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The Electrical Length of a Wire

Welcome to the 21st article in the "Circuit Intuitions" column series. As the title suggests, each article provides insights and intuitions into circuit design and analysis. These articles are aimed at undergraduate students but may serve the interests of other readers as well. If you read this article, I would appreciate your comments and feedback as well as your requests and suggestions for future articles in this series. Please e-mail me your comments: ali@ece.utoronto.ca.

We use wires to connect various electronic building blocks, for example, to connect microchips, such as microprocessors and memories, on a printed circuit board. These wires are expected to transfer a signal from one point on the board to another, without changing the signal. However, there is always a delay and attenuation associated with this transfer.

A delay occurs because the signal cannot travel at an infinite speed; in fact, it cannot travel faster than the speed of light. This is true even if one uses a superconducting material for the wire. The attenuation occurs because the wire (which may be made of copper) has a nonzero resistance and parasitic capacitances and inductances, and these will attenuate the signal (in addition to delaying it) at high frequencies.

As a result, a piece of wire may alter the signal at high frequencies with a transfer function that is not unity, and so it may not be considered a short circuit. In this article, we look at the circumstances under which a wire can and cannot be deemed a short circuit. We conclude intuitively that we need to design circuits to compensate for the signal attenuation of the wire.

Digital Object Identifier 10.1109/MSSC.2019.2922885
Date of publication: 27 August 2019

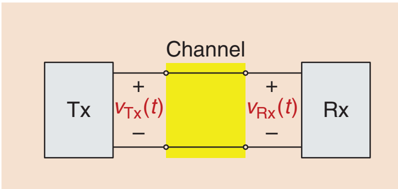

FIGURE 1: A simplified block diagram of a high-speed wireline transceiver.

Figure 1 shows a simplified block diagram of what is known as a wireline transceiver [transmitter (Tx) and receiver (Rx)]. In its simplest form, the Tx sends binary data to the channel, and the Rx reliably detects the data at the other end. As an example, consider a binary signal at 40 Gb/s being sent over a 1-cm piece of wire. At this speed, one bit period is 25 picoseconds (ps) in duration; that is, the period during which the Tx maintains a constant voltage corresponding to a digital zero or one is only 25 ps. However, the time of flight, which is the time it takes for this signal to propagate through 1 cm of wire to arrive at the Rx, is about 50 ps (this is close to the time it takes for light to travel the same distance).

Figure 2(a) presents a case where we send a digital one at $t = 0$ ps, followed by a digital zero at $t = 25$ ps. Because the time of flight is 50 ps, this sequence arrives at the Rx 50 ps later, as seen in Figure 2(b). The voltage along the 1-cm wire at exactly 50 ps, $v(x)$, where x is the distance from the Tx, looks like the waveform pictured in Figure 2(c). Note that, while the waveforms in Figure 2(a) and (b) relate to voltages as functions of time (at the ends of the wire), the waveform in Figure 2(c) is a voltage as a function of "distance" from the Tx. In other words, Figure 2(c) gives a snapshot of the voltage on the wire (at one instant of time, $t = 50$ ps). This figure clearly shows that the voltage along

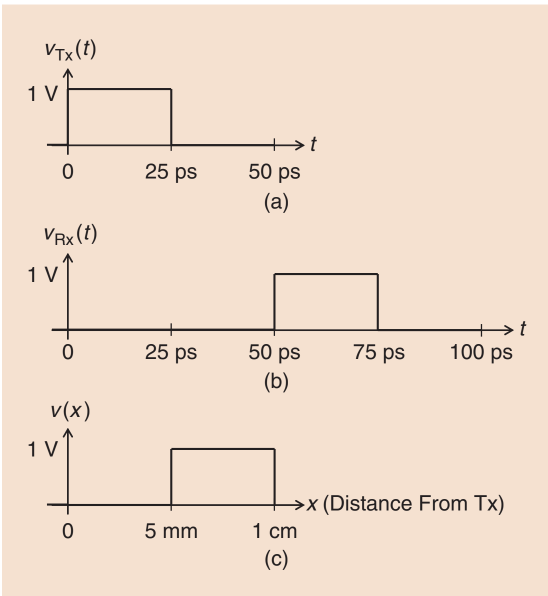

FIGURE 2: The voltage waveform at (a) the Tx and (b) the Rx. (c) The voltage as a function of distance from the Tx at t = 50 ps.

the 1 cm of wire is not constant; the voltage at 2.5 mm away from the Tx is 0 V, while the voltage at 7.5 mm away is 1 V. Therefore, the 1-cm wire is not considered short, as we cannot assume the two ends have the same voltage.

The preceding example may cast doubt on what we typically presume in our undergraduate studies, that is, assuming the two ends of a wire have the same voltage. Do we need to worry about this? The reader is encouraged to ponder the question before continuing.

To answer this, consider transmitting a sinusoidal signal at 1 MHz over the same 1 cm of wire, for instance, with the same time of flight of 50 ps. Assuming the wire is ideal (it does not attenuate the signal), the 1-MHz signal will arrive at the Rx intact but only 50 ps later. If we draw the Tx and Rx waveforms on the same time axis, as in Figure 3, we observe that the two waveforms are almost identical except for a shift in time of 50 ps, exaggerated in the time scale shown. The maximum difference between the two waveforms in the voltage domain is only 0.3 mV.

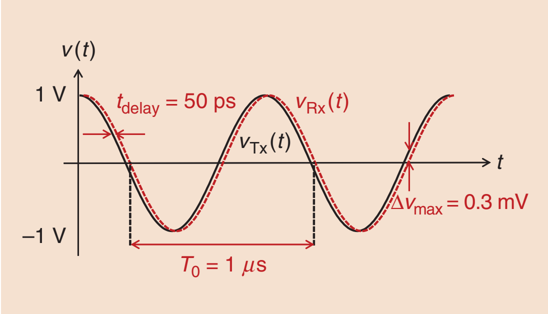

FIGURE 3: The voltage waveform at the Tx and Rx. The time delay between the waveform (50 ps) is negligible compared to the signal period (1 μs).

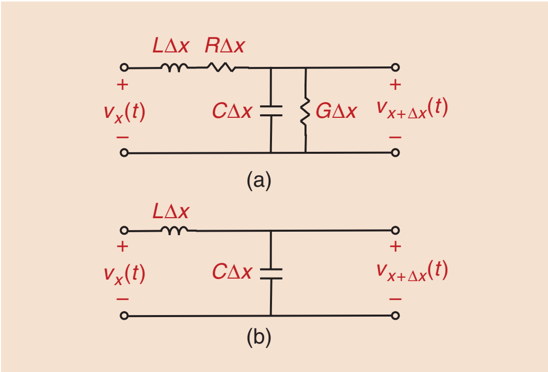

FIGURE 4: (a) A model for a section of a wire with the small length of Δx. (b) A simplified model where the resistances are ignored.

In general, if the time of flight is much less than one period of the signal waveform, we may ignore this shift in time and consider the wire to be short. Otherwise, we cannot ignore the time of flight.

Given that the time of flight is d/v, where d is the physical length of the wire, v is the propagation speed, and the signal period is T_0 $(= 1/f_0)$, we consider the wire to be short if the delay through it is much less than the period (for example, if $d/v \ll T_0$). Equivalently, we can write $d \ll vT_0$. The reader may recognize vT_0 (which is the distance traveled by the waveform in one period) as the signal wavelength, shown as λ. Therefore, we can write the condition for a short circuit as $d \ll \lambda$.

Given this, it makes sense to define the *electrical* length of a wire as the *physical* length of the wire in terms of λ. Accordingly, a wire with a length of $0.001\ \lambda$ or less may be considered short, while a longer wire (e.g., 0.25λ) may not be considered short. Also, while a wire may be considered short at one frequency, it may not be short at a higher frequency. For example, the 1-cm wire discussed previously is 0.00005λ (short) at 1 MHz, but it is 0.5λ (not short) at 10 GHz.

In the preceding discussion, we assumed an ideal wire, where it delayed but did not attenuate the signal. In reality, the wire does attenuate the signal, also as a function of frequency, as described next.

A piece of wire can be modeled as a distributed network of parasitic capacitors, resistors, and inductors, as shown in Figure 4. Even if we ignore the two parasitic resistors, the combination of inductors and capacitors will attenuate the signal at higher frequencies. This is simply because, at higher frequencies, the capacitors act as short circuits and the inductors as open circuits. Looking at one segment of the wire [Figure 4(b)], which consists of a voltage divider formed by an inductor and a capacitor, the voltage divider ratio is one at dc (since the inductor is short and the capacitor is open at dc) and zero at infinity (since the inductor is open and the capacitor is short at infinity). This, and other secondary effects, will cause the wire to exhibit a low-pass filter characteristic, allowing the signal to pass through at lower frequencies, while attenuating it at higher frequencies. Because a signal such as the binary waveform in Figure 2(a) has low- and high-frequency components, it will experience a frequency-dependent attenuation. Figure 5 presents a cartoon of the signal waveform at the Rx and shows how it is rounded where sharp transitions existed at the Tx.

If we use a wire longer than 1 cm in 40 Gb/s or a higher data rate with a 1-cm wire, we will subject the signal to higher loss and spreading, to a point where the adjacent bits may begin to merge or interfere

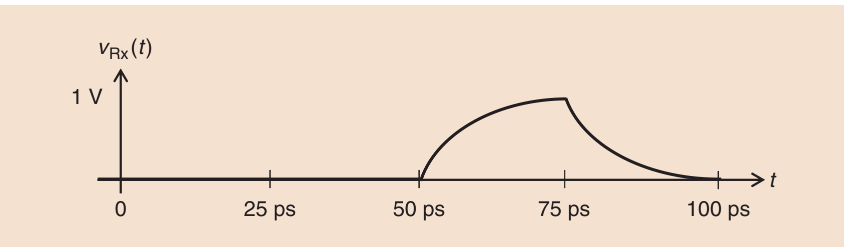

FIGURE 5: The voltage waveform at the Rx, subject to the high-frequency attenuation of the channel.

with each other. In this case, the task of reliably detecting the transmit signal (with a low error rate) becomes more challenging. Indeed, this has been the subject of a broad research field known as *high-speed signaling* or *wireline signaling*.

One way to combat the frequency-dependent attenuation of the channel (wire) is through the design of a circuit with a transfer function that is the inverse of the channel's transfer function: that is, a circuit which provides frequency-dependent amplification to cancel the corresponding frequency-dependent attenuation of the channel. As a result, the combined transfer function of the wire and this circuit will have a flat frequency response, at least up to some higher frequency. This circuit is called an *equalizer* because it equalizes the frequency attenuation across the frequency range of interest. We will discuss the details of an example equalizer in the next article of this series. Stay tuned!

In summary, to find out if a piece of wire can be considered an electrical short, we must express the wire length in terms of the signal wavelength (the length the signal travels in one period). If the electrical length is much less than λ, we can consider the wire as short; otherwise, it cannot be considered short and must be treated as a transmission line with a frequency-dependent transfer function.

A piece of wire may alter the signal at high frequencies with a transfer function that is not unity, and so it may not be considered a short circuit.

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are the IEEE Brokaw Award for Circuit Elegance, IEEE Solid-State Circuits Society New Frontier Award, IEEE Solid-State Circuits Society Innovation Education Award, and IEEE Solid-State Circuits Society Industry Impact Award. The nominations period for these awards is now open. More information about these awards is on page 110 of this issue. I encourage you to nominate deserving candidates.

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