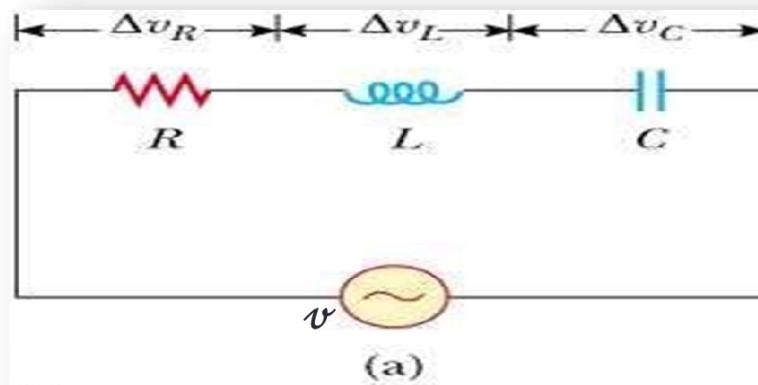


Let us consider as R-L-C series circuit

We know that the impedance in R-L-C series circuit is

$$| Z | = \sqrt{R^2 + (X_L - X_C)^2}$$

Where  $X_L = 2\pi fL$  &  $X_C = \frac{1}{2\pi fL}$



Such a circuit shown in figure is connected to an a.c. source of constant supply voltage V but having variable frequency. The frequency can be varied from zero, increasing and approaching infinity.

- Since  $X_L$  and  $X_C$  are functions of frequency, at a particular frequency of the applied voltage,  $X_L$  and  $X_C$  will become equal in magnitude.

$$\text{Since } X_L = X_C$$

$$X_L - X_C = 0$$

$$\therefore Z = \sqrt{R^2 + 0} = R$$

The circuit, when  $X_L = X_C$  and hence  $Z = R$ , is said to be in resonance. In a series circuit current I remains the same throughout we can write,

$$IX_L = IX_C$$

$$\text{i.e. } V_L = V_C$$

So, at resonance  $V_L$  and  $V_C$  will cancel out each other.

$\therefore$  the supply voltage

$$V = \sqrt{V_L^2 + (V_L - V_C)^2}$$

$$V = \sqrt{V_R^2}$$

$$\therefore V = V_R$$

i.e. The entire supply voltage will drop across the resistor R

# Resonant frequency

- At resonance  $X_L = X_C$

$$\therefore 2\pi f_r L = \frac{1}{2\pi f_r C} \quad (f_r \text{ is the resonant frequency})$$

$$\therefore f_r^2 = \frac{1}{(2\pi)^2 LC}$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}$$

Where L is the inductance in henry, C is the capacitance in farad  
and  $f_r$  the resonant frequency in Hz

*Under resonance condition the net reactance is zero . Hence the impedance of the circuit.*

$$Z = \sqrt{R^2 + X^2} = R \left[ \because X = 0 \text{ or } X_L - X_C = 0 \right]$$

*This is the minimum possible value of impedance. Hence, circuit current is maximum for the given value of R and its value is given by*

$$\boxed{I_m = \frac{V}{Z} = \frac{V}{R}} \quad [\because Z = R]$$

*The circuit behaves like a pure resistive circuit because net reactance is zero . So, the current is in phase with applied voltage .obviously, the power factor of the circuit is unity under resonance condition.*

*as current is maximum it produces large voltage drop across L and C.*

*Voltage across the inductance at resonance is given by*

$$V_L = I_m X_L = I_m \frac{\omega_r}{L}$$

$$= I_m (2\pi f_r L)$$

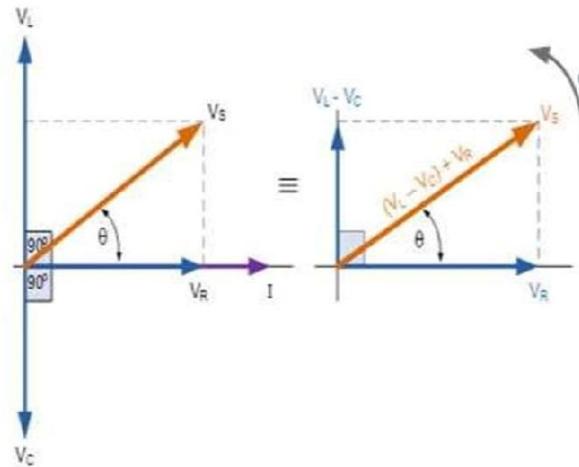
$$= I_m 2\pi \times \frac{1}{2\pi\sqrt{LC}} L = I_m \frac{L}{\sqrt{LC}} = I_m \sqrt{\frac{L^2}{LC}}$$

$$= I_m \sqrt{\frac{L}{C}}$$

*At resonance, the current flowing in the*

*circuit is equal to  $\frac{V}{R}$*

$$V_L = \frac{V}{R} \sqrt{\frac{L}{C}} = V \sqrt{\frac{L}{CR^2}}$$



*Similarly voltage across capacitance at resonance is given by*

$$V_C = I_m X_C$$

$$= I_m \frac{1}{\omega_r C} = I_m \frac{1}{2\pi f C}$$

$$= I_m \frac{1}{2\pi \times \frac{1}{2\pi \sqrt{LC}} \times C} = \frac{I_m \sqrt{LC}}{C} = I_m \sqrt{\frac{LC}{C^2}}$$

$$= I_m \sqrt{\frac{L}{C}} = \frac{V}{R} \sqrt{\frac{L}{C}}$$

*Thus voltage drop across L and C are equal and many times the applied voltage. Hence voltage magnification occurs at the resonance condition. so series resonance condition is often refers to as voltage resonance.*

# Q-FACTOR IN R-L-C SERIES CIRCUIT

**Q-FACTOR:** In case of R-L-C series circuit Q-Factor is defined as the voltage magnification of the circuit at resonance. Current at resonance is given by

$$I_m = \frac{V}{R} \Rightarrow V = I_m R$$

And voltage across inductance or capacitor is given by =

$$I_m X_L \text{ OR } I_m X_C$$

Voltage magnification = voltage across L or C / applied voltage

$$= \frac{V_L}{V} \quad \text{OR} \quad = \frac{V_C}{V} \quad [\because V_L = V_C]$$

$$= \frac{I_m X_L}{I_m R} \quad \text{OR} \quad \frac{I_m X_C}{I_m R}$$

$$\boxed{= \frac{X_L}{R} \text{ OR } = \frac{X_C}{R}}$$

Thus Q-factor

$$\begin{aligned} &= \frac{X_L}{R} OR = \frac{X_C}{R} \\ &= \frac{\omega_r L}{R} OR = \frac{1}{\omega_r C R} \\ &= \frac{2\pi f_r L}{R} OR = \frac{1}{2\pi f_r C R} \\ &= \frac{2\pi L}{2\pi \sqrt{LC} R} OR = \frac{1}{2\pi \times \frac{1}{2\pi \sqrt{LC}} C R} \\ &= \frac{1}{R} \sqrt{\frac{L^2}{LC}} OR = \frac{1}{R} \sqrt{\frac{LC}{C^2}} \\ &= \frac{1}{R} \sqrt{\frac{L}{C}} \end{aligned}$$

$$Q - factor = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{2\pi f_r L}{R} = \frac{1}{2\pi f_r C R}$$

# Bandwidth, $\beta$

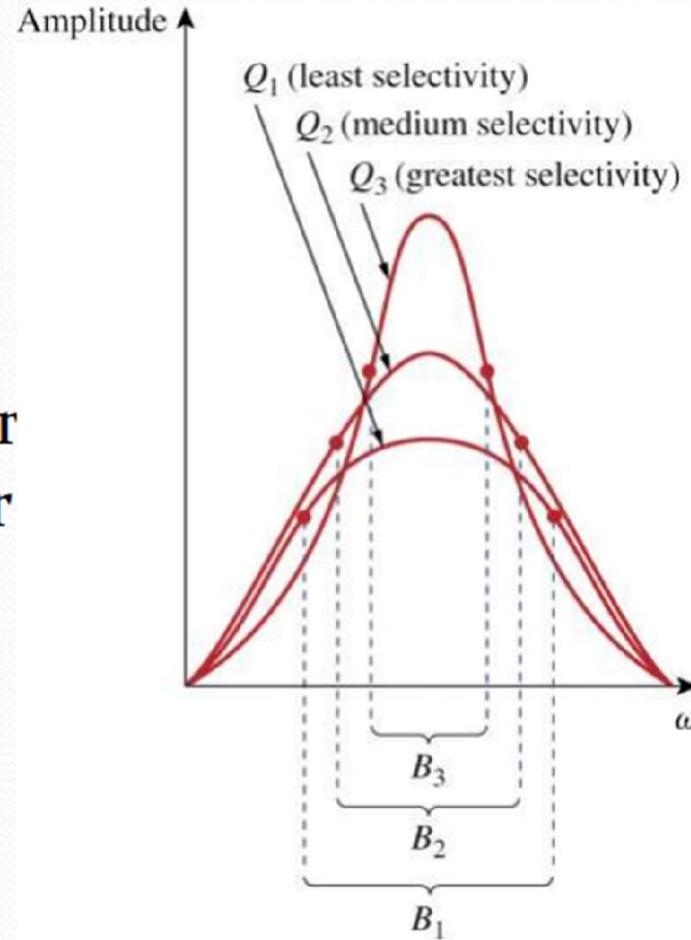
Bandwidth,  $\beta$  is define as the difference between the two half power frequencies.

The width of the response curve is determine by the bandwidth.

$$\beta = (\omega_{c2} - \omega_{c1}) \text{rad/s}$$

$$\beta = \frac{R}{L} \text{rad/s}$$

- Higher value of  $Q$ , smaller the bandwidth. (Higher the selectivity)
- Lower value of  $Q$  larger the bandwidth. (Lower the selectivity)



## Effects of series resonance

1. When a series in R-L-C circuit attains resonance  $X_L = X_C$  i.e., the net reactance of the circuit is zero.
2.  $Z = R$  i.e., the impedance of the circuit is minimum.
3. Since  $Z$  is minimum,  $I = \frac{V}{Z}$  will be minimum.
4. Since  $I$  is maximum, the power dissipated would be maximum  $P = I^2 R$ .
5. Since  $V_L = V_C$ ,  $V = V_R$ . i.e., the supply voltage is in phase with the supply current

# Problem on Resonance Frequency

A coil having resistance  $\Sigma$ , inductance of  $5\Omega$  &  $32mH$ , respectively is connected in series with a  $796-pF$  capacitor.

Determine resonance frequency of the circuit

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$
$$= \frac{1}{2\pi\sqrt{32 \times 10^{-3} \times 796 \times 10^{-12}}}$$
$$= \underline{31.53 \text{ kHz}}$$

A  $10\text{-}\Omega$  resistor,  $10\text{-mH}$  inductor, and  $10\text{-}\mu\text{F}$  capacitor are connected in series with a  $10\text{-kHz}$  voltage source. The rms current through the circuit is  $0.20\text{ A}$ . Find the rms voltage drop across each of the 3 elements

What is not a frequency for ac current?

- a) 50 Hz
- b) 55 Hz
- c) 0Hz
- d) 60 Hz

# Power and power factor in AC circuits

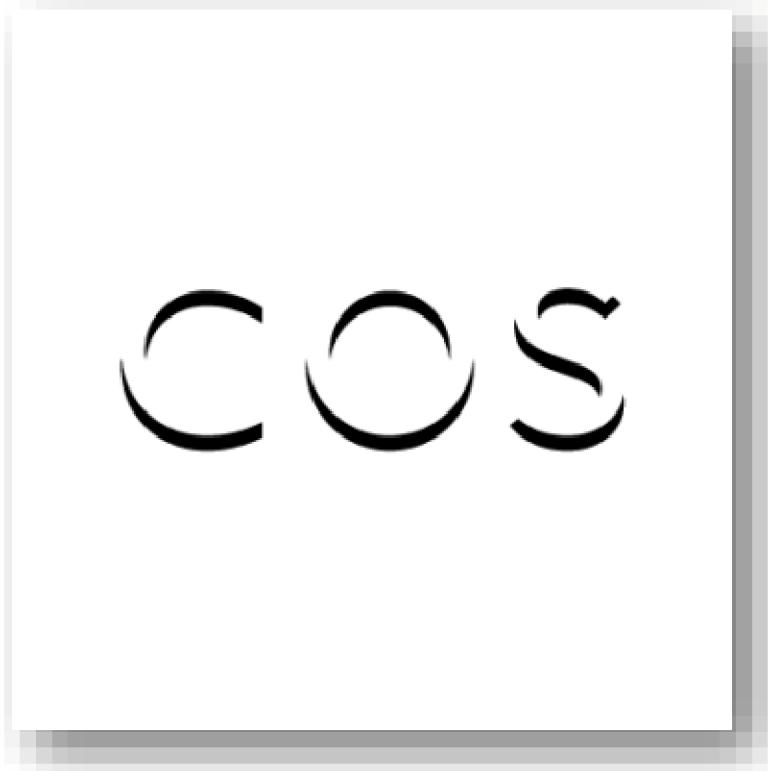
Which of the following is not an expression  
power?

- a)  $P=VI$
- b)  $P=I^2R$
- c)  $P=V^2/R$
- d)  $P=I/R$

- To understand power factor, we'll first start with the definition of some basic terms:
- **KW** is Working Power (also called Actual Power or Active Power or Real Power). It is the power that actually powers the equipment and performs useful work.
- **KVAR** is Reactive Power. It is the power that magnetic equipment (transformer, motor and relay) needs to produce the magnetizing flux.
- **KVA** is Apparent Power. It is the “vectorial summation” of KVAR and KW.

## Definition - 1

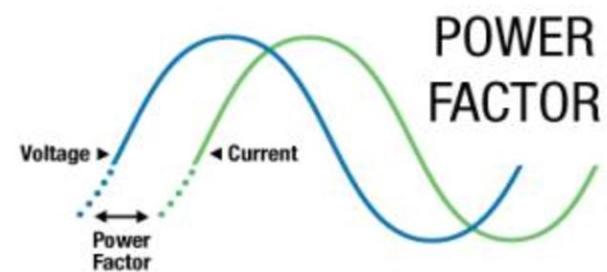
- It is the cosine of the phase angle between the applied voltage and resulting current of the circuit



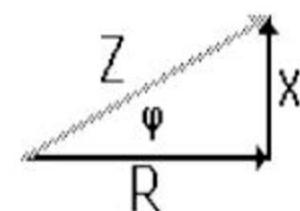
COS

## Definition - 2

- It is defined as the ratio of resistance of the circuit to the impedance of the circuit.
- For a purely resistive AC circuit,  $R=Z$  and the power factor = 1.

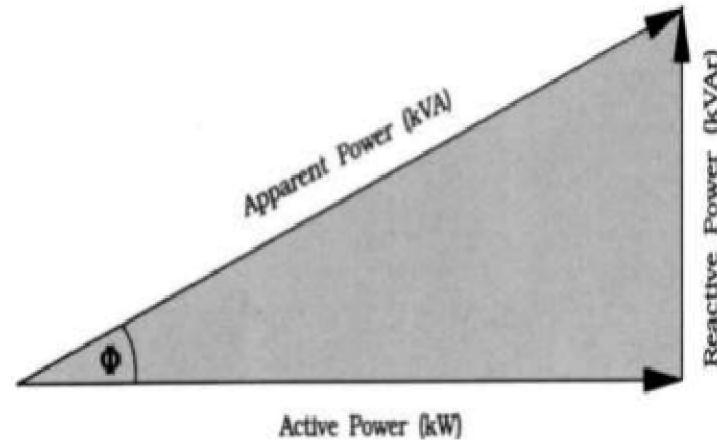


$$\text{POWER FACTOR} = \cos\varphi = \frac{R}{Z}$$



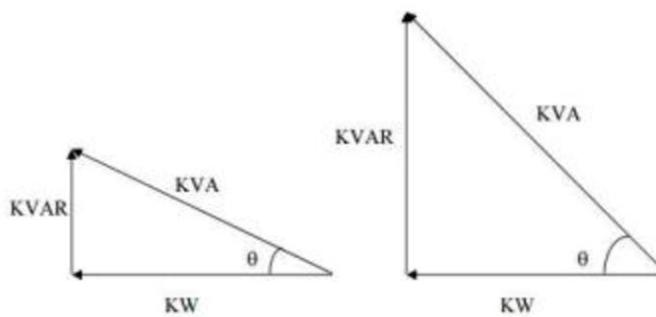
## Definition – 3

- Power Factor (P.F.) is the ratio of Active Power to Apparent Power.
- $P.F. = KW / KVA$
- $P.F. = KW / KW + KVAR$



# What Causes Low Power Factor?

- Since power factor is defined as the ratio of KW to KVA, we see that low power factor results when KW is small in relation to KVA.
- What causes a large KVAR in a system? The answer is...inductive loads.
- These inductive loads constitute a major portion of the power consumed in industrial complexes.
- Reactive power (KVAR) required by inductive loads increases the amount of apparent power (KVA) in your distribution system.
- This increase in reactive and apparent power results in a larger angle  $\theta$  (measured between KW and KVA). As  $\theta$  increases, cosine  $\theta$  (or power factor) decreases.



## Importance of Power Factor

- A power factor of one or "unity power factor" is the goal of any electric utility company since if the power factor is less than one, they have to supply more current to the user for a given amount of power use. In so doing, they incur more line losses.
- They also must have larger capacity equipment in place than would be otherwise necessary.
- As a result, an industrial facility will be charged a penalty if its power factor is much different from 1.

## Importance of Power Factor

- Industrial facilities tend to have a "lagging power factor", where the current lags the voltage (like an inductor). This is primarily the result of having a lot of electric induction motors - the windings of motors act as inductors as seen by the power supply.
- Capacitors have the opposite effect and can compensate for the inductive motor windings. Some industrial sites will have large banks of capacitors strictly for the purpose of correcting the power factor back toward one to save on utility company charges.

$$P = \text{true power} \quad P = I^2 R \quad P = \frac{E^2}{R}$$

*Measured in units of **Watts***

$$Q = \text{reactive power} \quad Q = I^2 X \quad Q = \frac{E^2}{X}$$

*Measured in units of **Volt-Amps-Reactive (VAR)***

$$S = \text{apparent power} \quad S = I^2 Z \quad S = \frac{E^2}{Z} \quad S = IE$$

*Measured in units of **Volt-Amps (VA)***

The power factor is the ratio of \_\_\_\_\_ power  
to the \_\_\_\_\_ power.

- a) average, apparent
- b) apparent, reactive
- c) reactive, average
- d) apparent, average

For the circuit shown in Fig. 10.8a , calculate (a) the impedance, (b) the current, (c) the phase angle, (d) the voltage across each element, (e) the power factor, (f) the apparent power, and (g) the average power. Also, draw the phasor diagram for the circuit.

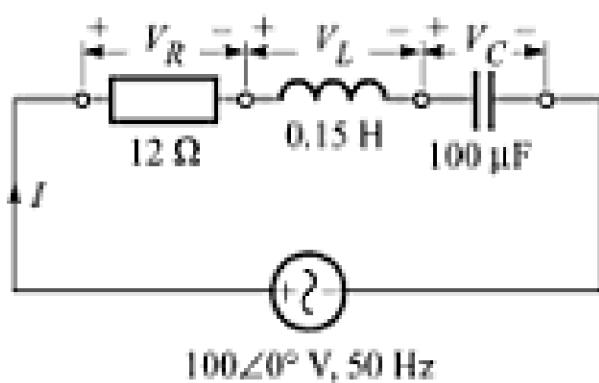
**Solution**  $X_L = \omega L = 2\pi fL = 100\pi \times 0.15 = 47.1 \Omega$ ;  $X_C = 1/\omega C = 1/(100\pi \times 100 \times 10^{-6}) = 31.8 \Omega$

(a) The impedance,  $Z = R + j(X_L - X_C) = 12 + j(47.1 - 31.8) = (12 + j15.3) \Omega$

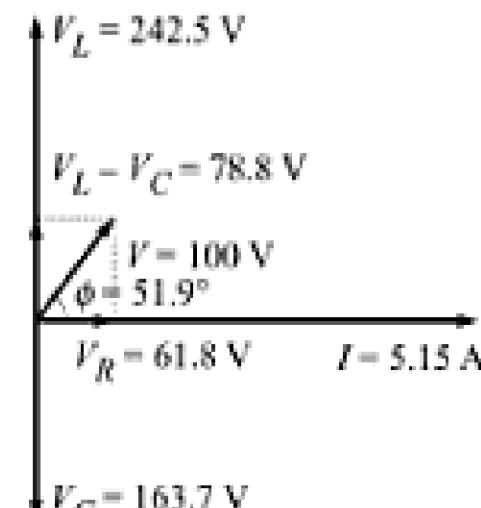
$$= \sqrt{12^2 + 15.3^2} \angle \tan^{-1}(15.3/12) = 19.4 \angle 51.9^\circ \Omega$$

(b) The current,  $I = \frac{V}{Z} = \frac{100 \angle 0^\circ}{19.4 \angle 51.9^\circ} = 5.15 \angle -51.9^\circ A$

(c) The phase angle,  $\phi = -51.9^\circ$



(a) The circuit.



(b) Phasor diagram.

**Fig. 10.8 Series RLC circuit.**

- (d) The voltage,  $V_R = IR = 5.15 \times 12 = 61.8 \text{ V}$ ;  $V_L = IX_L = 5.15 \times 47.1 = 242.5 \text{ V}$ ;  
 $V_C = IX_C = 5.15 \times 31.8 = 163.7 \text{ V}$
- (e) The power factor,  $pf = \cos 51.9^\circ = 0.617$  lagging
- (f) The apparent power  $P_{app} = VI = 100 \times 5.15 = 515 \text{ VA}$
- (g) The average power  $P_{avg} = VI \cos 51.9^\circ = 317.75 \text{ W}$

The phasor diagram is given in Fig. 10.8b.

# RESONANCE IN R-L-C SERIES CIRCUIT

Thus Q-factor

$$\begin{aligned} &= \frac{X_L}{R} OR = \frac{X_C}{R} \\ &= \frac{\omega_r L}{R} OR = \frac{1}{\omega_r C R} \\ &= \frac{2\pi f_r L}{R} OR = \frac{1}{2\pi f_r C R} \\ &= \frac{2\pi L}{2\pi \sqrt{LC} R} OR = \frac{1}{2\pi \times \frac{1}{2\pi \sqrt{LC}} C R} \\ &= \frac{1}{R} \sqrt{\frac{L^2}{LC}} OR = \frac{1}{R} \sqrt{\frac{LC}{C^2}} \\ &= \frac{1}{R} \sqrt{\frac{L}{C}} \end{aligned}$$

$$Q - factor = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{2\pi f_r L}{R} = \frac{1}{2\pi f_r C R}$$

If the resonant frequency in a series RLC circuit is 50kHz along with a bandwidth of 1kHz, find the quality factor.

- a) 5
- b) 50
- c) 100
- d) 500

Reactive power is expressed in?

- a) Watts (W)
- b) Volt Amperes Reactive (VAR)
- c) Volt Ampere (VA)
- d) No units

The power factor is the ratio of \_\_\_\_\_ power  
to the \_\_\_\_\_ power.

- a) average, apparent
- b) apparent, reactive
- c) reactive, average
- d) apparent, average

Resonance frequency occurs when

---

- a)  $X_L = X_C$
- b)  $X_L > X_C$
- c)  $X_L < X_C$
- d) Cannot be determined

# Q-FACTOR IN R-L-C SERIES CIRCUIT

**Q-FACTOR:** In case of R-L-C series circuit Q-Factor is defined as the voltage magnification of the circuit at resonance. Current at resonance is given by

$$I_m = \frac{V}{R} \Rightarrow V = I_m R$$

And voltage across inductance or capacitor is given by =

$$I_m X_L \text{ OR } I_m X_C$$

Voltage magnification = voltage across L or C / applied voltage

$$= \frac{V_L}{V} \quad \text{OR} \quad = \frac{V_C}{V} \quad [\because V_L = V_C]$$

$$= \frac{I_m X_L}{I_m R} \quad \text{OR} \quad \frac{I_m X_C}{I_m R}$$

$$\boxed{= \frac{X_L}{R} \text{ OR } = \frac{X_C}{R}}$$

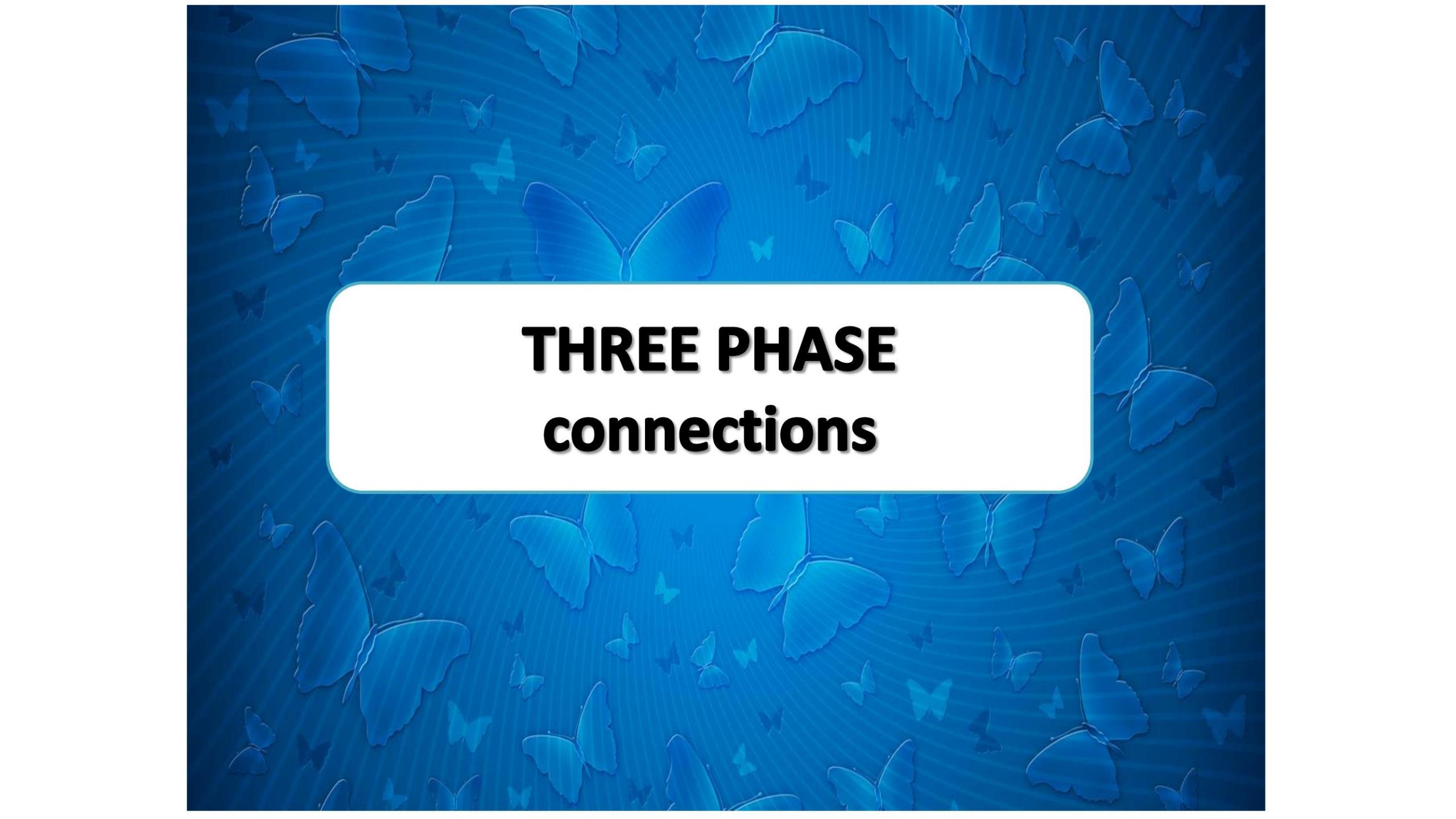
Thus Q-factor

$$\begin{aligned} &= \frac{X_L}{R} OR = \frac{X_C}{R} \\ &= \frac{\omega_r L}{R} OR = \frac{1}{\omega_r C R} \\ &= \frac{2\pi f_r L}{R} OR = \frac{1}{2\pi f_r C R} \\ &= \frac{2\pi L}{2\pi \sqrt{LC} R} OR = \frac{1}{2\pi \times \frac{1}{2\pi \sqrt{LC}} C R} \\ &= \frac{1}{R} \sqrt{\frac{L^2}{LC}} OR = \frac{1}{R} \sqrt{\frac{LC}{C^2}} \\ &= \frac{1}{R} \sqrt{\frac{L}{C}} \end{aligned}$$

$$Q - factor = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{2\pi f_r L}{R} = \frac{1}{2\pi f_r C R}$$

If the resonant frequency in a series RLC circuit is 50kHz along with a bandwidth of 1kHz, find the quality factor.

- a) 5
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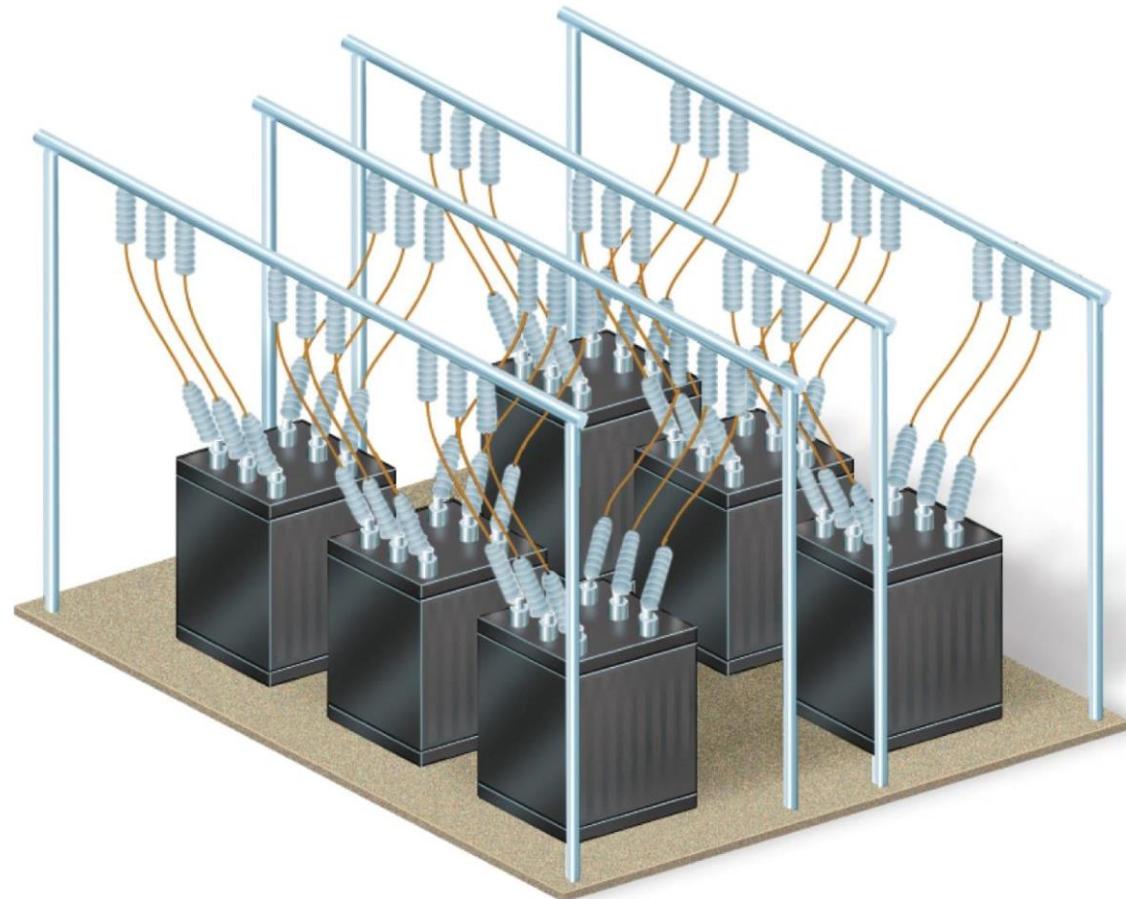


# **THREE PHASE connections**

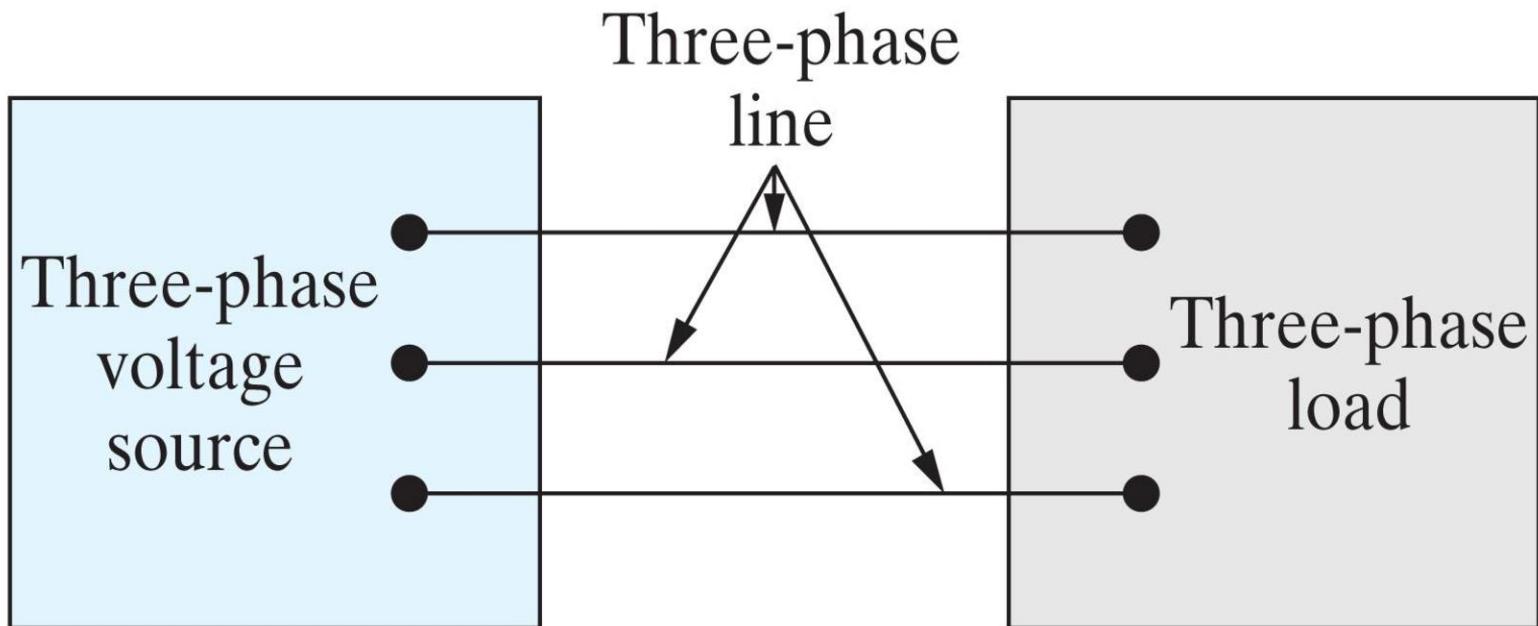
# Introduction to Three-Phase Power



# Typical Transformer Yard



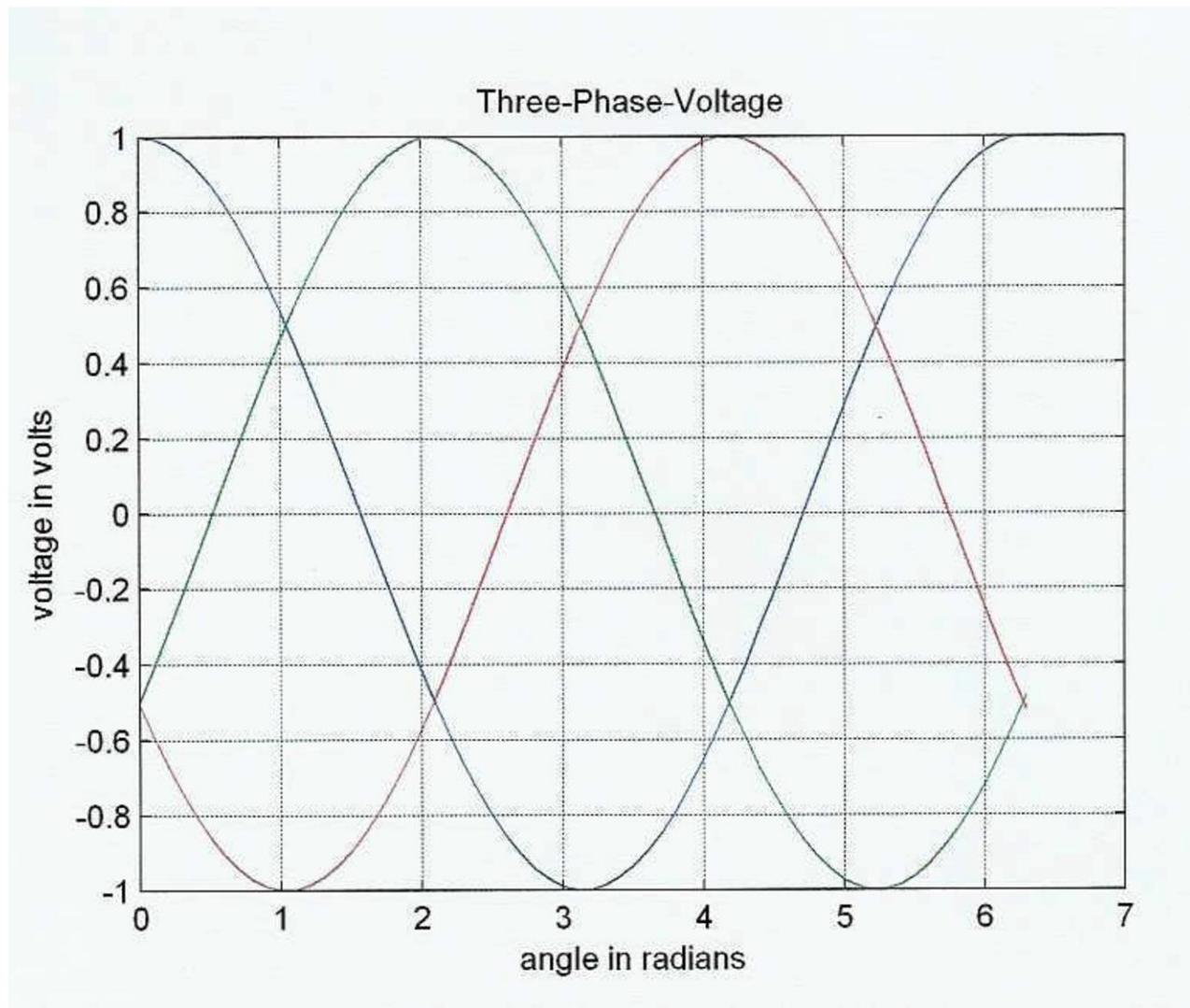
# Basic Three-Phase Circuit



# What is Three-Phase Power?

- Three sinusoidal voltages of equal amplitude and frequency out of phase with each other by  $120^\circ$ . Known as “balanced”.
- Phases are labeled A, B, and C.
- Phases are sequenced as A, B, C (positive) or A, C, B (negative).

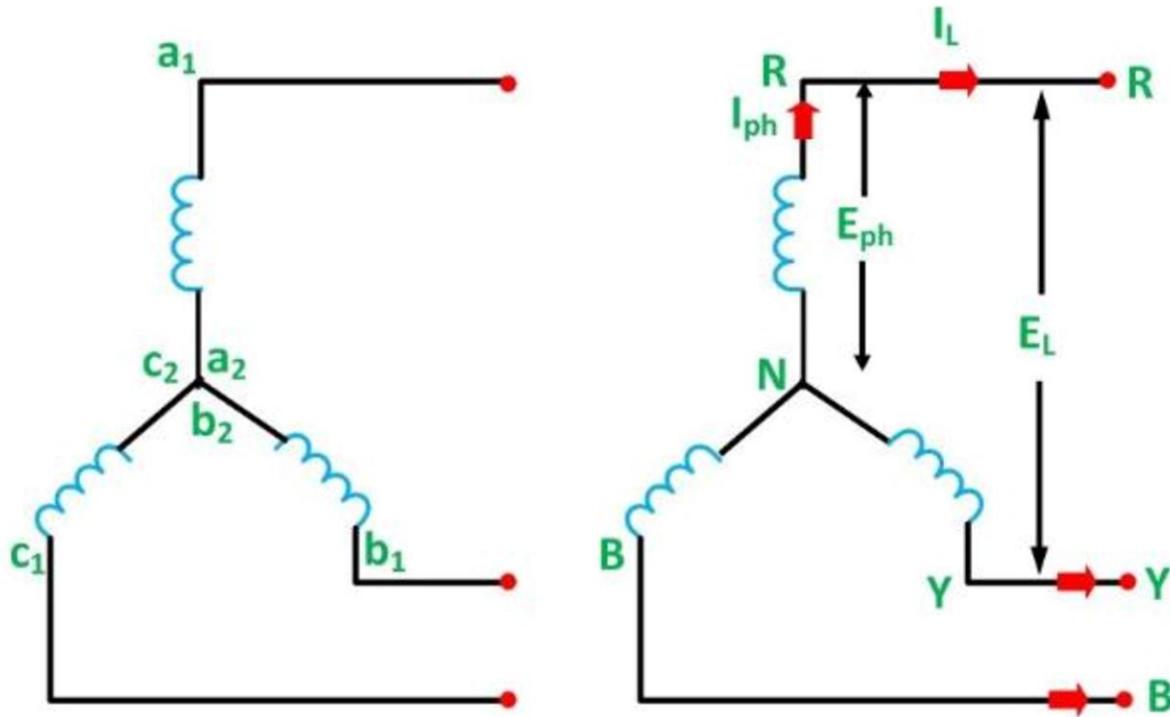
# Three-Phase Power



# Definitions

- 4 wires
  - 3 “active” phases, A, B, C
  - 1 “ground”, or “neutral”
- Color Code
  - Phase A            Red
  - Phase B            Black
  - Phase C            Blue
  - Neutral            White or Gray

The star connection is shown in the diagram below.



The current flowing through each phase is called **Phase current  $I_{ph}$** , and the current flowing through each line conductor is called **Line Current  $I_L$** .

Similarly, the voltage across each phase is called **Phase Voltage  $E_{ph}$** , and the voltage across two line conductors is known as the **Line Voltage  $E_L$** .