Ali Sheikholeslami

Equalizer Circuit

Welcome to the 23rd article in the "Circuit Intuitions" column series. As the title suggests, each article provides insights and intuitions into circuit design and analysis. These articles are aimed at undergraduate students but may serve the interests of other readers as well. If you read this article, I would appreciate your comments and feedback as well as your requests and suggestions for future articles in this series. Please email me your comments: ali@ece.utoronto.ca.

In a previous article [1] in this series, we discussed how a piece of wire connecting a data transmitter to a receiver attenuates the transmit signal at high frequencies, making the task of reliable data recovery by the receiver difficult. To compensate for the channel attenuation, we design an equalizer circuit and place it after the channel (Figure 1). The equalizer circuit has a transfer function that is the inverse of the wire's transfer function in the frequency range of interest, as shown in Figure 2. The equalizer is designed such that the combined transfer function of the wire and the equalizer circuit will be flat up to the Nyquist frequency f_N, defined as half of the baud rate. In this article, we review a common equalizer known as the *continuous-time linear equalizer.* We rely on the techniques described in the first article in this series [2] to find the transfer function of this equalizer.

Figure 3 presents a circuit diagram of an equalizer consisting of a differential pair with capacitive and resistive degeneration (C_s and R_s) and a parallel combination of resistive and capacitive load (R_L and C_L). The input to the differential pair is the differential received signal, V_{id}. This signal is essentially a low-pass-filtered version of the transmit signal, and we wish to use the equalizer to undo what the channel (the wire) has done to it. Intuitively, since the transmit signal is attenuated at high frequencies, we wish to amplify the received signal at high frequencies. Therefore, we need a high-pass filter; such a filter will amplify the high-frequency content of the signal or, equivalently, attenuate the low-frequency content. With this circuit, all frequency components of the transmit signal receive the same treatment and are, hence, equalized.

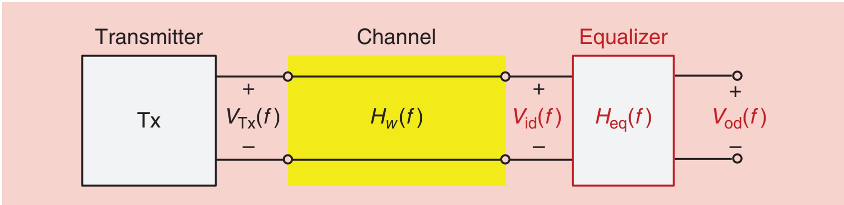

FIGURE 1: A simplified block diagram of a high-speed wireline transceiver. An equalizer at the receiver end of the channel compensates for the channel attenuation of the transmit signal.

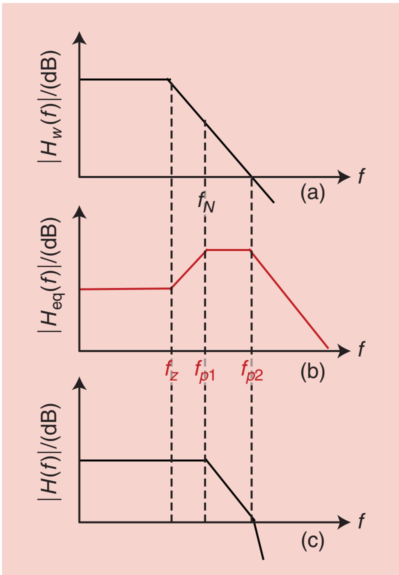

FIGURE 2: (a) The channel's (wire's) transfer function exhibits a low-pass characteristic. The attenuation at the Nyquist frequency f_N is highlighted. (b) The equalizer's transfer function exhibits a high-pass characteristic at the frequencies of interest. It has one zero and two poles. (c) The combined transfer function of the channel and the equalizer has a flat frequency response up to f_N.

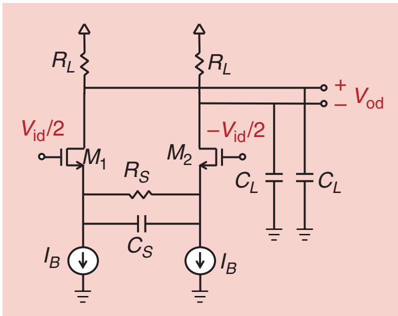

FIGURE 3: A differential circuit implementation of a continuous-time linear equalizer.

Let us first understand intuitively how this circuit performs equalization. If we assume the input is differential, that is, the left side sees $V_{id}/2$ and the right side $-V_{id}/2$, then, by symmetry, the circuit can be reduced to a half circuit, as demonstrated in Figure 4(a). Note that the transfer function

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of this circuit is identical to that of the differential one since both the input and the output are divided by two in the half circuit. We observe that the half circuit is an amplifier with source degeneration [3]. In general, the gain of a common-source amplifier is reduced when a resistor is added to its source terminal. In this case, at low frequencies, when C_s is considered open, a nonzero R_s does reduce the voltage gain of the circuit. At higher frequencies, however, the capacitor (C_s) begins to short R_s, reducing the degeneration and increasing the gain. So, in effect, this capacitor produces a frequency-dependent gain where the gain increases with frequency (as the capacitor becomes a short circuit with increasing frequency). This is the exact effect we wished to produce.

Let us now find V_o in the half circuit using the method described in [2]. We find the short circuit current at the output node (I_{scd}) and multiply it by the total impedance (Z_{eqd}) seen at the output.

Figure 4 presents the steps toward finding I_{scd}. In the first step [Figure 4(b)], we find V_s by multiplying its short circuit current (I_{scs}) and its equivalent impedance Z_{eqs}. We then use Figure 4(c) to find I_{scd}.

The short circuit current at the source node can be written as

$$I_{\text{scs}} = g_m V_i,$$

where g_m is the transistor's transconductance. Note that this is the short circuit current at the source node while the drain is also shorted to ground. The equivalent impedance at this node, while the drain is shorted, can be written as

$$Z_{\text{eqs}} = \frac{1}{g_m + 2/R_s + 2sC_s},$$

where we have ignored the body effect and the channel-length modulation (that is, we have assumed $g_{\text{me}} = g_m$ and $g_m r_o \gg 1$). The voltage at the source node, while the drain is shorted, can be written as the product of I_{scs} and Z_{eqs}:

$$V_s = \frac{g_m V_i}{g_m + 2/R_s + 2sC_s}.$$

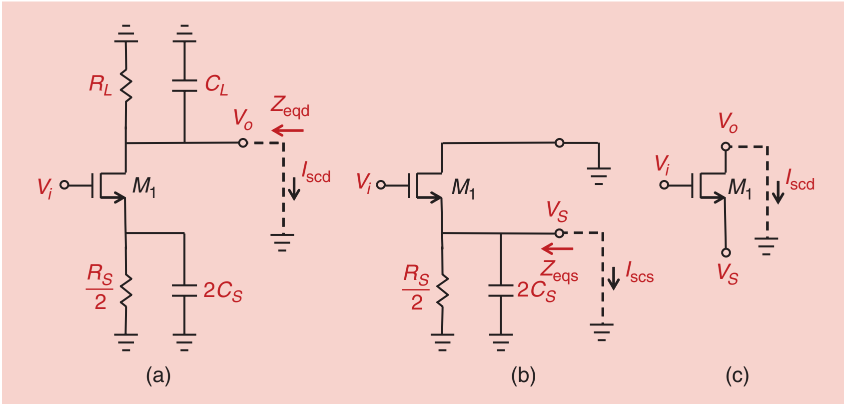

FIGURE 4: (a) A small-signal half circuit for the differential equalizer in Figure 3. We use this circuit to derive an expression for V_o. (b) A circuit model to derive an expression for V_s while the output node is shorted to ground. (c) Calculating the short circuit current at the output node given V_i and V_s.

We now use Figure 4(c) to find the output short circuit current

$$I_{\text{scd}} = g_m(V_s - V_i).$$

If we further divide I_{scd} by V_i, we find the short circuit transadmittance of this amplifier, which is

$$G(s) = \frac{-g_m}{\alpha} \frac{1 + sC_sR_s}{1 + sC_sR_s/\alpha},$$

where $\alpha = 1 + g_m R_s/2$ is the degeneration factor.

We note that $G(s)$ does not include the load. This is simply because we have shorted the output node to ground. This transfer function exhibits a gain at dc ($-g_m/\alpha$) and has one zero at $1/2\pi C_s R_s$ and one pole at $\alpha/2\pi C_s R_s$. The high-pass characteristic of this equalizer is due to the fact that the zero frequency is lower than the pole frequency (since $\alpha > 1$).

We now multiply this transfer function by the equivalent output impedance (Z_{eqd}) to find the overall voltage gain of this amplifier:

$$Z_{\text{eqd}} = \frac{R_L}{1 + sC_L R_L}.$$

In writing this equation, we have ignored the impedance looking down into the drain of the transistor as it is assumed to be much larger than the load. Finally, we can write

$$\frac{V_o}{V_i} = \frac{-g_m R_L}{\alpha} \frac{1 + sC_sR_s}{1 + sC_sR_s/\alpha} \frac{1}{1 + sC_L R_L}.$$

This expression clearly identifies one zero and two poles of the transfer function. More explicitly, in connection to the parameters shown on Figure 2, we have

$$f_z = 1/2\pi C_s R_s,$$
$$f_{p1} = \alpha/2\pi C_s R_s,$$
$$f_{p2} = 1/2\pi C_L R_L.$$

The reader can verify that, by changing C_s, we can move f_z and f_{p1} without changing f_{p2} or the equalizer's dc gain. Indeed, by making C_s tunable, this circuit can equalize a wider variety of channels, although an equalizer with only one zero may not be able to compensate for higher-order channel attenuations. In these cases, we may require two to three stages of equalization or a more sophisticated equalizer altogether.

In summary, an equalizer provides a transfer function in the signal path that is the inverse of the transfer function introduced by the wire connecting the transmitter and the receiver. In doing so, the equalizer compensates for the frequency-dependent loss caused by the wire and restores the original spectrum of the transmit signal.

References

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[2] A. Sheikholeslami, "Circuit intuitions: Looking into a node," *IEEE Solid-State Circuits Mag.*, vol. 6, no. 2, pp. 8–10, 2014. doi: 10.1109/MSSC.2014.2315062.

[3] A. Sheikholeslami, "Circuit intuitions: Source degeneration," *IEEE Solid-State Circuits Mag.*, vol. 6, no. 3, pp. 5–6, 2014. doi: 10.1109/MSSC.2014.2329233.

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