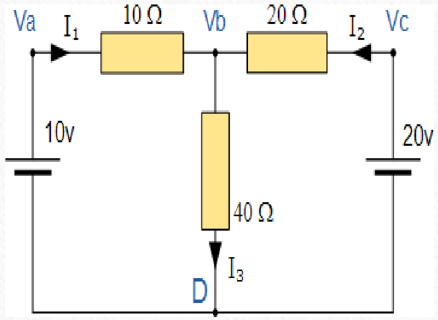
- Nodal analysis is generally used to determine______

  a) Voltage

  b) Current

  c) Resistance

  d) Power



In the above circuit, node D is chosen as the reference node and the other three nodes are assumed to have voltages, Va, Vb and Vc with respect to node D. For example;

$$\frac{(V_a - V_b)}{10} + \frac{(V_c - V_b)}{20} = \frac{V_b}{40}$$

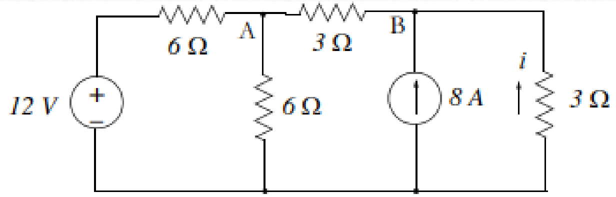
As Va = 10v and Vc = 20v , Vb can be easily found by:

$$\left(1 - \frac{Vb}{10}\right) + \left(1 - \frac{Vb}{20}\right) = \frac{Vb}{40}$$

$$2 = Vb\left(\frac{1}{40} + \frac{1}{20} + \frac{1}{10}\right)$$

$$Vb = \frac{80}{7}V$$

$$\therefore I_3 = \frac{2}{7} \text{ or } 0.286 Amps$$



At Node A

$$\frac{V_A - 12}{6} + \frac{V_A}{6} + \frac{V_A - V_B}{3} = 0$$

and at Node B

$$\frac{V_B - V_A}{3} + \frac{V_B}{3} = 8$$

These simplify to

$$\frac{2}{3}V_A - \frac{1}{3}V_B = 2$$

and

$$-\frac{1}{3}V_A + \frac{2}{3}V_B = 8$$

Multiplication of the last equation by 2 and addition with the first yields $V_B = 18$ and thus $i = -18/3 = -6\ A$.

Mesh analysis employs the method of

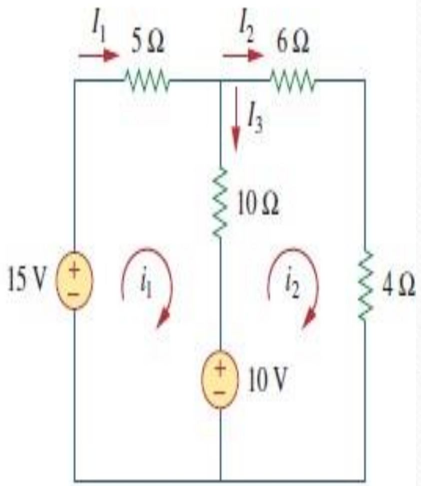
____

a) KVL
b) KCL
c) Both KVL and KCL
d) Neither KVL nor KCL

Mesh analysis is generally used to determine __________

a) Voltage
b) Current
c) Resistance
d) Power

For the circuit in Fig. 3.18, find the branch currents I_1, I_2, and I_3 using mesh analysis.



Solution:

We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \tag{3.5.1}$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \tag{3.5.2}$$

EXAMPLE-2

Use mesh analysis to find the current I_o in the circuit of Fig. 3.20.

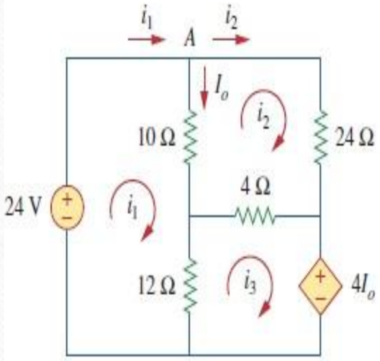


Figure 3.20

Solution:

We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12 \qquad \textbf{(3.6.1)}$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

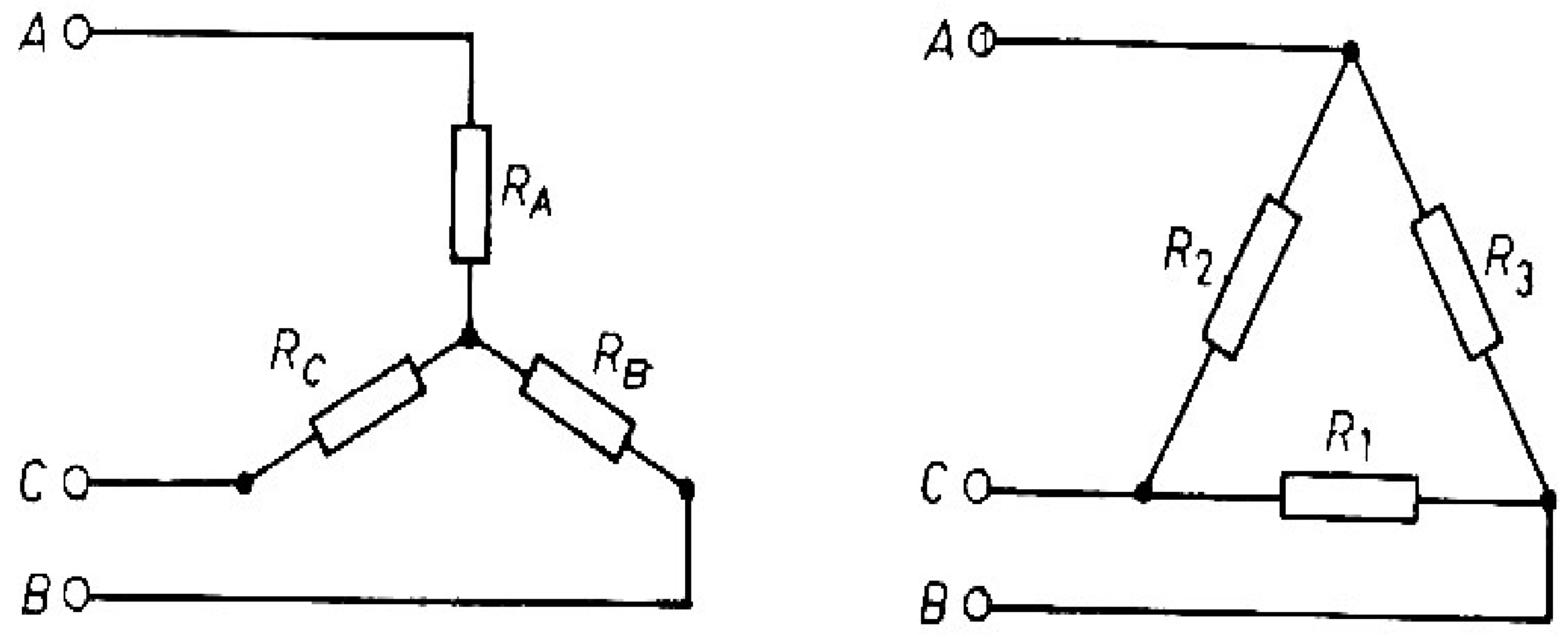
or

$$-5i_1 + 19i_2 - 2i_3 = 0 \qquad \textbf{(3.6.2)}$$

For mesh 3,

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

Star-Delta Transformation

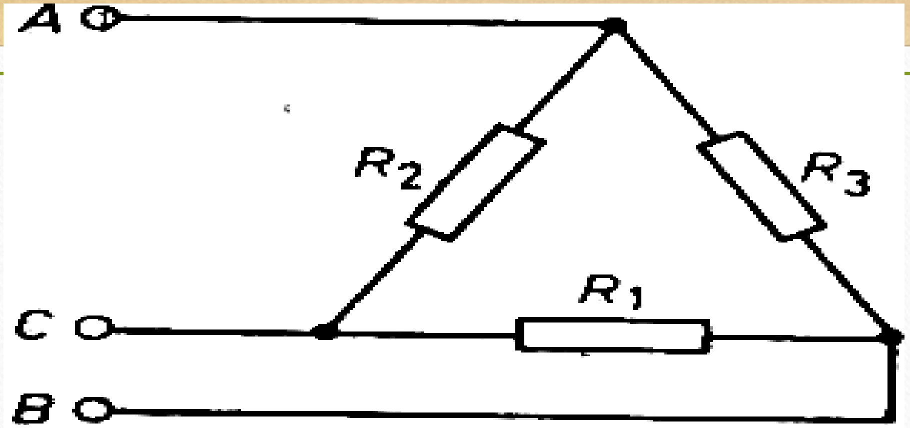


(a) Star (Y) section

(b) Delta or mesh (Δ) section

Equivalence

- **Equivalence can be found on the basis that the resistance between any pair of terminals in the two circuits have to be the same, when the third terminal is left open.**



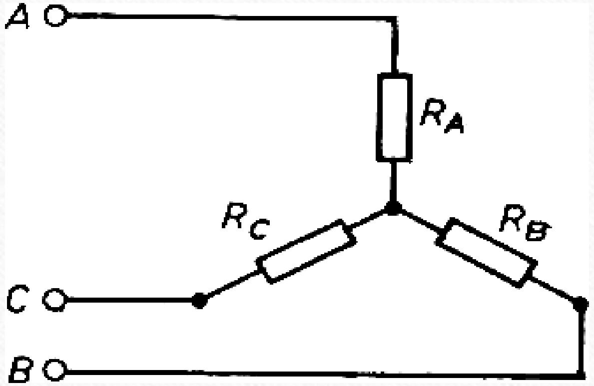
(b) Delta or mesh (Δ) section

- First take delta connection: between A and C, there are two parallel paths, one having a resistance of R_2 and other having a resistance of (R_1+R_3)

Hence resistance between terminal A and C is

$$= R_2.(R_1+R_3)/[R_2+(R_1+R_3)]$$

- Now take the star connection



The resistance between the same terminal A and C is (R_A+R_C)

Since terminal resistance have to be same so we must have

$$(R_A+R_C) = R_2.(R_1+R_3)/[R_2+(R_1+R_3)] \qquad (1)$$

Similarly for terminals A and B, B and C, we can have the following expression

$$(R_A+R_B) = R_3.(R_1+R_2)/[R_3+(R_1+R_2)] \qquad (2)$$

$$(R_B+R_C) = R_1.(R_2+R_3)/[R_1+(R_2+R_3)] \qquad (3)$$

DELTA to STAR

Now subtracting 2 from 1 and adding the result to 3, we will get the following values for R_1, R_2 and R_3.

$$R_A = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

How to remember?

Resistance of each arm of star is given by the product of the resistance of the two delta sides that meet at its ends divided by the sum of the three delta resistance

STAR to DELTA

Multiplying 1 and 2, 2 and 3 , 3 and 1 and adding them together and simplifying, we will have the following result.

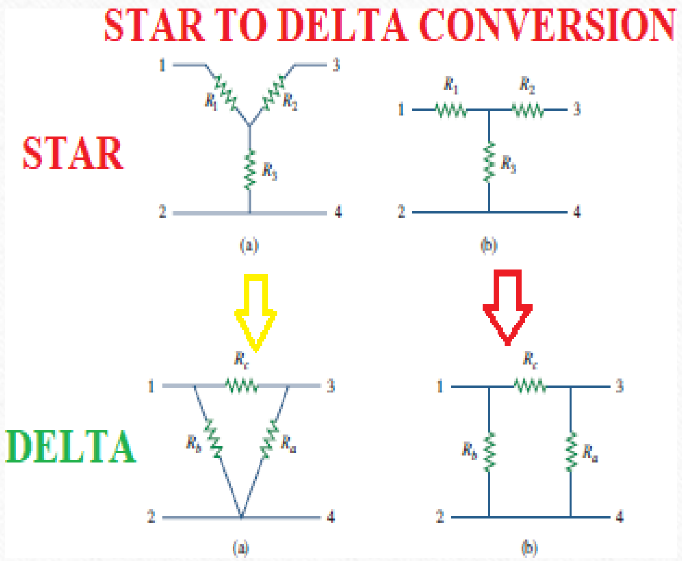
$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

How to remember: The equivalent delta resistance between any two point is given by the product of resistance taken two at a time divided by the opposite resistance in the star configuration.

STAR-DELTA TRANSFORMATION

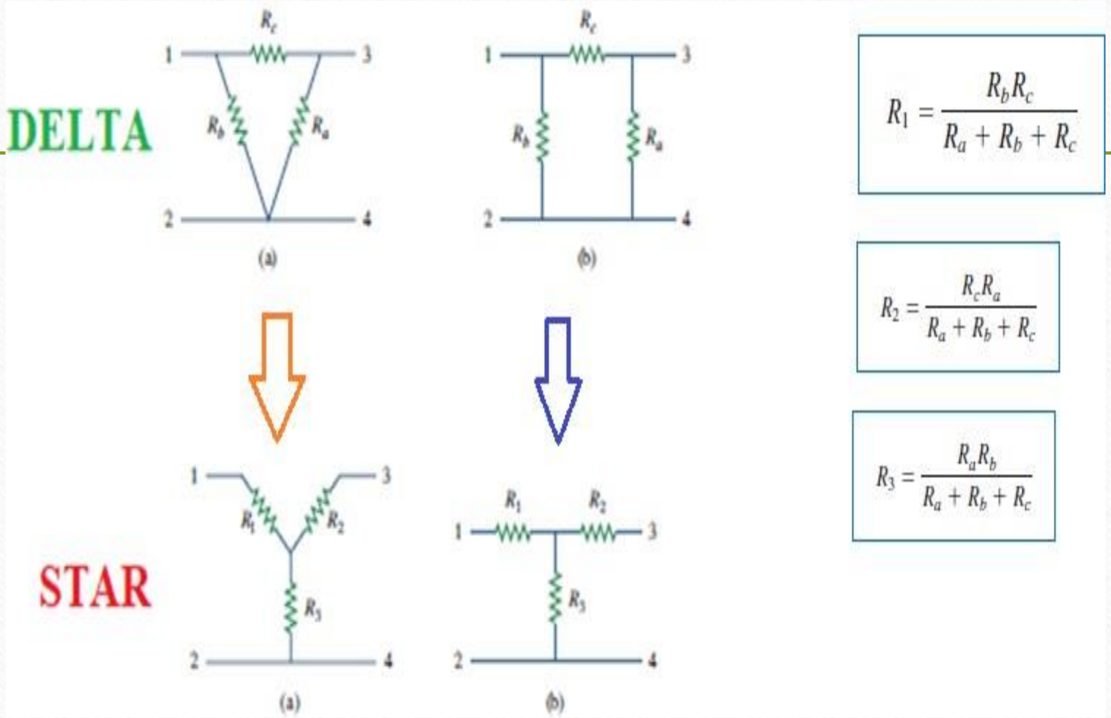


$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

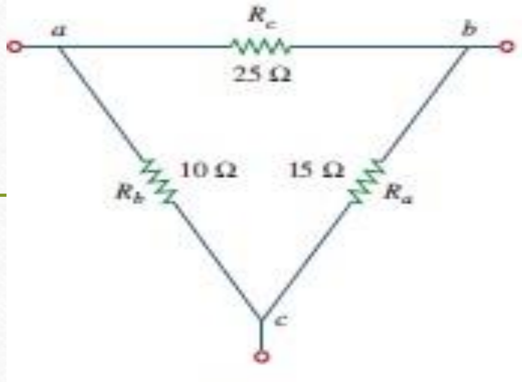
$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

DELTA-STAR TRANSFORMATION



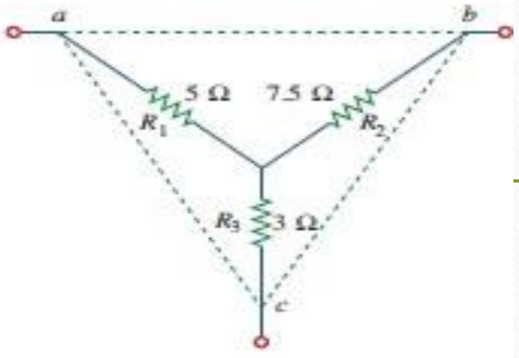
Example: Delta to Star

Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.



(a)

a) original Δ network,



(b)

(b) Y equivalent network.

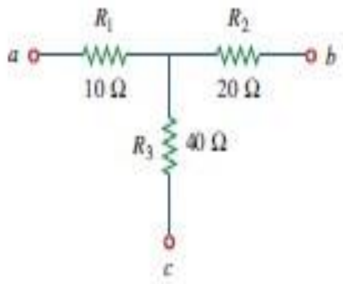
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

Example: Star to Delta

Transform the wye network in Fig. 2.51 to a delta network.



Answer: $R_a = 140\ \Omega$, $R_b = 70\ \Omega$, $R_c = 35\ \Omega$.

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Delta connection is also known as______________

a) Y-connection

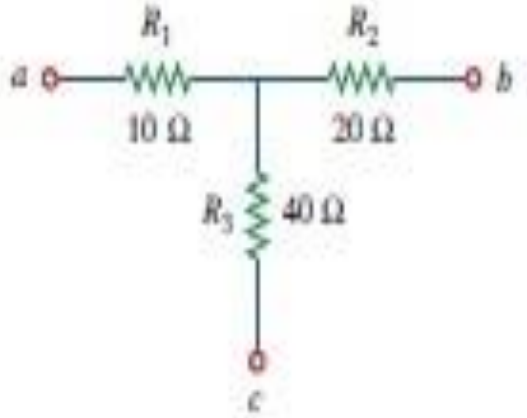
b) Mesh connection

c) Either Y-connection or mesh connection

d) Neither Y-connection nor mesh connection

PRACTICE PROBLEM

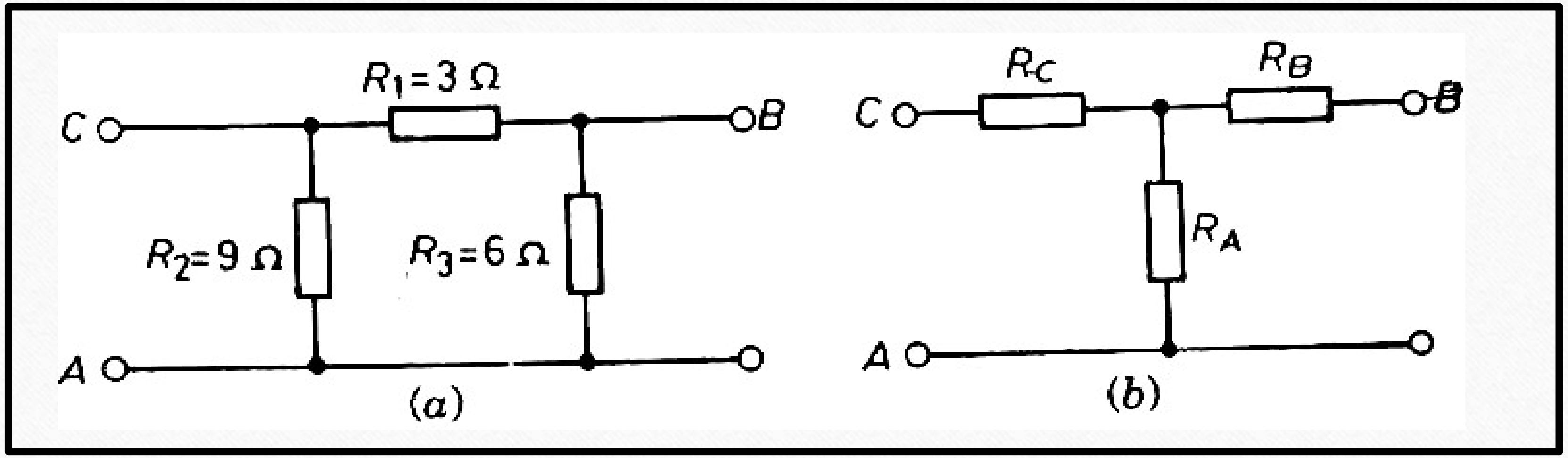
Transform the wye network in Fig. to a delta network.



Answer: $R_a = 140\ \Omega$, $R_b = 70\ \Omega$, $R_c = 35\ \Omega$.

Problem

- **A delta-section of resistors is given in figure. Convert this into an equivalent star-section.**



Ans. : $R_A = 3\,\Omega; \quad R_B = 1.0\,\Omega; \quad R_C = 1.5\,\Omega.$

SUPERPOSITION THEOREM

- If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis.
- Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the *superposition.*
- The idea of superposition rests on the linearity property.

STATEMENT

- "The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents

through) that element due to each independent source acting alone".

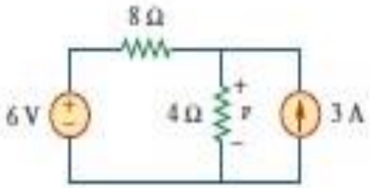
NOTE: Superposition is not limited to circuit analysis but is applicable in many fields where cause and effect bear a linear relationship to one another.

Procedure to Apply Superposition Principle/Theorem

- 1. Turn off all independent sources except one source.
- 2. Find the output (voltage or current) due to that active source using any techniques such as Ohm's Law, KCL, KVL, Nodal/Mesh Analysis etc.
- 3. Repeat step 1 for each of the other independent sources.
- 4. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Numerical

Use the superposition theorem to find v in the circuit of Fig. 4.6.

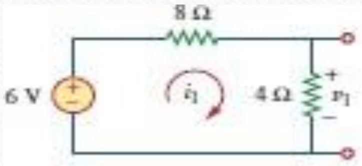


Solution:
Since there are two sources, let

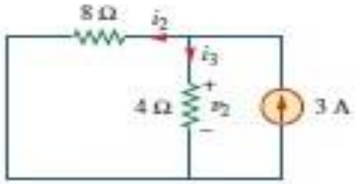
$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the 6-V voltage source and the 3-A current source, respectively. To obtain v_1, we set the current source to zero, as shown in Fig. 4.7(a). Applying KVL to the loop in Fig. 4.7(a) gives

$$12i_1 - 6 = 0 \quad \Rightarrow \quad i_1 = 0.5 \text{ A}$$



(a)



(b)

Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

To get v_2, we set the voltage source to zero, as in Fig. 4.7(b). Using current division,

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

Hence,

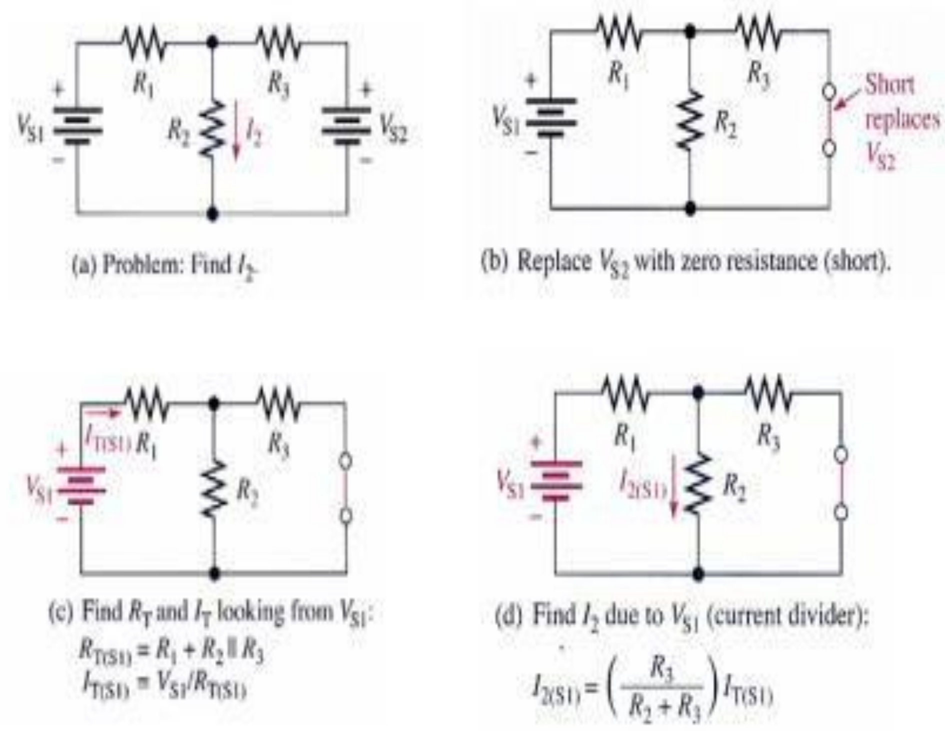
$$v_2 = 4i_3 = 8 \text{ V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

Superposition Theorem

The approach to superposition is demonstrated in the figure for a series-parallel circuit with two ideal voltage sources.



(a) Problem: Find I_2.

(b) Replace V_{S2} with zero resistance (short).

(c) Find R_T and I_T looking from V_{S1}:
$R_{T(S1)} = R_1 + R_2 \parallel R_3$
$I_{T(S1)} = V_{S1}/R_{T(S1)}$

(d) Find I_2 due to V_{S1} (current divider):

$$I_{2(S1)} = \left(\frac{R_3}{R_2 + R_3}\right) I_{T(S1)}$$

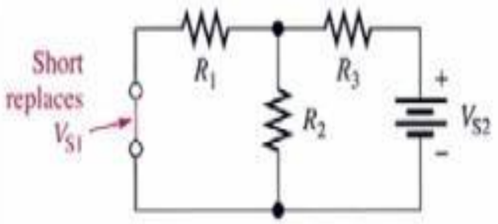
Superposition theorem does not work for __________

a) Current
b) Voltage
c) Power
d) Works for all: current, voltage and power

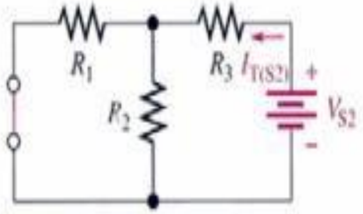
Explanation: Power across an element is not equal to the power across it due to all the other sources in the system. The power in an element is the product of the total voltage and the total current in that element.

Superposition Theorem

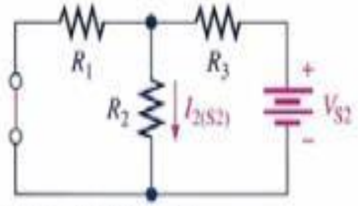
The approach to superposition is demonstrated in the figure for a series-parallel circuit with two ideal voltage sources.



(e) Replace V_{S1} with zero resistance (short).

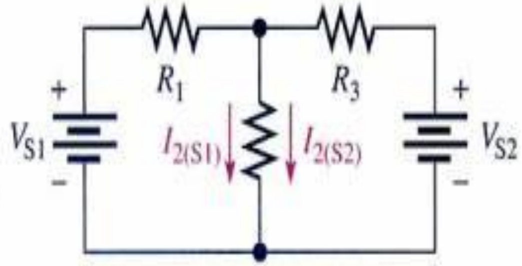


(f) Find R_T and I_T looking from V_{S2}:
$R_{T(S2)} = R_3 + R_1 \parallel R_2$
$I_{T(S2)} = V_{S2}/R_{T(S2)}$



(g) Find I_2 due to V_{S2}:

$$I_{2(S2)} = \left(\frac{R_1}{R_1 + R_2}\right) I_{T(S2)}$$

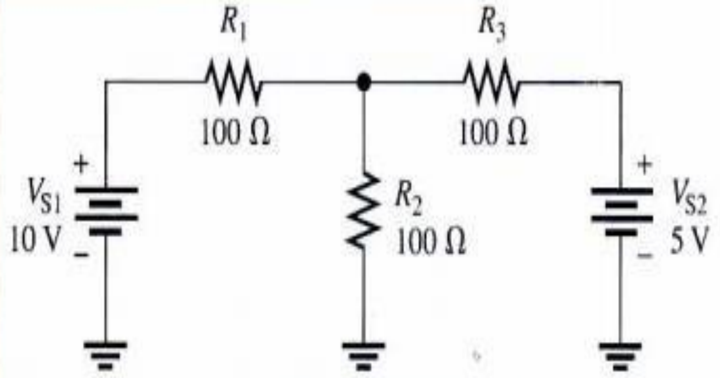


(h) Restore the original sources. Add $I_{2(S1)}$ and $I_{2(S2)}$ to get the actual I_2 (they are in same direction):

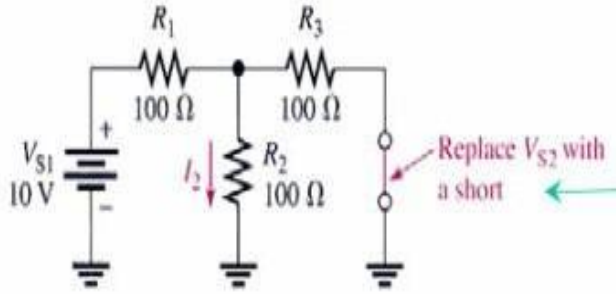
$$\boxed{I_2 = I_{2(S1)} + I_{2(S2)}}$$

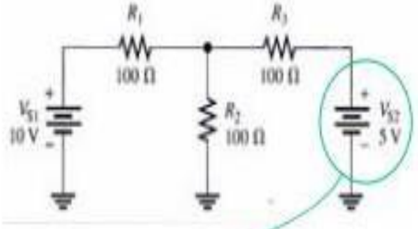
Superposition Theorem

EXAMPLE Use the superposition theorem to find the current through R_2.



Superposition Theorem





Solution **Step 1:** Replace V_{S2} with a short and find the current through R_2 due to voltage source V_{S1}, as shown in Figure 8–18. To find I_2, use the current-divider formula (Equation 6–6). Looking from V_{S1},

$$R_{T(S1)} = R_1 + \frac{R_3}{2} = 100\,\Omega + 50\,\Omega = 150\,\Omega$$

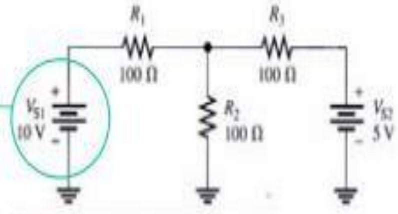
$$I_{T(S1)} = \frac{V_{S1}}{R_{T(S1)}} = \frac{10\text{ V}}{150\,\Omega} = 66.7\text{ mA}$$

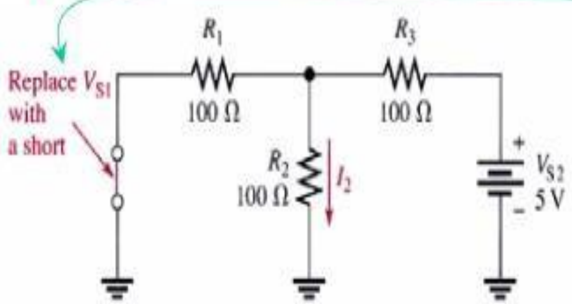
The current through R_2 due to V_{S1} is

$$I_{2(S1)} = \left(\frac{R_3}{R_2 + R_3}\right) I_{T(S1)} = \left(\frac{100\,\Omega}{200\,\Omega}\right) 66.7\text{ mA} = 33.3\text{ mA}$$

Note that this current is downward through R_2.

Superposition Theorem





Step 2: Find the current through R_2 due to voltage source V_{S2} by replacing V_{S1} with a short, as shown in Figure 8–19. Looking from V_{S2},

$$R_{T(S2)} = R_3 + \frac{R_1}{2} = 100\ \Omega + 50\ \Omega = 150\ \Omega$$

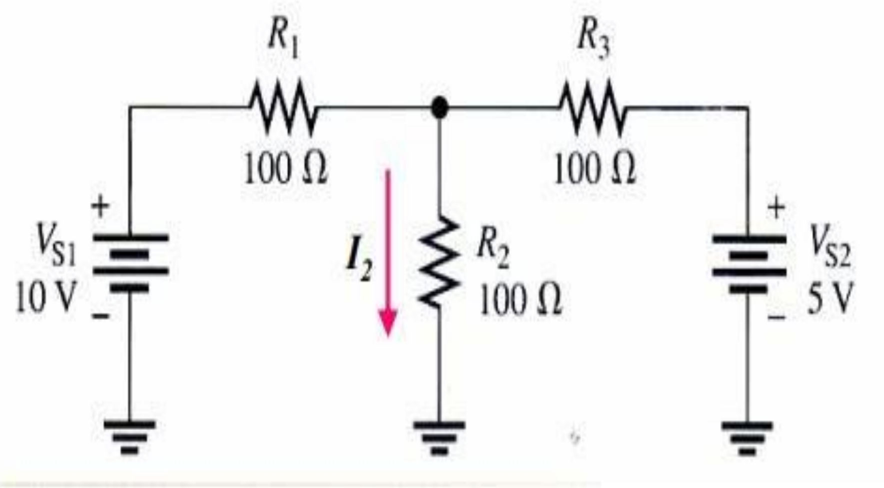
$$I_{T(S2)} = \frac{V_{S2}}{R_{T(S2)}} = \frac{5\ \text{V}}{150\ \Omega} = 33.3\ \text{mA}$$

The current through R_2 due to V_{S2} is

$$I_{2(S2)} = \left(\frac{R_1}{R_1 + R_2}\right) I_{T(S2)} = \left(\frac{100\ \Omega}{200\ \Omega}\right) 33.3\ \text{mA} = 16.7\ \text{mA}$$

Note that this current is downward through R_2.

Superposition Theorem



Step 3: Both component currents are downward through R_2, so they have the same algebraic sign. Therefore, add the values to get the total current through R_2.

$$I_{2(tot)} = I_{2(S1)} + I_{2(S2)} = 33.3\,\text{mA} + 16.7\text{mA} = \mathbf{50\,mA}$$

- Superposition theorem is valid for ___________

  a) Linear systems

  b) Non-linear systems

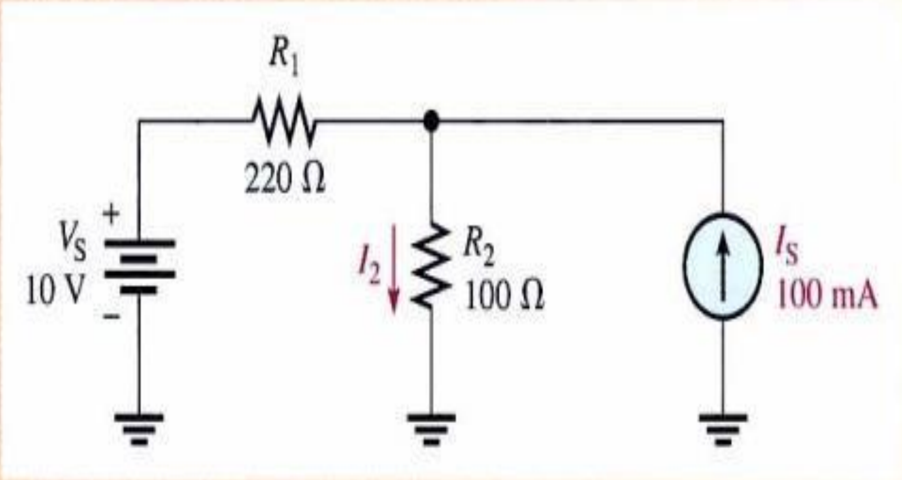
  c) Both linear and non-linear systems

  d) Neither linear nor non-linear systems

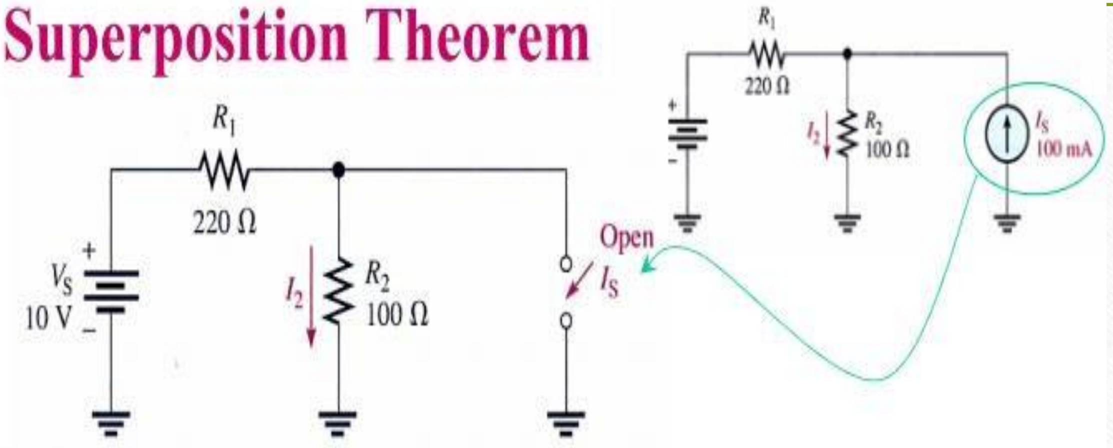
Explanation: Superposition theorem is valid only for linear systems because the effect of a single source cannot be individually calculated in a non-linear system.

Superposition Theorem

EXAMPLE Find the current through R_2 in the circuit.



Superposition Theorem



Solution

Step 1: Find the current through R_2 due to V_S by replacing I_S with an open.

Notice that all of the current produced by V_S is through R_2. Looking from V_S,

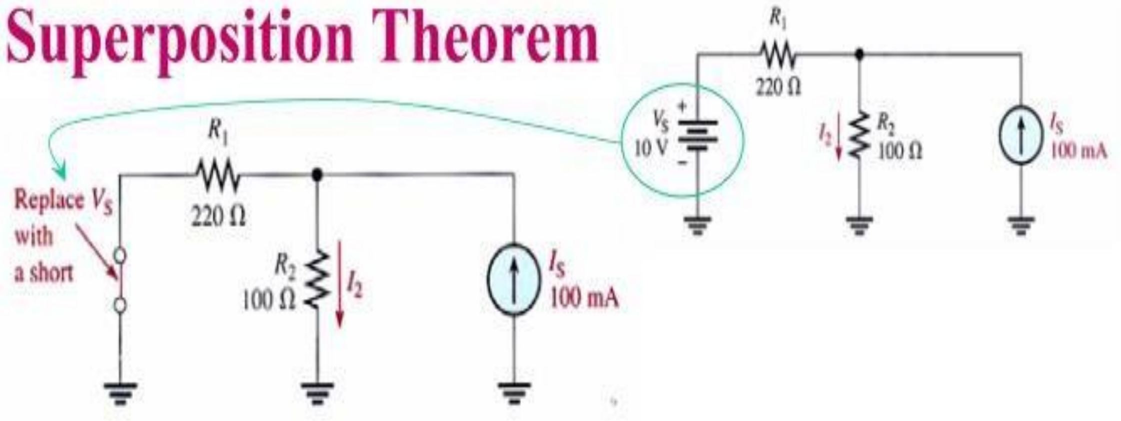
$$R_T = R_1 + R_2 = 320\ \Omega$$

The current through R_2 due to V_S is

$$I_{2(V_S)} = \frac{V_S}{R_T} = \frac{10\text{ V}}{320\ \Omega} = 31.2\text{ mA}$$

Note that this current is downward through R_2.

Superposition Theorem

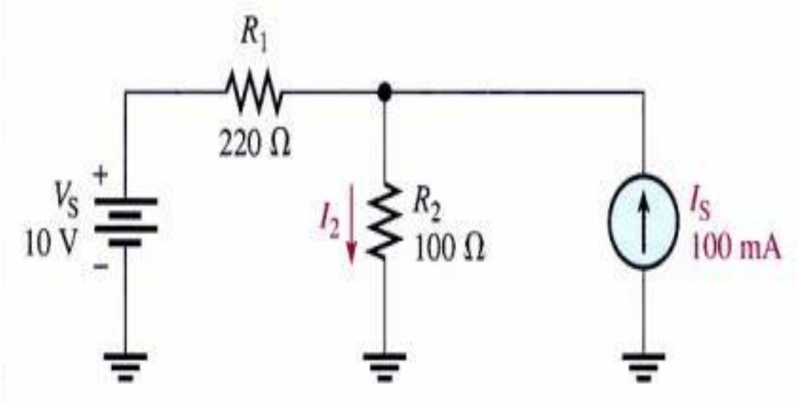


Step 2: Find the current through R_2 due to I_S by replacing V_S with a short.
Use the current-divider formula to determine the current through R_2 due to I_S.

$$I_{2(I_S)} = \left(\frac{R_1}{R_1 + R_2}\right) I_S = \left(\frac{220\ \Omega}{320\ \Omega}\right) 100\ \text{mA} = 68.8\ \text{mA}$$

Note that this current also is downward through R_2.

Superposition Theorem

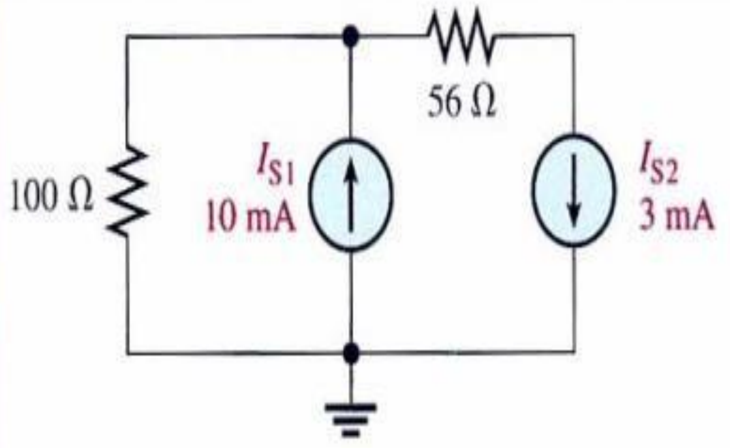


Step 3: Both currents are in the same direction through R_2, so add them to get the total.

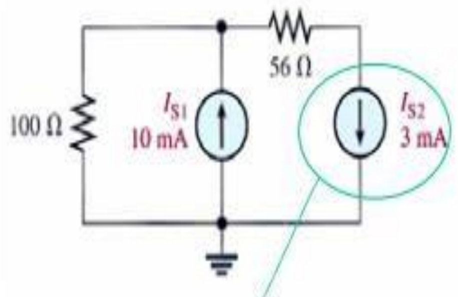
$$I_{2(\text{tot})} = I_{2(V_S)} + I_{2(I_S)} = 31.2\,\text{mA} + 68.8\,\text{mA} = \mathbf{100\,mA}$$

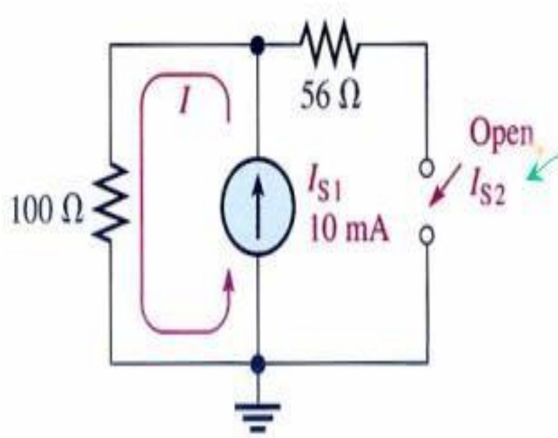
Superposition Theorem

EXAMPLE Find the current through the 100 Ω resistor.



Superposition Theorem



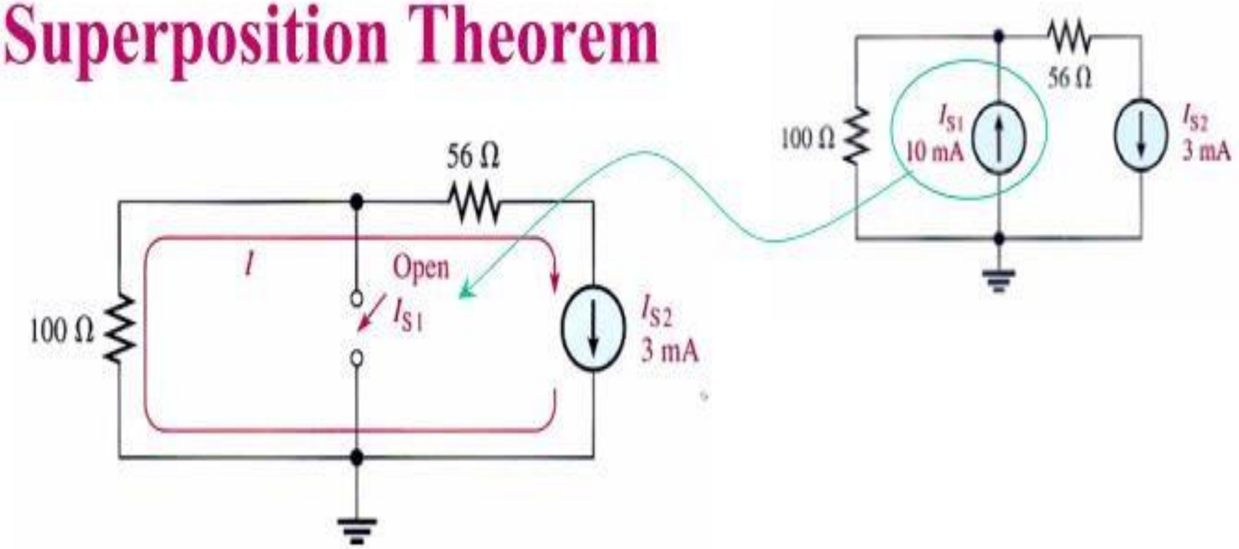


Solution

Step 1: Find the current through the 100 Ω resistor due to current source I_{S1} by replacing source I_{S2} with an open,

$$I_{S1} = 10\text{ mA}$$

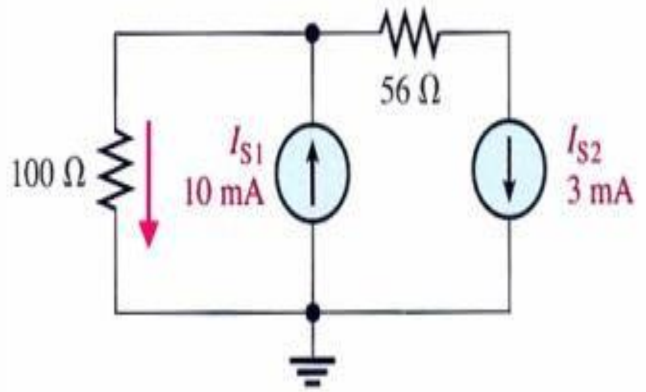
Superposition Theorem



Step 2: Find the current through the 100 Ω resistor due to source I_{S2} by replacing source I_{S1} with an open.

I_{S2} = 3 mA (upward through the 100 Ω resistor.)

Superposition Theorem



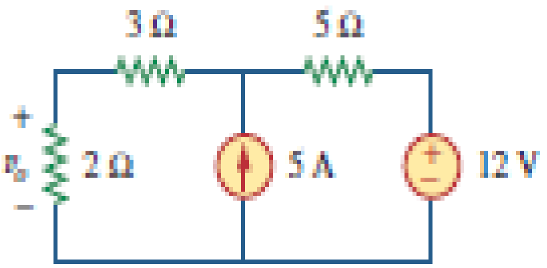
Step 3: To get the total current through the 100 Ω resistor, subtract the smaller current from the larger because they are in opposite directions. The resulting total current is in the direction of the larger current from source I_{S1}.

$$I_{100\Omega(\mathrm{tot})} = I_{100\Omega(I_{S1})} - I_{100\Omega(I_{S2})}$$
$$= 10\,\mathrm{mA} - 3\,\mathrm{mA} = \mathbf{7\,mA}$$

The resulting current is downward through the resistor.

PRACTICE PROBLEM

Using the superposition theorem, find v_o in the circuit of Fig. 4.8.



Answer: 7.4 V.

Thevenin’s Theorem

- Thevenin’s theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

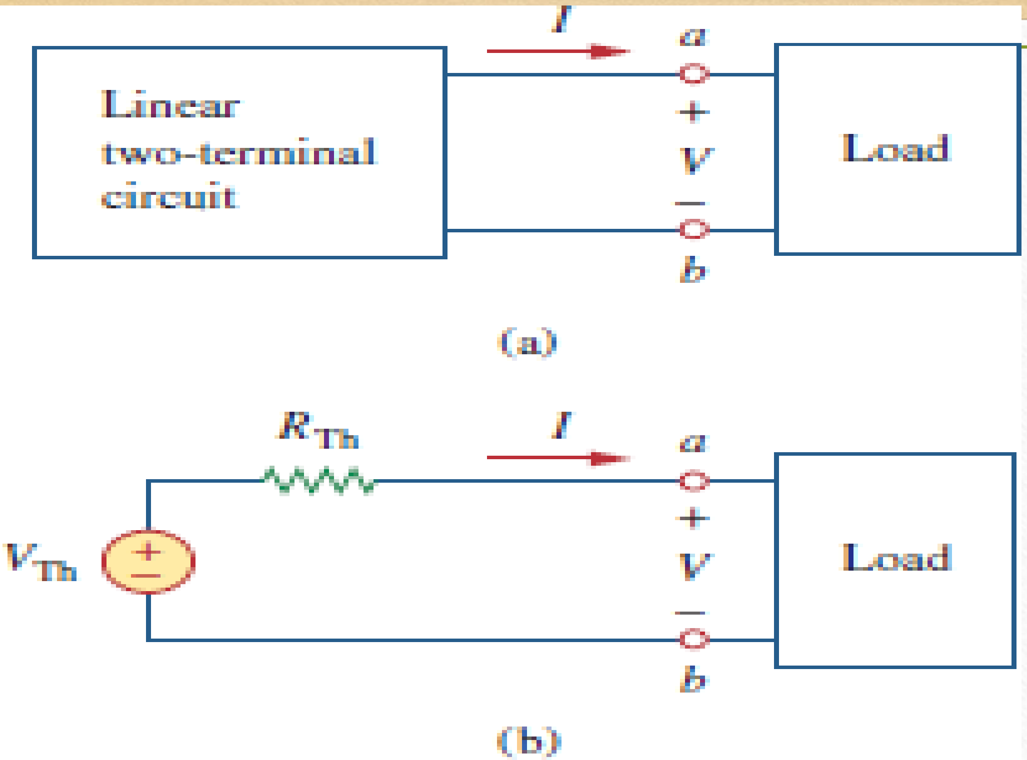
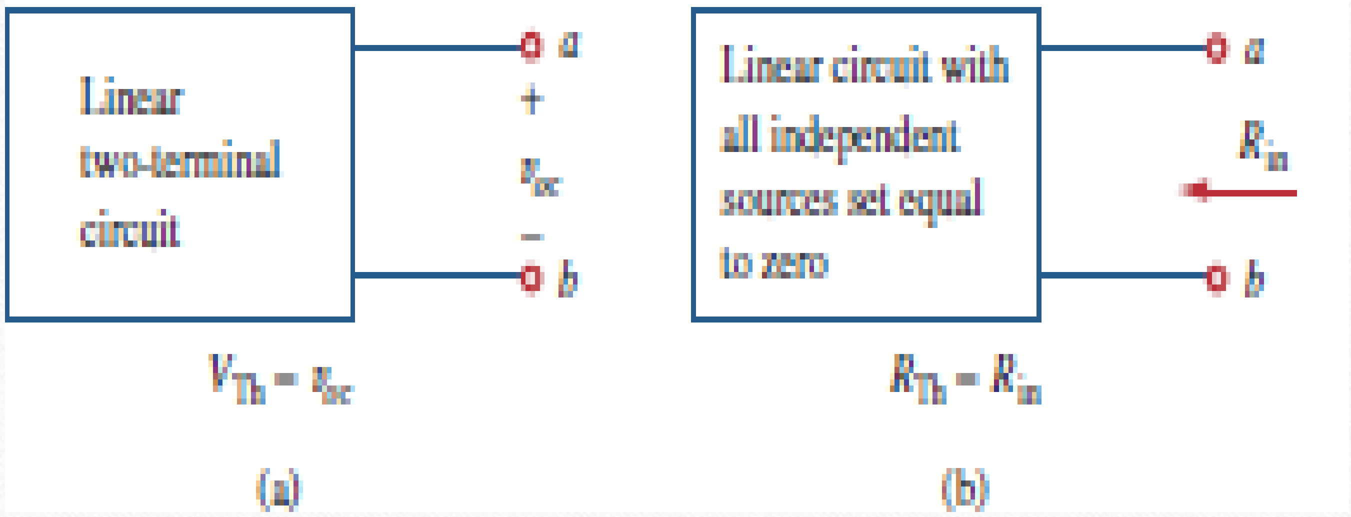


Figure 4.23
Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

$$V_{Th} = v_{oc} \tag{4.6}$$



Summary of Thevenin's Theorem

Step 1. Open the two terminals (remove any load) between which you want to find the Thevenin equivalent circuit.

Step 2. Determine the voltage (V_{TH}) across the two open terminals.

Step 3. Determine the resistance (R_{TH}) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened).

Step 4. Connect V_{TH} and R_{TH} in series to produce the complete Thevenin equivalent for the original circuit.

Step 5. Replace the load removed in Step 1 across the terminals of the Thevenin equivalent circuit. You can now calculate the load current and load voltage using only Ohm's law. They have the same value as the load current and load voltage in the original circuit.

Determining V_{TH} and R_{TH} by Measurement

You can find Thevenin's equivalent for an actual circuit by the following general measurement methods.

Step 1. Remove any load from the output terminals of the circuit.

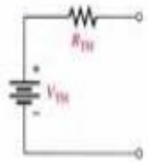
Step 2. Measure the open terminal voltage. The voltmeter used must have an internal resistance much greater (at least 10 times greater) than the R_{TH} of the circuit so that it has negligible loading effect. (V_{TH} is the open terminal voltage.)

Step 3. Connect a variable resistor (rheostat) across the output terminals. Set it at its maximum value, which must be greater than R_{TH}.

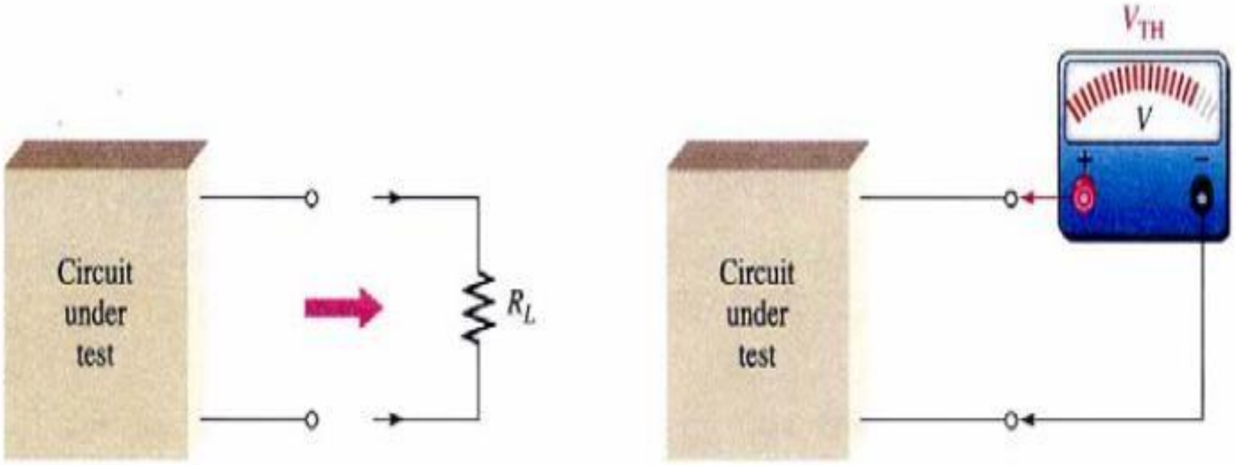
Step 4. Adjust the rheostat until the terminal voltage equals $0.5V_{TH}$. At this point, the resistance of the rheostat is equal to R_{TH}.

Step 5. Disconnect the rheostat from the terminals and measure its resistance with an ohmmeter. This measured resistance is equal to R_{TH}.

Determining V_{TH} and R_{TH} by Measurement



Measuring V_{TH}

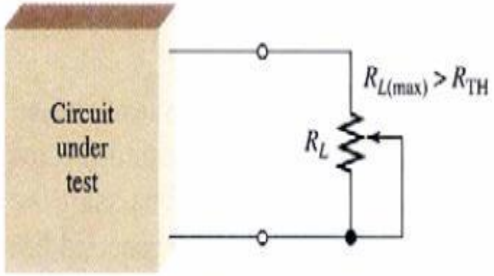


Step 1: Open the output terminals (remove load).

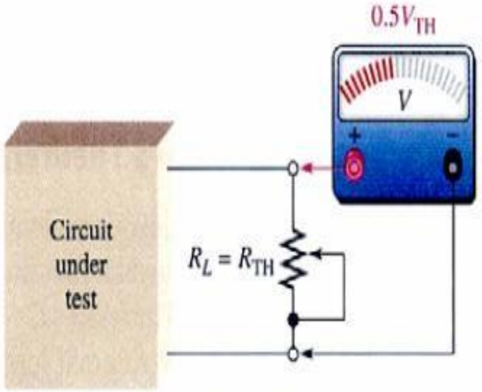
Step 2: Measure V_{TH}.

Determining V_{TH} and R_{TH} by Measurement

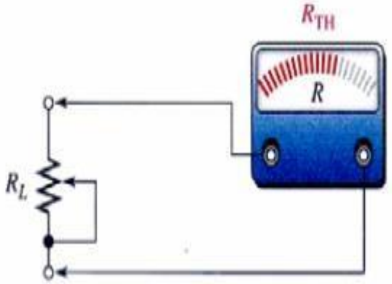
Measuring R_{TH}



Step 3: Connect variable load resistance set to its maximum value across the terminals.



Step 4: Adjust R_L until $V_L = 0.5V_{TH}$. When $V_L = 0.5V_{TH}$, $R_L = R_{TH}$.



Step 5: Remove R_L from the circuit under test and measure its resistance to get R_{TH}.

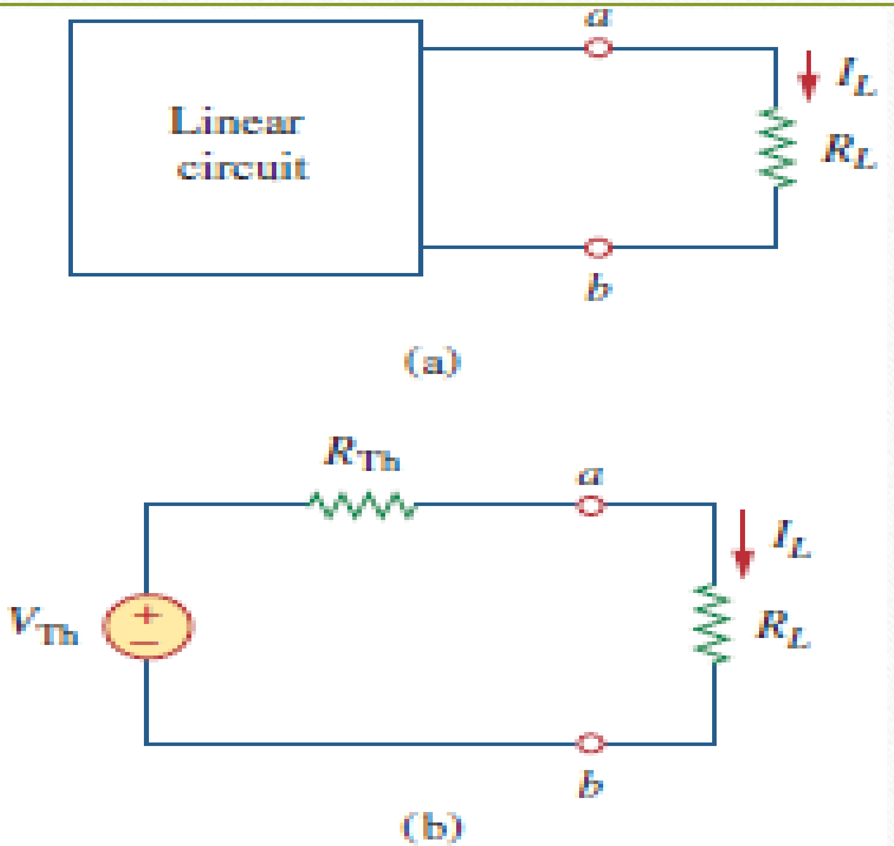
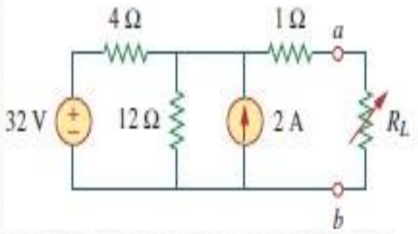


Figure 4.26
A circuit with a load: (a) original circuit, (b) Thevenin equivalent.



Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals *a*-*b*. Then find the current through $R_L = 6, 16,$ and $36\ \Omega$.

Solution:

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in Fig. 4.28(a). Thus,

$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

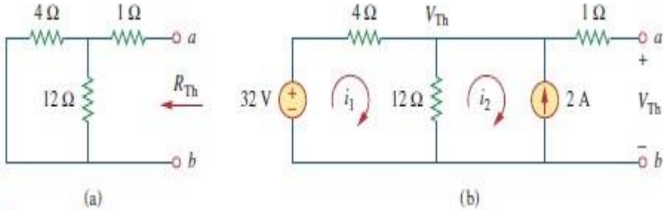


Figure 4.28
For Example 4.8: (a) finding R_{Th}, (b) finding V_{Th}.

To find V_{Th}, consider the circuit in Fig. 4.28(b). Applying mesh analysis to the two loops, we obtain

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\ \text{A}$$

Solving for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\ \text{V}$$

Alternatively, it is even easier to use nodal analysis. We ignore the 1-Ω resistor since no current flows through it. At the top node, KCL gives

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

or

$$96 - 3V_{Th} + 24 = V_{Th} \quad \Rightarrow \quad V_{Th} = 30 \text{ V}$$

as obtained before. We could also use source transformation to find V_{Th}.

The Thevenin equivalent circuit is shown in Fig. 4.29. The current through R_L is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

When $R_L = 6$,

$$I_L = \frac{30}{10} = 3 \text{ A}$$

When $R_L = 16$,

$$I_L = \frac{30}{20} = 1.5 \text{ A}$$

When $R_L = 36$,

$$I_L = \frac{30}{40} = 0.75 \text{ A}$$

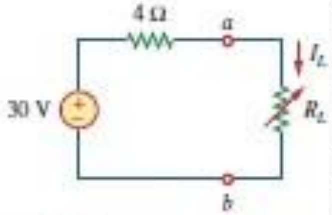
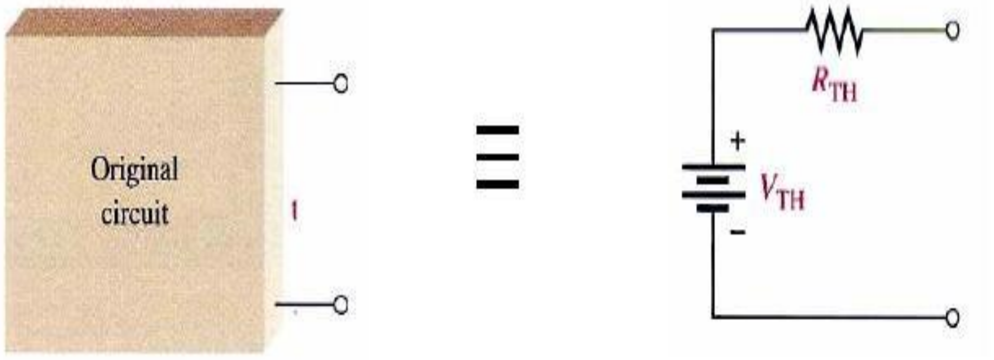


Figure 4.29
The Thevenin equivalent circuit for Example 4.8.

Thevenin's Theorem

The Thevenin equivalent form of any two-terminal resistive circuit consists of an equivalent voltage source (V_{TH}) and an equivalent resistance (R_{TH}),



In Thevenin’s theorem Vth is ___________

a) Sum of two voltage sources

b) A single voltage source

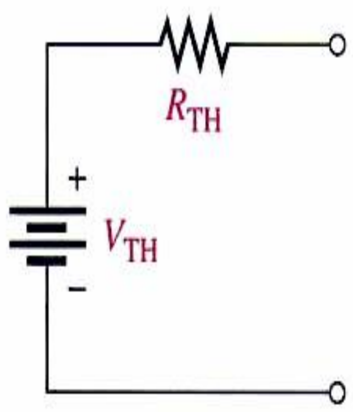
c) Infinite voltage sources

d) 0

Answer: b

Explanation: Thevenin’s theorem states that a combination of voltage sources, current sources and resistors is equivalent to a single voltage source V and a single series resistor R.

Thevenin's Theorem

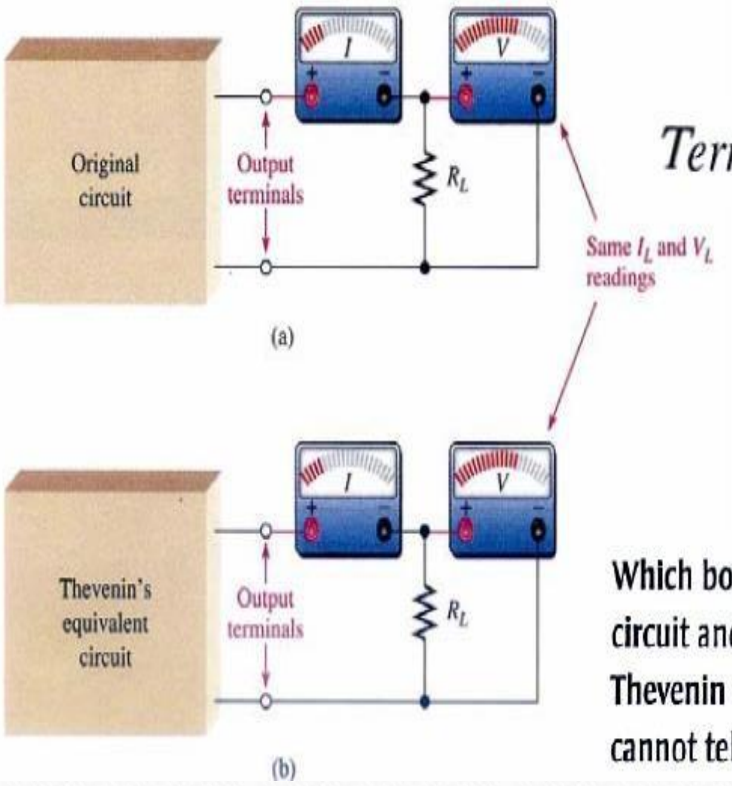


The Thevenin equivalent voltage (V_{TH}) is the open circuit (no-load) voltage between two output terminals in a circuit.

The Thevenin equivalent resistance (R_{TH}) is the total resistance appearing between two terminals in a given circuit with all sources replaced by their internal resistances.

Thevenin's Theorem

Although a Thevenin equivalent circuit is not the same as its original circuit, it acts the same in terms of the output voltage and current.

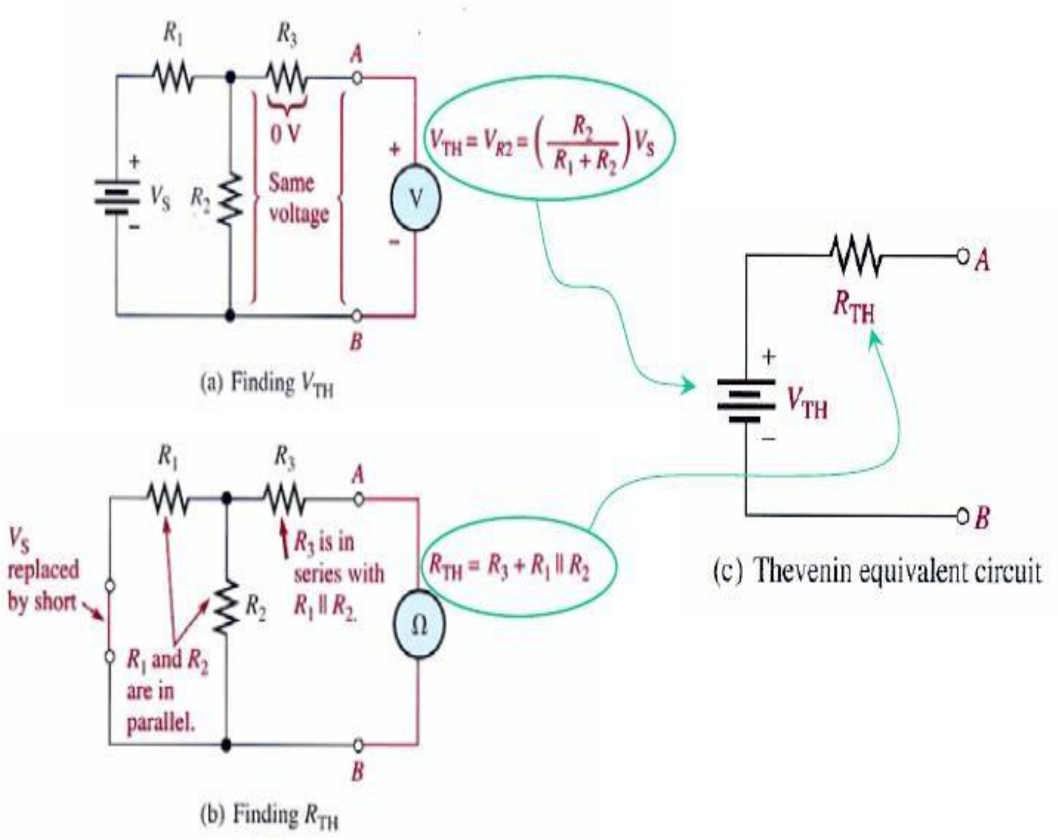


Terminal equivalency

Which box contains the original circuit and which contains the Thevenin equivalent circuit? You cannot tell by observing the meters.

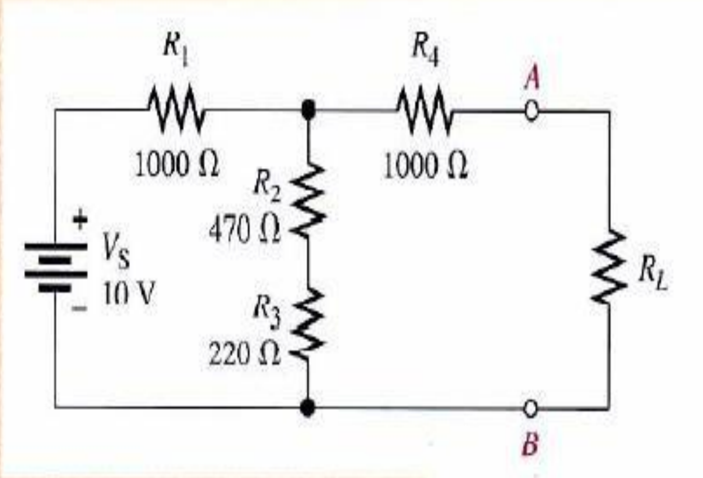
Thevenin's Theorem

Example of the simplification of a circuit by Thevenin's theorem.

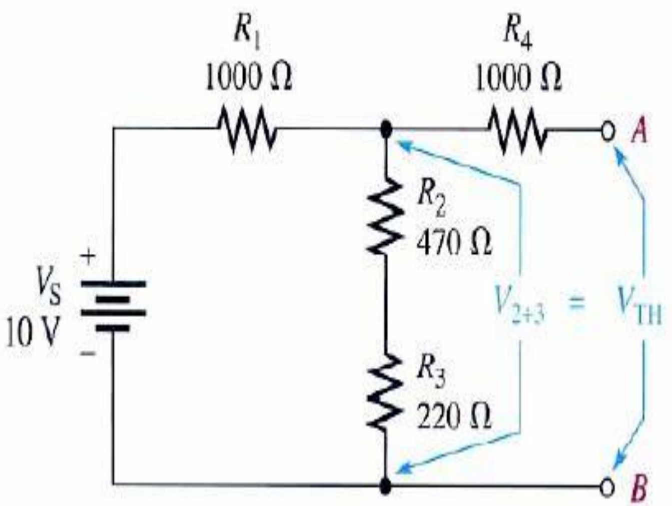


Thevenin's Theorem

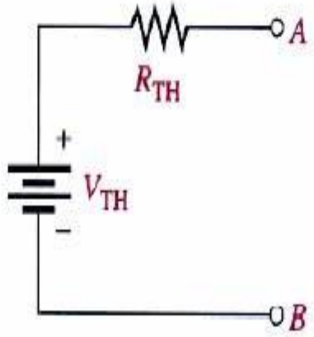
EXAMPLE Find the Thevenin equivalent circuit between *A* and *B* of the circuit.



Thevenin's Theorem



The voltage from A to B is V_{TH} and equals V_{2+3}.

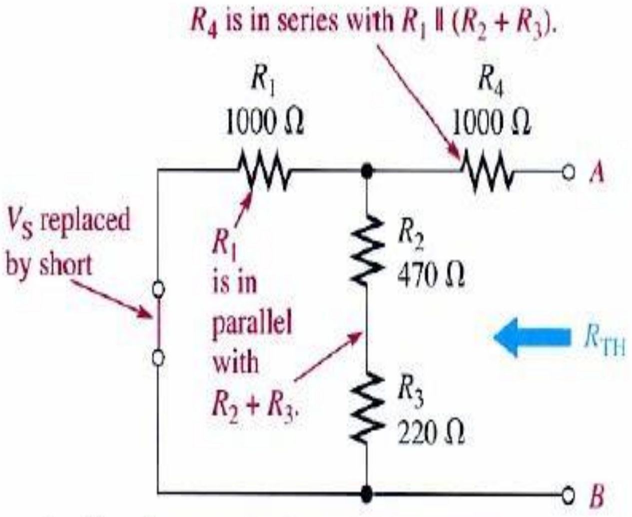


(c) Thevenin equivalent circuit

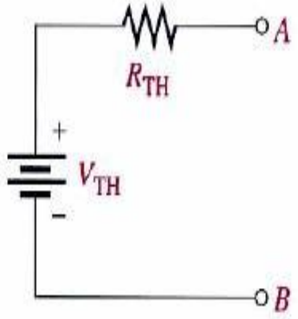
Solution First, remove R_L. Then V_{TH} equals the voltage across $R_2 + R_3$, because $V_4 = 0$ V since there is no current through it.

$$V_{TH} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right)V_S = \left(\frac{690\ \Omega}{1690\ \Omega}\right)10\text{ V} = \mathbf{4.08\ V}$$

Thevenin's Theorem



Looking from terminals A and B, R_4 appears in series with the combination of R_1 in parallel with $(R_2 + R_3)$.

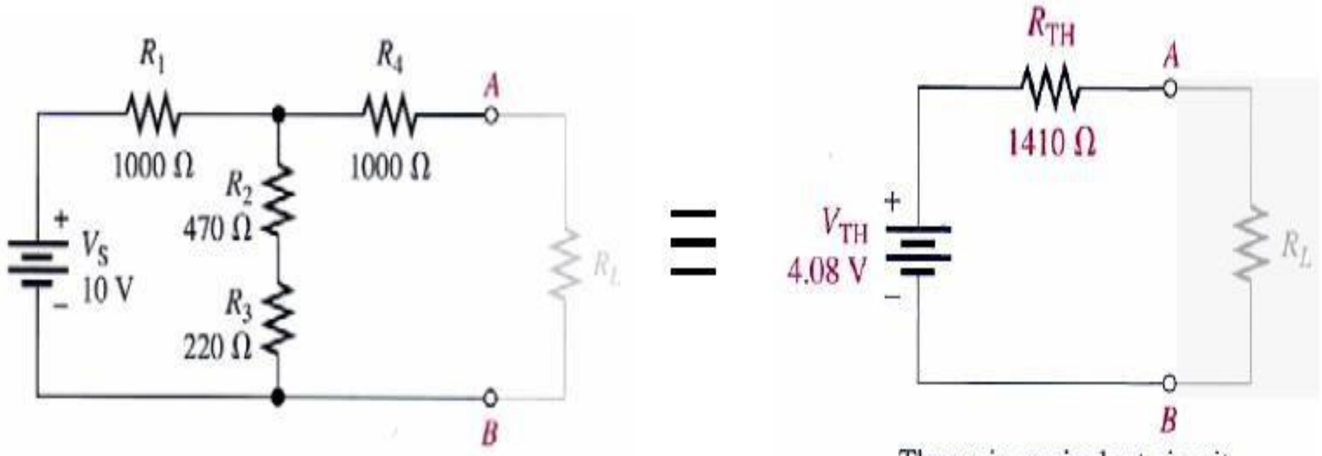


(c) Thevenin equivalent circuit

To find R_{TH}, first replace the source with a short to simulate a zero internal resistance. Then R_1 appears in parallel with $R_2 + R_3$, and R_4 is in series with the series-parallel combination of R_1, R_2, and R_3,

$$R_{TH} = R_4 + \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} = 1000\ \Omega + \frac{(1000\ \Omega)(690\ \Omega)}{1690\ \Omega} = \mathbf{1410\ \Omega}$$

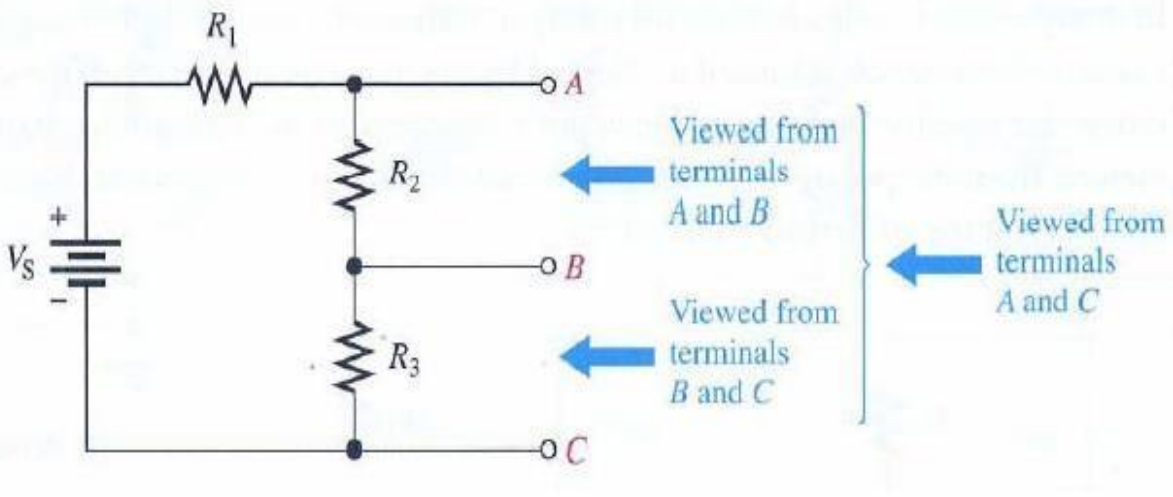
Thevenin's Theorem



Thevenin equivalent circuit

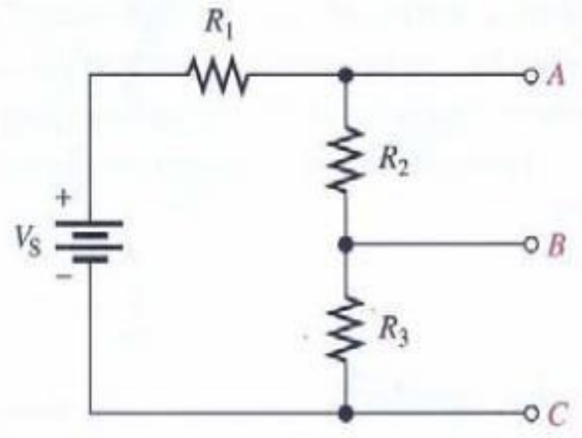
Thevenin's Theorem

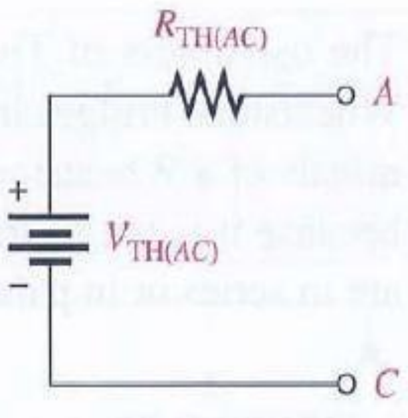
Thevenin Equivalency Depends on the Viewpoint



Thevenin's equivalent depends on the output terminals from which the circuit is viewed.

Thevenin's Theorem



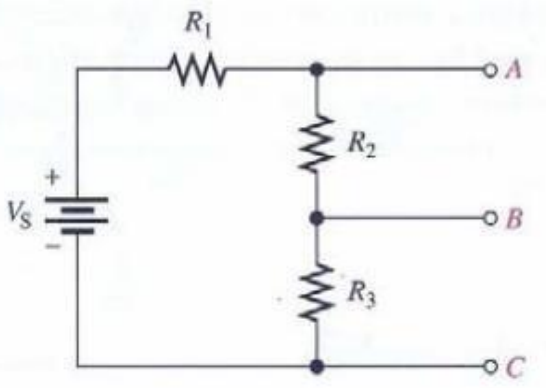


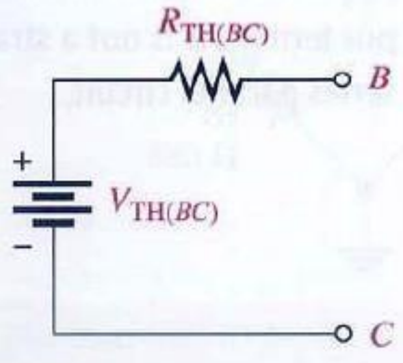
Thevenin equivalent

$$V_{\mathrm{TH}(AC)} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right)V_{\mathrm{S}}$$

$$R_{\mathrm{TH}(AC)} = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$$

Thevenin's Theorem





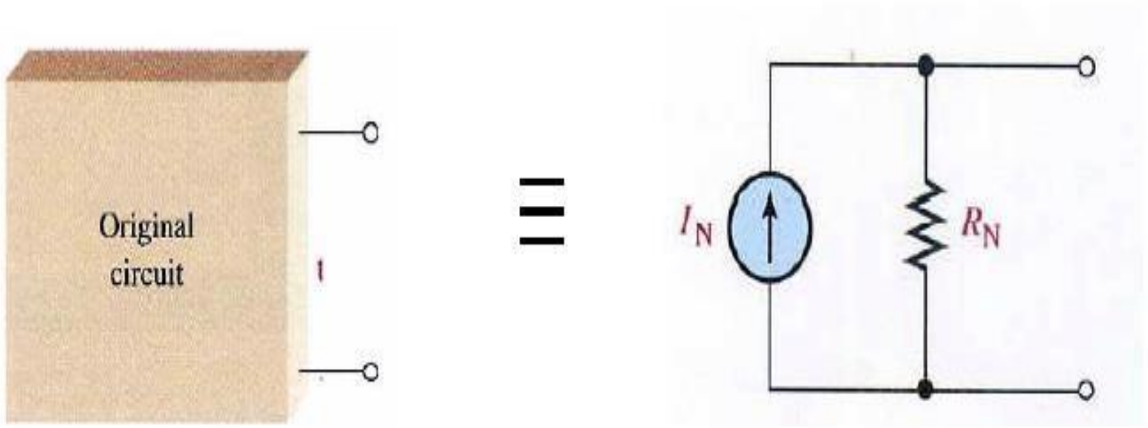
Thevenin equivalent

$$V_{TH(BC)} = \left(\frac{R_3}{R_1 + R_2 + R_3}\right)V_S$$

$$R_{TH(BC)} = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

Norton's Theorem

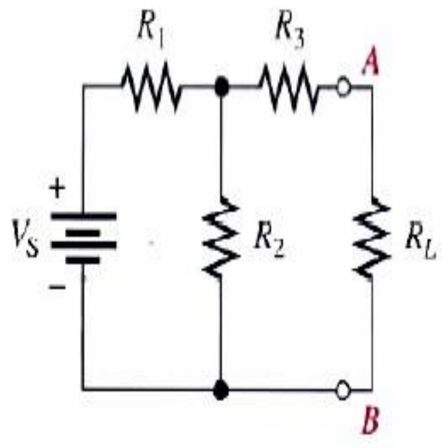
Like Thevenin's theorem, Norton's theorem provides a method of reducing a more complex circuit to a simpler equivalent form.



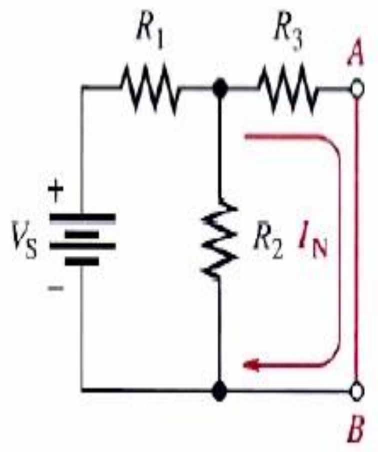
Norton's Theorem

Norton's Equivalent Current (I_N)

Norton's equivalent current (I_N) is the short-circuit current between two output terminals in a circuit.



(a) Original circuit

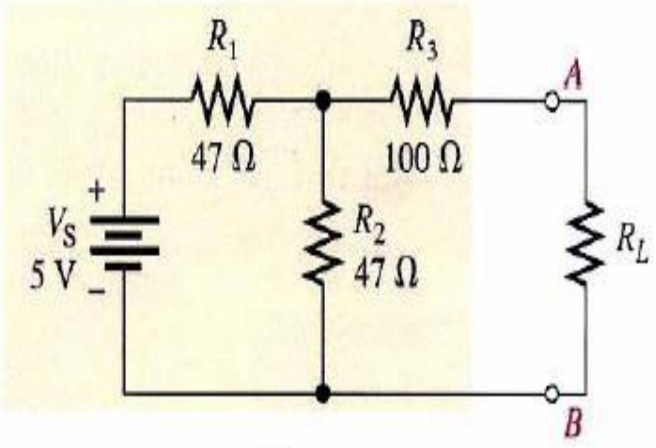


(b) Short the terminals to get I_N.

Norton's Theorem

Norton's Equivalent Current (I_N)

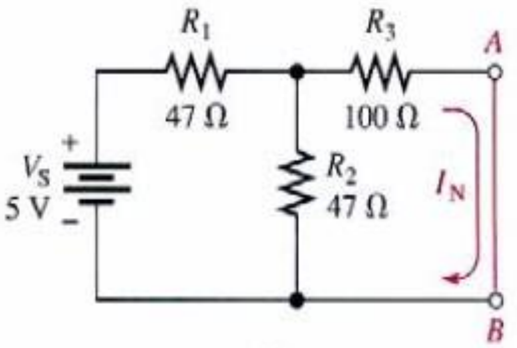
EXAMPLE Determine I_N for the circuit within the beige area.



(a)

Norton's Theorem

Norton's Equivalent Current (I_N)



(b)

Solution

Short terminals A and B. I_N is the current through the short. First, the total resistance seen by the voltage source is

$$R_T = R_1 + \frac{R_2R_3}{R_2 + R_3} = 47\ \Omega + \frac{(47\ \Omega)(100\ \Omega)}{147\ \Omega} = 79\ \Omega$$

The total current from the source is

$$I_T = \frac{V_S}{R_T} = \frac{5\ \text{V}}{79\ \Omega} = 63.3\ \text{mA}$$

Now apply the current-divider formula to find I_N (the current through the short).

$$I_N = \left(\frac{R_2}{R_2 + R_3}\right)I_T = \left(\frac{47\ \Omega}{147\ \Omega}\right)63.3\ \text{mA} = \mathbf{20.2\ mA}$$

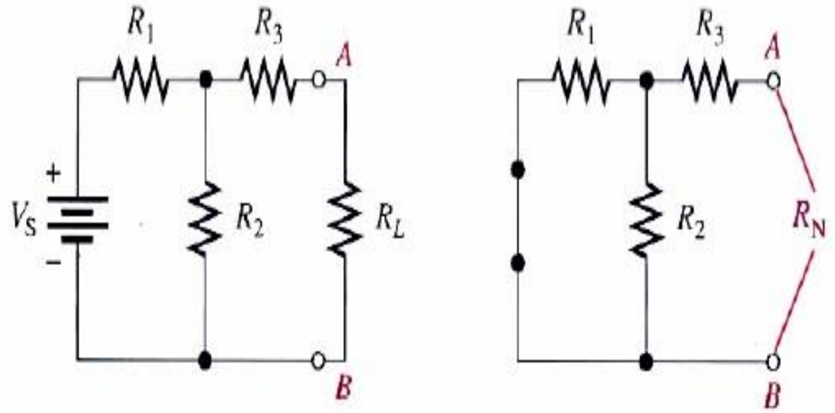
This is the value for the equivalent Norton current source.

Norton's Theorem

Norton's Equivalent Resistance (R_N)

Norton's equivalent resistance (R_N) is defined in the same way as R_{TH}.

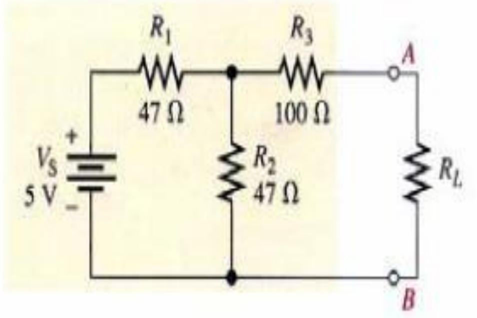
The Norton equivalent resistance, R_N, is the total resistance appearing between two output terminals in a given circuit with all sources replaced by their internal resistances.

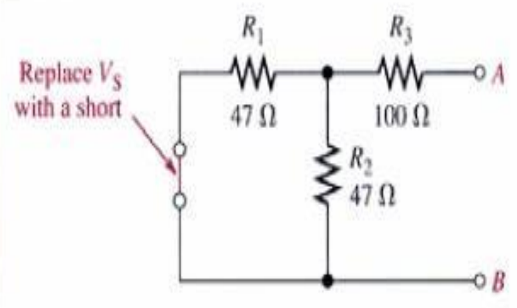


Norton's Theorem

Norton's Equivalent Resistance (R_N)

EXAMPLE Find R_N for the circuit within the beige area





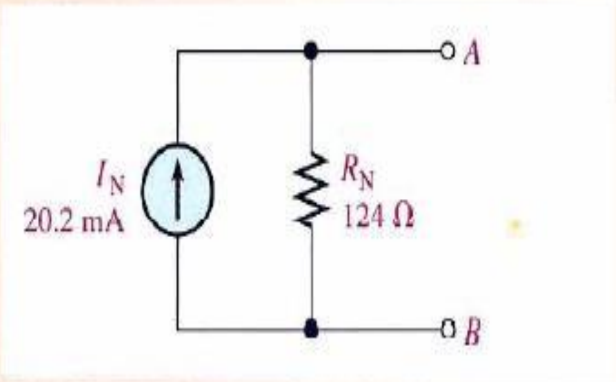
Solution

$$R_N = R_3 + \frac{R_1}{2} = 100\ \Omega + \frac{47\ \Omega}{2} = \mathbf{124\ \Omega}$$

Norton's Theorem

EXAMPLE Draw the complete Norton equivalent circuit for the original circuit that $I_N = 20.2$ mA and $R_N = 124\ \Omega$.

Solution



Norton's Theorem

Summary of Norton's Theorem

Step 1. Short the two terminals between which you want to find the Norton equivalent circuit.

Step 2. Determine the current (I_N) through the shorted terminals.

Step 3. Determine the resistance (R_N) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened). $R_N = R_{TH}$.

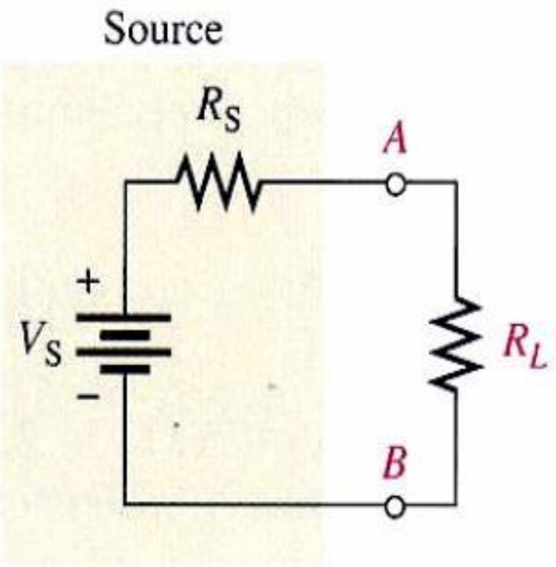
Step 4. Connect I_N and R_N in parallel to produce the complete Norton equivalent for the original circuit.

Maximum Power Transfer Theorem

The maximum power transfer theorem is important when you need to know the value of the load at which the most power is delivered from the source.

The **maximum power transfer** theorem is stated as follows:

For a given source voltage, maximum power is transferred from a source to a load when the load resistance is equal to the internal source resistance.



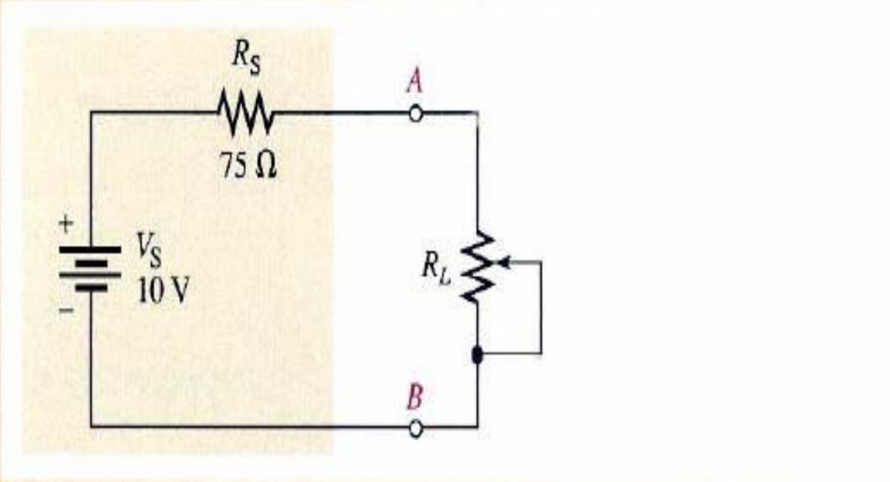
Maximum power is transferred to the load when $R_L = R_S$.

Maximum Power Transfer Theorem

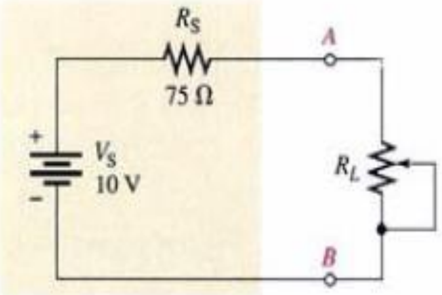
EXAMPLE The source has an internal source resistance of 75 Ω. Determine the load power for each of the following values of load resistance:

(a) 0 Ω (b) 25 Ω (c) 50 Ω (d) 75 Ω (e) 100 Ω (f) 125 Ω

Draw a graph showing the load power versus the load resistance.



Maximum Power Transfer Theorem



Solution

Use Ohm's law ($I = V/R$) and the power formula ($P = I^2R$) to find the load power, P_L, for each value of load resistance.

(a) For $R_L = 0\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{75\ \Omega + 0\ \Omega} = 133\text{ mA}$$

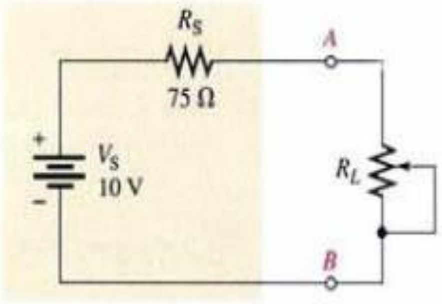
$$P_L = I^2R_L = (133\text{ mA})^2(0\ \Omega) = \mathbf{0\ mW}$$

(b) For $R_L = 25\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{75\ \Omega + 25\ \Omega} = 100\text{ mA}$$

$$P_L = I^2R_L = (100\text{ mA})^2(25\ \Omega) = \mathbf{250\ mW}$$

Maximum Power Transfer Theorem



(c) For $R_L = 50\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{125\ \Omega} = 80\text{ mA}$$

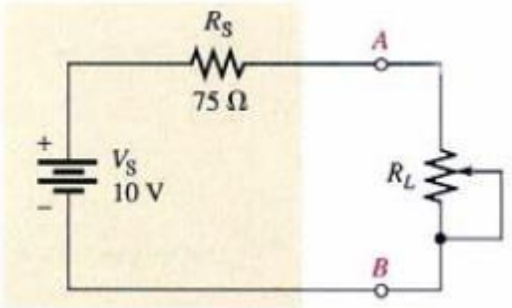
$$P_L = I^2R_L = (80\text{ mA})^2(50\ \Omega) = \mathbf{320\text{ mW}}$$

(d) For $R_L = 75\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\text{ V}}{150\ \Omega} = 66.7\text{ mA}$$

$$P_L = I^2R_L = (66.7\text{ mA})^2(75\ \Omega) = \mathbf{334\text{ mW}}$$

Maximum Power Transfer Theorem



(e) For $R_L = 100\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\ \text{V}}{175\ \Omega} = 57.1\ \text{mA}$$

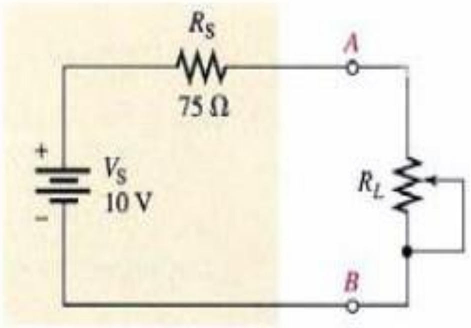
$$P_L = I^2 R_L = (57.1\ \text{mA})^2 (100\ \Omega) = \mathbf{326\ mW}$$

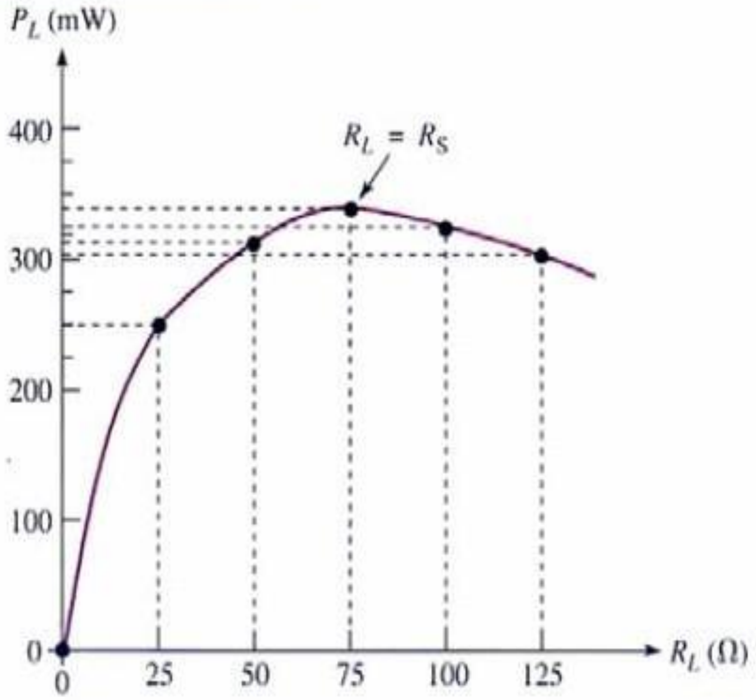
(f) For $R_L = 125\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\ \text{V}}{200\ \Omega} = 50\ \text{mA}$$

$$P_L = I^2 R_L = (50\ \text{mA})^2 (125\ \Omega) = \mathbf{313\ mW}$$

Maximum Power Transfer Theorem





The load power is greatest when $R_L = 75\ \Omega$, which is the same as the internal source resistance.

Thevenin’s Theorem

- Thevenin’s theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

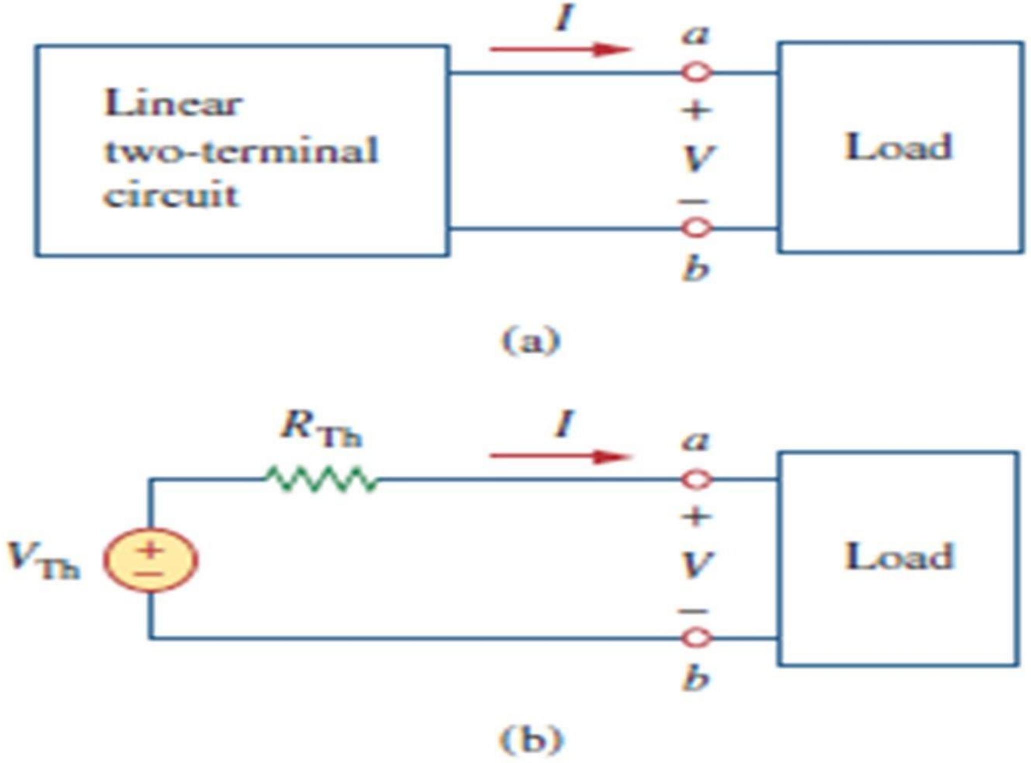
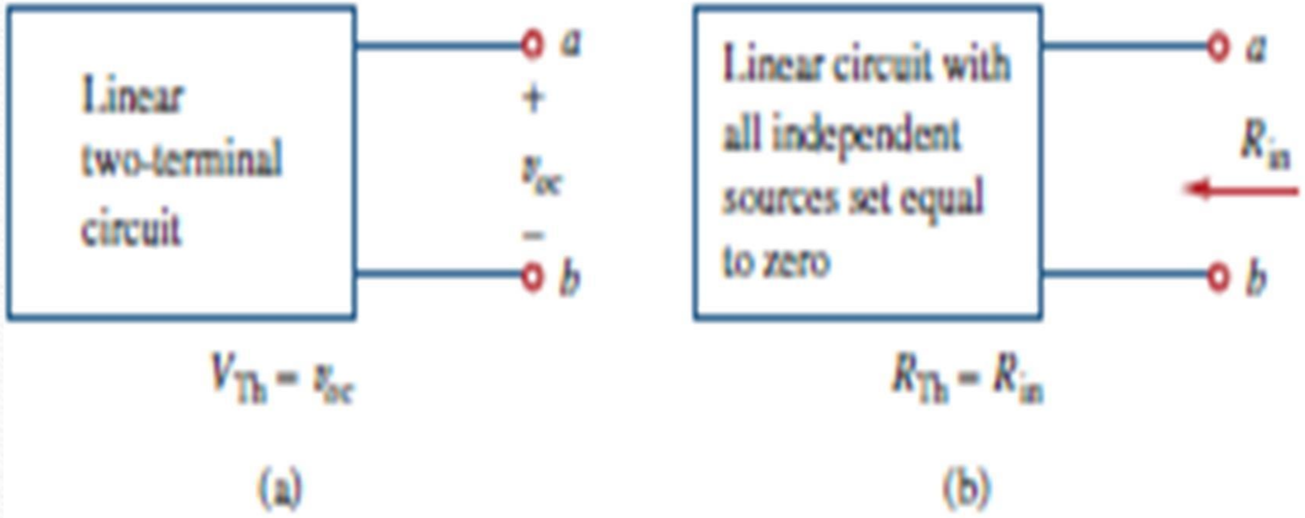


Figure 4.23
Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

$$V_{Th} = v_{oc} \tag{4.6}$$



Summary of Thevenin's Theorem

Step 1. Open the two terminals (remove any load) between which you want to find the Thevenin equivalent circuit.

Step 2. Determine the voltage (V_{TH}) across the two open terminals.

Step 3. Determine the resistance (R_{TH}) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened).

Step 4. Connect V_{TH} and R_{TH} in series to produce the complete Thevenin equivalent for the original circuit.

Step 5. Replace the load removed in Step 1 across the terminals of the Thevenin equivalent circuit. You can now calculate the load current and load voltage using only Ohm's law. They have the same value as the load current and load voltage in the original circuit.

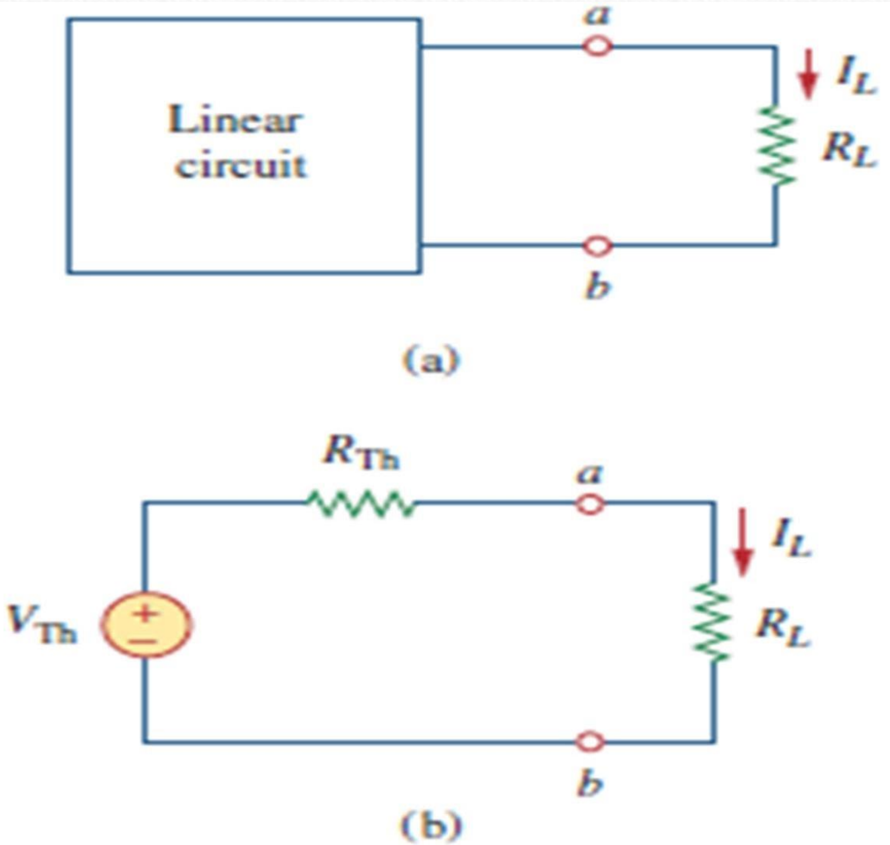


Figure 4.26
A circuit with a load: (a) original circuit, (b) Thevenin equivalent.

In Thevenin’s theorem Vth is ___________

a) Sum of two voltage sources

b) A single voltage source

c) Infinite voltage sources

d) 0