

Complexity:

Based on the information provided in the search results, the time complexities of various operations on binary search trees (BSTs) are as follows:

1. Searching in a BST:

- Average case: $O(\log n)$
- Worst case: $O(n)$

The worst-case scenario occurs when the BST is skewed (e.g., a linked list), and the search time becomes linear in the number of nodes ($O(n)$). However, for a balanced BST, the search time is logarithmic in the number of nodes ($O(\log n)$).

2. Insertion in a BST:

- Average case: $O(\log n)$
- Worst case: $O(n)$

Similar to searching, the insertion time is logarithmic in the average case for a balanced BST, but can degrade to linear time ($O(n)$) in the worst case for a skewed BST.

3. Deletion in a BST:

- Average case: $O(\log n)$
- Worst case: $O(n)$

Deleting a node in a BST involves searching for the node, finding its successor, and then performing the deletion. The time complexity follows the same pattern as searching and insertion.

4. Balancing an Unbalanced BST:

- Time complexity: $O(n)$

The search results indicate that the time complexity to balance an unbalanced BST is linear in the number of nodes ($O(n)$). This is typically achieved by first performing an inorder traversal to obtain a sorted array of the nodes, and then constructing a balanced BST from the sorted array.

In summary, the key points are:

- For a balanced BST, the time complexities of search, insertion, and deletion are $O(\log n)$ on average.
- For an unbalanced BST (e.g., a skewed tree), the time complexities can degrade to $O(n)$ in the worst case.
- Balancing an unbalanced BST can be done in $O(n)$ time.

The search results also mention that there are self-balancing binary search tree variants, such as AVL trees and Red-Black trees, which can guarantee logarithmic time complexities for all operations, even in the worst case.