Description

MATLAB is used to implement image processing techniques to analyze x-ray and MRI images from a chest cavity and brain tumor respectively. The implementation x-ray and MRI analysis can be found separately

Section 3.1

1. Two-Dimensional DFT
    a.

$$A = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 4 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

Note: $M = N = 3$, the result of the DFT at the coordinate (u, v) is

$$D(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} A(x,y) \cdot e^{-2\pi i(\frac{ux}{M}+\frac{vy}{N})}$$

$$D(0,0) = \sum_{x=0}^{2}\sum_{y=0}^{2} A(x,y) \cdot e^{-2\pi i\left(\frac{0\cdot x}{M}+\frac{0\cdot y}{N}\right)} = \sum_{x=0}^{2}\sum_{y=0}^{2} A(x,y) = 0$$

$$D(1,0) = \sum_{x=0}^{2}\sum_{y=0}^{2} A(x,y) \cdot e^{-2\pi i\left(\frac{1\cdot x}{M}+\frac{0\cdot y}{N}\right)} = \sum_{y=0}^{2} A(x,y)e^{-2\pi i\left(\frac{x}{3}\right)}$$

$$\text{If } A(x,y)e^{-2\pi i\left(\frac{x}{3}\right)} = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 4a & 0 \\ 0 & -1b & -1b \end{bmatrix} \text{ where a} = e^{-2\pi i\left(\frac{1}{3}\right)}\ and\ b = e^{-2\pi i\left(\frac{2}{3}\right)}$$

$$\text{If } A(x,y)e^{-2\pi i\left(\frac{x}{3}\right)} = -2 + 4a - 2b; \text{using } e^{ix} = \cos(x) + i\sin(x)$$

$$Real\{D(1,0)\} = -2 + 4\cos\left(\frac{-2\pi}{3}\right) - 2\cos\left(\frac{-4\pi}{3}\right) = -3$$

$$Imag\{D(1,0)\} = 2\sin\left(\frac{-2\pi}{3}\right) - \sin\left(\frac{-4\pi}{3}\right) = -5.1962$$

$$D(1,0) = -3 - i\,5.1962$$

$$D(0,1) = \sum_{x=0}^{2}\sum_{y=0}^{2} A(x,y) \cdot e^{-2\pi i\left(\frac{0\cdot x}{M}+\frac{1\cdot y}{N}\right)} = \sum_{x=0}^{2} A(x,y)e^{-2\pi i\left(\frac{y}{3}\right)}$$

$$\text{If } A(x,y)e^{-2\pi i\left(\frac{y}{3}\right)} = \begin{bmatrix} -1 & -1a & 0 \\ 0 & 4a & 0 \\ 0 & -1a & -1b \end{bmatrix} \text{ where a} = e^{-2\pi i\left(\frac{1}{3}\right)}\ and\ b = e^{-2\pi i\left(\frac{2}{3}\right)}$$

$$D(0,1) = -1 + 2a - b; \text{using } e^{iy} = \cos(y) + i\sin(y)$$

$$Real\{D(0,1)\} = -1 + 2\cos\left(\frac{-2\pi}{3}\right) - \cos\left(\frac{-4\pi}{3}\right) = -1.5$$

$$Imag\{D(0,1)\} = 2\sin\left(\frac{-2\pi}{3}\right) - \sin\left(\frac{-4\pi}{3}\right) = -2.5981$$
$$D(0,1) = -1.5 - i\ 2.5981$$

$$D(1,1) = \sum_{x=0}^{2}\sum_{y=0}^{2} A(x,y)\cdot e^{-2\pi i\left(\frac{0\cdot x}{M}+\frac{1\cdot y}{N}\right)} = \sum_{x=0}^{2}\sum_{y=0}^{2} A(x,y)e^{-2\pi i\left(\frac{y}{3}\right)}$$
$$\text{If } A(x,y)e^{-2\pi i\left(\frac{y}{3}\right)} = \begin{bmatrix} -1x_0y_0 & -1x_1y_0 & 0 \\ 0 & 4x_1y_1 & 0 \\ 0 & -1x_1y_2 & -1x_2y_2 \end{bmatrix}$$
$$\text{where } \begin{pmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \end{pmatrix} = \begin{pmatrix} 1 & e^{-2\pi i\left(\frac{1}{3}\right)} & e^{-2\pi i\left(\frac{2}{3}\right)} \\ 1 & e^{-2\pi i\left(\frac{1}{3}\right)} & e^{-2\pi i\left(\frac{2}{3}\right)} \end{pmatrix} = \begin{pmatrix} 1 & a & b \\ 1 & a & b \end{pmatrix}$$
$$D(0,1) = -1 - a + 4a^2 - ab - b^2; \text{using } e^{ix} = \cos(x) + i\sin(x)$$
$$a^2 = \left(e^{-2\pi i\left(\frac{1}{3}\right)}\right)^2 = e^{-2\pi i\left(\frac{2}{3}\right)} = b$$
$$b^2 = \left(e^{-2\pi i\left(\frac{2}{3}\right)}\right)^2 = e^{-2\pi i\left(\frac{8}{3}\right)} = a^4$$
$$D(0,1) = -1 - a + 4a^2 - a^3 - a^4$$
$$Real\{D(1,0)\} = -1 - \cos\left(\frac{-2\pi}{3}\right) + 4\cos\left(2\frac{-2\pi}{3}\right) - \cos\left(3\frac{-2\pi}{3}\right) - \cos\left(4\frac{-2\pi}{3}\right)$$
$$= -3$$
$$Imag\{D(1,0)\} = -\sin\left(\frac{-2\pi}{3}\right) + 4\sin\left(2\frac{-2\pi}{3}\right) - \sin\left(3\frac{-2\pi}{3}\right) - \sin\left(4\frac{-2\pi}{3}\right)$$
$$= 5.1962$$
$$D(1,1) = -3 + i\ 5.1962$$

… solving the rest of the DFT…
The result is:

$$D(u,v) = \begin{bmatrix} 0 & -1.5 - i\ 2.5981 & -1.5 + i\ 2.5981 \\ -3 - i\ 5.1962 & -3 + i\ 5.1962 & 3 \\ -3 + i\ 5.1962 & 3 & -3 - i\ 5.1962 \end{bmatrix}$$

b. Calculate the amplitude in (1-a)

$$|D(x,y)| = \sqrt{(Real\{D(x,y)\})^2 + (Imag\{D(x,y)\})^2}$$
$$|D(0,0)| = \sqrt{(0)^2 + (0)^2} = 0$$
$$|D(1,0)| = \sqrt{(-1.5)^2 + (-2.5981)^2} = 3$$
$$|D(0,1)| = \sqrt{(-3)^2 + (-5.1962)^2} = 6$$
$$|D(1,1)| = \sqrt{(-3)^2 + (-5.1962)^2} = 6$$

…

The amplitude spectrum is…

$$|D(u,v)| = \begin{bmatrix} 0 & 3 & 3 \\ 6 & 6 & 3 \\ 6 & 3 & 6 \end{bmatrix}$$