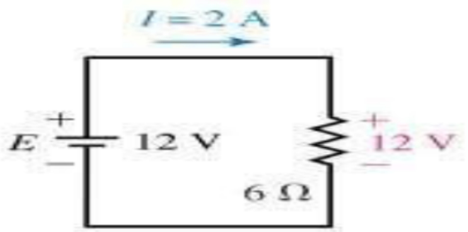
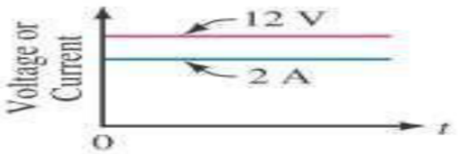
AC Fundamentals

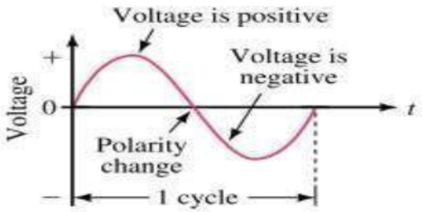
- Previously you learned that DC sources have fixed polarities and constant magnitudes and thus produce currents with constant value and unchanging direction





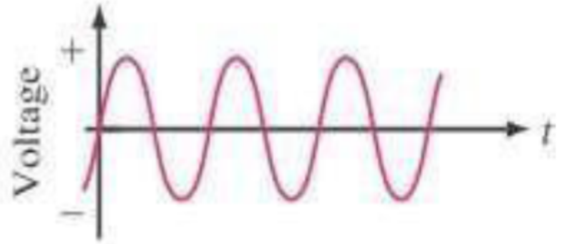
(b) Voltage and current versus time for dc

- In contrast, the voltages of ac sources alternate in polarity and vary in magnitude and thus produce currents that vary in magnitude and alternate in direction.



Sinusoidal ac Voltage

One complete variation is referred to as a cycle.



(b) A continuous stream of cycles

Starting at zero,
the voltage increases to a positive peak amplitude, decreases to zero, changes polarity,
increases to a negative peak amplitude,
then returns again to zero.

Since the waveform repeats itself at regular intervals, it is called a **periodic signal**.

Which of the following is not ac waveform?

a) sinusoidal

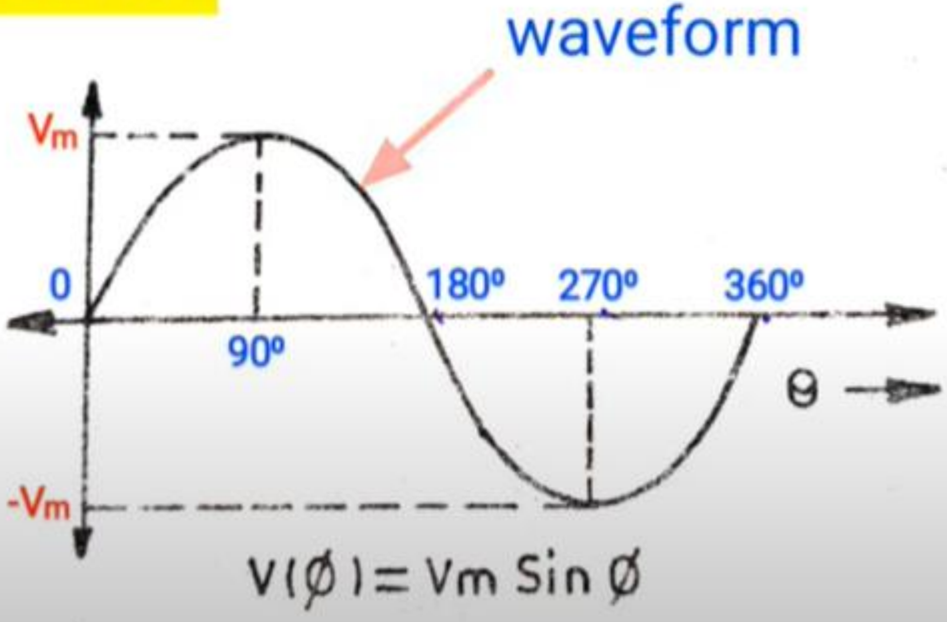
b) square

c) constant

d) triangular

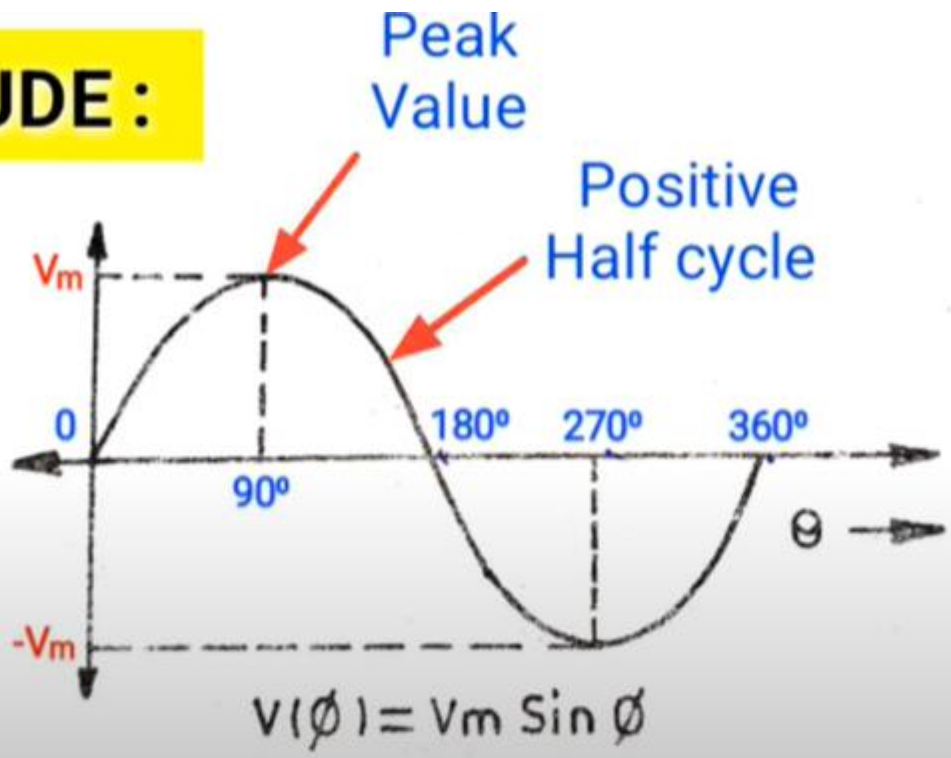
1) WAVEFORM :

The nature of graph of alternating quantity against time is known as waveform or waveshape.



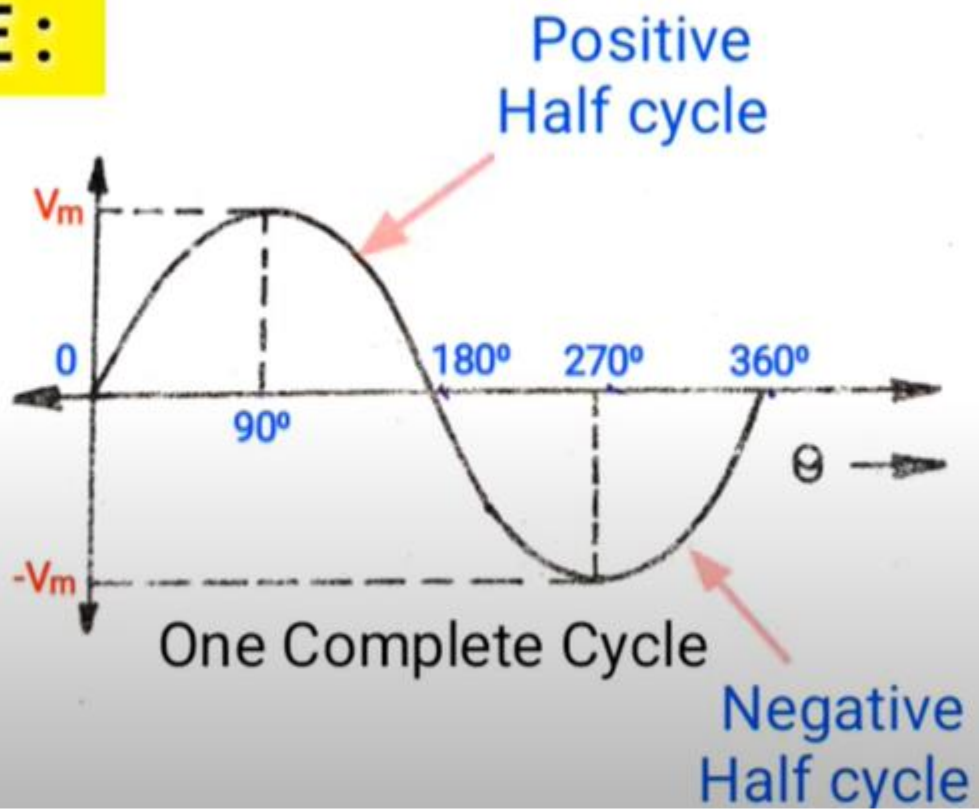
2) AMPLITUDE :

The maximum value (positive or negative) of an alternating quantity in half cycle is known as its amplitude. It is also known as peak value.



3) CYCLE :

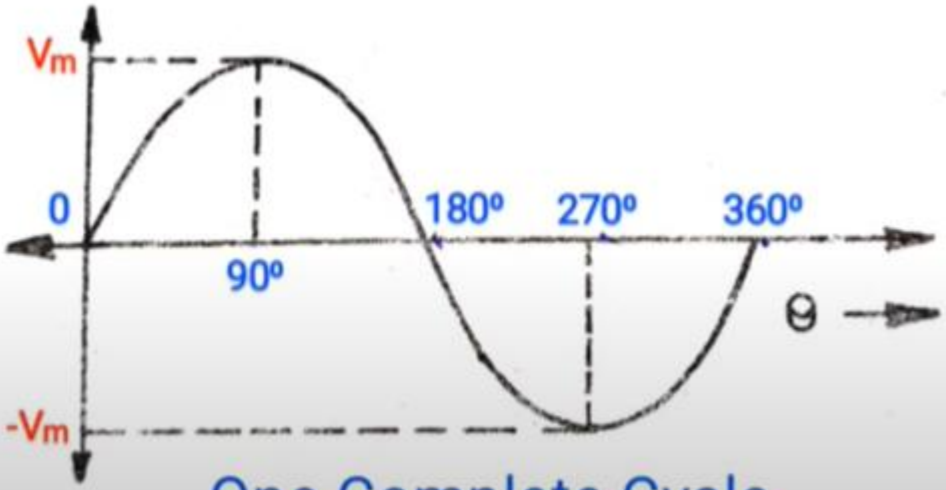
One complete set of positive and negative values of alternating quantity is known as a cycle.



4) TIME PERIOD :

Time required to complete one cycle of an alternating quantity is called time period.

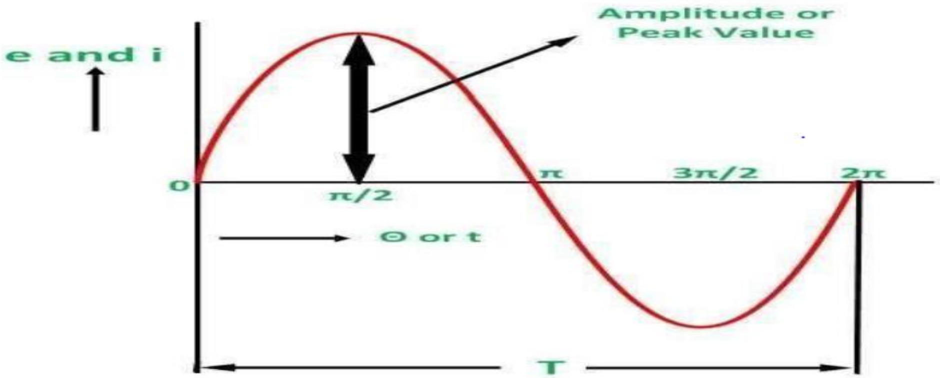
$$T = 1 / f$$



One Complete Cycle

Peak Value

- The maximum value attained by an alternating quantity during one cycle is called its **Peak value**. It is also known as the maximum value or amplitude.



❑ Consider the sinusoidal voltage,

$$\boxed{v(t) = V_m \sin \omega t}$$

Where, V_m = the *amplitude* of the sinusoid

ω = the *angular frequency* in radians/s

Numerical 3: Find the amplitude, phase, period, and frequency of the sinusoid $v(t) = 12\cos(50t + 10^{\circ})$ V.

Solution:

The amplitude is $V_m = 12$ V
The phase is $\phi = 10^{\circ}$
The angular frequency is $\omega = 50$ rad/s
The period $T = 2\pi/\omega = 2\pi/50 = 0.1257$ s
The frequency is $f = 1/T = 7.958$ Hz

RMS Value

8) ROOT MEAN SQUARE VALUE (RMS VALUE) :

It is that value of D.C. current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

$$\text{RMS value} = \frac{\text{Max.value}}{\sqrt{2}}$$

RMS value = 0.707 × Max.value

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

10) FORM FACTOR :

It is the ratio of RMS value to the Average value.

$$\text{Form Factor} = \frac{\text{RMS value}}{\text{Avg.value}}$$

$$\text{Form Factor} = \frac{\text{Max.value} / \sqrt{2}}{0.637 \times \text{Max.value}}$$

$$\boxed{\text{Form Factor} = 1.11}$$

11) CREST FACTOR OR AMPLITUDE FACTOR :

It is the ratio of Maximum value to the RMS value.

$$\text{Crest Factor} = \frac{\text{Max.value}}{\text{RMS value}}$$

$$\text{Crest Factor} = \frac{\text{Max.value}}{\frac{\text{Max.value}}{\sqrt{2}}}$$

$$\text{C.F.} = \sqrt{2}$$

$$\boxed{\text{C.F.} = 1.414}$$

Form Factor and Peak Factor

- Form Factor, $K_f = \frac{V_{rms}}{V_{avg}}$

- Peak Factor or Crest Factor, $K_p = \frac{V_m}{V_{rms}}$

Let us calculate these two factors for ***a sinusoidal voltage waveform***,

$$K_f = \frac{V_{rms}}{V_{av}} = \frac{V_m/\sqrt{2}}{2V_m/\pi} = \frac{0.707\,V_m}{0.637\,V_m} = \mathbf{1.11}$$

And

$$K_p = \frac{V_m}{V_{rms}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2} = \mathbf{1.414}$$

For a pure sinusoidal waveform the Form Factor will always be equal to:

A. $\frac{1}{\sqrt{2}}$

B. 0.637

C. 1.11

D. 1.414

Numerical 1: An alternating current of sinusoidal waveform has an RMS value of 10.0 A. What is the peak-to-peak value of this current?

Solution: $I_m = \frac{I}{0.707} = \frac{10}{0.707} = 14.14A$

The peak-to-peak value is therefore **14.14 − (-14.14) = 28.28 A**

Numerical 2: An alternating voltage has the equation $v = 141.4 \sin 377t$, what are the values of (a) RMS voltage (b) frequency (c) the instantaneous voltage when $t = 3$ ms?

Solution: The relation is of the form $v = V_m\, sin\omega t$ and by comparison,

$(a) V_m = 141.4V = \sqrt{2}V$ Hence, $V = \dfrac{141.4}{\sqrt{2}} = 100V$

(b) Also by comparison,

$$\omega = 377 rad/s = 2\pi f, f = \frac{377}{2\pi} = 60Hz$$

(c) Finally, $v = 141.4 \sin 377t$

when $t = 3 \times 10^{-3}$ sec,

$$v = 141.4 \sin(377 \times 3 \times 10^{-3}) = 141.4 \sin 1.131$$

$$= 141.4 \times 0.904 = 127.8V$$

Peak value divided by the rms value gives us?

a) Peak factor

b) Crest factor

c) Both peak and crest factor

d) Neither peak nor crest factor

Calculate the crest factor if the peak value of current is 10A and the rms value is 2A.

a) 5

b) 10

c) 5A

d) 10A

Question 1: Given the sinusoidal voltage $v(t) = 50 \cos(30t + 10^\circ)$ V, find: (a) the amplitude V_m, (b) the period T, (c) the frequency f, and (d) $v(t)$ at $t =$ 10 ms

Ans.1: (a) 50 V, (b) 209.4 ms, (c) 4.775 Hz, (d) 44.48 V, 0.3 rad

Question 2: A current source in a linear circuit has $i_s = 8\cos(500\pi t - 25^\circ)$ A

(a) What is the amplitude of the current?

(b) What is the angular frequency?

(c) Find the frequency of the current.

(d) Calculate *is* at $t = 2$ ms.

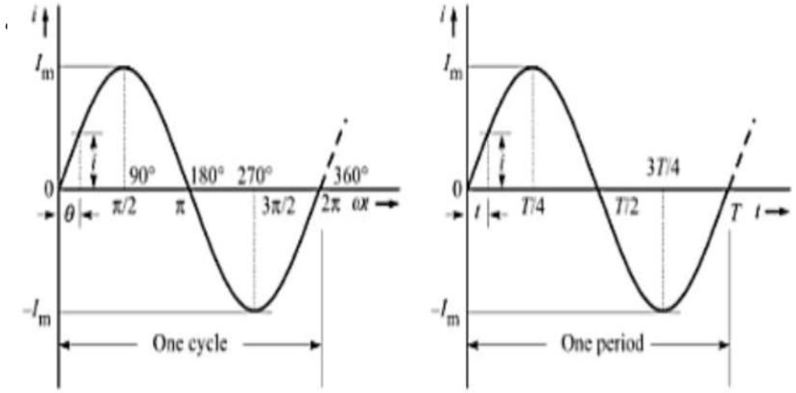
**Ans.2: (a) 8 A, (b) 1570.8 rad/s,
(c) 250 Hz, (d) -7.25 A**

Average Value

Algebraic sum of all the values divided by the total number of values.

$$V_{av} = \frac{\text{Area under full cycle}}{\text{Length of one cycle}} = \frac{\int_0^{2\pi} v\,d\theta}{2\pi} = \frac{1}{2\pi}\int_0^{2\pi} v\,d\theta = \frac{1}{2\pi}\int_0^{2\pi} v\,d(\omega t)$$

$$V_{av} = \frac{1}{T}\int_0^{T} v\,dt$$



(a) Current i versus angle ωt.
(b) Current i versus time t.

Thus, the average value over full cycle is ZERO

The average value of current in a sinusoidal signal over full cycle is:

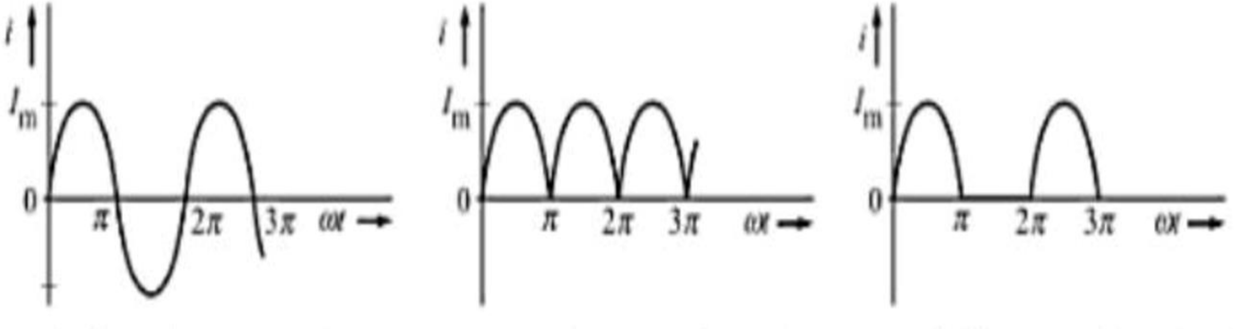
$A.\ I_m/2$

$B.\ I_m/\sqrt{2}$

C. 1

D. 0

However, an average value can be defined for the half-cycle (positive or negative) for a sinusoidal signal.



(a) Sinusoidal ac current. (b) Full-wave rectifier output. (c) Half-wave rectifier output.

RMS (Effective) Value

- The r.m.s value is defined in terms of heating effect.
- The r.m.s. value of an alternating current is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

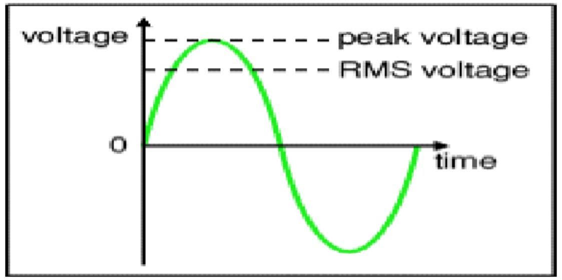
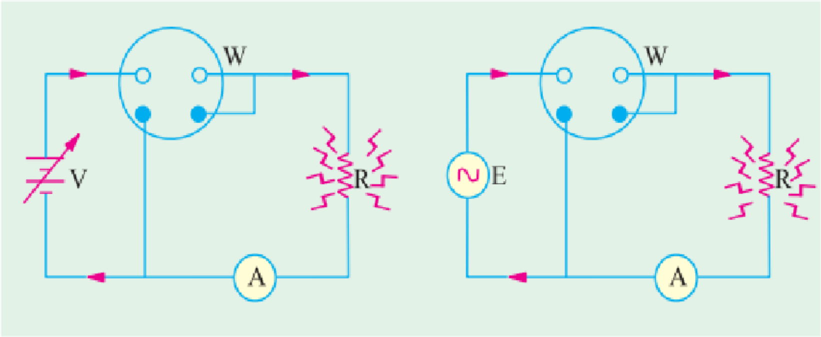


Figure- Difference between peak and RMS voltage



For a pure sinusoidal waveform the Form Factor will always be equal to:

A. $\frac{1}{\sqrt{2}}$

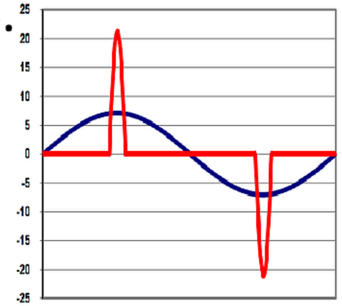
B. 0.637

C. 1.11

D. 1.414

Importance of Form Factor and Peak Factor

- Actually some of our meters are designed to measure the RMS values but that is of pure sinusoidal waveforms, if there comes any distortion in the waveform, the meter **won't give the correct** RMS value. For meter the waveform is still a sinusoidal but it doesn't detect the distortion that's why we use form factor to get accurate value of RMS by just multiplying form factor with the average value of that distorted waveform. It is helpful in finding the RMS values of waveforms other than pure sinusoidal.
- Similarly, Some loads, such as switching power supplies or lamp ballasts, have current waveforms that are not sinusoidal. They draw a high current for a short period of time, and their crest factors, therefore, can be quite a bit higher than 1.414.



___________ current is found by dividing the area enclosed by the half cycle by the length of the base of the half cycle.

a) RMS current

b) Average current

c) Instantaneous current

d) Total current

In a sinusoidal wave, average current is always _______ rms current.
a) Greater than
b) Less than
c) Equal to
d) Not related

Numerical 1: An alternating current of sinusoidal waveform has an RMS value of 10.0 A. What is the peak-to-peak value of this current?

Solution: $I_m = \dfrac{I}{0.707} = \dfrac{10}{0.707} = 14.14A$

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Which of the following is not ac waveform?
a) sinusoidal
b) square
c) constant
d) triangular

An ac voltage is mathematically expressed as $v = 141.42\sin(157.08t + \pi/2)$ volts.

Find its (*a*) effective value, (*b*) frequency, and (*c*) periodic time.

[**Ans.** (*a*) 100 V; (*b*) 25 Hz; (*c*) 40 ms]

An ac voltage is mathematically expressed as

$$v = 141.42 \sin(157.08t + \pi/2) \text{ Volts}$$

$$v = V_m \sin(\omega t + \theta)$$

$$\text{Effective value} = \frac{V_m}{\sqrt{2}} = \frac{141.42}{\sqrt{2}} = 100V$$

$$\omega = 157.08$$

$$2\pi f = 157.08$$

$$f = \frac{157.08}{2\pi} = 25 \text{ HZ}$$

$$T = \frac{1}{f} = \frac{1}{25} = 0.04 \text{ sec}$$

Peak value divided by the rms value gives us?
a) Peak factor
b) Crest factor
c) Both peak and crest factor
d) Neither peak nor crest factor

Calculate the crest factor if the peak value of current is 10A and the rms value is 2A.

a) 5

b) 10

c) 5A

d) 10A

Find the average value of current when the current that are equidistant are 4A, 5A and 6A.

a) 5A

b) 6A

c) 15A

d) 10A

Question 1: Given the sinusoidal voltage $v(t) = 50 \cos(30t + 10^\circ)$ V, find: (a) the amplitude V_m, (b) the period T, (c) the frequency f, and (d) $v(t)$ at $t =$ 10 ms

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(d) Calculate *is* at $t = 2$ ms.

Ans.2: (a) 8 A, (b) 1570.8 rad/s, (c) 250 Hz, (d) -7.25 A

Question 10: If the waveform of a voltage has a form factor of 1.15 and a peak factor of 1.5, and if the peak value is 4.5 kV, calculate the average and the r.m.s. values of the voltage.

Question11: An alternating current was measured by a DC milliammeter in conjunction with a full-wave rectifier. The reading on the milliammeter was 7.0 mA. Assuming the waveform of the alternating current to be sinusoidal, calculate: (a) the r.m.s. value; and (b) the maximum value of the alternating current.

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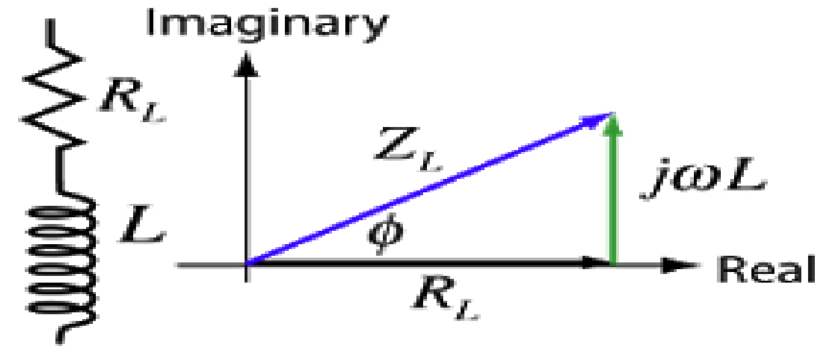
Impedance

- Impedance is a complex number, with the same units as resistance, for which the SI unit is the ohm (Ω). Its symbol is usually Z, and it may be represented by writing its magnitude and phase in the polar form $|Z|\angle\theta$. However, cartesian complex number representation is often more powerful for circuit analysis purposes. $Z=R+jX$

Impedance Representation

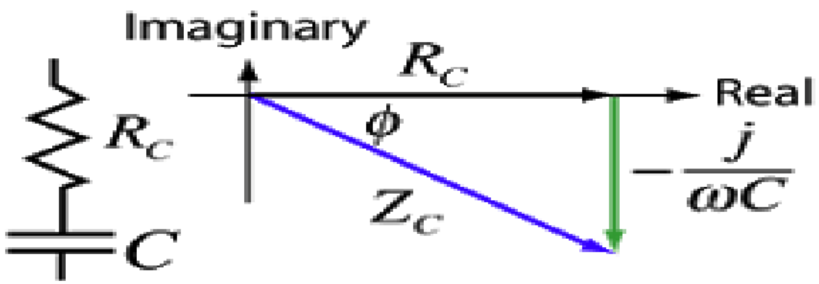


Cartesian form: $Z_L = R_L + j\omega L$

Polar form: $Z_L = |Z_L| e^{j\phi}$

where $|Z_L| = \sqrt{R_L^2 + \omega^2 L^2}$

$$\phi = \tan^{-1} \frac{\omega L}{R_L}$$



Cartesian form: $Z_C = R_C - \frac{j}{\omega C}$

Polar form: $Z_C = |Z_C| e^{j\phi}$

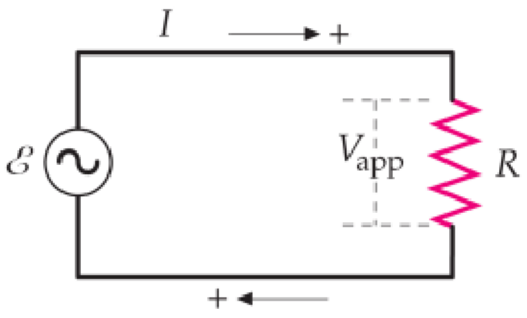
where $|Z_C| = \sqrt{R_C^2 + \left[\frac{-1}{\omega C}\right]^2}$

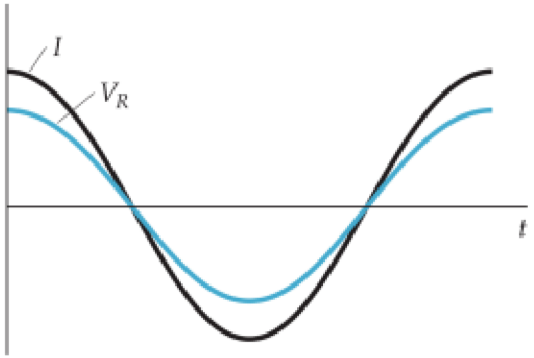
$$\phi = \tan^{-1} \frac{-1}{\omega C R_C}$$

Basic Formulae of Z

1. Impedance $Z = R$ *or* X_L *or* X_C *(if only one is present)*
2. Impedance **in series only** $Z = \sqrt{(R^2 + X^2)}$ *(if both R and one type of X are present)*
3. Impedance **in series only** $Z = \sqrt{(R^2 + (|X_L - X_C|)^2)}$ *(if R, X_L, and X_C are all present)*
4. Impedance **in any circuit** $= R + jX$ *(j is the imaginary number $\sqrt{(-1)}$)*
5. Resistance $R = \Delta V / I$
6. Inductive reactance $X_L = 2\pi f L = \omega L$
7. Capacative reactance $X_C = {}^{1} / {}_{2\pi f C} = {}^{1} / {}_{\omega C}$

Resistor in an AC Circuit





For the case of a resistor in an AC circuit the V_R across the resistor is in phase with the current I through the resistor.

In phase means that both waveforms peak at the same time.

CIRCUIT WITH PURE RESISTANCE ONLY

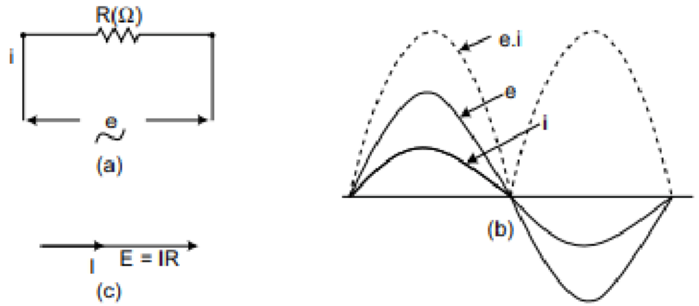
A pure resistance is that in which there is ohmic voltage drop only. Consider a circuit having a pure resistance R as shown is Fig. 2.24 below.

Let the instantaneous value of the alternating voltage applied be,

$$e = E_m \sin \omega t$$

The instantaneous value of current,

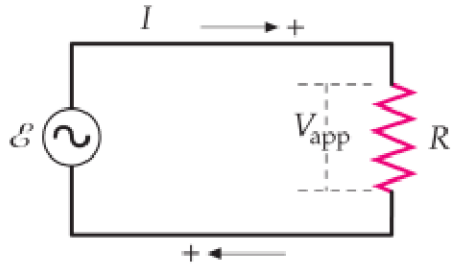
$$i = \frac{e}{R} = \frac{E_m}{R} \sin \omega t$$



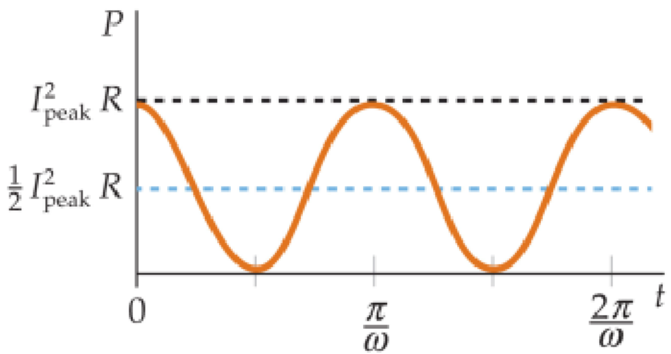
Resistor in an AC Circuit

$$P(t) = I^2(t)R = \left(I_p \cos\omega t\right)^2 R$$

$$P(t) = I_p^2 R\cos^2\omega t$$



The instantaneous power is a function of time. However, the average power per cycle is of more interest.



$$P_{avg} = \frac{1}{T}\int_0^T P(t)dt$$

$$P_{avg} = \frac{1}{T}\int_0^T I_p^2 R\cos^2\omega t dt$$

$$P_{avg} = \frac{I_p^2 R}{T}\int_0^T \cos^2\omega t dt = \frac{I_p^2 R}{\omega T}\pi = \frac{1}{2}I_p^2 R = \left(\frac{I_p}{\sqrt{2}}\right)^2 R = I_{RMS}^2 R$$

CIRCUIT WITH PURE Capcitance ONLY

- The circuit containing only a pure capacitor of capacitance C farads is known as a **Pure Capacitor Circuit**. The capacitors stores electrical power in the electric field, their effect is known as the capacitance. It is also called the **condenser**.

- The capacitor consists of two conductive plates which are separated by the dielectric medium. The dielectric material is made up of glass, paper, mica, oxide layers, etc. In pure AC capacitor circuit, the current leads the voltage by an angle of 90 degrees.

- When the voltage is applied across the capacitor, then the electric field is developed across the plates of the capacitor and no current flow between them. If the variable voltage source is applied across the capacitor plates then the ongoing current flows through the source due to the charging and discharging of the capacitor.

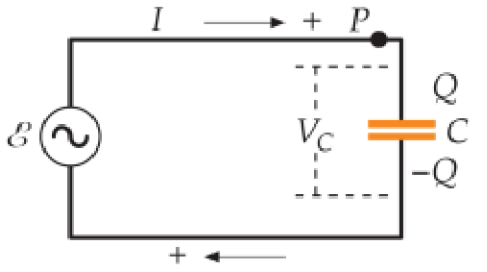
Capacitors in an AC Circuit

$$V_C = \mathcal{E}_p cos\omega t = V_{C\,p} cos\omega t$$

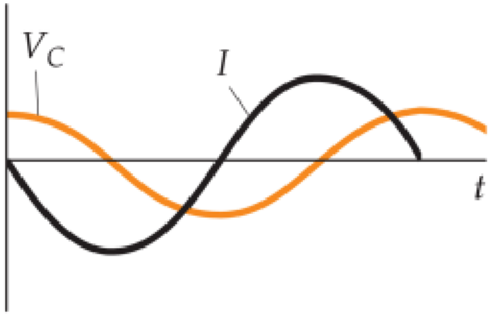
$$Q = V_C C = V_{C\,p} C cos\omega t = Q_p cos\omega t$$

$$I = \frac{dQ}{dt} = -\omega Q_p sin\omega t = -I_p sin\omega t$$

$$I = -\omega Q_p sin\omega t = I_p cos\left(\omega t + \pi/2\right)$$

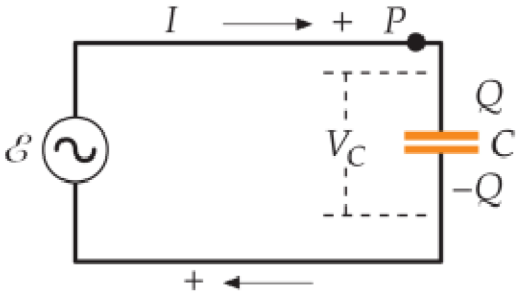


For the case of a capacitor in an AC circuit the V_C across the capacitor is 90^0 behind the current I on the capacitor.



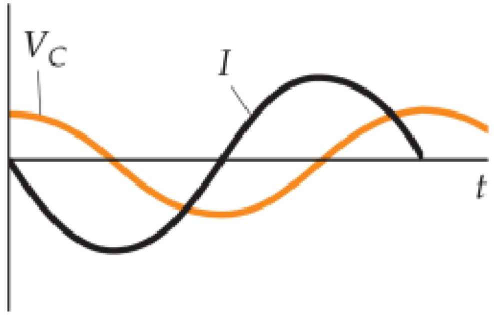
Capacitors in an AC Circuit

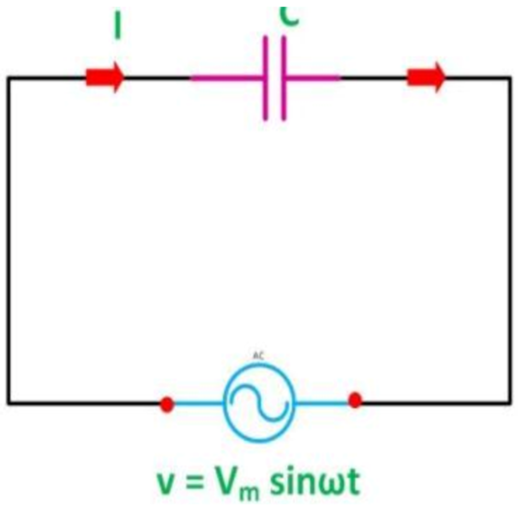
$$I_p = \omega Q_p = \omega C V_{Cp} = \frac{V_{Cp}}{1/\omega C} = \frac{V_{Cp}}{X_C}$$



$$X_C = \frac{1}{\omega C}$$

X_C is the capacitive reactance.





Let the alternating voltage applied to the circuit is given by the equation:

$$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

Charge of the capacitor at any instant of time is given as:

$$q = Cv \ \ldots \ldots \ldots \ldots (2)$$

Current flowing through the circuit is given by the equation:

$$i = \frac{d}{dt} q$$

Putting the value of q from the equation (2) in equation (3) we will get

$$i = \frac{d}{dt}(Cv) \ldots \ldots \ldots \ldots (3)$$

Now, putting the value of v from the equation (1) in the equation (3) we will get

$$i = \frac{d}{dt} C\, V_m \mathrm{Sin}\omega t = C\, V_m \frac{d}{dt} \sin\omega t \quad \text{or}$$

$$i = \omega\, C\, V_m \cos\omega t = \frac{V_m}{{}^{1}/_{\omega C}} \sin\left(\omega t + {}^{\pi}/_{2}\right) \quad \text{or}$$

$$i = \frac{V_m}{X_C} \sin\left(\omega t + {}^{\pi}/_{2}\right) \ldots\ldots\ldots\ldots(4)$$

Where Xc = 1/ωC is the opposition offered to the flow of alternating current by a pure capacitor and is called **Capacitive Reactance.**

The value of current will be maximum when sin(ωt + π/2) = 1. Therefore, the value of maximum current I_m will be given as:

$$I_m = \frac{V_m}{X_C}$$

Substituting the value of I_m in the equation (4) we will get:

$$i = I_m \sin\left(\omega t + {}^{\pi}/_{2}\right)$$

Power in Pure Capacitor Circuit

Instantaneous power is given by p = vi

$$P = (V_m \sin\omega t)(I_m \sin(\omega t + \pi/_2))$$

$$P = V_m I_m \sin\omega t \cos\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\,\omega t \quad \text{or}$$

$$P = 0$$

Hence, from the above equation, it is clear that the average power in the capacitive circuit is zero.

The average power in a half cycle is zero as the positive and negative loop area in the waveform shown are same.

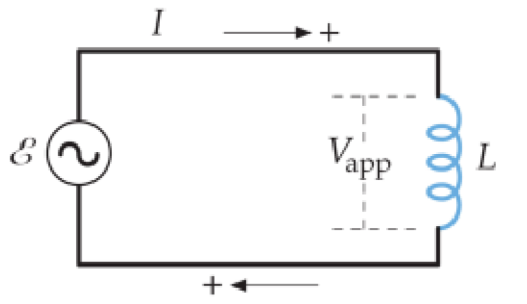
In the first quarter cycle, the power which is supplied by the source is stored in the electric field set up between the capacitor plates. In the another or next quarter cycle, the electric field diminishes, and thus the power stored in the field is returned to the source. This process is repeated continuously and, therefore, no power is consumed by the capacitor circuit.

Inductors in an AC Circuit

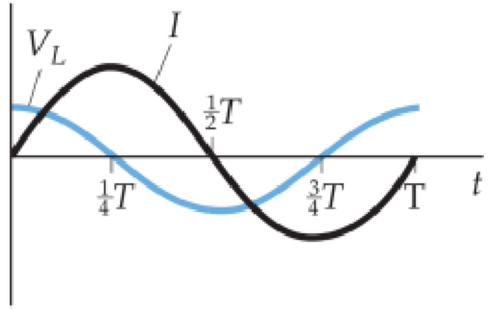
$$\varepsilon_{peak} \cos\omega t = V_{L\,peak} \cos\omega t = L\frac{dI}{dt}$$

$$I = \frac{V_{L\,peak}}{L}\int \cos\omega t\,dt = \frac{V_{L\,peak}}{\omega L}\sin\omega t$$

$$I = I_p \sin\omega t = I_p \cos\left(\omega t - \pi/2\right)$$



For the case of an inductor in an AC circuit the V_L across the inductor is 90^0 ahead of the current I through the inductor.



CIRCUIT WITH PURE INDUCTANCE ONLY

Let the applied voltage

$$e = E_m \sin \omega t$$

(1)

instantaneous value of self induced emf is e'

$$e' = -L\frac{di}{dt} = -e$$

$$di = \frac{1}{L}\, e\, dt$$

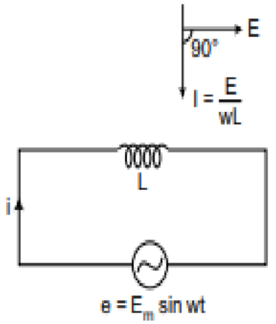


Fig. 2.25

integrating both side, we get

$$\int di = \frac{1}{L}\int E_m \sin \omega t . dt$$

$$i = \frac{E_m}{\omega L}(-\cos \omega t)$$

$$i = \frac{E_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \quad \left\{\begin{array}{l}\text{integration constant will} \\ \text{cancel out from both side}\end{array}\right.$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \qquad (2) \quad \left\{ I_m = \frac{E_m}{\omega L}\right.$$

The quantity ωL is called inductive reactance and is usually devoted by symbol X_L and units is ohm.

$$X_L = \omega L \text{ ohms}$$

where, L is in henry and ω is in rad/sec.

Wave diagram and Phasor diagram for Pure inductance

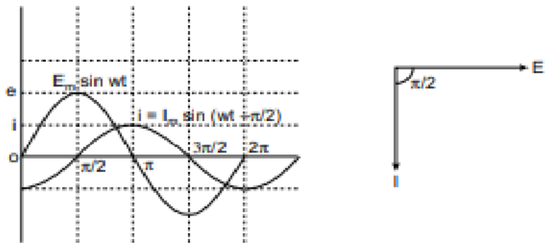


Fig. 2.26

Average Power→

$$P = \frac{1}{2\pi}\int_0^{2\pi} e\, i\, d(wt)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} E_m \sin wt . I_m \sin\left(wt - \frac{\pi}{2}\right) d(wt)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} -E_m\, I_m \sin wt . \cos wt . d(wt)$$

$$= -\frac{V_m\, I_m}{2\pi}\int_0^{2\pi}\frac{\sin 2wt}{2}.d(wt)$$

$$= 0$$

CIRCUIT WITH RESISTANCE AND INDUCTANCE IN SERIES

Let R = Resistance in ohms in the circuit.

L = Inductance in henries

X_L = Inductive reactance

$= \omega L$

E = Effective value of applied emf

I = Effective value of current in circuit.

Voltage drop across resistance,

$E_R = R.I$ in phase with current vector as shown in vector diagram of Fig. 2.30.

Voltage across reactance,

$E_L = I.\omega L = IX_L$, 90° ahead of vector I

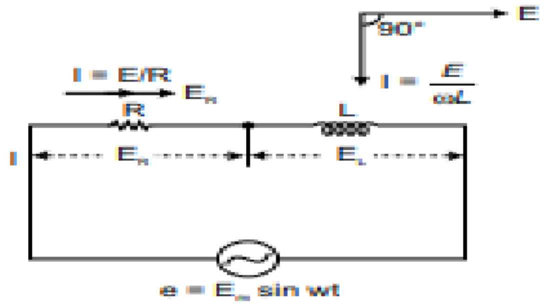


Fig. 2.29

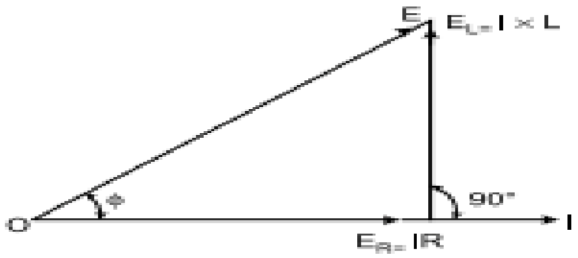


Fig. 2.30

$$Z = R + jX_L$$

$$= \sqrt{R^2 + X_L^2} \angle \tan^{-1} \frac{X_L}{R}$$

here $\phi = \tan^{-1} \dfrac{X_L}{R}$

and $$|Z| = \sqrt{R^2 + X_L^2}$$

$$Z = |Z| \angle \phi$$

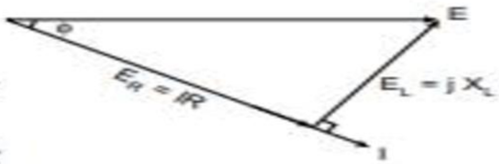
$$I = \frac{E}{Z} = \frac{E \angle \phi}{|Z| \angle \phi}$$

$$\boxed{I = \frac{E}{Z} \angle -\phi}$$

instantaneous value of current is, $i = I_m \sin(\omega t - \phi)$, where $I_m = \frac{E}{Z}$

hence in R-L circuit current lags the applied voltage by angle $\phi = \tan^{-1} \frac{X_L}{R}$

The applied voltage is therefore given by,



$\therefore$ $$E = \sqrt{E_r^2 + E_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$\therefore$ $$E = I\sqrt{R^2 + X_L^2} = IZ$$

$$\phi = \tan^{-1} \frac{X_L}{R} = \tan^{-1} \frac{\omega L}{R}$$

or, $$I = \frac{E}{\sqrt{R^2 + X_L^2}}$$

The quantity $\sqrt{R^2 + X_L^2}$ is called impedance.

Since, the power is consumed by the resistance only, so the power in the circuit is given by,

$$P = I^2 R = I.IR$$

$$= \frac{E}{\sqrt{R^2 + X_L^2}} . IR$$

or, $$P = E.I \frac{R}{\sqrt{R^2 + X_L^2}}$$

If ϕ is the angle between E and I, then

$$\cos \phi = \frac{E_R}{E} = \frac{IR}{I\sqrt{R^2 + X_L^2}} = \frac{R}{Z}$$