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Magnetic spacecraft attitude control: a survey and some new results

Enrico Silani*, Marco Lovera

Dipartimento di Elettronica e Informazione, Politecnico di Milano 32, via Ponzio 34, Piazza Leonardo da Vinci, Milano 20133, Italy

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Abstract

The problem of attitude stabilization of a small satellite using magnetic actuators is considered, a review of the existing approaches based on linear and nonlinear control theory (with particular emphasis on periodic control) is proposed and a solution to the problem in terms of model-based predictive control is presented and analyzed. Simulation results are also given, which show the feasibility of the predictive approach.

Keywords: Attitude control; Model predictive control; Periodic systems; Spacecraft magnetic control; Time-varying systems

1. Introduction

Attitude control systems (ACS) play a fundamental role in the operation of spacecraft as they constitutes a mandatory feature both for the survival of a satellite and for the satisfactory achievement of mission goals. While a number of possible approaches to the control of attitude dynamics has been developed through the years, a particularly effective and reliable one is constituted by the use of electromagnetic actuators, which turn out to be specially suitable in practice for low Earth orbit (LEO) satellites. Such actuators operate on the basis of the interaction between a set of three orthogonal, current-driven magnetic coils and the magnetic field of the Earth (Wertz, 1978; Sidi, 1997) and therefore provide a very simple solution to the problem of generating torques on board a satellite. The major drawback of this control technique is that the torques which can be applied to the spacecraft for attitude control purposes are constrained to lie in the plane orthogonal to the magnetic field vector. In particular, three axis magnetic stabilization is only possible if the considered orbit "sees" a variation of the magnetic field which is sufficient to guarantee the stabilizability of the spacecraft (see Bhat & Dham, 2003).

The use of magnetic coils for control purposes has been the subject of extensive study since the early years of satellite missions (see White, Shigemoto, & Bourquin, 1961), both for attitude control and for momentum management on spacecraft controlled with reaction wheels (see Camillo & Markley, 1980) for an overview of magnetic momentum dumping). Until recent years, however, only approximate solutions to the problem of dealing with such time-varying actuators were available (see, e.g., Stickler & Alfriend, 1976; Arduini & Baiocco, 1997). In particular, while periodic control has been already proved successful in various applications (see, e.g., Arcara, Bittanti, & Lovera, 2000), the feasibility of periodic techniques for the control of small satellites using magnetic actuators has become only recently a topic of active research (see, e.g., the recent works (Pittelkau, 1993; De Marchi, Rocco, Morea, & Lovera, 1999; Wisniewski and Blanke, 1999; Wisniewski and Markley, 1999, Wang and Shtessel, 1999; Lovera, De Marchi, & Bittanti, 2002).

In the recent literature, both the linear and the nonlinear attitude control problems have been investigated; the former has been studied mostly in an LQ perspective, while the latter has been investigated using Lyapunov methods and exploiting periodicity of the Earth magnetic field in order to apply Krasovski–La Salle Theorem for the stability analysis. No actual periodic nonlinear controllers have been developed so far, to the best knowledge of the authors. In the light of the above considerations, the aim of this paper is

*Corresponding author. Tel.: +39-0223-993-650; fax: +39-022-3993-412.

E-mail address: silani@elet.polimi.it (E. Silani).

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twofold. First of all to present a detailed survey of the (numerous) approaches to the analysis and design of magnetic attitude control systems which have been proposed in the last 10–15 years. In addition, a novel approach to the magnetic attitude control problem, based on a predictive approach is proposed in this paper. The novel idea is to consider the magnetically controlled spacecraft as a *time invariant* system and to incorporate the time-varying actuators in the problem formulation via an appropriate set of constraints. It is possible to show that the state feedback problem can be given a simple closed-form solution, and that actuator saturation constraints can also be dealt with in the same framework. The proposed control design methodology has been tested in a simulation study for a spacecraft of the MITA class (see Della Torre, Lupi, Sabatini, & Coppola, 1999) and the (satisfactory) results obtained so far have been also reported.

The paper is organized as follows: in Section 2, a description of the dynamics of a spacecraft will be presented, and a linearized dynamic model derived; Sections 3 and 4 offer an overview of the relevant results in linear and nonlinear control which are available in the literature for this problem; Section 5 presents the (novel) predictive approach to the magnetic attitude control problems, while in Section 6 the results obtained with the predictive magnetic controller in a simulation study are shown.

2. Dynamics of a magnetically controlled spacecraft

2.1. Attitude dynamics

The attitude dynamics of a rigid spacecraft can be expressed by the well-known Euler's equations, as follows (Wertz, 1978):

$$\mathbf{I}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega} + \mathbf{T}_{mag} + \mathbf{T}_d, \tag{1}$$

where $\boldsymbol{\omega} \in \mathbb{R}^3$ is the vector of spacecraft angular rate, expressed in body frame, $\mathbf{I} \in \mathbb{R}^{3\times 3}$ is the inertia matrix, $\mathbf{T}_{mag} \in \mathbb{R}^3$ is the vector of magnetic control torques and $\mathbf{T}_d \in \mathbb{R}^3$ the vector of external disturbance torques.

The most common parameterization for spacecraft attitude is given by the four Euler parameters, which lead to the following representation for the attitude kinematics:

$$\dot{\mathbf{q}} = \tfrac{1}{2}\mathbf{W}(\boldsymbol{\omega})\mathbf{q}, \tag{2}$$

where $\mathbf{q} \in \mathbb{R}^4$ is the unit norm vector of Euler parameters and

$$\mathbf{W}(\boldsymbol{\omega}) = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}. \tag{3}$$

For the ACS of an Earth pointing spacecraft in a circular orbit the following reference systems are adopted:

- Orbital axes (X_0, Y_0, Z_0). The origin of these axes is in the satellite center of mass. The X-axis points to the Earth's center; the Y-axis points in the direction of the orbital velocity vector. The Z-axis is normal to the satellite orbit plane and completes the right-handed orthogonal triad.
- Satellite body axes (X, Y, Z). The origin of these axes is in the satellite center of mass.

Under nominal pointing conditions, the satellite body axes coincide with the orbital axes.

2.2. Attitude control actuators

On board small satellites the usual choice for the generation of external torques falls on a set of three magnetic coils, aligned with the spacecraft principal axes, which generate torques according to the law

$$\mathbf{T}_{mag} = \mathbf{m} \times \mathbf{b} = \mathbf{B}(\mathbf{b})\mathbf{m}, \tag{4}$$

where $\mathbf{m} \in \mathbb{R}^3$ is the vector of magnetic dipoles for the three coils (which represent the actual control variables for the coils), $\mathbf{b} \in \mathbb{R}^3$ is the vector formed with the components of the Earth's magnetic field in the body frame of reference and

$$\mathbf{B}(\mathbf{b}) = \begin{bmatrix} 0 & b_z & -b_y \\ -b_z & 0 & b_x \\ b_y & -b_x & 0 \end{bmatrix}. \tag{5}$$

As is well known (see, for example, Wertz, 1978; Sidi, 1997), spacecraft which correspond to the so-called *momentum bias* configuration, run (usually only) one momentum wheel at constant speed in order to provide gyroscopic stiffness to one spacecraft axis, according to Eq. (6). In this paper, we will focus on spacecraft belonging to this category, so that it will be assumed in the following that an appropriate control loop has been designed in order to keep the momentum wheel at a constant angular velocity $\boldsymbol{\nu}$ with respect to the body frame of reference.

The presence of wheels also introduces additional gyroscopic terms in the system's dynamics, which must be modified as follows:

$$\mathbf{I}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times [\mathbf{I}\boldsymbol{\omega} + \mathbf{J}\boldsymbol{\nu}] + \mathbf{T}_{mag} + \mathbf{T}_d, \tag{6}$$

where $\mathbf{J} \in \mathbb{R}^{3\times 3}$ represents the overall wheels inertia matrix wrt the body frame.

2.3. Disturbance torques

External disturbance torques occur naturally and have different sources, such as gravity gradient, aerodynamics, solar radiation and residual magnetic dipoles.

Their size is a function of the mass properties (position of the center of mass and inertia matrix), orbit characteristics and mission lifetime. Regardless of the physical mechanism giving rise to them (i.e., magnetic, aerodynamic, solar, gravity gradient), they can be separated into a secular component (i.e., the part with nonzero mean around each orbit) and a cyclic component (i.e., the zero mean, periodic part, with a period given by the orbit period). See, e.g., (Annoni, Marchi, Diani, Lovera, & Morea, 1999; Lovera, 2003) for an overview of issues related to the simulation of spacecraft dynamics.

2.4. Linearized dynamics

Assuming a momentum biased, Earth pointing configuration for the spacecraft (i.e., one momentum wheel, aligned with the body z-axis) and defining the states for the linearized system as

$$\delta\mathbf{q} = [\,\delta q_1 \quad \delta q_2 \quad \delta q_3 \quad 0\,]', \tag{7}$$

$$\delta\boldsymbol{\omega} = \boldsymbol{\omega} - [\,0 \quad 0 \quad -\Omega\,]', \tag{8}$$

which corresponds to introducing small displacements from the nominal attitude quaternion $q_1 = q_2 = q_3 = 0$, $q_4 = 1$, and small deviations of the body rates from the nominal ones $\omega_x = \omega_y = 0$, $\omega_z = -\Omega$, Ω being the angular frequency associated with the orbit period. Introducing the state vector

$$\delta\mathbf{x} = [\,\delta q_1 \quad \delta q_2 \quad \delta q_3 \quad \delta\boldsymbol{\omega}'\,]' \tag{9}$$

the above Eqs. (2) and (6) can be linearized and the local linear dynamics for the system defined as

$$\dot{\delta\mathbf{x}} = \mathbf{A}\delta\mathbf{x} + \begin{bmatrix} 0 \\ \mathbf{I}^{-1} \end{bmatrix} (\mathbf{T}_c + \mathbf{T}_d), \tag{10}$$

where

$$\mathbf{A} = \frac{\partial \mathbf{f}(x,t)}{\partial \mathbf{x}}\Big|_{\mathbf{x}=\mathbf{x}_{nom}} = \begin{bmatrix} \frac{\partial \dot{\mathbf{q}}}{\partial \mathbf{q}} & \frac{\partial \dot{\mathbf{q}}}{\partial \boldsymbol{\omega}} \\ \frac{\partial \dot{\boldsymbol{\omega}}}{\partial \mathbf{q}} & \frac{\partial \dot{\boldsymbol{\omega}}}{\partial \boldsymbol{\omega}} \end{bmatrix}\Big|_{\mathbf{x}=\mathbf{x}_{nom}}. \tag{11}$$

Remembering now that the control torques generated by the magnetic coils are given by Eq. (4), the overall linearized model takes the form

$$\dot{\delta\mathbf{x}} = \mathbf{A}\delta\mathbf{x} + \begin{bmatrix} 0 \\ \mathbf{I}^{-1} \end{bmatrix} (\mathbf{B}(\mathbf{b})\mathbf{m} + \mathbf{T}_d). \tag{12}$$

Clearly, if the time variation of the magnetic field is fundamentally a periodic one, this model can be seen as a linear time-periodic one.

2.5. Periodic approximation of the magnetic field

A time history of the International Geomagnetic Reference Field (IGRF) model for the Earth's magnetic field (Wertz, 1978) along five orbits with respect to the orbital axes for a spacecraft in a circular, polar orbit (87^o inclination, 450 km altitude) is shown in Fig. 1 (solid lines).

A periodic approximant of the magnetic field can derived, by least-squares fitting of the output of the IGRF model to a simplified periodic structure such as the following:

$$\begin{aligned}\mathbf{b}(t) = {} & \mathbf{b}_o + \mathbf{b}_{1c}\cos(\Omega t) + \mathbf{b}_{1s}\sin(\Omega t) \\ & + \mathbf{b}_{2c}\cos(2\Omega t) + \mathbf{b}_{2s}\sin(2\Omega t).\end{aligned} \tag{13}$$

Using data from five orbits, corresponding to 28.070 s, the results of the least-squares fit are summarized in Fig. 1, where the time histories of the IGRF and approximate periodic magnetic field models are given. As can be seen, $b_x(t)$, $b_y(t)$ have a very regular and almost periodic behavior, while the $b_z(t)$ component is much less regular. This behavior can be easily interpreted by noticing that when the spacecraft is in the nominal attitude, the x and y body axes lie in the orbit plane while the z axis is normal to it. As a consequence, the x and y magnetometers sense only the variation of the magnetic field due to the orbital motion of the spacecraft (period equal to the orbit period) while the z-axis sensor is affected by the variation of $\mathbf{b}$ due to the rotation of the Earth (period of 24 h).

Combining the linearized dynamics derived in the previous section with the periodic approximation of the magnetic field obtained herein, one gets a complete periodic model of the local dynamics of the spacecraft.

2.6. Controllability issues

It is clear from Eq. (5) that the $\mathbf{B}$ matrix is structurally of rank two, so the choice of magnetic actuators implies that the spacecraft is instantaneously underactuated, i.e., at each time instant it is not possible to apply a control torque to all three spacecraft axes in an independent manner. Full controllability is only guaranteed provided that the variations in the direction of the magnetic field experienced by the spacecraft along each orbit are such that

$$\bigcup_{t=0,T} span_{col}(\mathbf{B}(t)) = R^3$$

that is, if the quasi-periodic variation of the geomagnetic field guarantees that it is possible, at least on average, to apply torques with arbitrary direction (see, e.g., Wisniewski, 2000; Bhat & Dham (2003) for additional details). This condition depends considerably on the inclination of the selected orbit: it turns out that magnetic actuators give rise to almost uncontrollable dynamics in equatorial orbit; controllability characteristics tend to improve with orbit inclination.

In particular, when considering an ideal circular polar orbit and the simple dipole approximation for the geomagnetic field (see, e.g., Wertz, 1978), it is possible to

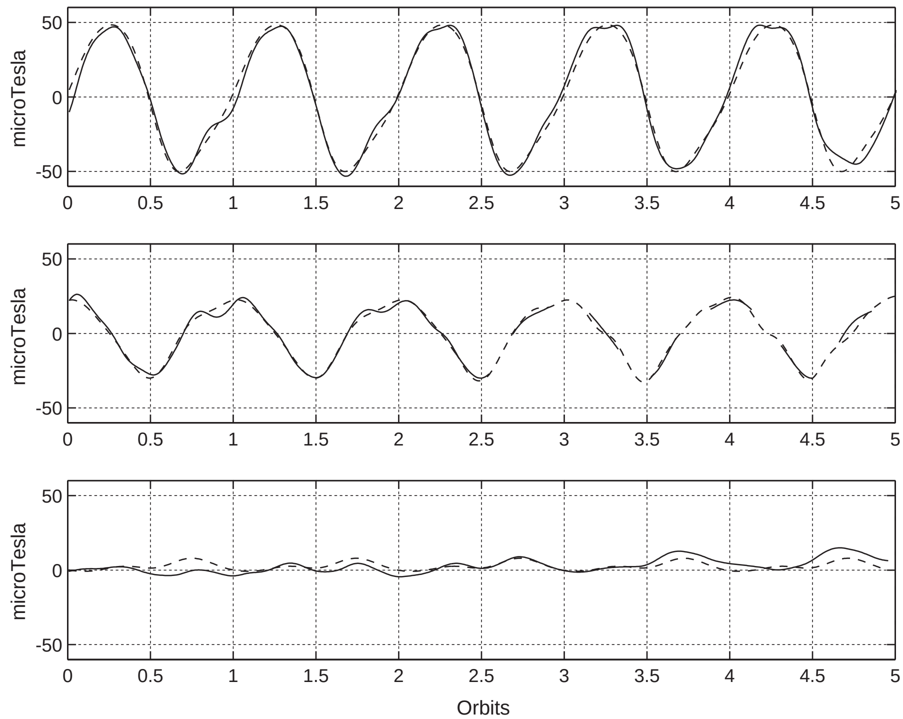

Fig. 1. Periodic approximation of the geomagnetic field in Pitch-Roll-Yaw coordinates, 87^o inclination orbit, 450 km altitude.

show that the controllability Gramian for the spacecraft dynamics is always nonsingular.

3. Linear design methods

The main issues in the design of a linear attitude controller based on magnetic actuators are the following:

(1) As shown in the previous section, the local dynamics of a magnetically controlled spacecraft is (approximately) time periodic, therefore the stabilization problem cannot be faced by means of conventional LTI design tools.
(2) The spacecraft is subject to external disturbances, which can be decomposed in a secular (i.e., constant) and a cyclic (i.e., periodic, with period equal to the orbit period $T = 2\pi/\Omega$); the controller must try and attenuate the effect of such disturbances on the attitude.
(3) Robustness issues: the dynamics of the spacecraft is subject to a number of sources of uncertainty, the most important of which are the moments of inertia of the spacecraft and the actual behavior of the magnetic field.
(4) Implementation issues: periodic gains are not easily implemented on board, as they give rise to a sensitive synchronization problem. The design of a periodic controller which can exploit on board measurements of the magnetic field vector remains an open problem.
(5) Actuator saturation: magnetic coils can be only driven with limited currents, so that saturation of the control inputs (magnetic dipoles) can be an issue, particularly when dealing with large attitude errors and angular rates.

In this section, the main contributions given so far in the literature (dealing with points 1 and 2 in the above list) will be reviewed and the application of periodic H_∞ control theory will be proposed, in order to deal with point 3 in the list.

3.1. Classical design methods

Magnetic coils have been used as actuators in spacecraft control systems for a long time (see Stickler & Alfriend (1976) and the references therein). In particular, control techniques for both attitude acquisition (see Section 4) and nominal attitude regulation have been developed. Concerning attitude regulation, linear, partial state feedback control laws have been extensively used, the closed-loop stability of which was usually checked a posteriori using Floquet theory (D' Angelo, 1970; Bittanti & Colaneri, 1999) by exploiting the quasiperiodicity of the geomagnetic field, or by carrying out a stability analysis for a time-invariant approximation of the magnetically controlled spacecraft (see Camillo & Markley, 1980; Hablani, 1995) for an overview of this approach to stability analysis).

The reason why approximate constant coefficient stability studies can provide useful results can be understood by looking at the operation of magnetic control laws in a geometric way, as originally noted in (Martel, Pal, & Psiaki, 1988) (see also Arduini & Baiocco, 1997). Assume initially that three independent control torques can be applied to the satellite and that, on the basis of this assumption, a conventional PD control law (which guarantees attitude regulation in the case of three controls, see (Wen & Kreutz-Delgado, 1991)) of the form

$$\mathbf{T}_{ideal} = k_p[\,\delta q_1 \quad \delta q_2 \quad \delta q_3\,]' + k_v\delta\boldsymbol{\omega} \tag{14}$$

has been designed. The constraint dictated by the magnetic actuators is such that only the component of $\mathbf{T}_{ideal}$ orthogonal to the magnetic field vector $\mathbf{b}$ can be actually applied to the spacecraft, so the best possible control policy is to apply the magnetic dipole given by

$$\mathbf{m} = \frac{1}{|\mathbf{b}|^2}\mathbf{B}(\mathbf{b})'\mathbf{T}_{ideal}, \tag{15}$$

which, according to Eq. (4), corresponds to the application of the magnetic control torque

$$\mathbf{T}_{mag} = \mathbf{B}(\mathbf{b})\mathbf{m} = \frac{1}{|\mathbf{b}|^2}\mathbf{B}(\mathbf{b})\mathbf{B}(\mathbf{b})'\mathbf{T}_{ideal}. \tag{16}$$

Eq. (16) can be also interpreted geometrically as a projection of $\mathbf{T}_{ideal}$ onto the plane orthogonal to the local direction of $\mathbf{b}$ (see Fig. 2) and provides some insight in the operation of the magnetic actuators.

Therefore, as long as the closed-loop dynamics is sufficiently slow, the stability of the time-varying closed-loop system can be approximately studied on the basis of its time-invariant approximations. This leads to a simple, though empirical, tuning rule: first choose the gains k_p and k_v such that the *ideal* closed-loop system is stabilized, with sufficiently slow closed-loop dynamics. Subsequently, check, e.g., by means of Floquet theory, that stability is preserved when dealing with the magnetic constraints.

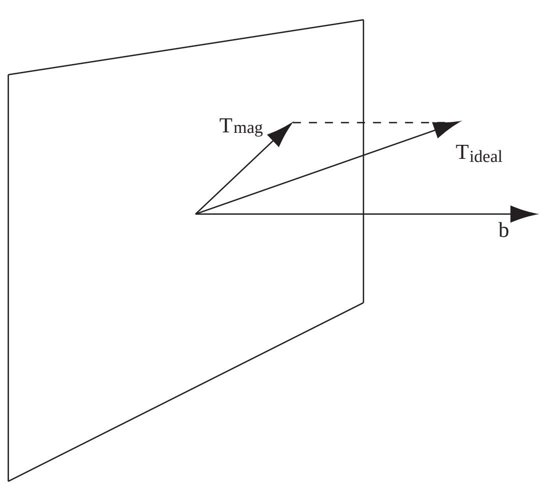

Fig. 2. Definition of the magnetic control torque $\mathbf{T}_{mag}$ by projection of the ideal torque $\mathbf{T}_{ideal}$.

More recently, the availability of accurate attitude sensors, such as star sensors, combined with modern control and filtering techniques for time-varying systems has shifted the attention to full state feedback formulations of the problem.

3.2. Optimal periodic control

Assuming that the periodic approximation for the geomagnetic field is satisfactory, the resulting periodic dynamics for the spacecraft can be stabilized either by state or output periodic feedback. A number of contributions have been given in the literature to the analysis of the LQ magnetic attitude control problem. In (Pittelkau, 1993) both output feedback stabilization and disturbance attenuation for a momentum biased spacecraft have been analyzed; in particular, the disturbance issue has been addressed by means of an internal model principle approach. In order to avoid the difficulties in the solution of the LQ problems due to the presence of uncontrollable modes with poles on the imaginary axis, the secular and cyclic components of the external disturbances have been modeled with stable models (first order with a large time constant fore the secular part and second order with small damping for the cyclic part). This gives rise to a time periodic filter and a time periodic state feedback. In Lovera et al. (2002), De Marchi et al. (1999) the problem is analyzed in a similar fashion, by using an extension of the periodic LQ control problem (initially proposed in Arcara et al. (2000)), by which it is possibile to include marginally stable disturbance models in the plant description. The resulting control design method is applied in a simulation study for the Italian spacecraft MITA. A similar approach has been also proposed in Wisniewski (1996), Wisniewski and Blanke (1999), Wisniewski and Markley (1999), where the sole state feedback problem is considered. More recently, an LMI approach to the design of H_2-optimal periodic controllers was proposed in Wisniewski & Stoustrup (2002).

A related line of work aims at taking into account the time variability of the system avoiding the explicit implementation of a periodic gain. In Wisniewski (2000) the use of a finite horizon controller is suggested, while other attempts at avoiding the use of a periodic control law by means of piecewise constant gains computed on board have been reported in Steyn (1994), Lagrasta and Bordin (1996), Curti and Diani (1999). While such techniques are interesting from an implementation point of view, they have the obvious drawback that no guarantees about closed-loop stability of the system can be obtained. A different approach to the problem of obtaining constant gains via optimal periodic control theory has been explored in Psiaki (2000, 2001). The idea is to analyze the asymptotic behavior of the solutions of the periodic Riccati equation in the limiting case of infinite control weighting, i.e., when in the LQ cost function

$$J = \int_0^\infty [\mathbf{x}(t)'\mathbf{Q}(t)\mathbf{x}(t) + \mathbf{u}(t)'\mathbf{R}(t)\mathbf{u}(t)]\,dt$$

one chooses

$$\mathbf{R}(t) = \frac{\mathbf{R}_0}{\varepsilon}$$

and $\varepsilon \to 0$. It turns out that, under suitable assumptions, the limiting solution is a constant one and it can be computed in an approximate way by means of an appropriate averaging scheme, for a small, but finite, ε. This approach has the advantage of providing a constant state feedback gain; unfortunately, the approximations which are introduced in the computation of the limiting solution of the Riccati equation do not guarantee that the computed gain is a stabilizing one, so that a posteriori verification via Floquet's theorem remains necessary. Similar ideas have been discussed in Varga and Pieters (1998).

3.3. The H_∞ approach

The H_∞ approach to the problem has been first proposed in Lovera (2000) for the design of state feedback attitude controllers and in Lovera (2001) for the implementation of momentum management control laws based on magnetic actuators. In the cited contributions, however, no attempt has been made to exploit the H_∞ setting in order to enforce robustness in the design procedure.

3.3.1. H_∞ periodic state feedback control

Given the T-periodic, linear, continuous-time system:

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}_1(t)\mathbf{w} + \mathbf{B}_2(t)\mathbf{u},$$

$$\mathbf{z} = \mathbf{C}(t)\mathbf{x} + \mathbf{D}(t)\mathbf{u} \tag{17}$$

with $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{u} \in \mathbb{R}^m$, under the assumptions that

(1) $\mathbf{C}(t)'\mathbf{D}(t) = 0$, $\forall t$.
(2) $\mathbf{D}(t)'\mathbf{D}(t) = I$, $\forall t$.
(3) $(\mathbf{A}(\cdot), \mathbf{B}_2(\cdot))$ is stabilizable

the state feedback H_∞ periodic control problem can be stated as follows: find the linear, time periodic state feedback control law, which stabilizes the system and guarantees that $\|\mathbf{T}_{zw}\|_\infty < \gamma$ (see, e.g., Bittanti & Colaneri 1999, Colaneri, 2000a,b) for the formal definition of the H_∞ norm for periodic systems).

This leads to the optimal control law $\mathbf{u}(t) = -\mathbf{B}_2(t)'\mathbf{P}_\infty(t)\mathbf{x}(t)$ where $\mathbf{P}_\infty(t)$ is the symmetric, positive definite solution of the H_∞ periodic Riccati equation

$$\mathbf{P}(t)\mathbf{A}(t) + \mathbf{A}(t)'\mathbf{P}(t) + \mathbf{P}(t)(\gamma^{-2}\mathbf{B}_1(t)\mathbf{B}_1(t)' + \mathbf{B}_2(t)\mathbf{B}_2(t)')\mathbf{P}(t) + \mathbf{C}(t)'\mathbf{C}(t) = -\dot{\mathbf{P}}(t).$$

An H_∞ filter can be designed, either for the sole spacecraft dynamics or for a model augmented with an appropriate description of the external disturbance torques (see Lovera (2000) for details). Clearly, the choice of an H_∞ approach in the formulation of the control problem makes it possible to take into account robustness issues in the tuning of the controller gain. Dealing with parametric and nonparametric uncertainties in the design of magnetic controllers will be the subject of the following section.

3.3.2. Robust H_∞ magnetic attitude control

Considering the linearized equations of motion (12) given in the previous Section, one has that the main sources of uncertainty in the model can be reduced to the following:

- Discrepancies between the ideal periodic behavior of the magnetic field and the actual one (see Fig. 1): $\mathbf{b}(t) = \bar{\mathbf{b}}(t) + \delta\mathbf{b}(t)$, where $\bar{\mathbf{b}}(t+T) = \bar{\mathbf{b}}(t)$ and $\|\delta\mathbf{b}(t)\| \leqslant \delta\mathbf{b}_{MAX}$. The bound $\delta\mathbf{b}_{MAX}$ can be constructed from high-order geomagnetic field models or in-flight measurements.
- Uncertainty in the measurement of the inertial properties of the spacecraft.

Other possible sources of uncertainty are mounting errors in sensors and actuators (which however can be taken into account by means of suitable calibration procedures) and the presence of unmodelled high-frequency dynamics such as due to large flexible appendages, which will not be considered herein.

Clearly, the above two parametric uncertainties can be dealt with in an H_∞ framework by writing the attitude equations in LFT form and introducing suitable scalings in order to meet the assumptions of classical H_∞ theory.

4. Nonlinear design methods

Nonlinear control plays a major role in the attitude stabilization problem, as every spacecraft, in the initial part of its life, will have to deal with an "attitude acquisition" phase, in which usually large rotations and high angular rates must be dealt with. Perhaps the most popular control law for spacecraft detumbling and initial acquisition using magnetic actuators is the so-called "b-dot" algorithm, which was originally proposed in Stickler and Alfriend (1976). The "b-dot" control law is given by

$$\mathbf{m} = -\mathbf{K}\dot{\mathbf{b}}, \tag{18}$$

where $\mathbf{K}$ is a positive definite gain matrix. Eq. (18) is equivalent to a (time-varying) dissipative control law, as, under some approximations, $\dot{\mathbf{b}}$ can be related to the spacecraft angular rate as follows:

$$\dot{\mathbf{b}} \simeq \mathbf{b} \times \boldsymbol{\omega} \tag{19}$$

so that it can be shown that this control law can be effectively used to the reduce the kinetic energy of the satellite. Note that (18) can be easily implemented on board as magnetometers are usually accurate enough to allow for numerical differentiation of the magnetic field measurements.

More recently, attention has been dedicated to the nonlinear magnetic attitude control problem in a state feedback setting. In particular, while no specific periodic attitude control laws have been developed so far in the literature, extensive use has been made of periodic systems theory in order to guarantee (at least locally) closed-loop stability for a number of attitude acquisition control laws. We mention the recent works (Wang & Shtessel, 1999; Wisniewski & Blanke, 1999). It is interesting to point out that the stability proof for the desired equilibrium state of the closed-loop system given in Wisniewski and Blanke (1999) relies on the 24 h periodicity of the geomagnetic field rather than on its orbital periodicity, as is commonly done in local linear analyses, in which, as mentioned in the previous section, variations of **b** due to the rotation of the Earth are rather treated as uncertain variations of the periodic parameters.

Also, while a stability analysis in the most general case is a very difficult task, some results have been proven concerning the following cases of practical interest:

- Case of an inertially pointing spacecraft: conditions for (almost) global attitude regulation have been obtained in Lovera and Astolfi (2001), Astolfi and Lovera (2002) for the case of full state feedback, and local results have been derived for the case of output (attitude only) feedback.
- Case of an Earth pointing spacecraft, subject to gravity gradient: an adaptive, globally convergent state feedback control law has been presented in Lovera and Astolfi (2003).

Finally, a related line of work has been recently devoted to the nonlinear analysis of a magnetic attitude control scheme based on the sole measurement of the magnetic field vector **b**. The achievable results have been recently reported in Bushenkov, Ovchinnikov, and Smirnov (2002).

5. Magnetic predictive attitude control

The aim of this section is to propose a novel approach to the magnetic attitude control problem based on the application of predictive control ideas. Precisely, the magnetically controlled spacecraft is modelled as a time-invariant system with three independent control inputs, and the presence of magnetic actuators is taken into account via a suitable set of constraints.

5.1. MPC: state-space formulation

The generic model-based predictive control design is based on a *receding-horizon* strategy in which at each sample instant k:

- the model is used to predict the output response to a certain set of future control signals;
- a function including the cost of future control actions and future deviations from the reference is optimized to obtain the 'best future control sequence';
- only the first control of the sequence is asserted, and the entire operations repeated at time $k+1$.

Let us consider a multi-variable process with n outputs and m inputs, described by the following state-space model:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{P}\mathbf{v}(k),$$
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{w}(k),$$

where $\mathbf{x}(k)$ is the state vector, $\mathbf{y}(k)$ is the output vector, $\mathbf{v}(k)$ and $\mathbf{w}(k)$ are the noise affecting the process and the output, respectively.

The optimal j-ahead prediction of the output is given by Camacho and Bordons (1998):

$$\hat{\mathbf{y}}(k+j\,|\,k) = \mathbf{C}\mathbf{A}^{j}\mathbb{E}[\mathbf{x}(k)] + \sum_{i=0}^{j-1} \mathbf{C}\mathbf{A}^{j-i-1}\mathbf{B}\mathbf{u}(k+i).$$

Let us consider a set of N_c (control horizon) j-ahead predictions:

$$\mathbf{Y} = \begin{bmatrix} \hat{\mathbf{y}}(k+1\,|\,k) \\ \hat{\mathbf{y}}(k+2\,|\,k) \\ \vdots \\ \hat{\mathbf{y}}(k+N_c\,|\,k) \end{bmatrix} = \begin{bmatrix} \mathbf{C}\mathbf{A}\mathbb{E}[\mathbf{x}(k)] + \mathbf{C}\mathbf{B}\mathbf{u}(k) \\ \mathbf{C}\mathbf{A}^2\mathbb{E}[\mathbf{x}(k)] + \sum_{i=0}^{1} \mathbf{C}\mathbf{A}^{1-i}\mathbf{B}\mathbf{u}(k+i) \\ \vdots \\ \mathbf{C}\mathbf{A}^{N_c}\mathbb{E}[\mathbf{x}(k)] + \sum_{i=0}^{N_c-1} \mathbf{C}\mathbf{A}^{N_c-1-i}\mathbf{B}\mathbf{u}(k+i) \end{bmatrix},$$

which can be expressed as

$$\mathbf{Y} = \mathbf{\Upsilon}\hat{\mathbf{x}}(k) + \mathbf{\Gamma}\mathbf{U},$$

where $\hat{\mathbf{x}}(k) = \mathbb{E}[\mathbf{x}(k)]$, $\mathbf{\Gamma}$ is a block-lower triangular matrix with its nonnull elements defined by $(\mathbf{\Gamma})_{ij}=$

$\mathbf{CA}^{i-j}\mathbf{B}$ and matrix $\mathbf{Y}$ is defined as

$$\mathbf{Y} = \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{N_c} \end{bmatrix}$$

and $\mathbf{U} = [\mathbf{u}(k)' \dots \mathbf{u}(k+N_c-1)']'$ is the vector of the $\dot{N}_c$ future control actions.

Remark 1. The prediction equation requires an unbiased estimation of the state vector $\mathbf{x}(k)$. If the state vector is not accessible, a Kalman filter is required.

At each step, the control action $\mathbf{u}(k)$ is obtained by minimizing with respect to the sequence $\mathbf{U}$ of future control moves the following performance index:

$$J(k) = (\mathbf{Y} - \mathbf{Y}^o)'\mathbf{Q}(\mathbf{Y} - \mathbf{Y}^o) + \mathbf{U}'\mathbf{R}\mathbf{U}, \tag{20}$$

where $\mathbf{Y}^o = [\mathbf{y}^o(k+1)' \cdots \mathbf{y}^o(k+N_c)']'$ is the vector of the future evolution of the reference trajectory, $\mathbf{R} > 0$ and $\mathbf{Q} \geqslant 0$.

5.2. Problem formulation

In this paper, the idea is to consider the system as a linear time-invariant one and to take into account magnetic actuators by inserting a *constraint* on the magnetic control torques, which guarantees the orthogonality between the geomagnetic field vector $\mathbf{b}$ and the magnetic control vector $\mathbf{T}_{mag}$.

To this purpose, let us consider the overall linearized system (12) together with the performance index (20), where the control variable is $\mathbf{u} = \mathbf{T}_{mag}$, and the constraints

$$\mathbf{b}'(k)\mathbf{T}_{mag}(k) = \mathbf{b}'(k)\mathbf{u}(k) = 0. \tag{21}$$

Eq. (21) can be expressed in term of $\mathbf{U}$ as

$$\mathbf{\Psi}\mathbf{U} = [\,\mathbf{b}'(k)0 \dots 0\,]\mathbf{U} = 0 \tag{22}$$

and by the use of Lagrange multipliers (Luenberger, 1984), the optimal solution of the constrained minimization problem defined by (12), (20) and (22) is given by

$$\begin{aligned} \mathbf{U}_{opt} = &[\mathbf{\Lambda}(I - \mathbf{\Psi}'(\mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}')^{-1}\mathbf{\Psi}\mathbf{\Lambda})] \\ &\mathbf{\Gamma}'\mathbf{Q}(\mathbf{Y}^o - \mathbf{Y}\hat{\mathbf{x}}) \end{aligned} \tag{23}$$

where

$$\mathbf{\Lambda} = (\mathbf{\Gamma}'\mathbf{Q}\mathbf{\Gamma} + \mathbf{R})^{-1}.$$

Note that $\mathbf{\Lambda}$ is always non singular, due to the choice of the weights $\mathbf{R} > 0$ and $\mathbf{Q} \geqslant 0$.

According to a receding horizon strategy, $\mathbf{U}^{opt}$ is computed at every sampling time, while only the first element of $\mathbf{U}^{opt}$, $\mathbf{u}^{opt}(k)$, is effectively used to control the system.

Finally, the vector of the coils' magnetic dipoles $\mathbf{m}(k)$ can be determined on the basis of $\mathbf{u}(k) = \mathbf{T}_{mag}(k)$ in the following way. First note that Eq. (4) implies

$$\begin{aligned} \mathbf{T}_{mag}(k)'\mathbf{T}_{mag}(k) &= [\mathbf{m}(k) \times \mathbf{b}(k)]'\mathbf{T}_{mag}(k) \\ &= [\mathbf{b}(k) \times \mathbf{T}_{mag}(k)]'\mathbf{m}(k). \end{aligned} \tag{24}$$

In addition, $\mathbf{m}(k)$ must satisfy orthogonality conditions with respect to $\mathbf{T}_{mag}(k)$ and $\mathbf{b}(k)$, so one must have

$$\begin{aligned} &\mathbf{T}_{mag}(k)'\mathbf{m}(k) = 0, \\ &\mathbf{b}(k)'\mathbf{m}(k) = 0. \end{aligned} \tag{25}$$

Note that the second of Eq. (25), which enforces the orthogonality between $\mathbf{b}(k)$ and $\mathbf{m}(k)$, is a design degree of freedom which can be used to minimize the norm of the vector of the coils' magnetic dipoles.

The vector of the coils' magnetic dipoles $\mathbf{m}(k)$ can be calculated directly from Eqs. (24) and (25) and is given by

$$\mathbf{m}^{opt}(k) = \begin{bmatrix} (\mathbf{b}(k) \times \mathbf{T}^{opt}_{mag}(k))' \\ \mathbf{T}^{opt}_{mag}(k)' \\ \mathbf{b}(k)' \end{bmatrix}^{-1} \begin{bmatrix} |\mathbf{T}^{opt}_{mag}(k)|^2 \\ 0 \\ 0 \end{bmatrix},$$

which can be equivalently written as

$$\mathbf{m}^{opt}(k) = \frac{\mathbf{b}(k) \times \mathbf{T}^{opt}_{mag}(k)}{|\mathbf{b}(k)|^2} = -\frac{\bar{\mathbf{B}}(\mathbf{b}(k))}{|\mathbf{b}(k)|^2}\mathbf{T}^{opt}_{mag}(k). \tag{26}$$

Remark 2. The control problem is well posed since the orthogonality between $\mathbf{u}(k)$ and $\mathbf{b}(k)$ is a necessary and sufficient condition for the existence of a magnetic dipole $\mathbf{m}^{opt}(k)$ such that $\mathbf{T}^{opt}_{mag}(k) = \mathbf{m}^{opt}(k) \times \mathbf{b}(k)$. In addition, this approach to magnetic control, which has been illustrated in the case of attitude stabilization only, can be easily applied in the more general case of combined attitude and momentum control.

Remark 3. Eq. (26) is equivalent to the projection of the vector $\mathbf{T}^{opt}_{mag}(k)$ onto the plane orthogonal to the local direction of the geomagnetic field vector $\mathbf{b}(k)$. In many applications, the attitude control problem using magnetic actuators is solved by computing an ideal optimal control vector and then by applying only the projection of this vector onto the plane orthogonal to $\mathbf{b}(k)$ (see e.g., Arduini & Baiocco, 1997), so that constraints (22) are not dealt with explicitly. Here the main difference is that at each time step the optimal magnetic control torque is a priori chosen among the set of all the admissible control vectors (i.e., the vectors orthogonal to the geomagnetic field).

Remark 4. From the inspection of Eqs. (22), (23) and (26) it is clear that the optimal magnetic dipole $\mathbf{m}^{opt}(k)$ is a function of the geomagnetic field vector. In particular, by plugging a suitable periodic magnetic field model into Eqs. (22) and (26) one obtains a conventional periodic magnetic control law.

5.3. Actuators' saturation

Constraints on the coils' magnetic dipoles play a major role in the formulation of the magnetic attitude control problem. Such constraints (which act componentwise on each of the control variables) are of the form

$$\mathbf{m}_{min} \leqslant \mathbf{m}(k) \leqslant \mathbf{m}_{\max},$$

which can be translated into constraints on the control variable $\mathbf{T}_{mag}$ using Eq. (26) to have

$$\mathbf{m}_{min} \leqslant \frac{\mathbf{b}(k) \times \mathbf{T}_{mag}}{|\mathbf{b}(k)|^2} \leqslant \mathbf{m}_{\max},$$

which in turn implies

$$\begin{bmatrix} -\bar{\mathbf{B}}(\mathbf{b}(k)) \\ \bar{\mathbf{B}}(\mathbf{b}(k)) \end{bmatrix} \mathbf{T}_{mag} \leqslant \begin{bmatrix} \mathbf{m}_{max} \\ \mathbf{m}_{min} \end{bmatrix} |\mathbf{b}(k)|^2. \tag{27}$$

One of the advantages of the predictive approach to magnetic attitude control is that constraints on the control action can be directly included in the formulation of the problem. Clearly, if one of these constraints becomes active then the control law becomes nonlinear and cannot be written in closed form.

5.4. Stability analysis

The proposed approach to the design of predictive attitude control laws does not guarantee closed-loop stability a priori, as is usually the case with finite horizon predictive control schemes (Camacho & Bordons, 1998).

The closed-loop stability of magnetic attitude control laws has been usually dealt with in the literature by exploiting the *quasi*periodic nature of the geomagnetic field and by resorting the theory of optimal periodic control (either LQ or H_∞, see e.g., (Pittelkau, 1993; Wisniewski & Markley, 1999; Psiaki, 2001; Lovera, 2001; Lovera et al. 2002)) to obtain *nominal* or *robust* stability results.

When dealing with the stability analysis of the predictive formulation of the magnetic control problem, therefore, many possibilities are available; we will discuss the possible choices by considering separately the unconstrained and the constrained cases. In the unconstrained case, the first possibility would be simply to check a posteriori, by means of Floquet theory (see, e.g., Bittanti & Colaneri, 1999), that the choice of weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ in the design of the predictive control laws guarantees that the *nominal* periodic closed-loop dynamics is asymptotically stable. This approach has been used, for example, in Psiaki (2001).

A less empirical approach to ensure closed-loop stability would be by means of an appropriate terminal constraint in the formulation of the control problem, as is common practice in the predictive control literature (see, for example, the classical works (Kwon & Pearson, 1978; Rawlings & Muske, 1993; Clarke & Scattolini, 1991). It is interesting to point out that the controllability assumptions which are needed in order to apply the results given in Kwon and Pearson (1978) are guaranteed for all practical purposes in the case of most orbits of interest (see Bittanti & Colaneri, 1999, for the definition of controllability for periodic systems and (Wisniewski, 1996; Lovera & Astolfi, 2001) for an analysis of the average controllability of magnetically controlled spacecraft).

A third possibility to carry out a stability analysis is to notice that the proposed approach leads to a control law which has a very similar (PD-like) structure to the one which has been studied in Lovera and Astolfi (2001), where conditions for continuous time, possibly saturated, global magnetic attitude regulation have been obtained for the case of an inertially pointing spacecraft. Future work will aim at bridging the gap between the continuous time analysis and the discrete time implementation of the controller.

Finally, concerning the constrained control problem: a large body of literature exists now on the stability of constrained MPC (see, e.g., Rawlings & Muske, 1993, Camacho & Bordons, 1998; Mayne, Rawlings, Rao, & Scokaert, 2000), mainly relating the closed-loop asymptotic stability of the desired equilibrium of the constrained control system to the feasibility of the corresponding optimization problem. For the specific case of attitude and momentum control, however, it is worth to point out that whenever feasibility problems arise, e.g., because of an excessive initial angular rate of the satellite, it is always possible to bring back the system to a feasible initial state by means of an auxiliary, usually nonlinear and dissipative control law (see the classical paper (Stickler & Alfriend, 1976)).

6. A simulation case study

In this section, some simulation results obtained by the application of the above-described model-based predictive control techniques to the dynamic of the MITA spacecraft are presented.

6.1. The simulation environment

A library of tools for the modelling and simulation of spacecraft dynamics based on the Modelica language is currently being developed (Lovera, 2003). Modelica is a language for hierarchical, object-oriented modelling of physical systems, which is being developed by the *Modelica Association*. The language has been developed in order to combine the flexibility of a high-level description language with an efficient development and code generation tool. As discussed in detail in Fritzson and Bunus (2002), the main features of Modelica are the following: first of all, the language is object oriented and is based on (acausal) equations rather than on assignments, which makes the description of physical systems much easier and natural; Modelica can be easily used to represent components which define interactions between different physical domains; finally, Modelica models are created by defining components and using the appropriate constructs to represent connections between components.

In particular, whenever a system model can be formulated in terms of connections/interactions of components belonging to predefined classes, Modelica provides the means of deriving a very natural description of the system itself. Modelica turns out to be specially suited for the modelling of spacecraft dynamics under many respects:

- Coordinate frames can be simply included in the model in terms of connectors, describing kinematic transformations from one coordinate system to another.
- Spacecraft dynamics is modelled by defining a Spacecraft class which can be (almost) directly implemented in terms of equations. The data structure to be used in representing all the quantities involved in a specific spacecraft model arises naturally during the modelling process.
- Specific Modelica constructs are available to deal with the modelling of physical fields and environmental quantities. This feature turns out to be extremely useful in modelling the space environment and representing the interaction between the environment and the spacecraft. In particular, with a suitable choice of the environment interfaces, models of increasing complexity for, e.g., the gravitational or the geomagnetic field can be defined.
- Sensors and actuators can also be easily represented in the Modelica paradigm. For example, the generation of magnetic torques is modelled in terms of the interaction with the geomagnetic field, while the momentum exchange between spacecraft and momentum/reaction wheels is modelled via a suitably defined mechanical connector.
- Packages of data sheets for each class can be constructed and components easily modified within each spacecraft model, using Modelica's advanced features (see, e.g., Otter & Olsson, 2002).
- Finally, as the components of the library are independent from each other, one can exploit this flexibility in order to build a simulation model of increasing complexity and accuracy according to the needs associated with each phase of the development process of an actual attitude control system.

The main components of the library are the following:

- A set of basic functions for operations on orbit parameters (Cartesian and orbit elements) and attitude parameters (attitude matrix, quaternions,

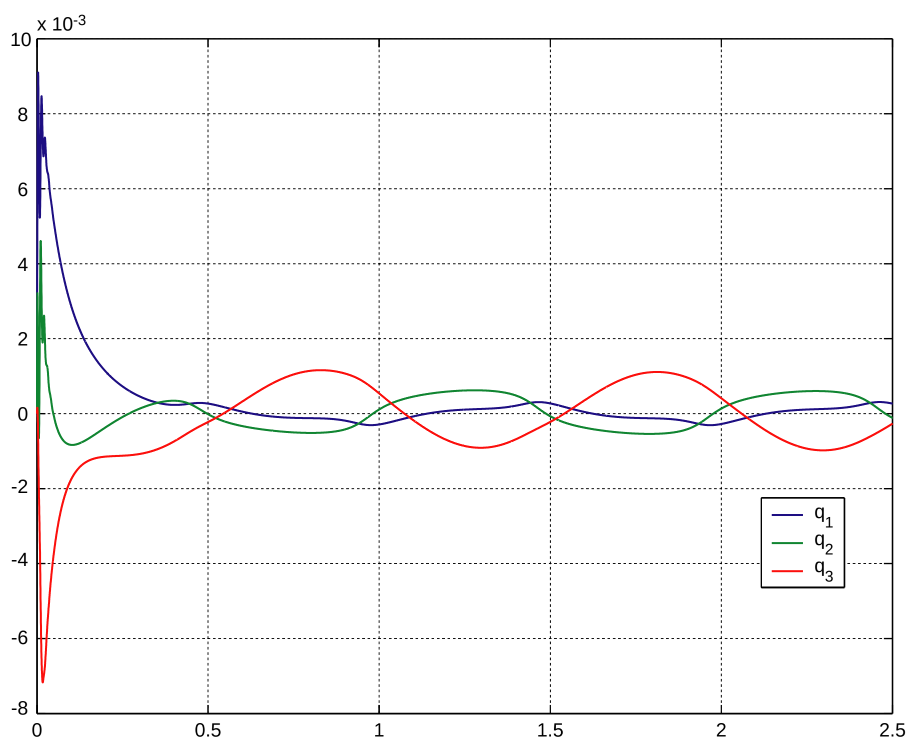

Fig. 3. Vector part of the attitude quaternion: simulation without constraints on the control variables.

Euler angles). The latter has been partially based on the rigid body kinematics Toolbox (see Schaub & Junkins, 1999).

- Class definitions for planet, orbit, spacecraft, and the most commonly used actuators and sensors.
- Environmental models of various complexity for gravitational field, magnetic field, air density and velocity, solar pressure.
- Data sheets for basic components, such as orbits, actuators and sensors.

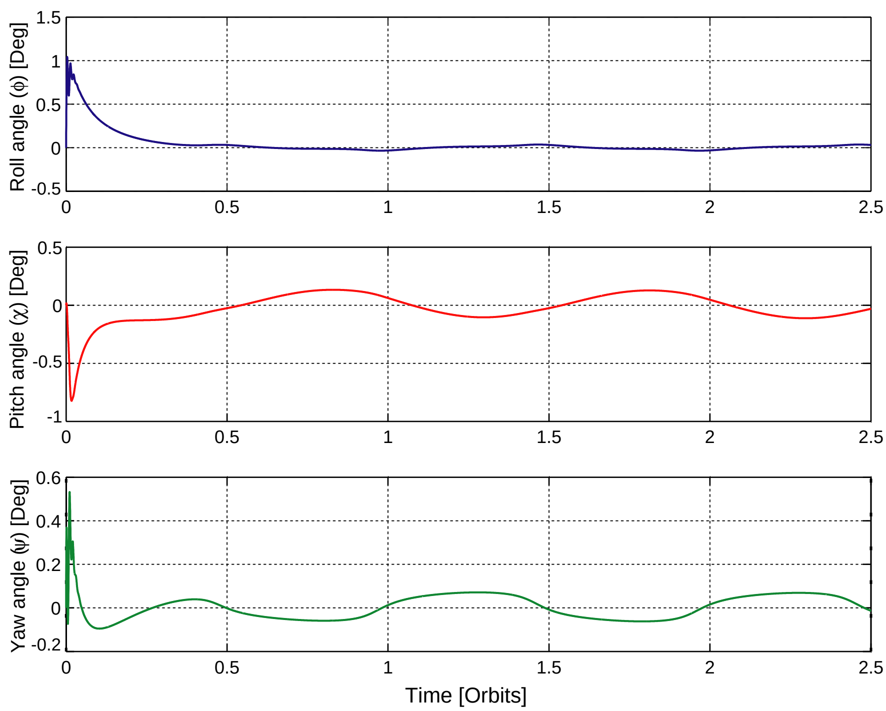

Fig. 4. Attitude angles: simulation without constraints on the control variables.

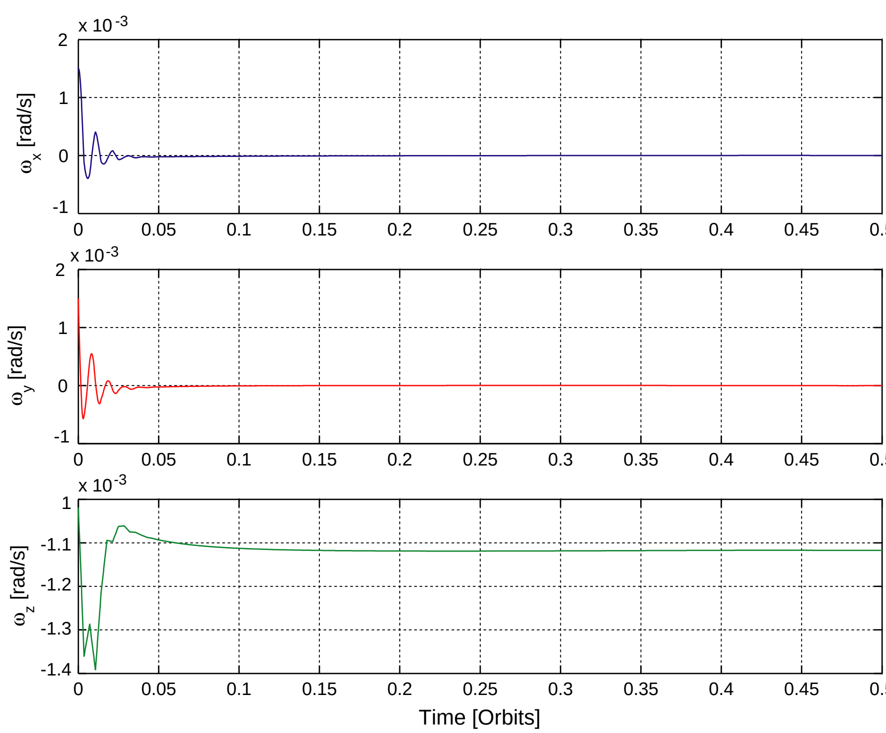

Fig. 5. Attitude angular rates: simulation without constraints on the control variables.

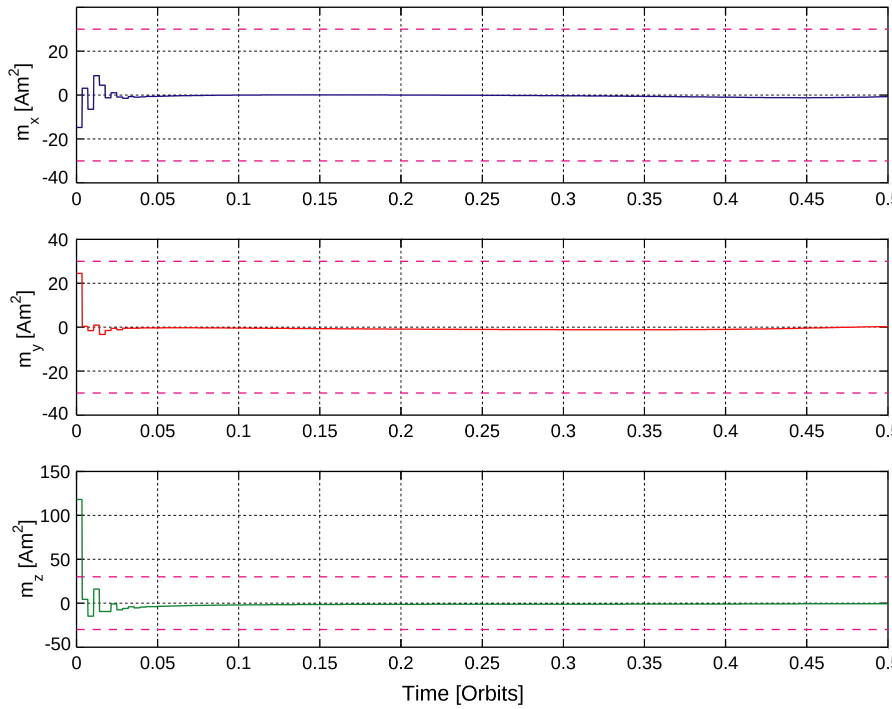

Fig. 6. Coils' magnetic dipoles: simulation without constraints on the control variables.

The library is being developed using the simulation environment Dymola, based on the Modelica language.

6.2. Overview of the considered spacecraft

MITA is a three axis stabilized satellite carrying on board NINA, a silicon spectrometer for charged particles developed by INFN (Istituto Nazionale di Fisica Nucleare). The selected orbit for the MITA mission is a circular one, with an altitude of 450 km and an inclination of 87.3^o, provided by a COSMOS launcher. The moments of inertia are $I_{xx} = 36$, $I_{yy} = 17$, $I_{zz} = 26$, $I_{xy} = 1.5$, $I_{xz} = I_{yz} = 0\ \mathrm{kgm}^2$ and the attitude control system shall ensure a three axis stabilization with the NINA detector always pointing in the opposite direction of the Earth (Zenith). For the purpose of the present study, the attitude control system is composed by the following sensors: 1 star sensor, 1 triaxial magnetometer, 5 coarse sun sensors. The attitude actuators are 1 momentum wheel, mounted along the z body axis, with moment of inertia $J = 0.01\ \mathrm{kg\,m}^2$ and angular velocity $v = 200\ \mathrm{rad/s}$ and 3 magnetic coils.

6.3. Controller design and simulation results

In the numerical calculations, the orbit is divided in 281 samples ($\Delta = 20$ s), the control horizon is $N_c = 30$ steps (which corresponds to a prediction horizon of about 10% of an orbit), the initial attitude is equal to the nominal one ($\mathbf{q}_0 = [0,\ 0,\ 0,\ 1]'$), the initial angular rate vector is $\boldsymbol{\omega}_0 = [0.0015,\ 0.0015,\ \Omega_0 - 0.0001]'$ and the weights are $\mathbf{R} = 10^4\,\mathrm{diag}\{1, 1, 0.1, 1, 1, 0.1\}\, I_{6N_c \times 6N_c}$ and $\mathbf{Q} = 10^6\,\mathrm{diag}\{1, 1, 1\}\, I_{3N_c \times 3N_c}$. The presence of an external torque due to a residual magnetic dipole of the spacecraft of an intensity of $1\ \mathrm{Am}^2$ along each body axis was also assumed.

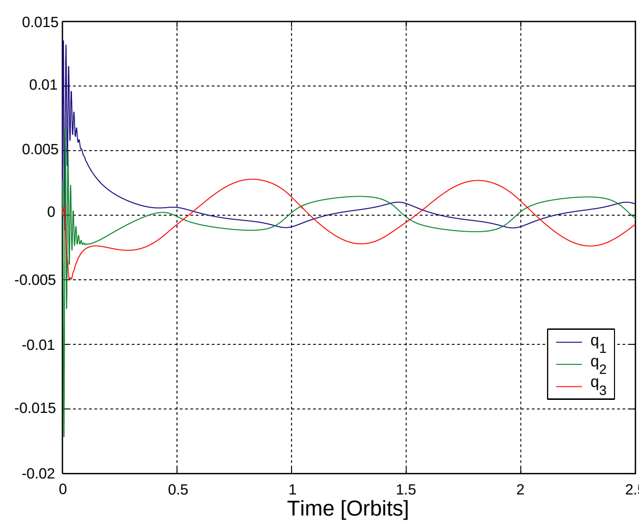

Fig. 7. Vector part of the attitude quaternion: simulation with constraints on the control variables.

A first set of simulation was carried out assuming no constraints on the control variables and the results are shown in Figs. 3–6. A second set of simulations was carried out assuming an amplitude limit of $\pm 30\ \mathrm{Am}^2$ in the control signal and the results are shown in Figs. 7–10. As can be seen, in the first set of

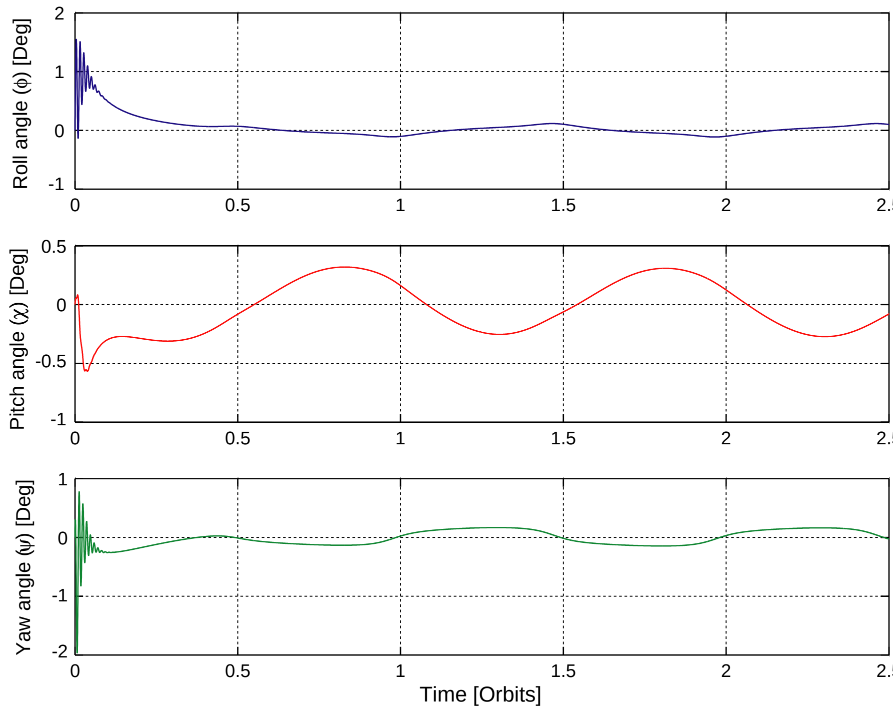

Fig. 8. Attitude angles: simulation with constraints on the control variables.

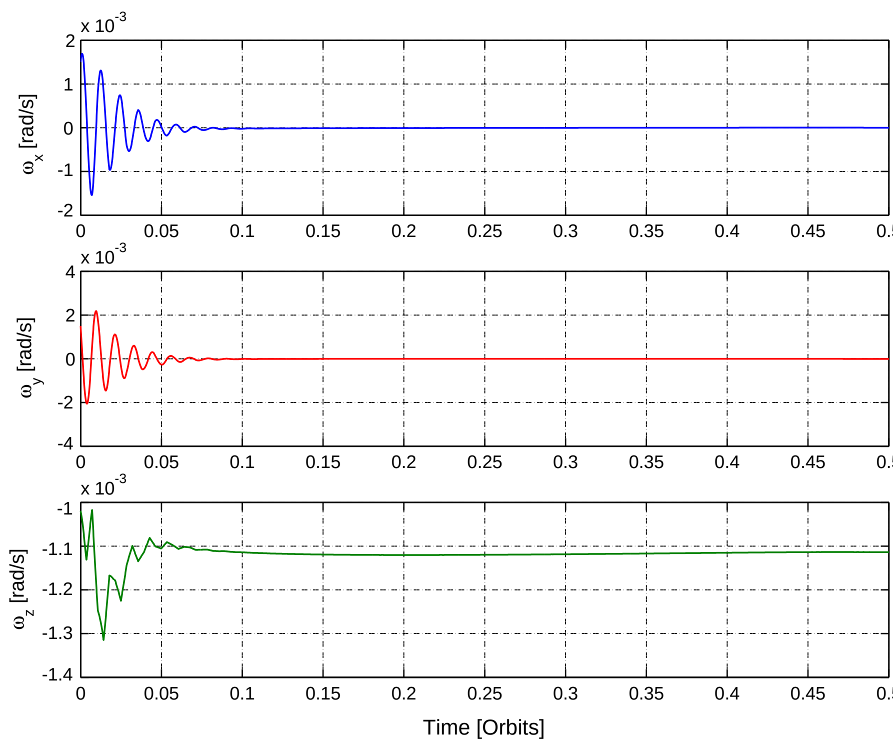

Fig. 9. Attitude angular rates: simulation with constraints on the control variables.

simulations the control variables violate the amplitude constraint, while in the second set of simulations constraint violation is avoided. Obviously a price has to be paid in term of performance of the controlled variables.

7. Concluding remarks

The attitude control problem for a spacecraft using magnetic actuators has been considered and analyzed in the framework of periodic control theory. A set of

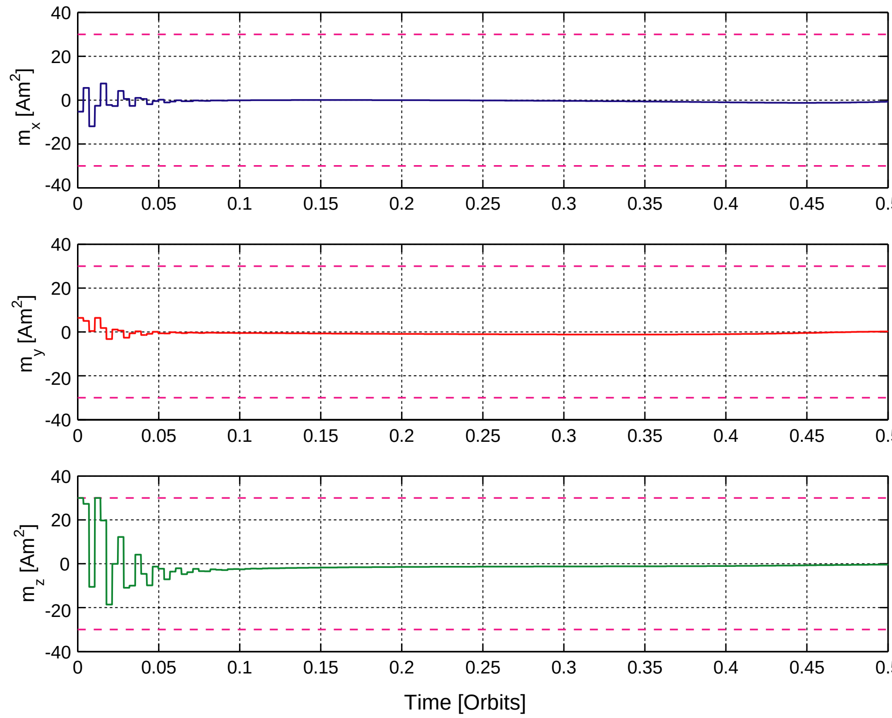

Fig. 10. Coils' magnetic dipoles: simulation with constraints on the control variables.

solutions in terms of classical LQ and H_∞ periodic control and nonlinear control have been discussed and extensions thereof, aiming at achieving robustness of the control law with respect to uncertain parameters have been proposed.

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