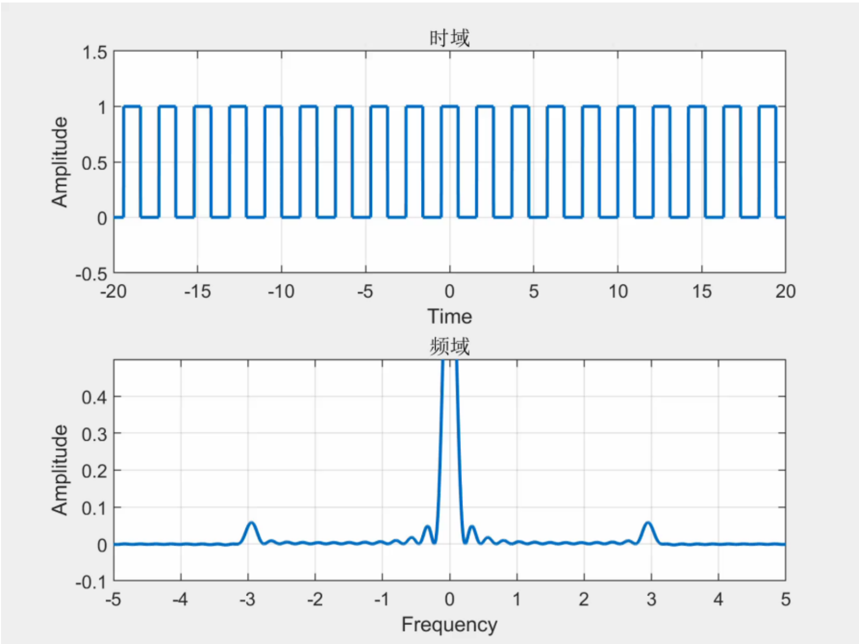
信号处理原理-05

刘华平

清华大学

回顾——从傅立叶级数到傅立叶变换

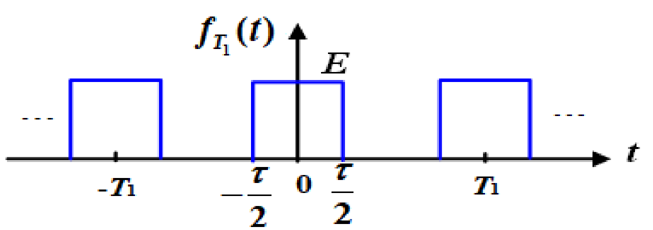


$$f_{T_1}(t)=\sum_{n=-\infty}^{\infty}F_n e^{jn\omega_1 t}$$

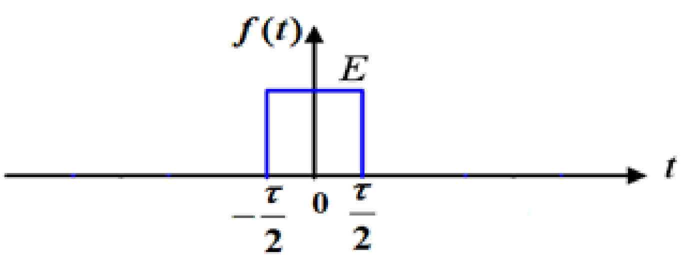
$$T_1 \to \infty$$

$$F_n=\frac{1}{T_1}\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} f(t)e^{-jn\omega_1 t}dt \to 0$$

$$F(\omega)=\lim_{T_1\to\infty}T_1F_n=\lim_{\Delta\omega\to 0}\frac{2\pi}{\Delta\omega}F_n$$



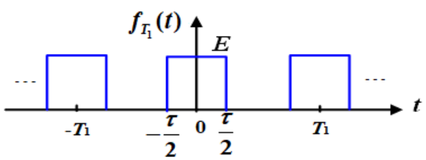
$$F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

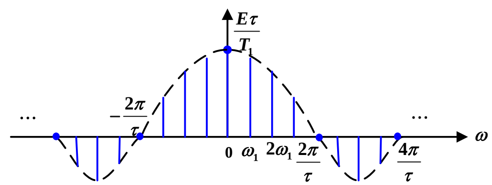


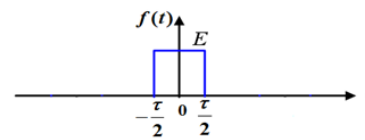
$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

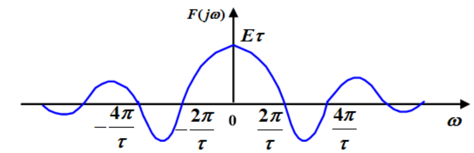
回顾——从傅立叶级数到傅立叶变换

$F(\omega)$ 与 F_n（周期信号频谱）的区别：









$$F_n = \frac{1}{T_1}\int_{t_0}^{t_0+T_1} f_{T_1}(t)e^{-jn\omega_1 t}dt$$

$$f_{T_1}(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_1 t}, \omega_1 = \frac{2\pi}{T_1}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

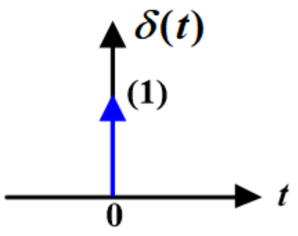
$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

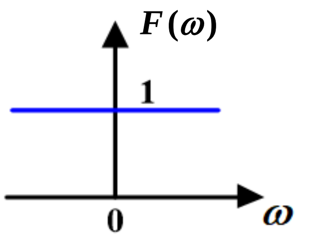
	FS	FT
被分析对象	周期信号	非周期信号
频率定义域	离散频率，谐波频率处	连续频率，整个频率轴
函数值意义	频率分量的数值	频率分量的密度值

回顾——傅立叶变换

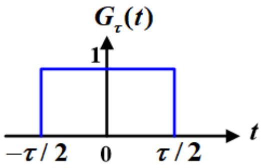
➢ 典型信号的傅立叶变换

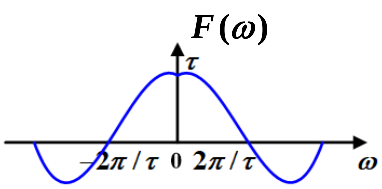
$$\delta(t) \leftrightarrow 1$$



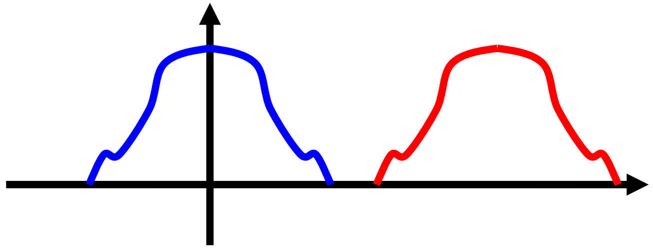


$$G_{\tau}(t) \leftrightarrow \tau \,\mathrm{Sa}\left(\frac{\omega\tau}{2}\right)$$





$$\mathcal{F}\left[f(t)e^{j\omega_0 t}\right] = F\left(\omega - \omega_0\right)$$



回顾——傅立叶变换

➢ 周期信号的傅立叶变换

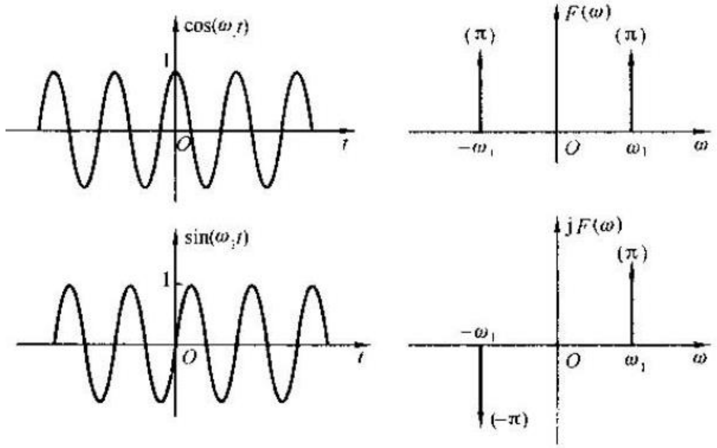
$$1 \leftrightarrow 2\pi\delta(\omega) \qquad e^{j\omega_0 t} \sim 2\pi\delta\left(\omega-\omega_0\right)$$

$$\cos\omega_0 t \overset{\text{Euler公式}}{=} \frac{1}{2}\left[e^{j\omega_0 t}+e^{-j\omega_0 t}\right]$$

$$\cos\omega_0 t \sim \pi\delta\left(\omega-\omega_0\right)+\pi\delta\left(\omega+\omega_0\right)$$

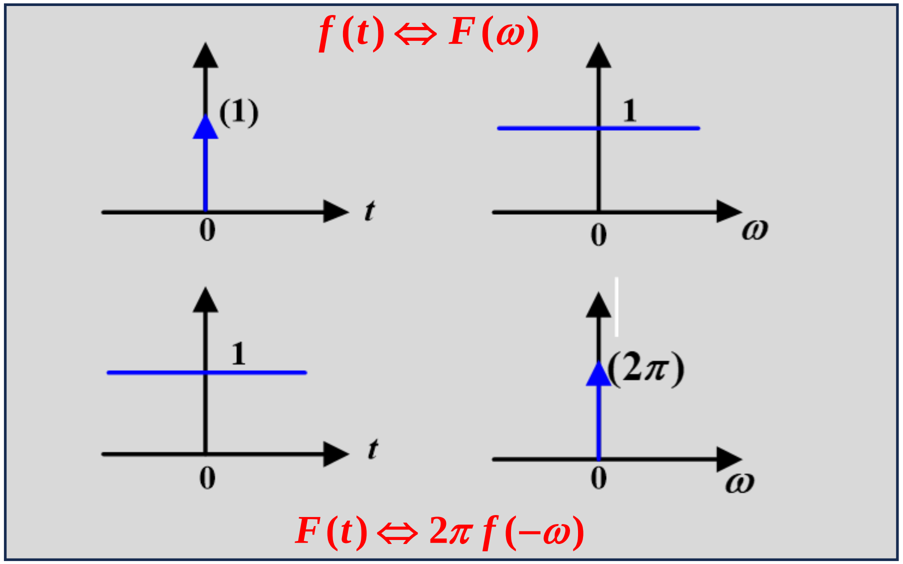
$$\sin\omega_0 t \overset{\text{Euler公式}}{=} \frac{1}{2j}\left[e^{j\omega_0 t}-e^{-j\omega_0 t}\right]=\frac{j}{2}\left[e^{-j\omega_0 t}-e^{j\omega_0 t}\right]$$

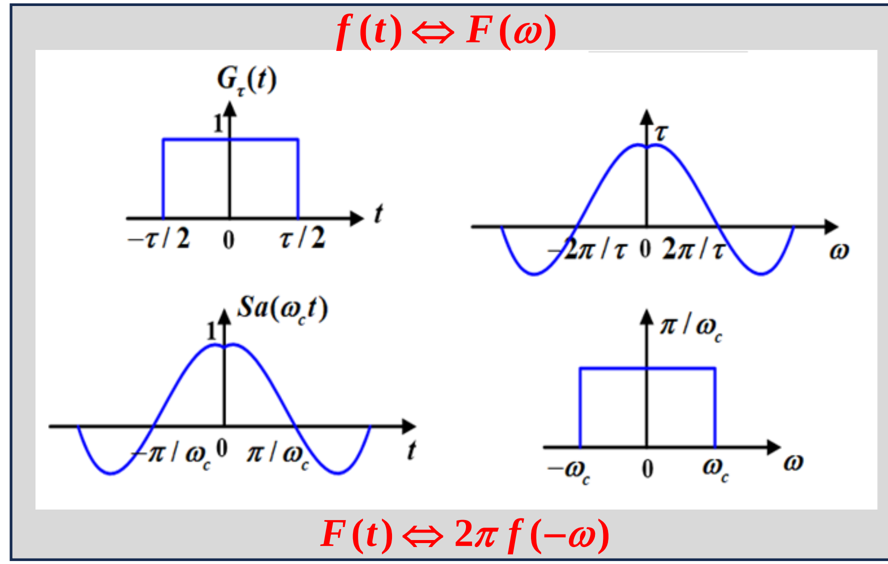
$$\sin\omega_0 t \sim j\pi\delta\left(\omega+\omega_0\right)-j\pi\delta\left(\omega-\omega_0\right)$$



回顾——傅立叶变换

➢ 对偶性质



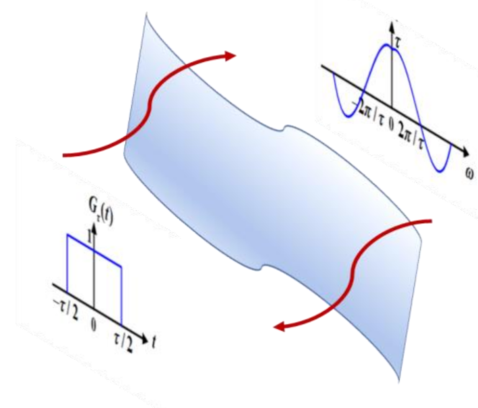


$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(F^*(\omega)e^{-j\omega t}d\omega\right)^*$$

$$= \frac{1}{2\pi}\left(\int_{-\infty}^{\infty} \left(F^*(\omega)e^{-j\omega t}d\omega\right)\right)^* = \frac{1}{2\pi}\left\{\mathcal{F}_\omega\left[F^*(\omega)\right]\right\}^*$$

两次傅里叶变换

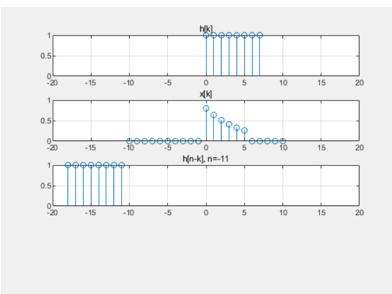


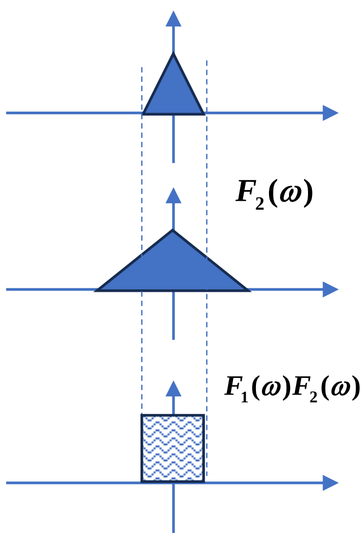
回顾——傅立叶变换

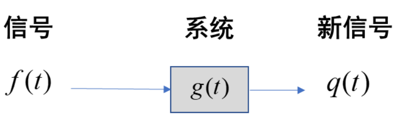
➤ 卷积性质

时域卷积定理
时域的卷积对应频域的乘积

$$\mathcal{F}\left[f_1(t) * f_2(t)\right] = F_1(\omega)\cdot F_2(\omega)$$



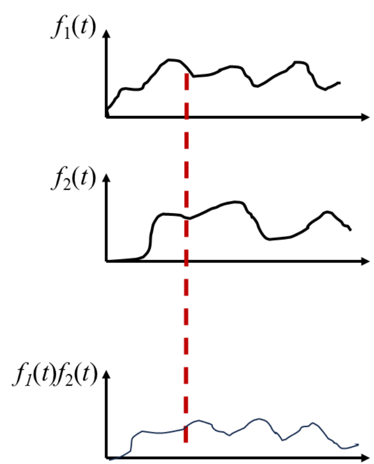


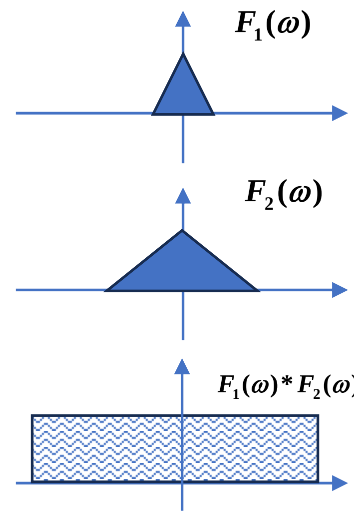


信号卷积不能产生新频率

频域卷积定理
时域的乘积对应频域的卷积

$$\mathcal{F}\left[f_1(t)\cdot f_2(t)\right] = \frac{1}{2\pi}F_1(\omega) * F_2(\omega)$$



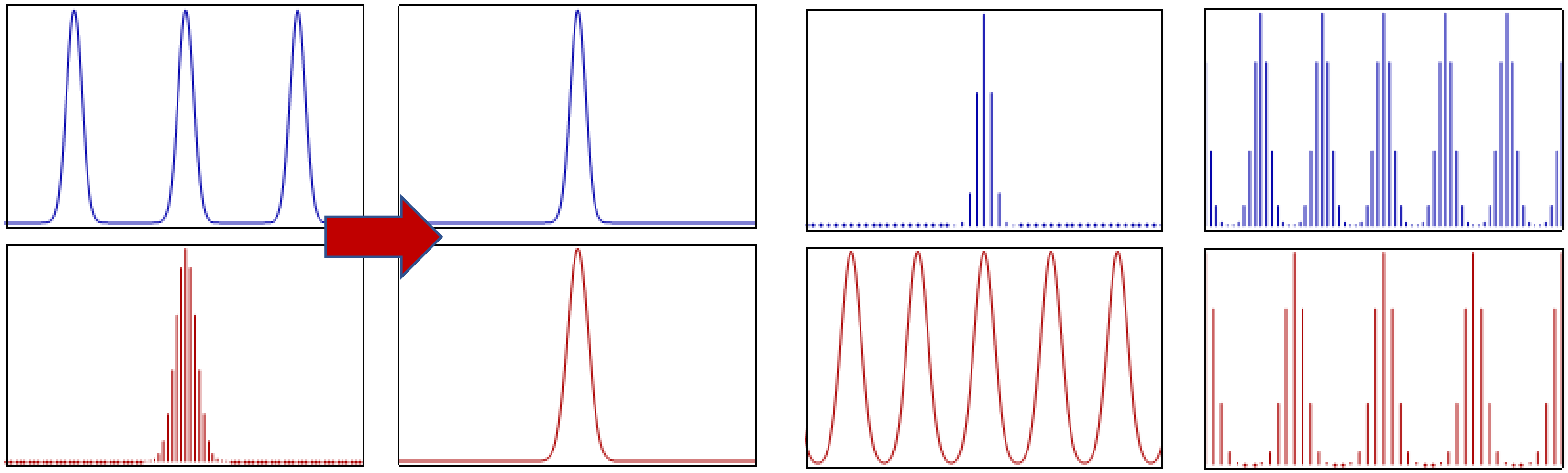


$$\sin\omega_1 t\cos\omega_2 t = \frac{1}{2}[\sin(\omega_1+\omega_2)t + \sin(\omega_1-\omega_2)t]$$

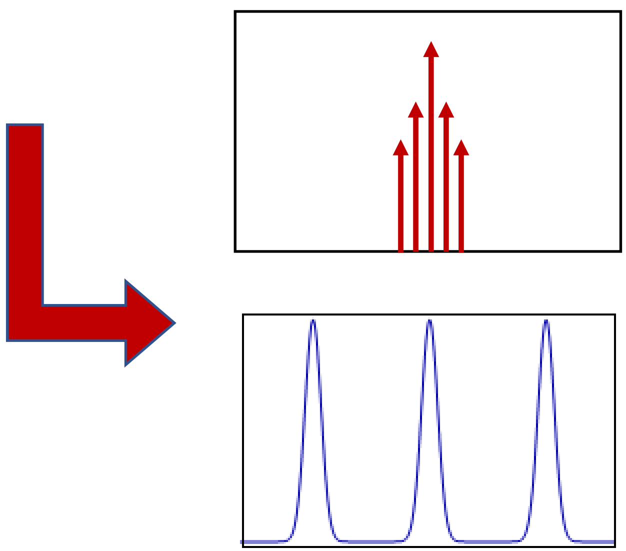
信号相乘可以产生新频率

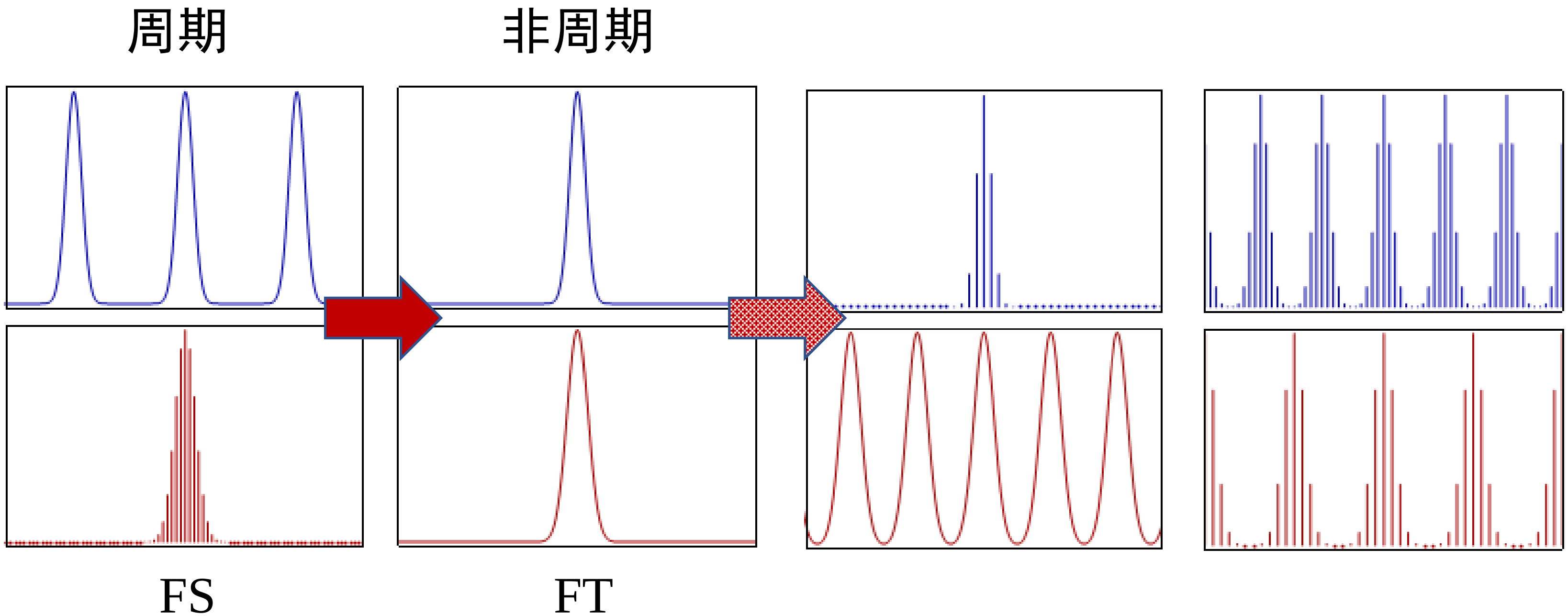
回顾——傅立叶变换

周期　非周期



FS　FT



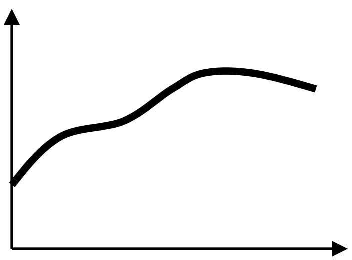


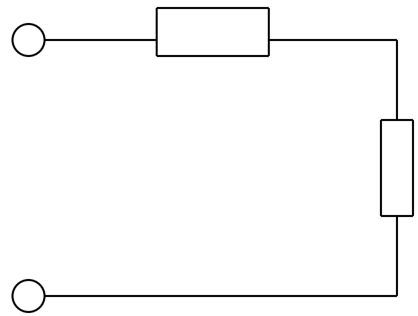
连续信号的离散化

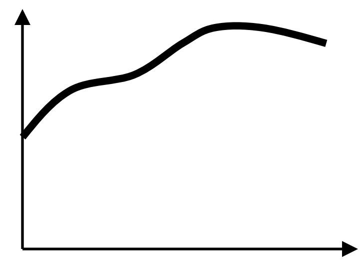
信号的采样与量化

信号的采样与量化

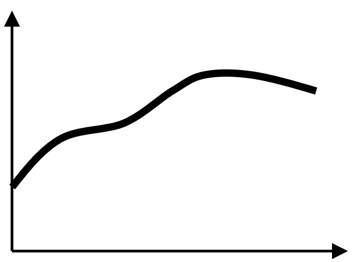
$e(t)$

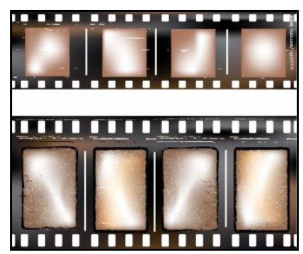




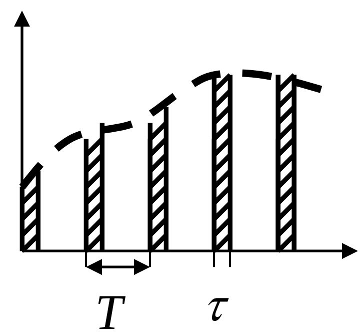


$e(t)$





$e(kT)$



采样周期越小，采样信号越接近原始信号。

信号的采样与量化

Spatial acuity

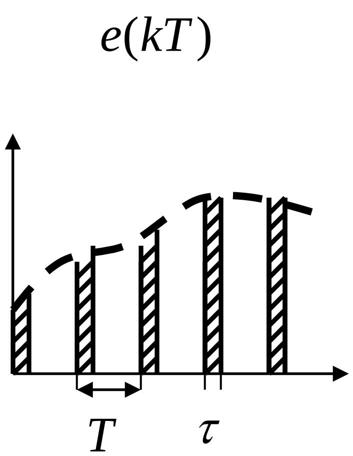
Vision > Tactile > Audition

Ex.: Fingertip: 1 mm, Belly: 30 mm

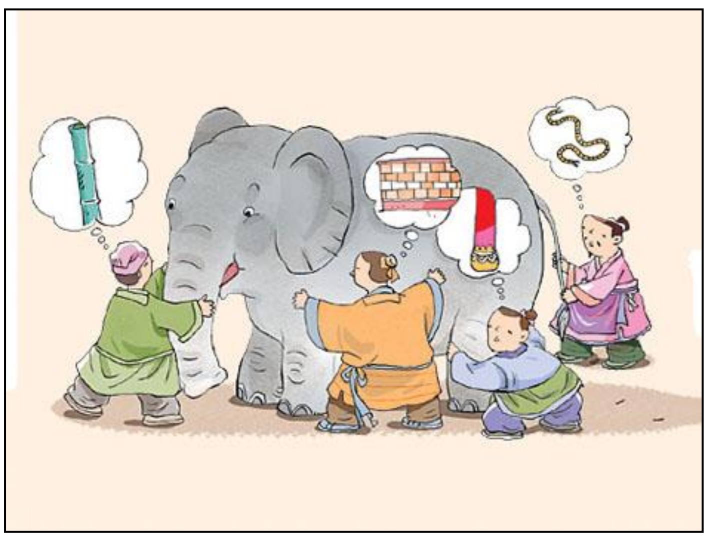
Temporal acuity

Audition (20kHz) > Tactile (700Hz) > Vision (50 Hz)

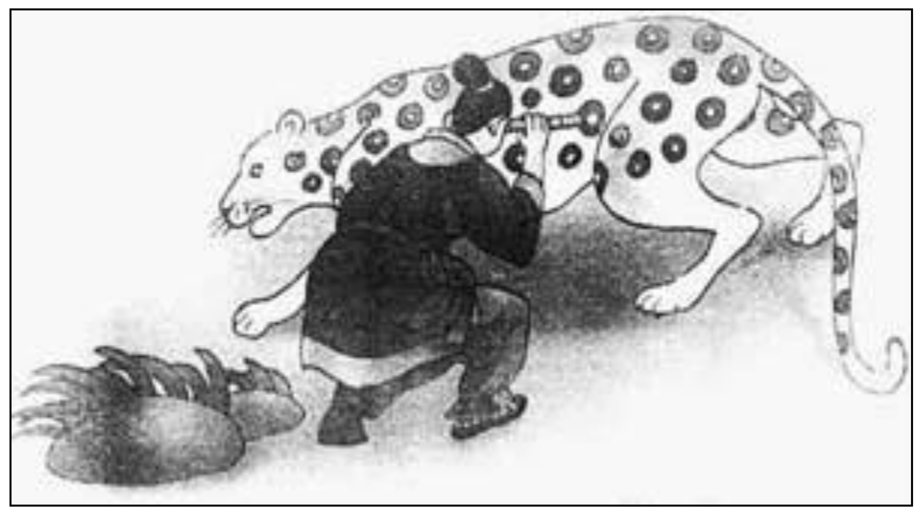
信号的采样与量化



采样周期越小，采样信号越接近原始信号。



盲人摸象



窥一斑而知全豹

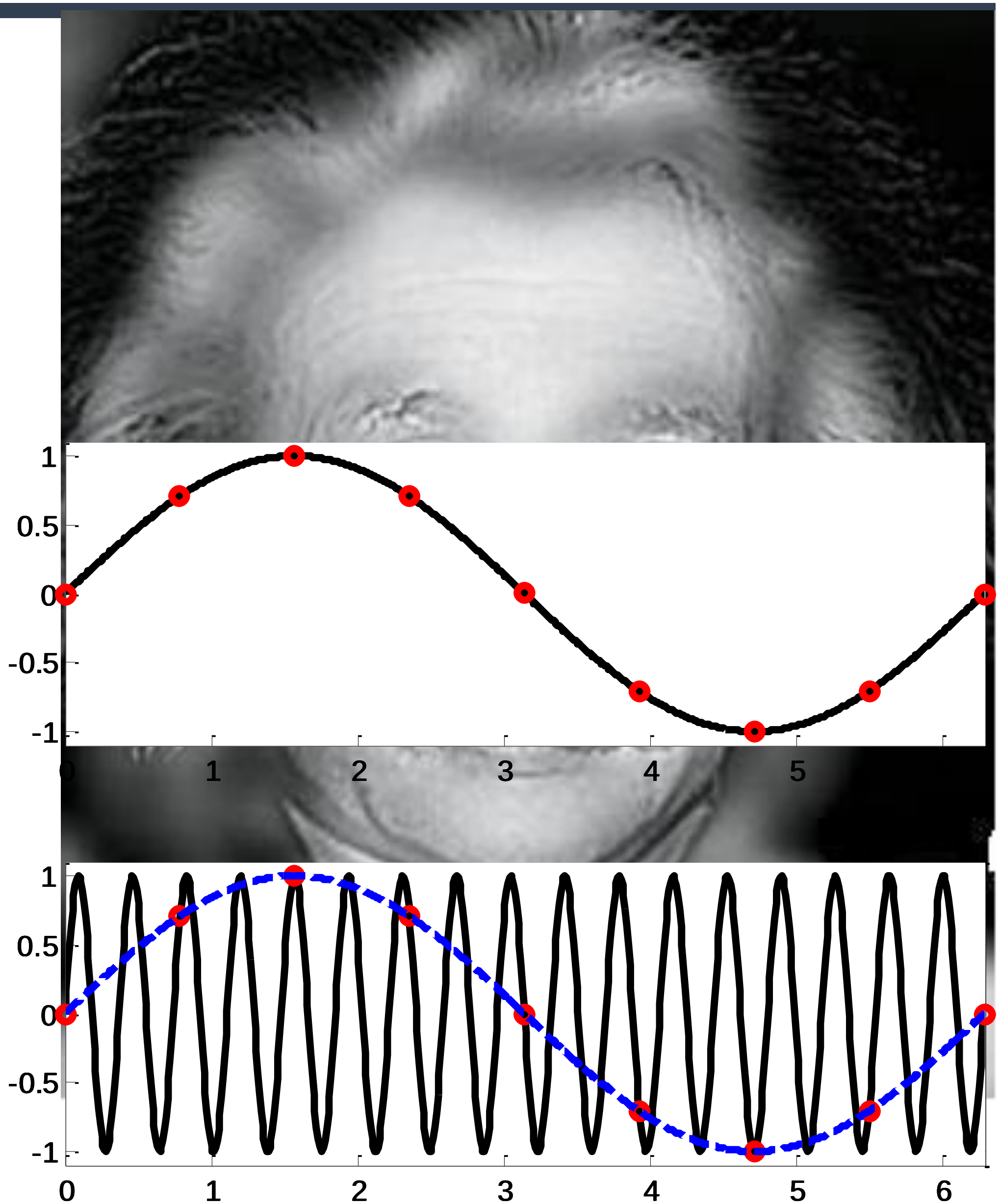
信号的采样与量化

Shannon定理 （1948）

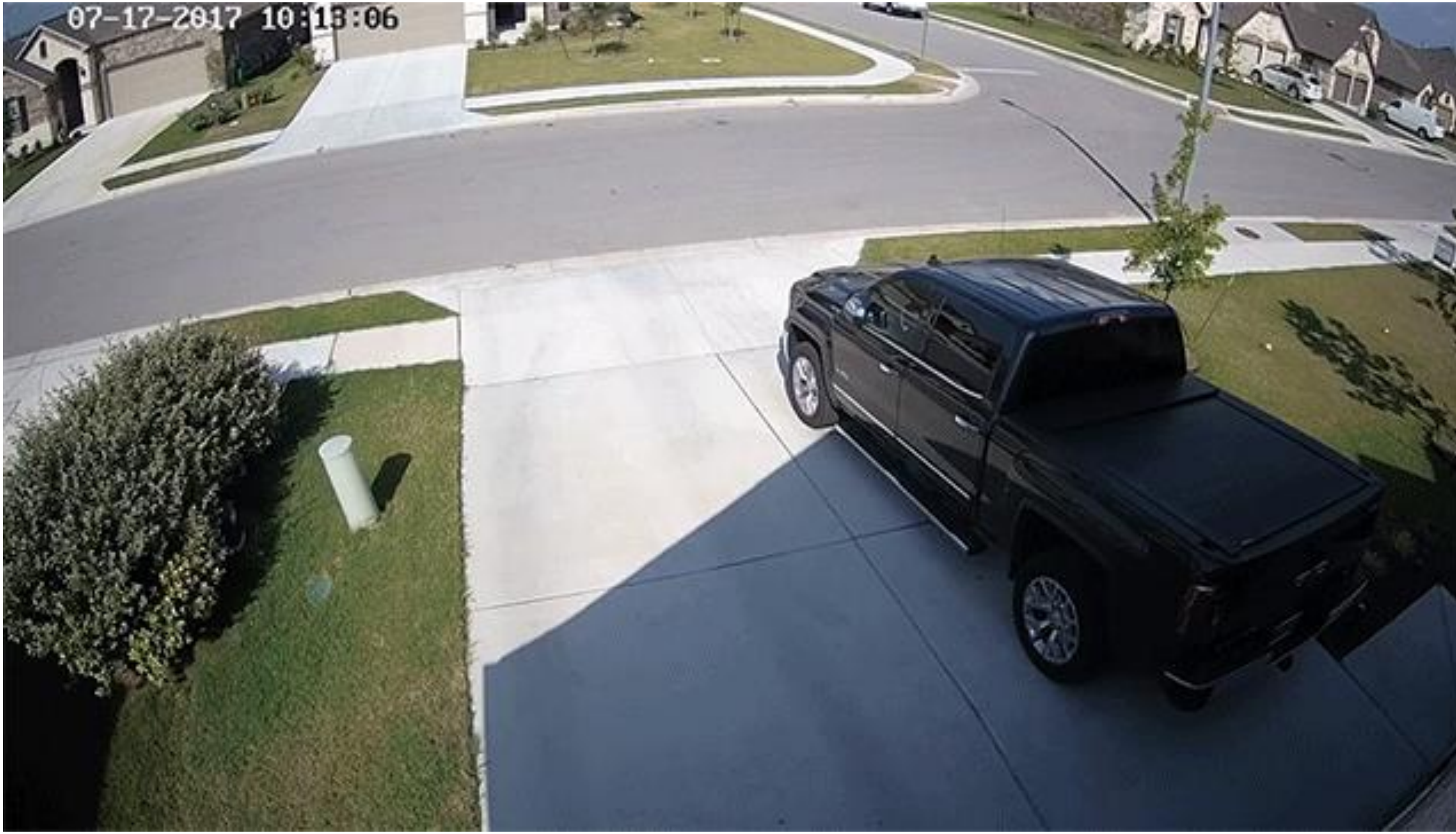
为了完美地重构信号，需要按照不小于2倍带宽采样率对信号进行采样

1889-1976

1916-2001

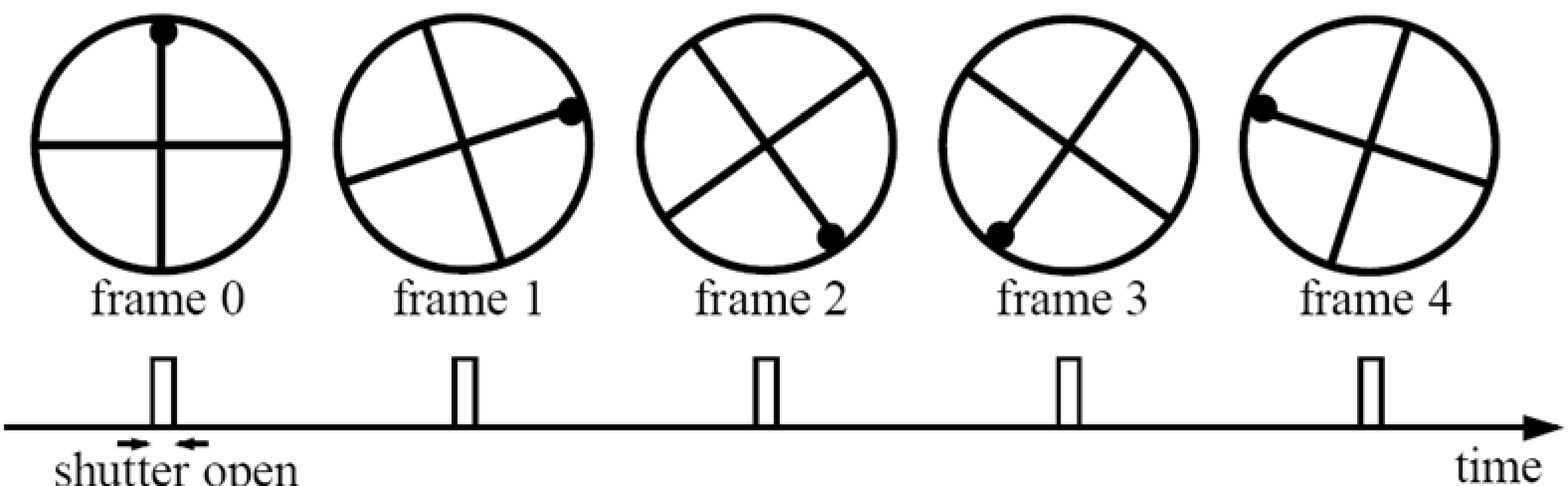


信号的采样与量化

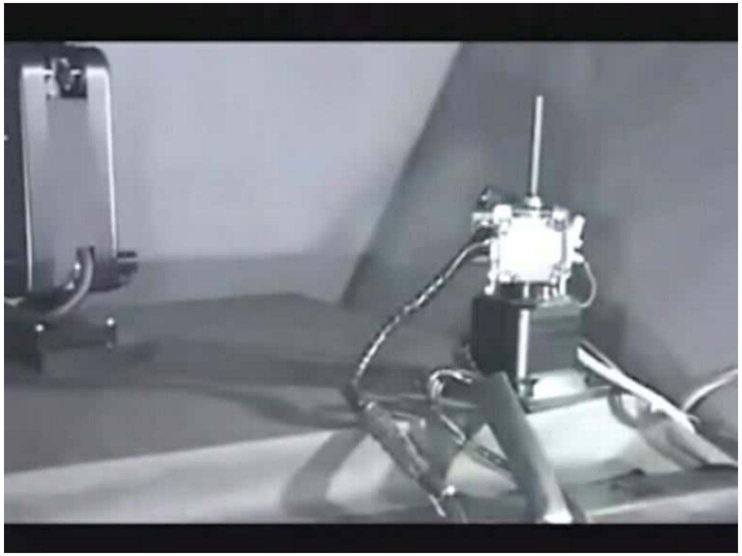


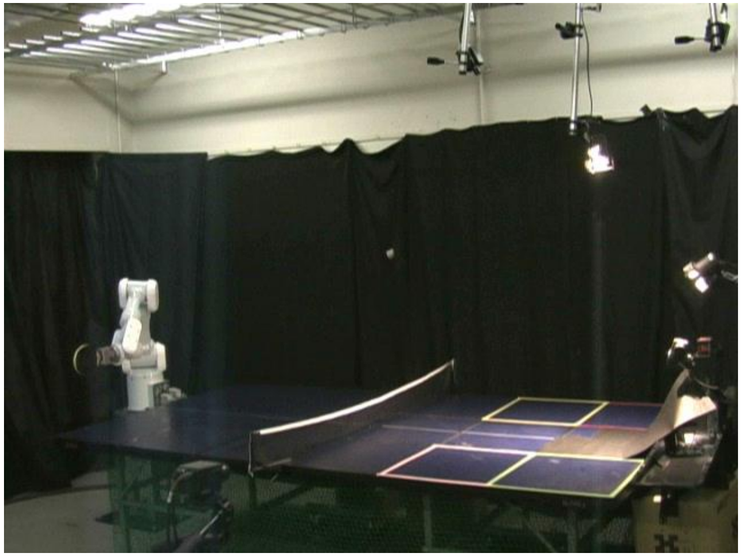
信号的采样与量化

信号的采样与量化



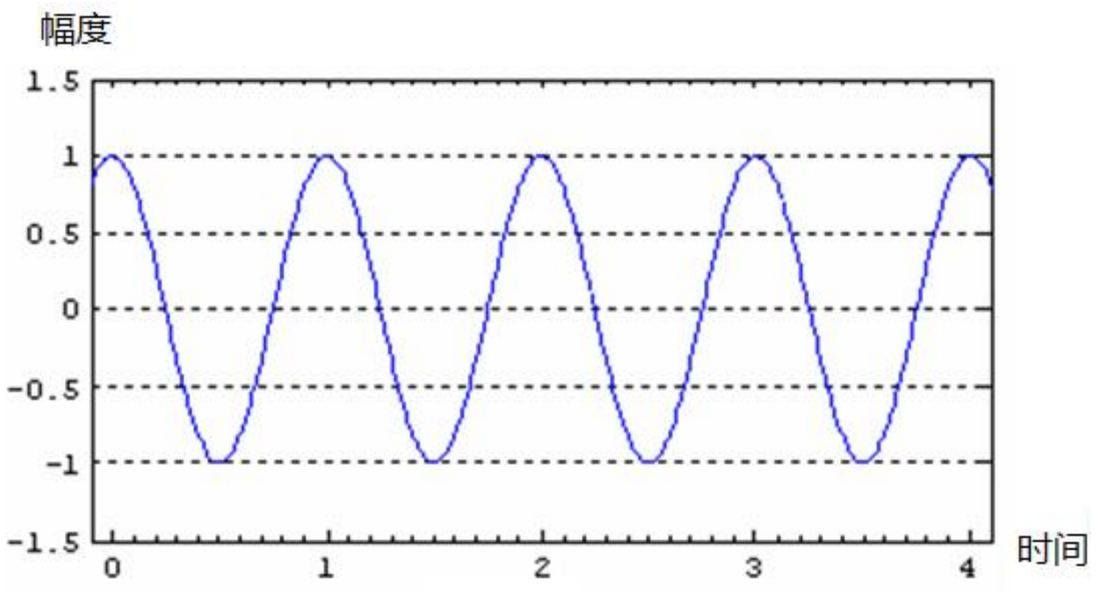
信号的采样与量化





信号的采样与量化

➢ 采样的概念

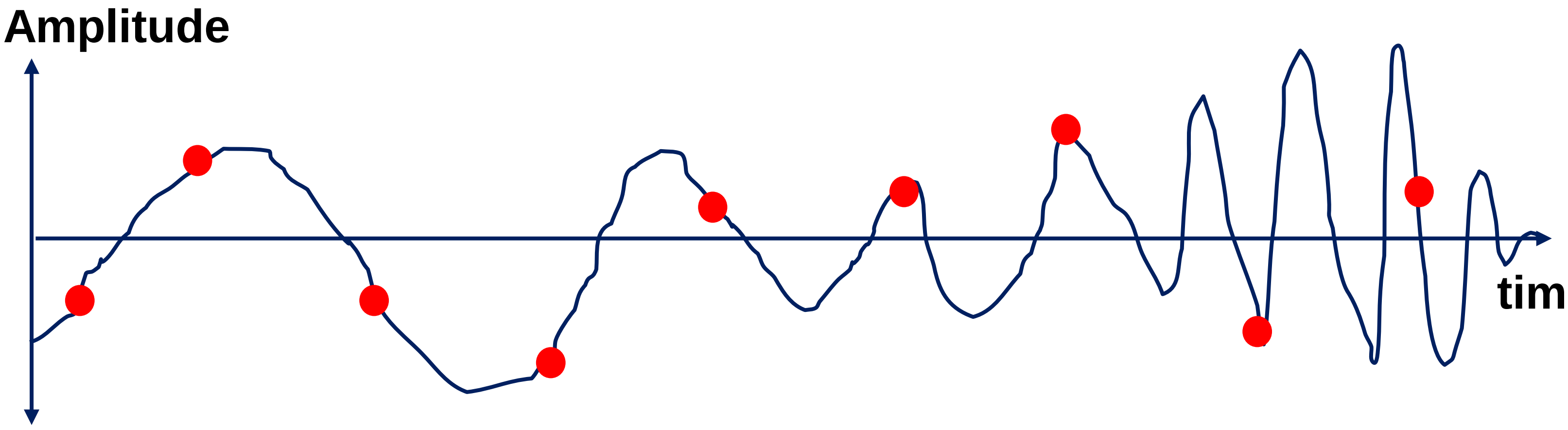


把模拟信号变成数字信号时，每隔一个时间间隔在模拟信号波形上抽取一个幅度值，这称之为采样。

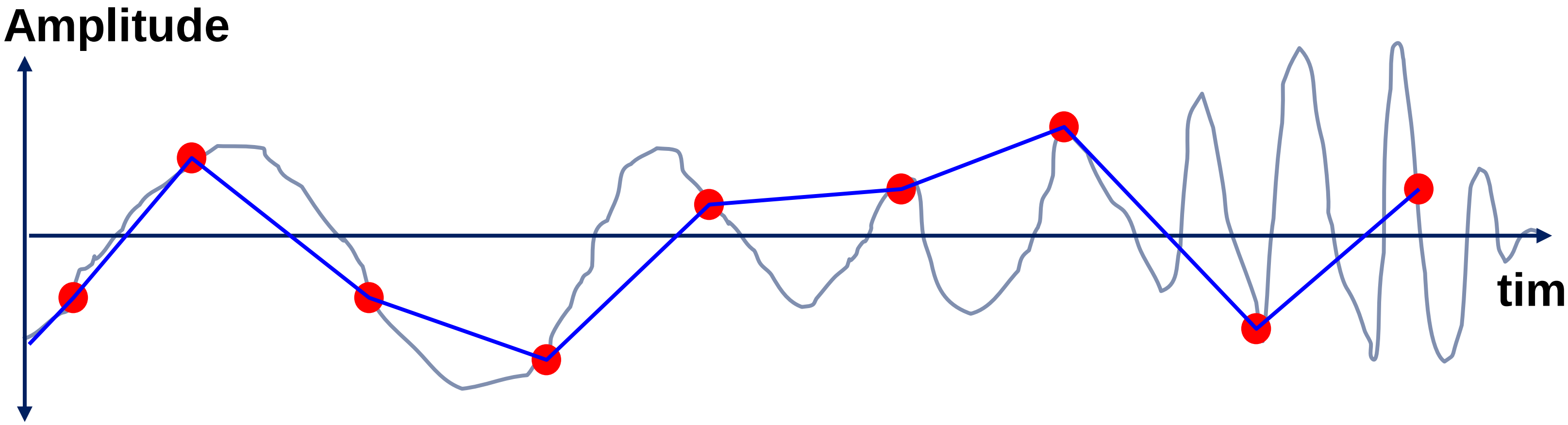
信号的采样与量化

➢ 采样的概念

Continuous-time Signal (real signal)

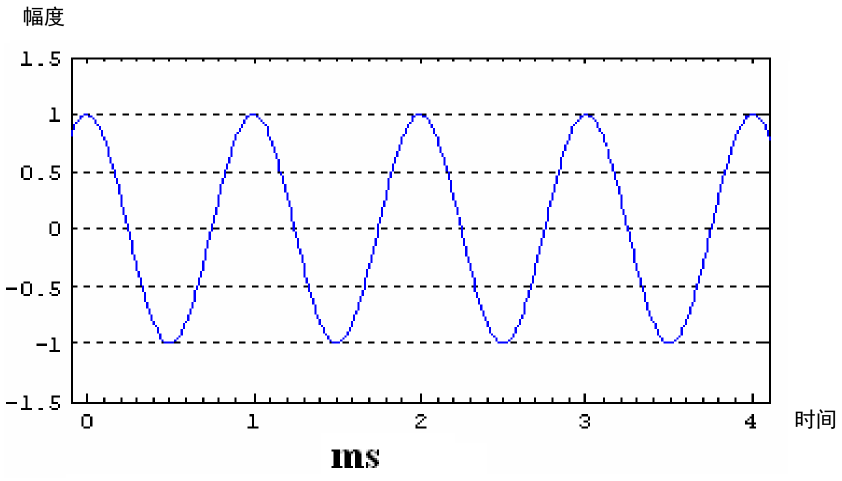


Discrete-time Signal



信号的采样与量化

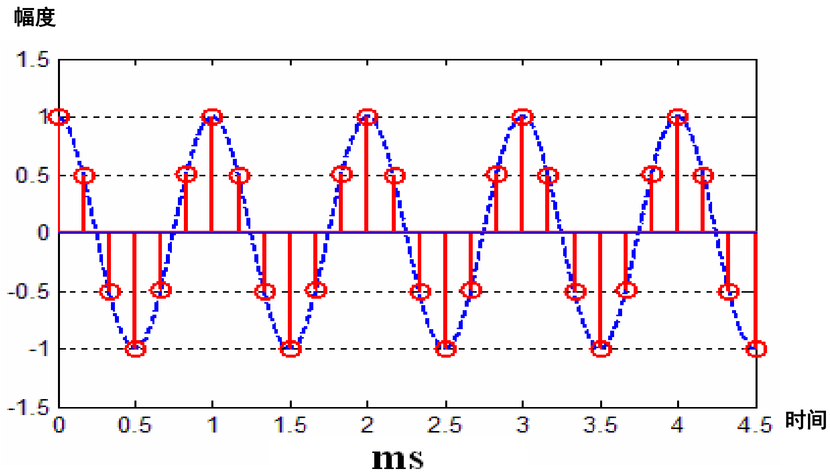
- 采样的概念



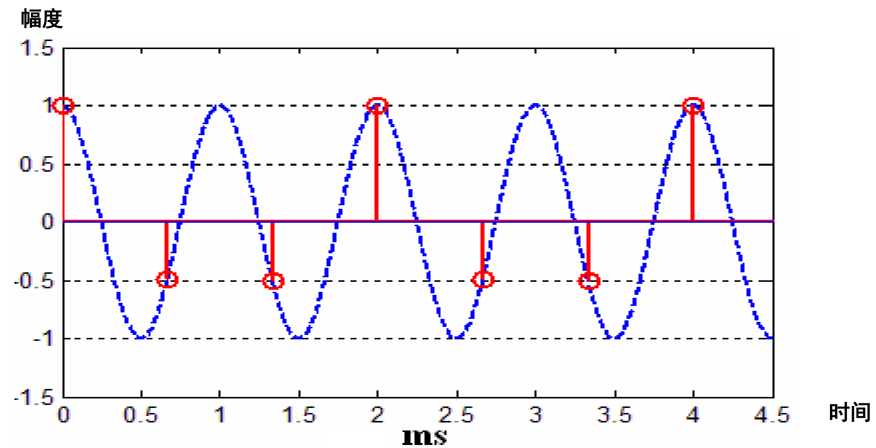
原始模拟信号

信号的采样与量化

➢ 采样的概念



采样后的信号A



采样后的信号B

采样的时间间隔称为**采样周期** T_s，

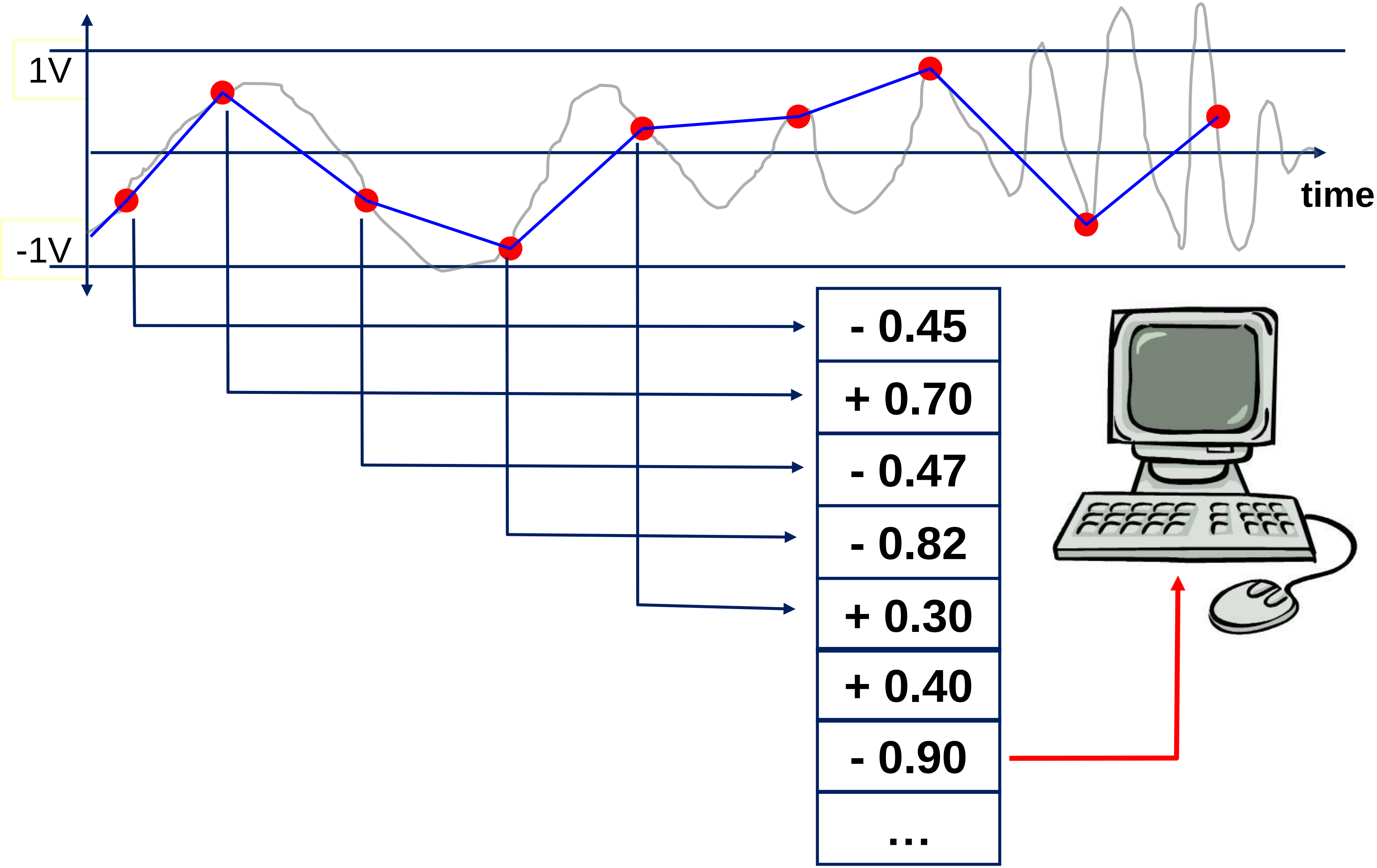
其倒数称为**采样频率** $f_s=1/T_s$（单位：次/秒，赫兹）。

$\omega_s=2\pi/T_s$ 称为采样角频率（单位：弧度/秒）。

注：在不发生混淆的情况下，ω_s**可简称为采样频率**。

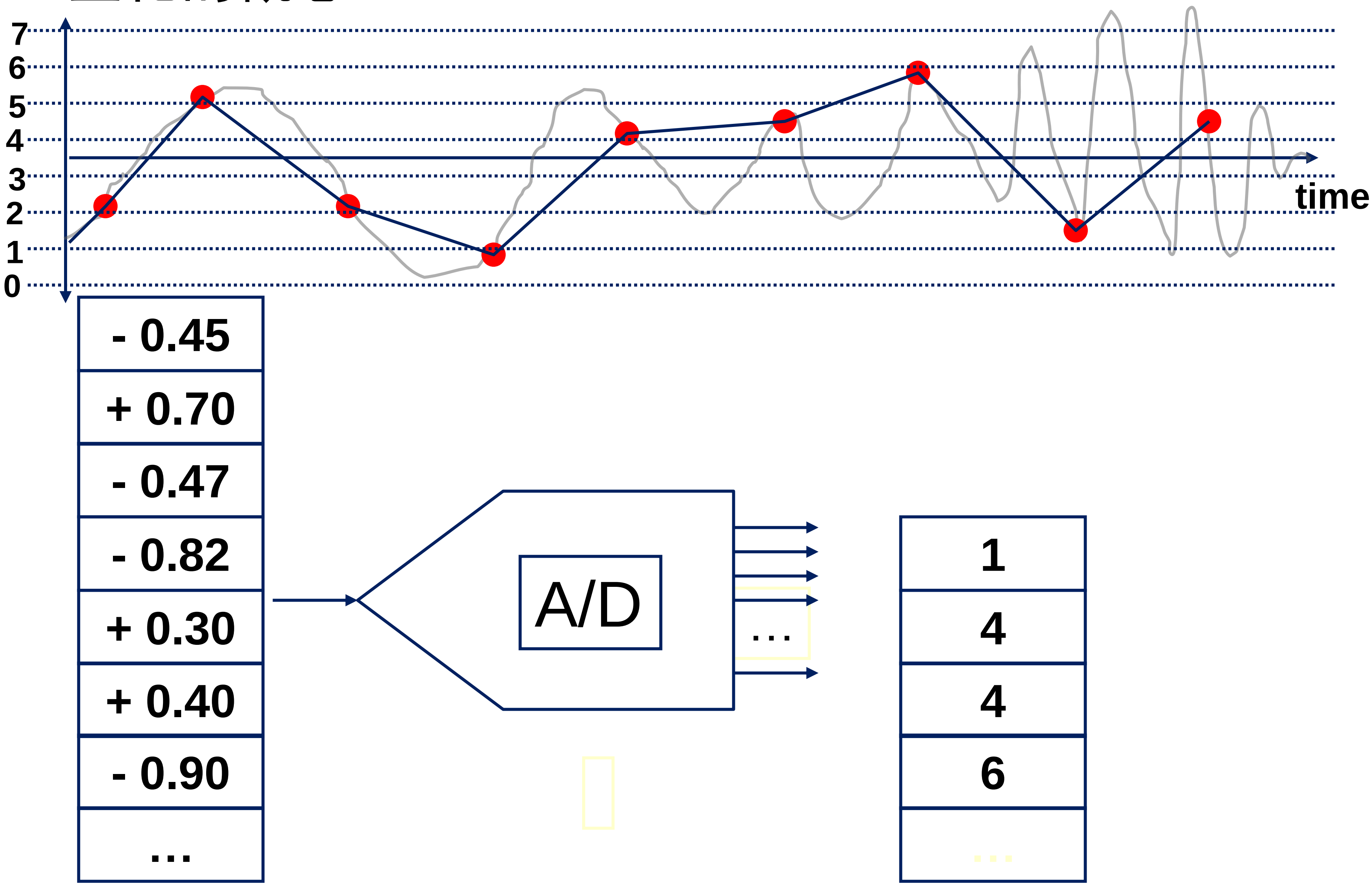
信号的采样与量化

- 量化的概念



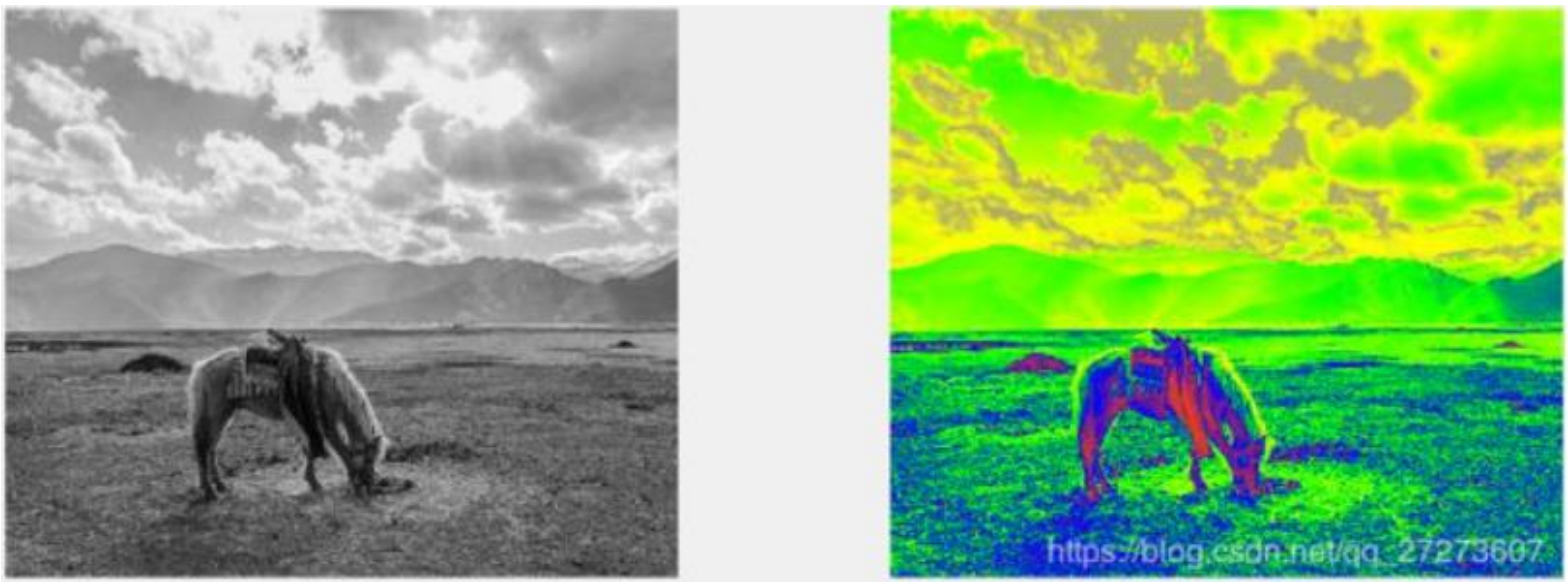
信号的采样与量化

➢ 量化的概念

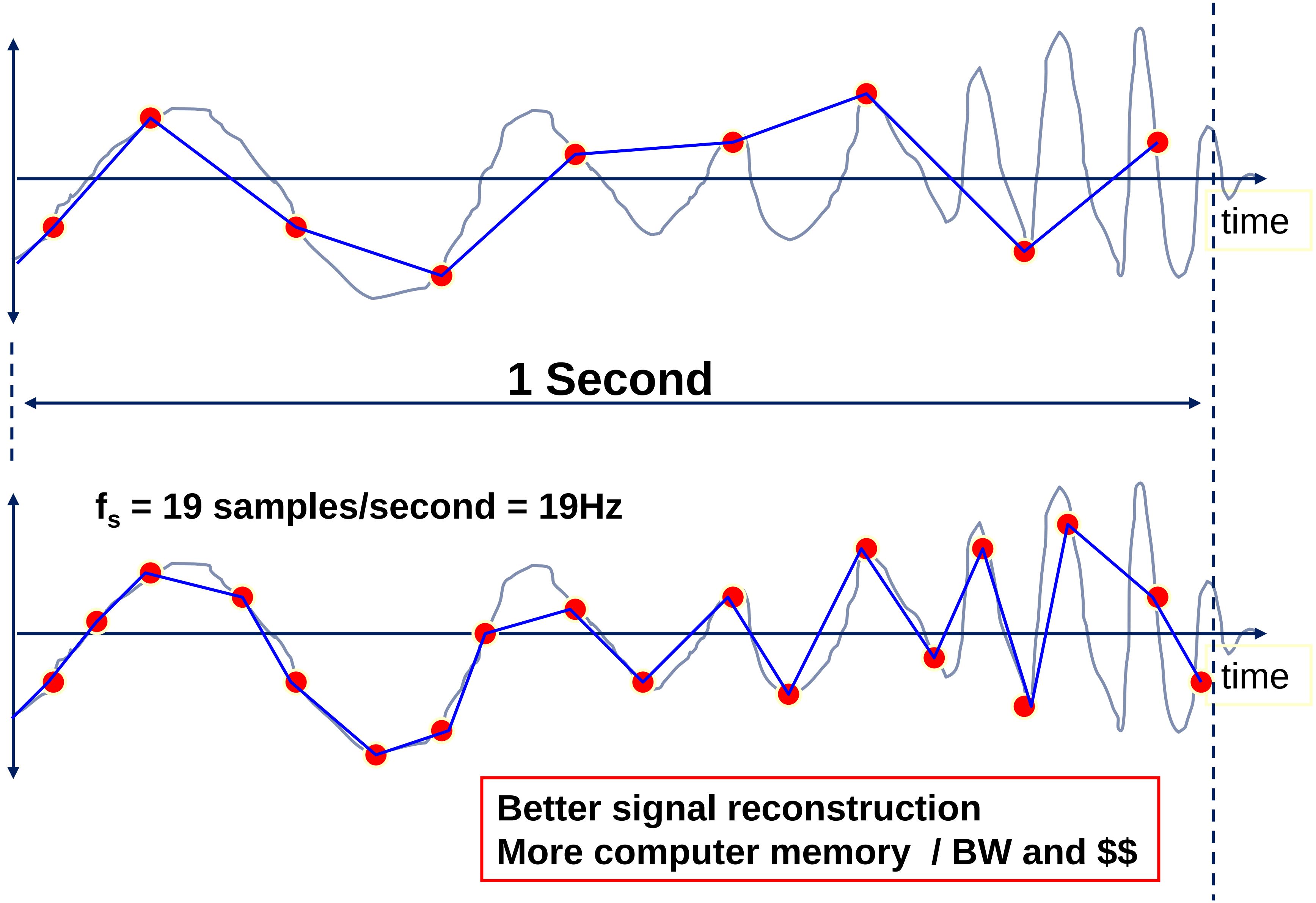


信号的采样与量化

➢ 量化的概念

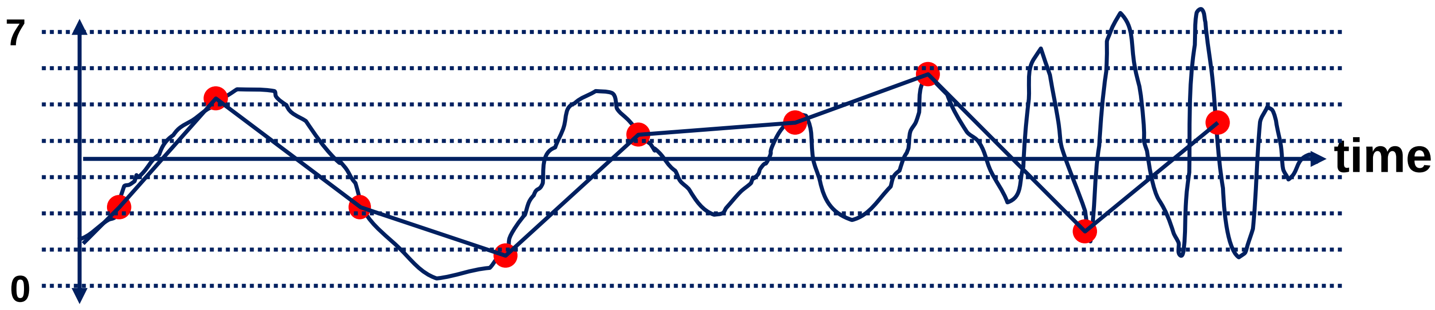


信号的采样与量化

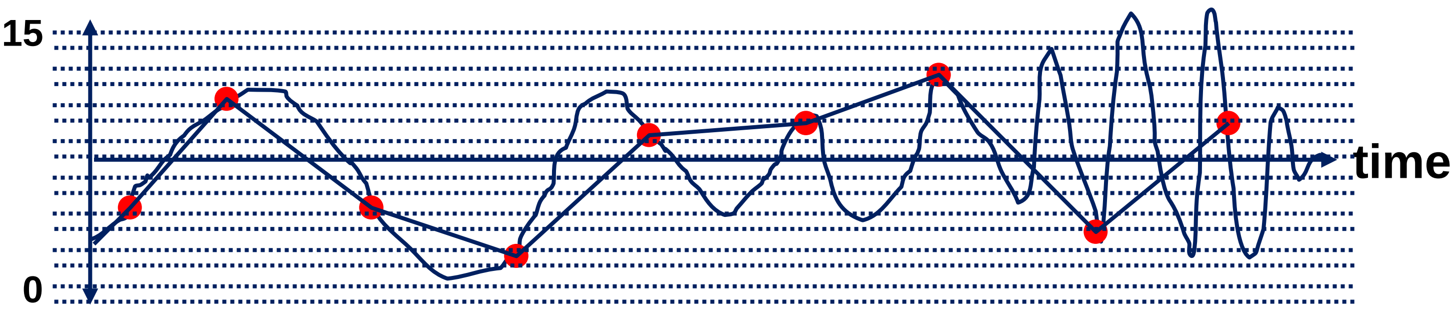

time
1 Second
f_s = 19 samples/second = 19Hz
time
Better signal reconstruction
More computer memory / BW and $$

信号的采样与量化

Resolution: 3bits, 2^3 = 8 combinations **Values from 0 to 7**



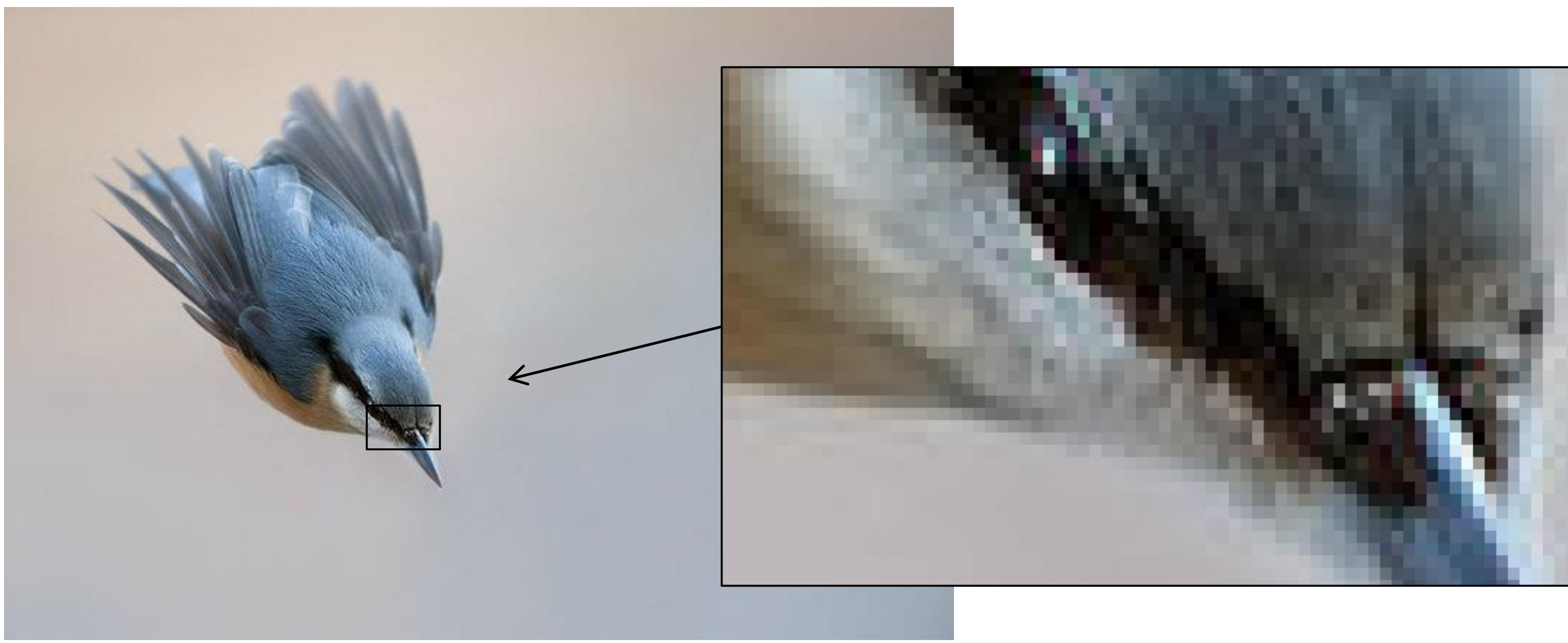
Resolution: 4bits, 2^4 = 16 combinations **Values from 0 to 15**



Better signal quantization
More computer memory and $$

信号的采样与量化

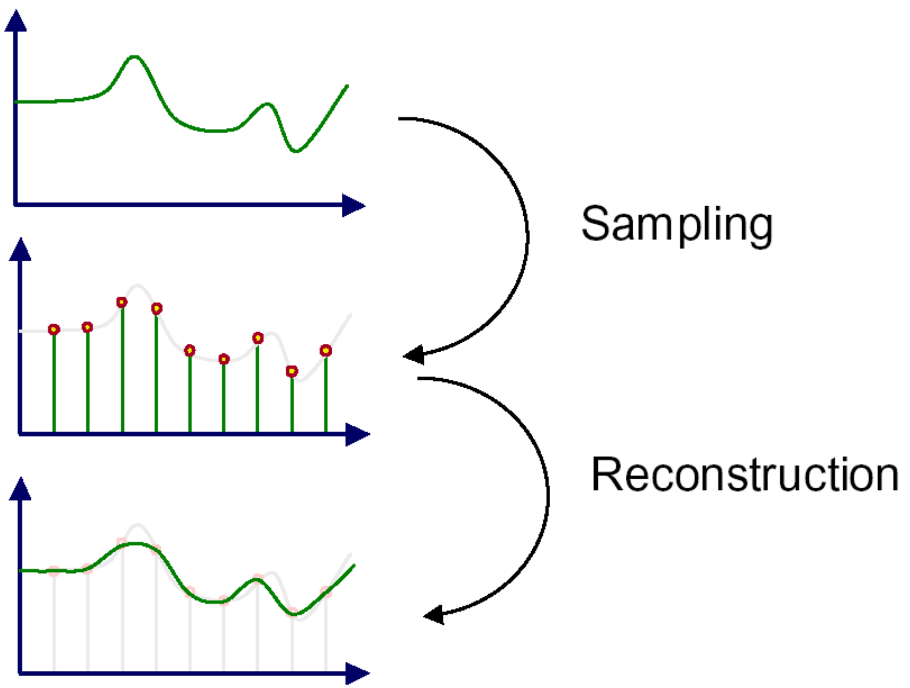
在日常生活中，常可以看到用离散时间信号表示连续时间信号的例子。如照片、屏幕的画面等等。在一定条件下，可以用离散时间信号代替连续时间信号。



研究连续时间信号与离散时间信号之间的关系主要包括 ：

- 在什么条件下，一个连续时间信号可以用它的离散时间样本来代替而不致丢失原有的信息？

- 如何从连续时间信号的离散时间样本不失真地恢复成原来的连续时间信号？

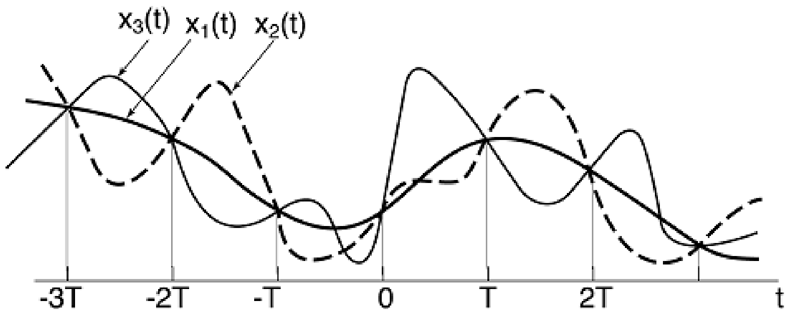


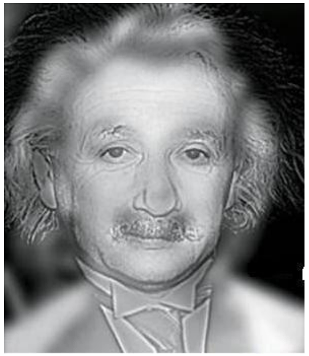
采样与采样定理

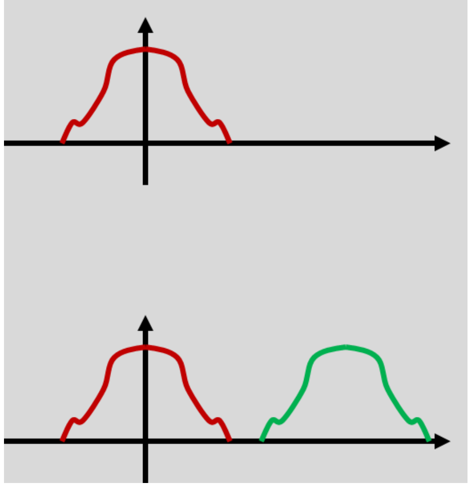
采样与采样定理

➢ 采样的定义

- 在某些离散的时间点上提取连续时间信号值的过程称为采样。



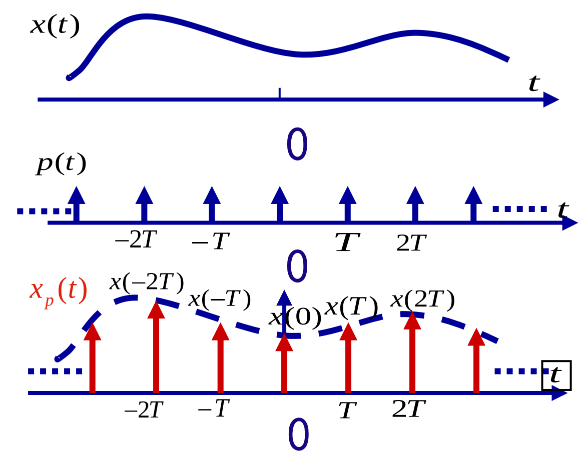




在没有任何条件限制的情况下，从连续时间信号采样所得到的样本序列不能唯一地代表原来的连续时间信号。

采样与采样定理

➢ 采样信号的时域表示



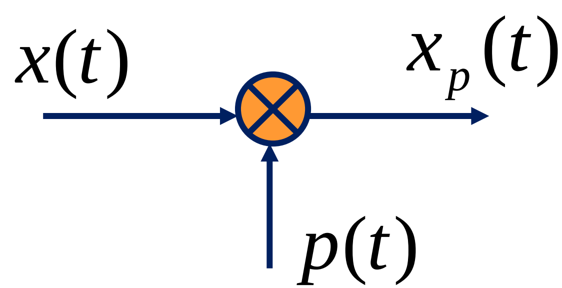
- 冲激串（理想采样）:

$$p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT_s)$$

T_s为采样周期

- 采样信号:

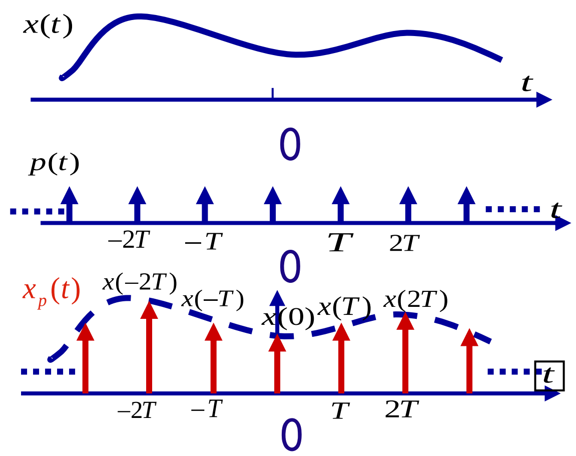
$$x_p(t)=x(t)p(t)=\sum_{n=-\infty}^{+\infty}x(nT_s)\delta(t-nT_s)$$



连续信号经采样后，变成幅度不等的等间隔脉冲串
（间隔为采样周期）

采样与采样定理

➢ 采样信号的时域表示



- 冲激串（理想采样）：

$$p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT_s)$$

T_s为采样周期

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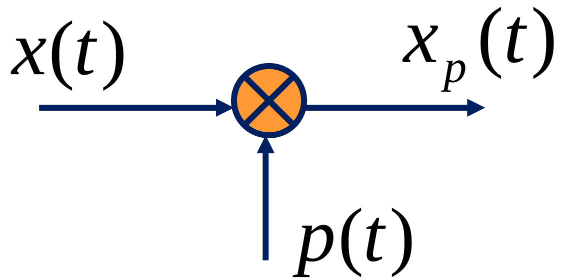
$$x_p(t)=x(t)p(t)=\sum_{n=-\infty}^{+\infty}x(nT_s)\delta(t-nT_s)$$



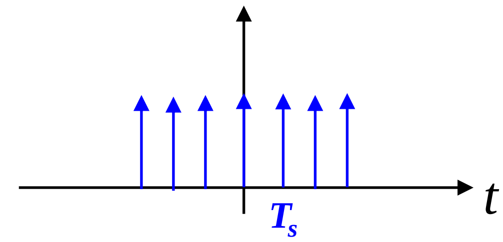
- 频域表示?

$$\boxed{\mathcal{F}[f_1(t)\cdot f_2(t)]=\frac{1}{2\pi}\mathcal{F}[f_1(t)]*\mathcal{F}[f_2(t)]}$$

$$X_p(\omega)=\frac{1}{2\pi}X(\omega)*P(\omega)$$

采样与采样定理

➢ 采样信号的频域表示



求频域 $P(\omega)$

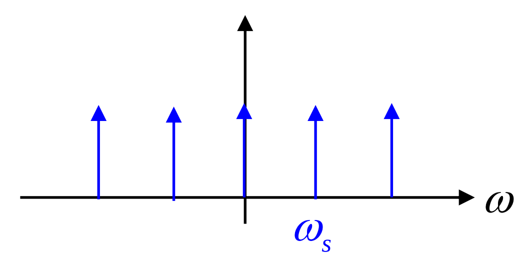
$$p(t)=\sum_{n=-\infty}^{\infty}\alpha_n e^{jn\omega_s t}$$

$$p(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$$

$$\alpha_n=\frac{1}{T_1}\int_{-T_1/2}^{T_1/2}p(t)e^{-jn\omega_s t}dt=\frac{1}{T_1}\int_{-T_1/2}^{T_1/2}\delta(t)e^{-jn\omega_s t}dt=\frac{1}{T_s}\int_{-\infty}^{\infty}\delta(t)e^{-jn\omega_s t}dt=\frac{1}{T_s}$$

$$p(t)=\frac{1}{T_s}\sum_{n=-\infty}^{\infty}e^{jn\omega_s t}$$

$$e^{jn\omega_s t}\sim 2\pi\delta(\omega-n\omega_s)\qquad P(\omega)\sim\frac{2\pi}{T_s}\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_s)$$



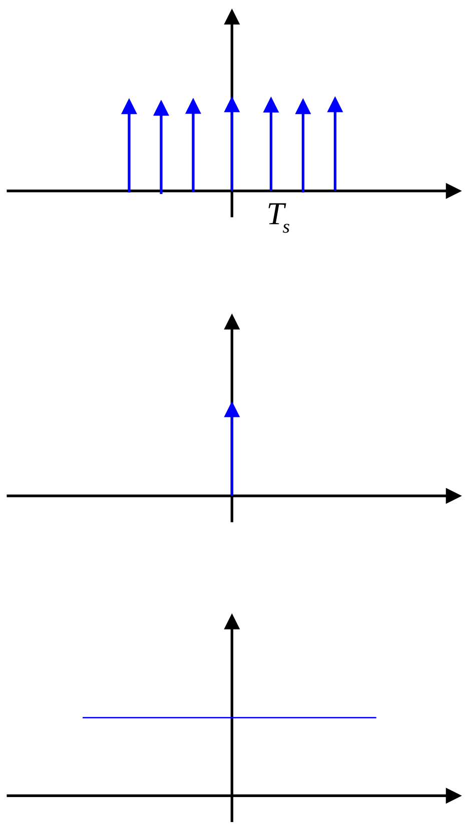
周期脉冲串的频域表示（傅里叶变换）仍然是周期脉冲串（周期不同）

采样与采样定理

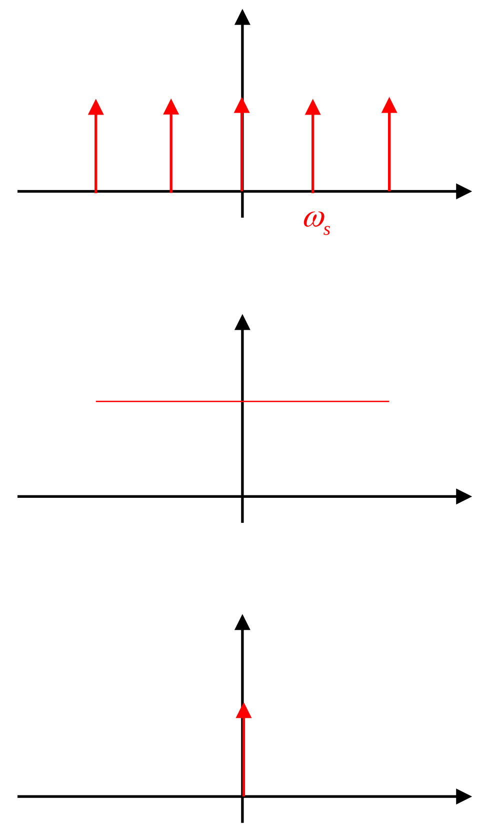
➢ 采样信号的频域表示

$$p(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$$

$$P(\omega)=\frac{2\pi}{T_s}\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_s)$$



$$\omega_s=\frac{2\pi}{T_s}$$



采样与采样定理 ★★★★★

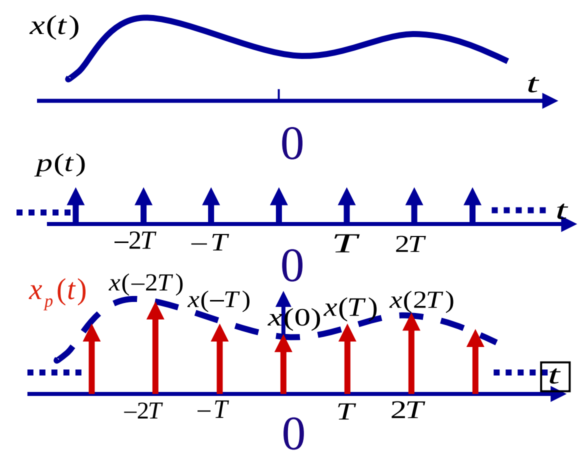
➢ 采样信号的频域表示

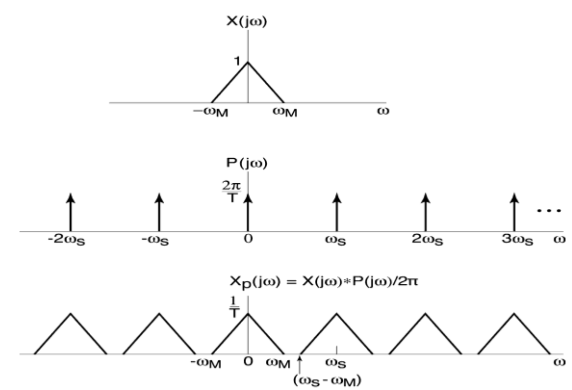
$$p(t) \leftrightarrow P(\omega) = \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$$

$$X_p(\omega) = \frac{1}{2\pi} X(\omega) * P(\omega)$$

$$= \frac{1}{2\pi} X(\omega) * \frac{2\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$$

$$= \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s) \qquad \omega_s = \frac{2\pi}{T_s}$$

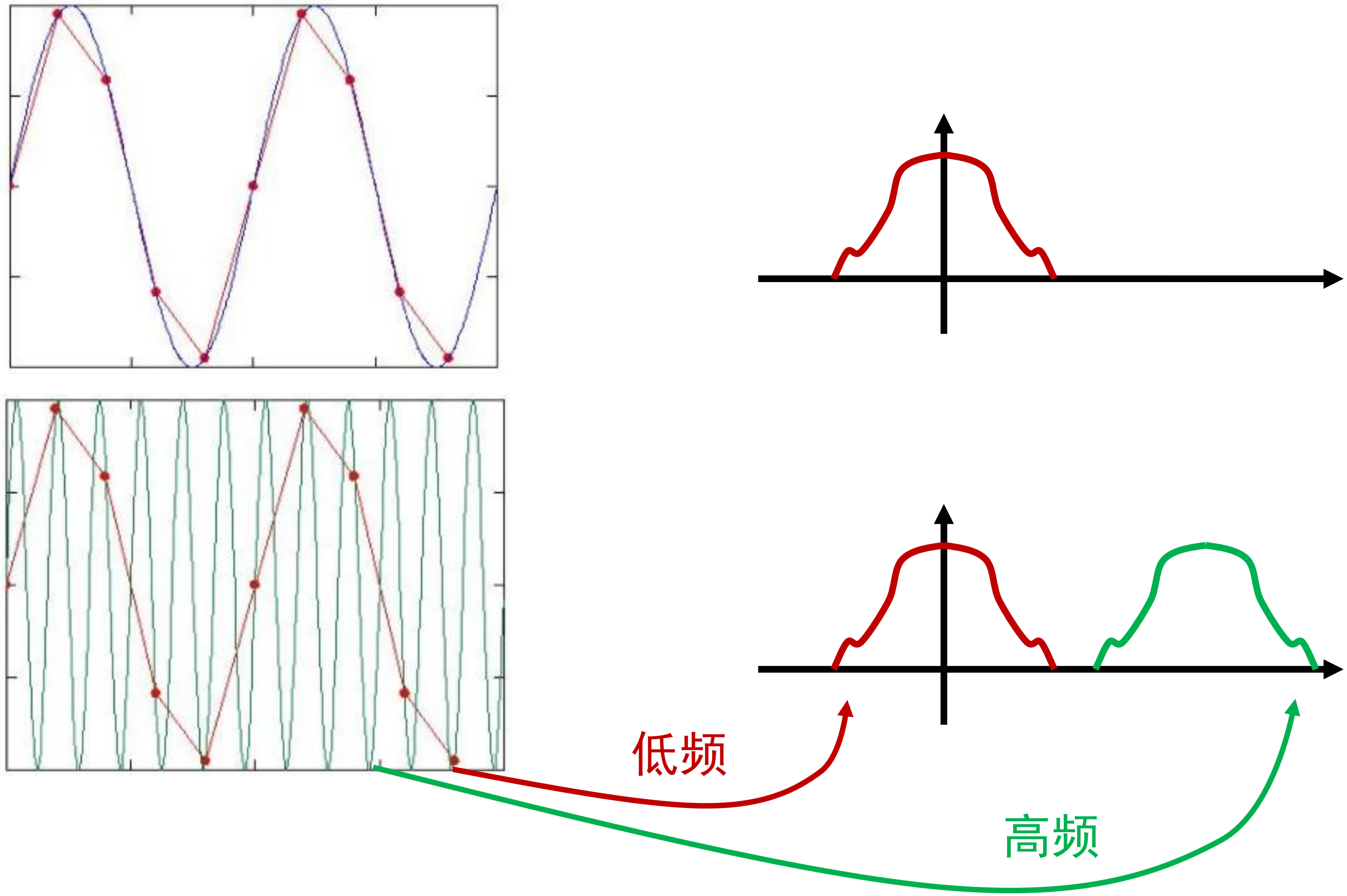




在时域对连续时间信号进行理想采样，就相当于在频域将连续时间信号的频谱以ω_s为周期进行延拓。

采样与采样定理

➢ 采样信号的频域表示

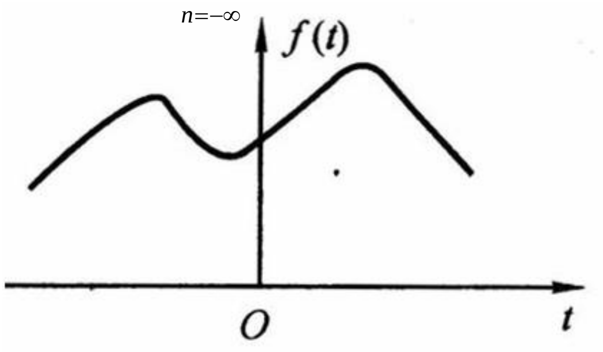


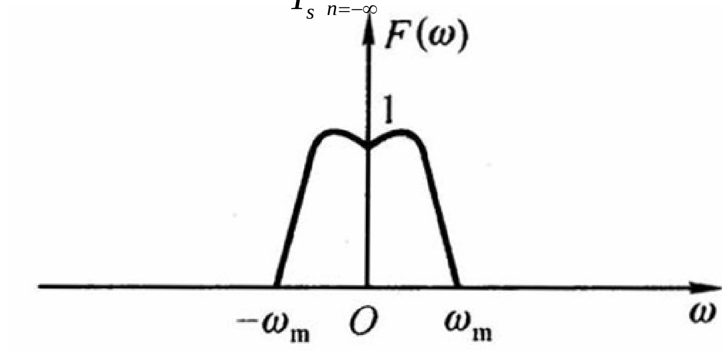
在时域对连续时间信号进行理想采样，就相当于在频域将连续时间信号的频谱以ω_s为周期进行延拓。

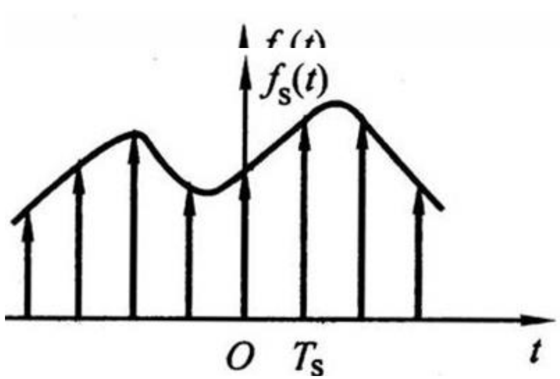
采样与采样定理

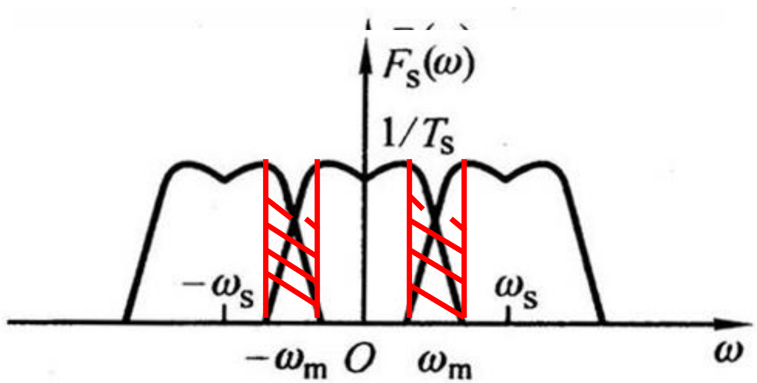
$$x_p(t)=\sum_{n=-\infty}^{+\infty}x(nT_s)\delta(t-nT_s)$$

$$X_p(\omega)=\frac{1}{T_s}\sum_{n=-\infty}^{\infty}X(\omega-n\omega_s)$$









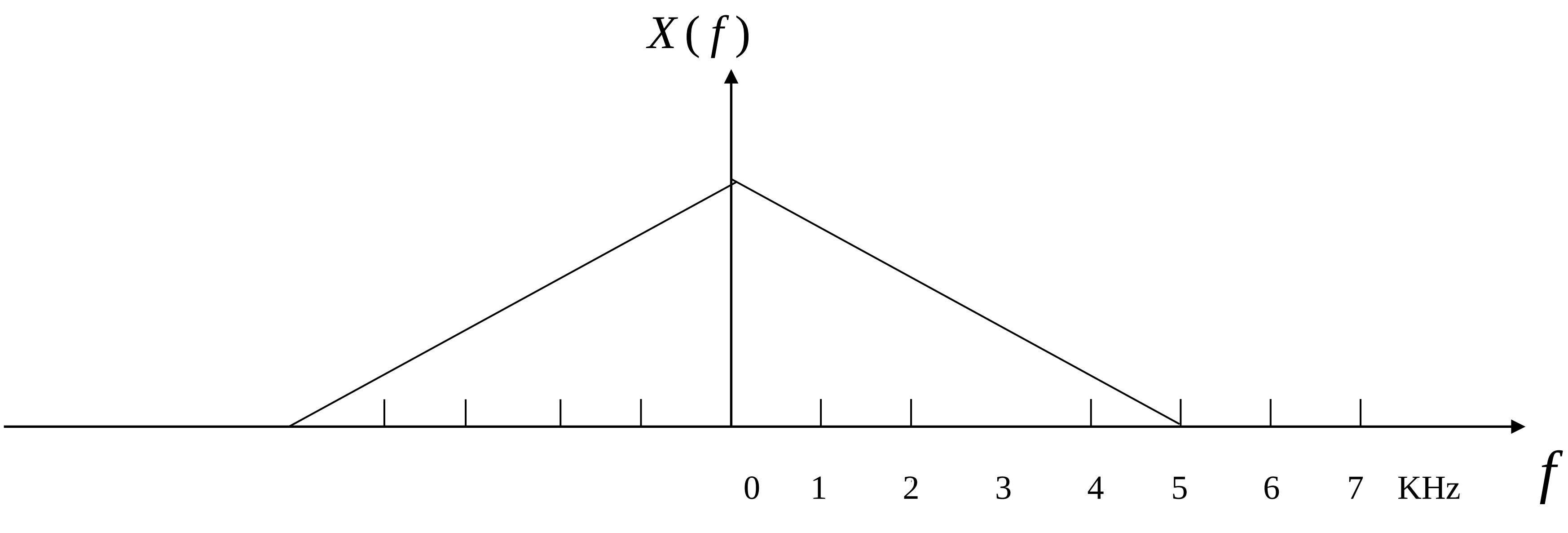
采样周期变大，频谱的周期变小，离散信号的谱发生相互重叠的现象：混叠

【课堂练习】

设模拟音频信号高频截至频率为5kHz,抽样频率为6kHz。

问题:

抽样后信号频谱与原信号频谱在2kHz处有什么差异?



采样与采样定理

【课堂练习】

设模拟音频信号高频截至频率为5kHz,抽样频率为6kHz。

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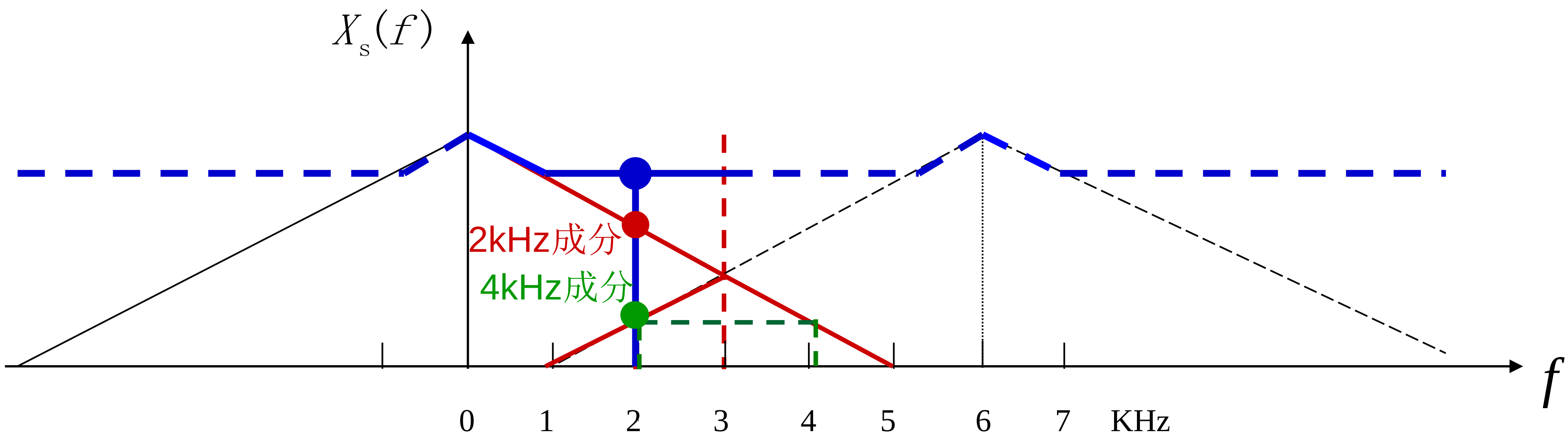
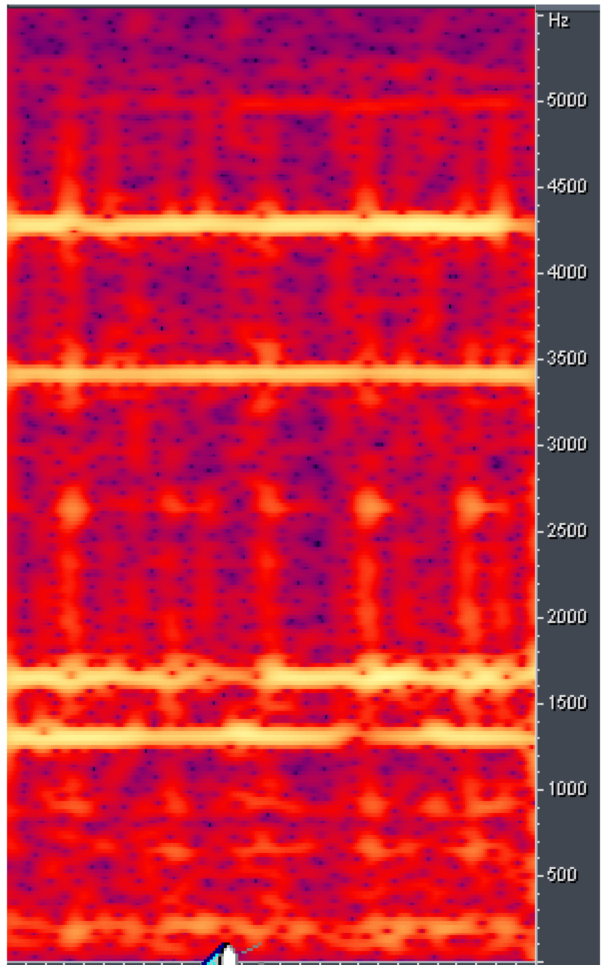
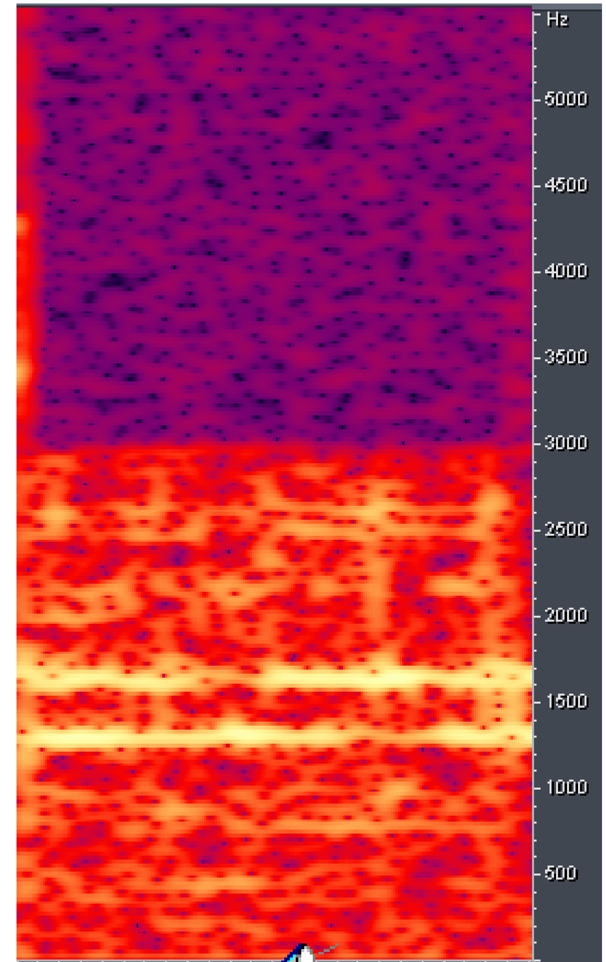


采样与采样定理

➢ 混叠





11KHz

6KHz

思考：如何防止混叠？

采样与采样定理

- 混叠

Original Signal → Prefilter → Sample → Reconstruction Filter → Reconstructed Signal

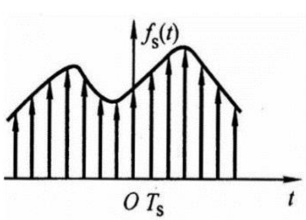
Result: aliasing replaced by blur

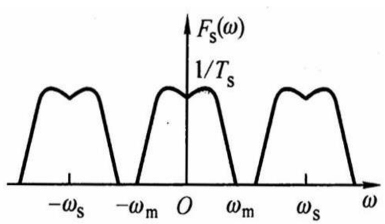
采样与采样定理

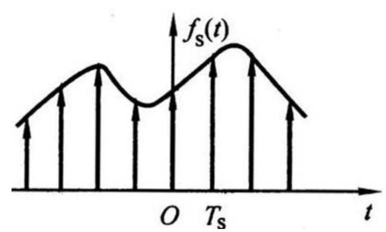
➢ 混叠

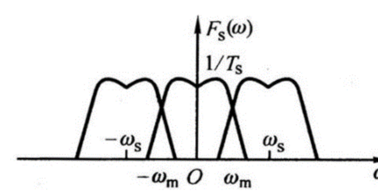
要想使采样后的信号样本能完全代表原来的信号，就意味着要能够从 $X_p(\omega)$ 中不失真地分离出 $X(\omega)$ 。这就要求在周期性延拓时不能发生频谱的混叠。为此必须要求：

- $x(t)$必须是带限的，最高频率分量为ω_M
- 采样频率必须满足 $\omega_s \geq 2\omega_M$









采样与采样定理

奈奎斯特
工程实践

香农
理论推导

抽样定理又称Nyquist采样定理，也称香农定理。

采样与采样定理

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K Krippendorff - Departmental Papers (ASC), 2009 - repository.upenn.edu
Claude Shannon's mathematical theory of communication concerns quantitative limits of mediated communication. The theory has a history in cryptography and of measuring telephone traffic. Paralleling work by US cybernetician Norbert Wiener and Soviet logician …
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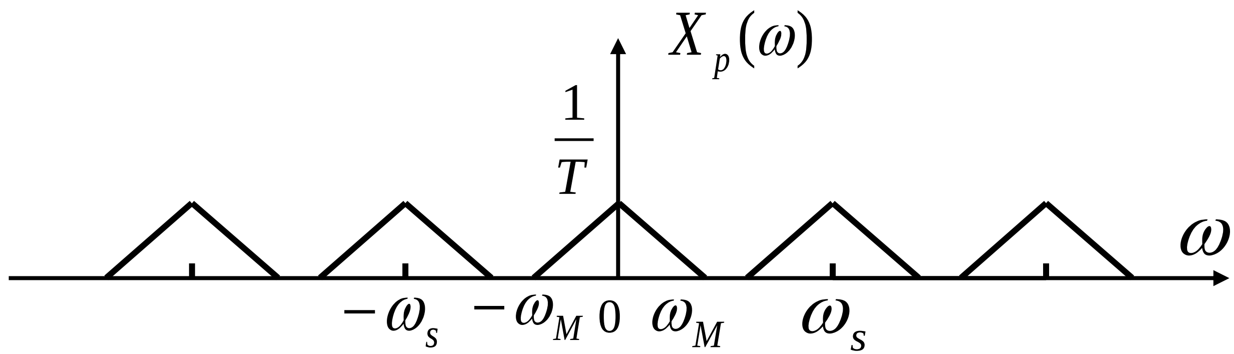
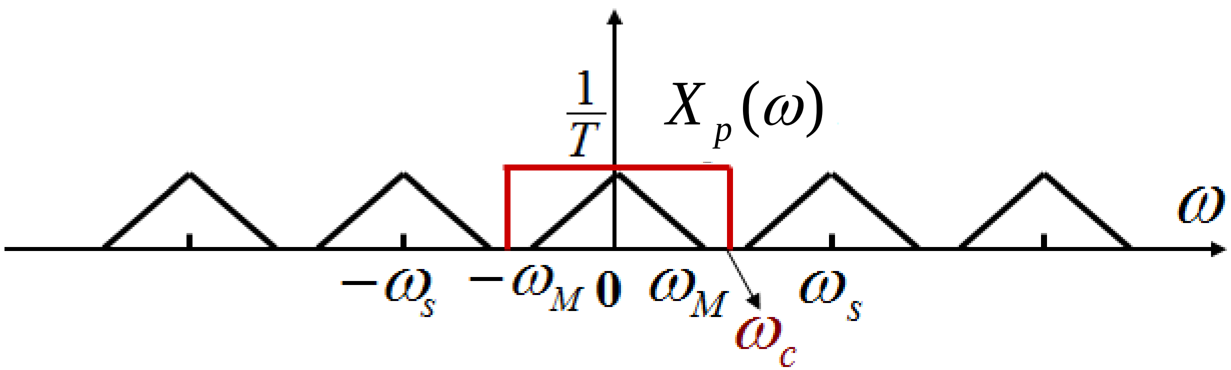
➢ Nyquist采样定理

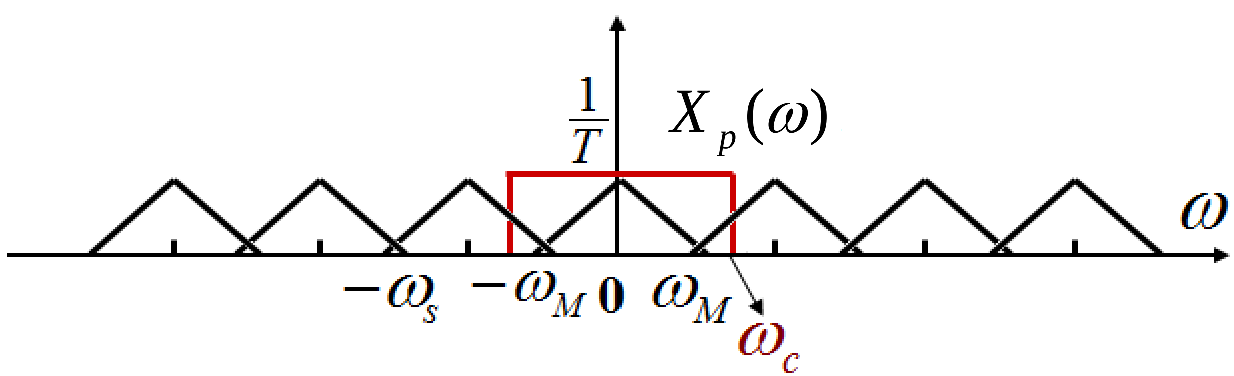


对带限于最高频率ω_M的连续时间信号$x(t)$，如果以$\omega_s \geq 2\omega_M$的频率进行理想采样，则$x(t)$可以唯一的由其样本$x(nT)$来确定。

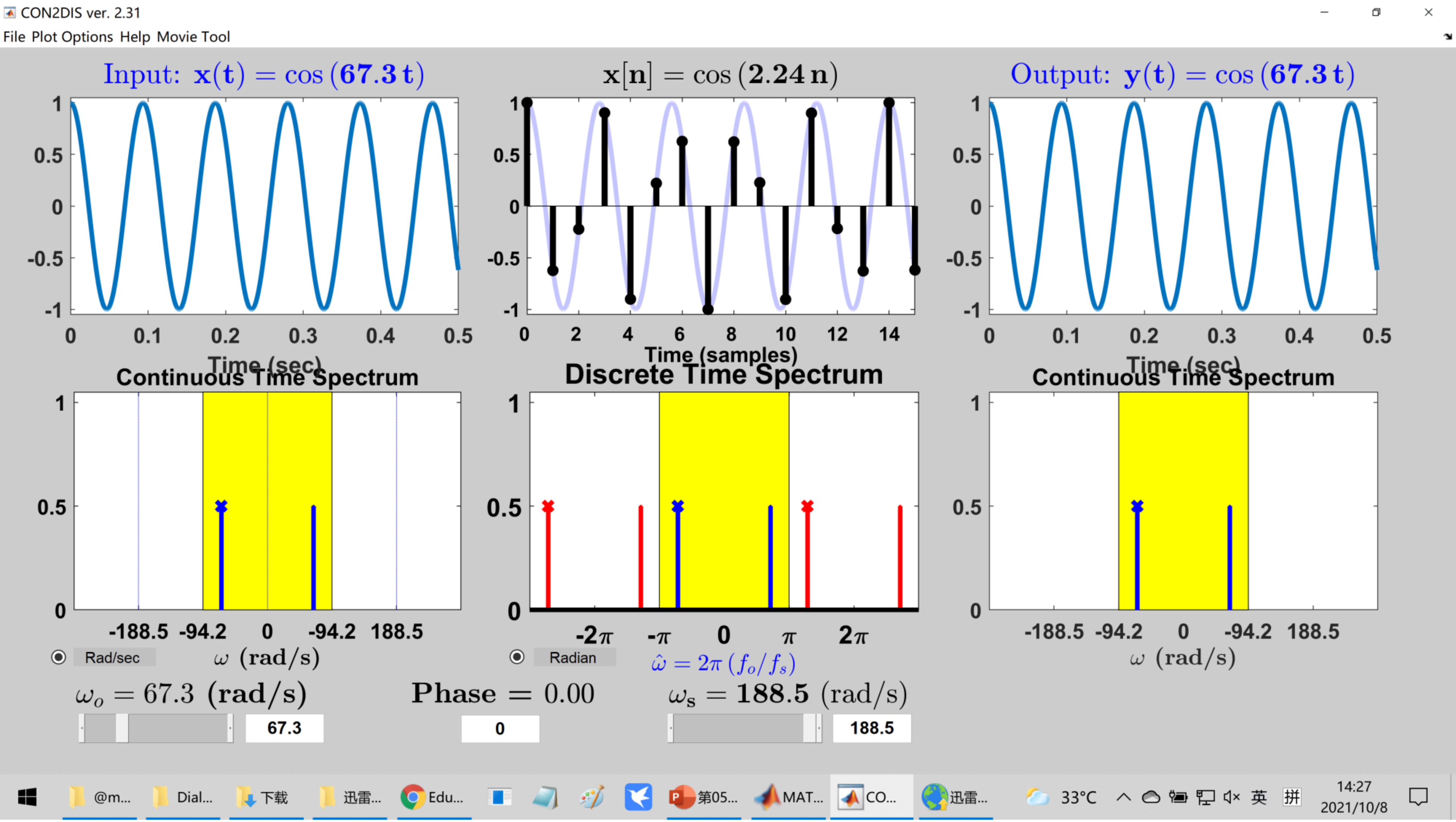
采样与采样定理

- Nyquist采样定理



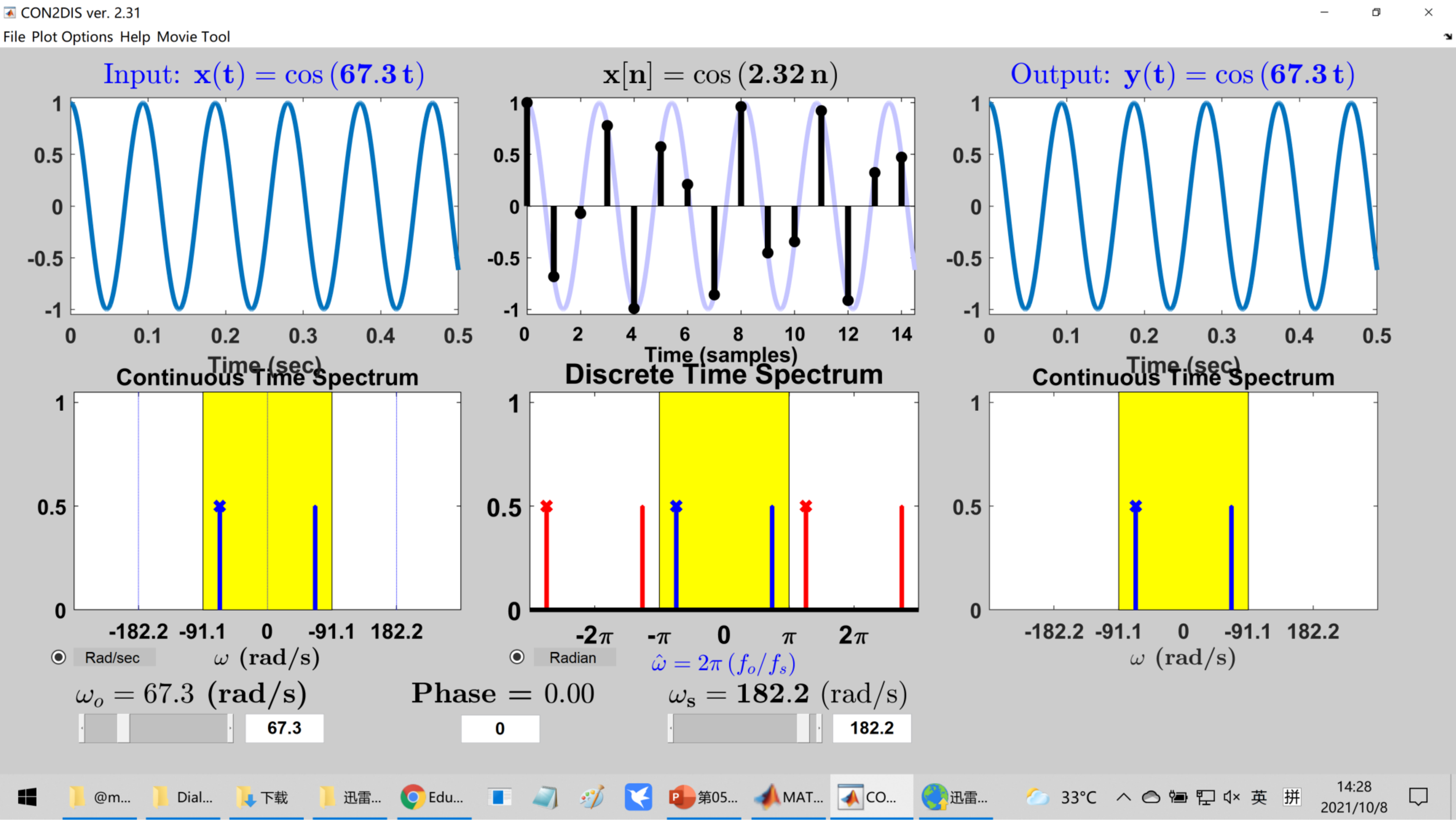


采样与采样定理



CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos(67.3 t)
x[n] = cos(2.24 n)
Output: y(t) = cos(67.3 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π(f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
67.3
Phase = 0.00
0
ω_s = 188.5 (rad/s)
188.5

采样与采样定理


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
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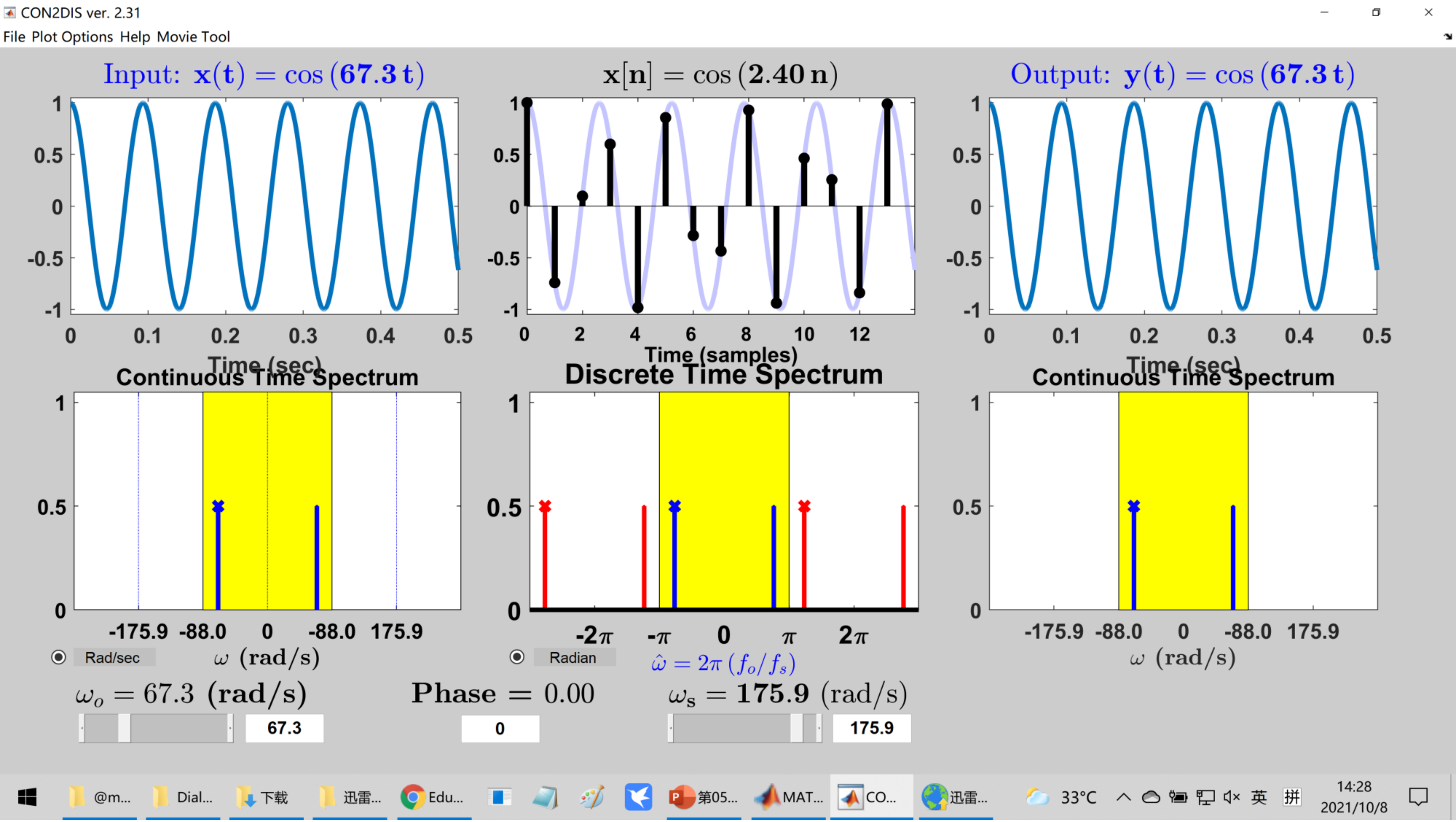
x[n] = cos(2.32 n)
Output: y(t) = cos(67.3 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
-182.2 -91.1 0 -91.1 182.2
-2π -π 0 π 2π
-182.2 -91.1 0 -91.1 182.2
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
Phase = 0.00
ω_s = 182.2 (rad/s)
67.3
0
182.2

采样与采样定理



CON2DIS ver. 2.31
File Plot Options Help Movie Tool
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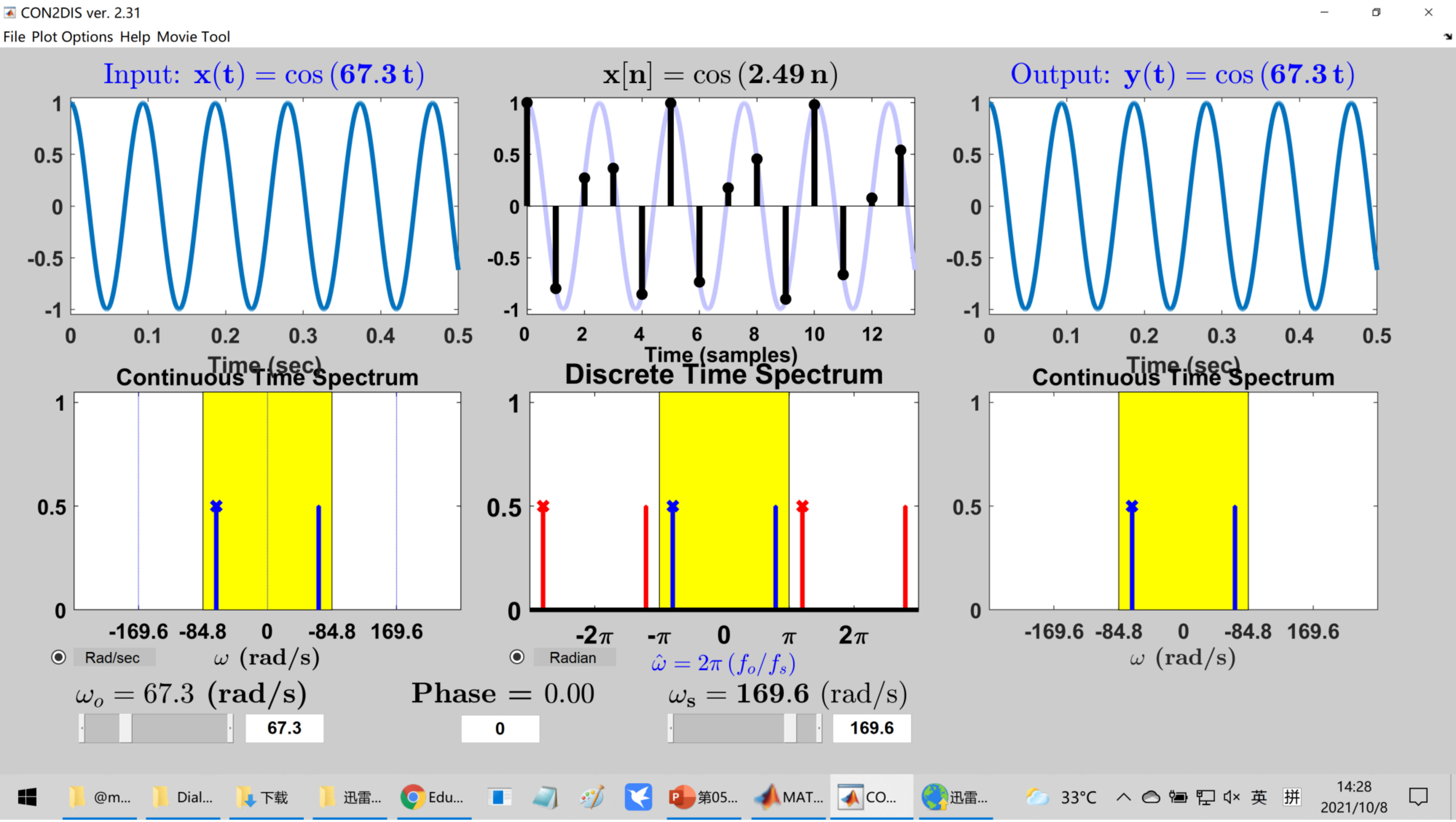
x[n] = cos(2.40 n)
Output: y(t) = cos(67.3 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
-175.9 -88.0 0 -88.0 175.9
-2π -π 0 π 2π
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω_o = 67.3 (rad/s)
Phase = 0.00
ω_s = 175.9 (rad/s)
67.3
0
175.9
14:28
2021/10/8

采样与采样定理


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
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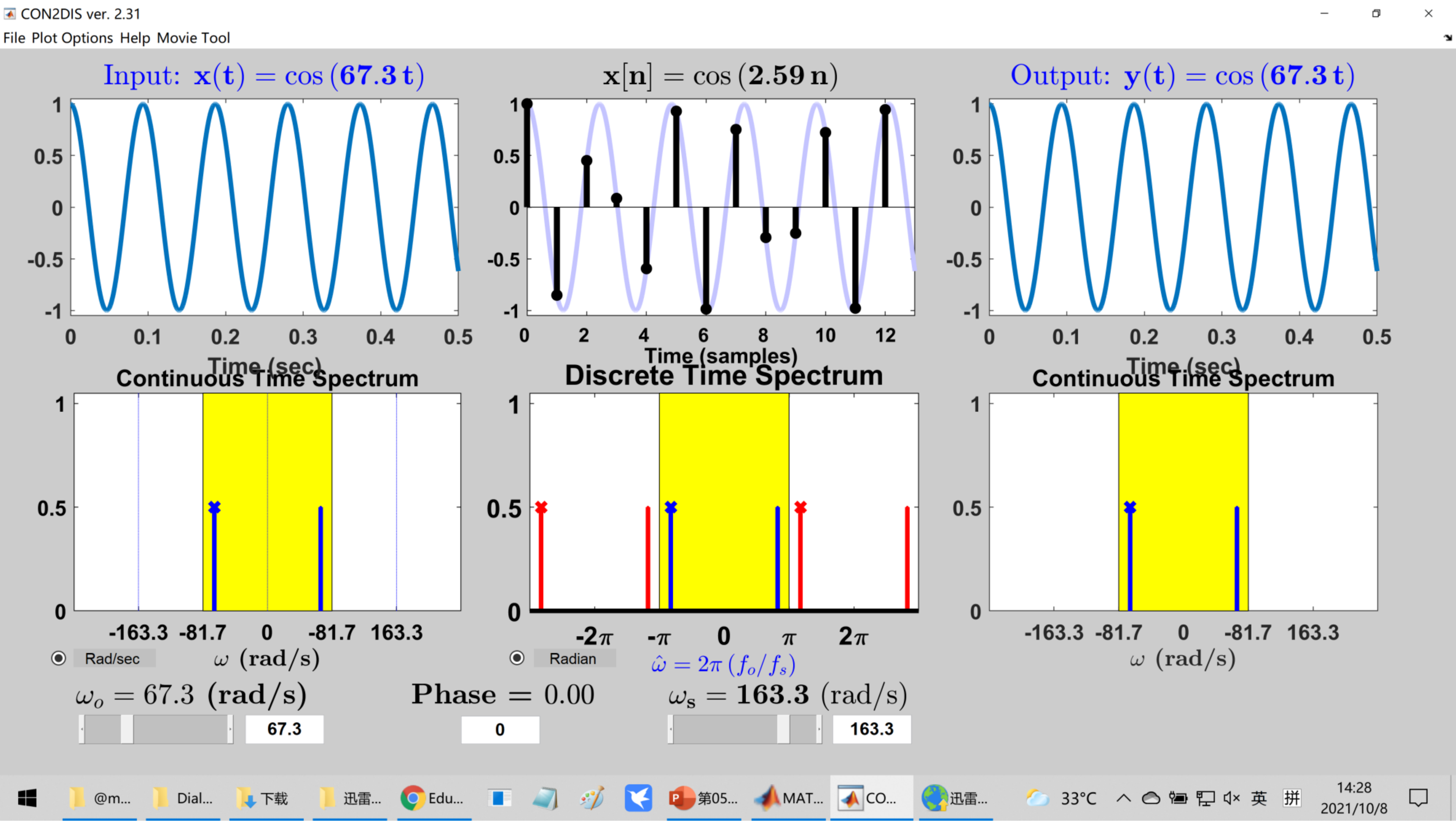
x[n] = cos(2.49 n)
Output: y(t) = cos(67.3 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
-169.6 -84.8 0 -84.8 169.6
-2π -π 0 π 2π
-169.6 -84.8 0 -84.8 169.6
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
Phase = 0.00
ω_s = 169.6 (rad/s)
67.3
0
169.6

采样与采样定理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File Plot Options Help Movie Tool
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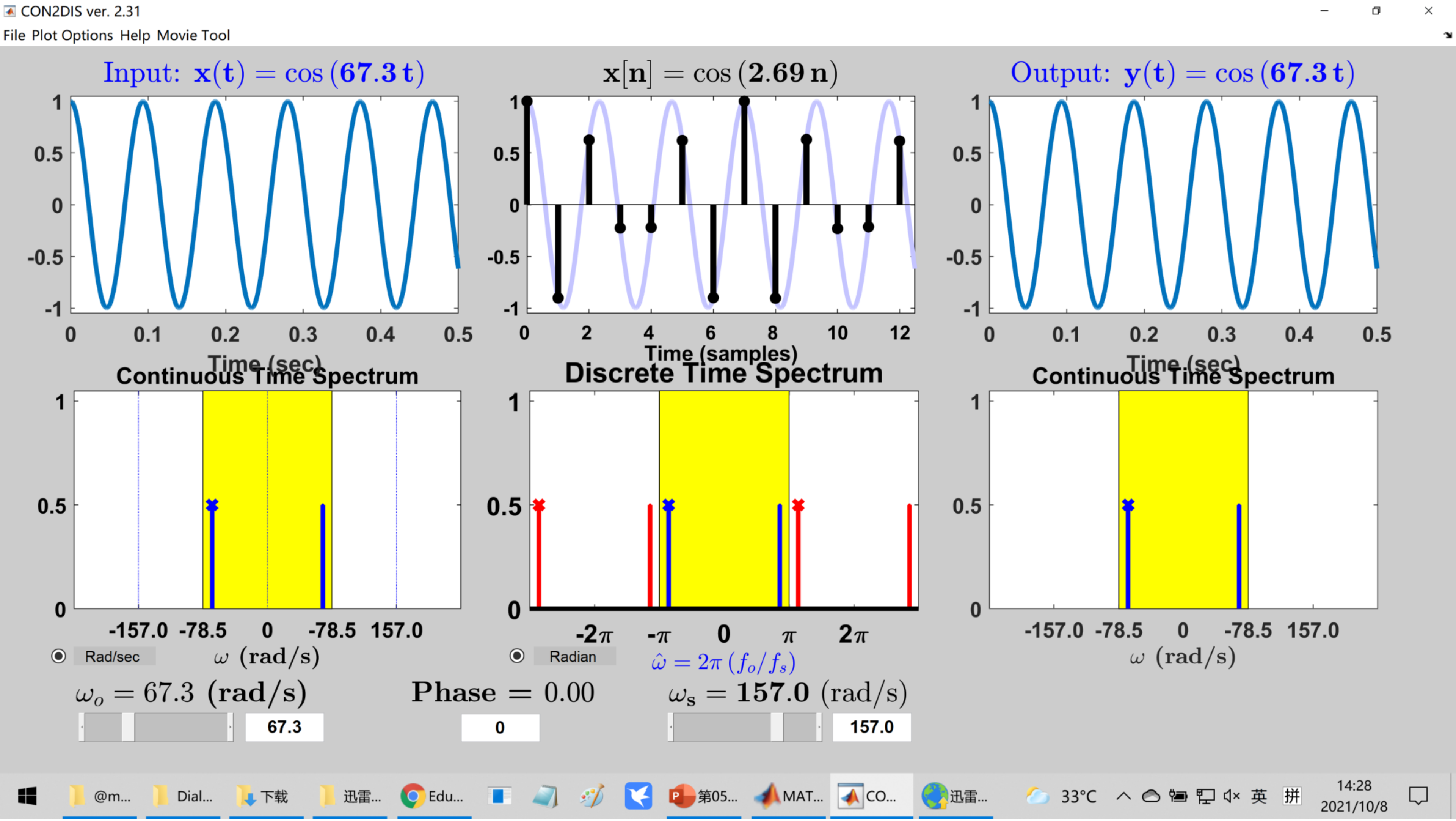
x[n] = cos(2.59 n)
Output: y(t) = cos(67.3 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
-163.3 -81.7 0 -81.7 163.3
-2π -π 0 π 2π
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
Phase = 0.00
ω_s = 163.3 (rad/s)
67.3
0
163.3
14:28
2021/10/8

采样与采样定理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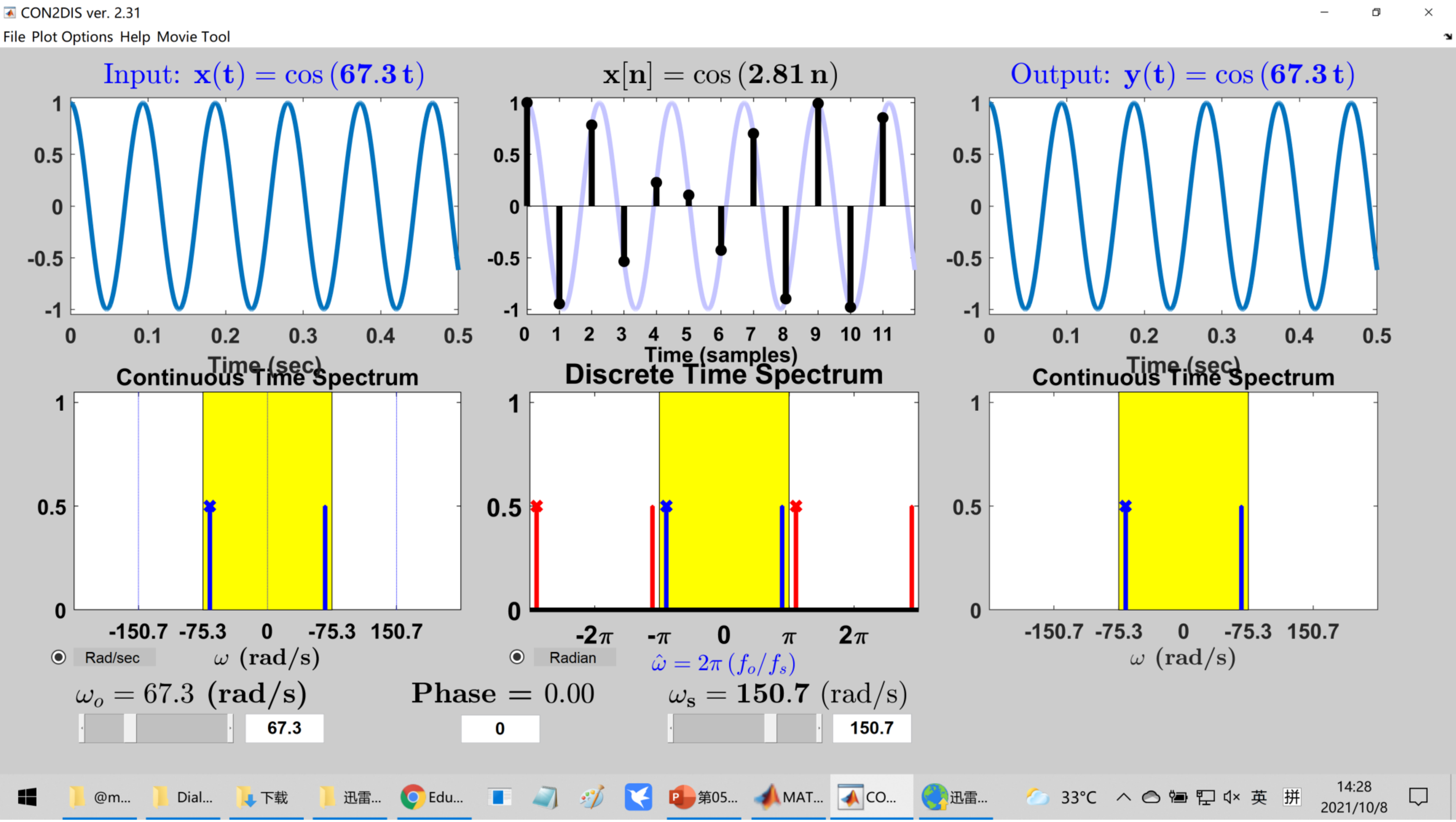
x[n] = cos (2.69 n)
Output: y(t) = cos (67.3 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
-157.0 -78.5 0 -78.5 157.0
-2π -π 0 π 2π
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
67.3
Phase = 0.00
0
ω_s = 157.0 (rad/s)
157.0
33°C
14:28
2021/10/8

采样与采样定理


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
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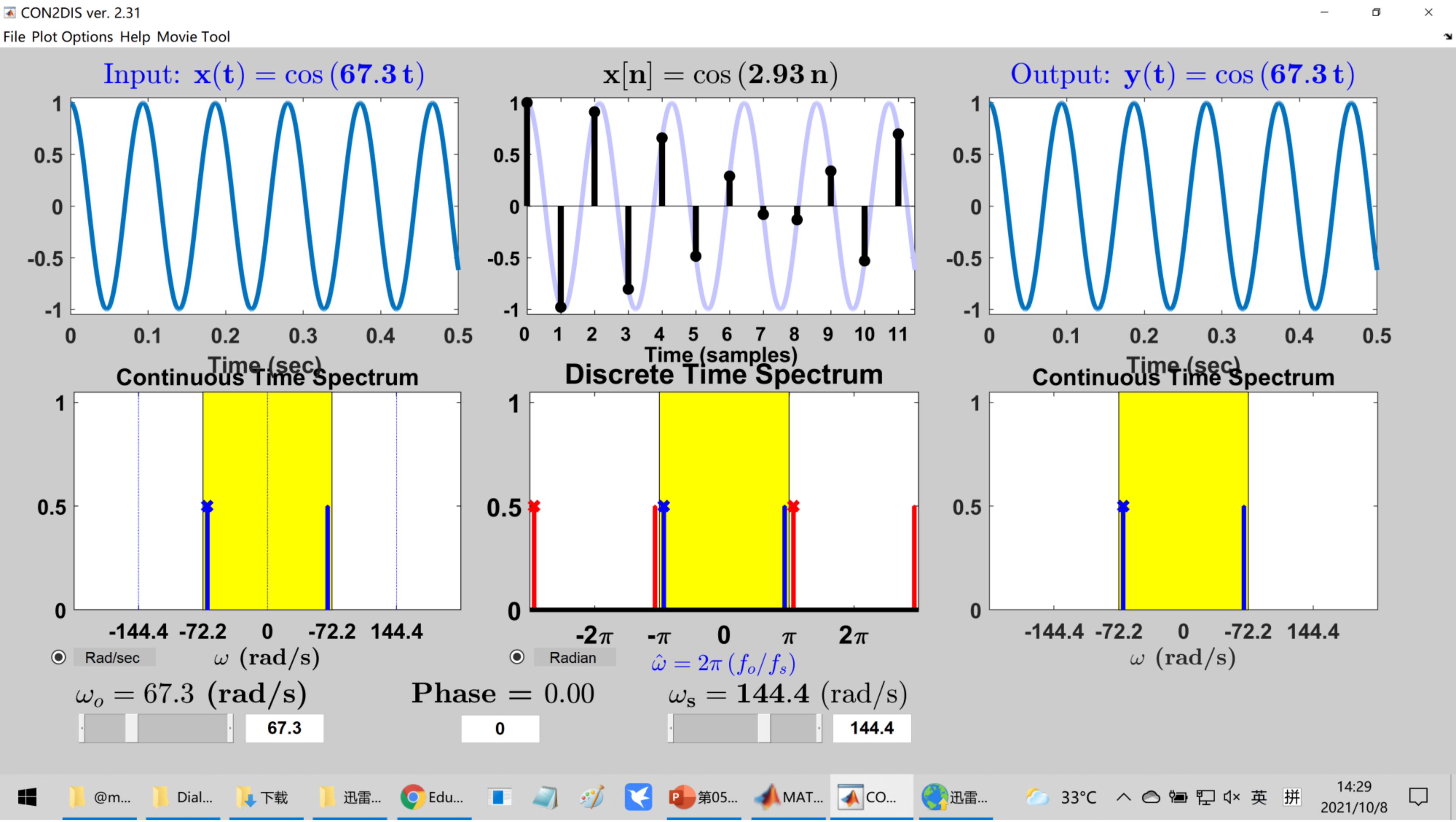
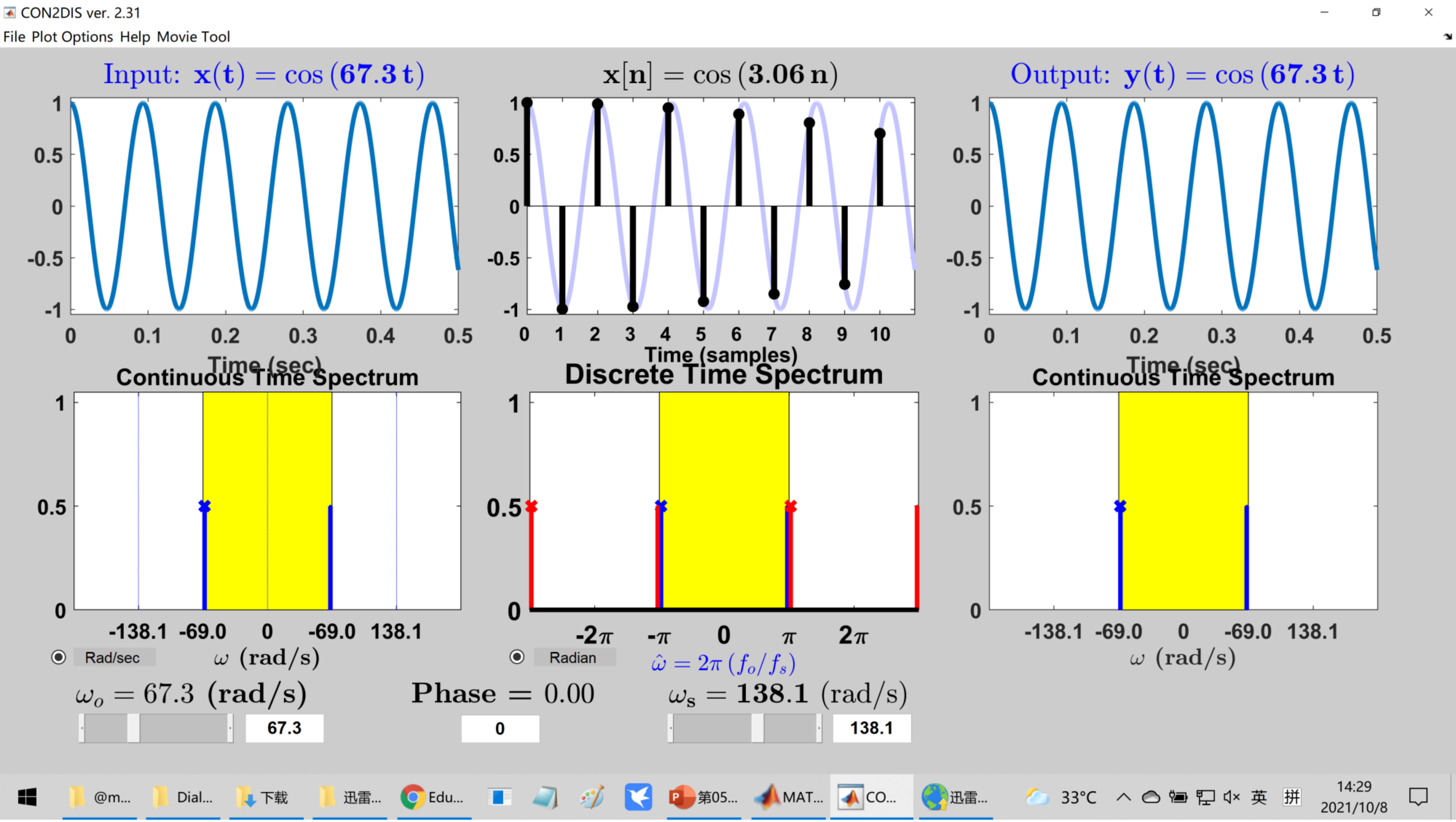
x[n] = cos(2.81 n)
Output: y(t) = cos(67.3 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
-150.7 -75.3 0 -75.3 150.7
-2π -π 0 π 2π
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
67.3
Phase = 0.00
0
ω_s = 150.7 (rad/s)
150.7
33°C
14:28
2021/10/8

采样与采样定理

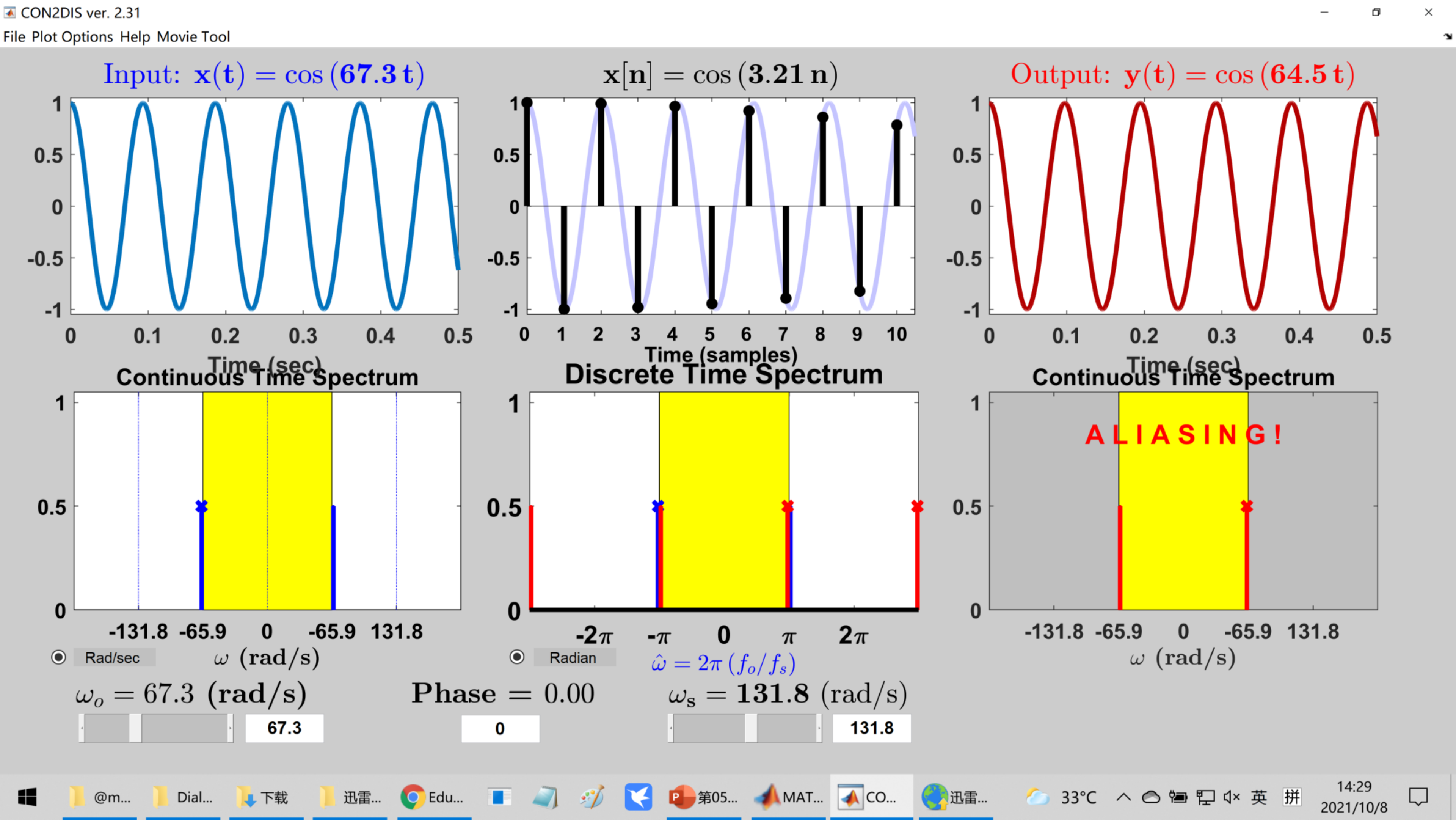

CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: $x(t) = \cos(67.3\,t)$
$x[n] = \cos(2.93\,n)$
Output: $y(t) = \cos(67.3\,t)$
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
-144.4 -72.2 0 -72.2 144.4
-2π -π 0 π 2π
Rad/sec
ω (rad/s)
Radian
$\hat{\omega} = 2\pi(f_o/f_s)$
$\omega_o = 67.3$ (rad/s)
Phase = 0.00
$\omega_s = 144.4$ (rad/s)
67.3
0
144.4
33°C
14:29
2021/10/8

采样与采样定理

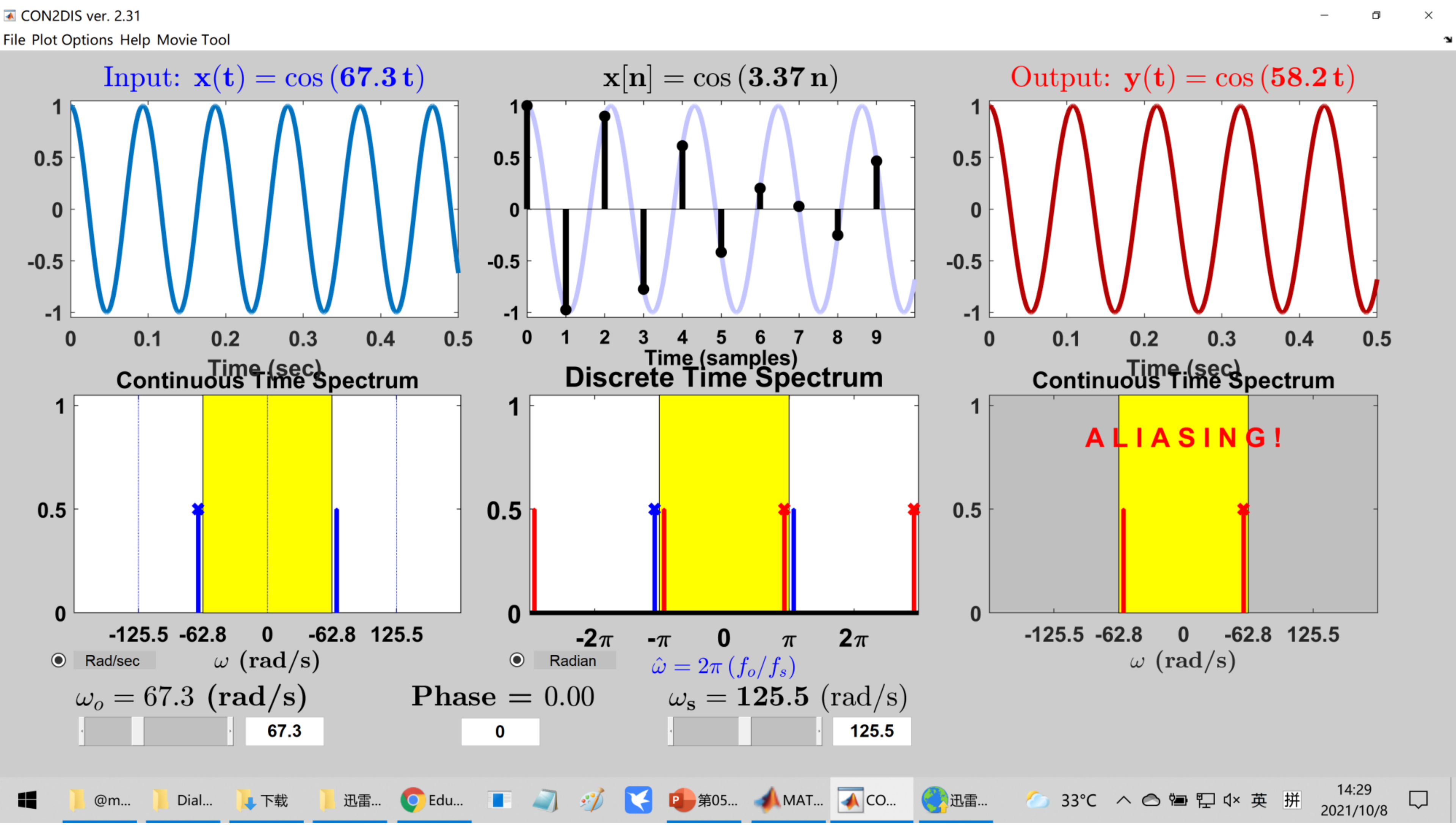


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos (67.3 t)
x[n] = cos (3.06 n)
Output: y(t) = cos (67.3 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
-138.1 -69.0 0 -69.0 138.1
Rad/sec
ω (rad/s)
-2π -π 0 π 2π
Radian
ω̂ = 2π (f_o/f_s)
-138.1 -69.0 0 -69.0 138.1
ω (rad/s)
ω_o = 67.3 (rad/s)
67.3
Phase = 0.00
0
ω_s = 138.1 (rad/s)
138.1
33°C
14:29
2021/10/8

采样与采样定理

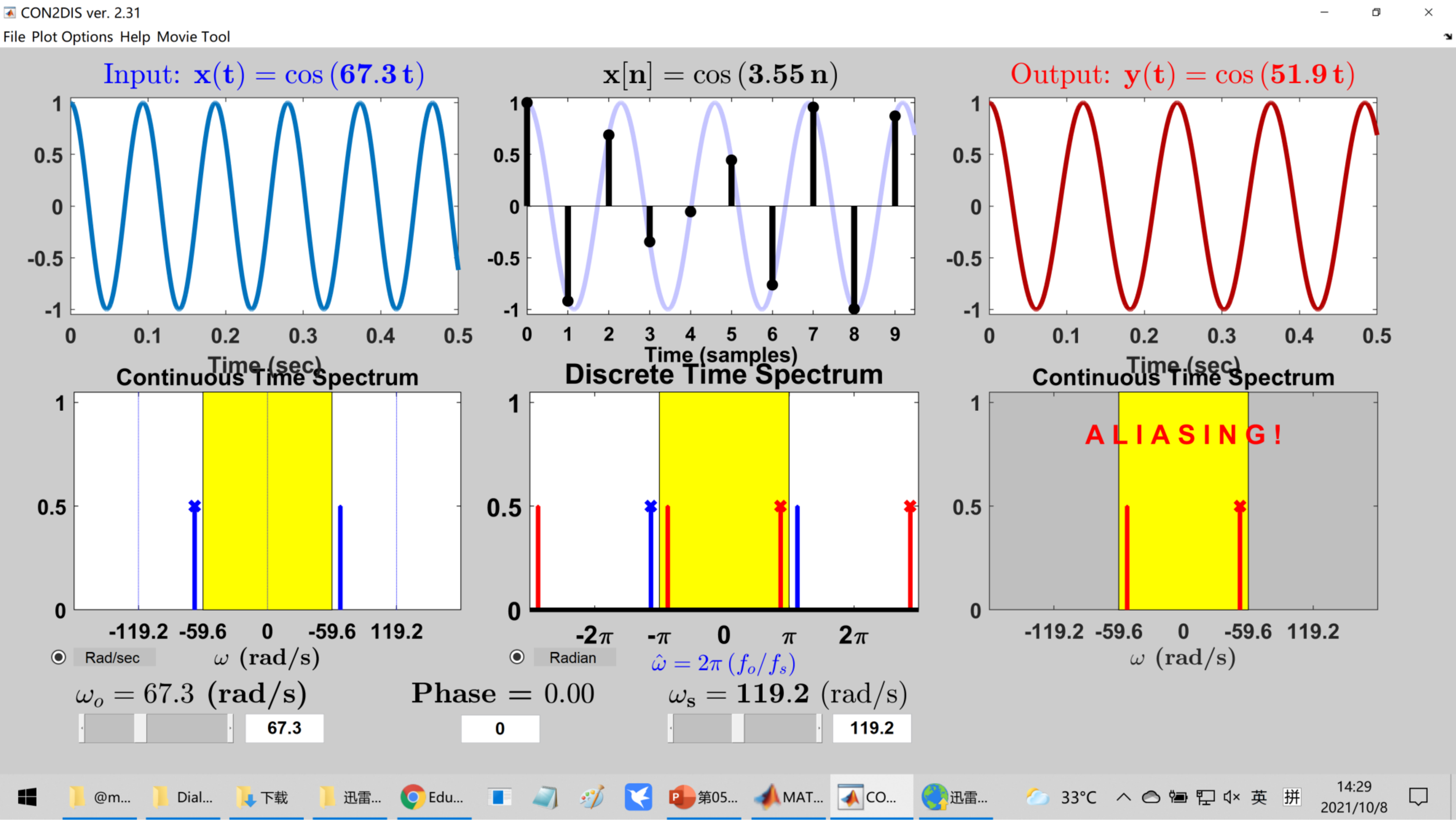

CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos (67.3 t)
x[n] = cos (3.21 n)
Output: y(t) = cos (64.5 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
ALIASING!
-131.8 -65.9 0 -65.9 131.8
-2π -π 0 π 2π
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω_o = 67.3 (rad/s)
Phase = 0.00
ω_s = 131.8 (rad/s)
67.3
0
131.8
14:29
2021/10/8

采样与采样定理



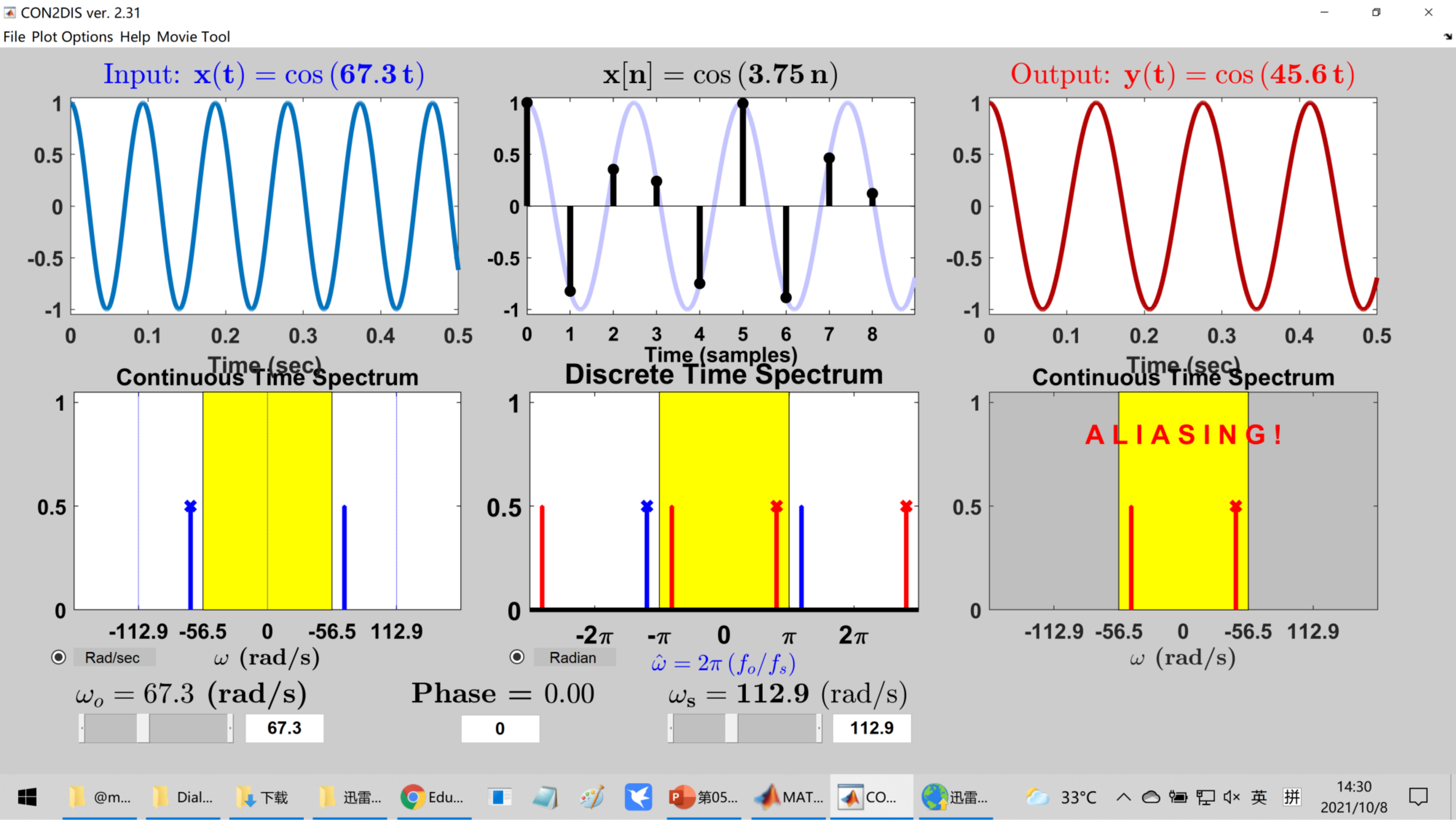
CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos(67.3 t)
x[n] = cos(3.37 n)
Output: y(t) = cos(58.2 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
ALIASING!
-125.5 -62.8 0 -62.8 125.5
-2π -π 0 π 2π
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
67.3
Phase = 0.00
0
ω_s = 125.5 (rad/s)
125.5
14:29
2021/10/8

采样与采样定理

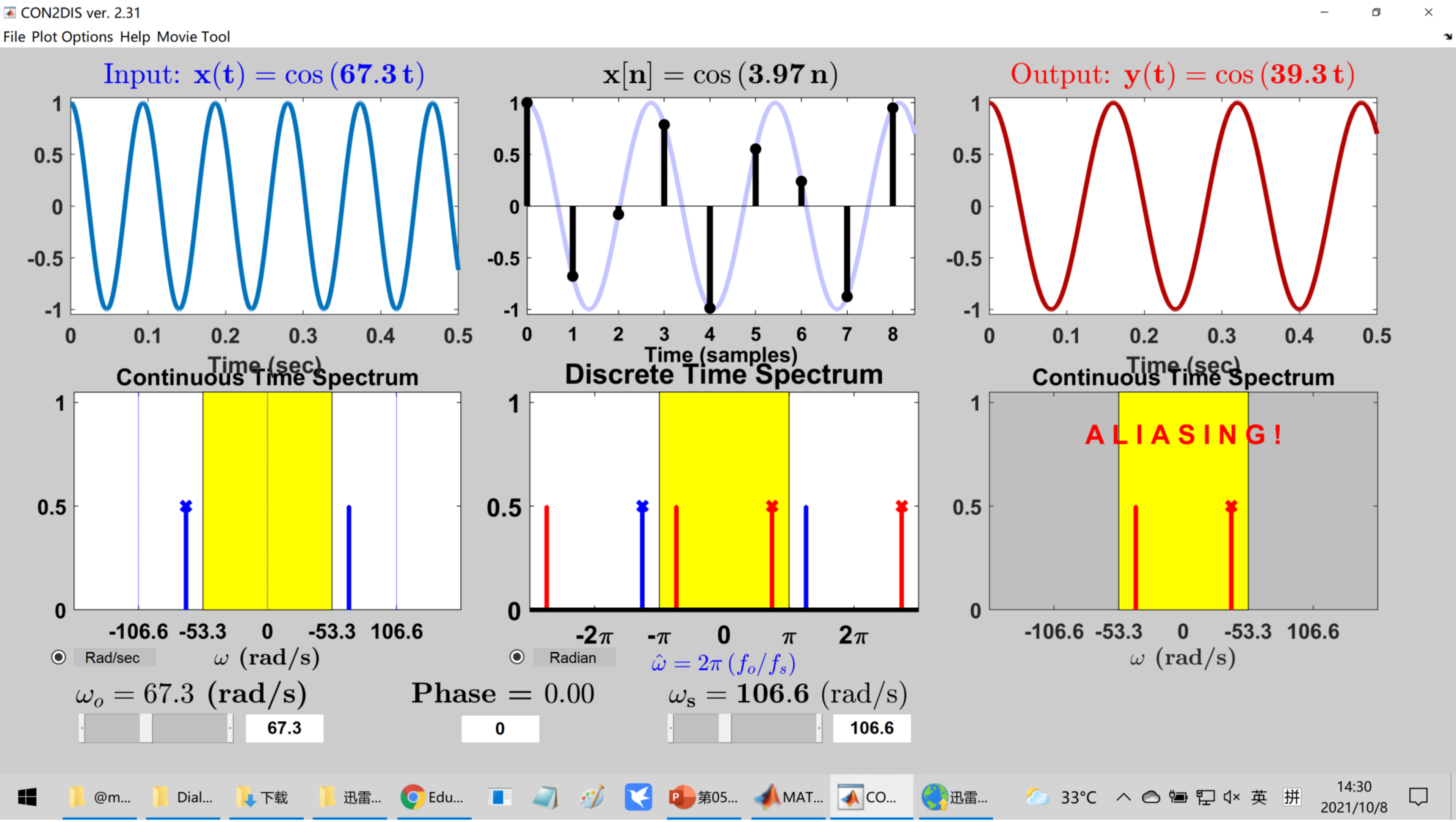


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos(67.3 t)
x[n] = cos(3.55 n)
Output: y(t) = cos(51.9 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
ALIASING!
-119.2 -59.6 0 -59.6 119.2
-2π -π 0 π 2π
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
67.3
Phase = 0.00
0
ω_s = 119.2 (rad/s)
119.2
33°C
14:29
2021/10/8

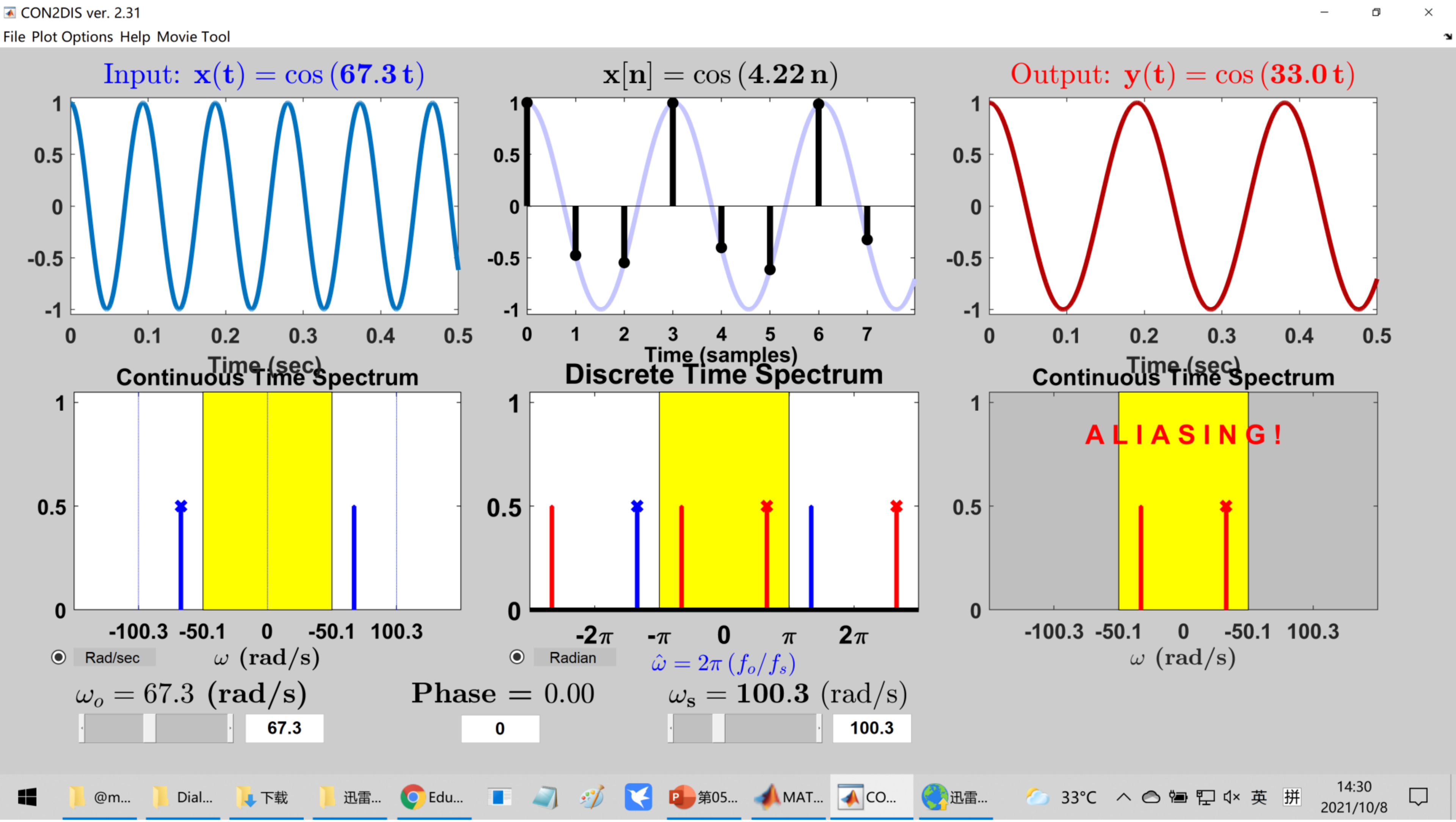
采样与采样定理


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos(67.3 t)
x[n] = cos(3.75 n)
Output: y(t) = cos(45.6 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
ALIASING!
-112.9 -56.5 0 -56.5 112.9
-2π -π 0 π 2π
-112.9 -56.5 0 -56.5 112.9
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
67.3
Phase = 0.00
0
ω_s = 112.9 (rad/s)
112.9
33°C
14:30
2021/10/8

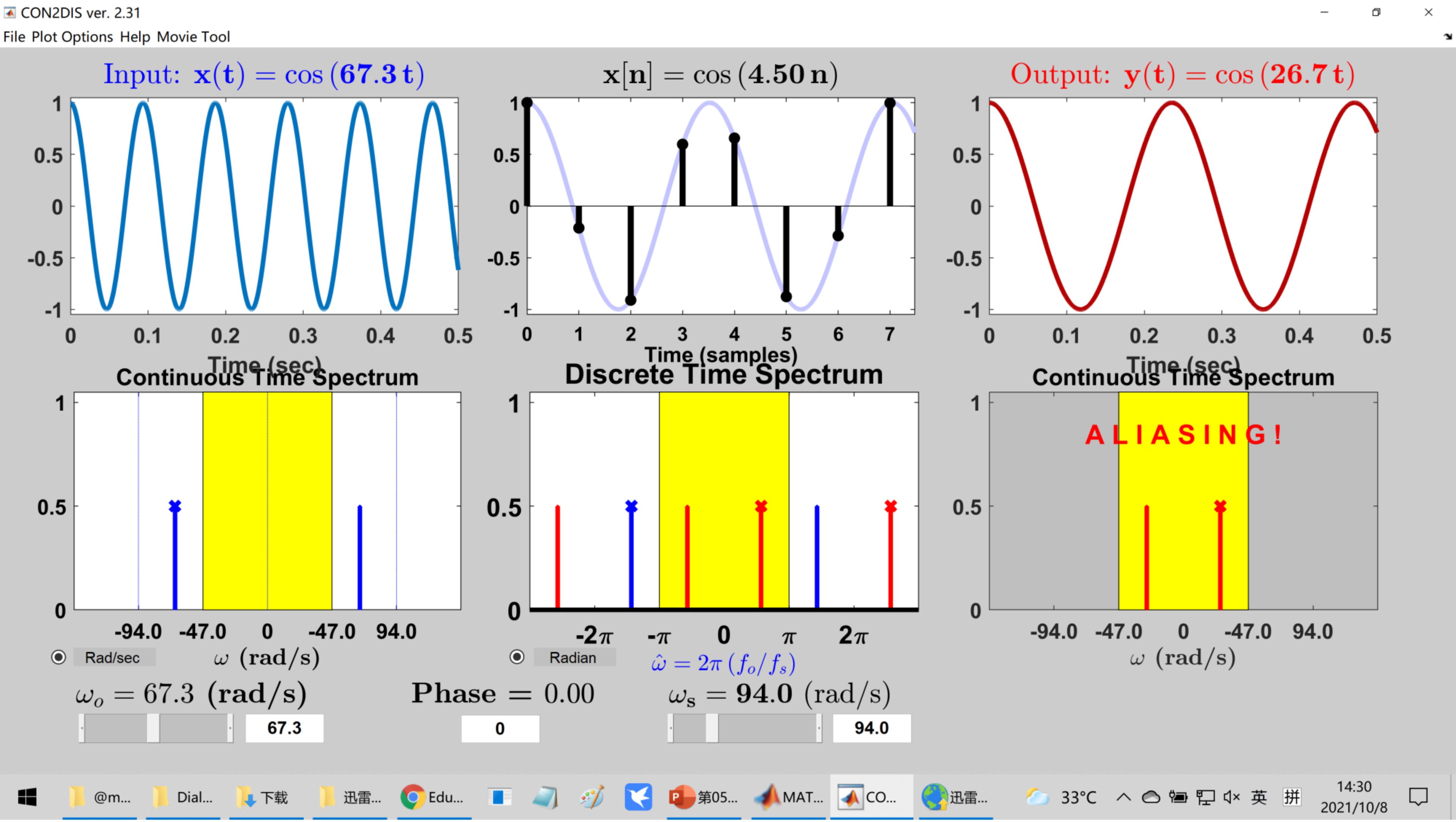
采样与采样定理


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos(67.3 t)
x[n] = cos(3.97 n)
Output: y(t) = cos(39.3 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
ALIASING!
-106.6 -53.3 0 -53.3 106.6
-2π -π 0 π 2π
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω_o = 67.3 (rad/s)
Phase = 0.00
ω_s = 106.6 (rad/s)
67.3
0
106.6
14:30
2021/10/8

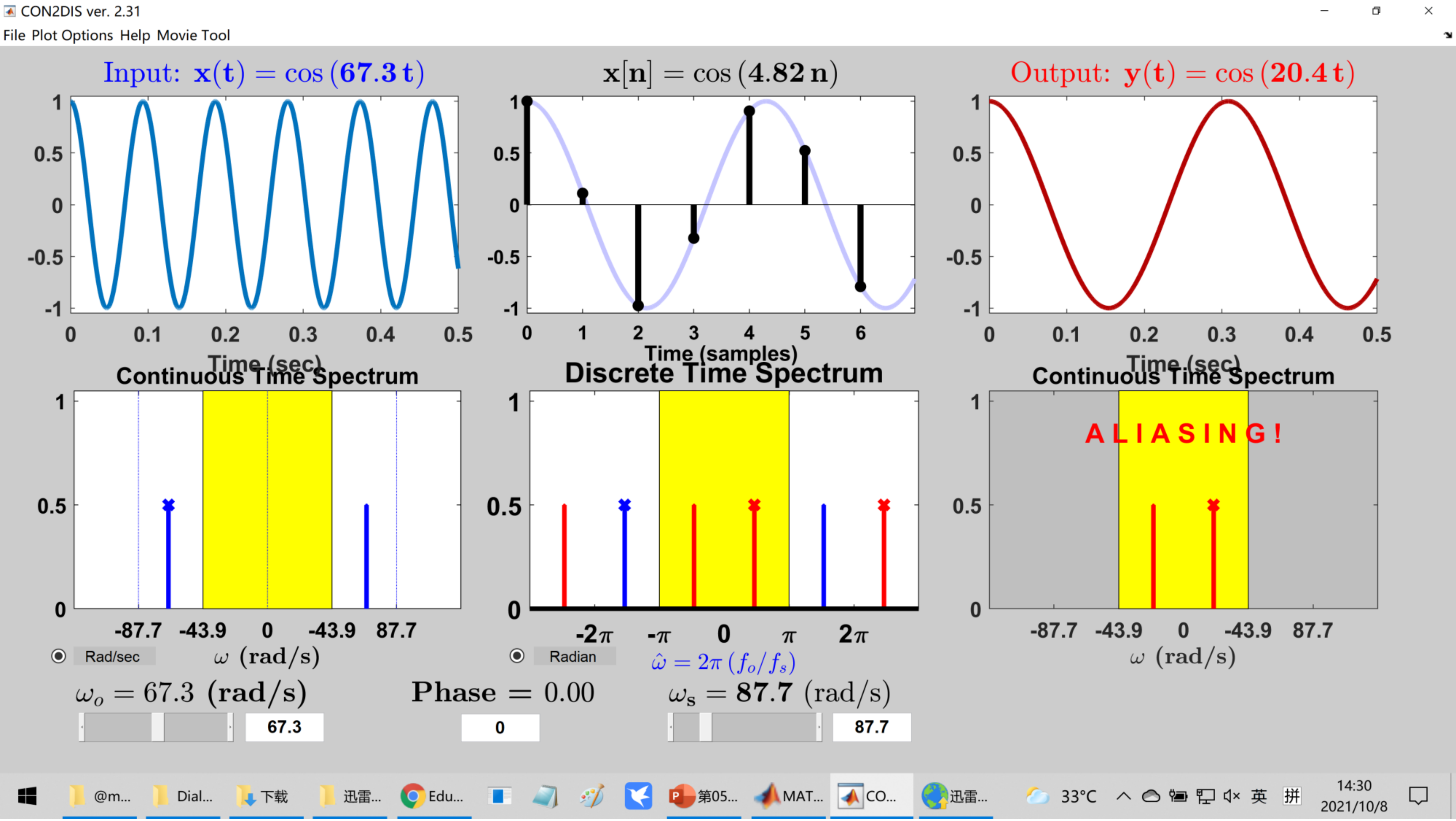
采样与采样定理


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos(67.3 t)
x[n] = cos(4.22 n)
Output: y(t) = cos(33.0 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
ALIASING!
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π(f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
Phase = 0.00
ω_s = 100.3 (rad/s)
67.3
0
100.3
14:30
2021/10/8

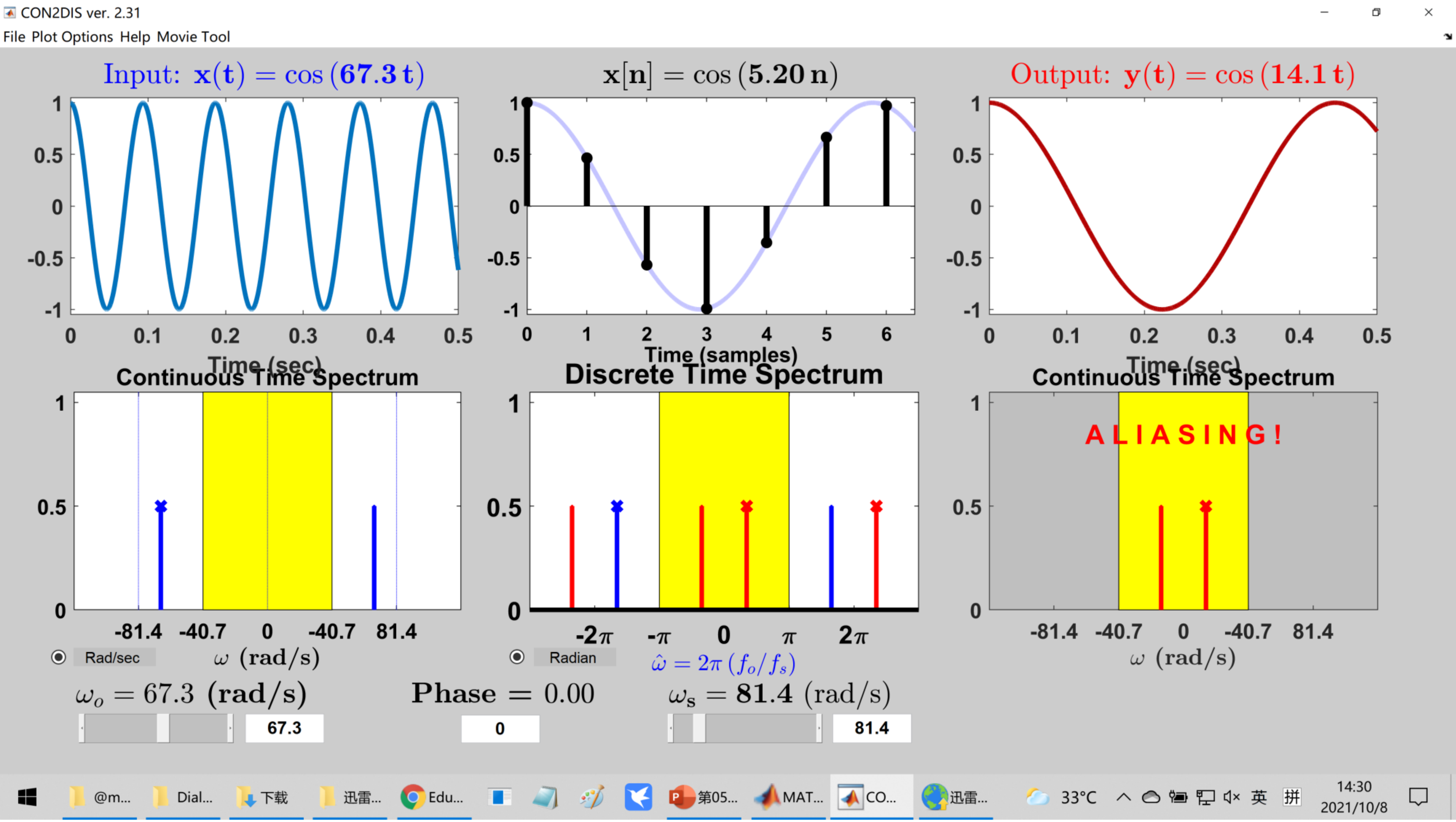
采样与采样定理


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos(67.3 t)
x[n] = cos(4.50 n)
Output: y(t) = cos(26.7 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
ALIASING!
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
67.3
Phase = 0.00
0
ω_s = 94.0 (rad/s)
94.0
33°C
14:30
2021/10/8

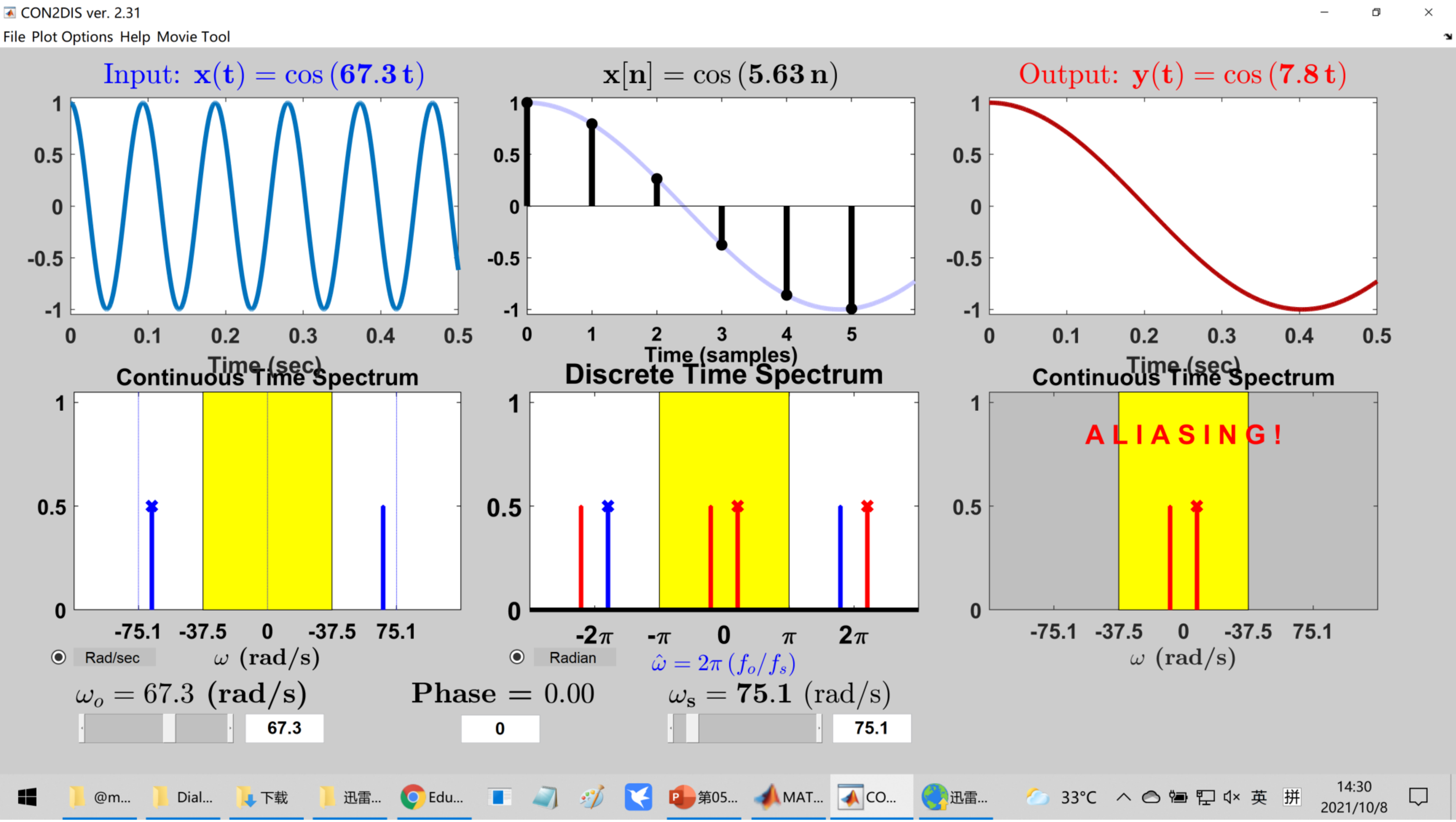
采样与采样定理


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos(67.3 t)
x[n] = cos(4.82 n)
Output: y(t) = cos(20.4 t)
Time (sec)
Time (samples)
Continuous Time Spectrum
Discrete Time Spectrum
ALIASING!
Rad/sec
Radian
ω (rad/s)
ω̂ = 2π(f_o/f_s)
ω_o = 67.3 (rad/s)
Phase = 0.00
ω_s = 87.7 (rad/s)
67.3
0
87.7

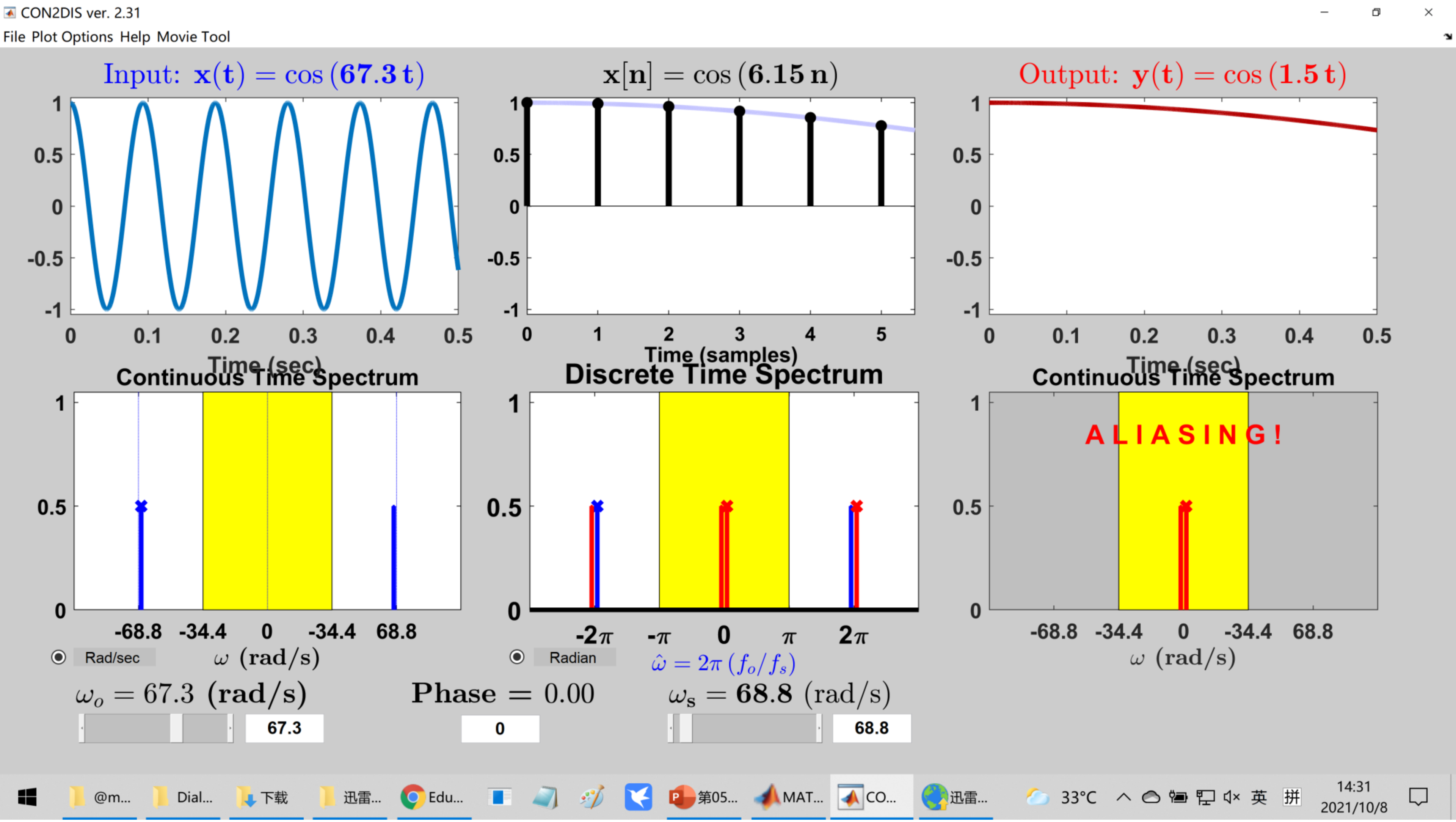
采样与采样定理


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos(67.3 t)
x[n] = cos(5.20 n)
Output: y(t) = cos(14.1 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
ALIASING!
-81.4 -40.7 0 -40.7 81.4
-2π -π 0 π 2π
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
Phase = 0.00
ω_s = 81.4 (rad/s)
67.3
0
81.4
33°C
14:30
2021/10/8

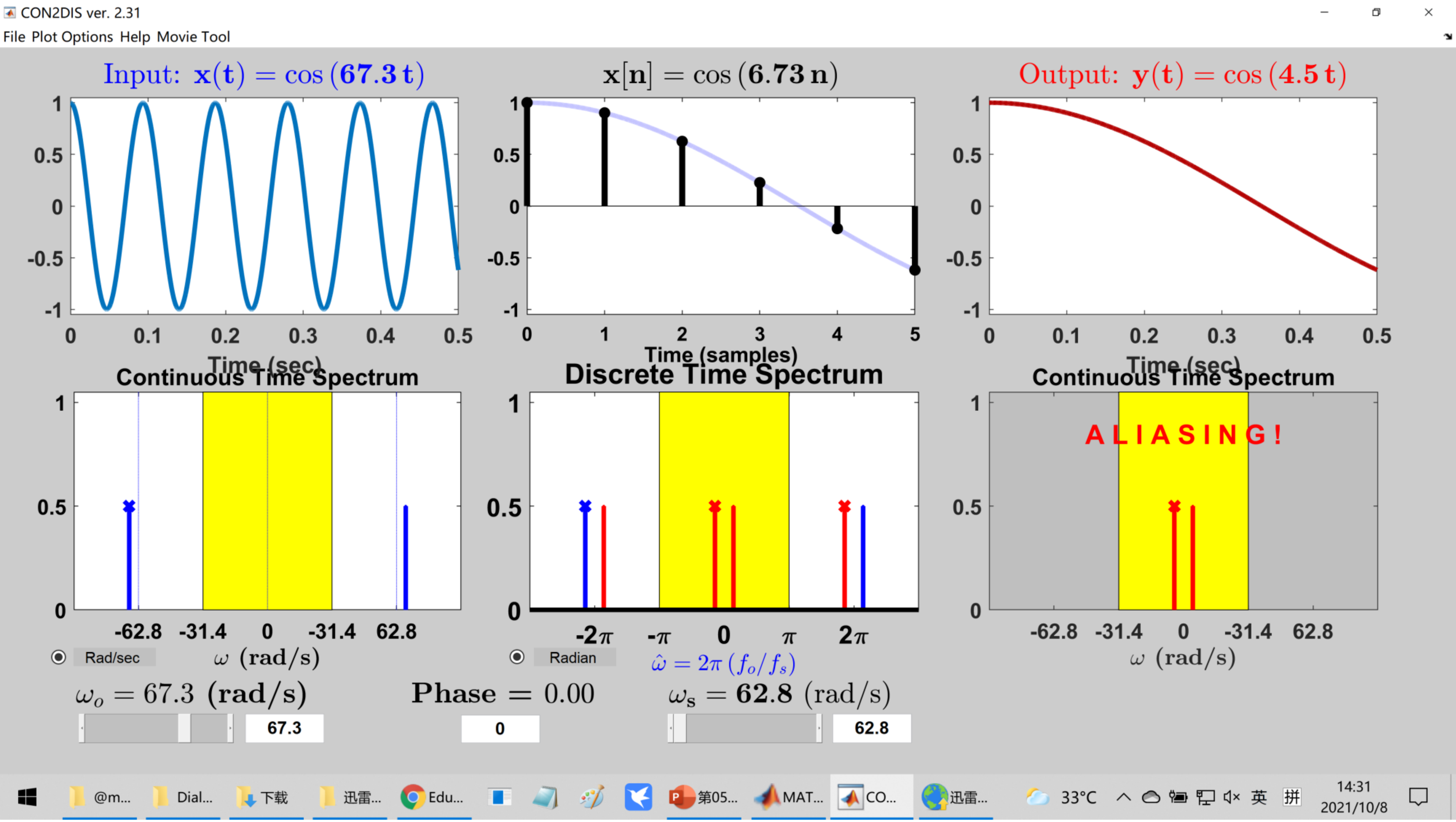
采样与采样定理


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos (67.3 t)
x[n] = cos (5.63 n)
Output: y(t) = cos (7.8 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
ALIASING!
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (fo/fs)
ω (rad/s)
ωo = 67.3 (rad/s)
67.3
Phase = 0.00
0
ωs = 75.1 (rad/s)
75.1
33°C
14:30
2021/10/8

采样与采样定理

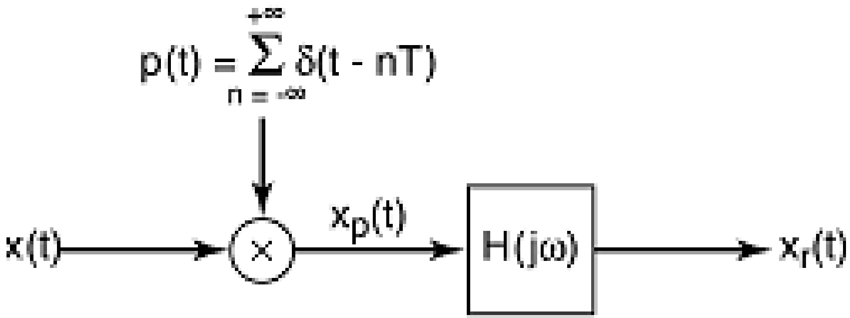

CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: $x(t) = \cos(67.3\,t)$
$x[n] = \cos(6.15\,n)$
Output: $y(t) = \cos(1.5\,t)$
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
ALIASING!
-68.8 -34.4 0 -34.4 68.8
-2π $-\pi$ 0 π 2π
Rad/sec
ω (rad/s)
Radian
$\hat{\omega} = 2\pi(f_o/f_s)$
ω (rad/s)
$\omega_o = 67.3$ (rad/s)
Phase = 0.00
$\omega_s = 68.8$ (rad/s)
67.3
0
68.8

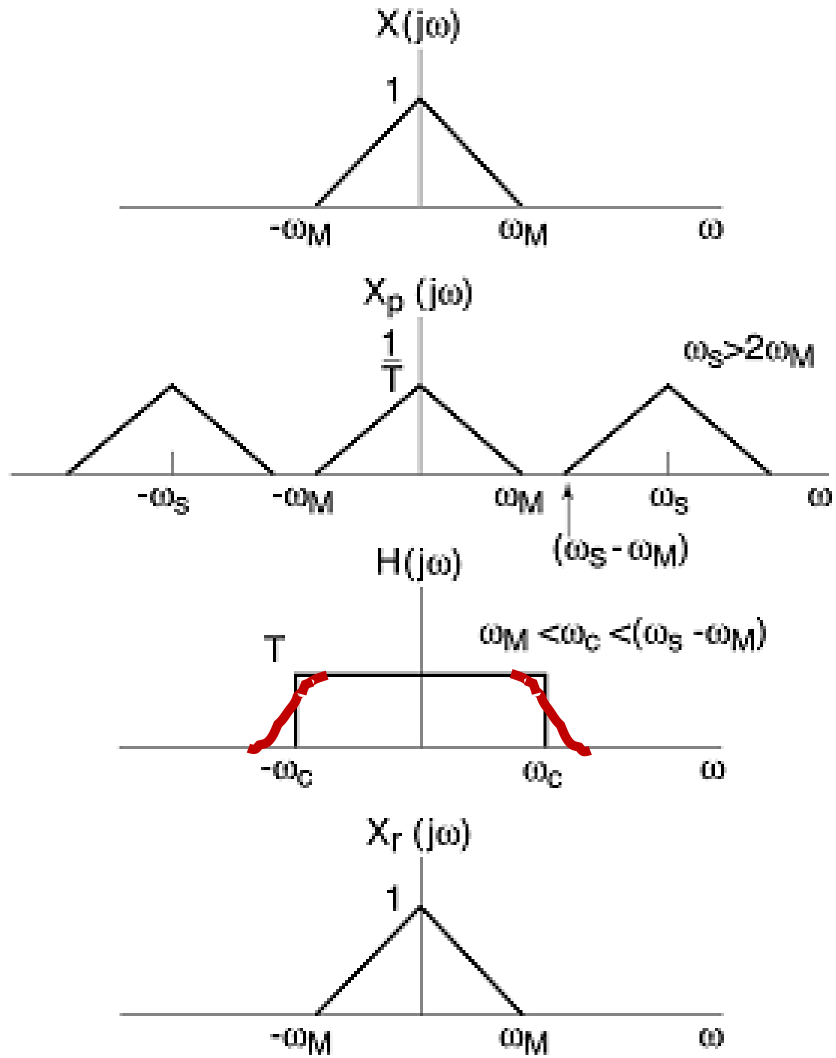
采样与采样定理


CON2DIS ver. 2.31
File Plot Options Help Movie Tool
Input: x(t) = cos(67.3 t)
x[n] = cos(6.73 n)
Output: y(t) = cos(4.5 t)
Time (sec)
Time (samples)
Time (sec)
Continuous Time Spectrum
Discrete Time Spectrum
Continuous Time Spectrum
ALIASING!
-62.8 -31.4 0 -31.4 62.8
-2π -π 0 π 2π
Rad/sec
ω (rad/s)
Radian
ω̂ = 2π (f_o/f_s)
ω (rad/s)
ω_o = 67.3 (rad/s)
67.3
Phase = 0.00
0
ω_s = 62.8 (rad/s)
62.8

采样与采样定理

➢ Nyquist采样定理





- 在工程实际应用中，理想滤波器是不可能实现的。而非理想滤波器一定有过渡带，因此，实际采样时, ω_s 必须大于$2\omega_M$。

采样与采样定理

- Nyquist采样定理

  - 通信中的信号采样

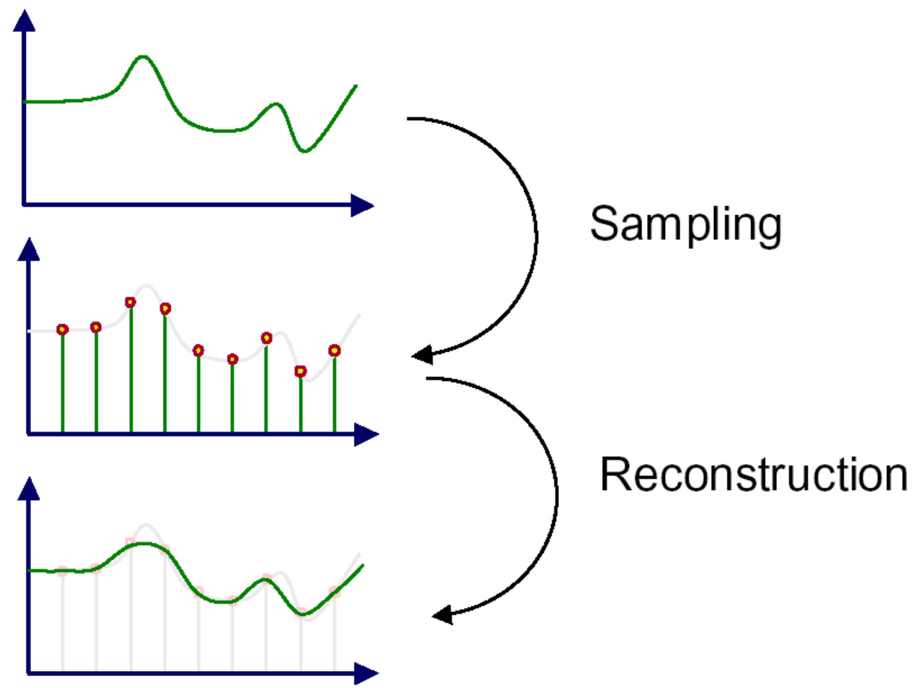
信号类型	语音信号	电话会议	自然声音	模拟电视
频带范围	300~3400Hz	50~7000Hz	0~20kHz	0~6MHz
采样频率	8kHz	16kHz	44.1kHz, 48kHz等	13.5MHz

- 根据采样定理，模拟话音采样频率 $\geq 2\times 3.4\text{kHz}$；
- CCITT规定模拟话音信号的采样频率：
  $f_s = 8kHz$ 其中有1.2kHz的保护频带。

研究连续时间信号与离散时间信号之间的关系主要包括 ：

- 在什么条件下，一个连续时间信号可以用它的离散时间样本来代替而不致丢失原有的信息？

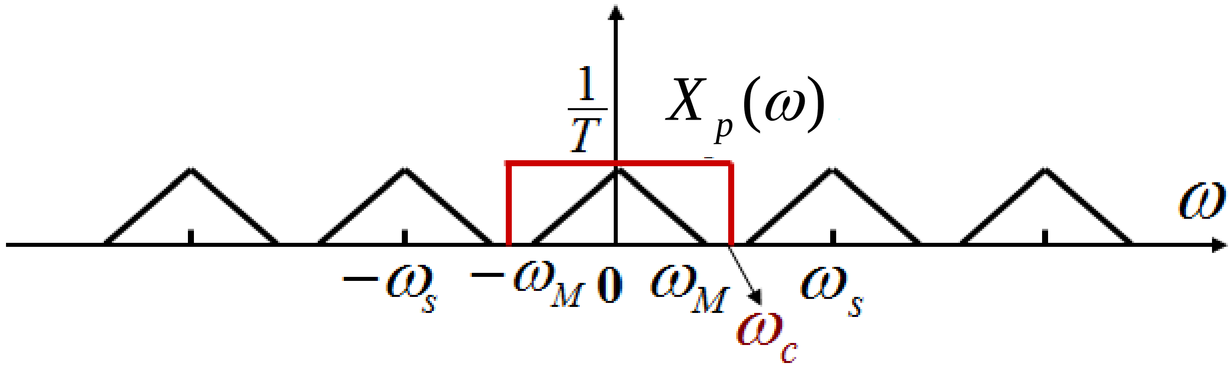
- 如何从连续时间信号的离散时间样本不失真地恢复成原来的连续时间信号？



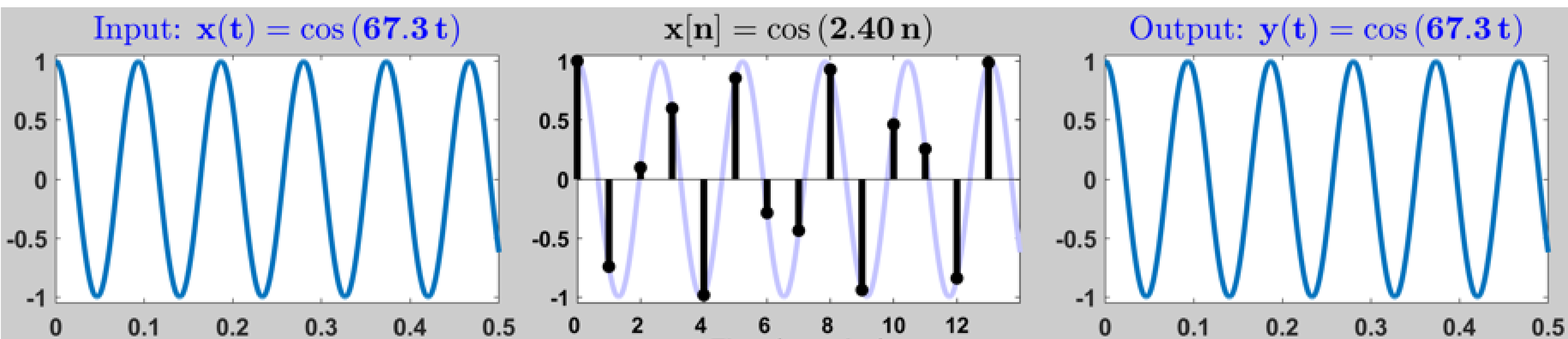
连续信号的采样重构

连续信号的采样重构

频域（分析）



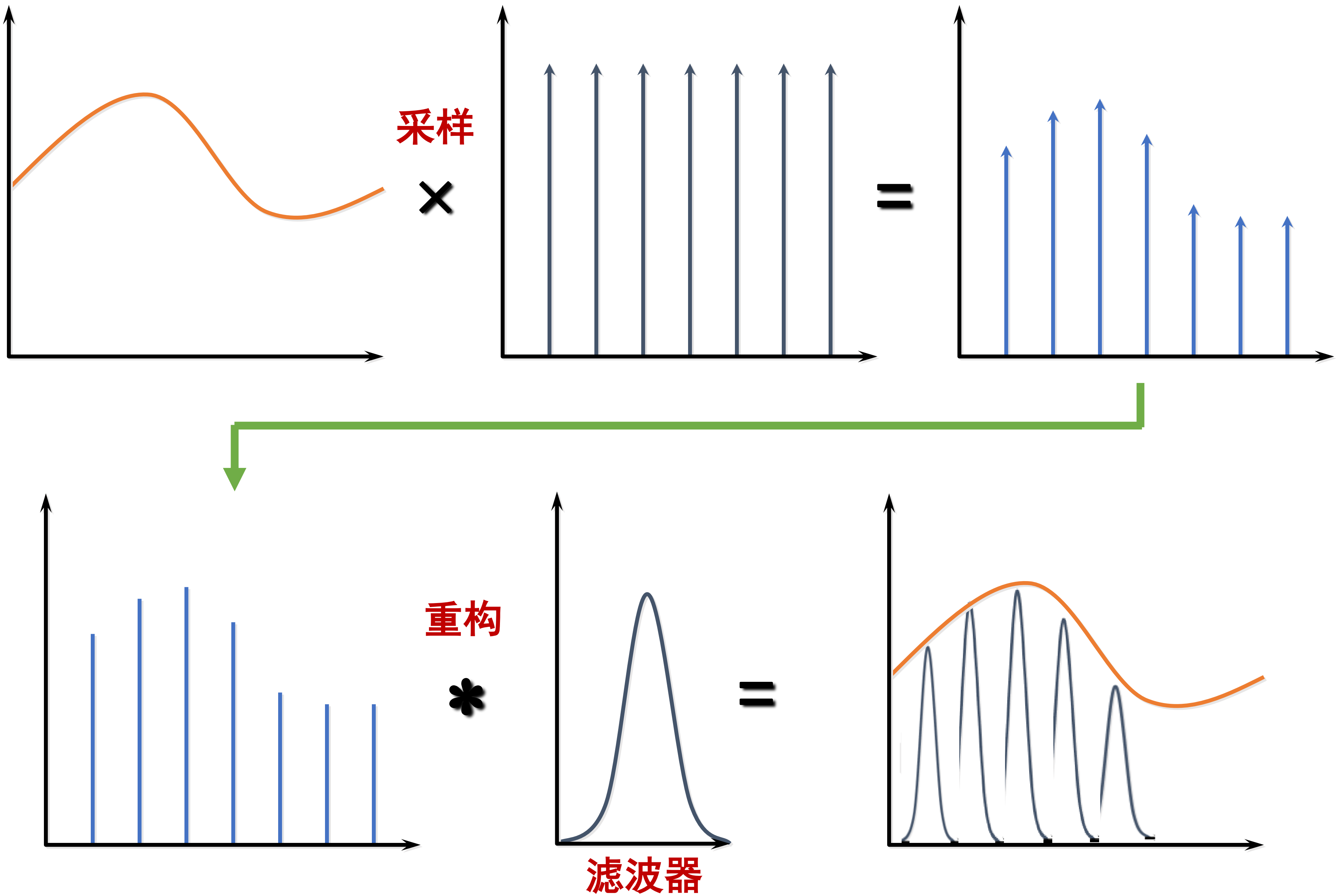
$$\omega_M < \omega_c < \omega_s - \omega_M$$



时域（实现）

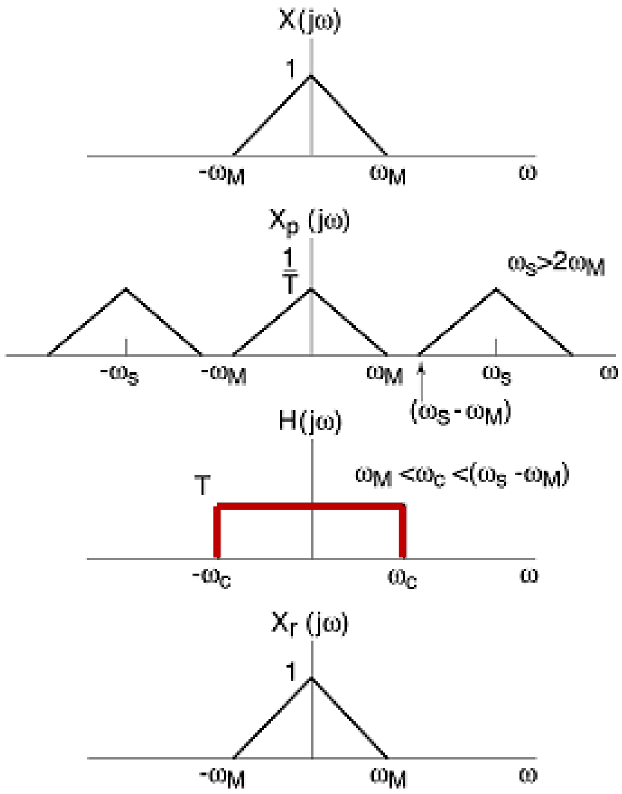
滤波器

连续信号的采样重构



连续信号的采样重构

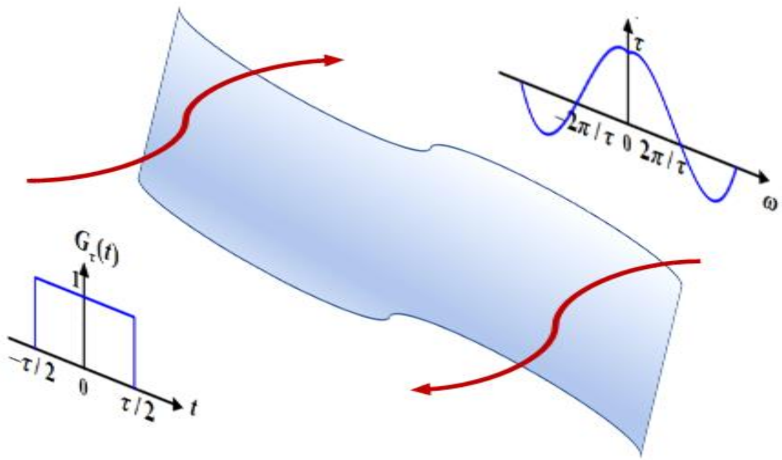
➢ 理想滤波器



以ω_c为截止频率的理想滤波器

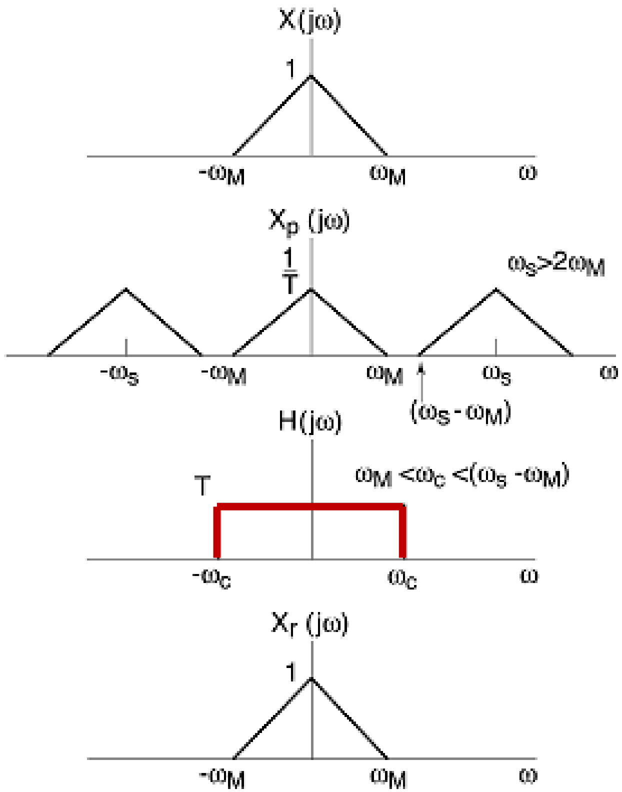
$$|H(\omega)| = \begin{cases} T_s & |\omega| \le \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

对偶性质



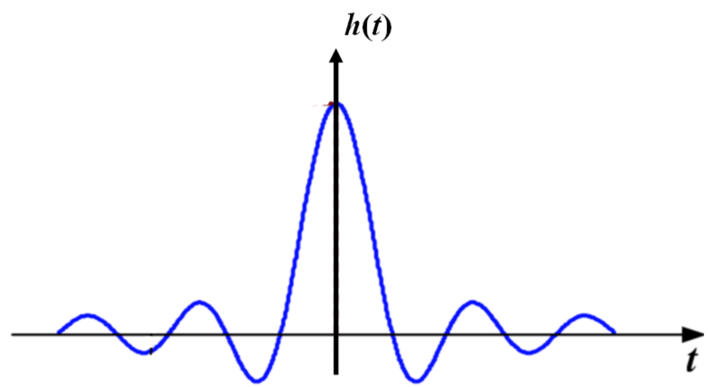
连续信号的采样重构

➢ 理想滤波器



以ω_c为截止频率的理想滤波器

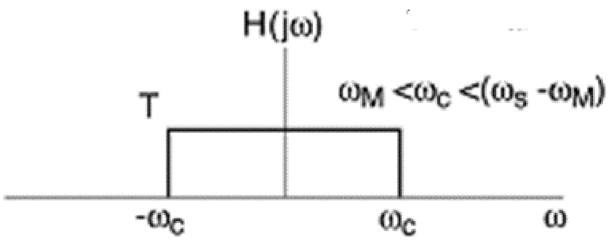
$$|H(\omega)| = \begin{cases} T_s & |\omega| \le \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$



$$h(t) = T_s \frac{\omega_c}{\pi} Sa(\omega_c t)$$

连续信号的采样重构

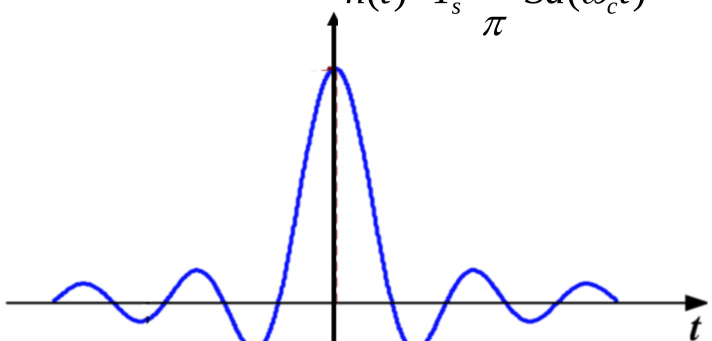
➢ 理想滤波器



$$H(\omega) \sim h(t)$$

$$1 \cdot H(\omega) \sim \delta(t) * h(t)$$

$$h(t)=T_s \frac{\omega_c}{\pi} Sa(\omega_c t)$$

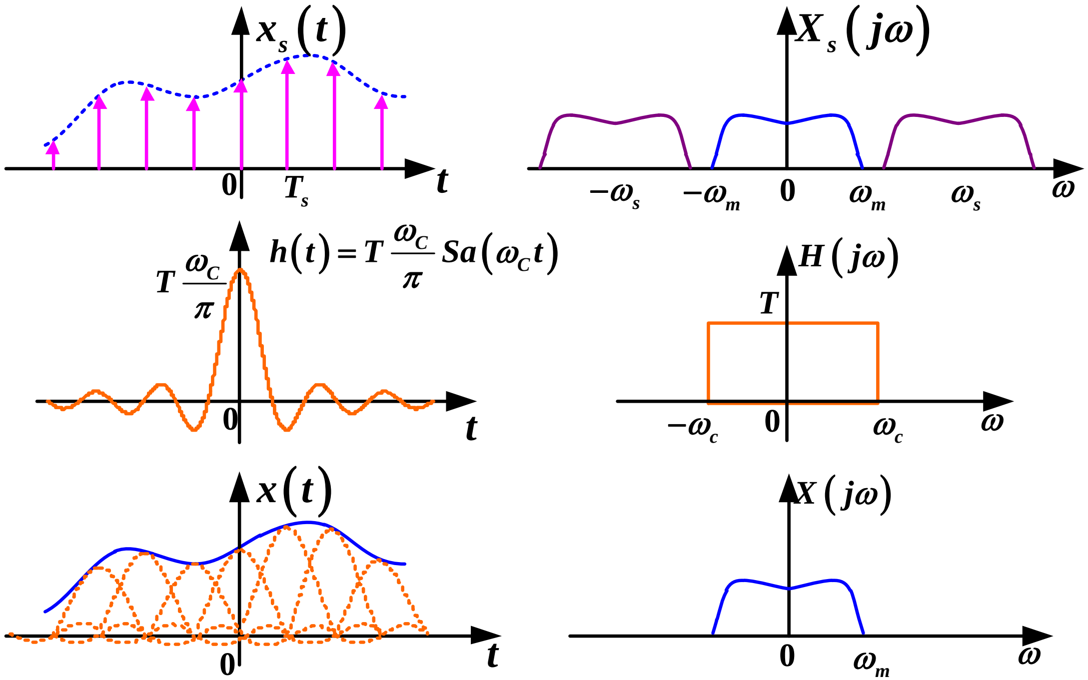


信号	系统	新信号
$f(t)$	$g(t)$	$q(t)$
$\delta(t)$		$h(t)$

$h(t)$可视为理想滤波器对应单位脉冲响应

连续信号的采样重构

- 内插：由样本值重建信号



$$x_p(t)=\sum_{n=-\infty}^{\infty}x(nT_s)\delta(t-nT_s)$$

$$h(t)=T_s\frac{\omega_c}{\pi}Sa(\omega_c t)$$

$$\hat{x}(t)=x_p(t)*h(t)=\sum_{n=-\infty}^{\infty}x(nT_s)\delta(t-nT_s)*h(t)$$

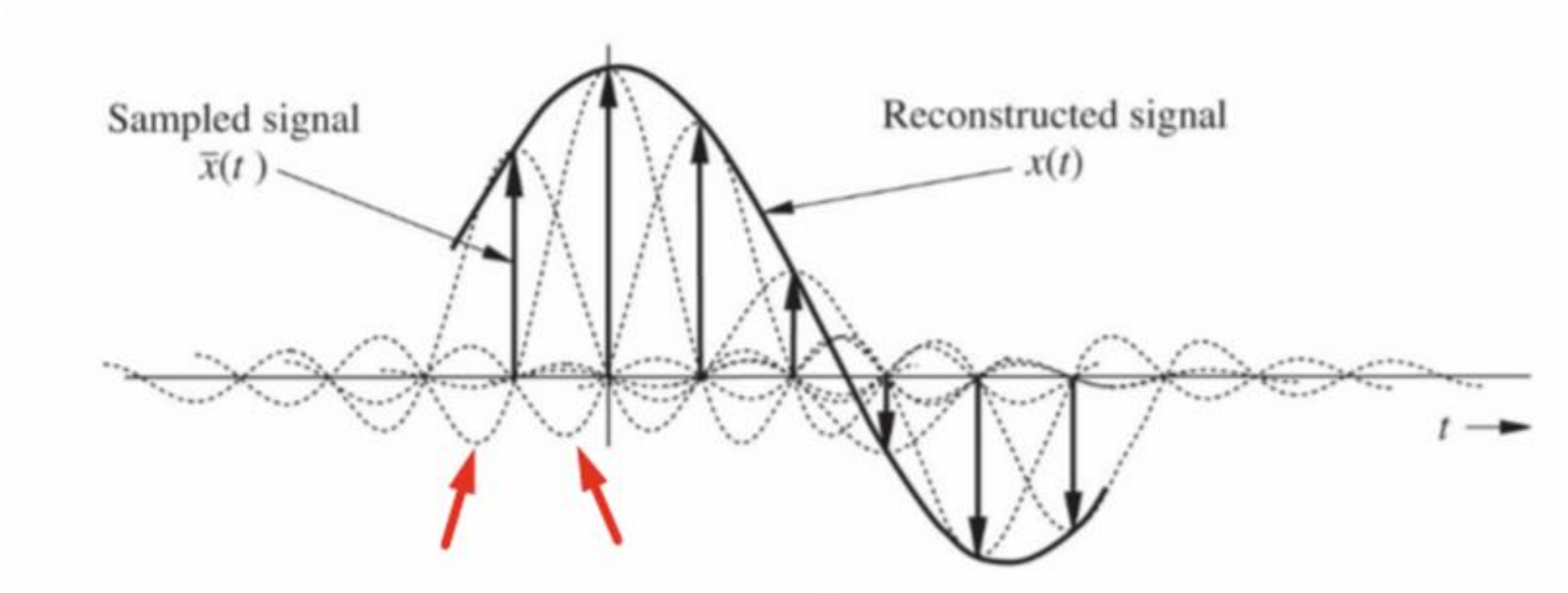
$$\hat{x}(t)=\frac{\omega_c T_s}{\pi}\sum_{n=-\infty}^{+\infty}x(nT_s)Sa(\omega_c(t-nT_s))$$

用滤波器函数对信号采样值进行内插来重建连续信号，相当于滤波器的脉冲响应与信号为权重的脉冲串的卷积。

连续信号的采样重构

➢ 内插：由样本值重建信号

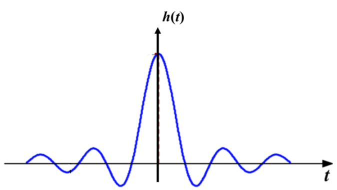
$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

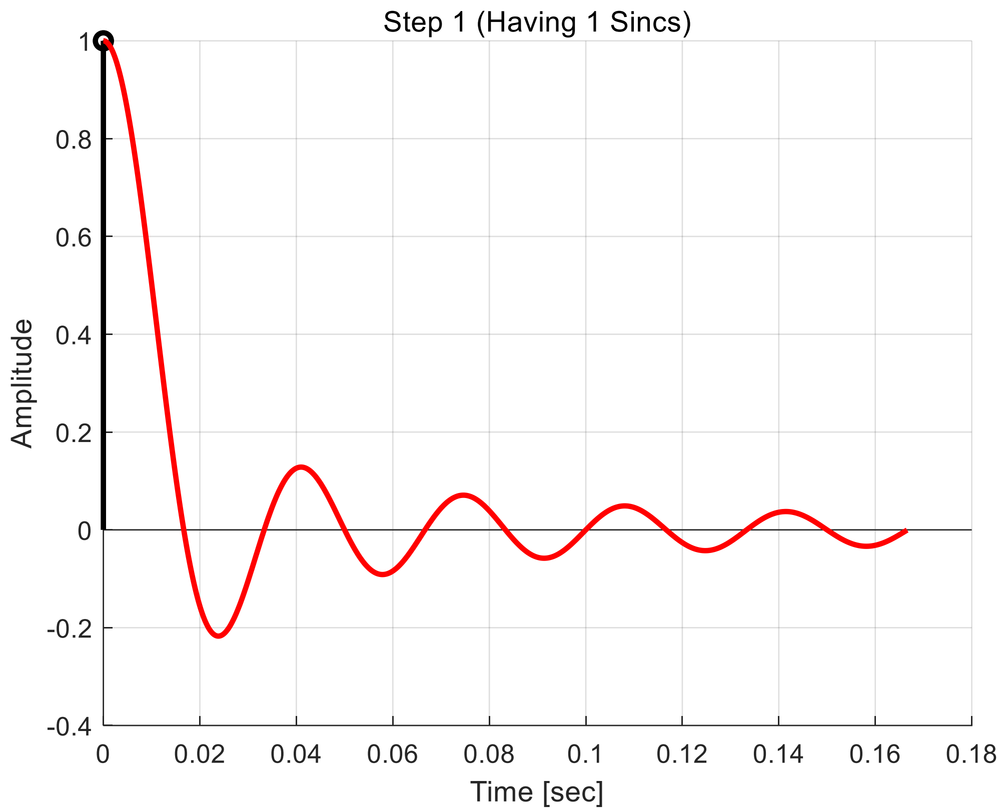


用滤波器函数对信号采样值进行内插来重建连续信号，相当于滤波器的冲击响应与信号为权重的脉冲串的卷积。

连续信号的采样重构

➢ 内插：由样本值重建信号

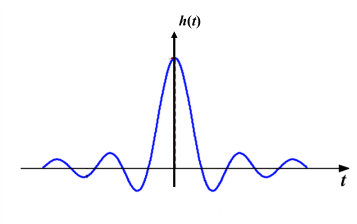


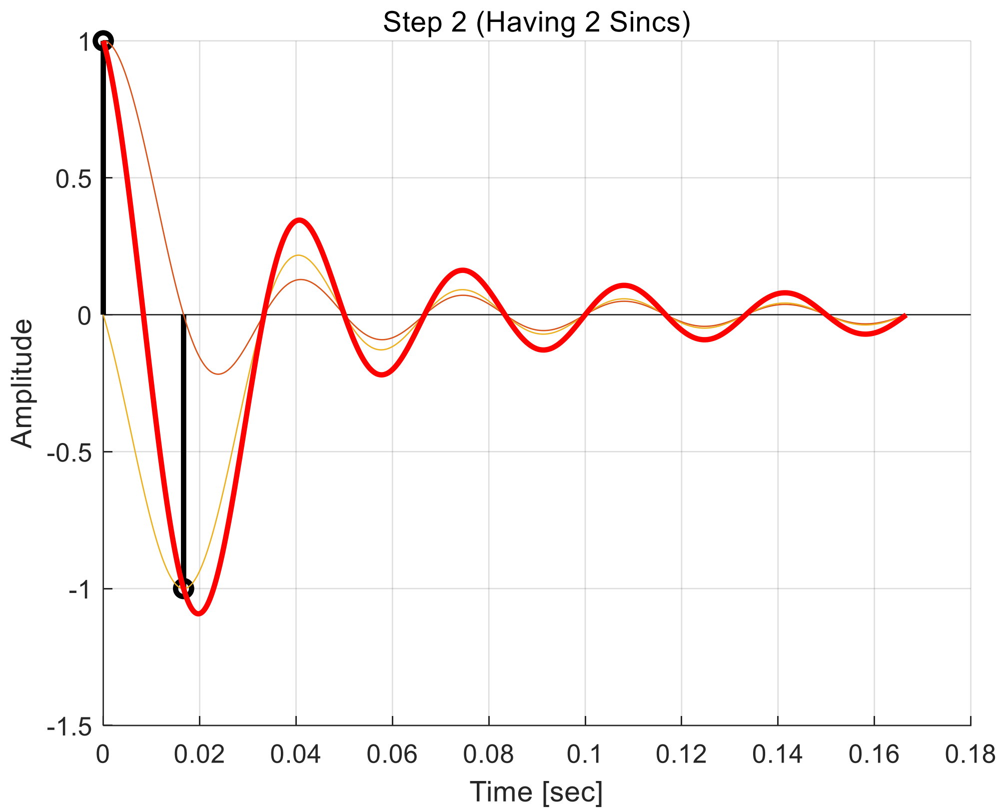


$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

连续信号的采样重构

➢ 内插：由样本值重建信号

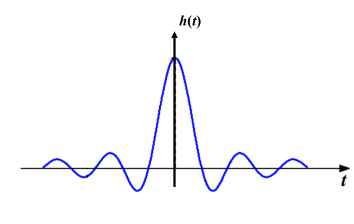


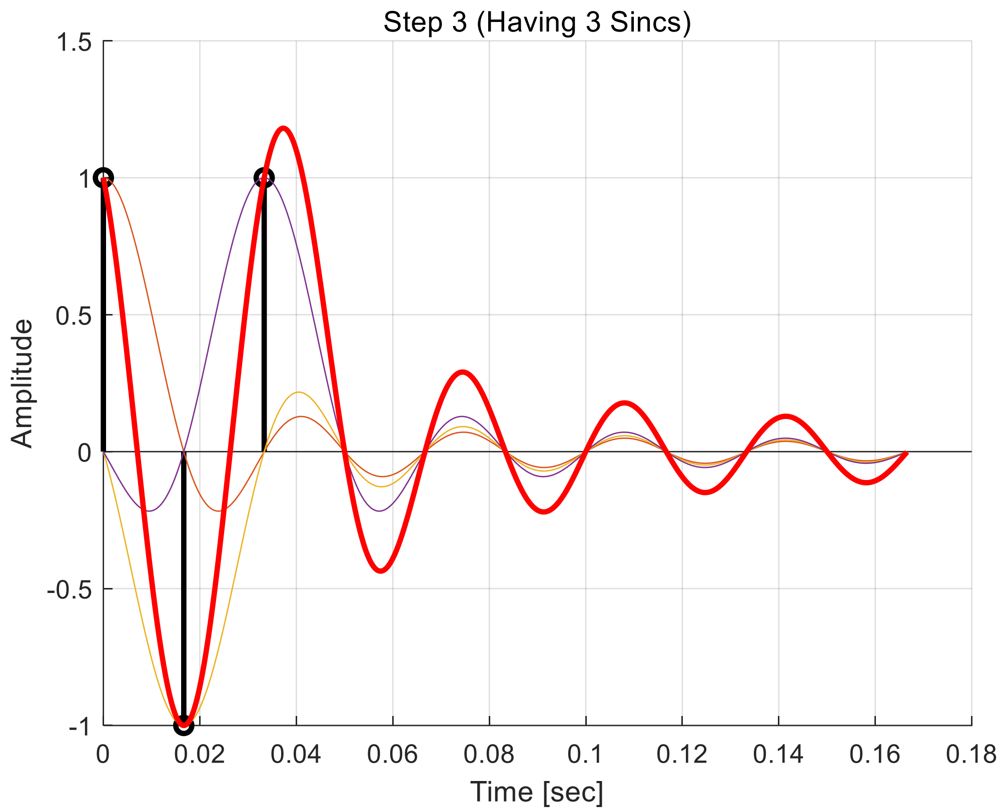


$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

连续信号的采样重构

- 内插：由样本值重建信号

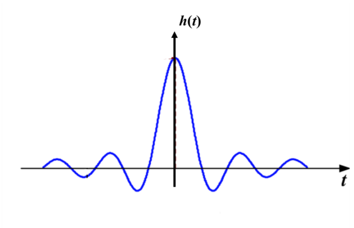


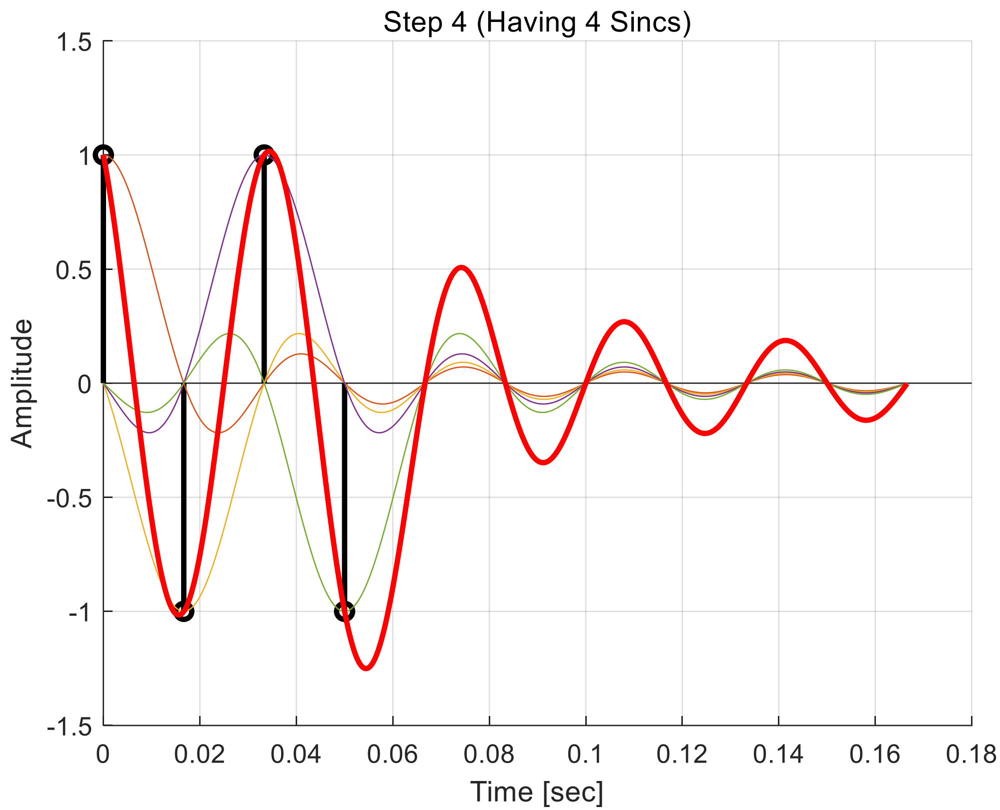


$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

连续信号的采样重构

➢ 内插：由样本值重建信号

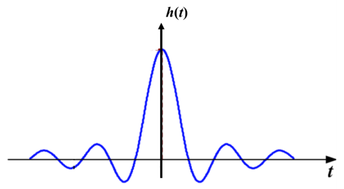


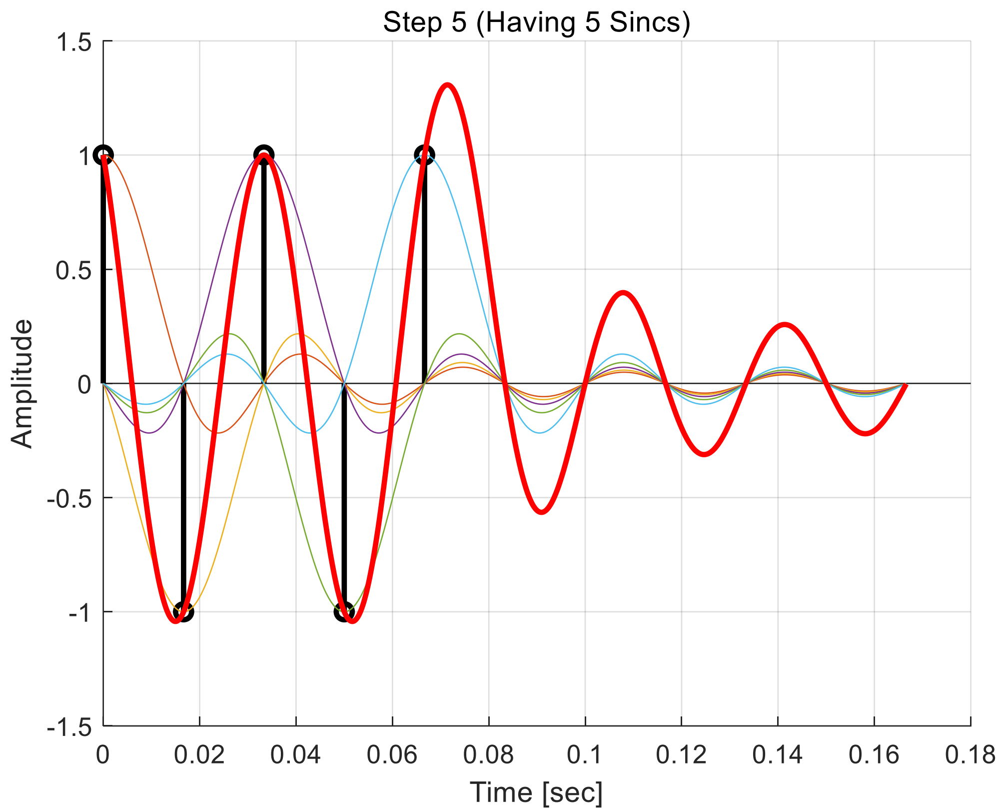


$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

连续信号的采样重构

➢ 内插：由样本值重建信号

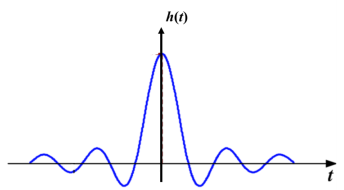


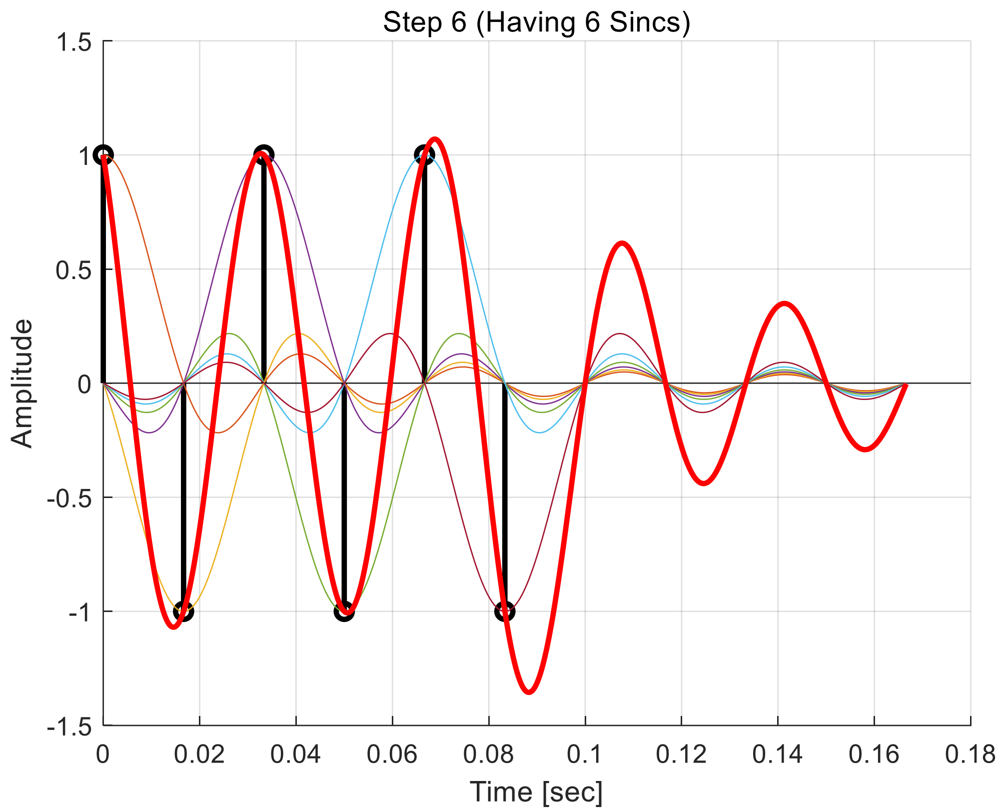


$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

连续信号的采样重构

➢ 内插：由样本值重建信号

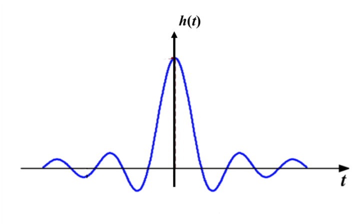


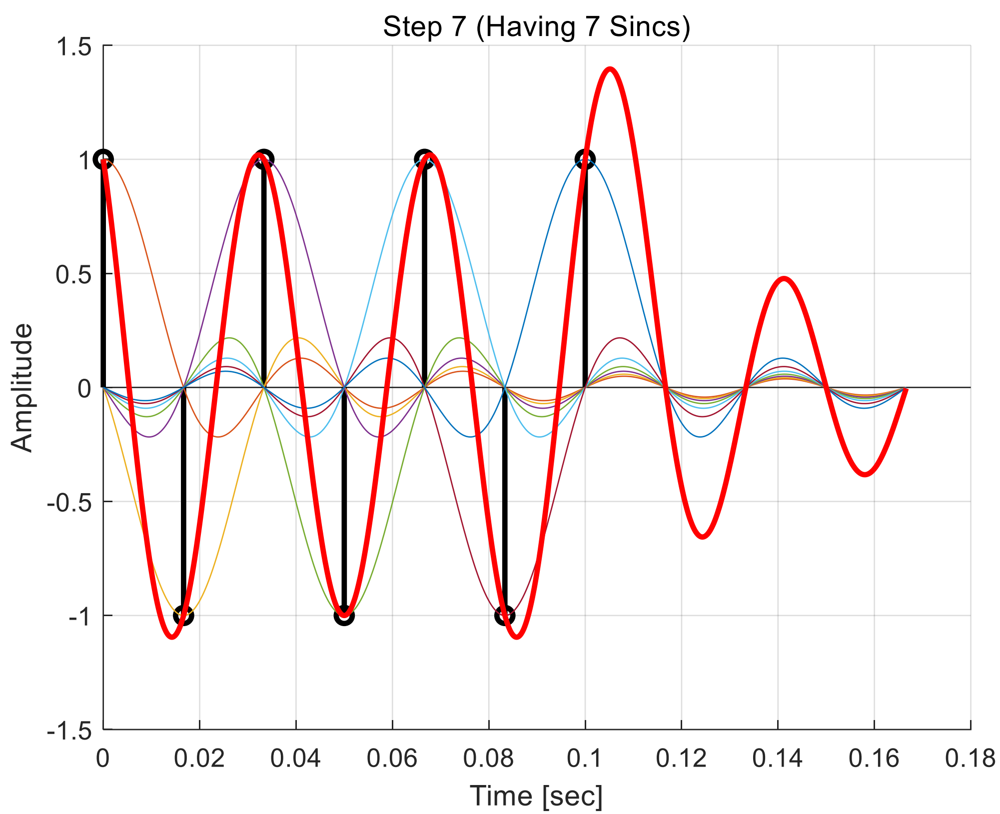


$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

连续信号的采样重构

➢ 内插：由样本值重建信号

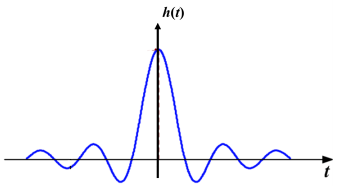


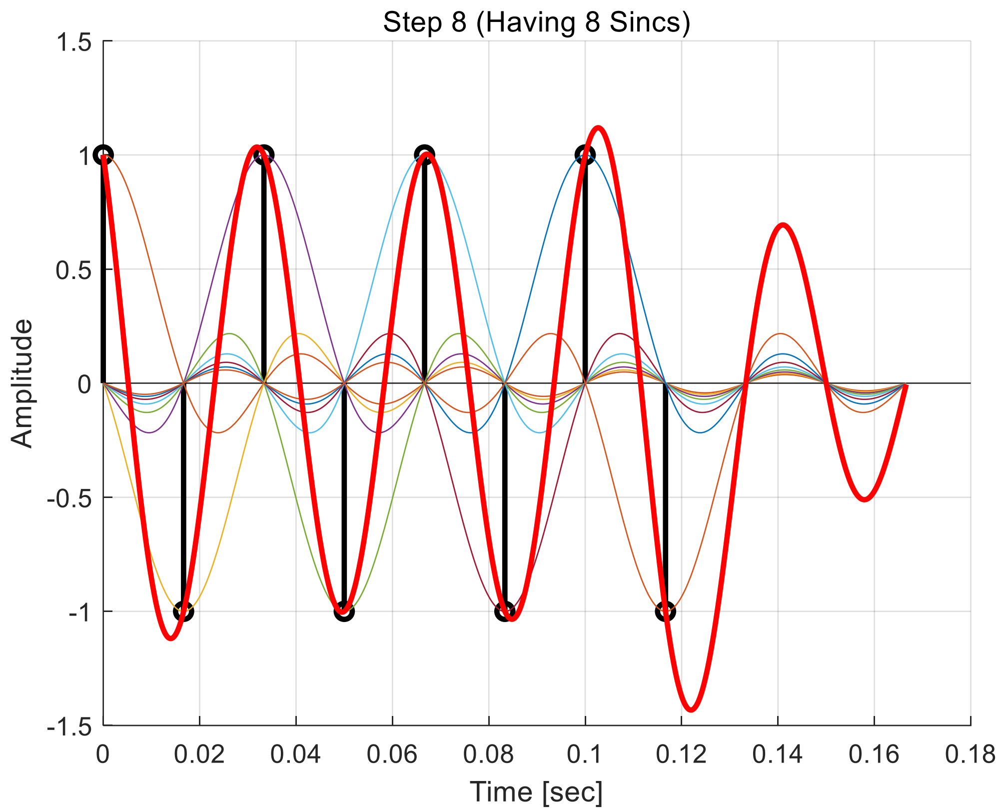


$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

连续信号的采样重构

➤ 内插：由样本值重建信号

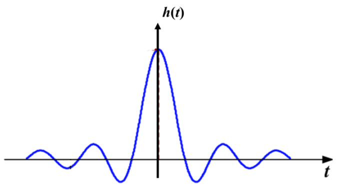


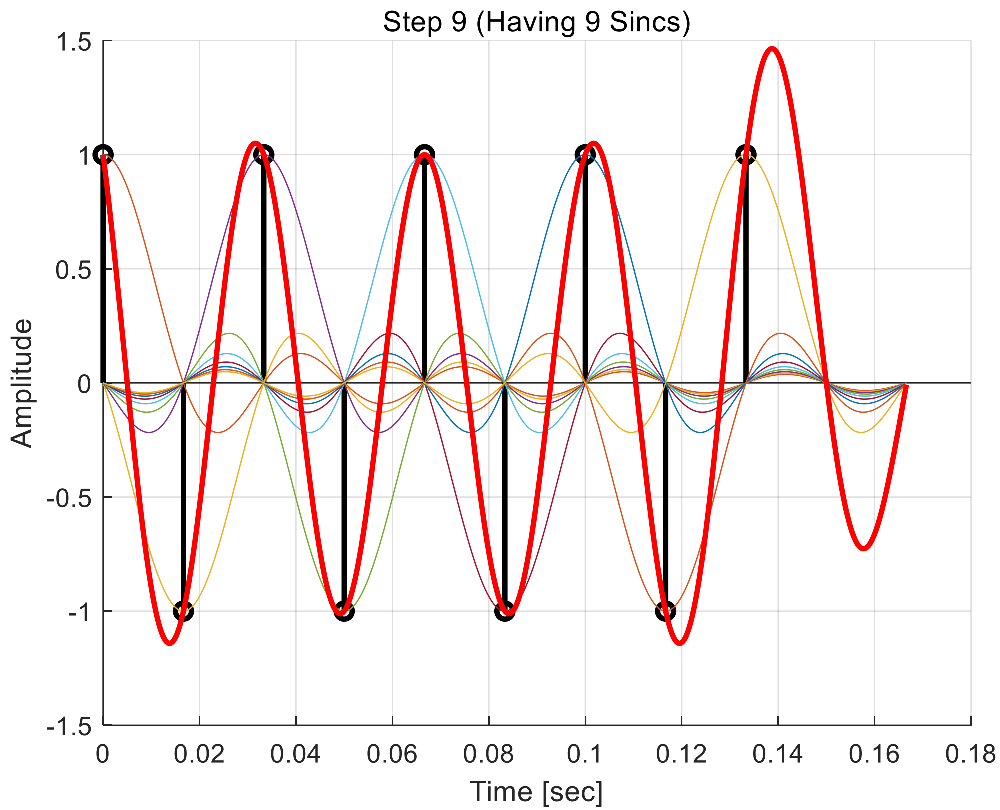


$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

连续信号的采样重构

➢ 内插：由样本值重建信号

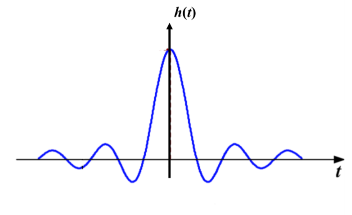


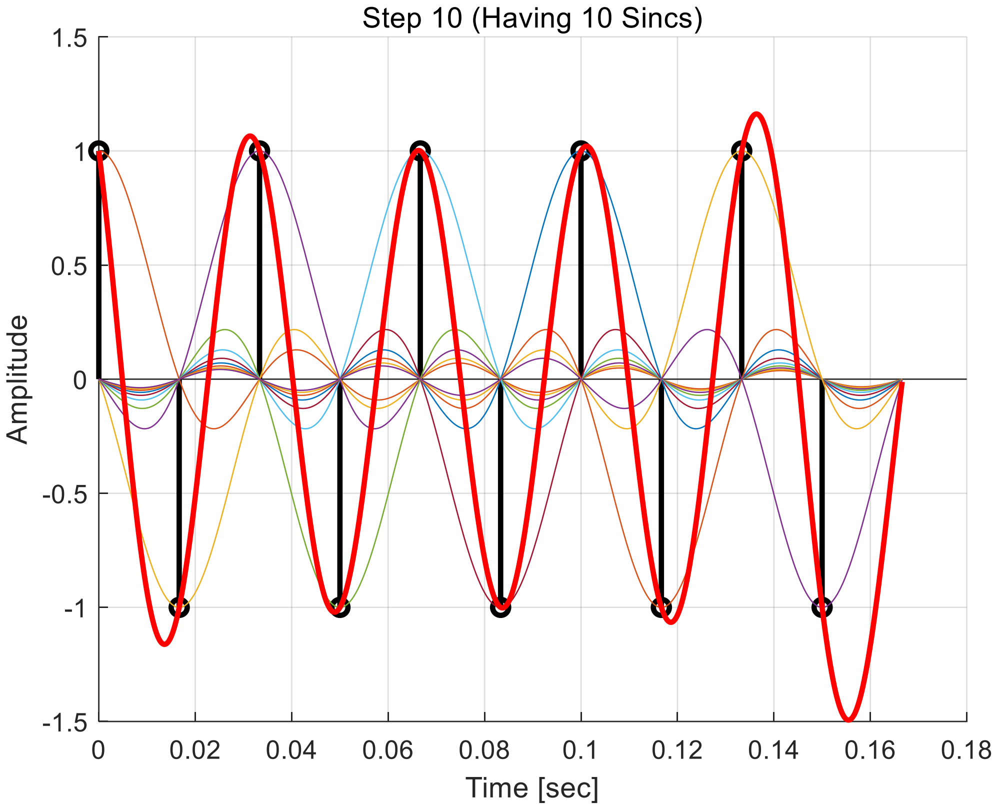


$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

连续信号的采样重构

➢ 内插：由样本值重建信号

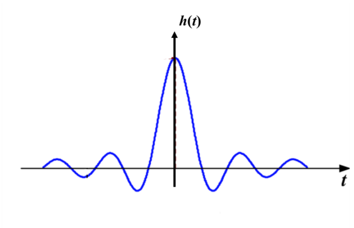


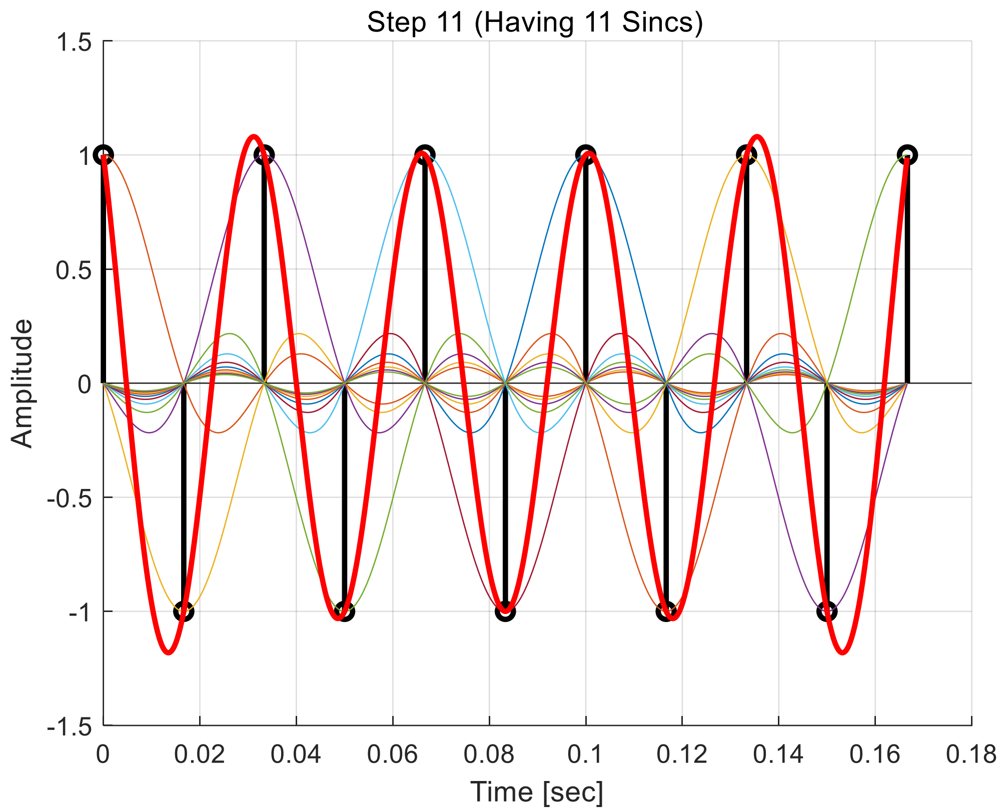


$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

连续信号的采样重构

- 内插：由样本值重建信号

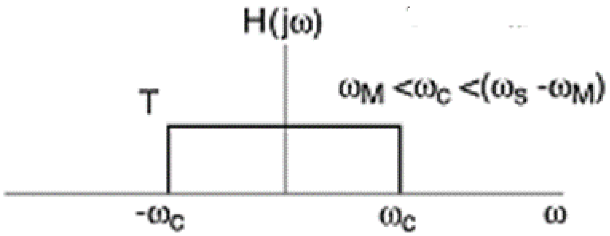


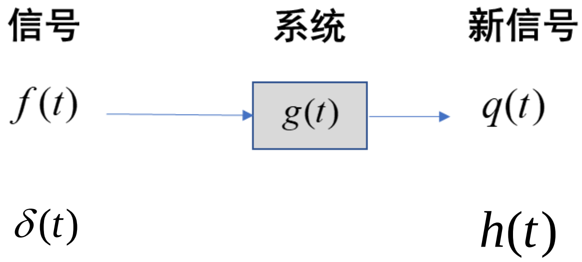


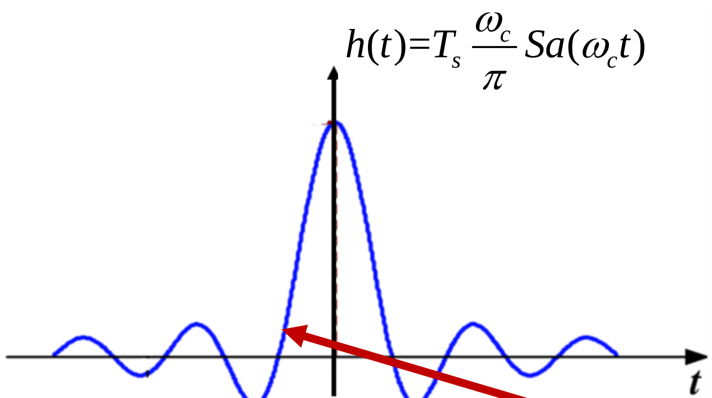
$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

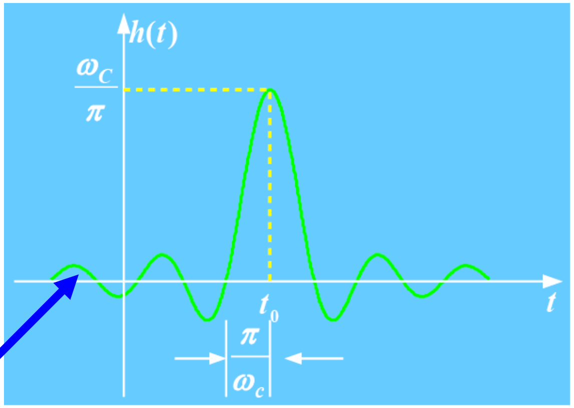
连续信号的采样重构

➢ 理想滤波器的可实现性









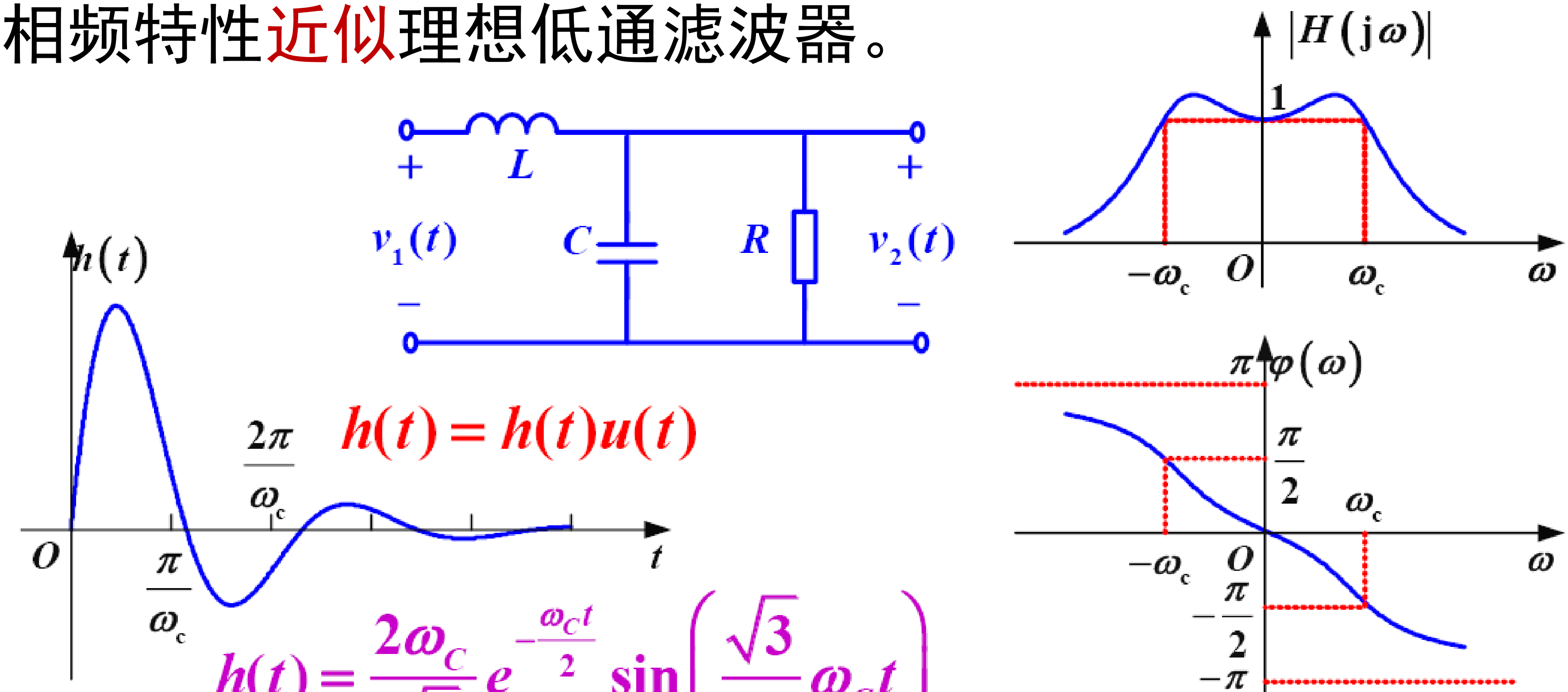
非因果的

$h(t)$可视为理想滤波器对应单位脉冲响应

连续信号的采样重构

➢ 理想滤波器的近似实现

当网络满足 $R=\sqrt{L/C}$，且令 $\omega_c=\dfrac{1}{\sqrt{LC}}$，其幅频特性与相频特性近似理想低通滤波器。



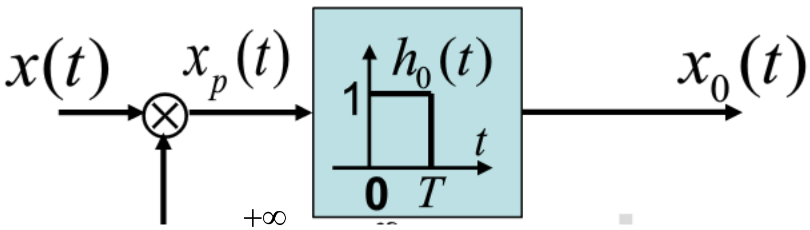
$$h(t)=h(t)u(t)$$

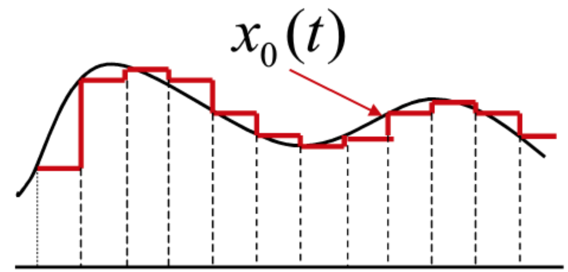
$$h(t)=\frac{2\omega_C}{\sqrt{3}}e^{-\frac{\omega_C t}{2}}\sin\left(\frac{\sqrt{3}}{2}\omega_C t\right)$$

连续信号的采样重构

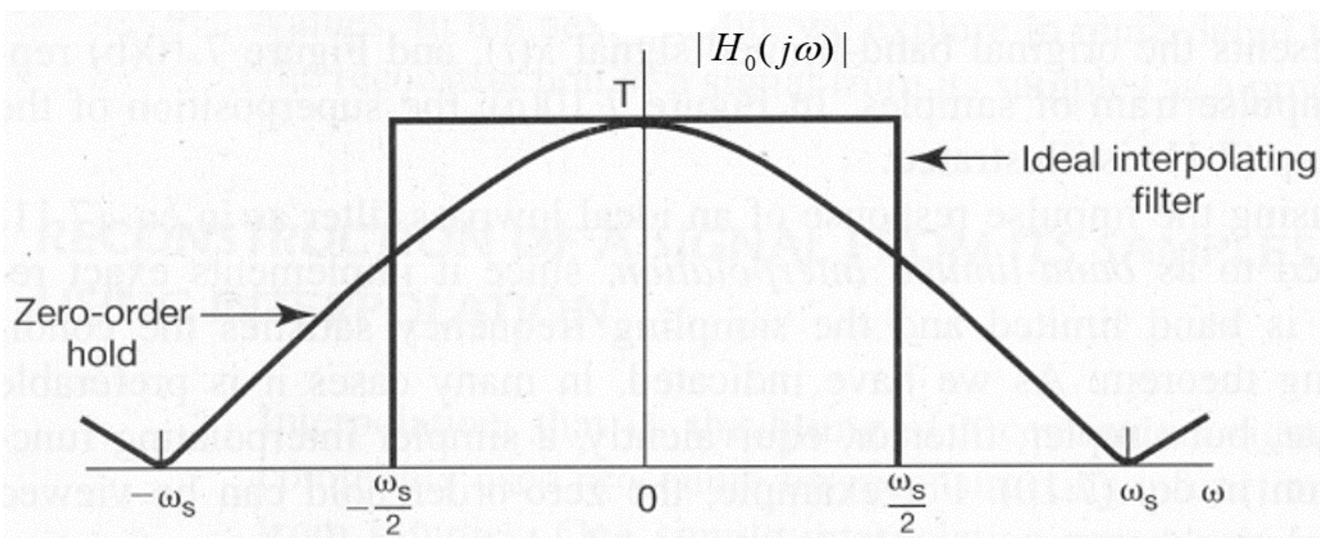
➢ 零阶保持插值

零阶保持内插的内插函数 $h_0(t)$ 是矩形脉冲



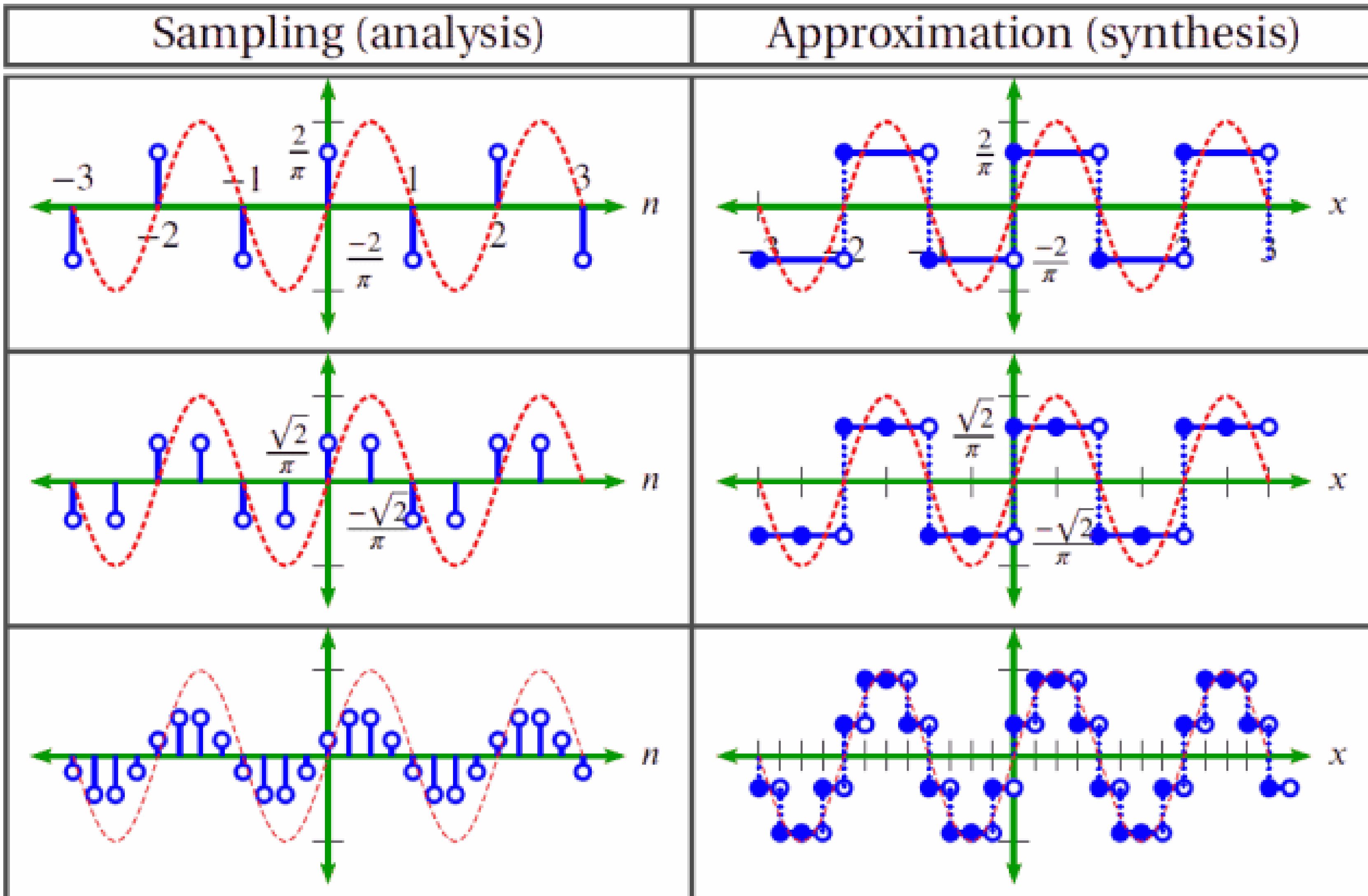


$$p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)$$



连续信号的采样重构

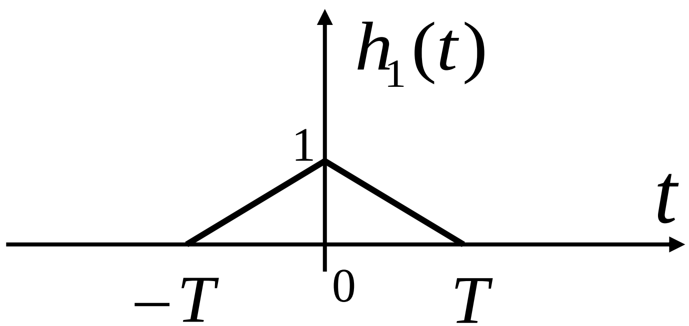
- ➢ 零阶保持插值

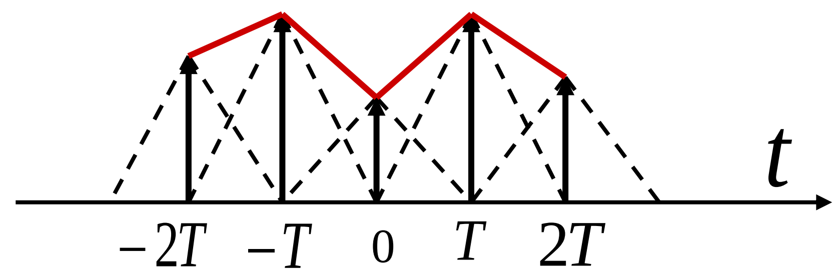


连续信号的采样重构

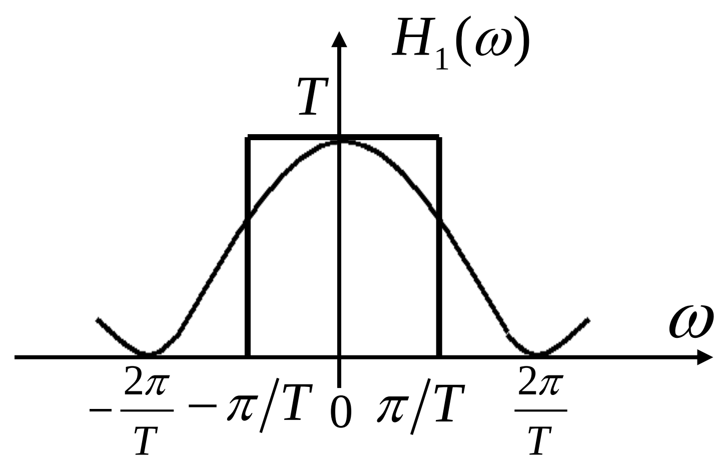
➢ 一阶保持插值（线性内插）

内插函数为三角形脉冲





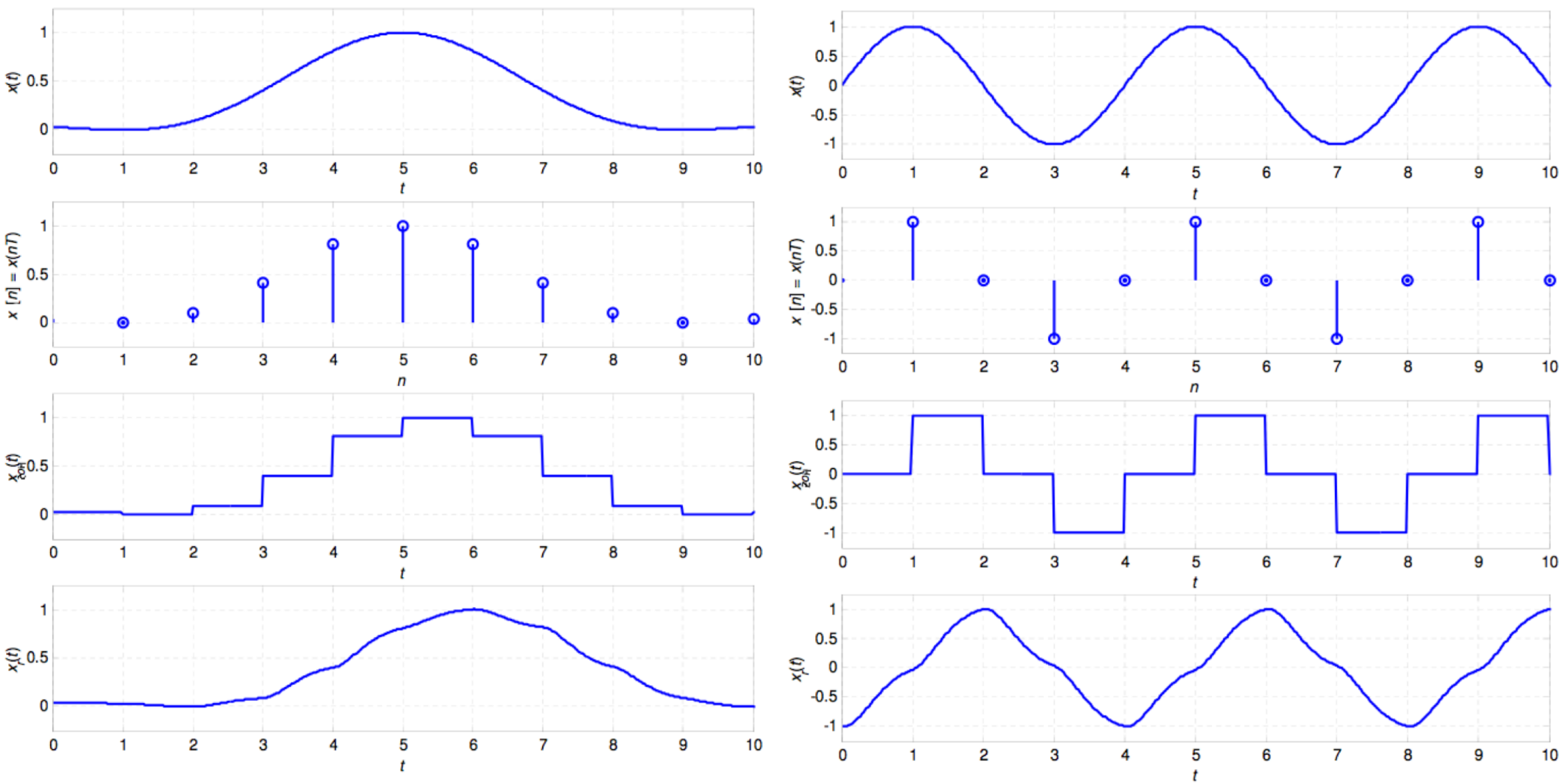
$$H_1(\omega) = T\left[\frac{\mathrm{Sin}\,\frac{\omega T}{2}}{\omega T/2}\right]^2 = \frac{1}{T}\left[\frac{\mathrm{Sin}\,\frac{\omega T}{2}}{\omega/2}\right]^2$$



需要延迟1步才能实现插值

连续信号的采样重构

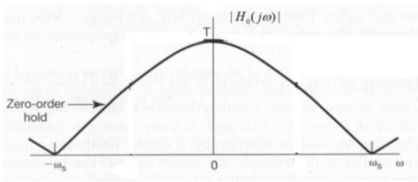
➢ 内插

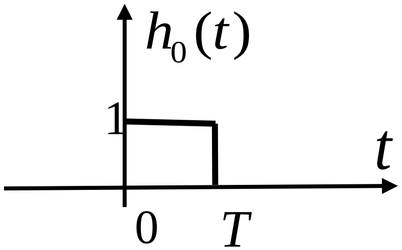


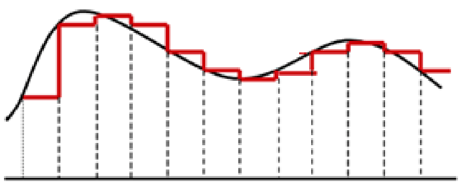
连续信号的采样重构 ★★★★

➢ 保持比较

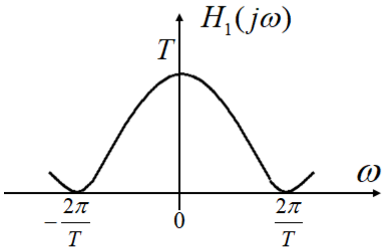
零阶

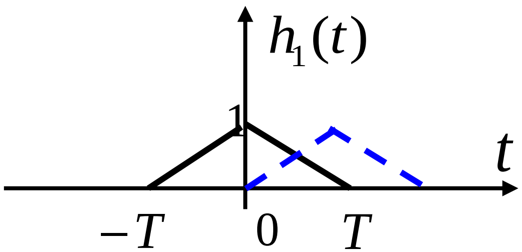


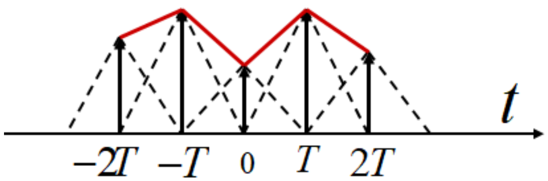




一阶

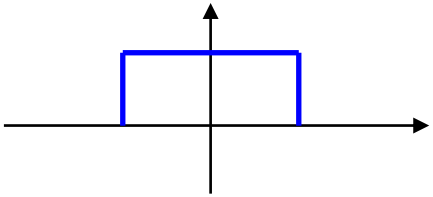


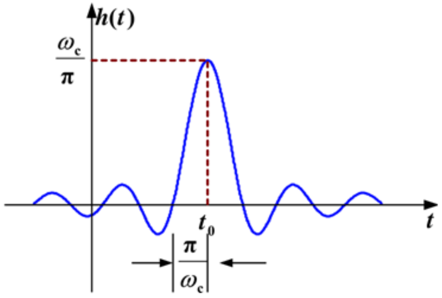


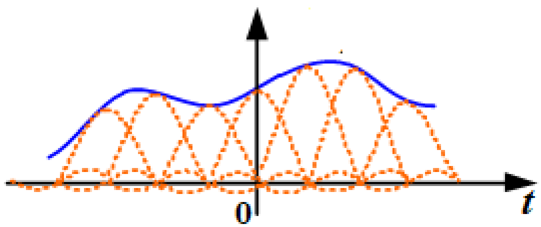


理想

∞阶







连续信号的采样重构

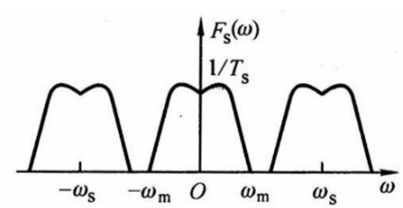
➢ 欠采样

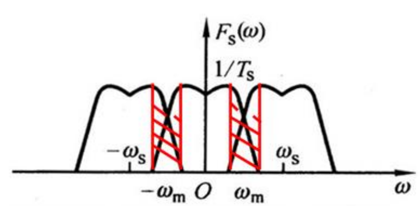
如果采样时，不满足采样定理的要求，就一定会在 $x(t)$ 的频谱周期延拓时，出现频谱混叠的现象。

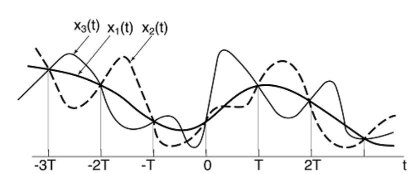
说明点一：

- 频谱混叠的情况下时域信号变了，但采样点处取值不变
- 此时，即使通过理想内插也得不到原信号。但是无论怎样，恢复所得的信号 $x_r(t)$ 与原信号 $x(t)$ 在采样点上将具有相同的值，即

$$x_r(nT) = x(nT)$$





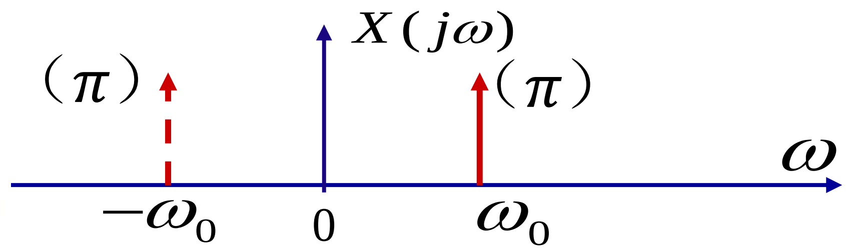


连续信号的采样重构

➢ 欠采样

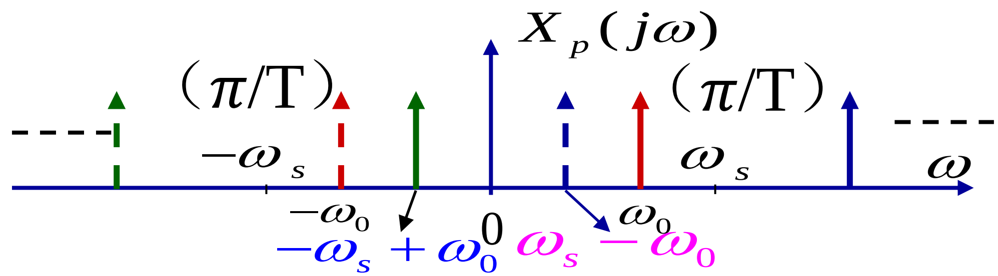
$$x(t) = \cos \omega_0 t$$

$$\mathscr{F}[\cos \omega_0 t] = \mathscr{F}\left[\frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}\right] = \pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$$



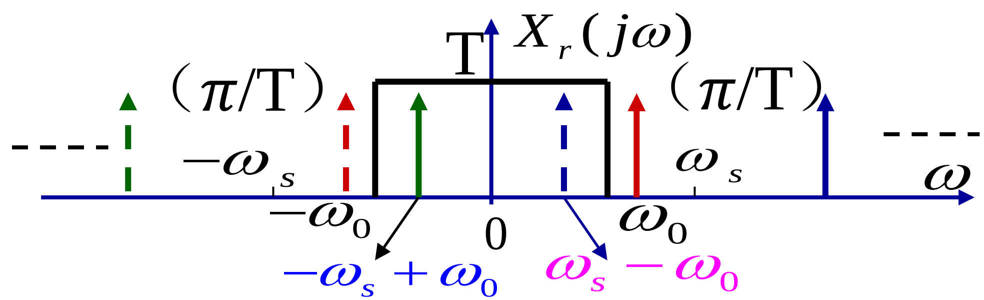
当 $\omega_0 < \omega_s < 2\omega_0$ 时,

产生频谱混叠。

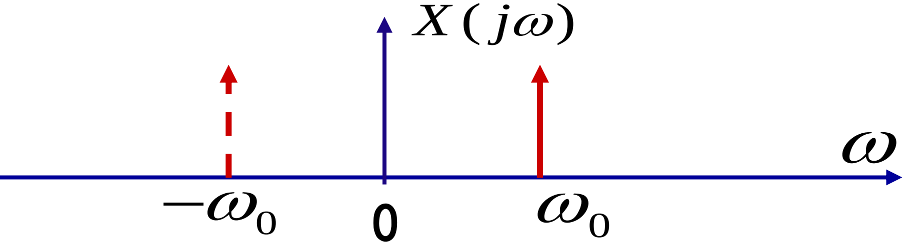


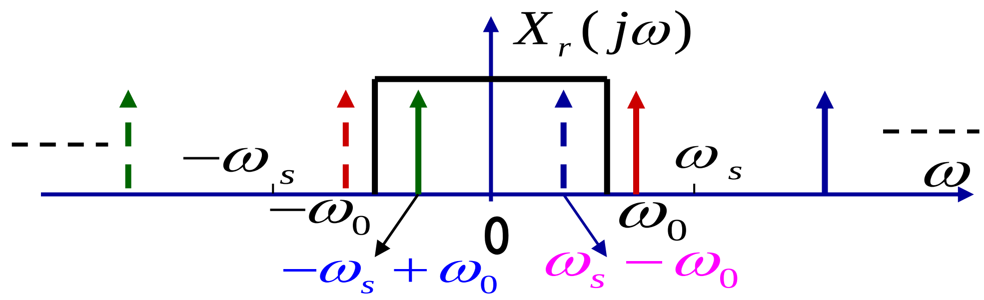
恢复的信号为

$$x_r(t) = \cos(\omega_s - \omega_0)t$$



- 欠采样





$$x(t)=\cos\omega_0 t$$

$$x_r(t)=\cos(\omega_s-\omega_0)t$$

$$t=nT$$

$$x_r(nT)=\cos(\omega_s-\omega_0)nT$$

$$=\cos\omega_s nT\cdot\cos\omega_0 nT+\sin\omega_s nT\cdot\sin\omega_0 nT$$

$$=\cos\omega_0 nT=x(nT)$$

$$\omega_s=2\pi/T$$

连续信号的采样重构

➢ 欠采样

说明点二：工程应用时，如果采样频率$\omega_s = 2\omega_M$将不足以从样本恢复原信号

例如　$x(t) = \cos(\omega_0 t + \varphi)$ 在 $\omega_s = \dfrac{2\pi}{T} = 2\omega_0$ 时

$$x(t) = \cos\varphi\cos\omega_0 t - \underline{\sin\omega_0 t\sin\varphi}$$

$$\sin\omega_0 t = \sin n\omega_0 T = \sin(2\pi n) = 0$$

$$x(nT) = \cos\varphi\cos\omega_0 nT$$

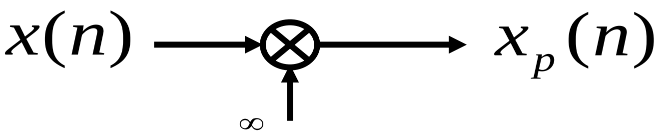
这和对 $x_1(t) = \cos\varphi\cos\omega_0 t$ 采样的结果一样。

从用样本代替信号的角度出发，出现欠采样的情况是工程应用中不希望的。

连续信号的采样重构

➢ 欠采样

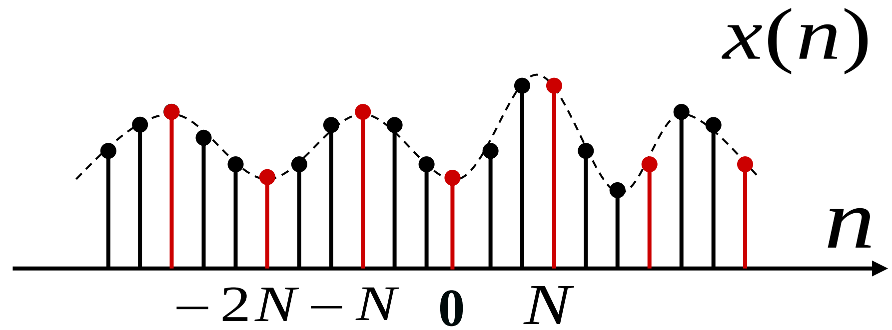
说明点三：采样之后又采样？

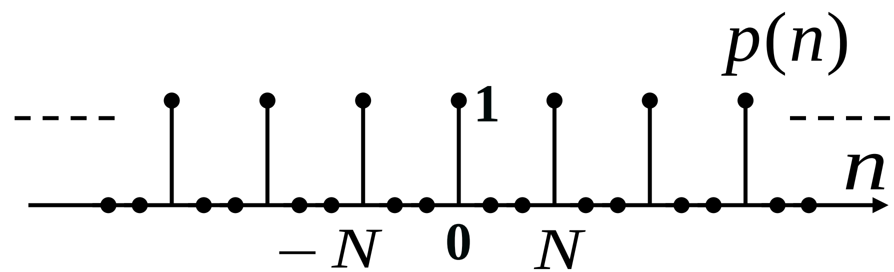


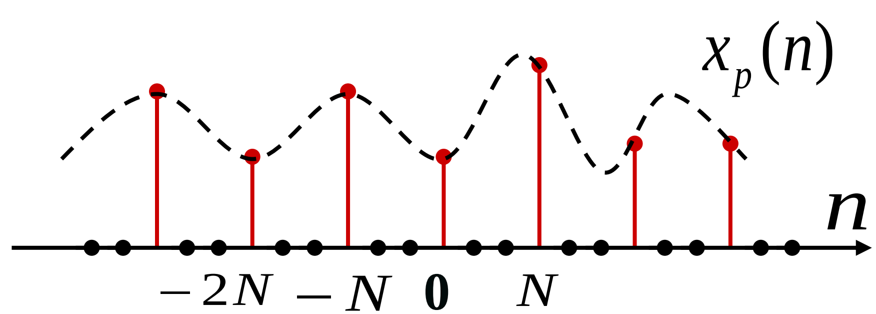
$$p(n)=\sum_{k=-\infty}^{\infty}\delta(n-kN)$$

$$x_p(n)=x(n)\cdot p(n)$$
$$=\sum_{k=-\infty}^{\infty}x(kN)\delta(n-kN)$$

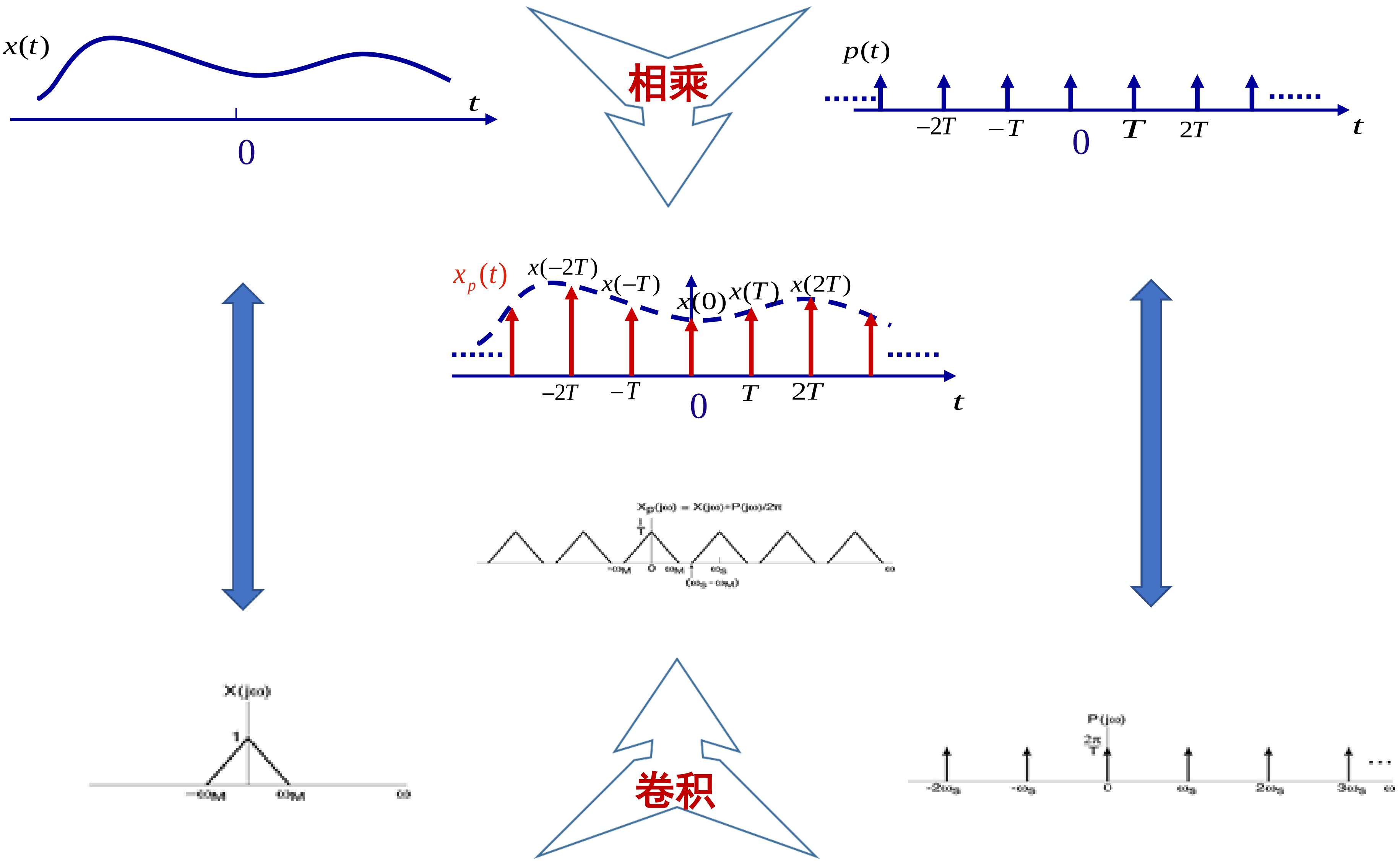
$$P(\omega)=\frac{2\pi}{N}\sum_{k=-\infty}^{\infty}\delta(\omega-\frac{2\pi}{N}k)$$







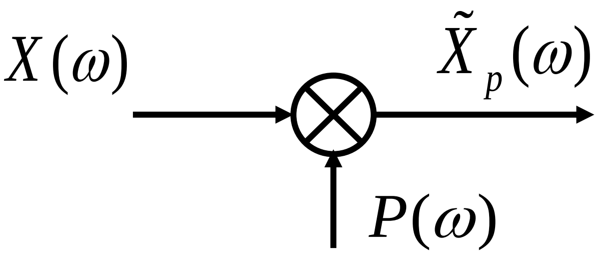
连续信号的采样重构

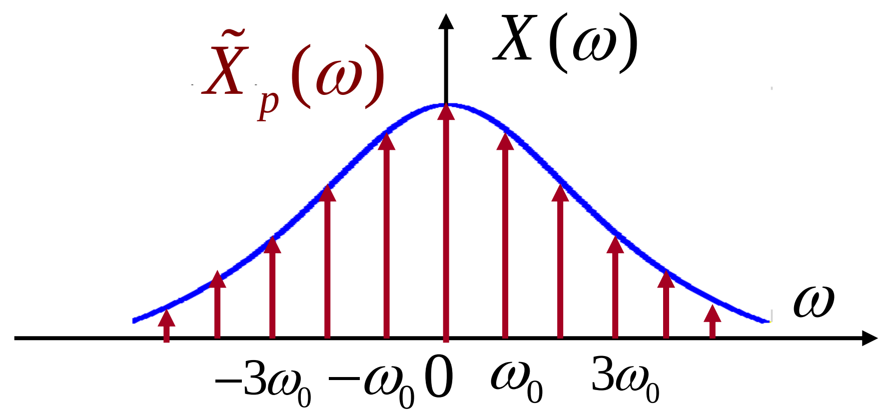


频域采样

频域采样

采样的本质是将连续变量的函数离散化。因此，在频域也可以对连续的频谱进行采样。这一过程与时域采样是完全对偶的。



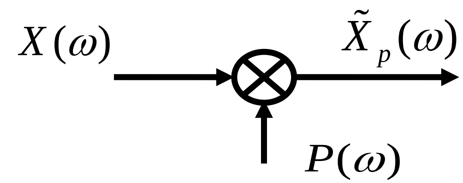


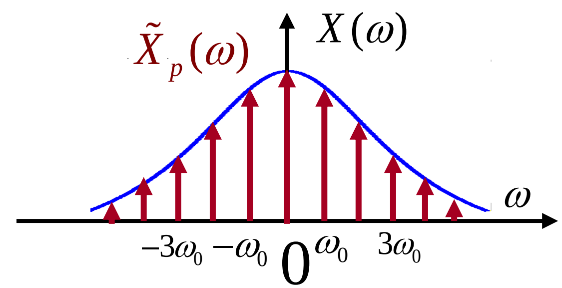
$$P(\omega)=\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0)$$

频域采样

在频域有： $P(\omega)=\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0)$

$$\tilde{X}_p(\omega)=X(\omega)P(\omega)\quad=\sum_{k=-\infty}^{\infty}X(k\omega_0)\delta(\omega-k\omega_0)$$





在时域有： $\tilde{x}_p(t)=x(t)*p(t)$

$$p(t)=\frac{1}{\omega_0}\sum_{k=-\infty}^{\infty}\delta(t-\frac{2\pi}{\omega_0}k)$$

$$\therefore\qquad \tilde{x}_p(t)=\frac{1}{\omega_0}\sum_{k=-\infty}^{\infty}x(t-\frac{2\pi}{\omega_0}k)$$

参考：

$$p(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$p(t)\leftrightarrow P(j\omega)=\frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\omega-\frac{2\pi}{T}n)$$

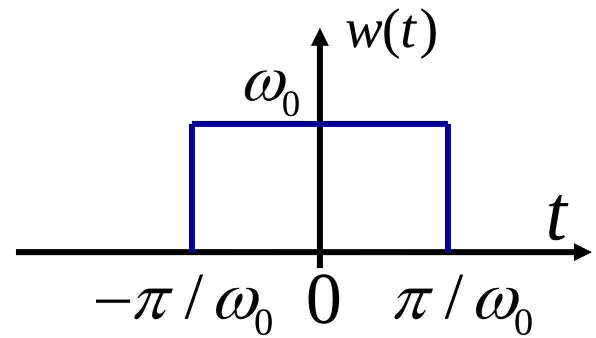
对信号的频谱在频域采样，相当于在时域周期无限延拓

（周期为 $2\pi/\omega_0$ ）

频域采样

此时，可以通过矩形窗从周期性延拓的信号中截取出原信号。

$$w(t)=\begin{cases}\omega_0 & |t|\le \pi/\omega_0 \\ 0 & |t|>\pi/\omega_0\end{cases}$$



$$x(t)=\tilde{x}_p(t)w(t)$$

在频域，从频谱的样本重建连续频谱时的频域时限内插过程是以矩形窗的频谱作为内插函数实现的。

频域采样

在频域有： $$X(\omega)=\frac{1}{2\pi}\tilde{X}_p(\omega)*W(\omega)$$

$$W(\omega)=2\pi\,\mathrm{sinc}(\omega/\omega_0)$$ —— **内插函数**

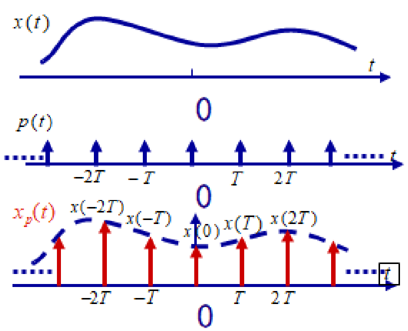
$$\therefore\quad X(\omega)=\sum_{k=-\infty}^{\infty}X(k\omega_0)\,\mathrm{sinc}\left(\frac{\omega-k\omega_0}{\omega_0}\right)$$

- 带限和时限没有必然的逻辑联系。请注意时域采样定理应用条件中的带限要求。频域采样同理。
- 因此，对带限信号在频域采样时，如果时域没有说明是时限，则不能保证频谱的样本可以恢复原信号。

总结

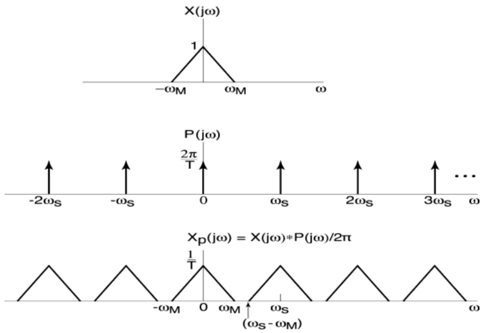
总结

➢ 连续信号的采样



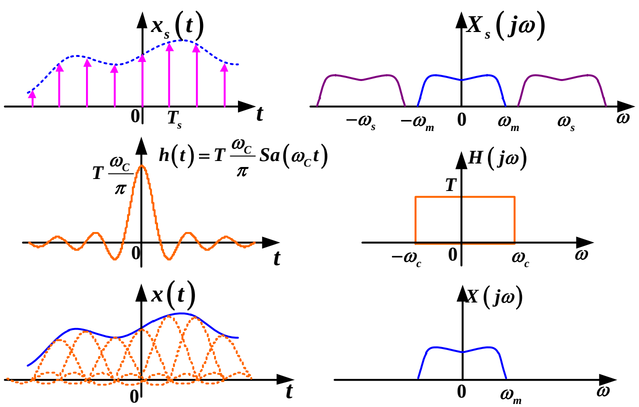
$$x_p(t)=x(t)p(t)=\sum_{n=-\infty}^{+\infty}x(nT_s)\delta(t-nT_s)$$

$$X_p(\omega)=\frac{1}{T_s}\sum_{n=-\infty}^{\infty}X(\omega-n\omega_s)$$



时域周期采样⟵⟶频域周期延拓

➢ 采样信号的重构



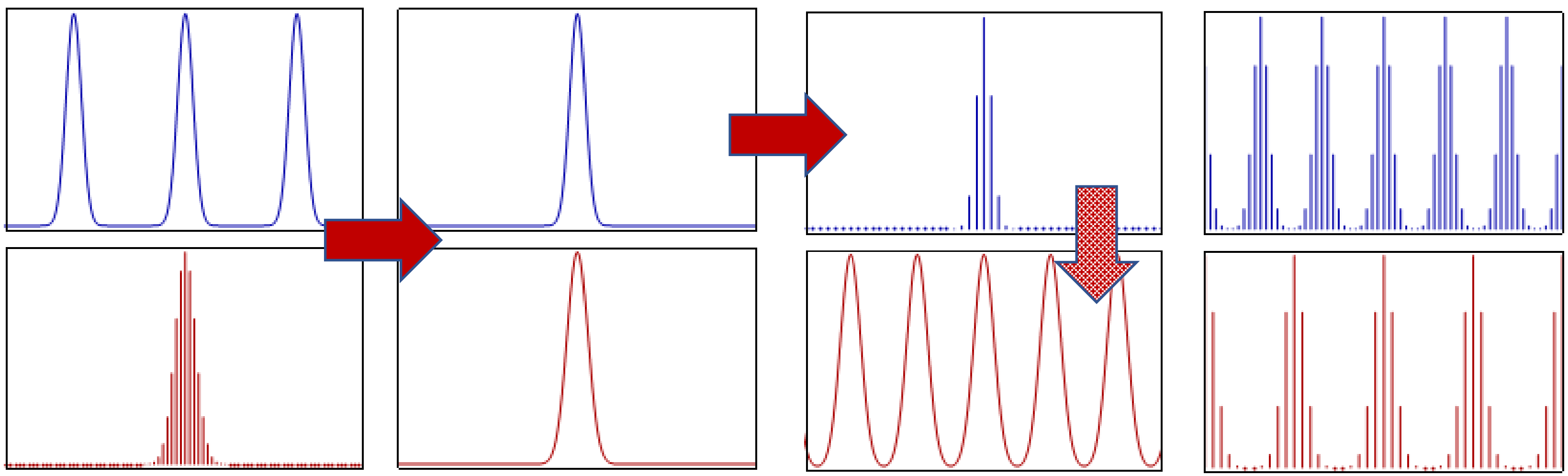
$$x_p(t)=\sum_{n=-\infty}^{\infty}x(nT_s)\delta(t-nT_s)$$

$$h(t)=T_s\frac{\omega_c}{\pi}Sa(\omega_c t)$$

$$\hat{x}(t)=x_p(t)*h(t)=\sum_{n=-\infty}^{\infty}x(nT_s)\delta(t-nT_s)*h(t)$$

$$\hat{x}(t)=\sum_{n=-\infty}^{\infty}\frac{T_s\omega_c}{\pi}x(nT_s)Sa[\omega_c(t-nT_s)]$$

周期　非周期　离散时间



FS　FT

$$X_p(\omega)=\sum_{n=-\infty}^{+\infty}X(\omega-n\omega_s)$$

其实什么都没干

作业

作业1：

说明：

如果 $x(t)=\cos(\omega_0 t+\varphi)$，证明欠采样时恢复的信号不仅频率降低，而且相位相反。

注：1. $\omega_0<\omega_s<2\omega_0$

2. 理想低通滤波器的带宽 ω_c，满足$\omega_s-\omega_0<\omega_c<\omega_0$

作业

作业2：

现有信号 $f(t)=e^{\frac{-t^2}{20}}$ 。为分析某时刻下的“局部频谱”，可选合适的窗函数 $w(t,t_0)$，并截取$f(t)$ 在 t_0 附近的信号，即

$$f_w(t,t_0)=f(t)w(t,t_0) .$$

a. 求信号 $f(t)$ 的FT。

b. 现不妨取窗函数 $w(t,t_0)=e^{\frac{-(t-t_0)^2}{2}}$ 。试分析 $t_0=0$时刻下对应的“局部频谱”，即求$f_w(t,0)$ 的FT。

c. 画出信号 $f(t)$ 的频谱图与信号 $f(t)$ 在 $t_0=0$ 时刻下的“局部频谱”图，并进行对比。

提示：若 $x\in R,c\in R$ ，$\int_{-\infty}^{+\infty}e^{-(x+jc)^2}dx=\sqrt{\pi}$ 。