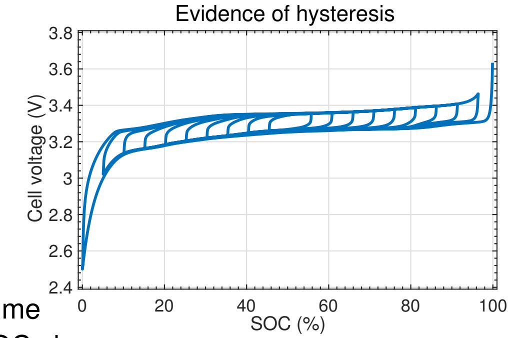
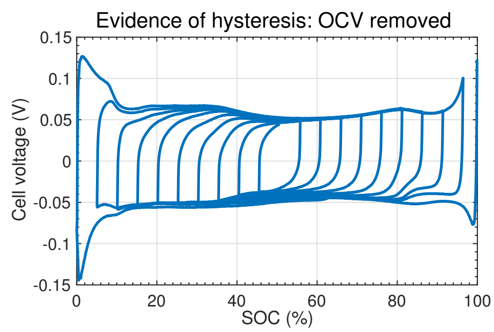
Hysteresis voltages

- If a cell is allowed to rest long enough, diffusion voltages decay to zero, so *model* voltage decays to OCV
- In a real cell, this doesn't happen
- For every SOC, we find a range of possible stable "OCV" values
- Plot shows evidence of hysteresis for C/30 (approximate equilibrium) test
- Ignoring it causes large prediction errors
- Note distinction between hysteresis and diffusion voltages:
  - Diffusion voltages change directly with time but hysteresis voltages change when SOC changes



Examining nature of hysteresis

- If we subtract OCV from the prior test results, we get a better idea of what this hysteresis looks like
- Appears there is a maximum plus/minus hysteresis, may be SOC dependent: $M(z)$
- Amount is positive if cell is presently charging; otherwise negative: $M(z,\dot{z})$
- Hysteresis, plotted versus z, "decays" toward $M(z,\dot{z})$ at a rate that depends on how close it presently is to that amount: indicates a differential equation in z

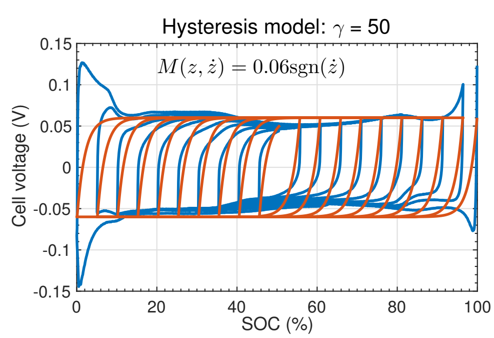


Simple differential equation in z

- Combining observations, we propose model

$$\frac{\mathrm{d}h(z,t)}{\mathrm{d}z} = \gamma\,\mathrm{sgn}(\dot{z})\big(M(z,\dot{z}) - h(z,t)\big)$$

- Max hysteresis $M(z,\dot{z})$ positive for charge ($\dot{z} > 0$), negative for discharge ($\dot{z} < 0$)
- $M(z,\dot{z}) - h(z,t)$ term causes hysteresis rate-of-change to be proportional to the distance away from major hysteresis loop
- Positive γ tunes rate of decay, and $\mathrm{sgn}(\dot{z})$ forces stability for both dis/charge
- Model is simple, but not fantastic



Simple differential equation in t

- To fit differential equation for $h(z,t)$ into cell model, must manipulate it to be a differential equation in time, not in SOC
- We accomplish this by multiplying both sides of the equation by $\mathrm{d}z/\mathrm{d}t$

$$\frac{\mathrm{d}h(z,t)}{\mathrm{d}z}\frac{\mathrm{d}z}{\mathrm{d}t} = \gamma\,\mathrm{sgn}(\dot{z})\big(M(z,\dot{z}) - h(z,t)\big)\frac{\mathrm{d}z}{\mathrm{d}t}$$

- Left side becomes $\dot{h}(t)$; on right side, note $\dot{z}\,\mathrm{sgn}(\dot{z}) = |\dot{z}|$ and $\dot{z}(t) = -\eta(t)i(t)/Q$

$$\dot{h}(t) = -\left|\frac{\eta(t)i(t)\gamma}{Q}\right| h(t) + \left|\frac{\eta(t)i(t)\gamma}{Q}\right| M(z,\dot{z})$$

Convert to discrete time

- Convert to discrete time using method from Lesson 2.1.5, assuming $i(t)$ and $M(z,\dot{z})$ are constant over sample period

$$h[k+1] = \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)h[k] + \left(1-\exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)\right)M(z,\dot{z})$$

  - Linear-time-varying system as factors multiplying state and input change with $i[k]$
- Simplest form, as shown on slide 3, is when $M(z,\dot{z}) = -M\,\mathrm{sgn}(i[k])$, when

$$h[k+1] = \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)h[k] - \left(1-\exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)\right)M\,\mathrm{sgn}(i[k])$$

- With this representation $-M \le h[k] \le M$ at all times, and $h[k]$ has units of volts

Unitless hysteresis state

- When optimizing model parameter values, helps to re-write in equivalent but slightly different representation, which has unitless hysteresis state $-1 \le h[k] \le 1$,

$$h[k+1] = \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)h[k] - \left(1-\exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)\right)\mathrm{sgn}(i[k])$$

$$v_h[k] = M\,h[k]$$

- Have simply moved M from the state equation to output equation
- M appears linearly in output equation, makes estimating M from lab-test data easier

Instantaneous hysteresis

- Dynamic hysteresis changes as SOC changes; instantaneous hysteresis changes when sign of $i[k]$ changes
- Define

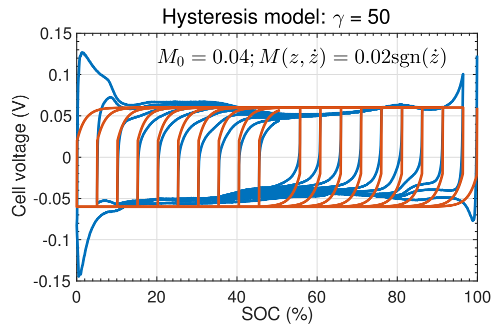
$$s[k] = \begin{cases} \operatorname{sgn}(i[k]), & |i[k]| > 0; \\ s[k-1], & \text{otherwise.} \end{cases}$$

- Instantaneous hysteresis is modeled as

$$h_i[k] = M_0 s[k],$$

and overall hysteresis is

$$v_h[k] = M_0 s[k] + M h[k]$$



Summary

- Hysteresis is a path-dependent voltage that does not decay to zero when the cell rests, unlike diffusion voltages
- Laboratory evidence indicates presence of both instantaneous and dynamic elements to hysteresis
- Have proposed a simple model that is not fantastic, but better than no model at all even if constant parameters γ, M, and M_0 are used
- Can improve model if SOC-dependent values are used