Elina Murtazina, homework 7

1 Task 1

1.1 Tools

Overleaf, drawing tool

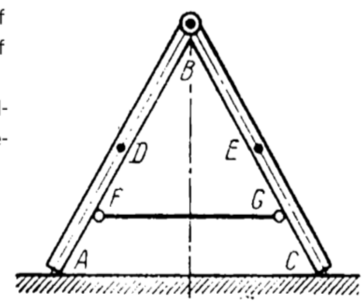
1.2 Task description

Task 1

A step ladder ABC, hinged at B, rests on a smooth horizontal floor, as shown on the figure. $AB = BC = 2l$.
The centres of gravity are at the midpoints D and E of the rods. The radius of gyration of each part of the ladder about the axis passing through the center of gravity is p.
The distance between B and the floor is h. At the certain moment the ladder collapses due to the rupture of a ling FG between the two halves of the ladder. Neglecting the effect of friction in the hinge, determine:

1. the velocity v_1 of the point B at the moment, when it hits the floor;
2. the velocity v_2 of point B at the moment, when it is at a distance $\frac{1}{2}h$ from the floor.

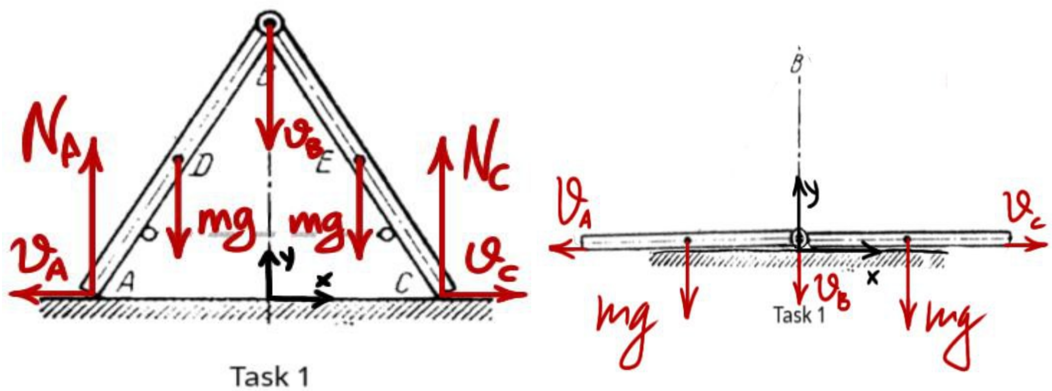
Answer: $v_1 = 2l\sqrt{\frac{gh}{l^2 + p^2}}$, $v_2 = \frac{1}{2}\sqrt{gh\frac{16l^2 - h^2}{2(l^2 + p^2)}}$.



Task 1

1.3 Task explanation

Here are two states of a system, the first one, when FG just broke(t=0) and the final one, then ladder collapsed completely, when point B reach the floor.



Since AB = BC by all parameters (length, mass, etc.) gravity acts in the same same way on both of them. To solve this use Theorem on the hange of Kinetic Energy of a System Newton-Euler equation.

Here's the table of changes in coordinates of points (coordinates of points which stays the same until the end: $x_B = 0; y_C = 0; y_A = 0$)

	Start	Finish
y_B	h	0
x_C	$\sqrt{4l^2 - h^2}$	$\sqrt{4l^2 - h^2} + h$
x_A	$-\sqrt{4l^2 - h^2}$	$-\sqrt{4l^2 - h^2} - h$

Force analysis: $N_A, N_C, m_{AB}g = m_{BC}g = mg$

Inertia of the rod:
$I = m_{AB}l^2 + m_{AB}p^2$ The formula combines contribution due to the mass being concentrated at the center of mass ($m_{AB}l^2$), and the contribution due to the distribution of mass around the axis of rotation ($m_{AB}p^2$) **Kinetic Energy:**

$T_{AB} = \frac{1}{2}I\omega_{AB}^2$
$T_{BC} = \frac{1}{2}I\omega_{BC}^2$ **Angular velocity of the rod:**

$v_B = \omega_{AB}2l => \omega_{AB} = \frac{v_B}{2l}$
$v_B = \omega_{BC}2l => \omega_{BC} = \frac{v_B}{2l}$ **Work:**

$A = mg\frac{h}{2} + mg\frac{h}{2} = mgh$ **Solution:**

$T_{AB} + T_{BC} = A$
$\frac{1}{2}I(\frac{v_B}{2l})^2 + \frac{1}{2}I(\frac{v_B}{2l})^2 = mgh$
$v_B = 2l\sqrt{\frac{gh}{l^2+p^2}}$

Solution for the second situation will look alike like for the first part of the task, let's review start and finish positions in the table ($x_B; y_C; y_A$ are still the same and equal to 0):

	Start	Finish
y_B	h	$\frac{h}{2}$
x_C	$\sqrt{4l^2-h^2}$	$\sqrt{4l^2-h^2}+\frac{h}{2}$
x_A	$-\sqrt{4l^2-h^2}$	$-\sqrt{4l^2-h^2}-\frac{h}{2}$

Angular velocity of the rod:
$v_B = \omega_{AB}\sqrt{4l^2-(\frac{h}{2})^2} => \omega_{AB} = \frac{v_B}{\sqrt{4l^2-(\frac{h}{2})^2}}$
$v_B = \omega_{BC}\sqrt{4l^2-(\frac{h}{2})^2} => \omega_{BC} = \frac{v_B}{\sqrt{4l^2-(\frac{h}{2})^2}}$ **Work:**
$A = mg\frac{h}{4} + mg\frac{h}{4} = mg\frac{h}{2}$ **Solution:**
$T_{AB} + T_{BC} = A$
$\frac{1}{2}I(\frac{v_B}{\sqrt{4l^2-(\frac{h}{2})^2}})^2 + \frac{1}{2}I(\frac{v_B}{\sqrt{4l^2-(\frac{h}{2})^2}})^2 = mg\frac{h}{2}$
$v_B = \frac{1}{2}\sqrt{\frac{gh(16l^2-h^2}{2(l^2+p^2)}}$
We came to the same answers as we seen at the task, so we can say we got it right

2 Task 2

2.1 Tools

Overleaf, Google Collab

2.2 Link to simulation:

Click here!

2.3 Task description

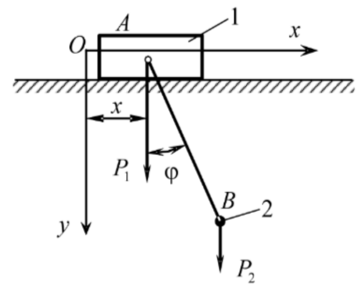
Task 2 (Coding)

System description

You have a a cart pole. Body 1 is a slider, mass m_1, it moves without friction.

AB is a massless rod with length l. Body 2 with mass m_2 is connected to AB in point B.

It's a 2 DoF system. You should take x and ϕ as a representation of this system. The origin of each coordinate should be the same as on the picture.



Task 2

Initial conditions:

1. $x = 0,\ \phi = 10°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$
2. $x = 0.5,\ \phi = 45°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$
3. $x = 0.5,\ \phi = -135°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$

Parameters: $m_1 = 5\,kg,\ m_2 = 1\,kg,\ l = 1\,m.$

Task 2 (Coding)

Tasks description

You should solve this problem using:

1. **Newton-Euler** method;
2. Model-oriented design applications (*SimInTech*, or MATLAB Simulink).

Tasks

1. To derive a differential equation of the motion, using **Newton-Euler** approach.
2. To create plots $x(t),\ \phi(t), \dot{x}(t), \dot{\phi}(t)$.
3. To make a simulation of this system. Show velocities and accelerations for 1, 2 bodies (coding approach).

Artifacts

1. Report in *.pdf* or in *.md*.
2. For **Newton-Euler** method — code, GIFs, plots.
3. For **SimInTech** — *.prt*, for **Simulink** — *.slx* file which contains a description of the system, GIFs, plots.

2.4 Task explanation

Let's start from deriving the Newton-Euler equations for this system:

$$\begin{cases} O_x : m_1\ddot{x} = T \cdot \sin(\phi) \\ O_y : 0 = m_1 g - N + t\cos(\phi) \end{cases}$$

$$
\begin{cases}
O_x : m_2(-l\sin(\phi)\dot{\phi}^2 + l\cos(\phi)\ddot{\phi} + \ddot{x}) = -T\sin(\phi) \\
O_y : m_2(-l\sin(\phi)\ddot{\phi} - l\cos(\phi)\dot{\phi}^2) = -T\cos(\phi) + m_2 g
\end{cases}
$$

$$
\begin{aligned}
\ddot{\phi} = &-\frac{g\cdot m_1\sin(\phi(t))}{l\cdot m_1\sin(\phi(t))^2 + l\cdot m_1\cos(\phi(t))^2 + l\cdot m_2\sin(\phi(t))^2} - \\
&\frac{g\cdot m_2\sin(\phi(t))}{l\cdot m_1\sin(\phi(t))^2 + l\cdot m_1\cos(\phi(t))^2 + l\cdot m_2\sin(\phi(t))^2} - \\
&\frac{l\cdot m_2\sin(\phi(t))\cdot\cos(\phi(t))\cdot(\dot{\phi})^2}{l\cdot m_1\sin(\phi(t))^2 + l\cdot m_1\cos(\phi(t))^2 + l\cdot m_2\sin(\phi(t))^2}
\end{aligned}
$$

$$
\begin{aligned}
\ddot{x} = &\frac{g\cdot m_2\sin(\phi(t))\cdot\cos(\phi(t))}{m_1\sin(\phi(t))^2 + m_1\cos(\phi(t))^2 + m_2\sin(\phi(t))^2} + \\
&\frac{l\cdot m_2\sin(\phi(t))^3\cdot(\dot{\phi})^2}{m_1\sin(\phi(t))^2 + m_1\cos(\phi(t))^2 + m_2\sin(\phi(t))^2} + \\
&\frac{l\cdot m_2\sin(\phi(t))\cdot\cos(\phi(t))^2\cdot(\dot{\phi})^2}{m_1\sin(\phi(t))^2 + m_1\cos(\phi(t))^2 + m_2\sin(\phi(t))^2}
\end{aligned}
$$

Now we simply go to Python to make a simulation and create plots

I didn't have enough time to do SimInTech or Simulink so no additional files (I'll do it for myself a little bit later after exam, I think).

Students at 21.03-22.03 night: