Overall procedure to determine parameter values

- The dynamic data $\{i[k], v[k]\}$ are used to identify all ESC model parameter values (except OCV vs. SOC relationship)
  1. First, compute η and Q from data, directly, as we did for the OCV test results
  2. Compute $z[k]$, OCV$(z[k])$ for every data sample; subtract OCV from $v[k]$
  3. Use *subspace system identification* technique to find R–C time constants
  4. Compute $s[k]$, $i_R[k]$ for every data sample
  5. Guess value for γ; using γ, compute $h[k]$ for every data sample
  6. "Unexplained" part of voltage is now linear in parameters—solve for these parameter values using least squares
  7. Compute rms voltage-prediction error of present model
  8. Adapt γ to minimize this error, iterating steps 5–8 until convergence is reached

1a. Compute coulombic efficiency

- First, use data set collected for test temperature 25 °C
- Since $\eta[k] = \eta(25\,°\text{C})$ in all steps, compute

$$\eta(25\,°\text{C}) = \frac{\text{total absolute ampere-hours discharged in all steps at } 25\,°\text{C}}{\text{total absolute ampere-hours charged in all steps at } 25\,°\text{C}}$$

- Next, for data sets collected where test temperature was not 25 °C, compute coulombic efficiency at test temperature T

$$\eta(T) = \frac{\text{total absolute ampere-hours discharged}}{\text{total absolute ampere-hours charged at temperature } T} - \eta(25\,°\text{C})\frac{\text{total absolute ampere-hours charged at } 25\,°\text{C}}{\text{total absolute ampere-hours charged at temperature } T}$$

1b. Compute total capacity

- First, use data set collected for test temperature 25 °C
- Sum over all data in steps 1–5, gives Q in ampere-seconds

$$Q(25\,°\text{C}) = \sum_{j=0}^{k-1} \eta[j]i[j]$$

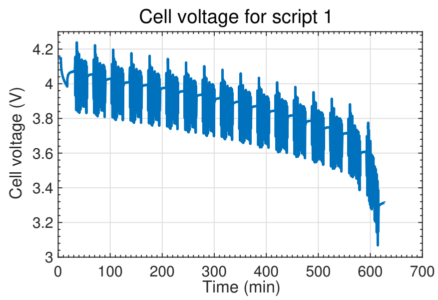
- For data sets collected for test temperatures different from 25 °C, sum over all data in steps 1–5, again gives Q in ampere-seconds

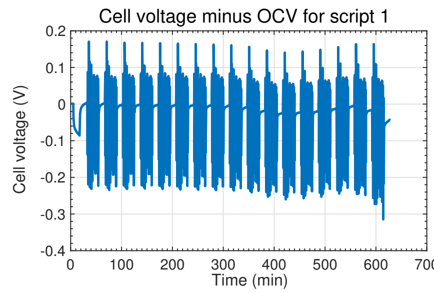
$$1 = \sum_{\text{data at } 25\,°\text{C}} \frac{\eta(25\,°\text{C})i[j]}{Q(25\,°\text{C})} + \sum_{\text{data at } T} \frac{\eta(T)i[j]}{Q(T)}$$

2. Subtract OCV from voltage

- Now, compute SOC for every point $z[k] = 1 - \sum_{j=0}^{k-1} \frac{\eta[k]i[k]}{Q}$
- Then, compute modified (as-yet unexplained) voltage

$$\tilde{v}[k] = v[k] - \text{OCV}(z[k], T[k])$$





3. Finding R–C time constants

- Now desire to find diffusion-voltages R–C time constants
- Could use nonlinear optimization to guess and adapt values (together with other unknown parameter values), to minimize rms voltage error
- Nonlinear optimization is slow, not guaranteed to find optimal solution
- *Subspace system identification* replaces nonlinear optimization with linear-algebra technique that finds solution in one step
- Unfortunately, while computationally simple, its derivation is too complex to teach in this specialization—provided sample code implements the solution, however
- For more info, cf. `http://mocha-java.uccs.edu/ECE5560`

4–5. Compute $i_R[k]$, $s[k]$, $h[k]$

- We don't know hysteresis rate constant γ, so we start by guessing a value, which we will later refine
- Can then compute all model states (cf. slide 12 for definitions of A_{RC}, B_{RC}, A_H)

$$i_R[k] = A_{RC}i_R[k-1] + B_{RC}i[k-1]$$

$$s[k] = \begin{cases} \text{sgn}(i[k]), & |i[k]| > 0; \\ s[k-1], & \text{otherwise} \end{cases}$$

$$h[k] = A_H[k-1]h[k-1] + (A_H[k-1] - 1)\text{sgn}(i[k-1])$$

- Generally initialize $i_R[0] = 0$; $h[0] = 1$; $s[0] = -1$

6. Solve for linear output parameters

- Unexplained part of the measured cell voltage is

$$\begin{aligned}\tilde{v}[k] &= v[k] - \text{OCV}(z[k], T[k]) \\ &= Mh[k] + M_0 s[k] - \textstyle\sum_j R_j i_{R_j}[k] - R_0 i[k]\end{aligned}$$

- We can solve this for the unknowns

$$\underbrace{\begin{bmatrix} \tilde{v}[1] \\ \tilde{v}[2] \\ \vdots \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} h[1] & s[1] & -i[1] & -i_{R_j}^T[1] \\ h[2] & s[2] & -i[2] & -i_{R_j}^T[2] \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} M \\ M_0 \\ R_0 \\ R_j \end{bmatrix}}_{X},$$

via least-squares solution `X = A\Y` in Octave/MATLAB

7. Compute rms voltage prediction error

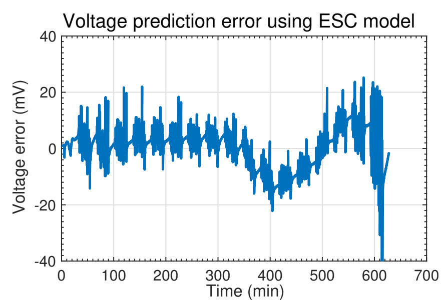
- Desire model prediction to approximate true voltage closely
- For present set of parameter values, model predicts voltage

$$\hat{v}[k] = \text{OCV}(z[k]) + Mh[k] + M_0 s[k] - \textstyle\sum_j R_j i_{R_j}[k] - R_0 i[k]$$

- Can use rms error as an indicator of model fit

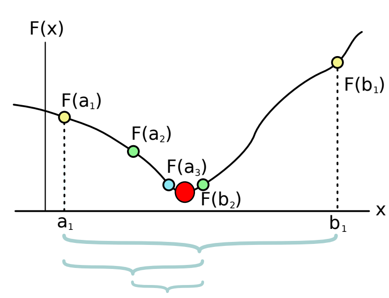
$$\text{rms} = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(v[k] - \hat{v}[k]\right)^2}$$

- Desire that rms be as small as possible (graph shown has rms error of 5.7 mV)



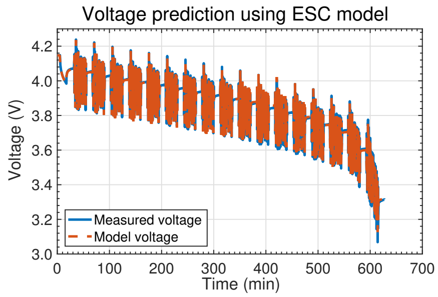
8. Iterate to find best γ

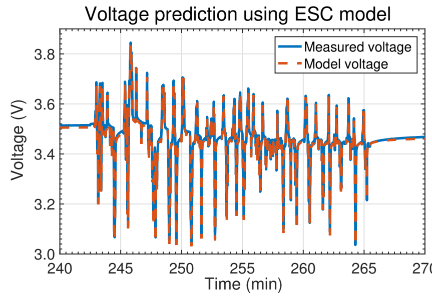
- Now, iterate to find "best" value for γ
- Bisection search is one simple possibility
  - Select bounded range for γ, between a_1 and b_1
  - Evaluate rms error for models computed with $\gamma = a_1$ and $\gamma = b_1$
  - Evaluate rms error for model computed with γ equal to the midpoint of a_1 and b_1
  - Replace endpoint a_1 or b_1 having worst rms error with midpoint
  - Iterate until satisfied "close enough" to best γ



Final result

- Final model is able to predict cell voltage very well
- Figures show model having 3 R–C pairs; rms error of 5.7 mV





- Improvement can be made with additional R–C pairs; however, error graph on slide 8 suggests that OCV refinement might provide greater benefit

Summary

- Have now seen how to process dynamic data to find all parameter values for ESC model
  1. Compute η and Q
  2. Compute $z[k]$, OCV$(z[k])$; subtract OCV from $v[k]$
  3. Use *subspace system identification* technique to find R–C time constants

  4–5. Guess value for γ; compute $s[k]$, $i_R[k]$, $h[k]$

  6. Solve for remaining parameter values using least squares

  7–8. Compute rms voltage-prediction error, adapt γ to minimize this error, iterating steps 5–8 until convergence is reached
- Final model predicts true cell voltage with high fidelity

Credits

Credits for photos in this lesson

- Bisection on slide 9, Tokuchan derivative work [CC-BY-SA-3.0 (`http://creativecommons.org/licenses/by-sa/3.0/`), via Wikimedia Commons, modified from `https://commons.wikimedia.org/wiki/File:Bisection_method.svg`

In the earlier slides, note that

$$A_{RC} = \begin{bmatrix} \exp\left(\frac{-\Delta t}{R_1 C_1}\right) & 0 & \cdots \\ 0 & \exp\left(\frac{-\Delta t}{R_2 C_2}\right) & \\ \vdots & & \ddots \end{bmatrix}, \qquad B_{RC} = \begin{bmatrix} \left(1 - \exp\left(\frac{-\Delta t}{R_1 C_1}\right)\right) \\ \left(1 - \exp\left(\frac{-\Delta t}{R_2 C_2}\right)\right) \\ \vdots \end{bmatrix}$$

$$A_H[k] = \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)$$