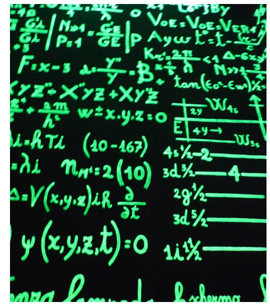
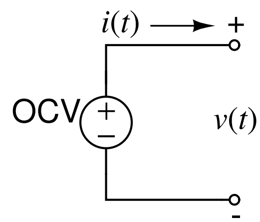
Open-circuit voltage and state of charge

- Models are (sets of) equations that describe something
- We begin to study battery models by building up behavioral/ phenomenological analogs using common circuit elements
- Resulting "equivalent circuit" models:
  - Help give feeling for how cells respond to different usage scenarios, and
  - Are the basis for the BMS algorithms studied in this specialization



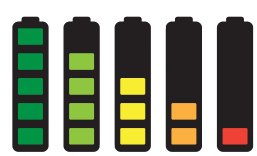
Open-circuit voltage

- We start with simplest possible model: ideal voltage source
- In this model, $v(t) = \text{OCV}$
  - Voltage is not a function of current
  - Voltage is not a function of past usage
  - Voltage is constant... period
- This model is inadequate, but provides a starting point
  - Batteries **do** supply a voltage to a load
  - And, when the cell is unloaded and in complete equilibrium (*i.e.*, "open circuit"), the voltage is fairly predictable
  - An ideal voltage source will be part of our ECM



State of charge and total capacity

- When a cell is fully charged, its open-circuit voltage is higher than when it is discharged
- So, can improve model by including dependence on cell's charge status
- We define the state of charge (SOC) $z(t)$ of a cell to be:
  - $z = 100\,\%$ when the cell is fully charged
  - $z = 0\,\%$ when the cell is fully discharged
- Also define total capacity Q to be total amount of charge removed when discharging from $z = 100\,\%$ to $z = 0\,\%$
  - Q is usually measured in Ah or mAh.



Modeling state of charge

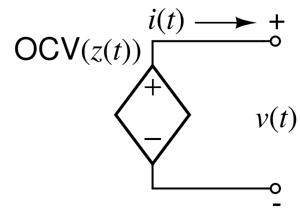
- Can model SOC as (where $\dot{z} = \mathrm{d}z/\mathrm{d}t$)

$$\dot{z}(t) = -i(t)/Q$$
$$z(t) = z(t_0) - \frac{1}{Q}\int_{t_0}^{t} i(\tau)\,\mathrm{d}\tau,$$

where the sign of $i(t)$ is positive on discharge

- In discrete time, if we assume that current is constant over sampling interval Δt,

$$z[k+1] = z[k] - \frac{\Delta t}{Q} i[k]$$



$$v(t) = \text{OCV}(z(t))$$
$$v[k] = \text{OCV}(z[k])$$

Coulombic efficiency

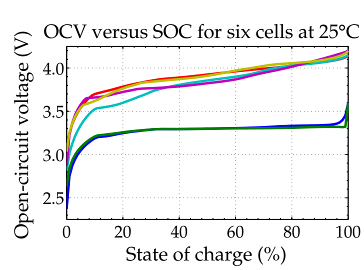
- Cells are not perfectly efficient: can accommodate by

$$\dot{z}(t) = -i(t)\eta(t)/Q$$
$$z[k+1] = z[k] - i[k]\eta[k]\Delta t/Q$$

  - "Coulombic efficiency" $\eta[k] \le 1$ on charge, as some charge is typically lost due to unwanted side reactions
  - We usually model $\eta[k] = 1$ on discharge
- Coulombic (or charge) efficiency $\neq$ energy efficiency
  - Coulombic efficiency $=$ (charge out)/(charge in), often around 99 % in Li-ion
  - Energy efficiency $=$ (energy out)/(energy in), is often closer to 95 %
    - Energy lost in resistive heating, but charge is not lost

Open-circuit voltage

- OCV plotted vs. SOC for six lithium-ion chemistries
- Note that OCV is also a function of temperature—we can include that in the model as OCV($z(t), T(t)$).
- Also note that "depth of discharge" or DOD is the inverse of SOC:
  - DOD $= 1 -$ SOC if expressed as a fraction
  - Sometimes expressed in Ah: DOD $= Q(1 -$ SOC)
- So, can plot OCV curves vs. DOD as well as SOC



Summary

- We have started to develop a model of lithium-ion cells
- So far, model has state-of-charge and voltage equations

$$\dot{z}(t) = -i(t)\eta(t)/Q \qquad \text{or} \qquad z[k+1] = z[k] - i[k]\eta[k]\Delta t/Q$$
$$v(t) = \text{OCV}(z(t)) \qquad\qquad v[k] = \text{OCV}(z[k])$$

- Capacity Q is amount of charge stored between $z = 0\,\%$ and $z = 100\,\%$
- Coulombic efficiency $0 \ll \eta \lesssim 1$, different from energy efficiency
- Open-circuit voltage (OCV) is at-rest voltage of cell at different SOCs, will depend on chemistry of cell being modeled

Credits

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