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Unified model simplification procedure applied to a single protection valve

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Abstract

A systematic approach is proposed in this paper for model simplification in order to reduce the number of state variables and parameters. The approach is applied for simplifying a lumped hybrid index-1 model of a single protection valve with electronic actuation that was originally developed from first engineering principles. The resulted simplified model preserves the engineering meaning of its state variables, while the number of state variables and parameters is decreased significantly.

Keywords: Brakes; Pneumatic systems; Nonlinear models; Model simplification; Model reduction; Reduced-order models

1. Introduction

Dynamic system models derived from systematic modelling approaches are usually too detailed and complex for controller design purposes. A model for control purpose should retain all major dynamic characteristics of the real plant (such as its stability and main time constants that are needed to be invariant under the simplification process) but omit all details that are weakly represented in the state variables and not related to the control aims.

Therefore a systematically derived model using first engineering principles should usually be simplified by reducing the dimension of its state vector and the complexity of its equation form by reducing the number of its parameters using engineering judgement and operating experience about the qualitative and quantitative behavior of the real world system.

In the engineering practice there is a frequently used complex nonlinear class of models, the discrete-continuous models, that is, the so called hybrid models. The dynamic equations in these models depend on the operation domain or on other factors of the system. A domain in the state space in which the same set of state equations with continuous input, state and output variables can be applied is called a hybrid mode. If the system switches into another hybrid mode then the underlying set of state equations are changing as well. Hybrid models are such composite models that are formed from all individual continuous models of the hybrid modes found in the entire state space. In case of such hybrid models, the number of independent hybrid modes should also be reduced as being a key complexity factor. Mosterman, Biswas, and Sztipanovits (1998) and Mosterman and Biswas (2000) have shown a general way for hybrid modelling and model verification of physical systems.

There are several methods proposed in the literature for performing model simplification and reduction in different ways to obtain a model with suitable size and complexity. These methods can be classified based on the underlying engineering knowledge used during model simplification.

The so called *model reduction* methods are entirely of black-box type: they apply state transformations to find

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Nomenclature

a	pressure distrib. factor [-; s/m]
A	area, surface [m^2]
α	contraction coefficient [-]
c	spring coefficient [N/m]
d	diameter [m]
$\mathbf{d}$	disturbance vector
ε	error [-]
F	force [N]
φ	angle [-]
I	electric current [A]
k	heat transm. coeff. [W/m^{2}K]
k	damping coefficient [Ns/m]
κ	adiabatic exponent [-]
m	mass [kg]
σ	air flow [kg/s]
μ	permeability [Vs/Am]
N	solenoid turns [-]
p	absolute pressure [Pa]
R	resistance [elec.-Ω; magn.-A/Vs]
R	specific gas constant [J/kgK]
t	time [s]
T	absolute temp., duration [K; s]
$\mathbf{u}$	input vector
U	voltage [V]
v	speed [m/s]
V	volume [m^3]
$\mathbf{y}$	output vector
x	stroke [m]
$\mathbf{x}$	state vector

Indices

0	initial state or vacuum
1	input chamber
2	output chamber
3	control chamber
PV	protection valve
MV	magnet valve
C	compressor
S	brake system
env	environment
crit	critical condition
in	input
out	output
exh	exhaust
max	maximum
ML	magnetic loop
Partial	one test case
Total	all test cases

out which combinations of the original state variables do not contribute significantly to the input–output behavior of the system and thus can be omitted. An example is the well-known `modred` procedure in MATLAB for LTI state-space models (The MathWorks Inc., 2002b; Hangos & Cameron, 2001b).

Tjärnström and Ljung (2000) and Tjärnström (2002) studied the variance aspects of L_2 model reduction that was applied first to finite impulse response linear models and later to general linear models.

The other class, called *model simplification* uses engineering insight and operation experience to leave out state variables to be omitted based on the dynamics of the original state variables with physical meaning. For example Leitold, Hangos, and Tuza (2000) and Hangos and Cameron (2001b) proposed a graph-theoretic method for structure simplification of lumped dynamic process models.

The aim of this paper is to propose a systematic approach to model simplification of lumped parameter nonlinear state space models that respects to pre-defined performance criteria and is based on physical insights. The approach is illustrated on a hybrid system of practical importance: on a single protection valve (SPV) that is used in pneumatic brake systems.

The ingredients of the paper are as follows. The next section describes the proposed model simplification method. Then the detailed model of the SPV is discussed briefly and the developed simplification approach is applied to the model in four simplification steps and the resulted model is presented. Finally conclusions are drawn on the simplification procedure and its results obtained on the SPV.

The detailed model simplification results of the SPV can be found in a technical report of Németh, Palkovics, and Hangos (2002b).

2. The model simplification approach

A dynamic state-space model (the simplified model) is considered simpler than another (the detailed one) with the same set of input and output variables, if

- the number of its state variables is less than that of the detailed one, and/or
- the algebraic form of its model equations is simpler, together with the number of the model parameters being less than that of the corresponding model elements in the detailed model.

Model simplification is performed by applying *model simplification assumptions* (Hangos & Cameron, 2001a) to the dynamic model. A model simplification assumption is formally described by a triplet of a model element (e.g. a model parameter), an operation (say "=") and a constant or another model element. A simple example is $a = 0$, where a is a model parameter.

2.1. The structure of the model elements

Before attempting to simplify a model that is systematically derived from first engineering principles, one has to consider the hierarchy of the model elements (Lakner, Hangos, & Cameron, 1999). A dynamic model can be seen as a hierarchically structured set of the following model elements:

- *balance volumes* over which conservation balances are constructed (the highest level),
- *balance equations*,
- *terms in balance equations corresponding to mechanisms*,
- *constitutive equations*,
- *variables and parameters* (the lowest level).

If one makes a simplification assumption to any of the model elements that will influence all the other elements on the lower level(s) that are related to it. For example, leaving out a balance volume from a model implies to leave out all the balance equations, their terms, constitutive equations, variables and parameters that belong to that particular balance volume.

Two simplifying assumptions are *not related* if they are hierarchically independent, i.e. they have no common elements in their sub-hierarchy. Naturally, one tries to perform model simplification by applying

- not related assumptions,
- assumptions in their descending order of hierarchy levels, i.e. to apply the most influential assumption first.

2.2. Sensitivity analysis

Another useful and necessary preparatory step of model simplification is to carry out sensitivity analysis (Hangos & Cameron, 2001b) of model parameters that are multiplicative factors of the influential model elements (for example flow rates or characteristic volumes) to see their effect on the input–output behavior of the system. If any of such effect is negligible then one can consider the corresponding model element as a potential target of model simplification (see Section 4.4.1).

This way one can combine engineering insight to the system which is highly domain specific with empirical (numerical) analysis of the model to be simplified.

2.3. Methodology

The systematic method proposed in this paper for model simplification consists of the following conceptual steps.

(1) *Test case selection*
The test cases to be used as reference cases for model simplification are determined based on the desired use of the model and its operation domain. These test cases should have characteristic samples from the entire operation domain and capture the most important dynamic behavior the system is able to produce. *Reference inputs are selected and reference output response to these inputs are generated using the detailed model* (i.e. the model to be simplified) under the test case conditions.

(2) *Decision criterion selection*
The modelling goal determines the desired accuracy of a model developed for a given purpose. Two models have the same performance with respect to a modelling goal, if their input–output behavior is the same within that accuracy. The accuracy is then usually specified in terms of a tolerance limit for a signal norm applied to the difference between the output response of the detailed and simplified models to the same reference input. *A simplification is then acceptable with respect to a modelling goal, if the simplified model produces the same output as the detailed one within the prescribed accuracy.*

$$\|\mathbf{y}_{\text{orig}} - \mathbf{y}_{\text{simpl}}\| \leqslant \varepsilon. \tag{1}$$

(3) *Simplification*
Having the test cases and the decision criterion, simplifications are carried out step-by-step repeating the same cycle as follows:
  3.1. select a simplification step based on engineering judgement and perform the model simplification to get a simplified model,
  3.2. generate the output response with the simplified model and perform the comparison with the response of the detailed model in the test cases,
  3.3. decide on the acceptance of the simplification step based on the decision criterion.

2.4. General applicability

This section is devoted to give the conditions of applying the above-described model simplification method in general cases. The necessary preparatory steps that one should perform on the actual system are as follows:

- construct the model hierarchy structure diagram,
- perform model parameter sensitivity analysis to preselect model elements from the hierarchy to be omitted,
- define model performance criteria that is used as decision criterion selection in accordance with the application aim.

The model hierarchy diagram is automatically obtained if the model is derived by a systematic procedure by using first engineering principles. This ingredient enables to apply a top-down simplification procedure.

The second ingredient is the result of model parameter sensitivity analysis that gives the parameters and terms in the model equations that have negligible effect on the dynamic behavior. This analysis complements and supports the needed engineering insight on which component are the candidates for elimination or complexity simplification.

The third element defines the comparison basis for the simplification method. In most cases of the control-related engineering applications, the model performance criteria can be given by using the input–output behavior (step response analogy). The application aim sets the domain and the type of the input–output behavior that is to be considered for the performance evaluation.

3. Model of the SPV

Protection valves (PVs) are often used in air brake systems for distributing the pneumatic energy into independent brake circuits. Future systems include electronic actuation for the PV that can be used for new functionalities like pressure limiting.

In a recent paper an engineering model has been proposed for an SPV with magnet valve (MV) actuation (Németh, Ailer, & Hangos, 2002a). This model included 11 state variables, 6 disturbance variables, 4 output variables (one from that as performance output), one input and 34 hybrid modes. Therefore this model has been found too complex for control design purpose, so a need for model simplification has been recognized.

3.1. System description

The SPV unit consists of the following elements (see Fig. 1):

- *Input chamber* (1) This chamber has an input air flow from the compressor and two output flows towards the PV and the MV.
- *Output chamber* (2) This chamber has an input air flow from the PV and an output towards the brake system or other consumers.
- *Control chamber* (3) This chamber has a single port that can be connected either to the input chamber or the ambient by the MV.
- *Input piping* (4) It connects the input chamber to the PV.
- *Output piping* (5) This is the connection between the PV and the output chamber.

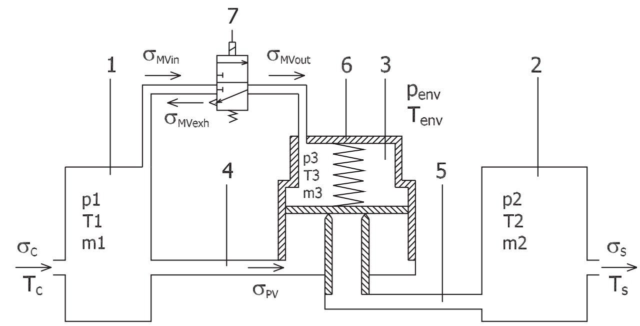

Fig. 1. Schematic of electro-magnetic SPV with the important variables.

- *Protection valve* (6) The valve has an input connection from the input chamber through the input pipe and an output to the output chamber through the output pipe.
- *Control magnet valve* (7) It is a 3/2-way valve with solenoid excitation with one input port connected to the input chamber and two output ports. The one is going to the control chamber and the other one is exhausting to the environment.

3.2. Modelling goal and accuracy requirements

The aim of the modelling is to use the developed model for control design. Therefore the resulted model should describe the dynamic behavior of the real process only within 10% desired accuracy (Hangos & Cameron, 2001b, Chapter 2), without being too complex both in the number of state variables and parameters.

3.3. Control aims

The following control aims are considered for circuit pressure limiting function of the electro-magnetic SPV:

- the circuit pressure has to be limited according to a target pressure with 500 mbar tolerance,
- the control has to be robust with respect to the external disturbances and the parameters of the simplified model.

3.4. Hybrid behavior

The system contains several parts that exhibit hybrid behavior. This means that the equations, which describe the dynamic behavior of the corresponding subsystem vary according to certain circumstances as discussed by Hangos and Cameron (2001b).

An example for this property is shown on the MV exhaust airflow equation. In this case two hybrid modes with different model equations are applied to the same term of $\sigma_{MV\mathrm{exh}}$ depending on the pressure ratio between the two corresponding ports of the valve as:

Magnet valve exhaust airflow hybrid mode 1: When $1 \geqslant \frac{p_{\mathrm{env}}}{p_3} > \Pi_{\mathrm{crit}}$, the exhaust airflow of the MV is written as:

$$\sigma_{MV\mathrm{exh}} = \alpha_{MV} A_{MV\mathrm{exh}} \times \sqrt{2\frac{\kappa}{\kappa-1}\frac{p_3 m_3}{V_3}\left[\left(\frac{p_{\mathrm{env}}}{p_3}\right)^{2/\kappa} - \left(\frac{p_{\mathrm{env}}}{p_3}\right)^{(\kappa+1)/\kappa}\right]} \tag{2}$$

Magnet valve exhaust airflow hybrid mode 2: When $\frac{p_{\mathrm{env}}}{p_3} \leqslant \Pi_{\mathrm{crit}}$, the exhaust airflow of the MV is obtained as follows:

$$\sigma_{MV\mathrm{exh}} = \alpha_{MV} A_{MV\mathrm{exh}} \times \sqrt{2\frac{\kappa}{\kappa-1}\frac{p_3 m_3}{V_3}[\Pi_{\mathrm{crit}}^{2/\kappa} - \Pi_{\mathrm{crit}}^{(\kappa+1)/\kappa}]}, \tag{3}$$

where Π_{crit} is the critical pressure ratio referring to the sonic limit.

To keep the model definition simple the state equations are shown in one dedicated hybrid mode only.

This dedicated hybrid mode corresponds to *the fill up procedure of the output chamber* (*brake circuit*). In this state the input chamber is filled by the compressor meanwhile the output chamber has lower pressure producing a positive direction air flow through the PV, where the PV stroke has an intermediate position (no stroke limiting). The streaming process is subsonic. The MV is not excited so air flow is not considered through that.

3.5. Model equations

A systematic modelling procedure using first engineering principles, described by Hangos and Cameron (2001b), was applied to develop the detailed model of the SPV. The developed model of the SPV is obtained in the form of a set of differential-algebraic equations (DAEs). This model is called index-1 because the set of algebraic equations is solvable for the algebraic variables and all the algebraic equations are substitutable into the differential ones (Pantelides, 1988; Petzold, Leimkuhler, & Gear, 1991). This section is devoted to briefly describe a lumped nonlinear state space model in its standard input-affine form from this engineering model.

The detailed model is based on six balance volumes. The three chambers form three balance volumes (1–3 in Fig. 1). There are two moving elements, the piston of the PV and the armature of the MV (inside components 3 and 7, respectively in Fig. 1), that form two additional balance volumes and finally the solenoid cross-section of the magnet valve forms the last balance volume.

The considered dynamic balance equations for the chamber balance volumes are the conservation of mass and energy of the compressed air inside. This gives six state variables for these three balance volumes. The energy equations are transformed into their intensive form resulting in an equation for the temperature change and finally for pressure change using the ideal gas equation (in consequence one obtains the state variables m_1–m_3 and p_1–p_3). The considered dynamic equations at the two moving elements were derived from Newton's second law that gives two state variables for each (x_{PV}, x_{MV} for positions and v_{PV}, v_{MV} for speeds). Finally the dynamic equation related to the magnetic circuit is based on Maxwell's second equation. This brings one state variable of the electric current (I_{MV}).

State vector of the nonlinear model: From the conservation equations, the state vector is composed of their differential variables as

$$\mathbf{x} = [m_1 \; p_1 \; m_2 \; p_2 \; m_3 \; p_3 \; x_{PV} \; v_{PV} \; x_{MV} \; v_{MV} \; I_{MV}]^{\mathrm{T}}. \tag{4}$$

Disturbance vector: The uncontrollable inputs form the disturbance vector including compressor flow rate and gas temperature, brake system consumption flow rate and gas temperature, ambient temperature and pressure, respectively

$$\mathbf{d} = \begin{bmatrix}\sigma_C & T_C & \sigma_S & T_S & T_{\mathrm{env}} & p_{\mathrm{env}}\end{bmatrix}^{\mathrm{T}}. \tag{5}$$

Input vector: The control input vector includes one member only, which is the excitation voltage of the MV

$$\mathbf{u} = [U]. \tag{6}$$

Measured output: The measured output includes the input, output, control chamber pressures and the solenoid current

$$\mathbf{y} = \begin{bmatrix}p_1 & p_2 & p_3 & I_{MV}\end{bmatrix}^{\mathrm{T}}. \tag{7}$$

The performance output is one member of the measured output that is the output chamber pressure: p_2.

3.5.1. State equation

Substituting the constitutive equations into the differential conservation balances the following state space model is obtained in input-affine form where the input matrix $\mathbf{B}$ is linear

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = f(\mathbf{x}, \mathbf{d}) + \mathbf{B}(\mathbf{x})\mathbf{u}. \tag{8}$$

The above state equation can be expanded as

$$\begin{bmatrix} \dot m_1 \\ \dot p_1 \\ \dot m_2 \\ \dot p_2 \\ \dot m_3 \\ \dot p_3 \\ \dot x_{PV} \\ \dot v_{PV} \\ \dot x_{MV} \\ \dot v_{MV} \\ \dot I_{MV} \end{bmatrix} = \begin{bmatrix} f_1(\mathbf{x},\mathbf{d}) \\ f_2(\mathbf{x},\mathbf{d}) \\ f_3(\mathbf{x},\mathbf{d}) \\ f_4(\mathbf{x},\mathbf{d}) \\ f_5(\mathbf{x},\mathbf{d}) \\ f_6(\mathbf{x},\mathbf{d}) \\ f_7(\mathbf{x},\mathbf{d}) \\ f_8(\mathbf{x},\mathbf{d}) \\ f_9(\mathbf{x},\mathbf{d}) \\ f_{10}(\mathbf{x},\mathbf{d}) \\ f_{11}(\mathbf{x},\mathbf{d}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{(R_{ML} + \frac{x_{MV}}{\mu_0 A_{MB}})}{N^2} \end{bmatrix} \mathbf{u}, \tag{9}$$

where the nonlinear state functions with all constitutive relations substituted are as:

$$f_1(\mathbf{x},\mathbf{d}) = \sigma_C - \alpha_{PV} d_2 \pi x_{PV} \xi(p_1,p_2,m_1) - \alpha_{MV} d_{MV\mathrm{in}} \pi x_{MV} \xi(p_1,p_3,m_1), \tag{10}$$

$$f_2(\mathbf{x},\mathbf{d}) = \frac{\kappa \sigma_C T_C m_1 \mathsf{R} - \kappa \alpha_{PV} d_2 \pi x_{PV} \xi(p_1,p_2,m_1) p_1 V_1}{m_1 V_1} + \frac{k_1 A_1 T_{\mathrm{env}} m_1 (\kappa-1) - k_1 A_1 p_1 V_1 (\kappa-1)/\mathsf{R}}{m_1 V_1} - \frac{\kappa \alpha_{MV} d_{MV} \pi x_{MV} \xi(p_1,p_3,m_1) p_1 V_1}{m_1 V_1}, \tag{11}$$

$$f_3(\mathbf{x},\mathbf{d}) = \alpha_{PV} d_2 \pi x_{PV} \xi(p_1,p_2,m_1) - \sigma_S, \tag{12}$$

$$f_4(\mathbf{x},\mathbf{d}) = \frac{k_2 A_2 T_{\mathrm{env}} m_2 (\kappa-1) - \kappa \sigma_S p_2 V_2 - k_2 A_2 p_2 V_2 (\kappa-1)/\mathsf{R}}{m_2 V_2} + \frac{\kappa \alpha_{PV} d_2 \pi x_{PV} \xi(p_1,p_2,m_1) p_1 V_1}{m_1 V_2}, \tag{13}$$

$$f_5(\mathbf{x},\mathbf{d}) = \alpha_{MV} d_{MVin} \pi x_{MV} \xi(p_1,p_3,m_1), \tag{14}$$

$$f_6(\mathbf{x},\mathbf{d}) = \frac{\kappa \alpha_{MV} d_{MV} \pi x_{MV} \xi(p_1,p_3,m_1) p_1 V_1 m_3}{m_3 V_3 m_1} + \frac{k_3 A_3 m_1 T_{\mathrm{env}} m_3 (\kappa-1) - k_3 A_3 m_1 p_3 V_3}{m_3 V_3 m_1}, \tag{15}$$

$$f_7(\mathbf{x},\mathbf{d}) = v_{PV}, \tag{16}$$

$$f_8(\mathbf{x},\mathbf{d}) = \frac{\frac{p_1}{4}(d_1^2 - (d_2 - 2x_{PV} \tan(\zeta(T_1,p_1,p_2)))^2)\pi}{m_{PV}} + \frac{\frac{p_2}{4}(d_2 - 2x_{PV} \tan(\zeta(T_1,p_1,p_2)))^2 \pi}{m_{PV}} - \frac{\frac{p_{\mathrm{env}}}{4} d_1^2 \pi + c_{PV}(x_{PV} + x_{0PV}) + \mathsf{k}_{PV} v_{PV}}{m_{PV}}, \tag{17}$$

$$f_9(\mathbf{x},\mathbf{d}) = v_{MV}, \tag{18}$$

$$f_{10}(\mathbf{x},\mathbf{d}) = \frac{\dfrac{N^2 I_{MV}^2}{2(R_{ML} + \frac{x_{MV}}{\mu_0 A_{MB}})^2 \mu_0 A_{MB}} - c_{MV}(x_{MV} + x_{0MV}) - \mathsf{k}_{MV} v_{MV}}{m_{MV}}, \tag{19}$$

$$f_{11}(\mathbf{x},\mathbf{d}) = \frac{I_{MV} v_{MV}}{(R_{ML} + \frac{x_{MV}}{\mu_0 A_{MB}})\mu_0 A_{MB}} - \frac{R I_{MV}(R_{ML} + \frac{x_{MV}}{\mu_0 A_{MB}})}{N^2}, \tag{20}$$

where

$$\xi(p_1,p_2,m_1) = \sqrt{\frac{2\kappa p_1 m_1 ((p_2/p_1)^{2/\kappa} - (p_2/p_1)^{(\kappa+1)/\kappa})}{(\kappa-1)V_1}},$$

$$\xi(p_1,p_3,m_1) = \sqrt{\frac{2\kappa p_1 m_1 ((p_3/p_1)^{2/\kappa} - (p_3/p_1)^{(\kappa+1)/\kappa})}{(\kappa-1)V_1}},$$

$$\zeta(T_1,p_1,p_2) = a_1 + a_2 \sqrt{2\,\kappa/(\kappa-1) R T_1 [1 - (p_2/p_1)^{(\kappa-1)/\kappa}]}.$$

3.5.2. Output equation

Since the output is linear with respect to the state vector, the output equation can be written as:

$$\mathbf{y} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}. \tag{21}$$

4. Simplification of the SPV model

4.1. Test cases and decision making

Four pressure limiting test cases, that are similar to the base control cycles of circuit pressure limiting using an SPV, are used for the model simplification case study (see Fig. 2). The first three test cases investigate the short term dynamic behavior with two test pulses of different duration (deactivations are 38, 45 and 50 ms long, respectively). The fourth one checks the long-term dynamics with one test pulse (deactivation is 38 ms long). The test cases were selected to include all the hybrid modes covered by the operation domain of a pressure limiting controller, since the test case selection is a critical part of this approach.

Individual errors are calculated for each test case based on the entries of the output vector of

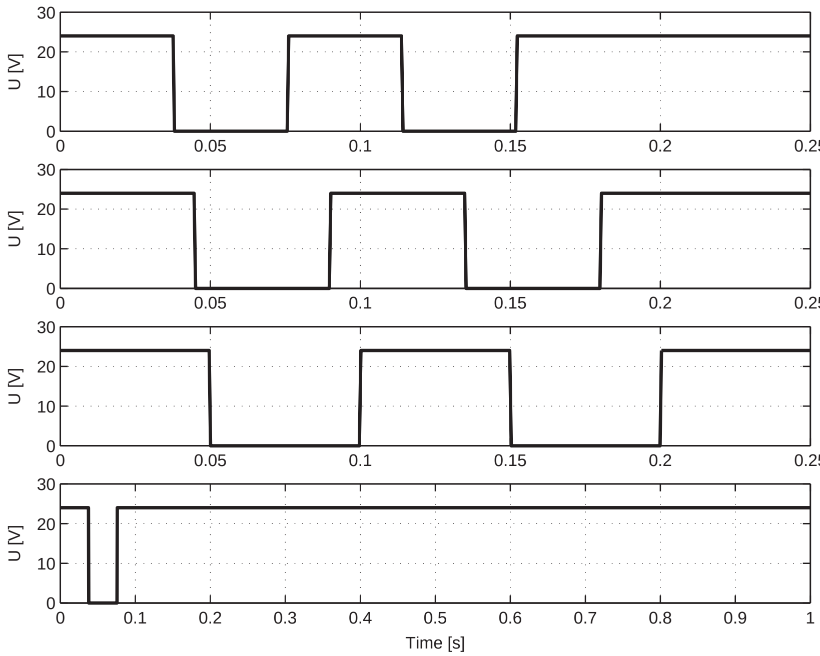

Fig. 2. Excitation voltage (system input) as function of time in the four test cases.

the SPV as follows:

$$\varepsilon_{p_1} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{p_1(t) - p_{1s}(t)}{p_1(t)}\right)^2 \mathrm{d}t}, \tag{22}$$

$$\varepsilon_{p_2} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{p_2(t) - p_{2s}(t)}{p_2(t)}\right)^2 \mathrm{d}t}, \tag{23}$$

$$\varepsilon_{p_3} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{p_3(t) - p_{3s}(t)}{p_3(t)}\right)^2 \mathrm{d}t}, \tag{24}$$

$$\varepsilon_{I_{MV}} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{I_{MV}(t) - I_{MVs}(t)}{I_{MV}(t)}\right)^2 \mathrm{d}t}, \tag{25}$$

where the suffix s refers to the corresponding output vector entries of the simplified model and T is the duration of the test case. Each individual error is a Euclidian signal norm of the prediction error in the particular output compared to response of the detailed model. A partial error is calculated based on these individual error terms for each test case as

$$\varepsilon_{\text{Partial}} = \sqrt{\varepsilon_{p_1}^2 + \varepsilon_{p_2}^2 + \varepsilon_{p_3}^2 + \varepsilon_{I_{MV}}^2}. \tag{26}$$

This error shows the total error of the corresponding test case. Finally the total error is calculated on the individual errors of the four test cases using the Euclidean norm as follows:

$$\varepsilon_{\text{Total}} = \sqrt{\sum_{k=1}^{4}(\varepsilon_{p_1,k}^2 + \varepsilon_{p_2,k}^2 + \varepsilon_{p_3,k}^2 + \varepsilon_{I_{MV},k}^2)}, \tag{27}$$

where the index k refers to the corresponding testing case.

A simplification step is accepted if the total error remains below the pre-specified tolerance of 10%. Otherwise it is refused. If a simplification step is accepted then the next step will be applied to the already simplified model and the resulted behavior is compared to the original detailed model. This way the error bound is accumulated by all the previous and the current simplification assumptions.

4.2. Simplification steps applied to the balance volumes

4.2.1. Simplification 1: input chamber dynamics neglected

The first simplification step was done in the form of problem reformulation by focusing only on system members that are strongly related to the control aims.

Doing so, the pressure limiting problem is only related to the performance output pressure of p_2. The means to influence this target pressure are: solenoid current, MV stroke and velocity for setting the control pressure to apply the desired PV piston velocity and stroke.

The only missing element in this analysis is the input pressure that can be found in the state vector. But this member serves rather as disturbance to the control problem and not as real state variable. In conclusion the following assumption can be made.

*A*1. Remove input chamber dynamics from the model and apply the input pressure as disturbance.

This assumption removes the input pressure p_1 (mass m_1 is removed as well) from the state vector and adds it to the disturbance vector but it also removes the compressor air flow and temperature (σ_C, T_C) terms from $\mathbf{d}$.

This simplification basically does not affect the model error unless the input chamber pressure as disturbance is not known exactly. Since this signal is measurable the assumption can be made.

Due to the role change of p_1 from state variable to disturbance variable its effect (ε_{p_1}) is not more considered into the error bound caused by the simplifying assumptions.

4.3. Simplification steps applied to the balance equations

4.3.1. Simplification 2: gas temperature dynamics neglected

Gas temperatures tend to the environment temperature due to the presence of heat transfer. When one assumes steady state temperatures in the gas chambers it means a significant simplification on the number of state variables and on the complexity of the conservation equations.

Based on the results of the sensitivity analysis, the first simplification assumption to the conservation equations on gas dynamics is as follows:

*A*2. The variation of gas temperatures is neglected over the whole time, pressure and temperature domain. So $\frac{dT_i}{dt} = 0 \Rightarrow T_i = T_{\text{env}}$ for $i = 2..3$.

According to assumption *A*2 the pressure change of the output chamber can be written as follows using the ideal gas equation ($pV = m\mathsf{R}T$) and the mass balance equation (12):

$$\frac{dp_2}{dt} = \frac{\mathsf{R}T_{\text{env}}}{V_2}(\sigma_{PV} - \sigma_S). \tag{28}$$

Using the same approach the control chamber pressure variation can be expressed as

$$\frac{dp_3}{dt} = \frac{\mathsf{R}T_{\text{env}}}{V_3}\sigma_{MV\text{out}}. \tag{29}$$

One has to mention that this assumption eliminates two state variables (m_2 and m_3) since the above equations are derived from the gas mass balances only and the gas energy balance equation is not needed anymore, so the number of total state variables is reduced to seven. Furthermore the brake system gas temperature (T_S) is also eliminated from the disturbance vector. The dimension of the disturbance vector is reduced to four.

The original pressure change equations are based on the energy balance equation including convective terms that exhibited hybrid behavior (the convective term is depending on the energy (temperature) of the gas where it is coming from, i.e. streaming direction). This caused two hybrid modes per port, so four hybrid modes for the output chamber and two hybrid modes for the control chamber. Since the above two simplified state equations are based on the mass balance only all these six hybrid modes are also eliminated.

4.3.2. Effect analysis of simplification 2

Having applied the above-simplified state equations the comparison results are shown in Table 1. The results show significant error compared to the original model. The difference is seen on the p_2 and p_3 output members. Furthermore qualitative error is observed in cases Test1 and Test4. An error is called qualitative error if a key behavior of the system is suppressed or eliminated (e.g. the valve does not open at all contrary to the detailed model). The reason for this qualitative error is the smaller decrease of the p_3 control pressure due to the absence of the gas temperature decrease. The two test cases showing qualitative error use the shortest solenoid deactivation, where the PV does not reach its opening situation by the smaller control pressure decrease.

An additional attempt of applying Simplification 2 was made based on the idea to emulate an increase on the MV air flows since the presence of the temperature change has a similar effect to that. The tool for that was the MV contraction coefficient parameter (α_{MV}) that was increased to 0.94 from its original value of 0.7.

This modified parameter loses then its real world meaning so it is rather a fictive one. The comparison results with this tuned simplified model are shown in Table 2.

The results show significant improvement concerning the level of total error. The error on the p_3 output member is reduced below 2%. An additional improvement is the absence of the qualitative error in each case. The dynamic behavior of the simplified system is very similar to the detailed one.

The same tuning could be made to the PV contraction coefficient (α_{PV}) that could further decrease the total

Table 1
Result comparison of the error terms in Simplification 2 in percent, untuned case

Case	ε_{p2}	ε_{p3}	$\varepsilon_{I_{MV}}$	$\varepsilon_{\text{Partial}}$	$\varepsilon_{\text{Total}}$	Qualitative error
Test1	12.01	7.505	0	14.16		Present
Test2	25.99	9.153	0	27.55		Not present
Test3	16.32	10.95	0	19.65		Not present
Test4	17.21	2.466	0	17.39		Present
All					40.6	

Table 2
Result comparison of the error terms in Simplification 2 in percent, tuned case

Case	ε_{p2}	ε_{p3}	$\varepsilon_{I_{MV}}$	$\varepsilon_{\text{Partial}}$	$\varepsilon_{\text{Total}}$	Qualitative error
Test1	2.809	0.858	0	2.937		Not present
Test2	5.185	1.324	0	5.351		Not present
Test3	5.006	1.835	0	5.332		Not present
Test4	2.632	0.378	0	2.659		Not present
All					8.53	

error below 3%. But this is not really necessary because the long term behavior of the output chamber shows that the error is asymptotically reducing to zero.

In conclusion: Simplification 2 can be accepted if the parameter α_{MV} is adjusted properly.

4.4. Simplification steps applied to the constitutive equations

4.4.1. Simplification 3: pitch angle is small

Gas forces affecting to the PV piston from the valve seat side (F_{PV1}, F_{PV2}) include a cone pitch angle in tangent function. Based on the results of the parameter sensitivity analysis for this angle the following assumption was made:

*A*3. The φ angles are considered not too big, such that $\tan\varphi \approx \varphi$ is applicable.

In this case the tangent can be linearized around $\varphi = 0$ as follows:

$$F_{PV1} = p_1\left(\frac{d_1^2}{4} - \frac{(d_2 - 2x_{PV}\varphi)^2}{4}\right)\pi, \tag{30}$$

and

$$F_{PV2} = p_2\frac{(d_2 - 2x_{PV}\varphi)^2}{4}\pi. \tag{31}$$

4.4.2. Simplification 4: Sonic MV exhaust air flow

The control chamber pressure is often close to the input chamber pressure that is usually the highest pressure in the system. Therefore the MV exhaust air flow is a sound speed limited flow down to Π_{crit} pressure ratio times the environment pressure. This leads to the following assumption.

*A*4. Magnet valve exhaust air flow is considered as sonic flow, i.e. $\frac{p_{\text{env}}}{p_3} \leqslant \Pi_{\text{crit}}$.

Since this limit is rarely exceeded the subsonic hybrid mode can be eliminated and the sonic is retained as (see Eqs. (2) and (3)):

$$\sigma_{MV\text{exh}} = \alpha_{MV}A_{MV\text{exh}} \times \sqrt{2\kappa/(\kappa-1)\frac{p_3 m_3}{V_3}[\Pi_{\text{crit}}^{2/\kappa} - \Pi_{\text{crit}}^{(\kappa+1)/\kappa}]}. \tag{32}$$

4.4.3. Effect analysis of simplifications 3–4

The results of simplification errors for the case when both the Simplifications 3–4 are applied can be seen in Table 3.

In conclusion: Simplifications 3–4 can be accepted.

4.5. The resulted simplified model

The system schematic of the simplified model of the SPV with the remaining important variables are shown in Fig. 3. The model parameter set can be found in Table 4.

Table 3
Result comparison of the error terms in Simplification 3–4 in percent

Case	ε_{p2}	ε_{p3}	$\varepsilon_{I_{MV}}$	$\varepsilon_{\text{Partial}}$	$\varepsilon_{\text{Total}}$	Qualitative error
Test1	2.808	0.857	0	2.936		Not present
Test2	5.185	1.323	0	5.351		Not present
Test3	5.006	1.834	0	5.331		Not present
Test4	2.632	0.378	0	2.659		Not present
All					8.53	

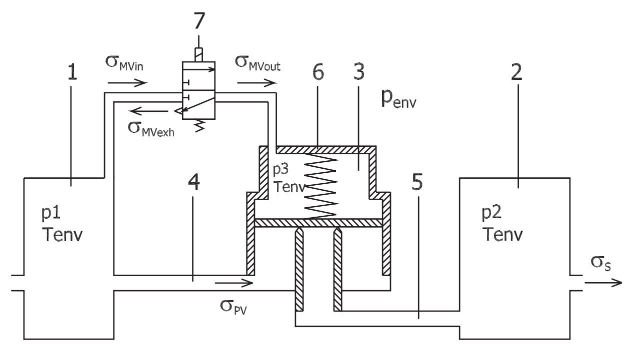

Fig. 3. Schematic of simplified electro-magnetic SPV with the important variables.

Table 4
Model parameters

Parameter name	Symbol	Unit	Value
Adiabatic exponent	κ	—	1.4
Permeability of vacuum	μ_0	Vs/Am	$4\pi \times 10^7$
Specific gas constant	R	J/kgK	287.14
Stiffness of spring	c_{MV}	N/m	1100
Stiffness of spring	c_{PV}	N/m	15000
Diameter of PV piston	d_1	m	0.023
Valve seat diameter of PV	d_2	m	0.012
MV body diameter	d_{MB}	m	0.008
MV inlet diameter	$d_{MV\text{in}}$	m	0.001
MV exhaust diameter	$d_{MV\text{exh}}$	m	0.001
Mass of MV body	m_{MV}	kg	0.002
Mass of PV piston	m_{PV}	kg	0.02
Number of solenoid turns	N	—	2200
Electric resistance of MV	R	Ω	30
Output chamber volume	V_2	m^3	0.001
Control chamber volume	V_3	m^3	0.000005
Spring preset stroke of MV	x_{MV0}	m	0.0025
Maximal MV stroke	$x_{MV\max}$	m	0.0007
Spring preset yield stroke	x_{PV0}	m	0.0125
Maximal PV stroke	$x_{PV\max}$	m	0.0025
Pressure distribution param1 of PV	a_1	rad	0
Pressure distribution param2 of PV	a_2	rad s/m	0.0002
MV contraction coefficient	α_{MV}	—	0.7
Contraction coefficient	α_{PV}	—	0.8
Damping coefficient of MV	k_{MV}	Ns/m	10
Damping coefficient of PV	k_{PV}	Ns/m	50
Magnetic loop resistance	R_{ML}	A/Vs	1.9×10^7

State vector: The number of the state vector entries are reduced to 7 as

$$\mathbf{x} = \begin{bmatrix} p_2 & p_3 & x_{PV} & v_{PV} & x_{MV} & v_{MV} & I_{MV} \end{bmatrix}^{\mathrm{T}}. \tag{33}$$

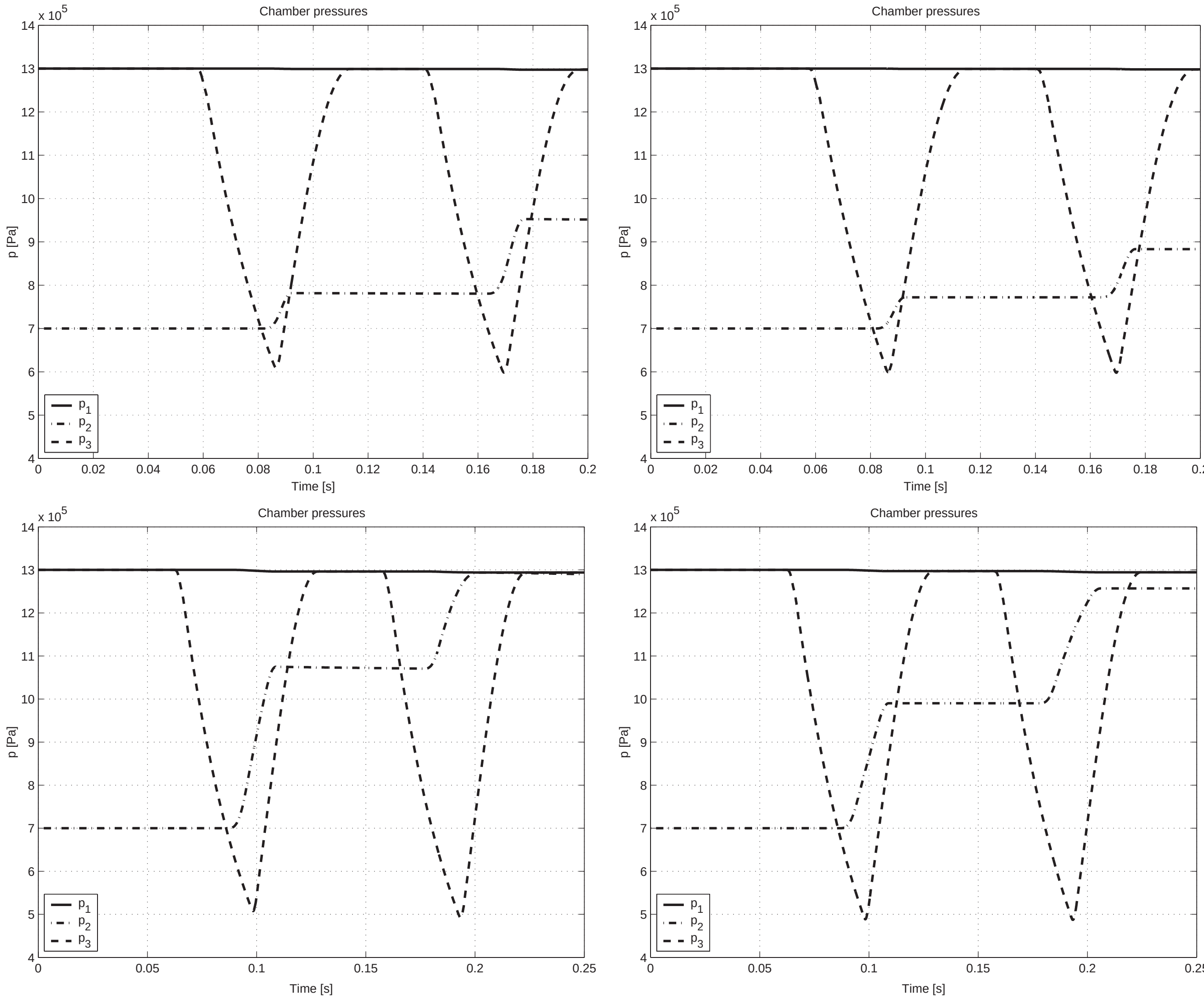

Fig. 4. Comparison of the detailed (left column) and simplified (right column) model responses—chamber pressures.

Disturbance vector: The compressor and system gas temperatures and compress air flow and environment pressure terms are removed here. The input chamber pressure was added to the disturbance vector to have

$$\mathbf{d} = \begin{bmatrix} p_1 & \sigma_S & T_{\text{env}} \end{bmatrix}^{\mathrm{T}}. \tag{34}$$

Input vector: Similarly to the original detailed model the control input vector includes one member only, which is the excitation voltage of the MV

$$\mathbf{u} = [U]. \tag{35}$$

Measured output: The output vector is not modified, too:

$$\mathbf{y} = \begin{bmatrix} p_1 & p_2 & p_3 & I_{MV} \end{bmatrix}^{\mathrm{T}}. \tag{36}$$

4.5.1. State equation

The simplified state space model is obtained in input-affine form as well as follows:

$$\begin{bmatrix} \dot{p}_2 \\ \dot{p}_3 \\ \dot{x}_{PV} \\ \dot{v}_{PV} \\ \dot{x}_{MV} \\ \dot{v}_{MV} \\ \dot{I}_{MV} \end{bmatrix} = \begin{bmatrix} f_1(\mathbf{x},\mathbf{d}) \\ f_2(\mathbf{x},\mathbf{d}) \\ f_3(\mathbf{x},\mathbf{d}) \\ f_4(\mathbf{x},\mathbf{d}) \\ f_5(\mathbf{x},\mathbf{d}) \\ f_6(\mathbf{x},\mathbf{d}) \\ f_7(\mathbf{x},\mathbf{d}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{(R_{ML} + \frac{x_{MV}}{\mu_0 A_{MB}})}{N^2} \end{bmatrix} \mathbf{u}, \tag{37}$$

where the nonlinear state functions with all constitutive relations substituted are as:

$$f_1(\mathbf{x},\mathbf{d}) = \frac{\mathrm{R}T_{env}}{V_2}(\alpha_{PV} d_2 \pi x_{PV} p_1 \xi(p_1, p_2, m_1) - \sigma_S), \tag{38}$$

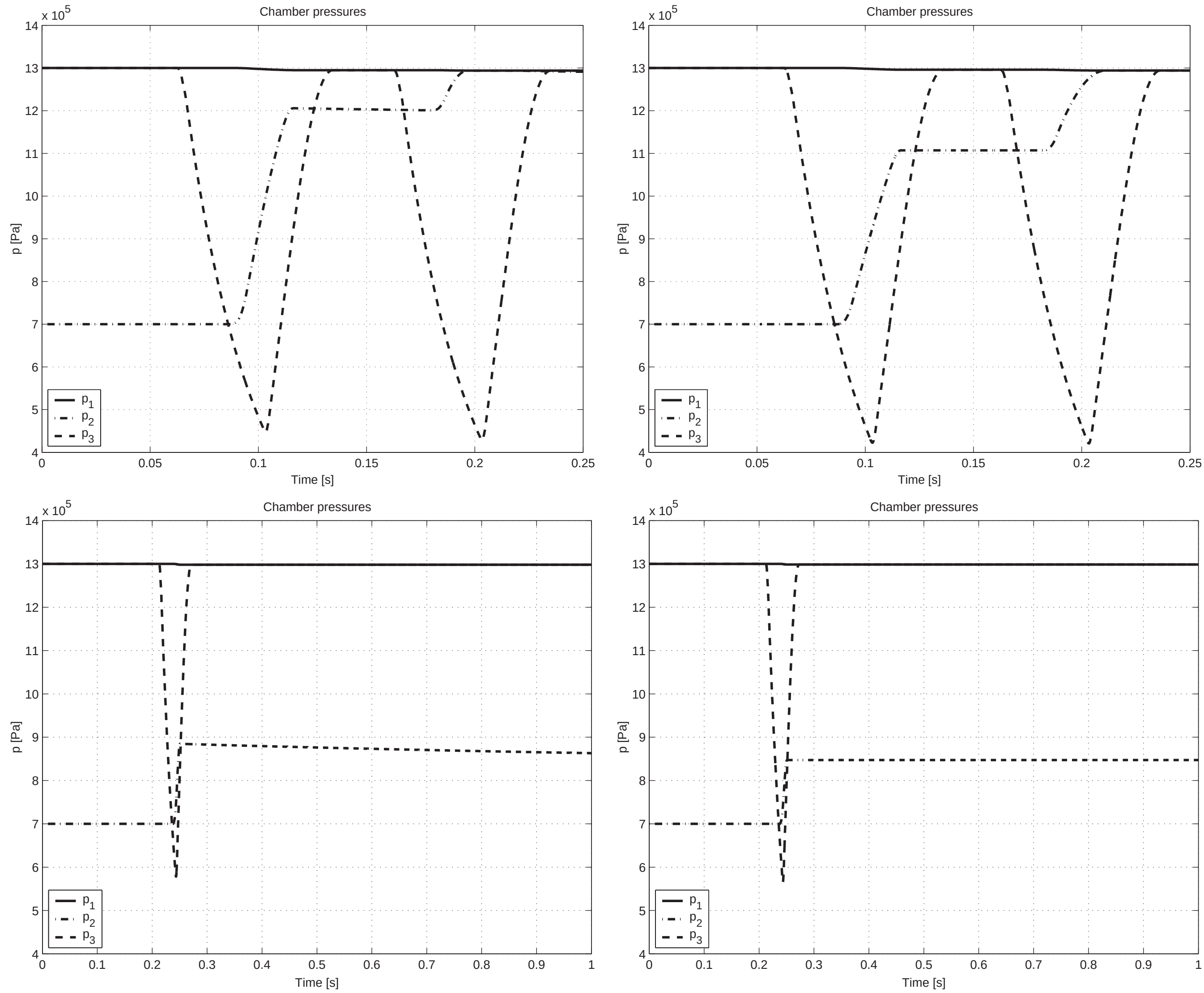

Fig. 4 (*continued*).

$$f_2(\mathbf{x},\mathbf{d}) = \frac{\mathsf{R}T_{env}}{V_3}\alpha_{MV} d_{MV\text{in}} \pi x_{MV} p_1 \xi(p_1, p_3, m_1), \tag{39}$$

$$f_3(\mathbf{x},\mathbf{d}) = v_{PV}, \tag{40}$$

$$f_4(\mathbf{x},\mathbf{d}) = \frac{\frac{p_1}{4}(d_1^2 - (d_2 - 2x_{PV}\zeta(T_1,p_1,p_2))^2)\pi}{m_{PV}} + \frac{\frac{p_2}{4}(d_2 - 2x_{PV}\zeta(T_1,p_1,p_2))^2\pi}{m_{PV}} + \frac{-\frac{p_{\text{env}}}{4}d_1^2\pi - c_{PV}(x_{PV}+x_{0PV}) - \mathsf{k}_{PV}v_{PV}}{m_{PV}}, \tag{41}$$

$$f_5(\mathbf{x},\mathbf{d}) = v_{MV}, \tag{42}$$

$$f_6(\mathbf{x},\mathbf{d}) = \frac{\frac{N^2 I_{MV}^2}{2(R_{ML}+\frac{x_{MV}}{\mu_0 A_{MB}})^2\mu_0 A_{MB}} - c_{MV}(x_{MV}+x_{0MV}) - \mathsf{k}_{MV}v_{MV}}{m_{MV}}, \tag{43}$$

$$f_7(\mathbf{x},\mathbf{d}) = \frac{I_{MV}v_{MV}}{(R_{ML}+\frac{x_{MV}}{\mu_0 A_{MB}})\mu_0 A_{MB}} - \frac{RI_{MV}(R_{ML}+\frac{x_{MV}}{\mu_0 A_{MB}})}{N^2}. \tag{44}$$

The meaning of $\xi(p_1,p_2,m_1)$, $\xi(p_1,p_3,m_1)$ and $\zeta(T_1,p_1,p_2)$ remained the same.

4.5.2. Output equation

The simplified output equation is written as

$$\mathbf{y} = \begin{bmatrix} 0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0 \end{bmatrix}\mathbf{d}. \tag{45}$$

See Fig. 4 for comparison on the simulated step response functions of the detailed and simplified models.

5. Conclusions

A systematic approach is proposed in this paper for model simplification using engineering insight to find model elements to be left out or simplified and a decision method based on the analysis of the effect of the simplification on the input–output behavior of the model using performance norms.

The approach is applied for simplifying a lumped hybrid index-1 model of an SPV with electronic actuation that was originally developed from first engineering principles. The resulted simplified model preserves the engineering meaning of its state variables, while the number of state variables decreased from 11 to 7, and the number of parameters to be estimated was reduced to 7 from 13. Some of the original parameters changed slightly their meaning and value due to the lumping and simplification effects. The number of necessary hybrid modes was also significantly reduced from 34 to 21.

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