Elina Murtazina, homework 8

1 Task 1

1.1 Tools

Overleaf, drawing tool, Google Collab

1.2 Task description

Task 1 (Coding)

A mechanical system under the gravity force moves from the rest. Define the velocity of object A if it travels distance s from the rest. The masses of the non-deformable ropes are ignored. Neglect the masses of links FK, KC and the piston K.

The task is to:

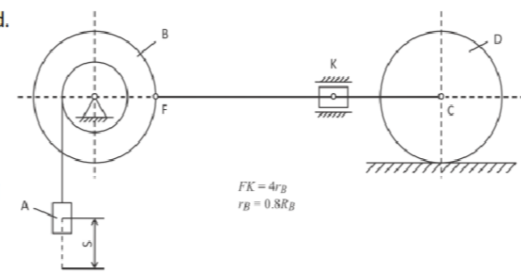
1. make a plot $v_A(s)$;
2. What will change if we omit the last sentence (Neglect ...). (Explain it and show on equations). Why Yablonskii made these constraints?

Needed variables:

$m_A = 1,\ m_B = 3,\ m_D = 20$ (kg);

$R_B = 20,\ R_D = 20,\ i_{Bx} = 18$ (cm), i_{Bx} - radii of gyration of the body;

$\psi = 0.6$ (cm), where ψ is rolling friction.



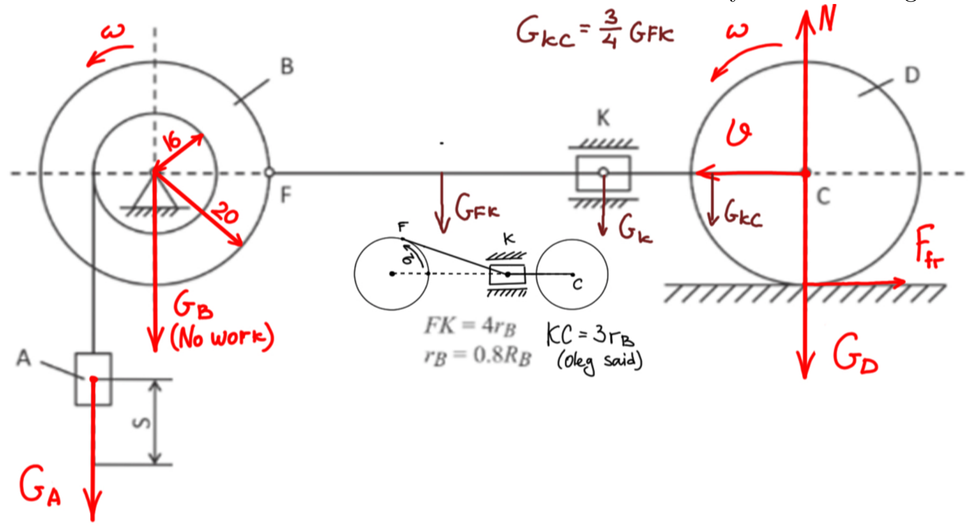
Task 1
(Yablonskii (eng) D6)

1.3 Link to simulation:

Click here!

1.4 Task explanation

Here is the scheme with all forces drawn + the sketch of how system is moving



Now, since we use Euler-Lagrange method we have to find differences of all kinetic energies and make them equal to the sum of the work done by all the parts of the system. The final equation should look like this: $T_2 - T_1 = \sum A_i$, but since the system is at rest at t=0 we can say that $T_2 = \sum A_i$.

Now let's look at the kinetic energies. Please allow me to just pin hand-written equations bc it look presentable enough:

$$T_2 - T_1 = \sum_i A_i$$

$$h = \sin\left(\frac{s \cdot 180}{\pi r_B}\right) \cdot R_B$$

$T_1 = 0$ (since system is at rest at t=0)

$$T_2 = T_{A2} + T_{B2} + T_{D2} + T_{K2} + T_{KC_2} + T_{FK2} = \frac{m_A \vartheta_A^2}{2} + \frac{m_B \cdot i_{Bx}^2 \cdot \omega_B^2}{2} + \frac{m_D \vartheta_C^2}{2} + \frac{1}{4} m_D R_D^2 \cdot \omega_D^2 + \frac{m_K \vartheta_C^2}{2} + \frac{m_{FK} \vartheta_F^2}{2} + \frac{m_{KC} \vartheta_C^2}{2} = \frac{m_A \vartheta_A^2}{2} + \frac{m_B i_{Bx}^2 \vartheta_A^2}{2 r_B^2} + (m_D + m_K + m_{KC}) \frac{\vartheta_A^2 R_B^2}{2 r_B^2} \cdot \frac{(FK^2 - h^2)}{FK^2} + \frac{m_{FK} R_B^2 \vartheta_A^2}{2 r_B^2} = 0{,}5\, \vartheta_A^2 \left(m_A + \frac{m_B i_{Bx}^2}{R_B^2} + (m_D + m_K + m_{KC}) \frac{R_B^2 \left(FK^2 - \sin^2\left(\frac{s \cdot 180^\circ}{\pi r_B}\right) R_B^2\right)}{r_B^2\, FK^2} + \frac{m_{FK} R_B^2}{r_B^2}\right)$$

$$T_{A2} = \frac{m_A \vartheta_A^2}{2}$$

$$T_{B2} = \frac{J_{Bx} \omega_B^2}{2} \quad (J_{Bx} = m_B \cdot i_{Bx}^2)$$

$$T_{D2} = \frac{m_D \vartheta_C^2}{2} + \frac{J_C \omega_D^2}{2} \quad \text{homogeneous solid cylinder } \left(J_C = \frac{1}{2} m_D R_D^2\right)$$

$$T_{K2} = m_K \vartheta_C^2 / 2 \qquad T_{FK} = \frac{m_{FK} \vartheta_F^2}{2}$$

$$\omega_B = \frac{\vartheta_A}{r_B}$$

$$\vartheta_C = \vartheta_A \cdot \frac{R_B}{r_B} \cdot \frac{\sqrt{FK^2 - h^2}}{FK}$$

$$\omega_D = \frac{\vartheta_C}{R_D}$$

$$\vartheta_F = \vartheta_A \cdot \frac{R_B}{r_B}$$

Where black writing used for both cases and red ones - neglected in the first case. The same rule applies to the next hand-written formulas:

$$F_{fr} = N_D\psi = G_D\psi = m_D g\psi \Rightarrow A_{Fr} = m_D g\psi s\cdot\frac{\sqrt{FK^2-h^2}}{FK}$$

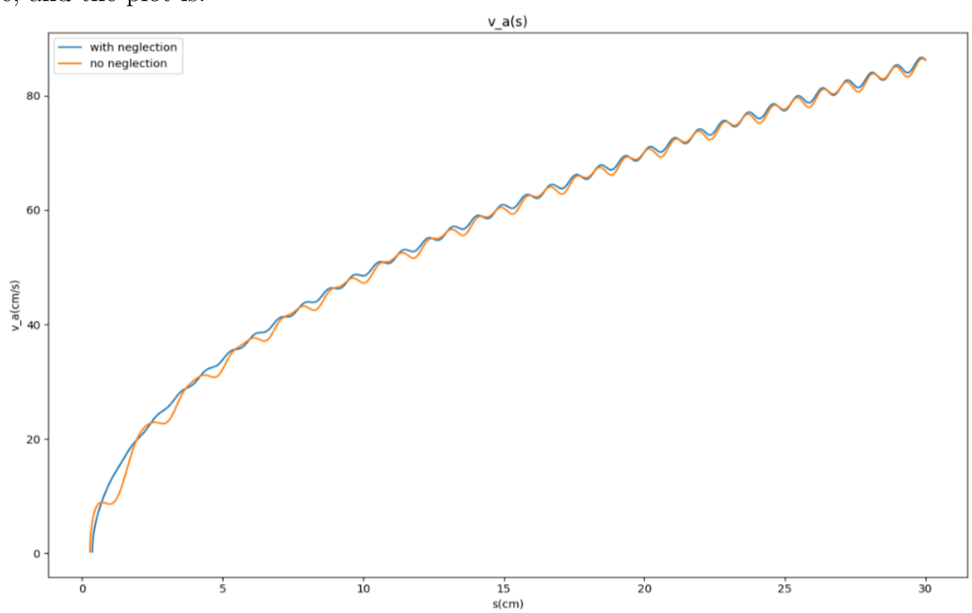
$$A_{GA} = m_A g S;\quad A_{N_i}=0\quad A_{GD}=0$$

$$A_{Gkc}=0\quad A_{G_k}=0\quad A_{G_{FK}} = m_{FK}\cdot g\cdot h/2$$

$$\sum A_i = sg\left(m_D\psi\frac{\sqrt{FK^2 - \sin^2\left(\frac{s\cdot 180}{\pi r_B}\right)R_B^2}}{FK}+m_A\right) + m_{FK}g\cdot\sin\left(\frac{s\cdot 180}{\pi r}\right)R\cdot\frac{1}{2}$$

Where works of gravity forces of bodies D, KC, K and B equal to zero because there is no moving along y axis. Therefore, work of N forces also equals to 0.

About how we got h: we used formula the length of the arc of the circle: $l = \frac{\pi r\alpha}{180}$ where $l = s$ at the moment and $r = r_B$, because it's the radius along which moves A. That way we can find α so we later can find h to solve all the rest. After all of that we come back to the $T_2 = \sum A_i$ form and just put it all into python for it to solve and the work done. The masses of firstly neglected bodies we take randomly, only remembering that $r_B = \frac{3}{4}R_B$. Again, solution is here, and the plot is:



I think Yablonskii made such constrains Moreover, if we change mass of a piston K to a small one, the "waves" of the orange plot eventually will get as close to blue plot as possible. The difference at the start of a plot is held by the masses of FK and KC. Telling practically, these constrains were made so the links won't affect the dynamics of body D.

Here is the meme for this task (about the plot specifically):

2 Task 1

2.1 Tools

Overleaf, Google Collab

2.2 Task description

Task 2 (Coding)

System description

You have a a cart pole. Body 1 is a slider, mass m_1, it moves without friction.

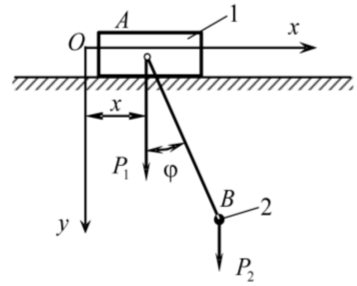
AB is a massless rod with length *l*. Body 2 with mass m_2 is connected to *AB* in point *B*.

It's a 2 DoF system. You should take *x* and ϕ as a representation of this system. The origin of each coordinate should be the same as on the picture.

Initial conditions:

1. $x = 0,\ \phi = 10°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$
2. $x = 0.5,\ \phi = 45°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$
3. $x = 0.5,\ \phi = -135°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$

Parameters: $m_1 = 5\,kg,\ m_2 = 1\,kg,\ l = 1\,m.$



Task 2

Task 2 (Coding)

Tasks description

You should solve this problem using **Euler-Lagrange** method;

Tasks

1. To derive a differential equation of the motion, using **Euler-Lagrange** approach.
2. To create plots $x(t),\ \phi(t), \dot{x}(t), \dot{\phi}(t)$.
3. To make a simulation of this system. Show velocities and accelerations for 1, 2 bodies (coding approach).
4. Compare the obtained results from previous lab (*Newton-Euler* and *Model-oriented design*).

2.3 Link to simulation:

same link, need to scroll lower

2.4 Task explanation

$x_1 = x; \dot{x_1} = \dot{x}; x_2 = x + lsin\phi; \dot{x_2} = \dot{x} + l\dot{\phi}\cos\phi; y_2 = -lcos\phi; \dot{y_2} = l\dot{\phi}sin\phi$

Euler Lagrange equations for this system is: $L = T + A$

$T = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2(\dot{x}^2 + 2l\dot{x}\dot{\phi}cos\phi + l^2\dot{\phi}^2cos^2\phi + l^2\dot{\phi}^2sin^2\phi) = \frac{1}{2}(m_1 + m_2)\dot{x} + m_2l\dot{x}\dot{\phi}cos\phi + \frac{1}{2m_2}l^2\dot{\phi}^2$

$A = -m_2glcos\phi$

$L = \frac{1}{2}(m_1 + m_2)\dot{x} + m_2 l\dot{x}\dot{\phi}cos\phi + \frac{1}{2m_2}l^2\dot{\phi}^2 + m_2 glcos\phi$

Now we can move to partial derivatives:

$\frac{dL}{dx} = 0$
$\frac{dL}{d\dot{x}} = (m_1 + m_2)\dot{x} + m_2 l\dot{\phi}cos\phi$
$\frac{dL}{d\phi} = -m_2 l\dot{x}\dot{\phi}sin\phi - m_2 glsin\phi$
$\frac{dL}{d\dot{\phi}} = m_2 l\dot{x}cos\phi + m_2 l^2\dot{\phi}$

Following Euler-Lagrange method we can get:

$\frac{d}{dt}(\frac{dL}{d\dot{x}} - \frac{dL}{dx} = 0 => (m_1 + m_2)\ddot{x} + m_2 l\ddot{\phi}cos\phi - m_2 l\dot{\phi}^2 sin\phi = 0$
$\frac{d}{dt}(\frac{dL}{d\dot{\phi}} - \frac{dL}{d\phi} = 0 => m_2 l\ddot{x}cos\phi + m_2 l^2\ddot{\phi} + m_2 glsin\phi = 0$

Then we go to the Python to substitute it and get plots. We managed to get plots one to one the same so it's all was done right. Good job

Meme with the reference to the Hamilton musical bc even if we're swearing we're people of the culture (and also bc "Non-Stop" party kept me sane this night)