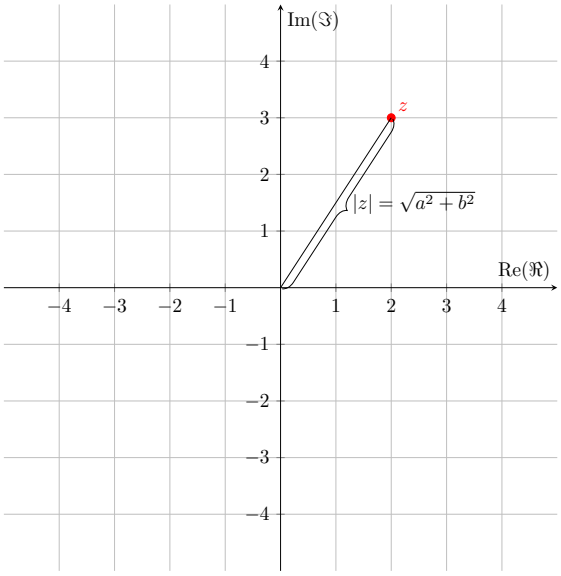
1 Arithmetik mit Komplexe Zahlen $\mathbb{C}$

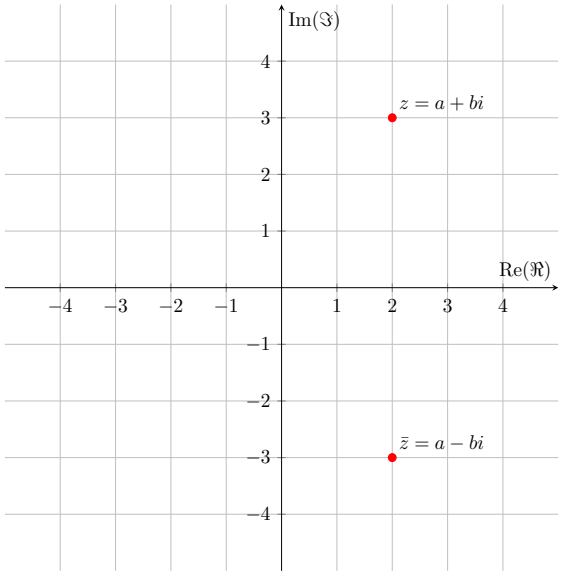
1.1 Addition, Subtraktion und Multiplikation

$$
\begin{aligned}
z_1 + z_2 &= (a + bi) + (c + di) = (a + c) + (b + d)i \\
z_1 - z_2 &= (a + bi) - (c + di) = (a - c) + (b - d)i \\
z_1 \cdot z_2 &= (a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i
\end{aligned}
$$

1.2 Betrag und Konjugation



(a) Betrag von Komplexen Zahlen



(b) Konjugation von Komplexen Zahlen

1.3 Division

$$
\begin{aligned}
z &= \frac{u}{v} \\
&= \frac{u}{v} \cdot \frac{\bar{v}}{\bar{v}} \\
&= \frac{u \cdot \bar{v}}{v \cdot \bar{v}} \\
&= \frac{(a + bi) \cdot (c + di)}{(c^2 + d^2)} \\
&= \frac{(ac + bd) + (ad - bc)i}{c^2 + d^2} \\
&= \frac{(ac + bd)}{c^2 + d^2} + \frac{(bc - ad)}{c^2 + d^2} i
\end{aligned}
$$

$$
\begin{aligned}
z &= \frac{1}{v} \\
&= \frac{1}{v} \cdot \frac{\bar{v}}{\bar{v}} \\
&= \frac{\bar{v}}{v \cdot \bar{v}} \\
&= \frac{a + bi}{(a^2 + b^2)} \\
&= \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2} i
\end{aligned}
$$

2 Aufgaben