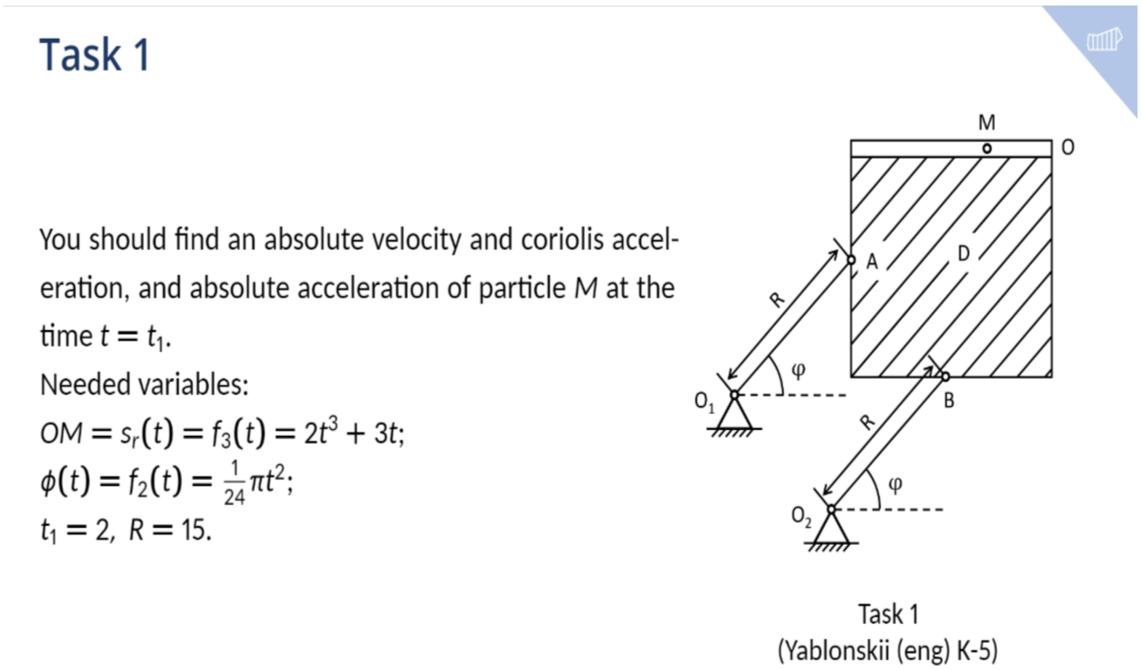
Elina Murtazina, homework 3

1 Task 1

1.1 Tools

Overleaf

1.2 Task description



1.3 Task explanation

As I understand, rigit body D moves in a translatory motion, since R from points O1 and O2 is the same value and both φ are the same. So the movement of point O is the same as the movement of a points A and B. Also this means that point M moves only in x axis. If ϕ is $\frac{1}{24}\pi t^2$ then $\omega = \phi' = \frac{1}{12}\pi t$. Since point M moves at itself in terms of point O, it gives us an option to treat motion of point M as a plane motion (rotation motion of point O around some stable point + translatory motion of point M depends on point O position). So using formula $r_B = r_A + A_\phi \rho_B$ where ρ_B is R, ϕ function is given, and r_B is what we need to find velocity and acceleration of a point M. Since it'll take long time to calculate meanwhile a lot of space too, I'll trust calculator on this one:

$$\begin{bmatrix} 2x^3+3x \\ 0 \end{bmatrix} + \begin{bmatrix} \cos\left(\frac{\pi x^2}{24}\right) & -\sin\left(\frac{\pi x^2}{24}\right) \\ \sin\left(\frac{\pi x^2}{24}\right) & \cos\left(\frac{\pi x^2}{24}\right) \end{bmatrix} \times \begin{bmatrix} 15\cos\left(\frac{\pi x^2}{24}\right) \\ 15\sin\left(\frac{\pi x^2}{24}\right) \end{bmatrix}$$

Упростите выражение

$$\begin{bmatrix} 2x^3+3x \\ 0 \end{bmatrix} + \begin{bmatrix} 15\cos\left(\frac{\pi x^2}{12}\right) \\ 15\sin\left(\frac{\pi x^2}{12}\right) \end{bmatrix}$$

Вычислить

Решение

$$\begin{bmatrix} 2x^3+3x+15\cos\left(\frac{\pi x^2}{12}\right) \\ 15\sin\left(\frac{\pi x^2}{12}\right) \end{bmatrix}$$

$$\dot{\xi}_B=\dot{\xi}_A - x_B\dot{\varphi}\sin\varphi - y_B\dot{\varphi}\cos\varphi$$
$$\dot{\eta}_B=\dot{\eta}_A + x_B\dot{\varphi}\cos\varphi - y_B\dot{\varphi}\sin\varphi$$
$$\dot{\zeta}_B=0$$

It is possible to use these equations directly

$\omega\times\rho$

$V_{BA}=\omega\times\rho$

$\boxed{V_B=V_A+\omega\times\rho}$ V_A is the pole velocity

$\boxed{V_B=V_A+V_{BA}}$ V_{BA} is the velocity of body rotation around pole

And now we can finally ~~cry~~ try to calculate absolute velocity by solution shown on the second picture upper. Here is solution:

$$\begin{bmatrix} \frac{d}{dx}\left(2x^3+3x\right) \\ 0 \end{bmatrix} + \begin{bmatrix} -\sin\left(\frac{\pi x^2}{24}\right)\times\frac{d}{dx}\left(\frac{\pi x^2}{24}\right) & -\cos\left(\frac{\pi x^2}{24}\right)\times\frac{d}{dx}\left(\frac{\pi x^2}{24}\right) \\ \cos\left(\frac{\pi x^2}{24}\right)\times\frac{d}{dx}\left(\frac{\pi x^2}{24}\right) & -\sin\left(\frac{\pi x^2}{24}\right)\times\frac{d}{dx}\left(\frac{\pi x^2}{24}\right) \end{bmatrix} \times \begin{bmatrix} 15\cos\left(\frac{\pi x^2}{24}\right) \\ 15\sin\left(\frac{\pi x^2}{24}\right) \end{bmatrix}$$

Дифференцировать

$$\begin{bmatrix} 6x^2+3 \\ 0 \end{bmatrix} + \begin{bmatrix} -\sin\left(\frac{\pi x^2}{24}\right)\times\frac{\pi x}{12} & -\cos\left(\frac{\pi x^2}{24}\right)\times\frac{\pi x}{12} \\ \cos\left(\frac{\pi x^2}{24}\right)\times\frac{\pi x}{12} & -\sin\left(\frac{\pi x^2}{24}\right)\times\frac{\pi x}{12} \end{bmatrix} \times \begin{bmatrix} 15\cos\left(\frac{\pi x^2}{24}\right) \\ 15\sin\left(\frac{\pi x^2}{24}\right) \end{bmatrix}$$

Вычислить

Решение

$$\begin{bmatrix} 6x^2+3 \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{\pi x\times\sin\left(\frac{\pi x^2}{24}\right)}{12} & -\frac{\pi x\times\cos\left(\frac{\pi x^2}{24}\right)}{12} \\ \frac{\pi x\times\cos\left(\frac{\pi x^2}{24}\right)}{12} & -\frac{\pi x\times\sin\left(\frac{\pi x^2}{24}\right)}{12} \end{bmatrix} \times \begin{bmatrix} 15\cos\left(\frac{\pi x^2}{24}\right) \\ 15\sin\left(\frac{\pi x^2}{24}\right) \end{bmatrix}$$

$$\begin{bmatrix} 24+3 \\ 0 \end{bmatrix} + \begin{bmatrix} -\pi\times2\times\frac{\sin\left(\frac{4}{24}\pi\right)}{12} & -\frac{\pi\times2\cos\left(\frac{4\pi}{24}\right)}{12} \\ 2\pi\times\frac{\cos\left(\frac{4\pi}{24}\right)}{12} & -\frac{2\pi\times\sin\left(\frac{\pi\times4}{24}\right)}{12} \end{bmatrix} \times \begin{bmatrix} 15\cos\left(\frac{4\pi}{24}\right) \\ 15\sin\left(\frac{4\pi}{24}\right) \end{bmatrix}$$

Найдите значение

$$\begin{bmatrix} 27-\frac{5\pi\sqrt{3}}{4} \\ \frac{5\pi}{4} \end{bmatrix}$$

To find $a^{cor} = 2\omega_{tr} \times v^{rel}$ and $v^{rel} = (\dot{x}_B i' + \dot{y}_B j' + \dot{z}_B k')$

2 Task 2

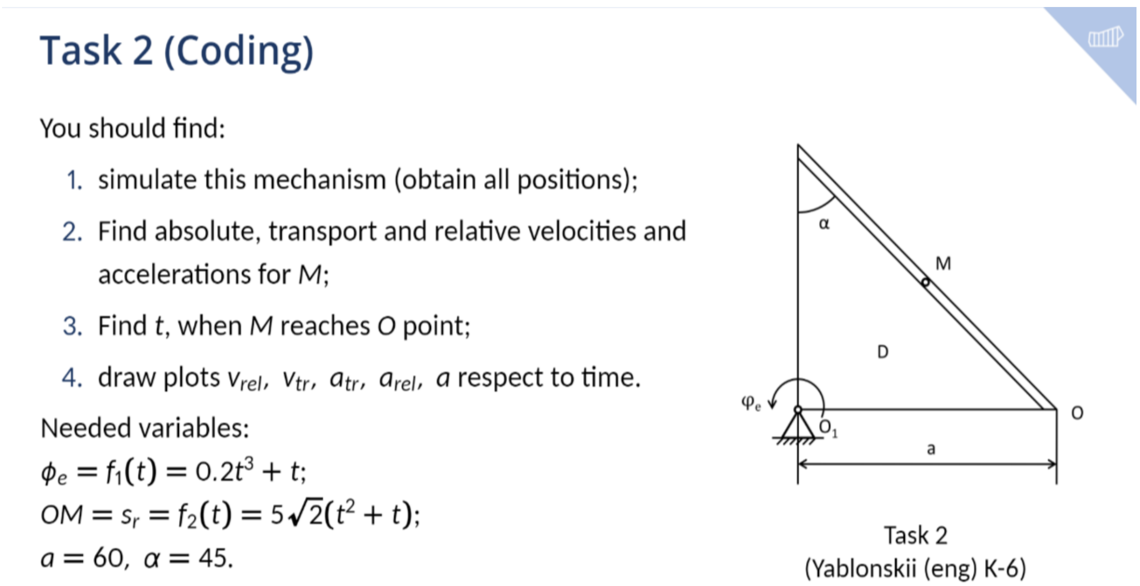
2.1 Tools

Overleaf, Google Collab

2.2 Link to simulation:

Click here!

2.3 Task description



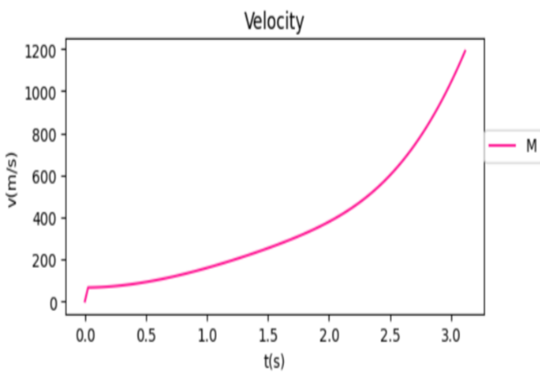
2.4 Task explanation

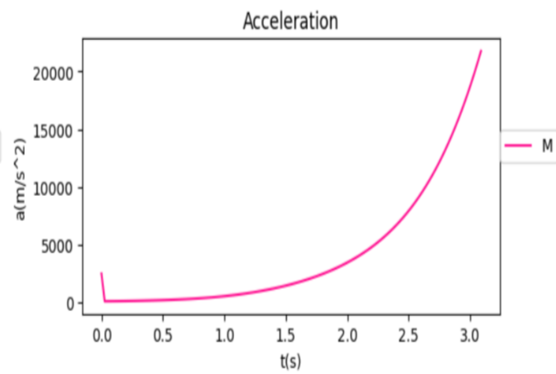
Since $\alpha = 45$, the triangle OQO1 is isosceles, so O1O = O1Q. The angle between O1Q and x-axis is 90. create movements for points Q and O knowing the distance a and ϕ. Connect points Q and O to complete body D. We know the changing of point M position along line OQ through the time so we find vector OQ and its' unit segment, then multiplying it by length OM at a certain time. And that's basically all the work regarding simulation.

2nd question is at which time point M will reach point Q and that's very easy, we just find constant distance OQ (since at this point OQ will be equal OM) with the usage of the Pythagorean theorem. $OQ = a\sqrt{2} = 60\sqrt{2}$. And now just solve the equation $f_2(t) = 5\sqrt{2}(t^2 + t) = 60\sqrt{2}$ which leads us to solution t = 3. We also have negative root but we don't talk about negative time.

About v_r, v_t, a_t and a_r where r - relative motion and t - transport motion. $v_B^t = v_A + \omega_t \times \rho_B$ and $v_B^r = (\dot{x}_B i' + \dot{y}_B j' + \dot{z}_B k')$, resulting in $v_B = v_B^r + v_B^t$. Now about accelerations: $a_B^t = a_A + a_{BA}^{\epsilon} + a_{BA}^{\omega}$ and $a_B^r = a_B^{\tau} + a_B^n$. It's easier to calculate a_B as the sum of $\dot{v}^t$ and $\dot{v}^r$

2.5 Plots





2.6 Screenshots