The enhanced self-correcting (ESC) cell model

- We can now summarize a cell model that combines
  - SOC-dependent open-circuit voltage
  - Ohmic resistance and diffusion voltages
  - Hysteresis
- "Enhanced self-correcting" (ESC) cell model does this
  - **Enhanced**: Model includes (poor) description of hysteresis (better than none)
  - **Self correcting**: Transient behavior imperfect, but steady-state correct
    - Voltage converges to OCV + hysteresis on rest,
    - Converges to OCV + hysteresis $-\,i\sum R$ on constant-current event

Using multiple R–C pairs

- Model can contain more than a single parallel R–C pair
- Can define vector valued

$$\begin{bmatrix} i_{R_1}[k+1] \\ i_{R_2}[k+1] \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} \exp\left(\frac{-\Delta t}{R_1C_1}\right) & 0 & \cdots \\ 0 & \exp\left(\frac{-\Delta t}{R_2C_2}\right) & \\ \vdots & & \ddots \end{bmatrix}}_{A_{RC}} \begin{bmatrix} i_{R_1}[k] \\ i_{R_2}[k] \\ \vdots \end{bmatrix} + \underbrace{\begin{bmatrix} \left(1-\exp\left(\frac{-\Delta t}{R_1C_1}\right)\right) \\ \left(1-\exp\left(\frac{-\Delta t}{R_2C_2}\right)\right) \\ \vdots \end{bmatrix}}_{B_{RC}} i[k]$$

$$i_R[k+1] = A_{RC}\, i_R[k] + B_{RC}\, i[k]$$

State and output equations of ESC model

- If we define $A_H[k] = \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)$, then

$$\underbrace{\begin{bmatrix} z[k+1] \\ i_R[k+1] \\ h[k+1] \end{bmatrix}}_{x[k+1]} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & A_{RC} & 0 \\ 0 & 0 & A_H[k] \end{bmatrix}}_{A[k]} \underbrace{\begin{bmatrix} z[k] \\ i_R[k] \\ h[k] \end{bmatrix}}_{x[k]} + \underbrace{\begin{bmatrix} -\eta[k]\Delta t/Q & 0 \\ B_{RC} & 0 \\ 0 & A_H[k]-1 \end{bmatrix}}_{B[k]} \underbrace{\begin{bmatrix} i[k] \\ \operatorname{sgn}(i[k]) \end{bmatrix}}_{u[k]}$$

- ESC "state equation" $x[k+1] = A[k]x[k] + B[k]u[k]$ describes all dynamic effects
- ESC "output equation" computes voltage

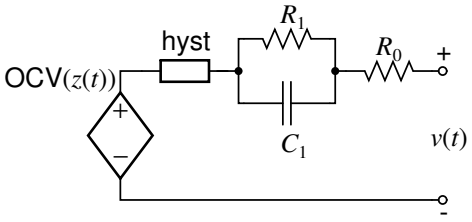
$$v[k] = \text{OCV}(z[k], T[k]) + M_0 s[k] + M h[k] - \textstyle\sum_j R_j i_{R_j}[k] - R_0 i[k]$$

- Can write as $v[k] = f(x[k], u[k])$ if we augment state $x[k]$ with $s[k]$
- Note that all model parameter values must be non-negative

Summary

- We have now fully derived ESC cell model: describes SOC-dependent OCV, ohmic and diffusion voltages, hysteresis
- Model be visualized as an equivalent circuit, where mathematically it comprises two coupled equations

$$x[k+1] = A[k]x[k] + B[k]u[k]$$
$$v[k] = f(x[k], u[k])$$



- This "state-space" model structure will be vitally important in Course 3 of the specialization, on SOC estimation