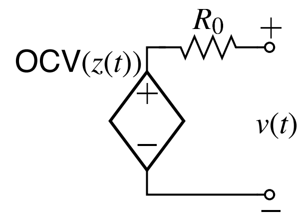
"Rint" model, incl. equivalent series resistance

- Polarization refers to any departure of the cell's terminal voltage away from OCV due to a passage of current
- For example, cell's voltage drops when it is under load
- This can be modeled, in part, as a resistance in series with the ideal voltage source (the "Rint" model)

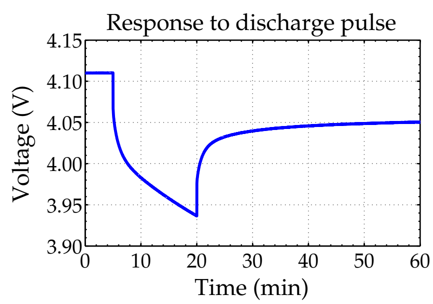
$$v(t) = \text{OCV}(z(t)) - i(t)R_0$$

  - $v(t) > \text{OCV}(z(t))$ on charge,
  - $v(t) < \text{OCV}(z(t))$ on discharge
- Power dissipated by R_0 as heat: energy efficiency imperfect



Diffusion voltages

- "Rint" model suffices for simple electronics designs, but not for advanced consumer electronics and xEV applications
- $i(t) \times R_0$ models instantaneous response to a change in input current
- In practice, we also observe dynamic (non-instantaneous) response to a current step
- Similarly, when cell rests, voltage doesn't immediately return to OCV: relaxes gradually
- Caused by slow diffusion processes in the cell, so I refer to this slowly-changing voltage as a diffusion voltage



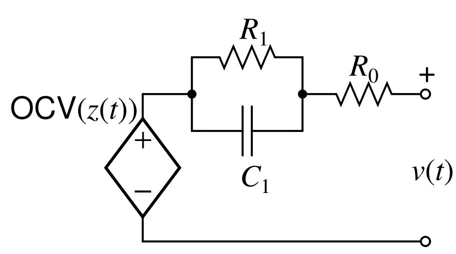
Thévenin model cell voltage

- Diffusion voltages can be closely approximated in a circuit using one or more parallel resistor-capacitor sub-circuits
- Cell voltage in "Thévenin model" is

$$v(t) = \text{OCV}(z(t)) - v_{C_1}(t) - i(t)R_0$$

- Process to identify parameter values from test data simpler if we write voltage in terms of element currents instead:

$$v(t) = \text{OCV}(z(t)) - R_1 i_{R_1}(t) - R_0 i(t)$$



Thévenin model resistor R_1 current

- To find an expression for $i_{R_1}(t)$, recognize that current through R_1 plus the current through C_1 must equal $i(t)$
- Further, $i_{C_1}(t) = C_1 \dot{v}_{C_1}(t)$, which gives

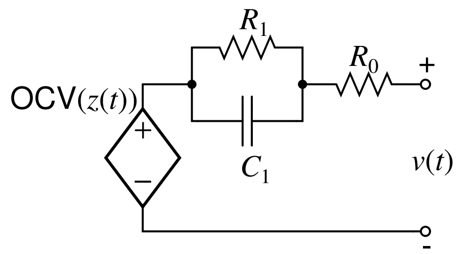
$$i_{R_1}(t) + C_1 \dot{v}_{C_1}(t) = i(t)$$

- Then, since $v_{C_1}(t) = R_1 i_{R_1}(t)$,

$$i_{R_1}(t) + R_1 C_1 \frac{\mathrm{d}i_{R_1}(t)}{\mathrm{d}t} = i(t)$$

- Re-arranging into standard ODE form:

$$\frac{\mathrm{d}i_{R_1}(t)}{\mathrm{d}t} = -\frac{1}{R_1 C_1} i_{R_1}(t) + \frac{1}{R_1 C_1} i(t)$$



Summary

- Have improved a model of lithium-ion cells to include simple description of polarization
  - “Rint” model adds equivalent-series resistance (ESR) term R_0
  - “Thévenin” model adds resistor-capacitor sub-circuit(s) to model diffusion
- So far, continuous-time model has two “state” equations and one “output” equation

$$\frac{\mathrm{d}z(t)}{\mathrm{d}t} = -\frac{\eta(t)}{Q} i(t)$$
$$\frac{\mathrm{d}i_{R_1}(t)}{\mathrm{d}t} = -\frac{1}{R_1 C_1} i_{R_1}(t) + \frac{1}{R_1 C_1} i(t)$$
$$v(t) = \text{OCV}(z(t)) - R_1 i_{R_1}(t) - R_0 i(t)$$

- You will learn how to convert to discrete-time, shortly