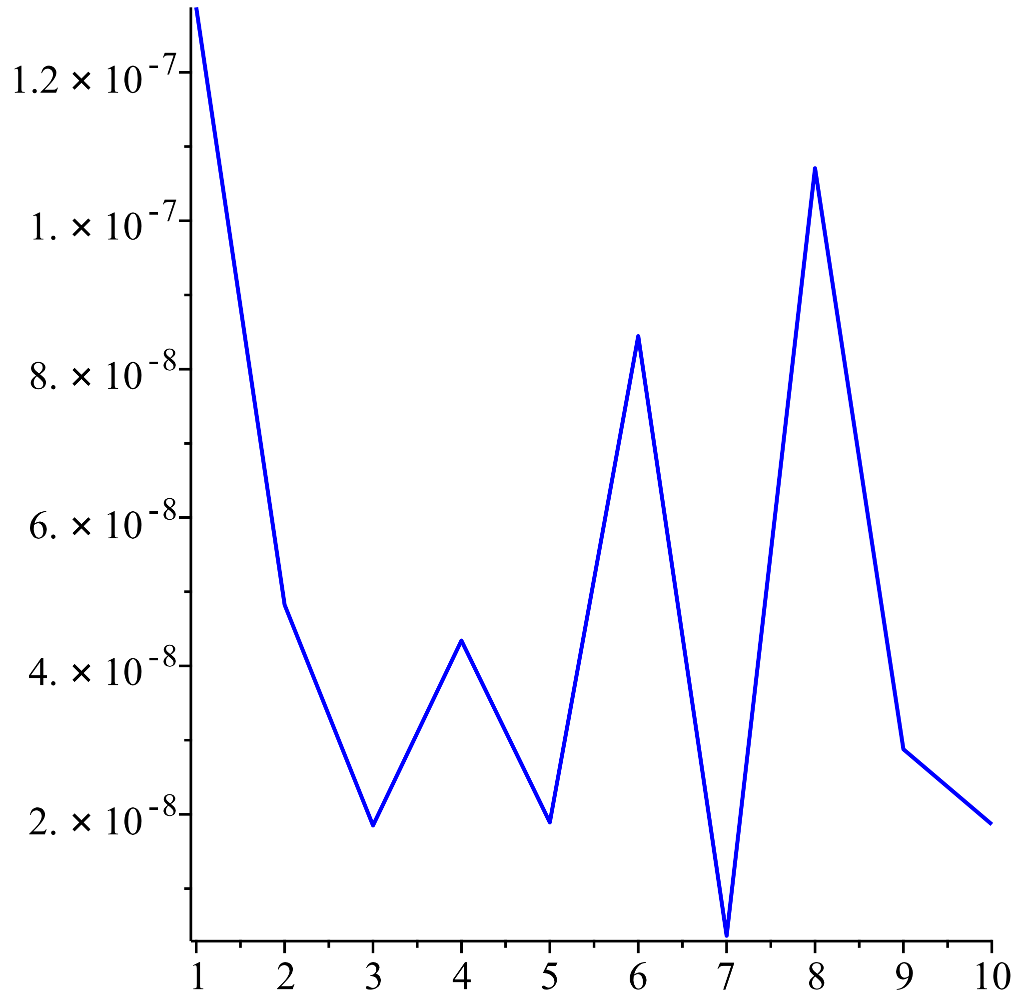
> $seq\_abc\_x := Matrix([seq([i], i=1..n)])$:

> $seq\_err\_x1 := Matrix([seq([\mathrm{abs}(x0[i]-x1[IMPI][i])], i=1..n)])$:

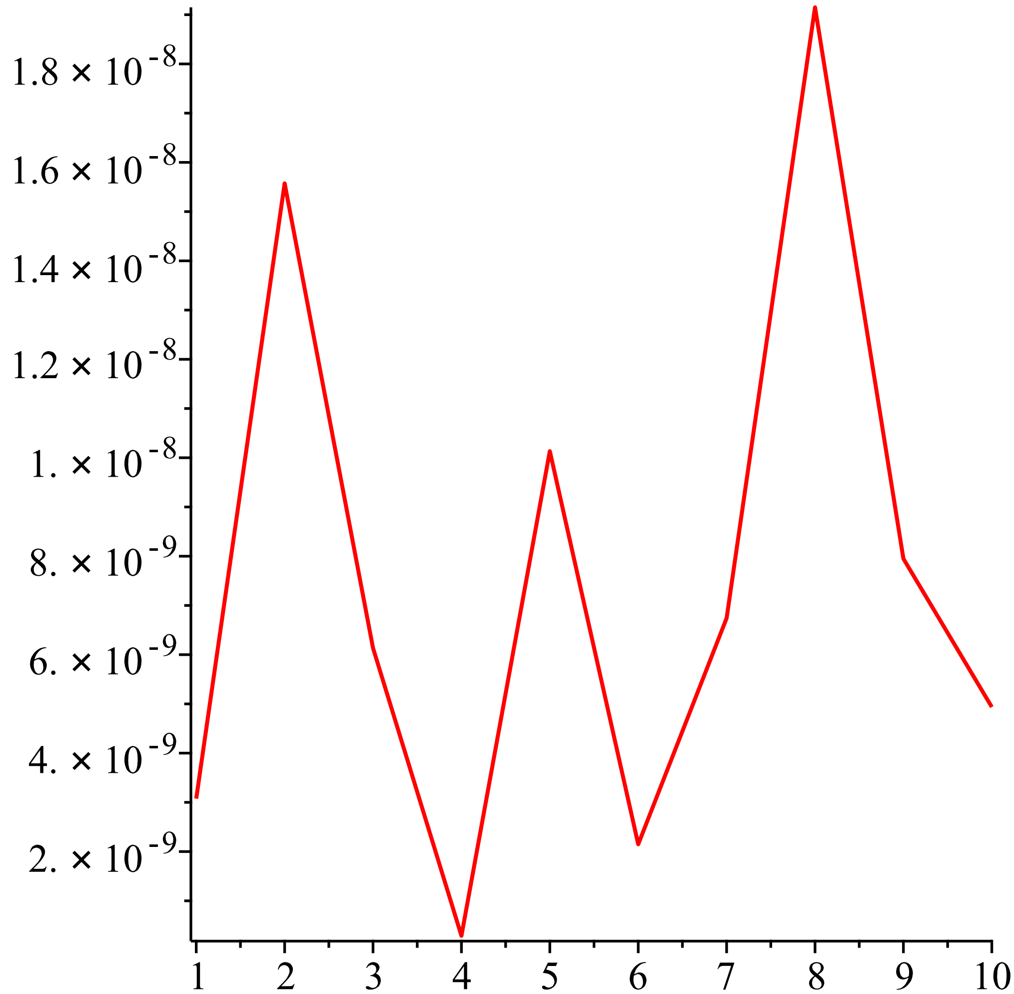
> $seq\_err\_x2 := Matrix([seq([\mathrm{abs}(x0[i]-x2[IMY][i])], i=1..n)])$:

> $seq\_err\_x3 := Matrix([seq([\mathrm{abs}(x0[i]-x3[IMZ][i])], i=1..n)])$:

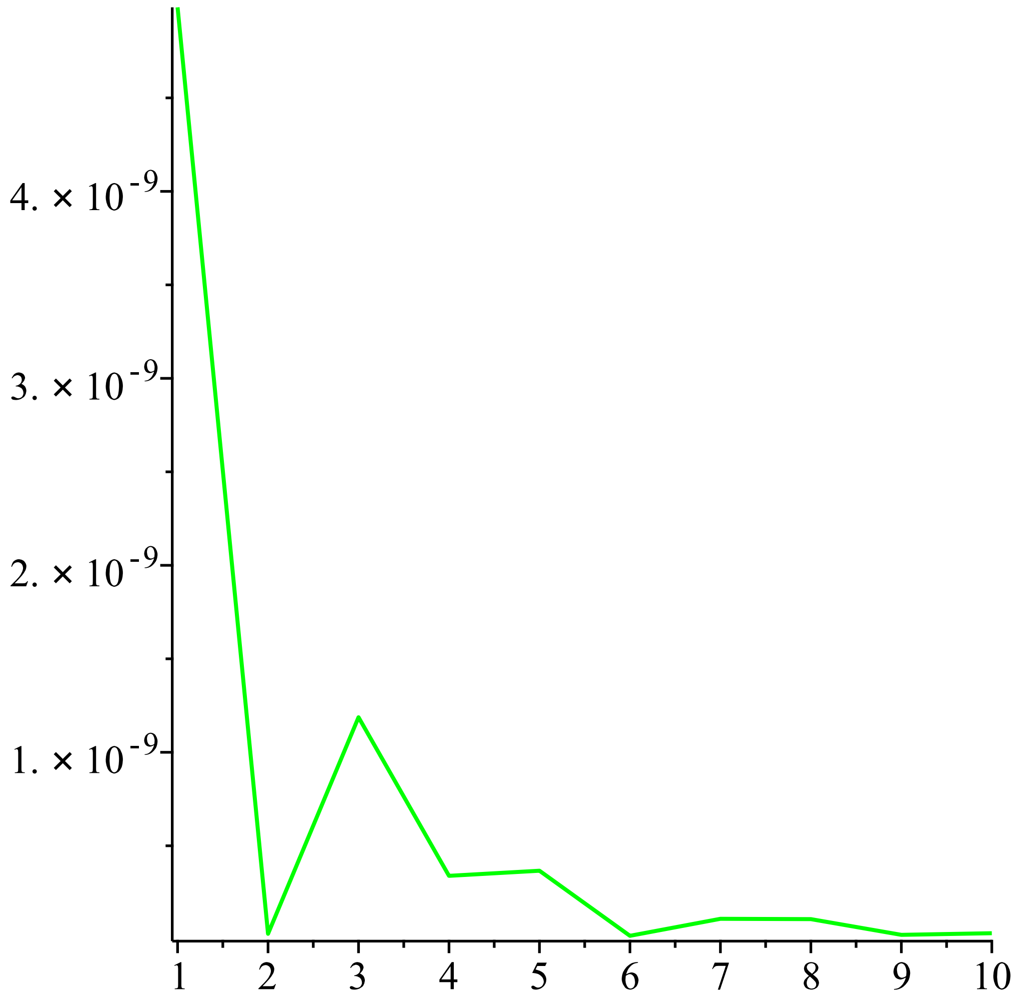
> $plot\_err\_x1 := plot(\langle seq\_abc\_x | seq\_err\_x1 \rangle, color=blue)$



> $plot\_err\_x2 := plot(\langle seq\_abc\_x | seq\_err\_x2 \rangle, color = red)$



> $plot\_err\_x3 := plot(\langle seq\_abc\_x | seq\_err\_x3 \rangle, color = green)$



4. × 10^-9
3. × 10^-9
2. × 10^-9
1. × 10^-9
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>