Kirchoff's Current And Voltage Law

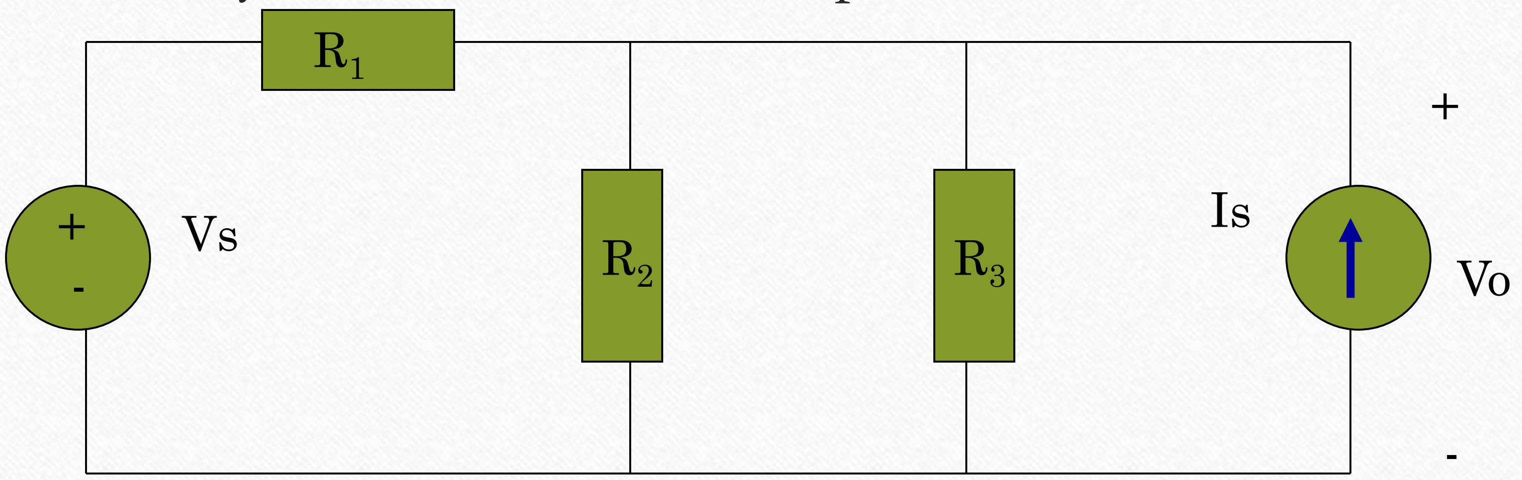
(KCL and KVL)

Circuit Definitions

- Node – any point where 2 or more circuit elements are connected together
  - Wires usually have negligible resistance
  - Each node has one voltage (w.r.t. ground)
- Branch – a circuit element between two nodes
- Loop – a collection of branches that form a closed path returning to the same node without going through any other nodes or branches twice

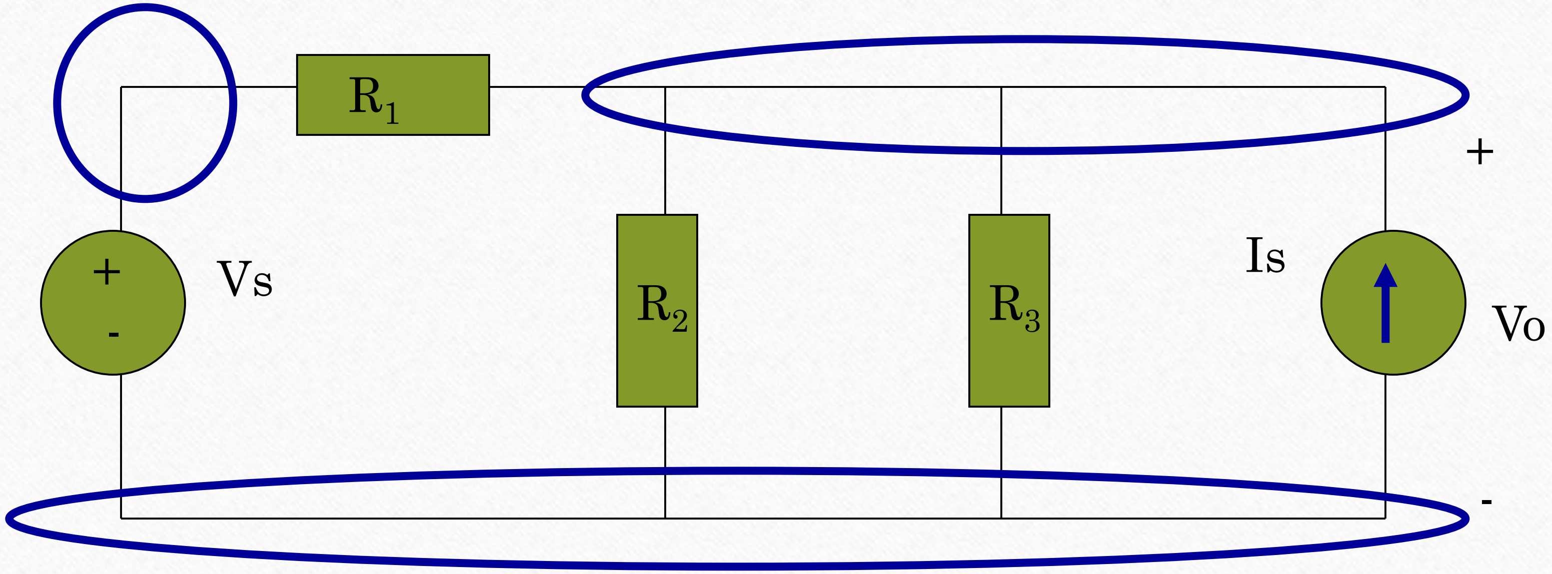
Example

- How many nodes, branches & loops?



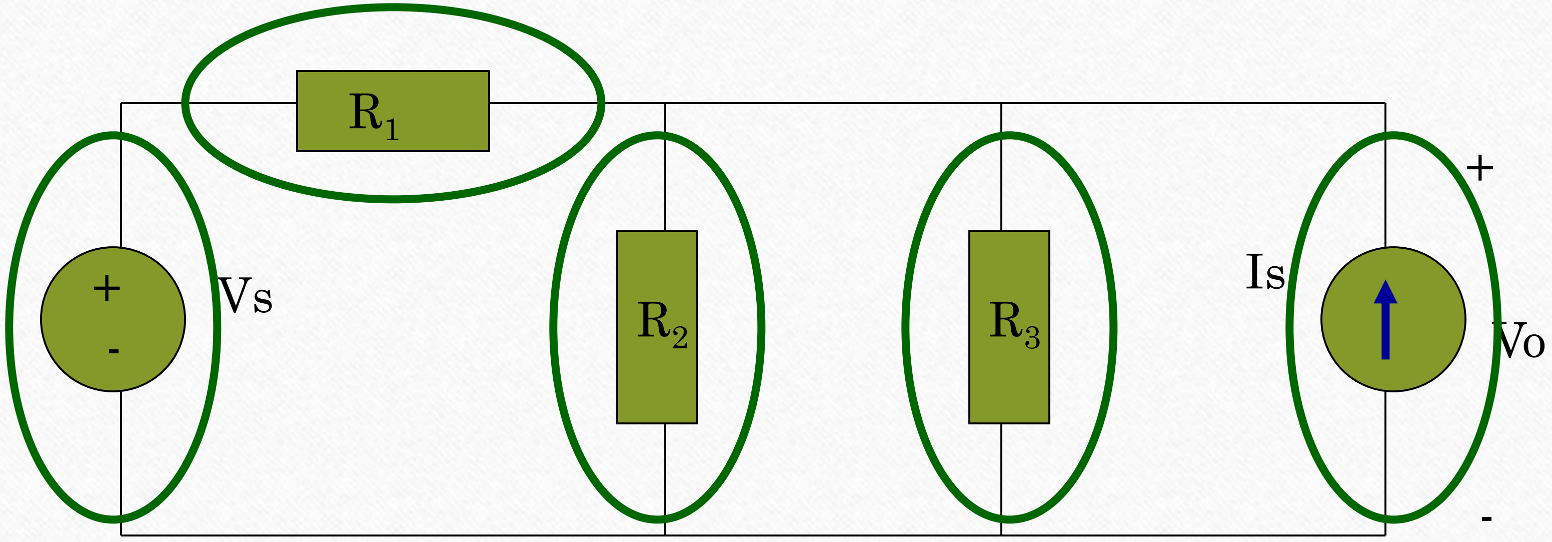
Example

- Three nodes



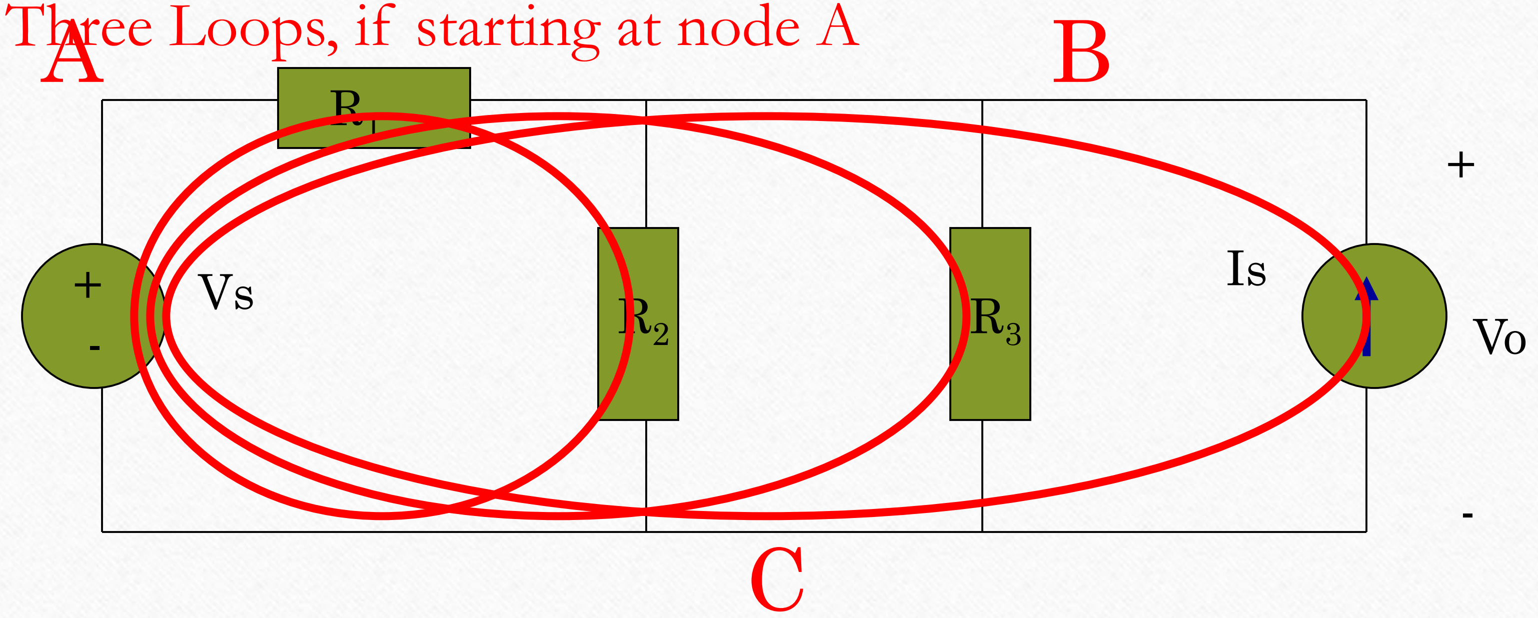
Example

- 5 Branches



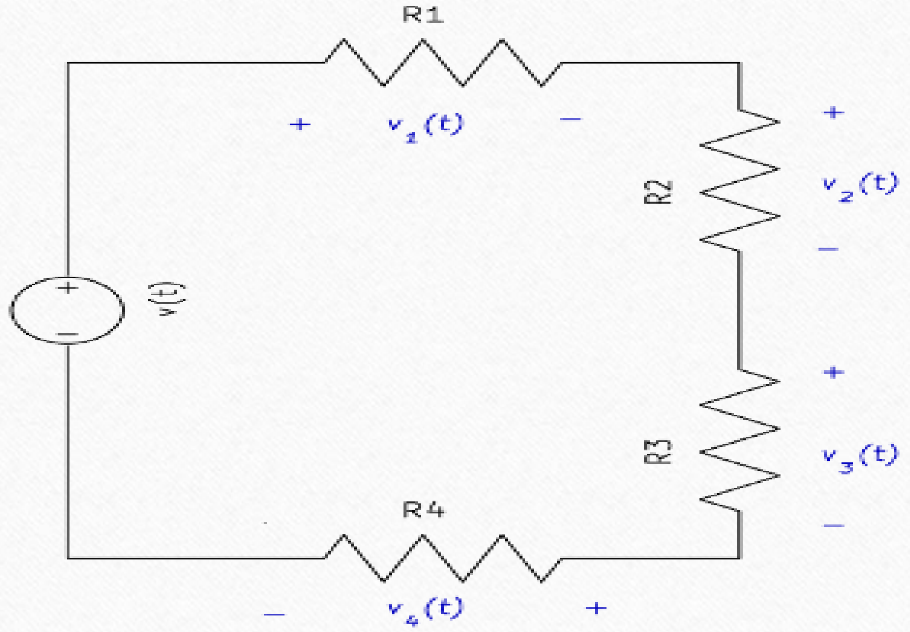
Example

- Three Loops, if starting at node A



Voltage division rule

- Voltage Division Rule: The voltage is divided between two series resistors in direct proportion to their resistance.

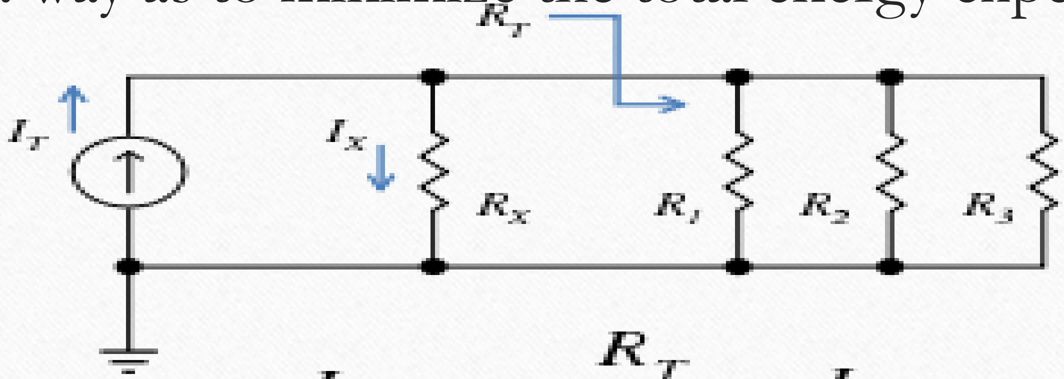


- Results

$$v_1(t) = \frac{R_1}{R_1+R_2+R_3+R_4} v(t),$$
$$v_2(t) = \frac{R_2}{R_1+R_2+R_3+R_4} v(t),$$
$$v_3(t) = \frac{R_3}{R_1+R_2+R_3+R_4} v(t),$$
$$v_4(t) = \frac{R_4}{R_1+R_2+R_3+R_4} v(t).$$

Current division rule

- **Current division** refers to the splitting of current between the branches of the divider. The currents in the various branches of such a circuit will always divide in such a way as to minimize the total energy expended.



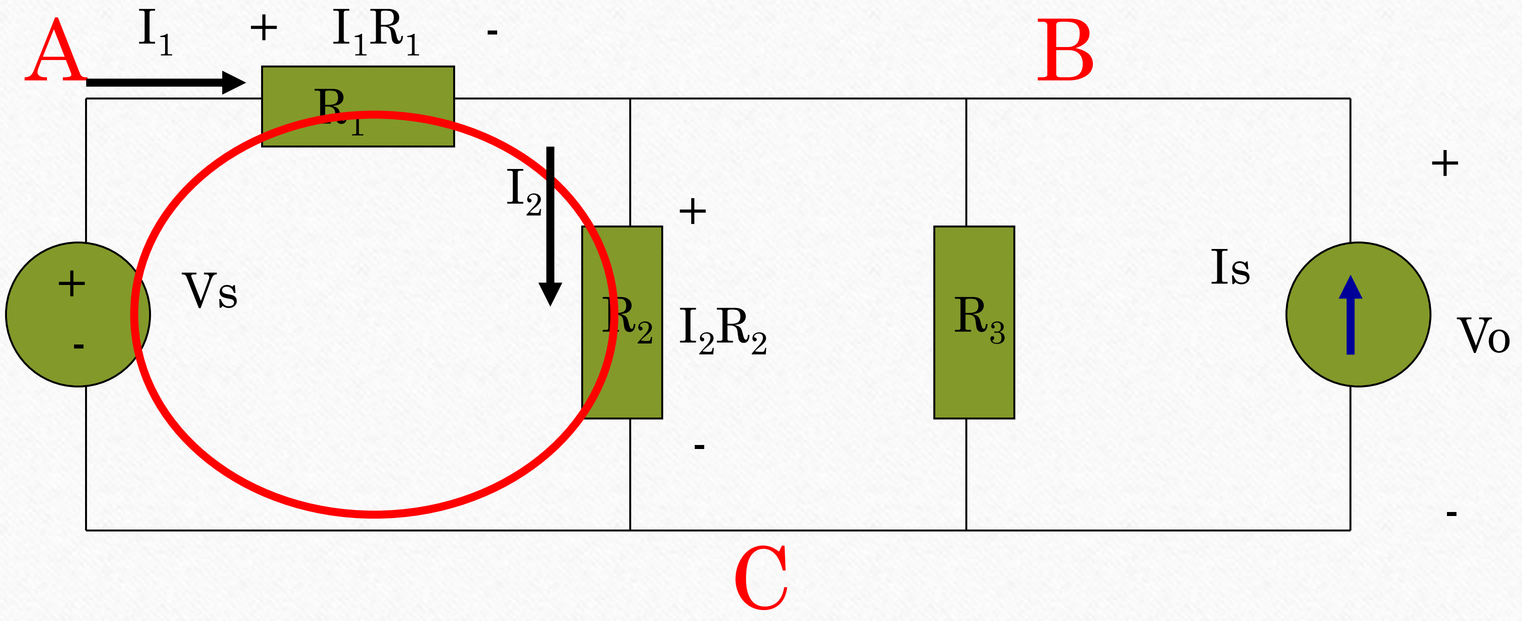
$$I_X = \frac{R_T}{R_X + R_T} I_T$$

Kirchoff's Voltage Law (KVL)

- The algebraic sum of voltages around each loop is zero
  - Beginning with one node, add voltages across each branch in the loop (if you encounter a + sign first) and subtract voltages (if you encounter a – sign first)
- Σ voltage drops - Σ voltage rises = 0
- Or Σ voltage drops = Σ voltage rises

Example

- Kirchoff's Voltage Law around 1^{st} Loop

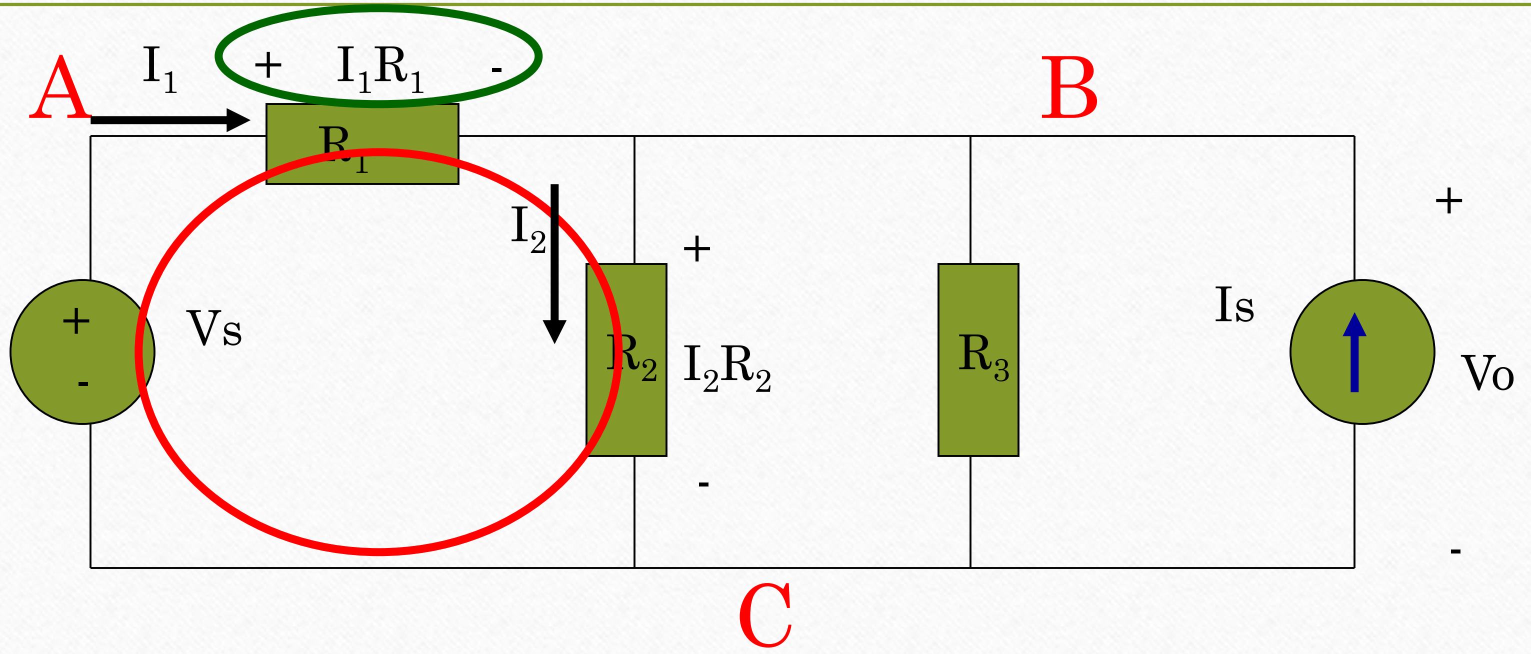


Assign current variables and directions

Use Ohm's law to assign voltages and polarities consistent with passive devices (current enters at the + side)

Example

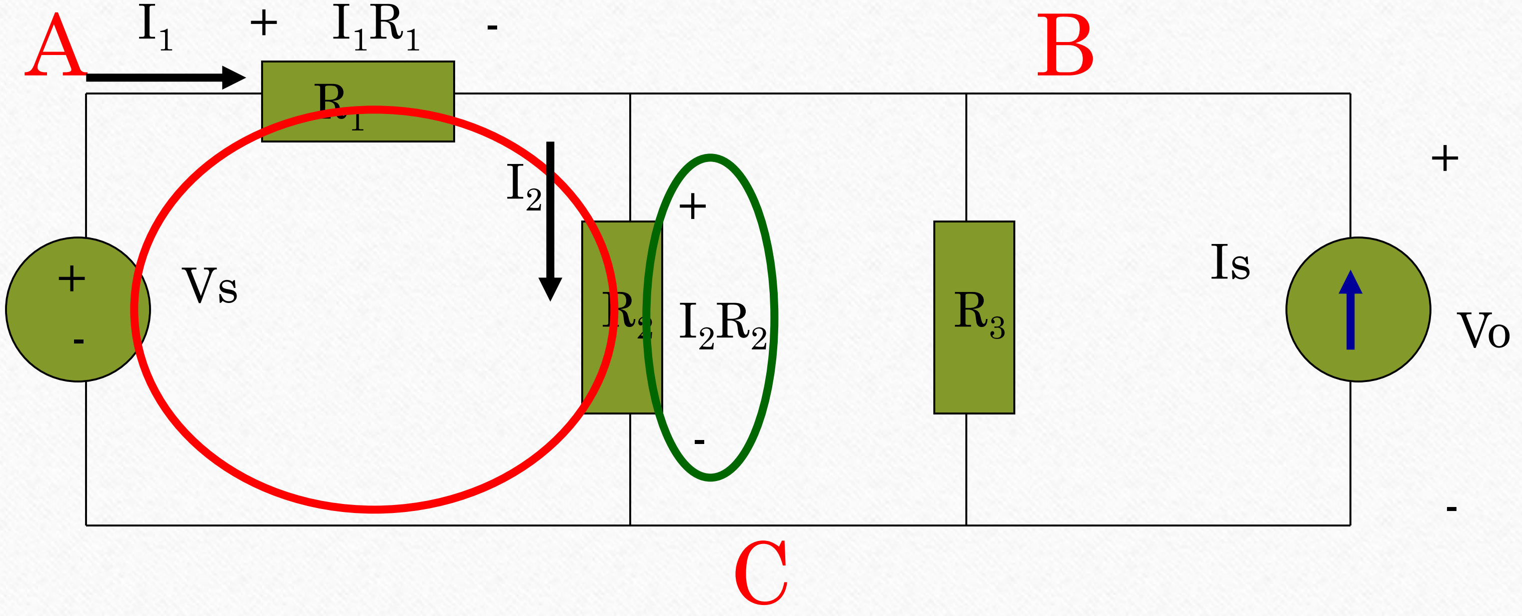
- Kirchoff's Voltage Law around 1st Loop



Starting at node A, add the 1st voltage drop: $-I_1R_1$

Example

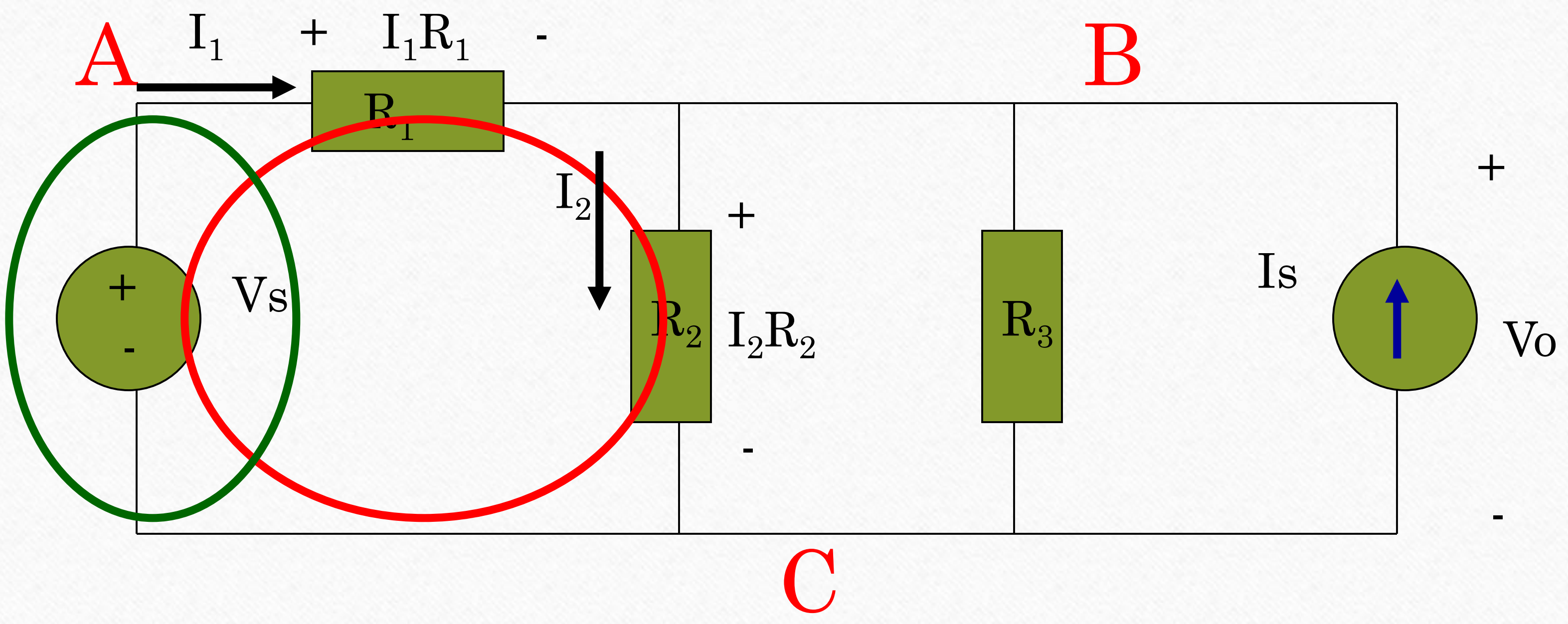
- Kirchoff's Voltage Law around 1[st] Loop



Add the voltage drop from B to C through R_2: $-I_1R_1$ $-I_2R_2$

Example

- Kirchoff's Voltage Law around 1st Loop



Subtract the voltage rise from C to A through Vs: $-I_1R_1 - I_2R_2 + Vs = 0$

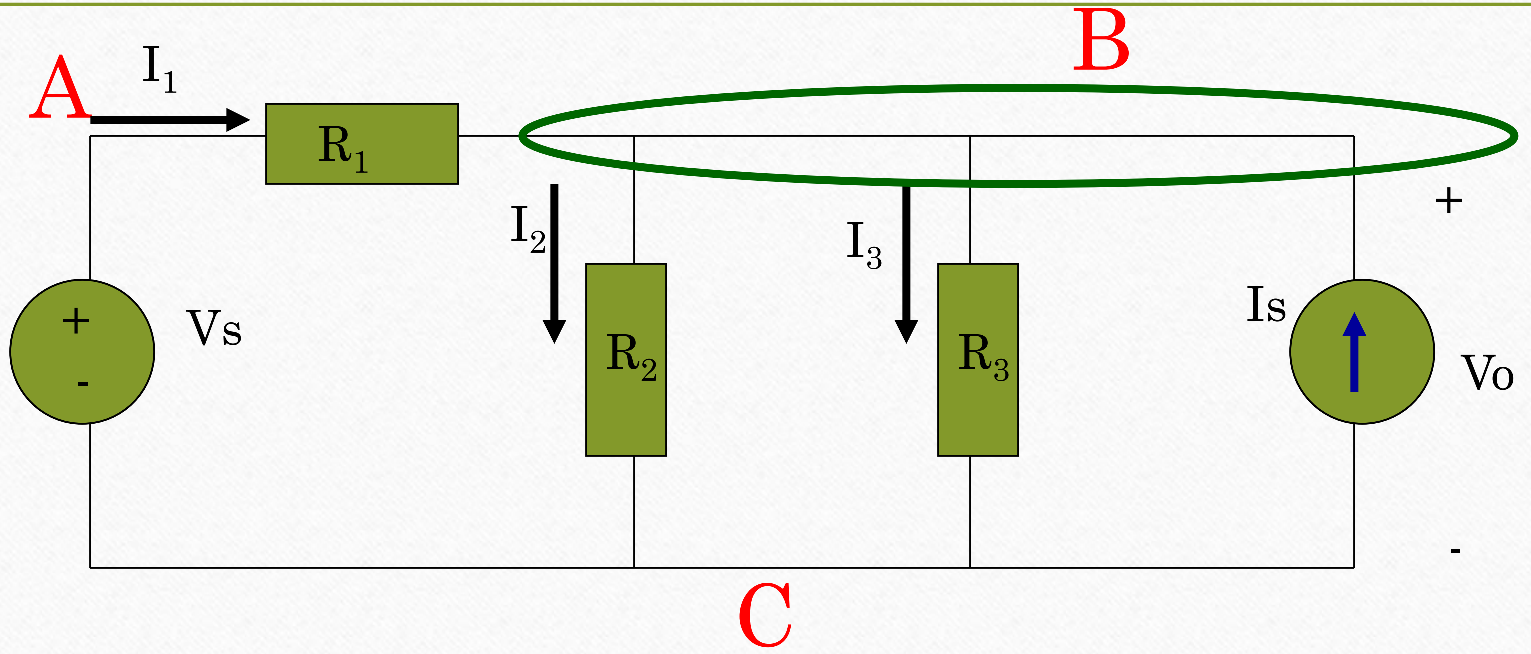
Notice that the sign of each term matches the polarity encountered 1st

Kirchoff's Current Law (KCL)

- The algebraic sum of currents entering a node is zero
  - Add each branch current entering the node and subtract each branch current leaving the node
- Σ currents in - Σ currents out = 0
- Or Σ currents in = Σ currents out

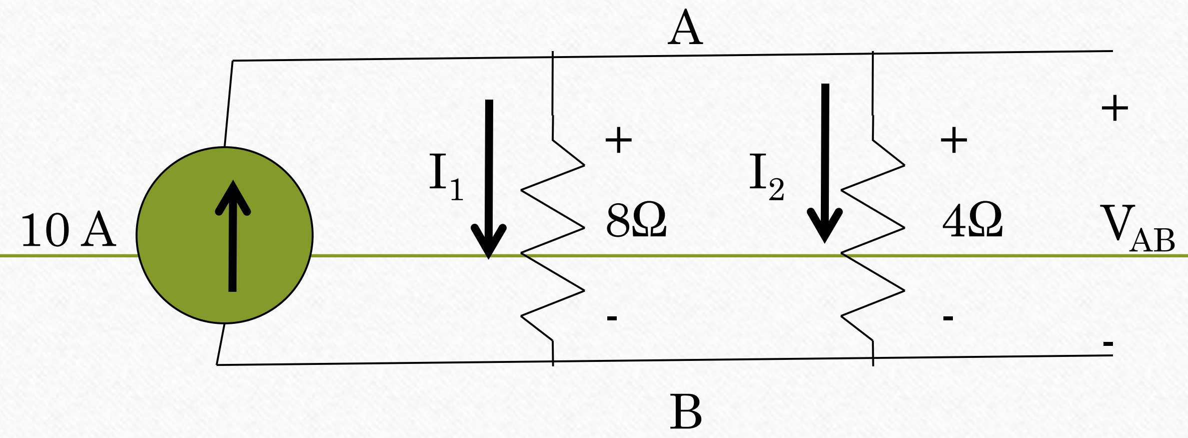
Example

- Kirchoff's Current Law at B



Assign current variables and directions

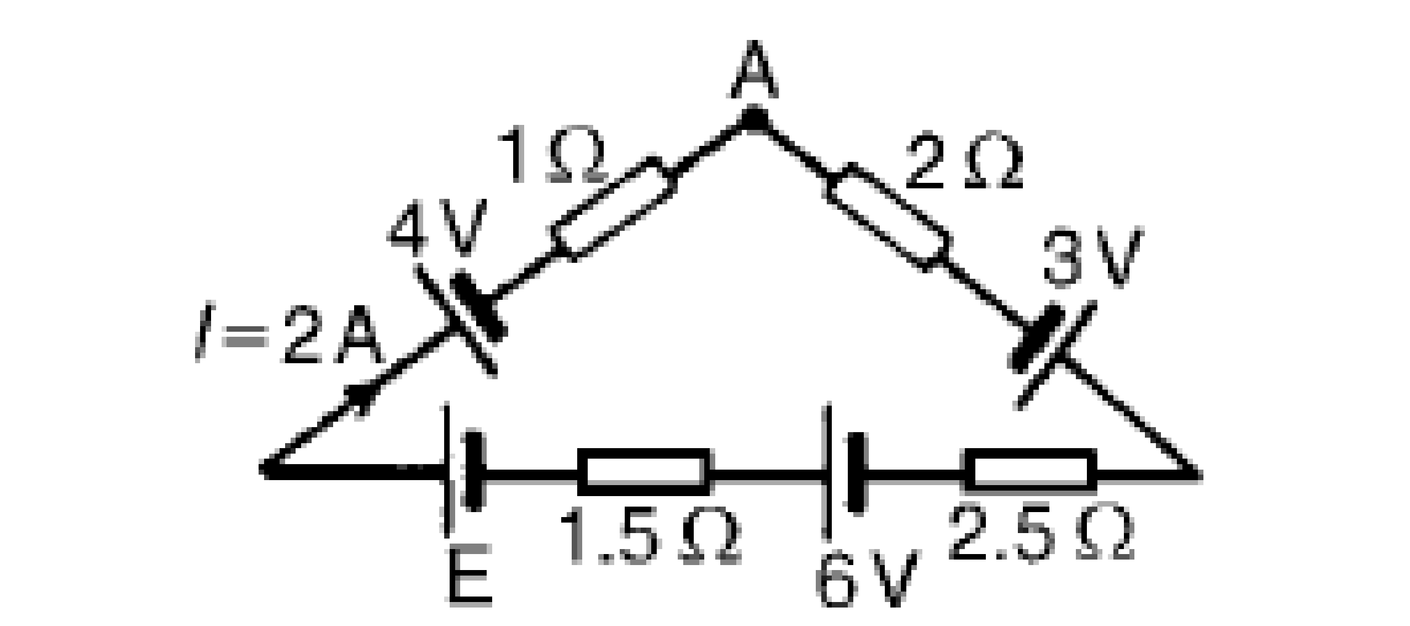
Add currents in, subtract currents out: $I_1 - I_2 - I_3 + Is = 0$


A
10 A
I_1
+
8Ω
-
I_2
+
4Ω
-
+
V_{AB}
-
B

- By KVL: $- I_1 \cdot 8\Omega + I_2 \cdot 4\Omega = 0$
- Solving: $I_2 = 2 \cdot I_1$
- By KCL: $10A = I_1 + I_2$
- Substituting: $10A = I_1 + 2 \cdot I_1 = 3 \cdot I_1$
- So $I_1 = 3.33$ A and $I_2 = 6.67$ A
- And $V_{AB} = 26.33$ volts

KVL and KCL Problems

Determine the value of e.m.f. E



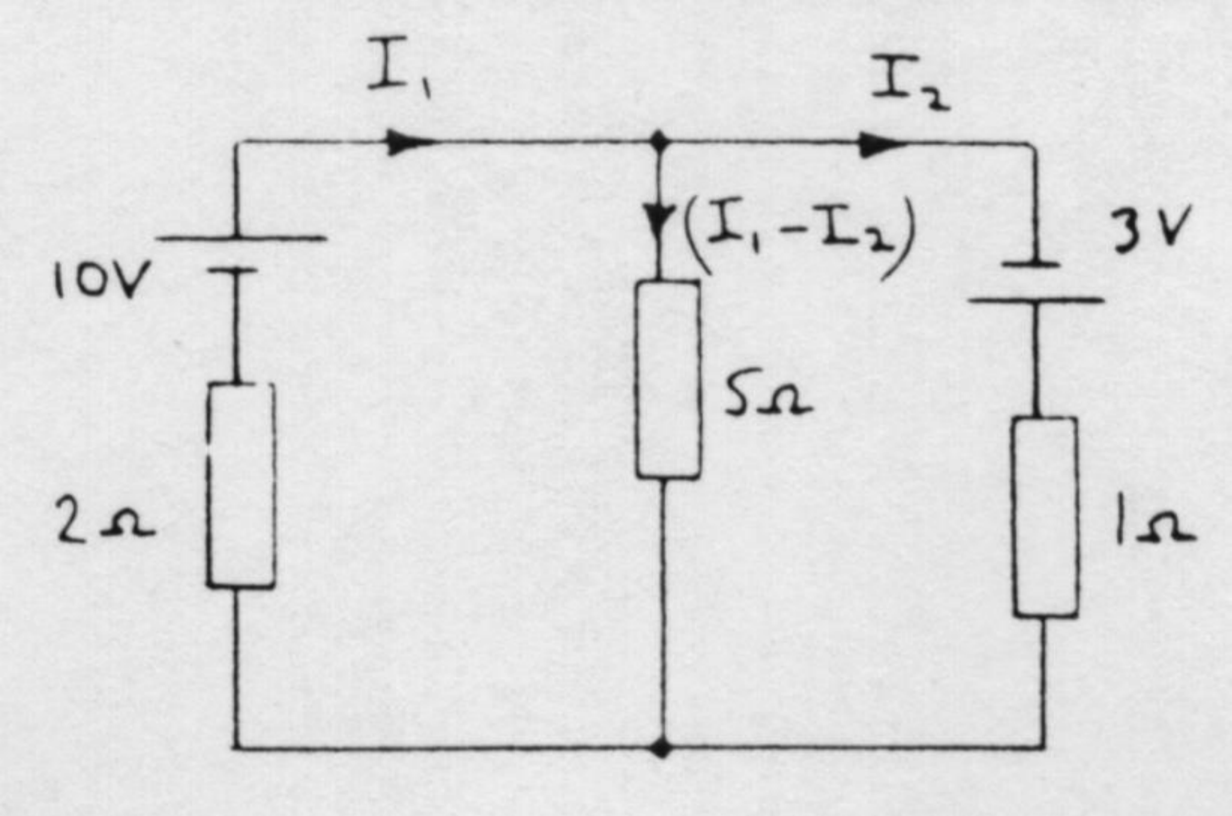
Applying Kirchhoff's voltage law and moving clockwise around the loop of Figure 13.3(b) starting at point A:

$$3 + 6 + E - 4 = (I)(2) + (I)(2.5) + (I)(1.5) + (I)(1)$$
$$= I(2 + 2.5 + 1.5 + 1)$$

i.e. $5 + E = 2(7)$, since $I = 2$ A

Hence $E = 14 - 5 = \mathbf{9\ V}$

Find the current flowing in the 5 Ω resistor of the circuit shown



Ans= **0.2353 A**

Solution

From the left-hand loop: $10 = 2I + 5(I - I\ )$

i.e. $7I - 5I\ = 10$ (1)

From the right-hand loop: $3 = I - 5(I - I\ )$

i.e. $-5I + 6I\ = 3$ (2)

5 × equation (1) gives: $35I - 25I\ = 50$ (3)

7 × equation (2) gives: $-35I + 42I\ = 21$ (4)

Equation (3) + equation (4) gives: $17I = 71$

from which, $I\ =\ = 4.1765$ A

From equation (1) $7I - 5(4.1765) = 10$

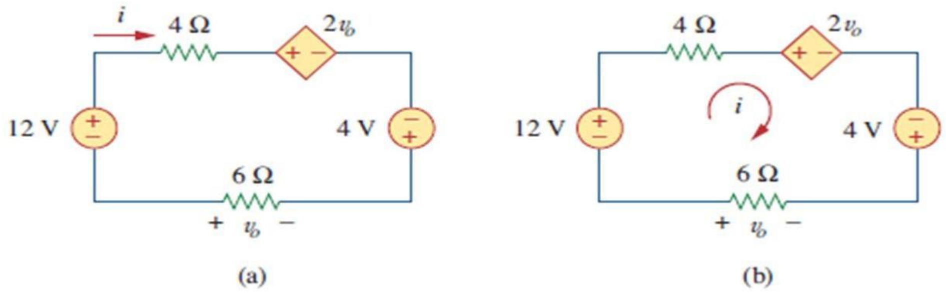
i.e. $7I = 10 + 20.8825 = 30.8825$

and $I\ =\ = 4.4118$ A

Hence, **the current in the 5 Ω resistor** = $I - I\ = 4.4118 - 4.1765 =$ **0.2353 A**

Example-4 for KVL

Determine v_o and i in the circuit shown in Fig.



Solution:

We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \qquad \textbf{(2.6.1)}$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \qquad \textbf{(2.6.2)}$$

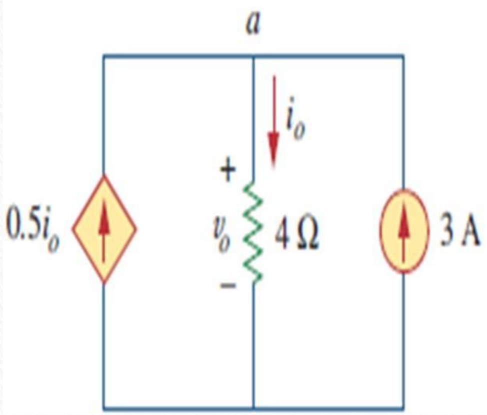
Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8 \text{ A}$$

and $v_o = 48$ V.

Example-5 for Ohm's Law and KVL

Find current i_o and voltage v_o in the circuit shown in Fig.



Solution:

Applying KCL to node a, we obtain

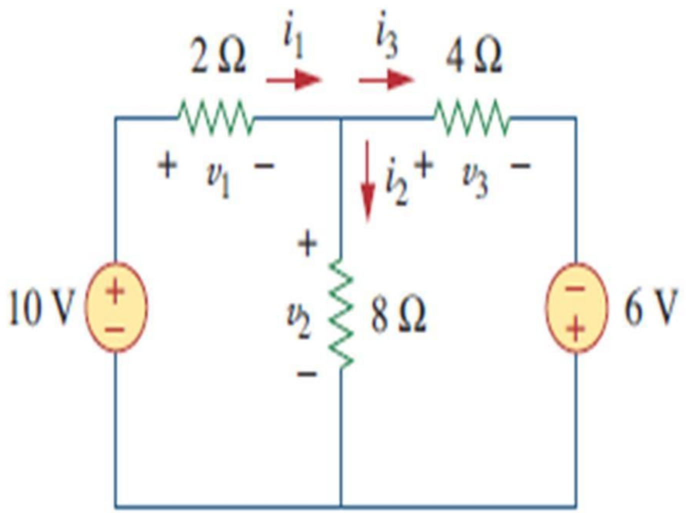
$$3 + 0.5i_o = i_o \quad \Rightarrow \quad i_o = 6\text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24\text{ V}$$

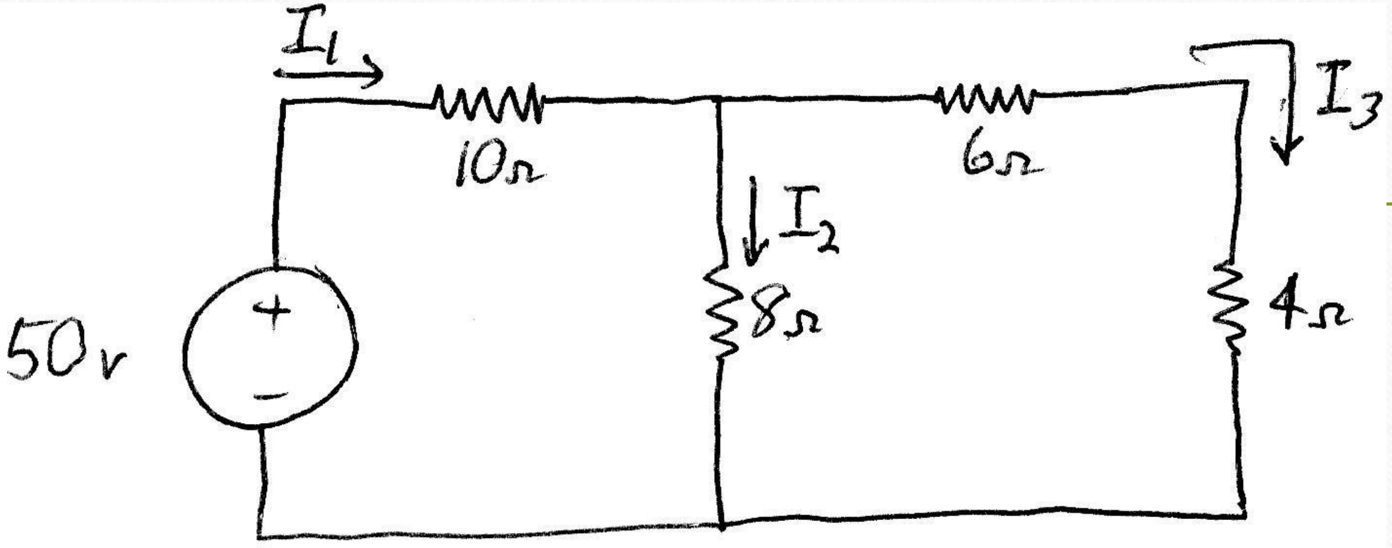
Practice Problem 3

Find the currents and voltages in the circuit shown in Fig.



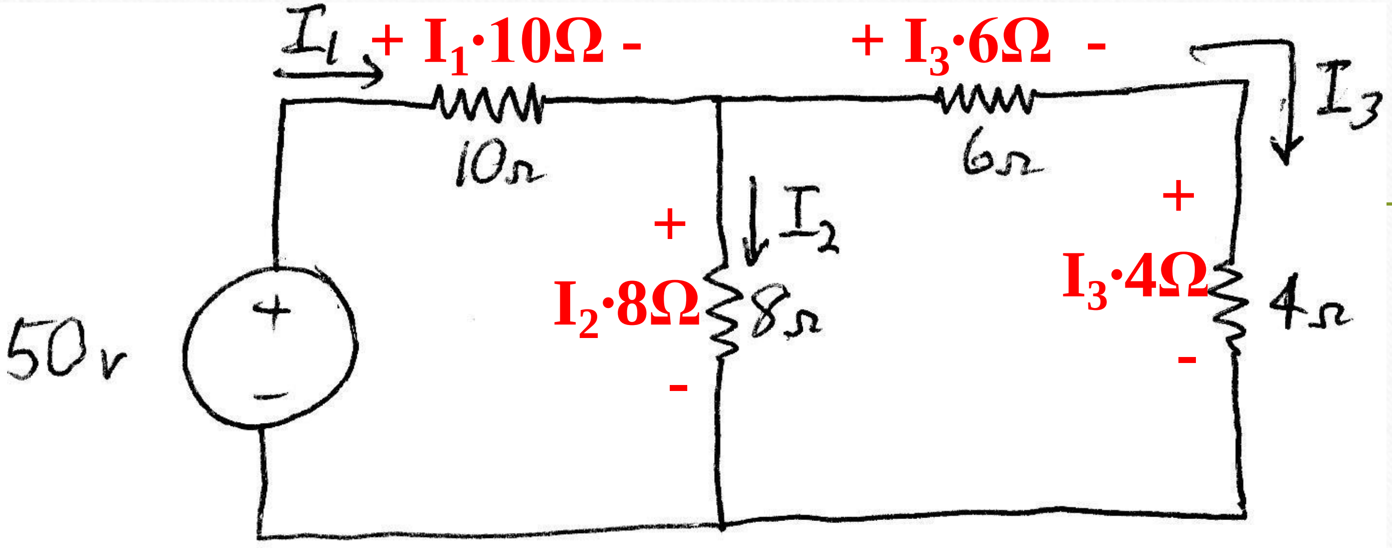
Answer: $v_1 = 6$ V, $v_2 = 4$ V, $v_3 = 10$ V, $i_1 = 3$ A, $i_2 = 500$ mA, $i_3 = 1.25$ A.

Example Circuit



Solve for the currents through each resistor
And the voltages across each resistor

Example Circuit



Using Ohm's law, add polarities and expressions for each resistor voltage

Example Circuit

- We now have 3 equations in 3 unknowns, so we can solve for the currents through each resistor, that are used to find the voltage across each resistor
- Since $I_1 - I_2 - I_3 = 0$, $I_1 = I_2 + I_3$
- Substituting into the 1st KVL equation

$$-50\text{ v} + (I_2 + I_3)\cdot 10\Omega + I_2\cdot 8\Omega = 0$$

$$\text{or } I_2\cdot 18\ \Omega + I_3\cdot 10\ \Omega = 50\text{ volts}$$

Example Circuit

- But from the 2nd KVL equation, $I_2 = 1.25 \cdot I_3$
- Substituting into 1st KVL equation:

$(1.25 \cdot I_3) \cdot 18\ \Omega + I_3 \cdot 10\ \Omega = 50$ volts

Or: $I_3 \cdot 22.5\ \Omega + I_3 \cdot 10\ \Omega = 50$ volts

Or: $I_3 \cdot 32.5\ \Omega = 50$ volts

Or: $I_3 = 50$ volts/$32.5\ \Omega$

Or: $I_3 = 1.538$ amps

Example Circuit

- Since $I_3 = 1.538$ amps

  $I_2 = 1.25 \cdot I_3 = 1.923$ amps
- Since $I_1 = I_2 + I_3,$ $I_1 = 3.461$ amps
- The voltages across the resistors:

  $I_1 \cdot 10\Omega = 34.61$ volts

  $I_2 \cdot 8\Omega = 15.38$ volts

  $I_3 \cdot 6\Omega = 9.23$ volts

  $I_3 \cdot 4\Omega = 6.15$ volts

1. Kirchhoffs current law is applicable to only
   (A) Closed loops in a network (B) Electronic circuits (C) Junctions in a network (D) Electric circuits

2. Kirchhoff's laws are applicable to.........
   (A) AC circuits (B) DC circuits (C) Both a and b (D) None

3. Kirchhoffs voltage law is concerned with
   (A) IR drops (B) battery e.m.fs. (C) junction voltages (D) both (a) and (b)

4. According to KVL, the algebraic sum of all IR drops and e.m.f.s in any closed loop of a network is always
   (A) zero (B) positive
   (C) negative (D) determined by battery e.m.fs.

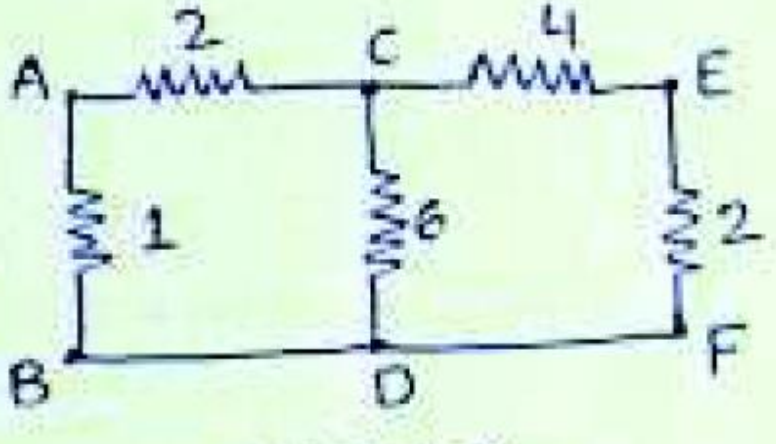
5. The algebraic sign or polarity of an IR drop is primarily dependent upon the
   (A) amount of current flowing through it (B) value of R
   (C) direction of current flow (D) battery connection.

6. Maxwell's loop current method of solving electrical networks
   (A) uses branch currents (B) utilizes Kirchhoff's voltage law
   (C) is confined to single-loop circuits (D) is a network reduction method.

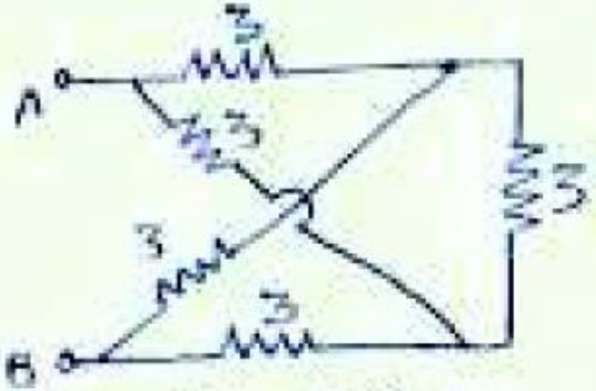
7. The sign or polarity of the battery e.m.f. is.......... of the direction of the current through that branch.
   (A) independent (B) dependent (C) either a or b (D) None

8. The sign or polarity of voltage drop across a resistor ....on the direction of current through that resistor but is ....of the polarity of any other source of e.m.f. in the circuit under consideration.
   (A) depends, dependent (B) depends, independent (C) independent, dependent (D) independent, independent

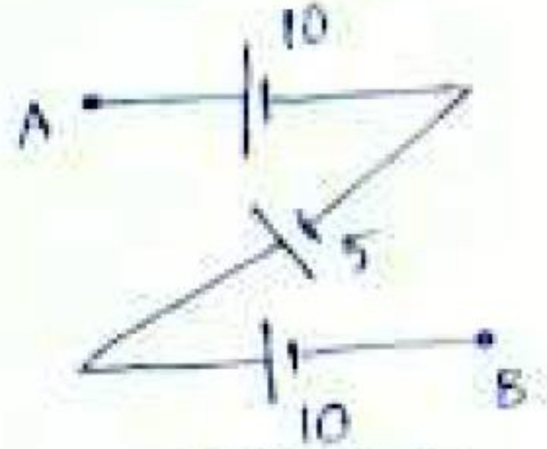
1	C
2	C
3	D
4	A
5	C
6	B
7	A
8	B



CIRCUIT-1



CIRCUIT-2



CIRCUIT-3

64. What is the equivalent value of resistance between A and B terminals in above circuit1?

(A) 1.5 Ω (B) $\frac{5}{6}$ Ω (C) $\frac{4}{3}$ Ω (D) $\frac{6}{5}$ Ω

65. What is the equivalent value of resistance between C and D terminals in above circuit1?

(A) 1.5 Ω (B) $\frac{5}{6}$ Ω (C) $\frac{4}{3}$ Ω (D) $\frac{6}{5}$ Ω

66. What is the equivalent value of resistance between A and C terminals in above circuit1?

(A) 1.5 Ω (B) $\frac{5}{6}$ Ω (C) $\frac{4}{3}$ Ω (D) $\frac{6}{5}$ Ω

67. What is the equivalent value of resistance between A and B terminals in above circuit2?

(A) 3 Ω (B) 1.5 Ω (C) 5.2 Ω (D) 6 Ω

68. What is the voltage between terminals A and B in above circuit3?

(A) 25 V (B) 15 V (C) -15 V (D) None

64	B
65	A
66	C
67	A
68	B
69	A