COT5615 Homework 1

due date: Sep 5

For answering the following questions you can invoke any theorem from the textbook, Principles of Mathematical Analysis by Rudin. If you are using any other non-standard theorem or lemma, please give a proper reference. Your submission must be uploaded on Canvas in a legible pdf format.

1. Let E be a nonempty subset of an ordered set; suppose α is a lower bound of E and β is an upper bound E. Prove that $\alpha \leq \beta$.

2. Prove that there are infinitely many primes.

3. Let A be a nonempty set of real numbers which is bounded below. Let -A be the set of all numbers $-x$, where $x \in A$. Prove that

$$inf A = -sup(-A)$$

4. if $z_1, ..., z_n$ are complex, prove that

$$|z_1 + z_2 + ... + z_n| \leq |z_1| + |z_2| + ... + |z_n|$$

5. Prove that the subset A of all rationals Q defined as:

$$A = \{x | x^2 + x \geq 1, x \geq 0, x \in Q\}$$

   does not have a greatest lower bound in Q.