Reducing Risk with Imperfectly Correlated Stocks

We just noted that when the correlation is less than one $(\rho < 1)$, the portfolio standard deviation is less than the weighted average of the individual standard deviations:
$\sigma_{p,\rho<1} < x_A\sigma_A + x_B\sigma_B$.
Let's walk through this together to see how this helps us as investors.

First, we notice that if the standard deviation of a portfolio is less than the standard deviation of another, then the variance of the first portfolio is also less than that of the second.
$\sigma_{p1} < \sigma_{p2} \Leftrightarrow \sigma_{p1}^2 < \sigma_{p2}^2$

So let's compare the variance of a portfolio where correlation is +1, and compare it to another portfolio where correlation is less than 1 (let's just say 0.9).

$\sigma_{p,\rho=1.0}^2 = x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2x_Ax_B\sigma_A\sigma_B\rho_{r_Ar_B}$
where $\rho_{r_Ar_B} = 1$

Versus
$\sigma_{p,\rho=0.9}^2 = x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2x_Ax_B\sigma_A\sigma_B\rho_{r_Ar_B}$
where $\rho_{r_Ar_B} = 0.9$

If we cancel all of the identical terms in both equations, we can compare the third term in each:
$2x_Ax_B\sigma_A\sigma_B \times 1 > 2x_Ax_B\sigma_A\sigma_B \times 0.9$. Or more simply: 1 > 0.9

So we can show that the variance of the imperfectly correlated portfolio is less than the variance of the perfectly correlated one.
$\sigma_{p,\rho=1.0}^2 = (x_A\sigma_A + x_B\sigma_B)^2 = x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2x_Ax_B\sigma_A\sigma_B \times 1$
$> x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2x_Ax_B\sigma_A\sigma_B \times 0.9 = \sigma_{p,\rho=0.9}^2$

In other words: $\sigma_{p,\rho=1.0}^2 > \sigma_{p,\rho=0.9}^2$
which implies that $\sigma_{p,\rho=1.0} > \sigma_{p,\rho=0.9}$

The nice benefit of putting two stocks into a portfolio is that, as long as they're not perfectly correlated, we'll end up with a portfolio whose risk is less than the the weighted sum of the individual risks. A key benefit of portfolio diversification is that it helps us to reduce risk!