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Adaptive flight control design for nonlinear missile

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Abstract

The focus of this work is to investigate the possible benefits of modern nonlinear control design methods for missile autopilot design. The basic requirement for an autopilot is firstly fast responses because of the short amount of time involved in the end-game. A slow response could easily cause a miss if the target has the capacity to perform high-g evasive manoeuvers. Secondly, minimum error is an obvious requirement if the missile is to achieve a kill. Finally, robustness to model uncertainties is important in order for the missile to achieve its objective in the physical environment.

In the first part of this paper input–output approximate linearisation of a nonlinear missile has been studied. A method for controlling the nonlinear system that is input–output linearisable is examined that retains the order of the system in the linearisation process, hence producing a linearised system with no internal or zero dynamics. Desired tracking performance for lateral acceleration of the missile is achieved by using a nonlinear control law that has been derived by selecting the lateral velocity as the linearisation output. Simulation results are shown to exercise the final design and show that the linearisation and controller design are satisfactory. Then an adaptive nonlinear controller is designed that guarantees tracking performance when the uncertain parameters vary within a stability bound. An autopilot combining an indirect adaptive controller with approximate feedback linearisation is proposed in order to achieve asymptotic tracking. Adaptation is introduced to enhance closed-loop robustness, while approximate feedback linearisation is used to overcome the problem of unstable zero dynamics. Computer simulations show that this approach offers a possible autopilot design for nonlinear missiles with uncertain parameters.

Keywords: Nonlinear control; Feedback linearisation; Flight control; Missile system; Adaptive control

1. Introduction

The performance of aerospace systems such as aircraft, spacecraft and missiles is highly dependent on the capabilities of the guidance, navigation and control systems. To achieve improved performance in such aerospace systems, it is important that more sophisticated control systems be developed and implemented. In particular, as the performance envelope is expanded, the control schemes must become adaptive and nonlinear, to provide performance over a greater range, in the face of uncertain or changing operating conditions.

The tracking performance of a missile is also dependent on the location within the flight envelope and varies with factors such as Mach number and dynamic pressure. Several approaches, including adaptive control (Lin & Cloutier, 1991), nonlinear control (White, Tsourdos, & Blumel, 1998) and gain scheduling (Shamma & Cloutier, 1993) have been used to alleviate these tracking problems. While gain scheduling is conceptually simple and has been proven successful, it has virtually no guarantee of stability in the transitional periods between operating points and relies on the fact that the scheduling variables should only change slowly. Furthermore, there is a heavy design overhead due to the large number of linear controllers which must be derived and, as the performance demands of modern-day missile systems become more stringent, alternatives to linear control are of increasing practical significance.

Feedback linearisation is a popular method used in nonlinear control applications, and there have been several flight control demonstrations (Snell, 1992). Dynamic model inversion is the feedback linearisation method employed to design the missile autopilot. This

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method is very effective in applications to aircrafts and missiles. The main drawback of dynamic model inversion is the need for high-fidelity nonlinear force and moment models that must be invertible in real time, which implies a detailed knowledge of the plant dynamics, and the approach tends to be computationally intensive. In general, dynamic model inversion is sensitive to modelling errors. The application of robust and/or adaptive control can alleviate this sensitivity and, therefore, the need for detailed knowledge of nonlinearities.

In this paper an adaptive nonlinear control design technique is applied to the autopilot for the missile model which is aerodynamically controlled. Missile motion is modelled to be nonlinear with unknown parameters. Based on the model, we adopt a design procedure similar to Sastry and Bodson (1989), basically an adaptive feedback linearisation method. In this scheme, unknown parameters are estimated and based on these estimates, control parameters are updated. Computer simulations show that this approach is very promising to apply the autopilot design for the missiles which are highly nonlinear in aerodynamics with unknown parameters.

The missile model can be represented in the general nonlinear state space

$$\dot{x}(t) = f(x,\theta) + g(x,\theta)u,$$
$$y(t) = h(x,\theta). \tag{1}$$

Typically the control law is based on a vector $\hat{\theta}$ which is an online estimate of the true parameter vector θ. The update laws for these adjusted parameters are determined as part of the design and shall be such that the closed loop system stability is preserved. The convergence of these parameters estimate to their true value θ is a necessary condition in the indirect schemes of adaptive control.

An indirect adaptive controller consists of a parameter identification scheme and a controller whose gains are calculated on-line based on estimates of the plant model parameters. The structure of the plant is assumed a priori, but the coefficients or parameters involved are estimated based on the available input/output information. Fig. 1 shows the schematic diagram for indirect adaptive control schemes. The identification block estimates the plant parameters from the control signal and the output measurement. The estimated parameters are then used to update the controller gains according to one of the several control methodologies.

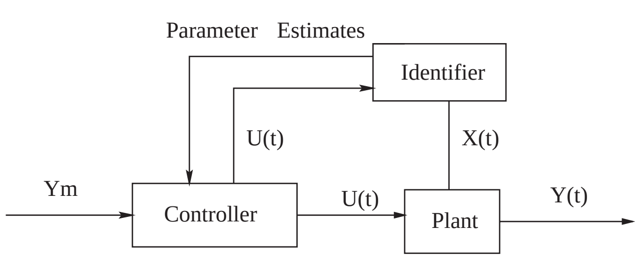

Fig. 1. Schematic diagram for indirect adaptive control schemes.

The parameter identifier is used in the outer loop design and continuously adjusts the parameter estimates based on observation error. The certainty equivalence principle suggests that these parameter estimates that are converging to their true values may be employed to asymptotically achieve the desired objective as parameter estimates converge to their true value. The adaptive scheme developed for the lateral missile flight control system is presented in the following sections.

Other adaptive schemes such as direct adaptive control schemes are discussed in details in Sastry and Bodson (1989). In the schemes of that form, parameters do not need to converge to their true value but they are required to stay bounded and converge to some constant. Typically, if the system is persistently exciting, then all the parameters will converge to their true values.

2. Missile model

The missile model used in this study derives from a nonlinear model produced by Horton of Matra-British Aerospace (Horton, 1992). This study will look at the reduced problem of a 2 DOF controller for the pitch and yaw planes without roll coupling. The angular and translational equations of motion of the missile airframe are given by

$$\dot{r} = \frac{1}{2} I_{yz}^{-1} \rho V_0 S d \left(\frac{1}{2} d C_{nr} r + C_{nv} v + V_0 C_{n\zeta} \zeta \right),$$
$$\dot{v} = \frac{1}{2m} \rho V_0 S (C_{yv} v + V_0 C_{y\zeta} \zeta) - Ur, \tag{2}$$

where the variables are defined in Fig. 2 and Tables 1 and 2. Eqs. (2) describe the dynamics of the body rates and velocities under the influence of external forces (e.g. C_{yv}) and moments (e.g. C_{nr}), acting on the frame. These forces and moments are derived from wind tunnel measurements and by using polynomial approximation algorithms, C_{yv}, $C_{y\zeta}$, C_{nr}, C_{nv} and $C_{n\zeta}$ (Horton, 1992) can be represented by polynomials which can be fitted to the set of curves taken from look-up tables for different flight conditions. A detailed description of the model can be found in Horton (1992).

The aerodynamic forces and moments acting on the airframe are nonlinear functions of Mach number, longitudinal and lateral velocities, control surface deflection, aerodynamic roll angle and body rates. Control of the missile will be accomplished in this paper by controlling an augmented version of lateral acceleration.

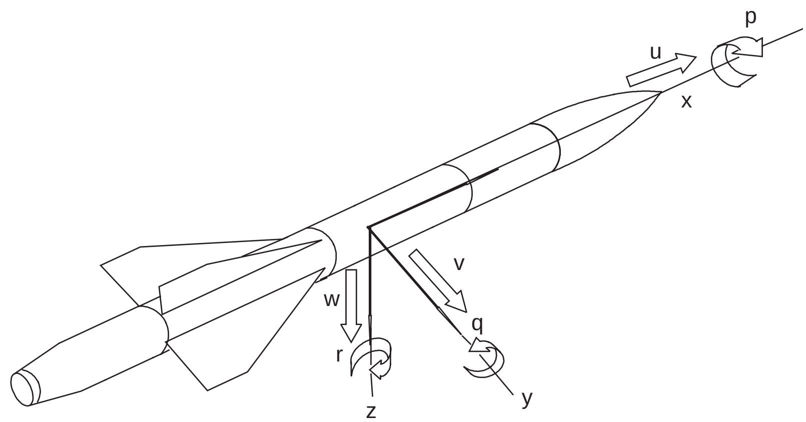

Fig. 2. Airframe axes.

Table 1
Nomenclature of missile model

	Nomenclature
f	Lateral acceleration
σ	Incidence angle
r	Yaw rate
M	Mach number
U	Velocity along the roll axis
x	Force along the roll axis
y	Force along the pitch axis
d	Missile diameter
V_0	Total velocity
ζ	Rudder angle

Table 2
Coefficients of missile model

C_{y_v}	$0.5[(-25+M-60\lvert\sigma\rvert)(1+\cos 4\lambda) + (-26+1.5M-30\lvert\sigma\rvert)(1-\cos 4\lambda)]$
C_{y_ζ}	$10+0.5[(-1.6M+2\lvert\sigma\rvert)(1+\cos 4\lambda) + (-1.4M+1.5\lvert\sigma\rvert)(1-\cos 4\lambda)]$
C_{n_r}	$-500-30M+200\lvert\sigma\rvert$
C_{n_v}	$s_m C_{y_v}$, where $s_m = d^{-1}[1.3+0.1M+0.2(1+\cos 4\lambda)\lvert\sigma\rvert + 0.3(1-\cos 4\lambda)\lvert\sigma\rvert-(1.3+m/500)]$
C_{n_ζ}	$s_f C_{y_\zeta}$, where $s_f = d^{-1}[2.6-(1.3+m/500)]$

The dynamic equation for lateral acceleration can be derived (White et al., 1998) and is given by

$$\begin{aligned}
f &= \dot{v} + Ur, \\
f &= V^0(C_{yv}v + V_0 C_{y\zeta}\zeta) \\
&= V^0[(C_{yv_0} + C_{yv_M}M + C_{yv_\sigma}|\sigma|)v \\
&\quad + V_0(C_{y\zeta_0} + C_{y\zeta_M}M + C_{y\zeta_\sigma}|\sigma|)\zeta] \\
&= V^0[(\bar{C}_{yv_0}v + \bar{C}_{yv_\sigma}|v|)v + V_0\bar{C}_{y\zeta_0}\zeta + V_0\bar{C}_{y\zeta_\sigma}|v|\zeta] \\
&= y_v v + y_\zeta \zeta \\
&= \phi(v) + \psi(v,\zeta),
\end{aligned} \tag{3}$$

where the Mach number M, and the total velocity V_0 are slowly varying.

From the viewpoint of autopilot design, the usual output for control purposes is the lateral acceleration of the centre of gravity f. This is not always possible to measure directly as the accelerometer is usually placed ahead of the c.g. and will thus pick up some angular acceleration as well as c.g. acceleration. It will, however, be assumed that it is directly measurable for this study and no significant difficulty is seen in the design process as a consequence of this choice. Indeed, the use of the accelerometer moment arm is beneficial in removing the nonminimum phase zero that is inherent in the c.g. acceleration measurement. Another output that can be considered is the sideslip velocity v. This can be obtained by measurement of incidence and knowledge of forward speed. This has some advantages over lateral acceleration in that the resulting system is minimum phase, but the relationship between sideslip velocity and acceleration is nonlinear and thus acceleration is not as accurately controlled as a direct measurement. Finally, an augmented acceleration signal $\bar{f}$ is considered. This produces a synthetic signal that removes the nonminimum phase attributes of other lateral acceleration measurements.

3. Augmented lateral acceleration

The conditions for feedback linearisation of nonlinear systems are restrictive and it is of practical interest to investigate situations where these conditions are met approximately. Such a case is the design of autopilot for nonlinear missiles. Continuing the work of Krener (1984), who gave conditions for approximate full state linearisation of nonlinear multi-input systems, Hauser, Satry, and Kokotovic (1992) discuss approximate input–output linearisation of single-input single-output systems which fail to have relative degree. In contrast to the extended linearisation (Baumann & Rugh, 1986) and pseudo-linearisation (Reboulet & Champetier, 1984), the technique presented in Hauser et al. (1992) does not approximate the system by a linear system or a family of linear systems but rather by a single nonlinear system that is input–output linearisable.

This approach to the tracking problem differs from the work of Isidori and Byrnes (1990) who provide (fragile) conditions under which one can exactly track the output of a finite-dimensional exosystem. In contrast, Hauser et al. (1992) provide approximate tracking of a large class of output signals that is valid under a wide class of nonlinear perturbations in the system model.

3.1. Modelling

To reduce the zero problem, consider using an augmented acceleration output $\bar{f}$, where

$$\bar{f} = f - y_\zeta \zeta. \tag{4}$$

The use of $\bar{f}$ can be justified on the grounds that the acceleration provided directly by the rear control surface $y_\zeta \zeta$ is usually small in comparison to the main lift acceleration f. It also causes the minimum phase zero to occur as the lift force developed by the fin appears directly on the c.g. acceleration variable. By removing this term, the effect of the fin is cancelled from the accelerometer signal. Using $\bar{f}$, we now have

$$\begin{aligned}\bar{f} &= f - y_\zeta \zeta \\ &= y_v v + y_r r.\end{aligned} \tag{5}$$

This is not a direct function of the control surface ζ, hence the relative degree is increased to 1. Differentiating the output $\bar{f}$ gives

$$\begin{aligned}\frac{\mathrm{d}\bar{f}}{\mathrm{d}t} &= y_v \dot{v} + y_r \dot{r} \\ &= y_v[y_v v + (y_r - U)r + y_\zeta \zeta] + y_r[n_r r + n_v v + n_\zeta \zeta] \\ &= [y_v^2 + y_r n_v]v + [y_v(y_r - U) + y_r n_r]r + [y_v y_\zeta + y_r n_\zeta]\zeta \\ &= a_v^{\bar{f}} v + a_r^{\bar{f}} r + b_\zeta^{\bar{f}} \zeta,\end{aligned} \tag{6}$$

where

$$\begin{aligned}a_v^{\bar{f}} &= [y_v^2 + y_r n_v], \\ a_r^{\bar{f}} &= [y_v(y_r - U) + y_r n_r], \\ b_\zeta^{\bar{f}} &= [y_v y_\zeta + y_r n_\zeta].\end{aligned} \tag{7}$$

This is now a direct function of the input ζ and hence the relative degree is 1. Substituting for v, as before, gives

$$\frac{\mathrm{d}\bar{f}}{\mathrm{d}t} = \frac{a_v^{\bar{f}}}{y_v} f + \left(a_r^{\bar{f}} - \frac{a_v^{\bar{f}}}{y_v} y_r\right) r + \left(b_\zeta^{\bar{f}} - \frac{a_v^{\bar{f}}}{y_v} y_\zeta\right)\zeta. \tag{8}$$

In order to achieve an order of 2 for the dynamics and hence eliminate the zero dynamics, another differentiation can be carried out to give

$$\begin{aligned}\frac{\mathrm{d}^2\bar{f}}{\mathrm{d}t^2} &= a_v^{\bar{f}} \dot{v} + a_r^{\bar{f}} \dot{r} + b_\zeta^{\bar{f}} \dot{\zeta} \\ &= a_v^{\bar{f}}[y_v v + (y_r - U)r + y_\zeta \zeta] \\ &\quad + a_r^{\bar{f}}[n_r r + n_v v + n_\zeta \zeta] + b_\zeta^{\bar{f}} \dot{\zeta} \\ &= [a_v^{\bar{f}} y_v + a_r^{\bar{f}} n_v]v + [a_v^{\bar{f}}(y_r - U) + a_r^{\bar{f}} n_v]r \\ &\quad + [a_v^{\bar{f}} y_\zeta + a_r^{\bar{f}} n_\zeta]\zeta + b_\zeta^{\bar{f}} \dot{\zeta} \\ &= a_v^{2\bar{f}} v + a_r^{2\bar{f}} r + b_\zeta^{2\bar{f}} \zeta + b_\zeta^{\bar{f}} \dot{\zeta},\end{aligned} \tag{9}$$

where

$$\begin{aligned}a_v^{2\bar{f}} &= [a_v^{\bar{f}} y_v + a_r^{\bar{f}} n_v] \\ &= [(y_v^2 + y_r n_v)y_v + (y_v(y_r - U) + y_r n_r)n_v], \\[1ex] a_r^{2\bar{f}} &= [a_v^{\bar{f}}(y_r - U) + a_r^{\bar{f}} n_v] \\ &= [(y_v^2 + y_r n_v)(y_r - U) + (y_v(y_r - U) + y_r n_r)n_r], \\[1ex] b_\zeta^{2\bar{f}} &= [a_v^{\bar{f}} y_\zeta + a_r^{\bar{f}} n_\zeta] \\ &= [(y_v^2 + y_r n_v)y_\zeta + (y_v(y_r - U) + y_r n_r)n_\zeta].\end{aligned} \tag{10}$$

Substituting for v, as before, gives

$$\begin{aligned}\frac{\mathrm{d}^2\bar{f}}{\mathrm{d}t^2} &= \frac{a_v^{2\bar{f}}}{y_v} f + \left(a_r^{2\bar{f}} - \frac{a_v^{2\bar{f}}}{y_v} y_r\right) r \\ &\quad + \left(b_\zeta^{2\bar{f}} - \frac{a_v^{2\bar{f}}}{y_v} y_\zeta\right)\zeta + b_\zeta^{\bar{f}} \dot{\zeta}\end{aligned} \tag{11}$$

or

$$\frac{\mathrm{d}^2\bar{f}}{\mathrm{d}t^2} = \bar{a}_f^{2\bar{f}} f + \bar{a}_r^{2\bar{f}} r + \bar{b}_\zeta^{2\bar{f}} \zeta + b_\zeta^{\bar{f}} \dot{\zeta}, \tag{12}$$

where

$$\begin{aligned}\bar{a}_f^{2\bar{f}} &= \frac{a_{2v}}{y_v}, \\ \bar{a}_r^{2\bar{f}} &= \left(a_{2r} - \frac{a_{2v}}{y_v} y_r\right), \\ \bar{b}_\zeta^{2\bar{f}} &= \left(b_{2\zeta} - \frac{a_{2v}}{y_v} y_\zeta\right).\end{aligned} \tag{13}$$

These equations now give the dynamics of the augmented output $\bar{f}$.

3.2. Augmented acceleration feedback control

These equations can be used to control the dynamics of $\bar{f}$ by again constructing an error function

$$\bar{f}_e = \bar{f}_d - \bar{f}, \tag{14}$$

where $\bar{f}_d$ is the desired lateral acceleration. With this definition, the following feedback can be used

$$\bar{b}_\zeta^{2\bar{f}} \zeta + b_\zeta^{\bar{f}} \dot{\zeta} = -\bar{a}_f^{2\bar{f}} f - \bar{a}_r^{2\bar{f}} r + k_e \bar{f}_e + k_{\dot{e}} \frac{\mathrm{d}\bar{f}_e}{\mathrm{d}t} \tag{15}$$

or

$$\dot{\zeta} = \frac{1}{b_\zeta^{\bar{f}}}\left(-\bar{a}_f^{2\bar{f}} f - \bar{a}_r^{2\bar{f}} r - \bar{b}_\zeta^{2\bar{f}} \zeta + k_e \bar{f}_e + k_{\dot{e}} \frac{d\bar{f}_e}{dt}\right). \quad (16)$$

This gives the closed loop equation

$$\frac{d^2 \bar{f}}{dt^2} = k_e \bar{f}_e + k_{\dot{e}} \frac{d\bar{f}_e}{dt}. \quad (17)$$

If the acceleration component in $\bar{f}_d$ is small, then

$$\frac{d^2 \bar{f}}{dt^2} \approx -\frac{d^2 \bar{f}_e}{dt^2} \quad (18)$$

and hence

$$\frac{d^2 \bar{f}_e}{dt^2} + k_{\dot{e}} \frac{d\bar{f}_e}{dt} + k_e \bar{f}_e = 0, \quad (19)$$

where the constants k_e and $k_{\dot{e}}$ make (19) Hurwitz. Note that the relative degree of 1 for the dynamics has forced the use of the $b_\zeta \dot{\zeta}$ term.

The configuration for the controller is shown in Fig. 3.

It can be seen from the figure that an extra pole has been inserted in the forward path. This effectively cancels the zero that must be in the quasi-linear equations which are order two, relative degree one. The zero must be at the solution of

$$\bar{b}_\zeta^{2\bar{f}} \zeta + b_\zeta^{\bar{f}} \dot{\zeta} = 0 \quad (20)$$

which gives a zero at

$$s = -\frac{\bar{b}_\zeta^{2\bar{f}}}{b_\zeta^{\bar{f}}}. \quad (21)$$

From the definition of $\bar{b}_\zeta^{2\bar{f}}$ and $b_\zeta^{\bar{f}}$, we have

$$\frac{\bar{b}_\zeta^{2\bar{f}}}{b_\zeta^{\bar{f}}} = \frac{[y_v(y_r - U) + y_r n_r][n_\zeta - n_v/y_v y_\zeta]}{[y_v y_\zeta + y_r n_\zeta]}. \quad (22)$$

In most cases $y_r \approx 0$, and if y_r is assumed negligible, then

$$\frac{\bar{b}_\zeta^{\bar{f}}}{b_\zeta^{\bar{f}}} \approx -U\left(\frac{n_\zeta}{y_\zeta} - \frac{n_v}{y_v}\right). \quad (23)$$

The ratios of moment to force can also be approximated by

$$\frac{n_\zeta}{y_\zeta} \approx -l_f \frac{J_{yy}}{m}, \qquad \frac{n_v}{y_v} \approx -l_s \frac{J_{yy}}{m}, \quad (24)$$

where l_f is the fin moment arm, and l_s is the static margin. With these approximations, the stability of the zero can be established as

$$s \approx U(l_f - l_s)\frac{J_{yy}}{m}. \quad (25)$$

Hence a stable zero results if the fin moment arm is significantly greater than the static margin. This is usually the case in most agile missiles as the static margin is made as close to zero as possible for a stable missile and negative for an unstable missile. If the zero is large, then the $\dot{\zeta}$ term can be neglected, and hence (15) becomes

$$\bar{b}_\zeta^{2\bar{f}} \zeta = -\bar{a}_f^{2\bar{f}} f - \bar{a}_r^{2\bar{f}} r + k_e \bar{f}_e + k_{\dot{e}} \frac{d\bar{f}_e}{dt} \quad (26)$$

or

$$\zeta = \frac{1}{\bar{b}_\zeta^{2\bar{f}}}\left(-\bar{a}_f^{2\bar{f}} f - \bar{a}_r^{2\bar{f}} r + k_e \bar{f}_e + k_{\dot{e}} \frac{d\bar{f}_e}{dt}\right). \quad (27)$$

The configuration for the simplified controller is shown in Fig. 4.

3.3. Simulation results

Fig. 4 shows the nonlinear controller structure. A fast linear actuator with natural frequency of 250 rad/s has

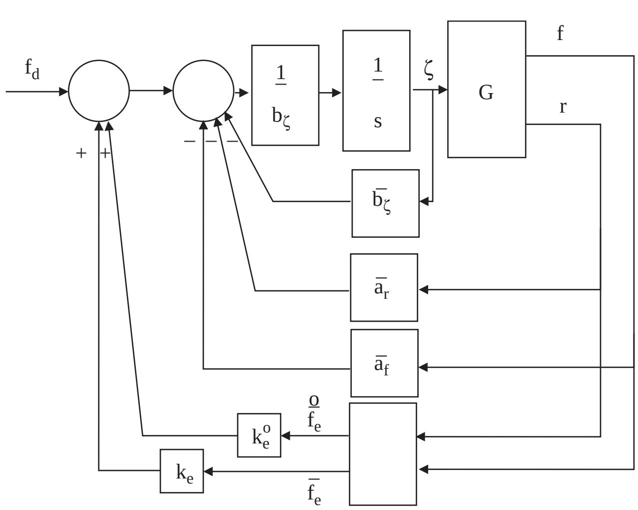

Fig. 3. Augmented acceleration controller configuration.

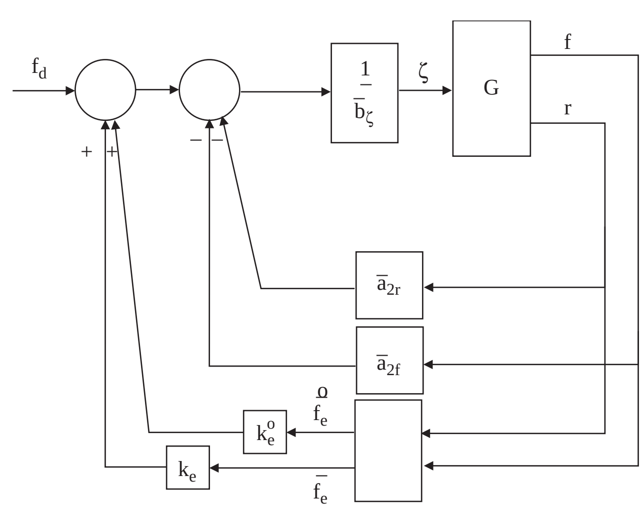

Fig. 4. Simplified augmented acceleration controller configuration.

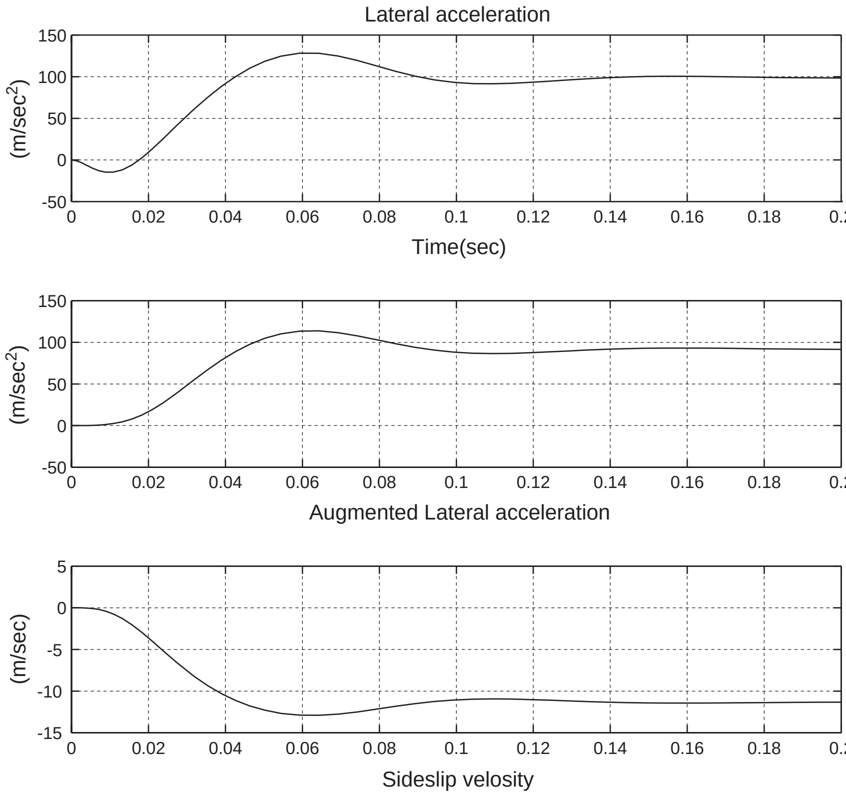

Fig. 5. Acceleration and lateral velocity for $a_d = 100$.

been included in the nonlinear system. The error dynamics are constructed using the a_d signal and the feedback of the actual states—velocity, rate and acceleration.

The error coefficients in Eq. (27) are chosen to satisfy a Hurwitz polynomial. For the second order error equation in each channel, $k_e = 2\zeta w_n$ and $k_{\dot{e}} = w_n^2$, where $w_n = 60(\text{rad/s})$ and $\zeta = 0.65$. The speed of response is significantly faster than the open loop missile response and so should exercise the dynamics of the nonlinear missile sufficiently for meaningful conclusions to be drawn.

The results of a $100\ \text{m/s}^2$ demand in acceleration is shown in Figs. 5 and 6 for the $100\ \text{m/s}^2$ case. The figures show almost identical step responses with some variation in peaks and steady state values for the body rate, the actuator movement and the lateral velocity. The difference between the lateral acceleration and the augmented acceleration shows that there is a good match between the two and that steady state values are very close. This illustrates the small effect that the fin force has on the missile acceleration and justifies the use of the augmented acceleration. The results also show that the actuator does not significantly affect the design. The nonlinear approach is also shown to be reasonably accurate, as the predicted and actual performance are very close.

4. Adaptive nonlinear control

The design presented in the previous section was for the nominal missile model. However, neither the Mach number nor the mass of the missile remain constant. As the flight conditions vary, both Mach number and mass vary. From Eq. (1) and Tables 1–3 it can easily shown that the different aerodynamic coefficients are multilinear functions of Mach number and mass. Hence, as these two variables vary, the aerodynamics coefficients vary. Since the feedback control law was design only for the nominal values of the aerodynamic coefficients, any variations of these will cause inexact cancellation of the system's nonlinearities, i.e. inexact decoupling.

Consider a SISO nonlinear system of form (2) under parametric uncertainty

$$\dot{x}(t) = F(x,\theta) + G(x,\theta)u,$$
$$y(t) = h(x,\theta), \tag{28}$$

where $x_1 = r$, $x_2 = v$, $u = \zeta$ and

$$F(x,\theta) = \begin{bmatrix} \frac{1}{2} I_{yz}^{-1} \rho V_0 Sd \frac{1}{2} dC_{nr} r & \frac{1}{2} I_{yz}^{-1} \rho V_0 SdC_{nv} v \\ \frac{1}{2m} \rho V_0 SC_{yv} v & -Ur \end{bmatrix}, \tag{29}$$

$$G(x,\theta) = \begin{bmatrix} \frac{1}{2} I_{yz}^{-1} \rho (V_0)^2 SdC_{n\zeta}\zeta) \\ \frac{1}{2m} \rho (V_0)^2 SC_{y\zeta}\zeta) \end{bmatrix}. \tag{30}$$

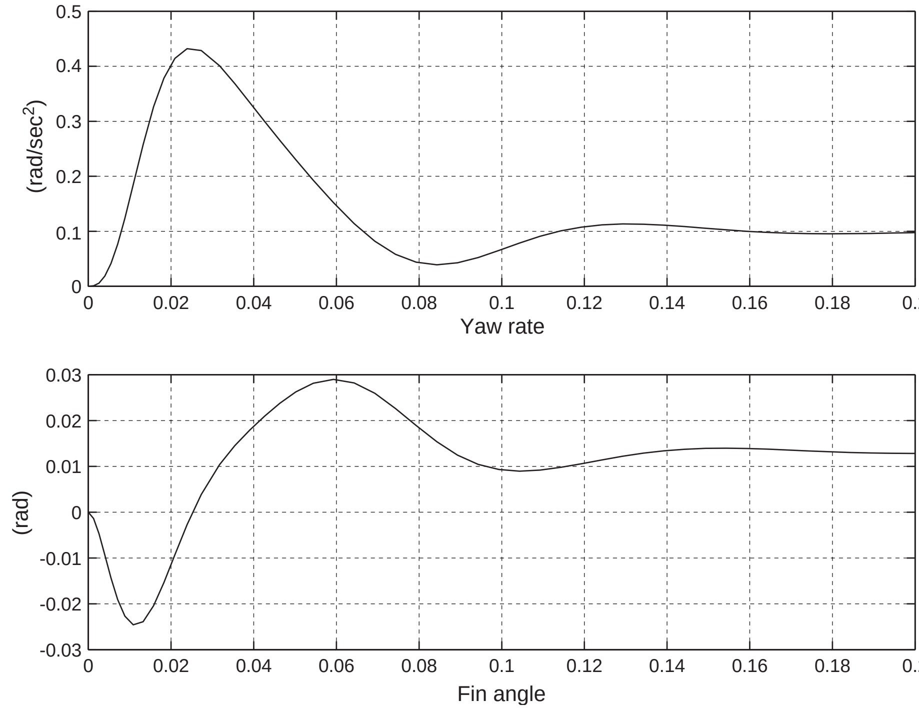

Fig. 6. Rate and fin angle for $a_d = 100$.

Further, assume $F(x)$ and $G(x)$ have the form

$$F(x,\theta) = \sum_{i=1}^{n} \theta_i^F F_i(x),$$
$$G(x,\theta) = \sum_{i=1}^{m} \theta_i^G G_i(x) \tag{31}$$

with θ^F and θ^G vectors of unknown parameters and $G_i(x)$ and $G_i(x)$ known functions. The estimates of these functions are given by

$$\hat{F}(x,\theta) = \sum_{i=1}^{n} \hat\theta_i^F F_i(x),$$
$$\hat{G}(x,\theta) = \sum_{i=1}^{m} \hat\theta_i^G G_i(x), \tag{32}$$

where $\hat\theta_j$ are the estimates of the unknown aerodynamic parameters θ_j and are multi-linear functions of Mach number and mass. $F_i(x)$ and $G_i(x)$ are obtained by $F(x)$ and $G(x)$ by parameterising them according to $\hat\theta_i^F$ and $\hat\theta_i^G$. Now let us replace the control law (27) by

$$u_{ad} = \frac{1}{\widehat{L_G L_F h}}\left[-\widehat{L_F^2 h} + v_{ad}\right] \tag{33}$$

with

$$v_{ad} = \frac{\mathrm{d}^2 \bar f_d}{\mathrm{d}t^2} + k_{\dot e}\left(\frac{\mathrm{d}\bar f_d}{\mathrm{d}t} - \hat\xi_2\right) + k_e(\bar f_d - \hat\xi_1), \tag{34}$$

where k_e, $k_{\dot e}$ are chosen as before in Eq. (27), $\xi_1 = \bar f$ and $\xi_2 = \mathrm{d}\bar f/\mathrm{d}t$ and $\hat\xi_{i-1} = L_F^i h$ are replaced by their estimates $\widehat{L_F^i h}$

$$\dot{\hat\xi}_1 = \widehat{L_F h} \doteq L_{\hat F}\hat h$$
$$\dot{\hat\xi}_2 = \widehat{L_F^2 h} + \widehat{L_G L_F h} \doteq L_{\hat F}^{i-1}\hat h + L_{\hat G} L_{\hat F}\hat h, \tag{35}$$

where from Eqs. (6) and (9)

$$L_F h = a_v^{\bar f} v + a_r^{\bar f} r,$$
$$L_F^2 h = a_v^{2\bar f} v + a_r^{2\bar f} r,$$
$$L_G L_F h = b_\zeta^{2\bar f}\zeta. \tag{36}$$

As in Sastry and Bodson (1989), since these estimates are not linear in the unknown parameters θ_i, we define each of the parameters products to be a new parameter. For example

$$\widehat{L_G L_F h} = \sum_{i=1}^{n}\sum_{i=1}^{m} \theta_i^F \theta_j^G L_G L_F h \tag{37}$$

and we let $\Theta \in \mathscr{R}^p$ be the large p-dimensional vector of all multi-linear parameter products $\theta_i^F, \theta_j^G, \theta_i^F\theta_j^G, \ldots$. The vector containing all the estimates is denoted by $\hat\Theta \in \mathscr{R}^p$ with $\Phi \doteq \Theta - \hat\Theta$ representing the parameter error.

Due to the indirect nature of our approach, this over-parametrisation does not increase the complexity of the closed loop system since a parameter identifier is to be used to estimate the unknown parameters θ_j. The parameter vector Θ is, however, constructed here in order to show the stability of the resulting adaptive

Table 3
Characteristics of the Horton model

ρ_0	Sea level air density	1.23 kg/m^3
ρ	Air density	$\rho_0 - 0.094h$
h	Altitude	in km
a_0	Sea level velocity of sound	340 m/s
a	Velocity of sound	$a_0 - 4h$ m/s
d	Reference diameter (calibre)	0.2 m
S	Reference area	$\pi d^2/4 = 0.0314$ m^2
l	Length of the missile	2.7 m
sp	Wing span	0.6 m
m	Mass	125 kg nominal 150 kg full 100 kg all burnt
I_z, I_y	Lateral inertia	67.5 kg m^2 nominal 75 kg m^2 full 60 kg m^2 all burnt
x_{cg}	Centre of gravity	1.6 m full 1.5 m all burnt $1.3 + \frac{m}{500}$
x_{cp}	Centre of pressure (body + wings)	$x_{cp}^0 = 1.3 + 0.1M + 0.2\lvert\sigma\rvert$ $x_{cp}^{45} = 1.3 + 0.1M + 0.3\lvert\sigma\rvert$
x_f	Centre of pressure (fins only)	2.6 m
x_r	Fin moment arm	$1.5\frac{d}{2}$m
s_m	Static margin (body + wings)	$x_{cp} - x_{cg}$
s_f	Fin moment arm for lateral motion	$x_f - x_{cg}$

system. The control law (33) now becomes

$$\begin{aligned}\dot{\xi}_2 &= L_F^2 h + [L_G L_F h - L_g \widehat{L_F h}]u_{ad} - \widehat{L_F^2 h} + v_{ad} \\ &= [L_F^2 - \widehat{L_F^2}] + [L_G L_F h - L_G \widehat{L_F h}]u_{ad} + v_{ad}. \end{aligned} \tag{38}$$

Subtracting v in error equation from both sides gives

$$\begin{aligned} e^2 + k_{\dot{e}}e + k_e e &= [L_G L_F^1 h - L_G \widehat{L_F^1} h]u_{ad} \\ &\quad + k_{\dot{e}}[L_F^2 - \widehat{L_F^2}] + k_{\dot{e}}\left(L_F - \widehat{L_F}\right) \\ &= \Phi^{\mathrm{T}} w(x, u_{ad}(x)), \end{aligned} \tag{39}$$

where $w^{\mathrm{T}} \doteq [L_G L_F h u_{ad}(x) | L_F^2 h | L_F h]$. Therefore, in the closed loop, for the approximate system, we have in compact form

$$\dot{e} = Ae + W^{\mathrm{T}}(x, u_{ad}(x))\Phi, \tag{40}$$

where A is a Hurwitz matrix and note that if $\phi \doteq \theta - \hat{\theta} \to 0$ as $t \to \infty$, then $\Phi \to 0$ as $t \to \infty$. To estimate the unknown parameters, we consider an observer-based identifier proposed in Taylor, Kokotovic, Marino, and Kanellakopoulos (1989), Kudva and Narendra (1973). First we rewrite Eq. (28) as

$$\begin{aligned} \dot{x} &= (F_1 \cdots F_n | G_1 u \cdots G_m u)\begin{pmatrix}\theta^F \\ \theta^G\end{pmatrix} \\ &\doteq Z^{\mathrm{T}}(x, u_{ad}(x))\theta. \end{aligned} \tag{41}$$

Consider the following identifier system:

$$\begin{aligned} \dot{x} &= \hat{A}(\hat{x} - x) + Z^{\mathrm{T}}(x, u_{ad}(x))\hat{\theta}, \\ \dot{\hat{\theta}} &= -Z(x, u)P(\hat{x} - x), \end{aligned} \tag{42}$$

where $\hat{A}$ is a Hurwitz matrix, $\hat{x}$ is the observer state, x is the plant state in Eq. (28), and $P > 0$ is a solution to the Lyapunov equation $\hat{A}P + P\hat{A} = -\lambda I$ with $\lambda > 0$. We assume that all the states x in Eq. (28) are available and hence $\hat{x}$ and $\hat{\theta}$ are given by Eq. (42). We also assume that θ is a vector of constant but unknown parameters. Then

$$\begin{aligned} \dot{\hat{e}} &= \hat{A}\hat{e} + Z^{\mathrm{T}}(x, u)\phi, \\ \dot{\phi} &= -Z(x, u)P\hat{e} \end{aligned} \tag{43}$$

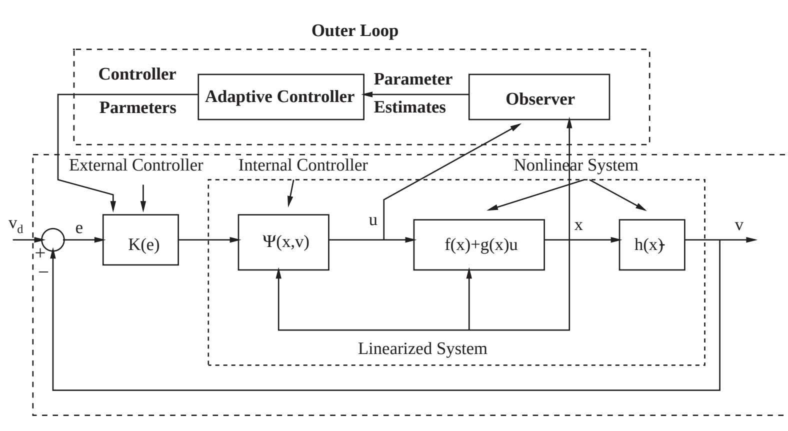

Fig. 7. Adaptive tracking control design.

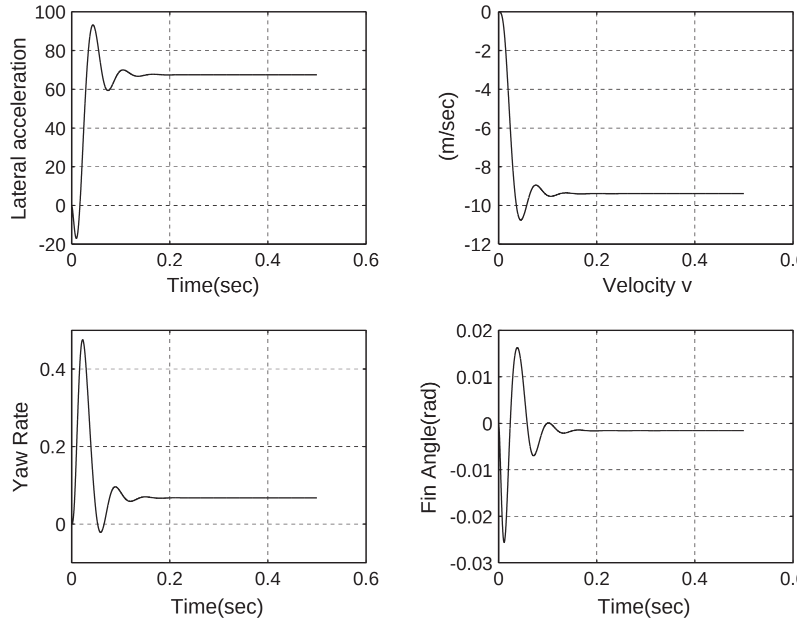

Fig. 8. Nonadaptive with uncertainty 35% in Mach number and mass for acceleration demand $a_d = 100$.

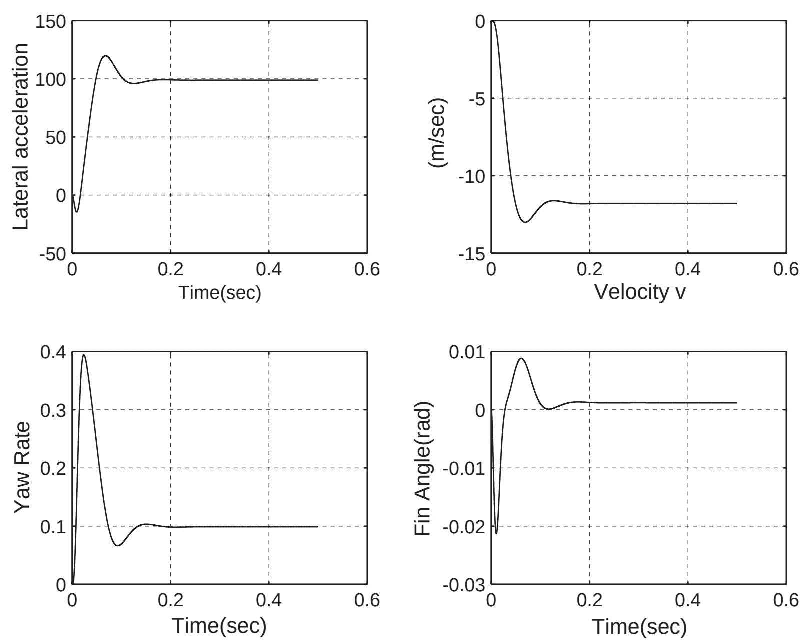

Fig. 9. Adaptive with uncertainty 35% in Mach number and mass for acceleration demand for $a_d = 100$.

is the observer error system where $\hat{e} \doteq \hat{x} - x$ is the observer state error and $\phi = \hat{\theta} - \theta$ is the parameter error.

Properties of the observer-based identifier in Eq. (43) are given in Sastry and Bodson (1989) and are

(1) $\phi \in \mathscr{L}_\infty$,
(2) with $\hat{e}(0) = 0, \phi(t) \leqslant \phi(0)\ \forall t \geqslant 0$,
(3) $\hat{e} \in \mathscr{L}_\infty \cap \mathscr{L}_2$,
(4) if $Z^{\mathrm{T}}(x, u_{ad})$ is bounded then $\hat{e} \in \mathscr{L}_\infty$ and $\hat{e} \to 0$ as $t \to \infty$,
(5) $\hat{e}$ and ϕ converge exponentially to zero if $Z(x, u)$ is sufficiently rich (i.e. $\exists \delta_1, \delta_2, \sigma > 0$ such that $\forall t$: $\delta_1 I \leqslant \int_t^{t+\sigma} ZZ^{\mathrm{T}}\, \mathrm{d}\tau \leqslant \delta_a I$.

However, since $Z(x, u)$ is a function of state x, condition 5 cannot be verified ahead of time.

The block diagram of our adaptive lateral flight control design for the nonlinear missile is shown in Fig. 7 while results of the nonadaptive and adaptive schemes are shown in Figs. 8 and 9, respectively. Good tracking performance for variation up to 35% in Mach number and mass is achieved.

5. Conclusions

Dynamic model inversion is the feedback linearisation method employed to design the missile autopilot. The main drawback of dynamic model inversion is the need for high-fidelity nonlinear force and moments models that must be invertible in real time, which implies a detailed knowledge of the plant dynamics, and the approach tends to be computationally intensive. In general, dynamic model inversion is sensitive to modelling errors. In this paper an adaptive nonlinear control design technique is applied to the autopilot for the missile model which is aerodynamically controlled. Missile motion is modelled to be nonlinear with unknown parameters. In the adaptive scheme used in this paper, unknown parameters are estimated and based on these estimates, control parameters are updated. Computer simulations show that this approach is very promising to apply the autopilot design for the missiles which are highly nonlinear in aerodynamics with unknown parameters.

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