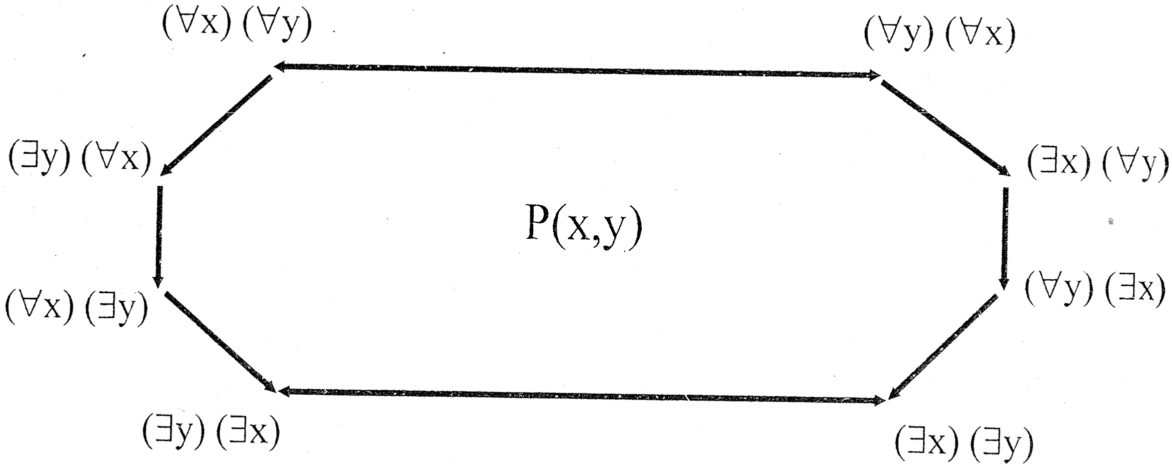
ACE
ENGINEERING ACADEMY
Estd : 1995

DISCRETE MATHEMATICS



FOR GATE, DRDO & PSUs

COMPUTER SCIENCE & ENGINEERING

REVISED EDITION – 2010

CH. PRAVALLIKA

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GATE – 2008 TOPPERS
COMPUTER SCIENCE ENGINEERING

S.No.	Name of the Student	H.T.Number	Rank	Score	Percentile
01.	Pimpalikar Rakesh Ramesh Rao	CS156551	1st	935	99.99
02.	S L N P Teja	CS154275	15th	768	99.92
03.	Kaushik B V	CS156241	27th	731	99.84
04.	K. Mohan Rao	CS151351	54th	659	99.70
05.	Varun Kumar Reddy	CS140043	54th	659	99.70
06.	Ganesh Naryana Murthy	CS154601	58th	651	99.67
07.	Vaddi Krishna Chaitanya	CS157563	71st	635	99.59
08.	Edukondalu Chappidi	CS154015	77th	630	99.57
09.	Siva Reddy Indela	CS650124	80th	625	99.50
10.	G. Sri Krishna	CS151407	83rd	622	99.53
11.	Kuriakose Mathew	CS154465	109th	604	99.39
12.	Ladha Akhilesh Yatiraj	CS151609	113th	601	99.38
13.	C. Guru Sreekanth	CS715337	114th	598	99.37
14.	Kiran T.V.S	CS156505	115th	596	99.34
15.	Karthik V	CS717107	123rd	590	99.29
16.	Ponaka Vishnu Prateek	CS709028	132nd	585	99.25
17.	Palvali Teja B	CS157349	145th	574	99.14
18.	P. Shekhar Reddy	CS156091	145th	574	99.14
19.	K. Bala Murali Krishna	CS736175	145th	574	99.14
20.	Harish Kumar	CS689229	160th	569	99.09
21.	A. Harish Kumar	CS689229	160th	569	99.09
22.	Maulica Harish	CS157049	167th	567	99.05
23.	Iffat Jahan Afreen	CS155385	174th	564	99.03
24.	Mahesh V	CS157439	178th	561	98.99
25.	Anil Shukla	CS156439	202nd	551	98.87
26.	Naga Jyothi Panidrapu	CS154353	207th	548	98.82
27.	R.S.S.K. Anil	CS707541	216th	545	98.79
28.	K. Parikshith	CS154113	230th	540	98.71
29.	Amarnath Bargade Santosh	CS154249	237th	537	98.68
30.	Kokkula Samrat	CS151069	242nd	535	98.65
31.	Mahale Rahul Machindra	CS157587	254th	529	98.56
32.	Pavan Agrawal	CS157305	271st	524	98.48
33.	N Kalyan	CS157419	304th	514	98.30
34.	D Sowmya	CS156537	310th	511	98.26
35.	A Abhinav Reddy	CS157593	334th	503	98.12
36.	Subhan Alisha Sheik	CS634153	334th	503	98.12
37.	Pandu Goud Malkaapuram	CS154027	334th	500	98.06
38.	Kundurthy Maitreya	CS154579	354th	498	98.00
39.	Dhanya V	CS151255	366th	495	97.90
40.	Mudi Reddy Jagadeesh Babu	CS151293	383rd	492	97.82
41.	Vara Kalyan Kumar Maddi	CS707085	398th	490	97.76
42.	M. Varakalyan Kumar	CS707085	398th	490	97.76
43.	Gadhamsetty Thrivikram	CS703225	409th	487	97.68
44.	Venkatesh Velaga	CS156571	424th	484	97.24
45.	Venkatesh Avula	CS156493	431st	482	97.58

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GATE – 2007 TOPPERS

COMPUTER SCIENCE ENGINEERING

Sl.No.	Name of the Student	H.T.Number	Rank	Score
01.	B. BuchiReddy	CS 152467	01	882
02.	Hari Prasad Arisetty	CS 156617	07	812
03.	Senjalia Mayur Keshubhai	CS 156231	10	800
04.	M. Abhirama	CS 153539	11	793
05.	T. Vasu Babu	CS 153379	23	762
06.	Sai Krishna.B	CS 153387	27	747
07.	Tanaji Raghav	CS 156011	29	745
08.	I. Satish Reddy	CS 151857	31	738
09.	R. Prashanth Reddy	CS 156775	40	726
10.	Chaitanya.N	CS 156761	44	717
11.	K. Sunil	CS 165075	49	713
12.	Dupukuntla Rajesh	CS 151021	63	698
13.	Poornachandra Bharath.U	CS 683349	66	694
14.	Singre Pawan Kishor	CS 151313	72	683
15.	Ravindrababu Ravula	CS 706060	86	669
16.	Lakshmi Narayana Rajavolu	CS 151831	89	664
17.	Waghmare Raj Kumar Babasaheb	CS 156303	91	662
18.	M. Aparna	CS 152029	96	654
19.	Padariya Nilesh Shivlal	CS 152457	98	652

ACE ENGINEERING ACADEMY

GATE – 2008 TOPPERS

INFORMATION TECHNOLOGY

Sl.No.	Name of the Student	H.T.Number	Rank	Score
01	Sourav Nandy	IT 675354	34	556
02	R. Sowmya	IT 165288	37	553
03	A. Rakesh Kumar	IT 165584	43	538
04	N. Sandeep	IT 165608	54	520
05	Himanshu Maurya	IT 644160	96	490

ACE ENGINEERING ACADEMY

COMPUTER SCIENCE

GATE – SYLLABUS

BASIC MATHEMATICS

Mathematical Logic: Propositional Logic, First-order Logic;

Probability: Random variables and expectation, Conditional probability, Independent random variables, Distributions (Uniform, Normal, Exponential, Poisson, Binomial);

Discrete Mathematics: Sets, Relations, Functions, Groups, Lattice, Boolean algebra, Induction, Recurrence relations;

Combinatorics: Permutations, Combinations, Counting, Summation;

Elementary graph theory: Basic properties, Connectivity, Covering and matching, Independent Sets, Planarity; Isomorphism, Coloring, Cut Vertices & edges

Linear Algebra: Algebra of Matrices, Determinants, Systems of linear equations, Eigen values & Eigen vectors.

Numerical methods: LU decomposition for systems of linear equations; numerical solutions of non – linear algebraic equations by Secant, Bisection and Newton – Raphson Methods; Numerical integration by trapezoidal & Simpson's rules.

Calculus: Limit, Continuity & Differentiability, Mean value Theorems, Theorems of integral calculus, evaluation of definite & improper integrals, Partial derivatives, Total derivatives, Maxima & Minima.

Books Recommended:

1) Discrete Mathematical Structures - Tremblay & Manohar.
2) Discrete Mathematics for Computer Scientists & Mathematicians – Mott, Kandell & Baker.
3) Discrete Mathematics – Kenneth Rosen
4) Discrete Mathematics – C.L. Liu.
5) Fundamentals of Mathematical Statistics – Gupta & Kapoor (S. Chand & Co.)
6) Matrices – A. R. Vasishta.
7) Numerical Methods for Engg. & Scientific Computation – M.K. Jain, S.R.K. Iyengar, R.K. Jain.
8) Differential & Integral Calculus (Vol –I) - Piskunov

CONTENTS

Y.V. Gopala Krishna Murthy

Managing Director

Rules of inference (Tautological implications)

I_1	$(P \wedge Q) \Rightarrow P$	(simplification)
I_2	$(P \wedge Q) \Rightarrow Q$	
I_3	$P \Rightarrow (P \vee Q)$	(addition)
I_4	$Q \Rightarrow (P \vee Q)$	
I_5	$\sim P \Rightarrow (P \rightarrow Q)$	
I_6	$Q \Rightarrow (P \rightarrow Q)$	
I_7	$\sim(P \rightarrow Q) \Rightarrow P$	
I_8	$\sim(P \rightarrow Q) \Rightarrow \sim Q$	
I_9	$P, Q \Rightarrow P \wedge Q$	
I_{10}	$(\sim P, P \vee Q) \Rightarrow Q$	(disjunctive syllogism)
I_{11}	$(P, P \rightarrow Q) \Rightarrow Q$	(modus ponens)
I_{12}	$(\sim Q, P \rightarrow Q) \Rightarrow \sim P$	(modus tollens)
I_{13}	$(P \rightarrow Q, Q \rightarrow R) \Rightarrow (P \rightarrow R)$	(hypothetical syllogism)
I_{14}	$(P \vee Q, P \rightarrow R, Q \rightarrow R) \Rightarrow R$	(dilemma)
I_{15}	$(P \vee Q, P \rightarrow R, Q \rightarrow S) \Rightarrow R \vee S$	(constructive dilemma)
I_{16}	$(P \rightarrow R, Q \rightarrow S, \sim R \vee \sim S) \Rightarrow (\sim P \vee \sim Q)$	(destructive dilemma)
I_{17}	$(\sim(P \wedge Q), P) \Rightarrow \sim Q$	(conjunctive syllogism)

EQUIVALENCES

E_1	$\sim(\sim P) \Leftrightarrow P$	(double negation)
E_2	$P \wedge Q \Leftrightarrow Q \wedge P$	(commutative laws)
E_3	$P \vee Q \Leftrightarrow Q \vee P$	
E_4	$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$	(associative laws)
E_5	$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$	
E_6	$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$	(distributive laws)
E_7	$P \wedge (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$	
E_8	$\sim(P \wedge Q) \Leftrightarrow (\sim P \vee \sim Q)$	(De Morgan's law)
E_9	$\sim(P \vee Q) \Leftrightarrow (\sim P \wedge \sim Q)$	
E_{10}	$P \vee P \Leftrightarrow P$	
E_{11}	$P \wedge P \Leftrightarrow P$	
E_{12}	$R \vee (P \wedge \sim P) \Leftrightarrow R$	
E_{13}	$R \wedge (P \vee \sim P) \Leftrightarrow R$	
E_{14}	$R \vee (P \vee \sim P) \Leftrightarrow T$	
E_{15}	$R \wedge (P \wedge \sim P) \Leftrightarrow F$	
E_{16}	$(P \rightarrow Q) \Leftrightarrow (\sim P \vee Q)$	
E_{17}	$\sim(P \rightarrow Q) \Leftrightarrow (P \wedge \sim Q)$	
E_{18}	$(P \rightarrow Q) \Leftrightarrow (\sim Q \rightarrow \sim P)$	
E_{19}	$P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$	
E_{20}	$\sim(P \leftrightarrow Q) \Leftrightarrow (P \leftrightarrow \sim Q)$	
E_{21}	$P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$	

E_{22}	$(P \rightleftarrows Q) \Leftrightarrow (P \wedge Q) \vee (\sim P \wedge \sim Q)$
E_{23}	$P \vee (P \wedge Q) \Leftrightarrow P$
E_{24}	$P \wedge (P \vee Q) \Leftrightarrow P$
E_{25}	$P \vee \sim P \Leftrightarrow T$
E_{26}	$P \wedge \sim P \Leftrightarrow F$
E_{27}	$P \vee F \Leftrightarrow P$
E_{28}	$P \vee T \Leftrightarrow T$
E_{29}	$P \wedge T \Leftrightarrow P$
E_{30}	$P \wedge F \Leftrightarrow F$

OBJECTIVES

1) If P and Q are propositions then $((P \vee Q) \vee \sim P)$
   a) is a tautology b) is a contradiction
   c) is a contingency d) is equivalent to Q

2) The prepositional function $(\sim(P \vee Q) \vee (\sim P \wedge Q) \vee P)$ is
   a) a tautology b) a contradiction c) a contingency d) $\Leftrightarrow P$

3) $\sim(P \vee (\sim P \wedge Q)) \Leftrightarrow$ (is equivalent to)
   a) $P \vee Q$ b) $P \wedge Q$ c) $\sim P \vee \sim Q$ d) $\sim P \wedge \sim Q$

4) The simplest form of $((P \rightarrow Q) \rightleftarrows (\sim P \vee Q)) \wedge R$ is
   a) T b) F c) R d) $P \rightarrow R$

$\sim(\sim P \vee Q)$
$P \wedge \sim Q$

5) $\sim(P \rightarrow Q) \Leftrightarrow$ (is equivalent to)
   a) $\sim P \vee Q$ b) $P \wedge \sim Q$ c) $P \vee Q$ d) $P \wedge Q$

6) $(P \rightarrow (Q \rightarrow R))$ is equivalent to
   a) $(P \wedge Q) \rightarrow \sim R$ b) $(P \vee Q) \rightarrow R$ c) $(P \vee Q) \rightarrow \sim R$ d) $(P \wedge Q) \rightarrow R$

7) $((P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R)) \Rightarrow$
   a) P b) Q c) R d) T

$\sim P \vee (Q \rightarrow P)$

8) Consider the following propositional functions S_1 and S_2
   S_1: $(P \vee \sim(P \wedge Q))$
   S_2: $((P \wedge Q) \wedge \sim(P \wedge Q)$ then which of the following statement is correct.
   a) S_1 is false and S_2 is true b) S_1 is true and S_2 false
   c) S_1 and S_2 both are true d) S_1 is false and S_2 is false

9) The number of non equivalent propositional functions in 'n' propositional variables $p_1, p_2 \ldots . p_n$ =
   a) 2^n b) n^2 c) 2^{n^2} d) 2^{2^n}

10) If a statement formula contains only three propositioal variables then its truth table is one of the k possible truth tables. Which of the following statement is correct.
   a) k = 16 b) k = 8 c) k = 64 d) k = 256

11) $(P \wedge (\sim P \vee Q)) \Leftrightarrow$

a) $P \wedge Q$ b) $P \vee Q$ c) T d) F

12) Which of the following is (are) not logical implication(s)

a) $(P \leftrightarrow Q) \Rightarrow (P \to Q)$
b) $(P \wedge Q) \Rightarrow (P \leftrightarrow Q)$
c) $(P \leftrightarrow Q) \Rightarrow (P \to \sim Q)$
d) $(P \leftrightarrow \sim Q) \Rightarrow (P \to Q)$

13) Which of the following is not a tautology

a) $(P \wedge Q) \to (P \vee Q)$
b) $\sim P \to (P \to Q)$
c) $(P \wedge Q) \to (P \to Q)$
d) $P \to (P \wedge Q)$

14) Which of the following is not a tautology

a) $\sim(P \to Q) \to P$
b) $\sim(P \to Q) \to \sim Q$
c) $(\sim P \wedge (P \vee Q)) \to Q$
d) $((P \to Q) \wedge Q) \to P$

15) Which of the following propositions is a tautology ?

a) $(P \vee Q) \to P$ b) $P \vee (Q \to P)$ c) $P \vee (P \to Q)$ d) $P \to (P \to Q)$

16) Which of the following arguments is valid

a) $(R \to S, \sim S) \vdash R$
b) $(R \to S, \sim R) \vdash \sim S$
c) $(R \to S, S) \vdash R$
d) $(R \to S, \sim S) \vdash \sim R$

17) Which of the following arguments is not valid

a) $(P \to Q, Q \to R, P) \vdash R$
b) $(P \to Q, Q \to R, \sim R) \vdash \sim P$
c) $(P \to Q, Q \to R, \sim P) \vdash \sim R$
d) $(P \to Q, Q \to R) \vdash (P \to R)$

18) Which of the following is false ?

a) $\sim P \Rightarrow (P \to Q)$ b) $Q \Rightarrow (P \to Q)$ c) $\sim(P \to Q) \Rightarrow P$ d) $\sim Q \Rightarrow (P \to Q)$

19) Which of the following is false ?

a) $\sim(P \to Q) \Rightarrow \sim Q$
b) $(P \wedge (P \to Q)) \Rightarrow Q$
c) $(\sim Q \wedge (P \to Q)) \Rightarrow \sim P$
d) $(\sim P \wedge (P \vee Q)) \Rightarrow \sim Q$

20) Consider the following arguments

Arg. 1 P_1 : If it rains then Erik will be sick
P_2 : It did not rain
C : $\therefore$ Erik was not sick

Arg. 2 P_1 : If it rains, Erik will be sick
P_2 : Erik was not sick
C : $\therefore$ It did not rain

Which of the following is correct

a) Arg 1 is valid and Arg 2 is not valid
b) Arg 1 is invalid and Arg 2 is valid
c) Both the arguments are valid
d) Both the arguments are invalid

21) Consider the following inferences

I_1 : $\{P \to (R \to S), (\sim R \to \sim P), P\} \vdash S$
I_2 : $\{(\sim T \to \sim R), \sim S, (T \to W), (R \vee S)\} \vdash W$

Which of the following is correct

a) I_1 is valid and I_2 is valid
b) I_1 is valid and I_2 is invalid
c) I_1 is invalid and I_2 is valid
d) I_1 and I_2 both are invalid

22) Consider the following inferences

$I_1 : (R \to S,\ P \to Q,\ R \vee P) \vdash (S \vee Q)$

$I_2 : \{\sim R \to (S \to \sim T),\ \sim R \vee W,\ \sim P \to S, \sim W\} \vdash (T \to P)$

Which of the following is correct

a) I_1 is valid and I_2 is not valid
b) I_1 is invalid and I_2 is valid
c) Both I_1 and I_2 are valid
d) Both I_1 and I_2 are invalid

23) If H_1, H_2 are premises and C is conclusion then consider following arguments

I $H_1 : P \to (Q \to R)$
$H_2 : P \wedge Q$
$C : R$

II $H_1 : \sim P$, $H_2 : P \vee Q$, $C : P \wedge Q$

Which of the following is correct

a) I is valid, II is not valid
b) I is not valid, II is valid
c) Both I and II are valid
d) Both I and II are invalid

24) In which of the following cases the premises are not inconsistent (A set of premises is said to be inconsistent, if they cannot all be true simultaneously)

a) $\{ P \to Q,\ P \to R,\ Q \to \sim R,\ P\}$
b) $\{R \vee M,\ \sim R \vee S, \sim M,\ \sim S\}$
c) $\{P \leftrightarrow Q,\ Q \to R,\ \sim R \vee S,\ \sim P \to S,\ \sim S\}$
d) $\{Q \to P,\ \sim R,\ P \to S,\ Q \vee R \}$

25) Consider the following arguments in which H_1, H_2..... are premises and C is conclusion

Argument I :

H_1 : If joe is a mathematician then he is ambitious
H_2 : If joe is an early riser, then he does not like out meal
H_3 : If joe is ambitious, then he is an early riser
C : If joe is a mathematician then he does not like out meal

Argument II :

H_1 : If cliffton does not live in France then he does not speak French
H_2 : Cliffton does not drive a datsun
H_3 : If cliffton lives in France then he rides a bicycle.
H_4 : Either cliffton speaks French or he drives a datsun
C : Cliffton rides a bicycle.

Which of the following is correct

a) Arg.I is valid, Arg.II is not valid
b) Arg.I is invalid and Arg.II is valid
c) Both the arguments are valid
d) Both the arguments are invalid.

26) $((\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)) \Leftrightarrow$

a) P b) Q c) T d) R

27) $(((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)) \Leftrightarrow$

a) P b) Q c) R d) T

28) $(\sim(P \wedge Q) \to (\sim P \vee (\sim P \vee Q))) \Leftrightarrow$

a) $(\sim P \vee Q)$ b) $(P \vee \sim Q)$ c) $(P \vee Q)$ d) $(\sim P \vee \sim Q)$

29) $((P \vee Q) \wedge (\sim P \wedge (\sim P \wedge Q))) \Leftrightarrow$

a) $(\sim P \wedge Q)$ b) $(P \wedge \sim Q)$ c) $(P \wedge Q)$ d) $(\sim P \wedge \sim Q)$

30) Which of the following is not a logical implication

a) $(P \wedge Q) \Rightarrow (P \rightarrow Q)$ b) $P \Rightarrow (Q \rightarrow P)$

c) $(P \rightarrow (Q \rightarrow R)) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$ d) $Q \Rightarrow (Q \rightarrow P)$

31) Which of the following equivalences is not correct

a) $((P \rightarrow Q) \wedge (R \rightarrow Q)) \Leftrightarrow ((P \vee R) \rightarrow Q)$ b) $((P \wedge Q) \vee (P \wedge \sim Q)) \Leftrightarrow P$

c) $\sim(P \rightarrow Q) \Leftrightarrow (P \wedge \sim Q)$ d) $(P \vee \sim Q) \Leftrightarrow (P \rightarrow Q)$

32) Which of the following equivalences is not valid

a) $\sim(P \leftrightarrow Q) \Leftrightarrow ((P \vee Q) \wedge \sim(P \wedge Q))$ b) $((P \vee Q) \wedge (P \vee \sim Q)) \Leftrightarrow P$

c) $\sim(P \leftrightarrow Q) \Leftrightarrow ((P \wedge \sim Q) \vee (\sim P \wedge Q))$ d) $(P \leftrightarrow Q) \Leftrightarrow (P \vee Q) \wedge (P \wedge Q)$

33) Which of the following implications is not valid

a) $(P \rightarrow Q) \Rightarrow (P \rightarrow (P \wedge Q))$ b) $((P \rightarrow Q) \rightarrow Q) \Rightarrow (P \vee Q)$

c) $(((P \vee \sim P) \rightarrow Q) \rightarrow ((P \vee \sim P) \rightarrow R)) \Rightarrow (Q \rightarrow R)$ d) $(P \rightarrow \sim Q) \Rightarrow (P \rightarrow Q)$

34) Consider the following

S_1 : If $A \Rightarrow B$ and $A \Rightarrow C$ then $A \Rightarrow (B \wedge C)$

S_2 : If $(H_1, H_2, \ldots, H_m$ and $P)$ implies Q, then $(H_1, H_2, \ldots H_m)$ implies $(P \rightarrow Q)$

Which of the following statement is correct

a) S_1 is true and S_2 is false b) S_1 is false and S_2 true

c) S_1 and S_2 both are true d) S_1 and S_2 both are false

35) Which of the following sets of connective is not functionally complete

a) $\{\vee, \sim\}$ b) $\{\wedge, \sim\}$ c) $\{\vee, \wedge\}$ d) none of these

36) Which of the following equivalences is false.

a) $(A \rightarrow (P \vee C)) \Leftrightarrow ((A \wedge \sim P) \rightarrow C)$ b) $((P \rightarrow C) \wedge (Q \rightarrow C)) \Leftrightarrow ((P \vee Q) \rightarrow C)$

c) $((P \wedge (Q \wedge S)) \vee (\sim P \wedge (Q \wedge S))) \Leftrightarrow (Q \wedge S)$ d) None of the above

37) Which of the following statements is false

a) $(Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q))$ is a tautology b) $(P \wedge (\sim P \wedge Q)) \Rightarrow R$

c) $(P \vee (\sim P \wedge Q)) \Leftrightarrow (P \vee Q)$ d) $(P \vee (\sim P \vee Q)) \Rightarrow R$

38) In which of the following cases the conclusion C do follow logically from the premises H_1 and H_2

a) H_1: $(P \rightarrow Q)$, H_2 : $\sim P$, C : Q b) H_1: $\sim P$, H_2: $(P \vee Q)$, C : $(P \wedge Q)$

c) H_1: $(P \rightarrow Q)$ C : $(P \rightarrow (P \wedge Q))$ d) H_1 : $(P \rightarrow Q)$, H_2 : $\sim P$, C : $\sim Q$

39) In which of the following arguments the conclusion is ***invalid***

a) $H_1 : P \rightarrow Q$, H_2 : $\sim(P \wedge Q)$, C : $\sim P$ b) H_1 : $\sim P$, H_2 : $P \leftrightarrow Q$, C : $\sim(P \wedge Q)$

c) $H_1 : (\sim P \vee Q)$, $H_2 : \sim(Q \wedge \sim R)$, $H_3 : \sim R$, C : $\sim P$ d) H_1 : $(P \vee \sim Q)$, H_2 : P, C : Q

40) In which of the following arguments the conclusion is ***invalid***

a) $H_1 : (P \vee Q)$, $H_2 : (P \rightarrow R)$, $H_3 : (Q \rightarrow R)$, C: R

b) $H_1 : P \rightarrow (Q \rightarrow R)$, $H_2 : (P \wedge Q)$, C : R

c) $H_1 : P \rightarrow (Q \rightarrow R)$, H_2 : R, C : P

d) $H_1 : P \wedge Q \wedge R$, $H_2 : (Q \rightarrow R) \rightarrow (L \vee M)$, C : $L \vee M$

41) Which of the following arguments is invalid

a) $\{C \vee D, (C \vee D) \rightarrow \sim H, \sim H \rightarrow (A \wedge \sim B), (A \wedge \sim B) \rightarrow (R \vee S) \quad \vdash R \vee S$

b) $(P \vee Q,\ P \rightarrow R,\ Q \rightarrow S) \quad \vdash (S \vee R)$

c) $(P \vee Q,\ Q \rightarrow R,\ P \rightarrow M,\ \sim M) \quad \vdash R$

d) None of the above

42) The following argument $(P \rightarrow (Q \rightarrow S), \sim R \vee P,\ Q)) \quad \vdash (R \rightarrow S)$

a) is valid only where R is true
b) is valid only where R is false
c) is valid
d) is not valid

KEY

1.a 2. a 3. d 4. c 5. b 6. d 7. c 8. b 9. d 10. d 11. b 12. c, d

13. d 14. d 15. c 16. d 17. c 18. d 19. d 20. b 21. a 22. c 23. a 24. d

25. c 26. d 27. d 28. a 29. a 30. d 31. d 32. d 33. d 34. c 35. c 36. d

37. d 38. c 39. d 40. c 41. d 42. c

PREVIOUS GATE QUESTIONS

01. The binary operation □ is defined as follows

P	Q	P □ Q
T	T	T
T	F	T
F	T	F
F	F	T

Which one of the following is equivalent to P V Q? **GATE – 2009**

(a) ~Q □ ~P (b) P □ ~Q (c) ~P □ Q (d) ~P □ ~Q

02. Consider the following well formed formulae

I $\sim\{\forall_x (P(x))\}$
II $\sim\{\exists x (P(x))\}$
III $\sim\{\exists x (\sim P(x))\}$
IV $\exists x (\sim P(x))$

Which of the above are equivalent? **GATE – 2009**

(a) I and III (b) I and IV (c) II and III (d) II and IV

03. P and Q are two propositions. Which of the following logical expressions are equivalent?

I $P \vee \sim Q$
II $\sim(\sim P \wedge Q)$
III $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$
IV $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$ **GATE – 2009**

(a) only I & II (b) only I, II & III (c) only I, II & IV (d) all of I, II, III & IV

04. A set of Boolean connectives is functionally complete if all Boolean functions can be synthesized using those. Which of the following sets of connectives is not functionally complete? **GATE – 2008**
(a) Ex-NOR (b) implication, negation (c) OR, negation (d) NAND

05. Let p, q, r and s be four primitive statements. Consider the following arguments :
$P : [(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$
$Q : [(\sim p \wedge q) \wedge (q \rightarrow (p \rightarrow r))] \rightarrow \sim r$
$R : [((q \wedge r) \rightarrow p) \wedge (\sim q \vee p)] \rightarrow r$
$S : [p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$
Which of the following arguments are valid? **GATE – 2007**
(a) P and Q only (b) P and R only (c) P and S only (d) P, Q, R and S

06. Consider the following propositional statements :
$P1 : ((A \wedge B) \rightarrow C) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$
$P2 : ((A \vee B) \rightarrow C) \equiv ((A \rightarrow C) \vee (B \rightarrow C))$
Which one of the following is true? **GATE – 2006**
(a) P1 is a tautology, but not P2
(b) P2 is a tautology, but not P1
(c) P1 and P2 are both tautologies
(d) Both P1 and P2 are not tautologies

07. A logical binary relation * is defined as follows

A	B	A * B
True	True	True
True	False	True
False	True	False
False	False	True

Let ~ be the unary negation operator, with higher precedence than *. Which one of the following is equivalent is $A \wedge B$? **GATE - 2006**
(a) $(\sim A * B)$ (b) $\sim (A * \sim B)$ (c) $\sim(\sim A * \sim B)$ (d) $\sim(\sim A * B)$

08. The following propositional statement $(P \rightarrow (Q \vee R)) \rightarrow ((P \wedge Q) \rightarrow R)$ is **GATE – 2004**
(a) Satisfiable but not valid
(b) Valid
(c) a contradiction
(d) none of the above

09. Consider the following well formed formulae in propositional logic
$F_1 : P \rightarrow \sim P$ $F_2 : (P \rightarrow \sim P) \vee (\sim P \rightarrow P)$
Which of the following is correct? **GATE – 2001**
(a) F_1 is satisfiable, F_2 is valid
(b) F_1 is unsatisfiable, F_2 is satisfiable
(c) F_1 is unsatisfiable, F_2 is valid
(d) F_1 and F_2 are both unsatisfiable

10. Let a, b, c, d be propositions. Assume that the equivalences $a \leftrightarrow (b \vee \sim b)$ and $b \leftrightarrow c$ hold. Then the truth value of the formulae $(a \wedge b) \rightarrow ((a \wedge c) \vee d)$ is always **GATE – 2000**
(a) true
(b) false
(c) same as truth value of b
(d) same as truth value of d

11. Which of the following is false? **GATE – 1996**
(a) $((x \rightarrow y) \wedge x) \rightarrow y$
(b) $((\sim x \rightarrow y) \wedge (\sim x \wedge \sim y)) \rightarrow x$
(c) $(x \rightarrow (x \vee y))$
(d) $((x \vee y) \leftrightarrow (\sim x \rightarrow \sim y))$

Key:

01. b 02. b 03. b 04. a 05. c 06. d 07. d 08. a 09.a 10.a 11. d

SENTENCE	ABBREVIATED MEANING
$\forall x, F(x)$	all true
$\exists x,. F(x)$	at least one true
$\sim[\exists x, F(x)]$	none true"
$\forall x, [\sim F(x)]$	all false
$\exists x, [\sim F(x)]$	at least one false
$\sim\{\exists x, [\sim F(x)]\}$	none false
$\sim\{\forall x, [F(x)]\}$	not all true
$\sim\{\forall x, [\sim F(x)]\}$	not all false

" all true"	$\forall x, F(x) \equiv \sim[\exists x, \sim F(x)]$	" none false "
" all false "	$\{\forall x, [\sim F(x)]\} \equiv \{\sim[\exists x, F(x)]\}$	" none true "
" not all true "	$\{\sim[\forall x, F(x)]\} \equiv \{\exists x, [\sim F(x)\}$	" at least one false "
" not all false "	$\{\sim[\forall x, \sim F(x)]\} \equiv \exists x, F(x)$	" at least one true "

STATEMENT		NEGATION	
" all true"	$\forall x, F(x)$	$\exists x, [\sim F(x)]$	"at least one false "
" at least one false"	$\exists x, [\sim F(x)]\}$	$\forall x, F(x)$	" all true "
" all false "	$\forall x, [\sim F(x)]$	$\exists x, F(x)$	" at least one true "
" at least one true "	$\exists x, F(x)$	$\forall x, [\sim F(x)]$	" all false "

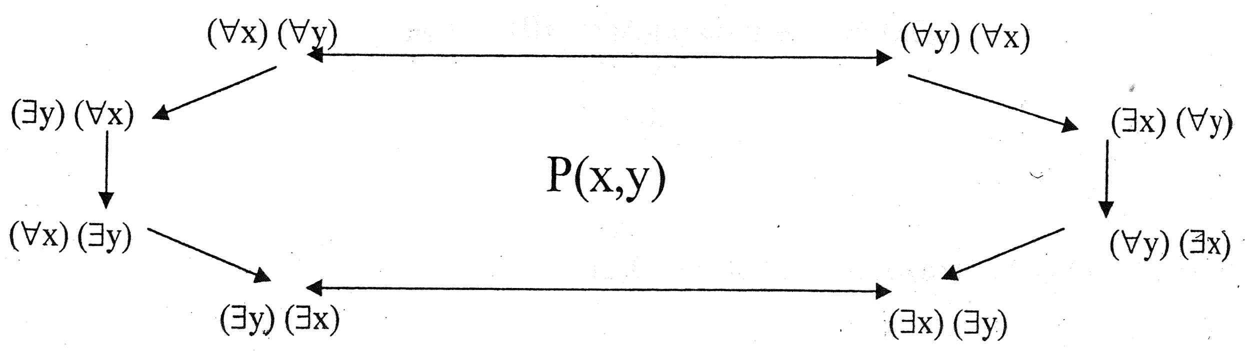
We see that *to form the* negation of a statement involving one quantifier we need only change the quantifier from universal to existential, or from existential to universal, the negate the statement which it quantifies.

Sentences with Multiple quantifiers :

In general, if P (x,y) is any predicate involving the two variables x and y, then the following possibilities exist:

$(\forall x)(\forall y)\, P(x,y)$	$(\forall x)(\exists y)\, P(x,y)$
$(\exists x)(\forall y)\, P(x,y)$	$(\exists x)(\exists y)\, P(x,y)$
$(\forall y)(\forall x)\, P(x,y)$	$(\exists y)(\forall x)\, P(x,y)$
$(\forall y)(\exists x)\, P(x,y)$	$(\exists y)(\exists x)\, P(x,y)$

There are logical relationships between sentences with two quantifiers if the same predicate is involved in each sentence. We depict these relationships in the following diagram:



RULES FOR INFERENCE FOR QUANTIFIED PROPOSITIONS :

"*Universal specification*": If a statement of the form $\forall x, P(x)$ is assumed to be true then the universal quantifier can be dropped to obtain P(c) is true for an arbitrary object c in the universe. This rule may be represented as $\dfrac{\forall x, P(x)}{\therefore P(c) \text{ for all } c.}$

Universal generalization: If a statement P(c) is true for each element c of the universe, then the universal quantifier may be prefixed to obtain $\forall x, P(x)$. In symbols, this rule is

$$\frac{P(c) \text{ for all } c}{\therefore \forall x, P(x)}$$

This rule holds provided we know P(c) is true for each element c in the universe.

Existential specification: If $\exists x, P(x)$ is assumed to be true, then there is an element c in the universe such that P(c) is true. This rule takes the form

$$\frac{\exists x, P(x)}{\therefore P(c) \text{ for some } c}$$

Existential generalization:. If P(c) is true for some element c in the universe, then $\exists x, P(x)$ is true. In symbols, we have $\dfrac{P(c) \text{ for some } c}{\therefore \exists x, P(x)}$

Generally speaking, in order to draw conclusions from quantified premises, we need to remove quantifiers properly, argue with the resulting propositions, and then properly prefix the correct quantifiers.

$(\exists x)(A(x) \vee B(x)) \Leftrightarrow (\exists x)A(x) \vee (\exists x)B(x)$	E_{23}
$(x)(A(x) \wedge B(x)) \Leftrightarrow (x)A(x) \wedge (x)B(x)$	E_{24}
$\sim(\exists x) A(x) \Leftrightarrow (x) \sim A(x)$	E_{25}
$\sim(x)(A(x) \Leftrightarrow (\exists x) \sim A(x)$	E_{26}
$(x)A(x) \vee (x)B(x) \Rightarrow (x)(A(x) \vee B(x))$	I_{15}
$(\exists x)(A(x) \wedge B(x)) \Rightarrow (\exists x)A(x) \wedge (\exists x)B(x)$	I_{16}

$(x)(A \vee B(x)) \Leftrightarrow A \vee (x)B(x)$	E_{27}
$(\exists x)(A \wedge B(x)) \Leftrightarrow A \wedge (\exists x)B(x)$	E_{28}
$(x) A(x) \to B \Leftrightarrow (\exists x)(A(x) \to B)$	E_{29}
$(\exists x) A(x) \to B \Leftrightarrow (x)(A(x) \to B)$	E_{30}
$A \to (x) B(x) \Leftrightarrow (x)(A \to B(x))$	E_{31}
$A \to (\exists x) B(x) \Leftrightarrow (\exists x)(A \to B(x))$	E_{32}

1) Consider the following

S_1: $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$

S_2: $(\exists x)(P(x) \wedge (\exists x)Q(x) \Rightarrow (\exists x) P(x) \wedge Q(x)$

Which of the following statement is correct

a) S_1 is true and S_2 is false
b) S_1 is false and S_2 is true
c) S_1 is false and S_2 is false
d) S_1 is true and S_2 is true

2) Let Q(x,y) denote $x + y = 0$ where x and y are real numbers. Consider the following quantifications

$Q_1 : \exists y\ \forall x\ Q(x,,y)$ $\quad$ $Q_2 : \forall x\ \exists y\ Q(x,.y)$

Which of the following is correct

a) Q_1 is true and Q_2 is false
b) Q_1 is false and Q_2 is true
c) Q_1 is false and Q_2 is false
d) Q_1 is true and Q_2 is true

3) Which of the following is not a logical implication

a) $(\exists y)\ (\forall x)\ P(x,y) \Rightarrow (\forall x)\ (\exists y)\ P(x,y)$
b) $(\exists x)\ (\forall y)\ P(x,y) \Rightarrow (\forall y)\ (\exists x)\ P(x,y)$
c) $(\forall x)\ (\exists y)\ P(x,y) \Rightarrow (\exists y)\ (\exists x)\ P(x,y)$
d) $(\forall y)\ (\exists x)\ P(x,y) \Rightarrow (\exists x)\ (\forall y)\ P(x,y)$

4) The negation of the sentence

$(\forall x)\ (\exists y)\ [F(x,y) \to G(x,y) \vee H(x,y)]$ is

a) $(\exists x)\ (\forall y)\ [F(x,y) \wedge \sim G(x,y) \wedge \sim H(x,y)]$
b) $(\forall x)\ (\exists y)\ [F(x,y) \wedge \sim G(x,y) \wedge \sim H(x,y)]$
c) $(\exists x)\ (\forall y)\ [\sim F(x,y) \vee G(x,y) \vee H(x,y)]$
d) $(\forall x)\ (\exists y)\ [\sim F(x,y) \vee G(x,y) \vee H(x,y)]$

5) The negation of the sentence

$\forall x, [C(x) \wedge B(x) \to \sim P(x)]$ is

a) $\forall x, [C(x) \vee B(x) \vee P(x)]$
b) $\exists x, [C(x) \wedge B(x) \wedge P(x)]$
c) $\forall x, [C(x) \wedge B(x) \vee P(x)]$
d) $\exists x, [C(x) \vee B(x) \vee P(x)]$

6) The sentence $\forall x, [\ B(x) \to I(x)\ ]$ is equivalent to

a) $\exists x, [B(x) \wedge \sim I(x)]$
b) $\exists x, [\sim B(x) \wedge I(x)]$
c) $\sim[\exists x, \{B(x) \wedge \sim I(x)\}\ ]$
d) $\sim[\exists x, \{\sim B(x) \wedge I(x)\}]$

7) The sentence $\exists x, [N(x) \wedge \sim R(x)]$ is equivalent to

a) $\forall x, [N(x) \to Rx)]$
b) $\sim[\forall x[N(x) \to R(x)]]$
c) $\forall(x)[\ \sim N(x) \to R(x)]$
d) $\sim[\forall x[R(x) \to N(x)]]$

8) Determine the truth value of each of the following statements if the universe of discourse of each variable is the set of all integers

$S_1: \forall n\ \exists m\ (n + m = 5)$ $\quad$ $S_2: \exists\ n\ \forall m\ (nm = m)$

a) S_1 is true and S_2 is false
b) S_1 is false and S_2 is true
c) S_1 and S_2 are both false
d) S_1 and S_2 are both true

9) Consider $\sim \forall x[\exists y \forall z\ P(x,y,z)\ \wedge \exists z \forall y\ P(x,y,z)]$

If we rewrite the above statement so that negation appear only within predicates (i.e.. so that no negation is out side a quantifier or an expression involving logical connectives) then it is equivalent to

a) $\exists x[\forall y\ \exists z \sim P(x,y,z) \vee \forall z\ \exists y \sim P(x,y,z)]$
b) $\forall x[\exists y\ \exists z \sim P(x,y,z) \vee \exists z\ \forall y \sim P(x,y,z)]$
c) $\forall x[\exists y\ \exists z \sim P(x,y,z) \wedge \exists z\ \forall y \sim P(x,y,z)]$
d) $\exists x[\forall y\ \exists z \sim P(x,y,z) \wedge \forall z\ \exists y \sim P(x,y,z)]$

10) Consider the statements

$S_1: \forall x\ P(x) \vee \forall x\ Q(x) \Leftrightarrow \forall x\ (P(x) \vee Q(x))$

$S_2: \exists x\ P(x) \wedge \exists x\ Q(x) \Leftrightarrow \exists x\ (P(x) \wedge Q(x))$

Which of the following is correct

a) S_1 is true and S_2 is false
b) S_1 is false and S_2 is true
c) S_1 and S_2 both are false
d) S_1 and S_2 both are true

11) Consider the following statements

S_1: $\forall x\, P(x) \wedge \exists x\, Q(x) \Leftrightarrow \forall x\, \exists y\, (P(x) \wedge Q(y))$
S_2: $\forall x\, P(x) \vee \exists x\, Q(x) \Leftrightarrow \forall x\, \exists y\, (P(x) \vee Q(y))$

a) S_1 is true and S_2 is false b) S_1 is false and S_2 is true
c) S_1 is true and S_2 is true d) S_1 is false and S_2 is false

12) Consider the following statements

S_1: $\exists x\, (P(x) \rightarrow Q(x)) \Leftrightarrow \forall x\, P(x) \rightarrow \exists x\, Q(x)$
S_2: $[\exists x\, P(x) \rightarrow \forall x\, Q(x)] \Leftrightarrow \forall x\, [P(x) \rightarrow Q(x)]$

Which of the following statements is correct

a) S1 is true and S2 is false b) S1 is false and S2 is true
c) S1 is true and S2 is true d) S1 is false and S2 is false

13) Consider the following Arguments

Arguments 1:

H_1 : All clear explanations are satisfactory.
H_2 : Some excuses are unsatisfactory
C : Some excuses are not clear explanations.

Argument 2:

All men are fallible.
All kings are men.
There fore, all kings are fallible.

Which of the following is correct

a) Argument 1 is valid and Argument 2 is not valid
b) Argument 1 is invalid and Argument 2 is valid
c) Both the arguments are valid
d) Both the arguments are invalid

14) Consider the following arguments :

Argument 1 :

H_1) Babies are illogical. H_2) No body is despised who can manage a crocodile.
H_3) Illogical persons are despised. **C:** Babies can not manage crocodiles.

Argument 2:

H_1) Every living thing is a plant or animal.
H_2) David's dog is alive and it is not a plant.
H_3) All animals have hearts.
C) David's dog has a heart.

Which of the following is true ?

a) Argument 1 is valid and argument 2 is invalid.
b) Argument 1 is invalid and argument 2 is valid.
c) Both the arguments are valid.
d) Both the arguments are invalid.

15) Let Q(x,y,z) be the statement 'x + y = z'. What are the truth values of the statements given below :

$S_1 : \forall_x \forall_y \exists_z\, Q(x,y,z)$ $S_2 : \exists_z \forall_x \forall_y\, Q(x,y,z)$

Where x, y, z are real numbers.

a) S_1 is true S_2 is false. b) S_1 is false S_2 is true.
c) S_1 is false S_2 is false. d) S_1 is true S_2 is true.

16) Let $L(x,y)$ be the statement "x loves y" where the universe of discourse for both x and y is the set of all people in the world.

Consider the statement

"There is some body whom no one loves"
which of the following represent the above statement.

a) $\forall x\, \exists y\, \sim L(x,y)$ b) $\exists x\, \forall y\, \sim L(y, x)$
c) $\sim(\forall x\, \exists y\, \sim L(x,y))$ d) $\sim(\exists x\, \forall y\, \sim L(x,y))$

17) Which of the following conclusions is validly derivable from the premises given

a) $\exists x\, (P(x) \wedge Q(x))$ C: $\forall x\, P(x)$
b) $\exists x\, P(x), \exists x\, Q(x)$ C: $\forall x\, (P(x) \wedge Q(x))$
c) $\forall x\, (P(x) \rightarrow Q(x)), \exists y\, P(y)$ C: $\exists z\, Q(z)$
d) $\forall x\, (P(x) \rightarrow Q(x)), \sim Q(a)$ C: $\forall x\, \sim P(x)$

18) Consider the following statements

S_1: $\forall_x (P(x) \vee Q(x)) \Rightarrow (\forall x) P(x) \vee (\exists x) Q(x)$
S_2: $\sim P(a,b)$ follows logically from $\sim W(a,b)$ and $\forall x\, \forall y\, (P(x,y) \rightarrow W(x,y))$

Which of the following is correct

a) S_1 is true and S_2 is false b) S_1 is false and S_2 is true
c) S_1 is true and S_2 is true d) S_1 is false and S_2 is false

19) Which of the following equivalences is false

a) $\exists x\, (A(x) \rightarrow B(x)) \Leftrightarrow (\forall x)\, A(x) \rightarrow (\exists x)\, B(x)$
b) $\exists x\, A(x) \rightarrow (\forall x)B(x) \Leftrightarrow (\forall x)(A(x) \rightarrow B(x))$
c) $\exists x\, (A(x) \vee B(x)) \Leftrightarrow (\exists x)\, A(x) \vee \exists x\, B(x)$
d) $\exists x\, (A(x) \wedge B(x)) \Leftrightarrow (\exists x)\, A(x) \wedge \exists x\, B(x)$

20) Which of the following is not valid

a) $\forall x\, (A(x) \wedge B(x)) \Leftrightarrow \forall x\, A(x) \wedge \forall x\, B(x)$ b) $\sim(\exists x)\, A(x) \Leftrightarrow \forall x\, \sim A(x)$
c) $\sim\forall x\, A(x) \Leftrightarrow \exists x\, \sim A(x)$ d) $\forall x\, A(x) \vee \forall x\, B(x)) \Leftrightarrow \forall x\, (A(x) \vee B(x))$

KEY

1. a 2. b 3. d 4. a 5. b 6. c 7. b 8. d 9. a 10. c 11. c 12. a 13. c 14. c 15. a

16. b 17. c 18. c 19. d 20. d

PREVIOUS GATE QUESTIONS

01. Suppose the predicate $F(x,y,t)$ is used to represent the statement that person x can fool person y at time t. Which one of the statements below expresses best the meaning of the formula $\forall x\, \exists y\, \exists t\, (\sim F(x,y,t))$? (GATE-2010)

a) Every one can fool some person at some time.
b) No one can fool everone all the time
c) Everone cannot fool some person all the time.
d) No one can fool some person at some time.

02. Which one of the following is the most appropriate logical formula to represent the statement "Gold and silver ornaments are precious" The following notations are used :

G(x) : x is a gold ornament
S(x) : x is a silver ornament
P(x) : x is precious **GATE – 2009**

(a) $\forall x\, ((P(x) \rightarrow (G(x) \wedge S(x)))$ (b) $\forall x\, ((G(x) \wedge S(x)) \rightarrow P(x))$
(c) $\exists x\, ((G(x) \wedge S(x)) \rightarrow P(x))$ (d) $\forall x\, ((G(x) \vee S(x)) \rightarrow P(x))$

03. Let fsa and pda be two predicates such that fsa(x) means x is a finite state automaton, and pda(x) means that x is a pushdown automaton. Let equivalent be another predicate such that equivalent of (a, b) means a and b are equivalent. Which of the following first order logic statements represent the following "Each finites state automaton has an equivalent pushdown automaton" **GATE – 2008**

(a) $(\forall x\ fsa(x)) \Rightarrow (\exists y\ \ pda(y) \wedge \text{equivalent}\ (x,y))$
(b) $\sim\forall y\ (\exists x\ \ fsa(x) \Rightarrow pda(y)\ \ \wedge \text{equivalent}\ (x,y))$
(c) $\forall y\ \exists x\ (fsa(x) \wedge pda(y) \wedge \text{equivalent}\ (x,y))$
(d) $\forall x\ \exists y\ (fsa(y) \wedge pda(x) \wedge \text{equivalent}\ (x,y))$

04. Which of the following is the negation of $[\forall x\ \alpha \rightarrow (\exists y, \beta \rightarrow \forall u, \exists v, \gamma)]$? **GATE – 2008**

(a) $[\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\exists u, \forall v, \gamma))]$ (b) $[\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow \exists u, \forall v, \sim\gamma)]$
(c) $[\forall x, \sim\alpha \rightarrow (\exists y, \sim\beta \rightarrow (\forall u, \exists v, \sim\gamma))]$ (d) $[\exists x, \alpha \wedge (\forall y, \beta \wedge (\exists u, \forall v, \sim\gamma))]$

05. Which of the following first order logic formulae is valid? **GATE – 2007**

(a) $\forall x\ [P(x) \rightarrow Q(x)] \Rightarrow [(\forall x\ P(x)) \rightarrow (\forall x Q(x))]$
(b) $\exists x\ [P(x) \vee Q(x)] \Rightarrow [(\exists x\ P(x)) \rightarrow (\exists x\ Q(x))]$
(c) $\exists x\ [P(x) \wedge Q(x)] \Leftrightarrow [(\exists x\ P(x)) \wedge (\exists x\ Q(x))]$
(d) $\forall x \exists y\ \ P(x, y)\ \Rightarrow \exists y\ \forall x\ \ P(x, y)$

06. Let P(x) and Q(x) be arbitrary predicates. Which of the following is always TRUE? **GATE – 2005**

(a) $[\forall x(P(x) \vee Q(x))] \Rightarrow [(\forall x\ P(x)) \vee (\forall x\ Q(x))]$
(b) $[\forall x(P(x) \rightarrow Q(x))] \Rightarrow [(\forall x\ P(x)) \rightarrow (\forall x\ Q(x))]$
(c) $[(\forall x\ P(x)) \rightarrow (\forall x\ Q(x))] \Rightarrow [\forall x\ (P(x) \rightarrow Q(x))]$
(d) $[(\forall x\ P(x)) \Leftrightarrow (\forall x\ Q(x))] \Rightarrow [\forall x\ (P(x) \Leftrightarrow Q(x))]$

07. Consider the following first order logic formula in which R is a binary relation symbol $\forall x\ \forall y\ (R(x, y) \Rightarrow R(y, x))$ the formula is **GATE – 2006**

(a) Satisfiable and valid (b) satisfiaboe and so its negation
(c) unsatisfiable but its negation is valid (d) satisfiable but its negation is unsatisfiable

08. Which one of the first order predicate calculus statements given below correctly express the following English statement? "Tigers and Lions attack if they are hungry or threatened" **GATE – 2006**

(a) $\forall x\ [(tiger(x) \wedge Lion(x)) \rightarrow \{(hungry(x) \vee threatened(x)) \rightarrow attacks\ (x)\}]$
(b) $\forall x\ [(tiger(x) \vee Lion(x)) \rightarrow \{(hungry(x) \vee threatened(x)) \wedge attacks\ (x)\}]$
(c) $\forall x\ [(tiger(x) \vee Lion(x)) \rightarrow \{attacks(x) \rightarrow (hungry(x) \vee threatened(x))\}]$
(d) $\forall x\ [(tiger(x) \vee Lion(x)) \rightarrow \{(hungry(x) \vee threatened(x)) \rightarrow attacks(x)\}]$

Key

01.d 02.d 03.c 04.d 05.a 06.d 07.b 08. d

The *probability* p of an event A has defined as follows. If A can occur in *s* ways out of a total of *n* equally likely ways then

$p = p(A) = s/n$

SAMPLE SPACE: The set S of all possible out comes of some given experiment is called the ***sample space.***

An *event* is a subset of the sample space. The empty set ϕ and S itself are events.

ϕ = Impossible event
S = Certain event

$A \cup B$ is the event that occurs if A occurs or B occurs (or both)
$A \cap B$ is the event that occurs if A occurs and B occurs

A^C, the ***complement*** of A , is the event that occurs if A does not occur

MUTUALLY EXCLUSIVE EVENTS: Two events A and B are called mutually exclusive, if they are disjoint i.e.. $A \cap B = \phi$. In other words, A and B are mutually exclusive if they cannot occur simultaneously.

Axioms Of Probability:

1. For every event A, $0 \le P(A) \le 1$ $0 \le P(A) \le 1$
2. $P(S) = 1$
3. If A and B are mutually exclusive events, then
   $P(A \cup B) = P(A \text{ or } B) = P(A) + P(B)$
4. If $A_1; A_2, \ldots\ldots A_n$ are mutually exclusive then
   $P(A_1 \cup A_2 \cup \ldots\ldots \cup A_n) = P(A_1) + P(A_2) + \ldots\ldots + P(A_n)$
5. If ϕ is the empty set, $P(\phi) = 0$
6. If A^C is the complement of an event A, then
   $P(A^C) = 1 - P(A)$
7. If $A \subset B$, then $P(A) \le P(B)$
8. If A and B are any two events then
   $P(A - B) = P(A) - P(A \cap B)$
9. ***Addition theorem of probability:***
   If A and B are any two events, then
   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
10. For any three events A, B, C
   $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$P(A) = 2P(B)$
$P(B)$

NOTE: A finite or countably infinite probability space is said to be *discrete* and an uncountable space is said to be ***non – discrete*** (continuous).

$A = 2B$
$B = 2C$

OBJECTIVES

01. Three horses A, Band C are in a race. A is twice as likely to win as B and B is twice as likely to win as C. What is the probability that B or C wins?
    a) 2/7 b) 3/7 c) 4/7 d) 6/7

02. A card is selected at random from an ordinary pack of 52 cards. Probability of selecting a spade card or a face card is
    a) 3/32 b) 23/52 c) 22/52 d) 25/52

13

B

52

03. Let two items be choosen from a lot containing 12 items of which 4 are defective. What is the probability that atleast one item is defective?
a) 19/33 b) 14/33 c) 1/11 d) 13/33

04. A number is selected at random from first 200 natural numbers. Find the probability that the number is divisible by 6 or 8?
a) 1/3 b) 1/4 c) 1/5 d)2/3

05. A point is selected at random inside a circle. Find the probability p that the point is closer to the centre of the circle than to its circumference?
a) 1/3 b) 1/4 c)1/5 d)2/3

06. Let A and B be events with $p(A) = 3/8$, $P(B) = 1/2$ and $P(A \cap B) = 1/4$ then which of the following is false.
a) $P(A^C \cup B^C) = 3/4$
b) $P(A^C \cap B^C) = 3/8$
c) $P(A \cap B^C) = 1/8$
d) $P(B \cap A^C) = 5/8$

07. Of 120 students, 60 are studying French, 50 are studying Spanish and 20 are studying French and Spanish. If a student is selected at random then which of the following is not correct.
a) Probability that the student is studying French and Spanish is 0.75.
b) Probability that the student is studying neither French nor Spanish is 0.25.
c) Probability that the student is studying Spanish but not French is 0.25.
d) Probability that the student is studying French but not Spanish is 0.3.

08. In a class of 100 students, 40 failed in mathematics, 30 failed in physics, 25 failed in chemistry, 20 failed in maths and physics, 15 failed in physics and chemistry, 10 failed in chemistry and maths, 5 failed in maths, physics and chemistry. If a student is selected at random then the probability that he passed in all three subjects is
a) 0.4 b) 0.45 c) 0.55 d) 0.65

Conditional Probability:

Let E be an arbitrary event in a sample space S with $P(E) > 0$. the Probability that an event A occurs once E has occurred or, in other words, ***Conditional Probability of 'A' given E,*** written P(A/E), is defined as follows

$$P(A/E) = \frac{P(A \cap E)}{P(E)}$$

NOTE: Let S be a finite equiprobable space with events a and E. Then

$$P(A/E) = \frac{\text{number of elements in } (A \cap E)}{\text{number of elements in } E}$$

Multiplication Theorem:

If A and B are any two events then

$$P(A \cap B) = P(A).\ P(B/A)$$
$$= P(B).\ P(A/B)$$

NOTE: If A and B are ***Independent events*** then
$P(A/B) = P(A)$ and $P(B/A) = P(B)$
$P(A \cap B) = P(A).\ P(B)$

09. Let a pair of dice be tossed. If the sum is 6, find the probability that one of the dice is a 2.
a) 1/5 b) 2/5 c) 3/5 d) 4/5

10. A man visits a couple who have two children. One of the children, a boy, comes in to the room . Find the probability p that the other is also a boy
a) 1/3 b) 2/3 c) 1/2 d) 3/4

11. Let A and B be events with P(A) = 3/8, P(B) = 5/8 and P(A $\vee$ B) = 3/4 . Find the conditional probability P(A/B)
a) 1/3 b) 2/5 c)3/4 d)1/2

12. In certain college, 25% of the students failed mathematics, 15% of the students failed in Chemistry, and 10% of the students failed in both maths and chemistry. A student is Selected at random. If he failed chemistry, what is the probability that he failed in maths?
a) 2/3 b)2/5 c) 3/5 d)1/5

13. A die is tossed. If the number appeared is odd, what is the probability that it is prime?
a) 1/3 b) 2/3 c) 3/4 d)1

14. In a certain college, 4% of the men and 1% of the women are taller than 1.8m. Further more, 60% of the students are women. Now if a student is selected at random and is taller than 1.8m, what is the probability that the student is a woman ?
a)3/11 b) 4/11 c) 5/11 d) 6/11

15. We are given three urns as follows. Urn A contains 3 red and 5 white marbles, Urn b contains 2 red and 1 white marble, Urn C contains 2 red and 3 white marbles. An urn is selected at and a marble is drawn from the urn. If the marble is red, what is the probability that it came from urn A?
a) 45/173 b) 37/165 c) 27/109 d) 39/185

16. A coin, weighted so that P(H) = 2/3 and P(T) = 1/3 is tossed. If heads appears, then a number is selected at random from the numbers 1 through 9. If tails appears, then a number is selected at random from the numbers 1 through 5. Find the probability P that an even number is selected.
a) 67/145 b) 58/135 c)74/157 d)43/142

17. A box contains three coins, two of them fair and one two headed. A coin is selected at random and tossed twice. If heads appears both times, what is the probability that the coin is two headed?
a) 2/3 b)1/3 c)3/4 d)1/2

18. Ann urn contains 3 red marbles and 7 white marbles. A marble is drawn from the urn and a marble of the colour is then put in to the urn. A second marble is drawn from the urn. If both marbles where of the same colour What is the probability P that they were both white?
a) 5/6 b)7/8 c)8/9 d)9/10

Random Variable And Expectation:

Suppose that to each point of a sample space we assign a number. We then have a function defined on the sample space. This function is called a ***random variable*** or more precisely a ***random function***. It is usually denoted by a capital letter such as X or Y.

A random variable which takes on a finite or countably infinite number of values is called a ***discrete random variable***. While one which takes on non countably infinite number of values is called a ***non – discrete or continuous random variable.***

Discrete Probability Distribution:

Let X be a discrete random variable and suppose that the possible values which it can assume are given by $x_1, x_2, \ldots\ldots$ are arranged in increasing order of magnitude. Suppose also that these values are assumed with probabilities given by

$$P(X = x_i) = f(x_i) \quad i = 1, 2, \ldots\ldots$$

Or
$$P(X = x) = f(x)$$

In general, f(x) is a probability function

If

1, $f(x) \geq 0$

2, $\sum_x f(x) = 1$ where sum is taken over all possible values of x.

CONTINUOUS PROBABILITY DISTRIBUTION

If X is a continuous distribution random variable then a function which satisfies the following requirements is called ***probability distribution or probability density function*** of X

1. $f(x) \geq 0$

2. $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$$P(a < X < b) = \int_a^b f(x)\, dx$$

MATHEMATICAL EXPECTATION:-

1. For a discrete random variable X having the possible values $x_1, x_2, \ldots\ldots\ldots, x_n$ the expectation of x is defined as

$$E(X) = x_1 \,.\, P(x_1) + x_2 \,.\, P(x_2) + \ldots.. + x_n P(x_n)$$
$$= \sum_{i=1}^{n} x_i . P(x_i)$$

2. For a continuous random variable X having density function f(x) the expectation of x is defined as

$$E(x) = \int_{-\infty}^{\infty} x . f(x)\, dx$$

3. If X is a discrete random variable having probability density function f(x), they

$$E\{g(x)\} = \sum_x [\, g(x). f(x)]$$

4. If X is a continuous random variable having p.d.f. g(x) they $E[g(x)] = \int_{-\infty}^{\infty} [g(x). f(x)]\, dx$

5. If c is any constant, then E(c X) = c. E(X)

6. If X and Y are any random variables, then E (x + y) = E(x) + E(y)

7. If X and Y are independent random variables then $E(XY) = E(X).\ E(Y)$

8. Variance & standard deviation

$Var(x) = E[(x - \mu)^2]$

S. D. of $X = \sigma x = \sqrt{var(x)}$

$Var(cx) = C^2\ Var(x)$

$\sigma^2 = E[(x - \mu)^2] = E(x^2) - [E(x)]^2$

$Var(x \pm y) = var(x) + var(y)$

19. Find the constant C such that the function

$f(x) = Cx^2,\ 0 < x < 3.$

$= 0.$ otherwise is a probability density function.

find $P(1 < x < 2)$

20. A random variable x has density function $f(x) = C/(x^2 + 1)$ where $-\infty < x < \infty$.

a) find the value of the constant C b) $P(1/3 \le x2 \le 1) = ?$

21. The density function of a random variable X is given by $f(x) = x/2 \quad 0 < x < 2$

$= 0$ otherwise

Then the mean and variance of X are

a) 4/3, 2/9 b) 2/3, 4/9 c) 4/3, 4/9 d) 2/3, 2/9

22. The expectation of discrete random variable X whose probability function is given by

$f(x) = (1/2)^x,\ x = 1, 2, 3, \ldots\ldots$ is

a) 1 b) 2 c) 3 d) 4

23. A continuous random variable X has probability density given by $f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x \le 0 \end{cases}$

then expectation of X

a) 0.1 b) 0.25 c) 0.5 d) 0.75

24. On a rainy day an umbrella salesman can earn Rs. 300, and on a fair day (no rain) he loses Rs. 60. What is his expectation if the probability for a rainy day is 0.3

a) Rs 24 b) Rs 36 c) Rs 48 d) Rs 64

25. In a lottery there are 200 prizes of Rs 5, 20 prizes of Rs. 25 and 5 prizes of Rs. 100. Assuming that 10,000 prizes tickets are to be issued and sold what is the fair price to pay for the ticket?(Or if some one purchases a lottery ticket his expectation is--------.)

a) Rs 0.2 b) Rs. 0.4 c) Rs. 0.5 d) Rs. 0.6

26. A random variable X has the following probability function

x :	0	1	2	3	4	5	6
P (x):	k	3k	5k	7k	9k	11k	13k

Then $P(3 < x \le 6) =$

a) 11/109 b) 22/49 c) 33/49 d) 44/49

27. Find the expectation of the sum of points in tossing three fair dice

a) 10 b) 10.5 c) 11 d) 11.5

28. A player tosses a fair dice. If a prime number occurs he wins that number of rupees, but if a non prime number occurs he loses that number of rupees. His expectation in rupees is
    a) 1/6 b) 1/2 c) –1/2 d) –1/6

29. The joint density function of two random variables X and Y is given by
$$f(x, y) = xy/96, \quad 0 < x < 4,\ 1 < y < 5$$
$$= 0, \quad \text{otherwise.}$$
Which of the following is false
    a) E (X) = 8/3 b) E(Y) = 31/9
    c) E(XY) = 248/27 d) E(2X + 3Y) = 47/3 e) none

30. Three machines A, B and C produce respectively 50%, 30%,and 20% of total number of items of a factory. The percentages of defective output of these machines are 3%, 4% and 5% respectively. If an item is selected at random and is found to be defective then the probability that it is produced by machine B is
    a) 15/37 b) 16/37 c) 14/34 d) 12/37

BINOMIAL DISTRIBUTION (BERNOULLI'S DISTRIBUTION)

Bernoulli trial: In an experiment if the probability will not change from one trial to the next (tossing a coin or die), such trials are called Bernoulli's trial

If p is the probability that an event will happen in any single Bernoulli's trial (called the probability of success). Then q = 1- p is the probability that the event will fail to happen (called the probability of failure)

The probability that the event will happen exactly x times in n trials (i.e., x successes and n-x failures will occur) is given by the probability function
$$f(x) = P(X = x) = C(n, x)\ p^x\ q^{n-x}$$
where the random variable X denotes the number of successes in n trials and x = 0, 1, 2,……..n.

For Binomial Distribution:

Mean = $\mu = n \cdot p$

Variance = $\sigma^2 = n\,p\,q$

S. D. = $\sigma = \sqrt{n\,p\,q}$

coefficient of skewnen = $\dfrac{q - p}{\sqrt{npq}}$

kurtosis = $3 + \dfrac{6 - 6pq}{npq}$

31. Ten coins are thrown simultaneously. Which of the following is wrong
    a) probability of getting atleast one head is 1023/1024
    b) probability of getting atmost 8 heads is 1013/1024
    c) probability of getting atleast 2 heads is 1013/1024
    d) probability of getting exactly 8 heads is 11/1024

32. Out of 2000 families with 4 children each, how many families would you expect to have atleast one boy?
    a) 1250 b) 1875 c) 1500 d) 1825

33. If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen, the number of defective bolts is less than 2
    a) 27/64 b) 81/256 c) 27/256 d) 512/625

34. The probability of getting a total of 7 atleast once in three tosses of a pair of fair dice is
a) 125/216 b) 91/216 c) 117/216 d) 99/216

35. If the probability of a defective bolt is 0.1, then the mean and standard deviation for the number of defective bolts in a total of 400 bolts are------ and -------
a) 40, 6 b) 36, 9 c) 36, 6 d) 40, 9

36. How many dice must be thrown so that there is better than even chances of getting a 6
a) 4 b) 5 c) 6 d) 7

37. Which of the following statements is true
a) The mean of the Binomial distribution is 5 and standard deviation is 3
b) For a Binomial distribution, mean is 6 and variance is 9
c) For a Binomial distribution mean is 3 and variance is 2
d) None of the above

POISSON DISTRIBUTION

Let X be a discrete random variable which can take on the values 0, 1, 2.....such that the probability of X is given by

$$f(x) = P(X = x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!} \quad x = 0, 1, 2, \ldots\ldots$$

Where λ is a given positive constant called the parameter of the distribution.

For Poisson Distribution:

Mean $= \mu = \lambda$ Variance $= \sigma^2 = \lambda$

S.D $= \sigma = \sqrt{\lambda}$

Coefficient of Skewness $= 1/\sqrt{\lambda}$ Coefficient of Kurtossis $= 3 + (1/\lambda)$

NOTE:

→ When n is large and p is small then Binomial distribution is very closely approximated by poisson distribution.

→ Poisson distribution is a limiting case of binomial distribution as $n \to \infty$ and $p \to 0$

38. If the probability that an individual suffers a bad reaction from injection of a serum is 0.001. Determine the probability that out of 2000 individuals, exactly 3 individuals suffer a bad reaction.
a) 0.12 b) 0.08 c) 0.18 d) 0.003

39. In the above problem, find the probability that more than 2 individuals will suffer a bad reaction
a) 0.823 b) 0.632 c) 0.523 d) 0.323

40. In problem (38), find the probability that atleast one individual suffer a bad reaction
a) 0.87 b) 0.64 c) 0.92 d) 0.47

41. If X follows poisson distribution such that P(X = 1) = P(X = 2) then P(X = 0) =
a) e^{-1} b) e^{-2} c) e^{-3} d) e^{-4}

42. In a certain factory of turning razor blades, there is a small chance (1/500) for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing atleast one defective blade in a consignment of 10,000 packets.
a) 9802 b) 198 c) 2 d)196

NORMAL DISTRIBUTION

Normal distribution is another limiting form of the binomial distribution under the following conditions
i) n, the number of trials is indefinitely large. ii)Neither p nor q is very small.

The normal probability (curve) density function with mean μ and standard deviation σ is given by the equation $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-(x-\mu)^2/2\sigma^2]$, $-\infty < x < \infty$

PROPERTIES:

1. The curve is bell shaped and symmetrical about the line $x = \mu$
2. Mean, Median and Mode of the distribution coincide.
3. As x increases numerically f(x) decreases rapidly.
4. The maximum probability occurs at the point $x = \mu$, and given by
   $$[P(x)]_{max} = \frac{1}{\sigma\sqrt{2\pi}}$$
5. x- axis is an asymptote to the curve.
6. Coefficient of skewness is zero.
7. Coefficient of kurtosis is 3.
8. Total area under the normal curve is unity.
9. Area property $P(\mu - \sigma < X < \mu + \sigma) = 0.6826$
   $P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9544$
   $P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$
10. Mean deviation (about mean) $\simeq (4/5)\,\sigma$
11. Standard normal distribution . If we let $Z = \frac{X-\mu}{\sigma}$
    then the mean of Z is 0 and variance is 1.
    The probability density function for Z is
    $$f(z) = \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)$$

$$P(a \le X \le b) = \int_a^b f(x)\,dx = \int_{z_1}^{z_2} f(z)\,dz$$
= Area under the standard normal curve between z_1 and z_2

$P(-1 \le Z \le 1) = 0.6827$, $P(-2 \le Z \le 2) = 0.9545$, $P(-3 \le Z \le 3) = 0.9973$

$P(-a \le Z \le a) = 2\,P(0 \le Z \le a) = 2$ (Area under the normal curve between $Z = 0$ and $Z = a$)

$P(-a \le Z \le b) = P(-a \le Z \le 0) + P(0 \le Z \le b)$
$= P(0 \le Z \le a) + P(0 \le Z \le b)$
= (Area under the normal curve between $Z = 0$ and $Z = a$)+ (Area between $Z = 0$ and $Z = b$)

$P(Z \ge a) = 0.5 - P(0 \le Z \le a)$
$= 0.5 -$ (Area between $Z = 0$ and $Z = a$)

$P(Z \le -a) = P(Z \ge a)$

$P(Z_1 \le Z \le Z_2) = P(0 \le Z \le Z_2) - P(0 \le Z \le Z_1)$

$P(-Z_1 \le Z \le -Z_2) = P(0 \le Z \le Z_1) - P(0 \le Z \le Z_2)$

$P(Z \le a) = 0.5 + P(0 \le Z \le a)$

$P(Z \ge -a) = P(Z \le a)$

43. Area under normal curve between Z = 0and Z = 1.2 is 0. 3849. Which of the following statements is false.
a) $P(Z > 1.2) = 0.1151$ b) $P(Z < 1.2) = 0.8849$
c) $P(-1.2 < Z < 1.2) = 0.7698$ d) $P(Z > -1.2) = 0.1151$

44. The mean inside diameter of a sample of 200 washers produced by a machine is 12mm and the standard deviation is 0.02mm. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 11.97 to 12.03mm. Other wise the washers are considered to be defective. Determine the percentage of non defective washers produced by the machine, assuming the diameters are normally distributed.(Area under the normal curve between Z = 0 and Z = 1.5 is 0.4332)
a) 43.32% b) 86.64% c) 93.32% d) 54.68%

45. Suppose that the temperature during june is normally distributed with mean 20^0 c and standard deviation 3.33^0. Find the probability P that the temperature is between 21.11^0c and 26.66^0c (Area under the normal curve between Z = 0 and Z = 2 is 0.4772 and between Z = 0 and Z = 0.33 is 0. 1293)
a) 0.3479 b) 0.6065 c) 0.8479 d) 0.1065

46. A die is tossed 180 times. Using normal distribution find the probability that the face 4 will turn up atleast 35 times (Area under the normal curve between Z = 0 and Z = 1 is 0. 3413)
a) 0.1587 b)0.8413 c) 0.6587 d)0.3413

47. Suppose the waist measurements of 500 boys are normally distributed with mean 66cm and standard deviation 5cm. Find the number of boys with waists ≤ 70cm (Area under the normal curve between z = 0 and Z = 0.8 is 0.2881)
a) 394 b) 288 c) 788 d) 112

48. Among 10,000random digits, find the probability P that the digit 3 appears at most 950 times.(Area under normal between Z = 0 and Z = 1.67 is 0.4525)
a) 0.4525 b) 0.9525 c) 0.91 d) 0.0475

UNIFORM (RECTANGULAR) DISTRIBUTION

A random variable X is said to have a continuous uniform distribution over an interval (a, b), if its probability density function (p. d. f) is constant (= K say) over the entire range of X. i.e., $f(x) = K, \quad a < x < b$
$= 0$ otherwise

since, total probability is always unity, we have

$$\int_a^b f(x)\,dx = 1 \Rightarrow K = 1/(b-a)$$

$$\therefore f(x) = 1/(b-a), \quad a < x < b$$
$$= 0, \quad \text{otherwise}$$

Note:-

1) a and b, $(a < b)$ are the two parameters of the uniform distribution on (a, b)
2) The distribution is also called rectangular distribution since the curve $y = f(x)$ describes a rectangle over the x-axis between $x = a$ and $x = b$
3) For a rectangular or uniform variate X in $(-a, a)$, p. d. f is given by

$$f(x) = 1/2a, \quad -a < x < a$$
$$= 0, \quad \text{otherwise.}$$

4) Mean $= \mu = (b + a)/2 = E(X)$.
5) $E(X^2) = \int_a^b x^2 f(x)\, dx = \dfrac{(b^2 + ba + a^2)}{3}$
6) Variance $= E(x^2) - [E(x)]^2 = \sigma^2 = \dfrac{(b-a)^2}{12}$
7) Moment generating function $M_X(t) = \int_a^b e^{tx}.\, f(x)\, dx = \dfrac{e^{bt} - e^{at}}{t(b-a)}$
8) Mean deviation about mean $= E(x - \mu) = (b - a)/4$

49. If a random variable X is uniformly distributed with mean 1 and variance 1/3, then $P(x < 1/2) =$
a) 1/2 b) 1/4 c) 2/3 d) 3/4

50. Subway trains on a certain line run every half hour between mid night and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait atleast twenty minutes?
a) 3/4 b) 1/3 c) 2/3 d) 3/5

51. For the rectangular distribution $f(x) = 1/2a$, $-a < x < a$ the moment generating function about the origin is
a) $\dfrac{\sinh(at)}{at}$ b) $\dfrac{\cosh(at)}{at}$ c) $\dfrac{\sin at}{at}$ d) $\dfrac{\cos at}{at}$

52. Suppose that X is uniformly distributed over $(-\alpha, \alpha)$, where $\alpha > 0$. Determine α so that $P(|X| < 1) = P(|X| > 1)$
a) 1 b) 2 c) 3 d) 4

53. In the previous problem find α so that $P(X > 1) = 1/3$
a) 1 b) 2 c) 3 d) 4

54. If X has a rectangular distribution in [0, 1] then which of the following is not true
a) mean of X is 1/2 b) variance = 1/12
c) mean deviation about mean = 1/4 d) the moment generating function is $e^t/2t$

55. If X is uniformly distributed in [-2, 2] then $P(|X - 1| \geq 1/2) =$
a) 1/4 b) 1/2 c) 3/4 d) 2/3

EXPONENTIAL DISTRIBUTION

A continuous random variable x assuming non negative values is said to have an exponential distribution with parameter $\theta > 0$, if its probability density function is given by

$f(x) = \theta . e^{-\theta x}, \quad x \geq 0$
$= 0$, otherwise

For exponential distribution,

a) mean $= E(x) = 1/\theta$

b) variance $= \sigma^2 = E(x^2) - E(x) = 1/\theta^2$

c) M G F $= M_x(t) = \Sigma\,(t/\theta)^r$

$\mu_r^1 = E(x^r) =$ coefficient of $t^r/r!$ in $M_x(t)$
$= r!\,/\,\theta^r, \quad r = 1, 2, \ldots\ldots\ldots$

d) The exponential distribution "lacks memory" i.e., if X has an exponential distribution, then for every constant $a \geq 0$, one has $P(Y \leq X / X \geq a) = P(X \leq x)$ for all x, where $Y = X - a$

56. If X has an exponential distribution with mean 1, then $P(1 < X < 2) =$
a) $\frac{e^{-2} - e^{-1}}{2}$ b) $e^{-1} - e^{-2}$ c) $\frac{e^{-1} - e^{-2}}{3}$ d) $\frac{2(e^{-1} - e^{-2})}{3}$

57. If a continuous random variable X has an exponential distribution with mean 0.5 then $P(x > 1/2) =$
a) e^{-1} b) $e^{-1/2}$ c) $(1/2)e^{-1}$ d) $(1/2)e^{-1/2}$

58. If x has a uniform distribution in [0, 1] then $y = -2 \log x$ has an exponential distribution with $\theta =$
a) 2 b) 1/2 c) –2 d) –1/2

59. If x has density function $f(x) = C.e^{-3x}, x > 0$
$= 0 \quad x \leq 0$
then $P(x < 1) =$
a) e^{-3} b) $1 - e^{-3}$ c) $(1/3)\,e^{-3}$ d) $(1 - e^{-3})/3$

KEY

1. b 2. c 3. a 4. b 5. b 6. d 7. d 8. b 9. b 10. a 11. b 12. a
13. b 14. a 15. a 16. b 17. a 18. b 19. C = 1/9, $P(1 < X < 2) = (7/27)$
20. $C = 1/\pi$, $P((1/3) \leq X^2 \leq 1) = 1/6$ 21. a 22. b 23.c 24. c 25. a 26. c 27. b
28. d 29. e 30. d 31. d 32. b 33. d 34. b 35. a 36. a 37. c 38. c 39. d
40. a 41. b 42. b 43. d 44. b 45. a 46. a 47. a 48. d 49. b 50. b 51. a
52. b 53. c 54.d 55. c 56.b 57. a 58. b 59. b

ADDITIONAL PROBLEMS

01. Let f(x) be the continuous probability function of a random variable X. The probability that $a < X < b$ is

a) $f(b - a)$ b) $f(b) - f(a)$ c) $\int_a^b f(x)\,dx$ d) $\int_a^b x \cdot f(x)\,dx$

02. Which one of the following statements is not true
a) The measure of skewness depends upon the amount of dispersion
b) In a symmetric distribution the values of mean, mode and median are the same
c) In a positively skewed distribution: mean > median > mode
d) In a negatively skewed distribution: mode > mean > median

03. A bag contains 10 blue marbles and 30 red marbles. A marble is drawn from the bag, its color recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same color is
(GATE'05[IT])
a) 1/36 b) 1/6 c) ¼ d) 1/3

04. If P and Q are two random events, then the following is true *(GATE'05[EE])*
a) Independence of P and Q implies that Probability$(P \cap Q) = 0$
b) Probability $(P \cap Q) \geq$ Probability(P) + Probability(Q)
c) If P and Q are mutually exclusive then they must be independent
d) Probability$(P \cap Q) \leq$ Probability(P)

05. A fair coin is tossed 3 times in succession. If the first toss produces a head, then the probability of getting exactly two heads in three tosses is *(GATE'05[EE])*
a) 1/8 b) ½ c) 3/8 d) ¾

06. Two dice are thrown simultaneously. The probability that the sum of numbers on both exceeds 8 is *(GATE'05[PI])*
a) 4/36 b) 7/36 c) 9/36 d) 10/36

07. A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is *(GATE'05[ME])*
a) 0.0036 b) 0.1937 c) 0.2234 d) 0.3874

08. A single die is thrown two times. What is the probability that the sum is neither 8 nor 9?
a) 1/9 b) 5/36 c) ¼ d) ¾ *(GATE'05[ME])*

09. The probability that there are 53 Sundays in a randomly chosen leap year is
a) 1/7 b) 1/14 c) 1/28 d) 2/7 *(GATE'05[IN])*

10. Find the probability of not getting a total of 7 or 11 on either of two tosses of a pair of fair dice?
a) 25/36 b) 21/49 c) 35/64 d) 49/81

11. Find the probability of a 4 turning up atleast once in two tosses of a fair dice?
a) 8/32 b) 10/36 c) 11/36 d) 13/36

12. A fair dice is rolled twice. The probability that an odd number will follow an even number is *(GATE'05[EC])*
a) ½ b) 1/6 c) 1/3 d) 1/4

13. In a population of N families, 50% of the families have three children, 30% of families have two children and the remaining families have one child. What is the probability that a randomly picked child belongs to a family with two children?
a) 3/23 b) 6/23 c) 3/10 d) 3/5 *(GATE'04[IT])*

14. If a fair coin is tossed 4 times, what is the probability that two heads and two tails will result?
a) 3/8 b) ½ c) 5/8 d) ¾ *(GATE'04[CS])*

15. An exam paper has 150 multiple choice questions of 1 mark each, with each question having four choices. Each incorrect answer fetches -0.25 marks. Suppose 1000 students choose all their answers randomly with uniform probability. The sum total of the expected marks obtained by all the students is *(GATE'04[CS])*
a) 0 b) 2550 c) 7525 d) 9375

16. In a class of 200 students, 125 students have taken programming language course, 85 students have taken data structures course, 65 students have taken computer organization course, 50 students have taken both programming languages and data structures, 35 students have taken both programming languages and computer organization, 30 students have taken both data structures and computer organization, 15 students have taken all the three courses. How many students have not taken any of the three courses?
a) 15 b) 20 c) 25 d) 35 *(GATE'04(IT)]*

17. Let P(E) denote the probability of an event E. Given P(A) =1, P(B) = ½ the values of P(A/B) and P(B/A) respectively are *(GATE'03[CSE})*
a) ¼, ½ b) ½, ¼ c) ½, 1 d) 1, ½

18. Four fair coins are tossed simultaneously. The probability that atleast one heads and atleast one tails turn up is *(GATE'02[CS])*
a) 1/16 b) 1/8 c) 7/8 d) 15/16

19. Seven car accidents occurred in a week, what is the probability that they all occurred on the same day ? *(GATE'01[CS])*
a) $1/7^7$ b) $1/7^6$ c) $1/2^7$ d) $7/2^7$

20. E_1 and E_2 are events in a probability space satisfying the following constraints
$P(E_1) = P(E_2)$; $P(E_1 \cup E_2) = 1$; E_1 & E_2 are independent
Then $P(E_1) =$ *(GATE'2000[CS])*
a) 0 b) ¼ c) ½ d) 1

21. Suppose that the expectation of a random variable X is 5. Which of the following statements is true? *(GATE'99[CS])*
a) There is a sample point at which X has the value 5
b) There is a sample point at which X has the value > 5
c) There is a sample point at which X has a value ≥ 5 d) none of the above

22. Consider two events E_1 and E_2 such that $P(E_1) = ½$, $P(E_2) = 1/3$ and $P(E_1 \cap E_2) = 1/5$. Which of the following statements is true? *(GATE'99[CS])*
a) $P(E_1 \cup E_2) = 2/3$ b) E_1 and E_2 are independent
c) E_1 and E_2 are not independent d) $P(E_1 / E_2) = 4/5$

23. A die is rolled three times. The probability that exactly one odd number turns up among the three outcomes is *(GATE'98[CS])*
a) 1/6 b) 3/8 c) 1/8 d) ½

24. The probability that it will rain today is 0.5. The probability that it will rain tomorrow is 0.6. The probability that it will rain either today or tomorrow is 0.7. What is the probability that it will rain today and tomorrow? *(GATE'97[CS])*
a) 0.3 b) 0.25 c) 0.35 d) 0.4

25. The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is *(GATE'95[CS])*
a) 16/25 b) $(9/10)^3$ c) 27/75 d) 18/25

26. The probability of an event B is p_1. The probability of events A and B occur together is p_2. While the probability that A and B occur together is p_3. The probability of event A in terms of p_1, p_2 and p_3 is
a) $p_1 + p_2$ b) $p_2 + p_3$ c) $p_3 + p_1$ d) $p_1 + p_3 - p_2$

27. Let A and B be any two arbitrary events then which one of the following is true?
a) $P(A \cap B) = P(A).\ P(B)$
b) $P(A \cup B) = P(A) + P(B)$
c) $P(A / B) = P(A \cap B)\ .\ P(B)$
d) $P(A \cup B) \leq P(A) + P(B)$

28. The probability that a new Airport will get an award for its design is 0.16. The probability that it will get an award for its efficient use of materials is 0.24 and probability that it will get both the awards is 0.11. What is the probability that it will get only one of the two awards?.
a) 0.29 b) 0.18 c) 0.21 d) 0.19

29. There are 27 students in a class. What is the probability that atleast 3 of them have their birthday in the same month?
a) 1/9 b) 1/12 c) ¼ d) 1

30. A jar has 5 marbles, one of each of the colors, red, white, blue, green and yellow. If 4 marbles are removed from the jar, what is the probability that the yellow one is removed?
a) 1/5 b) ½ c) 4/5 d) ¾

31. If the probability that a communication system has a high fidelity is 0.81 and the probability that it will have high fidelity and high selectivity is 0.18. What is the probability that a system with high fidelity will also have high selectivity?
a) 2/9 b) 7/9 c) 0.63 d) 0.37

32. A jar contains 4 marbles. 2 red and 2 white. Two marbles are chosen at random. If p_1 is the probability that the marbles chosen are of same color and p_2 is the probability that the marbles chosen be of different colors, then which of the following is true?
a) $p_1 = p_2$ b) $p_1 = 2p_2$ c) $p_2 = 2p_1$ d) $2p_1 = 3p_2$

33. Suppose that a large conference room for a certain company can be reserved for no more than 4 hours. However the use of the conference room is such that both long and short conferences occur quite often. In fact it can be assumed that the duration X of a conference has a uniform distribution on the interval [0, 4]. What is the probability that any given conference lasts atleast 3 hours?
a) ½ b) 1/3 c) ¼ d) ¾

34. Suppose that a system contains a certain type of component whose lifetime is T years. The random variable T is modeled nicely by the exponential distribution with mean time to failure is $\beta = 5$. The probability that a given component installed on a system is still working after 8 years is
a) 0.16 b) 0.17 c) 0.18 d) 0.2

35. In the previous example, if 5 of these components are installed in different systems. What is the probability that atleast 2 components are still functioning at the end of 8 years?
a) 0.4128 b) 0.3645 c) 0.3149 d) 0.2627

36. The average grade for an examination is 74 and the standard deviation is 7. If 12% of the class are given A's and the grades are curved to follow normal distribution then what is the lowest possible A? [The area under the standard normal curve to the left of Z = 1.175 is 0.88]
a) 79 b) 81 c) 83 d) 85

37. A and B play a game in which they toss a fair coin three times. The one obtaining heads first wins the game. If A tosses the coin first and if the total value of the stakes is Rs. 20. How much should be contributed by B in order that the game is fair?
a) Rs. 6.66 b) Rs. 7.50 c) Rs. 8 d) Rs. 8.25

38. An inefficient secretary places 5 different letters into 5 different addressed envelopes at random. Find the probability that atleast one of the letters will arrive at the proper destination.
a) 4/5 b) 1/120 c) 19/30 d) 119/120

39. Determine the probability p, that a non defective bolt will be found next, if out of 600 bolts already examined, 12 were defective?
a) 0.02 b) 0.04 c) 0.96 d) 0.98

40. Each of the three identical jewelry boxes has two shelves. In each shelf of the first box there is a gold watch. In each shelf of the second box there is a silver watch. In one shelf of the third box there is a gold watch while in the other there is a silver watch. If we select a box at random, open one of the shelves and find it to contain a silver watch. What is the probability that the other shelf of the box has the gold watch?
a) ½ b) 1/3 c) 1/4 d) ¾

41. Box P has 2 red balls and 3 blue balls and box Q has 3 red balls and 1 blue ball. A box is selected as follows.
i) Select a box
ii) Choose a ball from the selected box such that each ball in the box is equally likely to be chosen. The probabilities of selecting boxes P and Q are 1/3 and 2/3 respectively. Given that a ball selected in the above process is a red ball, the probability that it came from the box p is
a) 4/19 b) 5/19 c) 2/9 d) 19/30

KEY

1. c 2. d 3. b 4. d 5. b 6. d 7. b 8. d 9. d 10. d 11. c 12. d
13. b 14. a 15. d 16. c 17. d 18. c 19. b 20. d 21. c 22. c 23. b 24. d
25. d 26. b 27. d 28. b 29. d 30. c 31. a 32. c 33. c 34. d 35. d 36. c
37. b 38. c 39. d 40. b 41. a

MEAN / MEDIAN / MODE / SD

01. Calculate the arithmetic mean for following data
1600, 1560, 1440, 1530, 1670, 1860, 1750, 1910, 1490, 1800
(a) 1660 (b) 1661 (c) 1670 (d) 1560

02. Arithmetic mean of 24, 28, 29, 34, 18, 22, 26, 30, 32, 24, 20 is
(a) 26.09 (b) 26 (c) 24 (d) 34

03. Arithmetic mean of the natural numbers from 1 to n is
(a) $\frac{n(n+1)}{2}$ (b) $\frac{n+1}{2}$ (c) n/2 (d) none

04. If 10 is added to each and every item of a data, then the arithmetic mean
(a) Is increased by 10 times (b) is not increased
(c) is greater by 10 (d) none

05. The man of 25 values was calculated as 78.4, but while taking them an item 69 was misread as 96, the correct mean is
(a) 77.32 (b) 78.4 (c) 76 (d) 69

06. The median of 55, 100, 75, 80, 90, 85, 95, 45, 70, 70, 55
(a) 75 (b) 85 (c) 90 (d) none

07. The simplest measure of dispersion is
(a) S. D (b) Range (c) MD (d) QD

08. The measure of dispersion which is used to find more consistent data is
(a) SD (b) MD (c) QD (d) Range

09. Standard deviation of 27, 35, 40, 35, 36, 36, 29 is
(a) 17.14 (b) 4.14 (c) 34 (d) none

10. For a symmetrical distribution QD is
(a) 2/3 SD (b) SD (c) MD (d) 6/5 MD

11. Standard deviation of 3, 5, 7, 9, 11, 13 is
(a) 12 (b) 11 (c) 11.66 (d) 3.4

12. If the first and third quartiles of a data are 5, 10 then QD is
(a) 5/2 (b) 3 (c) 2 (d) 1

13. If the least and greatest values of a data are 5.95 then the coefficient of range is
(a) 10/9 (b) 9/10 (c) 1/10 (d) none

14. If standard deviation of a data is 3, mean is 20 then coefficient of variation is
(a) 156 (b) 3/20 (c) 20/3 (d) none

15.

Classes	0-10	10-20	20-30
Frequency	5	14	6

mean is
(a) 8.55 (b) 8 (c) 9 (d) 10

16. Range of 1, 4, 90, 100, 4 is
(a) 99 (b) 73 (c) 72 (d) 11

17. The mean of a set of numbers is '$\overline{X}$', if each number is increased by λ, the mean of new set is
(a) $\overline{X}$ (b) $\overline{X}+\lambda$ (c) $\lambda\overline{X}$ (d) λ

18. If mean = (3 Md – Mode) X, then the value 'X' is
(a) 1 (b) 2 (c) ½ (d) 3/2

19. From a frequency distribution C = 3, l = 65.5 f = 42, m = 23, N = 102 then median is
(a) 67.5 (b) 57.5 (c) 3.75 (d) 2.75

20. The variance of the first 'n' natural number is
(a) $\frac{n^2-1}{12}$ (b) $\frac{n^2-1}{6}$ (c) $\frac{n^2+1}{6}$ (d) $\frac{n^2+1}{12}$

21. If SD is 3 at $\overline{X}$ = 20 then coefficient of dispersion is 0.15
(a) 0.15 (b) 0.25 (c) 0.35 (d) 0.45

KEY

1. b 2. a 3. b 4. c 5. a 6. a 7. b 8. a 9. b 10. a 11. d 12. a
13. b 14. a 15. a 16. a 17. b 18. c 19. a 20. a 21. a

PREVIOUS GATE QUESTIONS - "PROBABILITY"

1. The probability that two friends share the same birth-month is ***(GATE'98)***
a) 1/6 b) 1/12 c) 1/144 d) 1/24

2. In a manufacturing plant, the probability of making a defective bolt is 0.1. The mean and standard deviation of defective bolts in a total of 900 bolts are respectively ***(GATE - 2000)***
a) 90 and 9 b) 9 and 90 c) 81 and 9 d) 9 and 81

3. Manish has to travel from A to D changing buses at stops B and C enroute. The maximum waiting time at either stop can be 8 minutes each, but any time of waiting up to 8 minutes is equally likely at both places. He can afford up to 13 minutes of total waiting time if he is to arrive at D on time, What is the probability that Manish will arrive late at D? ***(GATE'02)***
a) 8/13 b) 13/64 c) 119/128 d) 9/128

4. A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be ***(GATE'03)***
a) 100% b) 50% c) 49% d) none of these

5. A hydraulic structure has four gates which operate independently. The probability of failure of each gate is 0.2. Given that gate 1 has failed, the probability that both gates 2 and 3 will fail is ***(GATE'04)***
a) 0.240 b) 0.200 c) 0.040 d) 0.008

6. From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be kings, if the first card is NOT replaced? *(GATE'04)*

a) 1/26 b) 1/52 c) 1/169 d) 1/221

7. A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is *(GATE'05)*

(a) 0.0036 (b) 0.1937 (c) 0.2234 (d) 0.3874

KEY

1. b 2. a 3. 4. d 5. c 6. d 7. b

PREVIOUS GATE QUESTIONS

01. The life of a bulb (in hours) is a random variable with an exponential distribution $f(t) = \alpha e^{-\alpha t}$, $0 \le t \le \infty$. The probability that its value lies b/w 100 and 200 hours is
PI – 2005 – 2M
(a) $e^{-100\alpha} - e^{-200\alpha}$
(b) $e^{-100} - e^{-200}$
(c) $e^{-100\alpha} + e^{-200\alpha}$
(d) $e^{-200\alpha} - e^{-100\alpha}$

02. Assume that the duration in minutes of a telephone conversation follows the exponential distribution $f(x) = \frac{1}{5} e^{-x/5}$, $x \ge 0$. The probability that the conversation will exceed five minutes is
IN – 2007 – 2M
(a) $\frac{1}{e}$ (b) $1 - \frac{1}{e}$
(c) $\frac{1}{e^2}$ (d) $1 - \frac{1}{e^2}$

03. If the standard deviation of the spot speed of vehicles in a highway is 8.8 kmph and the mean speed of the vehicles is 33 kmph, the coefficient of variation in speed is
CE – 2007 – 2M
(a) 0.1517 (b) 0.1867
(c) 0.2666 (d) 0.3646

04. Let X and Y be two independent random variables. Which one of the relations b/w expectation (E), variance (V_{ar}) and covariance (C_{ov}) given below is FALSE? **ME – 2007 – 2M**
(a) $E(XY) = E(X)\,E(Y)$
(b) $C_{ov}(X, Y) = 0$
(c) $V_{ar}(X+Y) = V_{ar}(X) + V_{ar}(Y)$
(d) $E(X^2Y^2) = (E(X))^2\ (E(Y))^2$

05. Two cards are drawn at random in succession with replacement from a deck of 52 well shuffled cards Probability of getting both 'Aces' is
PI – 2007 – 1M
(a) $\frac{1}{169}$ (b) $\frac{2}{169}$
(c) $\frac{1}{13}$ (d) $\frac{2}{13}$

06. The random variable X taken on the values 1, 2 (or) 3 with probabilities $\frac{2+5P}{5}, \frac{1+3P}{5}$ and $\frac{1.5+2P}{5}$ respectively
The values of P and E(X) are respectively
PI – 2007 – 2M
(a) 0.05, 1.87 (b) 1.90, 5.87
(c) 0.05, 1.10 (d) 0.25, 1.40

07. If X is a continuous random variable whose probability density function is given by
$$f(x) = \begin{cases} k(5x - 2x^2), & 0 \le x \le 2 \\ 0 & \text{, otherwise} \end{cases}$$
Then $P(x > 1)$ is
PI – 2007 – 2M
(a) 3/14 (b) 4/5
(c) 14/17 (d) 17/28

08. If E denotes expectation, the variance of a random variable X is given by
EC – 2007- 1M
(a) $E(X^2) - E^2(X)$
(b) $E(X^2) + E^2(X)$
(c) $E(X^2)$
(d) $E^2(X)$

09. An examination consists of two papers, paper 1 and paper 2. The probability of failing in paper 1 is 0.3 and that in paper 2 is 0.2. Given that a student has failed in paper 2, the probability of failing in paper 1 is 0.6. The probability of a student failing in both the papers is **EC – 2007 – 2M**
(a) 0.5 (b) 0.18
(c) 0.12 (d) 0.06

10. X is uniformly distributed random variable that take values between 0 and 1. The value of $E(X^3)$ will be **EE – 2008 - 1M**

(a) 0 (b) 1/8
(c) 1/4 (d) 1/2

11. A random variable is uniformly distributed over the interval 2 to 10. Its variance will be **IN-2008-2M**

(a) 16/3 (b) 6
(c) 256/9 (d) 36

12. Consider a Gaussian distributed random variable with zero mean and standard deviation σ. The value of its cumulative distribution function at the origin will be **IN-2008-2M**

(a) 0 (b) 0.5
(c) 1 (d) $10\,\sigma$

13. $P_x(x) = Me^{(-2|x|)} + Ne^{(-3|x|)}$ is the probability density function for the real random variable X, over the entire x-axis M and N are both positive real numbers. The equation relating M and N is **EC-2008-2M**

(a) $M+\frac{2}{3}N=1$ (b) $2M+\frac{1}{3}N=1$
(c) $M+N=1$ (d) $M+N=3$

14. A coin is tossed 4 times. What is the probability of getting heads exactly 3 times? **ME-2008-1M**

(a) 1/4 (b) 3/8
(c) 1/2 (d) ¾

15. For a random variable $x(-\alpha < x < \alpha)$ following noımal distribution, the mean is $\mu = 100$. If the probability is $P = \alpha$ for $x \geq 110$. then the probability of x lying b/w 90 and 110 i.e. $P(90 \leq x \leq 110)$ will be equal to **PI-2008-1M**

(a) $1-2\alpha$ (b) $1-\alpha$
(c) $1-\alpha/2$ (d) $2\,\alpha$

16. In a game, two players X and Y toss a coin alternately. Whosever gets a 'head' first, wins the game and the game is terminated. Assuming that player X starts the game the probability of player X winning the game is **PI-2008-2M**

(a) 1/3 (b) 1/3
(c) 2/3 (d) ¾

17. The standard normal probability function can be approximated as

$$F(x_N) = \frac{1}{1+\exp\left(-1.7255\,x_N|x_N|^{0.12}\right)}$$

where x_N = standard normal deviate. If mean and standard deviation of annual precipitation are 102 cm and 27 cm respectively, the probability that the annual precipitation will be b/w 90 cm and 102 cm is **CE-2009-2M**

(a) 66.7 % (b) 50.0 %
(c) 33.3 % (d) 16.7 %

18. A fair coin is tossed 10 times. What is the probability that only the first two tosses will yield heads? **EC-2009-1M**

(a) $\left(\frac{1}{2}\right)^2$ (b) $10_{C_2}\left(\frac{1}{2}\right)^2$
(c) $\left(\frac{1}{2}\right)^{10}$ (d) $10_{C_2}\left(\frac{1}{2}\right)^{10}$

19. Consider two independent random variables X and Y with identical distributions. The variables X and Y take values 0, 1 and 2 with probabilities ½, ¼ and ¼ respectively. What is the conditional probability $P(X+Y=2/X-Y=0)$? **EC-2009-2M**

(a) 0 (b) 1/16
(c) 1/6 (d) 1

20. A discrete random variable X takes value from 1 to 5 with probabilities as shown in the table. A student calculates the mean of X as 3.5 and her teacher calculates the variance to X as 1.5. Which of the following statements is true?

k	1	2	3	4	5
P(X=k)	0.1	0.2	0.4	0.2	0.1

EC-2009-2M

(a) Both the student and the teacher are right
(b) Both the student and the teacher are wrong
(c) The student is wrong but the teacher is right
(d) The student is right but the teacher is wrong

21. A screening test is carried out to detect a certain disease. It is found that 12% of the positive reports and 15% of the negative reports are incorrect. Assuming that the probability of a person getting a positive report is 0.01, the probability that a person tested gets an incorrect report is

IN-2009-2M

(a) 0.0027 (b) 0.0173
(c) 0.1497 (d) 0.2100

22. If three coins are tossed simultaneously, the probability of getting at least one head is

ME-2009-1M

(a) 1/8 (b) 3/8
(c) ½ (d) 7/8

23. The standard deviation of a uniformly distributed random variable b/w 0 and 1 is **ME-2009-2M**

(a) $\frac{1}{\sqrt{12}}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{5}{\sqrt{12}}$ (d) $\frac{7}{\sqrt{12}}$

24. Assume for simplicity that N people, all born in April (a month of 30 days) are collected a room, consider the event of at least two people in the room being born on the same date of the month even if in different years e.g.1980 and 1985. What is the smallest N so that the probability of this event exceeds 0.5 is?

EE-2009-2M

(a) 20 (b) 7
(c) 15 (d) 16

25. A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that first removed ball is white, the probability that the 2^{nd} removed ball is red is

EE-2010-2M

(a) 1/3 (b) 3/7
(c) ½ (d) 4/7

26. A fair coin is tossed independently four times. The probability of the event "The number of times heads show up is more than the number of times tails show up" is **EC-2010-2M**

(a) 1/16 (b) 1/8
(c) 1/4 **(d)** 5/16

27. What is the probability that a divisor of 10^{99} is a multiple of 10^{96}?

CS-2010-2M

(a) 1/625 (b) 4/625
(c) 12/625 (d) 16/625

28. The diameters of 10,000 ball bearings were measured the mean diameter and standard deviation were found to be 10 mm and 0.05 mm respectively. Assuming Gaussian distribution of measurements, it can be expected that the number of measurements more than 10.15 mm will be

IN-2010-1M

(a) 230 (b) 115
(c) 15 (d) 2

29. Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty?

CS-2010-2M

(a) pq + (1 – p) (1 – q) (b) (1 – q) p
(c) (1 – p) q (d) pq

30. A box contains 2 washers, 3 nuts and 4 bolts. Items are drawn from the box at random one at a time without replacement. The probability of drawing 2 washers first followed by 3 nuts and subsequently the 4 bolts is

ME-2010-2M

(a) 2/315 (b) 1/630
(c) 1/1260 (d) 1/2520

31. Two coins are simultaneously tossed. The probability of two heads simultaneously appearing is

CE-2010-1M

(a) 1/8 (b) 1/6
(c) ¼ (d) ½

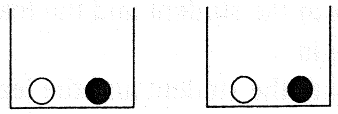
32. If a random variable X satisfies the poission's distribution with a mean value of 2-then the probability that $X \geq 2$ is

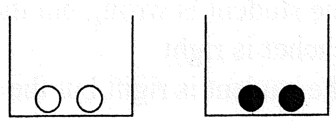
PI-2010-1M

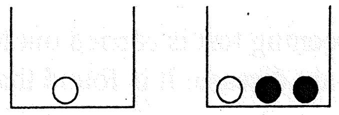
(a) $2e^{-2}$ (b) $1–2e^{-2}$
(c) $3e^{-2}$ (d) $1–3e^{-2}$

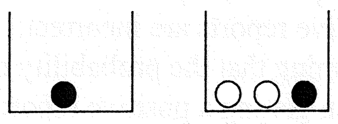
33. Two white and two black balls, kept in two bins, are arranged in four ways as shown below. In each arrangement, a bin has to be chosen randomly and only one ball needs to be picked randomly fron the chosen bin. Which one of the following arrangements has the highest probability for getting a white ball picked?

PI-2010-2M

(a)


(b)


(c)


(d)


KEY:

01. a	02. a	03. c
04. d	05. a	06. a
07. d	08. a	09. c
10. c	11. a	12. b
13. a	14. b	15. a
16. c	17. d	18. a
19. d	20. b	21. c
22. b	23. a	24. b
25. c	26. d	27. a
28. d	29. a	30. c
31. c	32. d	33. c

SUM RULE: If $E_1, E_2, \ldots\ldots, E_n$ are mutually exclusive events, and E_1 can happen in e_1 ways, E_2 can happen in e_2 ways, ..., E_n can happen in e_n ways, then E_1 or E_2 or ... or E_n can happen in $e_1 + e_2 + \ldots + e_n$ ways.

PRODUCT RULE: If events $E_1, E_2, \ldots\ldots, E_n$ can happen in $e_1, e_2, \ldots\ldots, e_n$ ways respectively, then the sequence of events E_1 first followed by E_2, ..., followed by E_n can happen in $(e_1 . e_2 . \ldots\ldots e_n)$ ways.

PERMUTATIONS:

$P(n, r) = \dfrac{n!}{n-r!}$ = The number of permutations of n objects taken r at a time (without any repetitions)

$= n(n-1)(n-2)\ldots\ldots(n-r+1)$

NOTE:

1) $P(n, n) = n!$ i.e.., there are $n!$ Permutations of n objects
2) There are $(n-1)!$ Permutations of n distinct objects in a circle.
3) A permutation of n objects taken r at a time (also called an r-permutation of n objects) is an ordered selection or arrangement of r of the objects.
4) $U(n, r)$ = The number of r-permutations of n objects with unlimited repetitions $= n^r$.
5) The number of permutations of n objects of which n_1 are alike, n_2 are alike, ..., n_r are alike is

$$\frac{n!}{n_1!\, n_2! \ldots\ldots n_r!}$$

6) The number of ordered partitions of a set S of type $(q_1, q_2, \ldots, q_z)$ where $|S| = n$ is

$$P(n; q_1, q_2, \ldots\ldots, q_z) = \frac{n!}{q_1!\, q_2! \ldots\ldots q_z!}$$

7) Enumeration of unordered partitions:

The number of unordered partitions of a set S of type (q_1, q_2, q_z), Where $|S| = n$ is

$$= \frac{n!}{(q_1!\, q_2! \ldots\ldots q_t!)\, .\, t!}$$

COMBINATIONS:

A combination of n-objects taken r at a time (called **an r-combination** of n objects) is an unordered selection of n objects.

$C(n, r)$ = The number of combinations of n-objects **taken r** at time (without repetition)

$$= \frac{n!}{r!(n-r)!}$$

NOTE:

1) $P(n, r) = r!\, C(n, r)$
2) $C(n, n) = 1$
3) $V(n, r)$ = The number of combinations of n distinct objects taken r at a time with unlimited repetitions.
4) $V(n, r) = C(n-1+r, r)$
$= C(n-1+r, n-1)$

$V(n, r)$ = The number of ways distributing r-similar balls in to n numbered boxes.

$V(n, r)$ = The number of non negative integral solutions to $x_1 + x_2 + \ldots\ldots + x_n = r$

$V(n, r)$ = The number of binary sequences with $n-1$ one's and r-zeros.

01. How many ways are there to distributive 10 different books among 15 people, if no person is to receive more than one book?
a) P(15, 10) b) C(15, 10) c) 10^{15} d) 15^{10}

02. How many binary sequences are there of length 15?
a) 2^{15} b) 15! c) P(15, 2) d) C(15, 2)

03. How many binary sequences are there of length 15 with exactly six one's?
a) P(15, 6) b) C(15, 6) c) 2^6 d) 2^9

04. A multiple choice test has 15questions and 4 choices for each answer. How many ways can the 15 questions be answered?
a) 15^4 b) 4^{15} c) C(15, 4) d) P(15, 4)

05. In the previous example, how many ways the 15 questions be answered so that exactly 3 answers are correct?
a) C(15, 3). 3^{12} b) C(15, 3). 4^{12} c) C(15, 3). 3^{15} d) P(15, 3)

06. The number of ways of placing 10 similar balls in six numbered boxes is
a) C(15, 5) b) P(15, 5) c) C(10, 6) d) P(10, 6)

07. The number of non negative integral solutions to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 10$ is
a) C(20, 5) b) P(20, 5) c) C(14, 4) d) C(15, 5)

08. The number of binary numbers with ten one's and five zeros is
a) C(15, 5) b) P(15, 5) c) 2^{10} d) 2^5

09. How many integral solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where $x_1 \geq 3$, $x_2 \geq 2$, $x_3 \geq 4$, $x_4 \geq 6$ and $x_5 \geq 0$
a) C(9, 4) b) P(9, 4) c) C(20, 5) d) P(20, 5)

10. In how many ways can 5 similar books be placed on 3 different shelves?
a) 15 b) 10 c) 21 d) 56

11. A professor wishes to give an examination with 10 questions. In how many ways can the test be given a total of 30 marks if each question is to be worth 2 or more marks?
a) C(30, 10) b) C(20, 10) c) C(19, 10) d) C(21,

12. How many 10-permutations are there of {3.a, 4.b, 2.c, 1.d} ?
a) 12, 600 b) 16, 200 c) 14, 620 d) 8, 400

13. How many 5 letter words can be formed from the word 'DADDY' ?
a) 20 b)120 c) 15 d) 80

14. Suppose that a set S has n distinct elements. How many n –part ordered partitions $(A_1, A_2, \ldots, A_n)$ are there in which each set A_i has exactly 1 element ?
a) 2^n b) n! c) $n(n-1)/2$ d) n^2

15. Suppose that there are 100 players entered in a single elimination tennis tournament. How many matches must be conducted to declare the winner ?
a) 101 b) 100 c) 99 d)

16. In how many ways can 10 persons be seated in a row so that a certain pair of them are not next to each other ?
    a) 9!   b) 2(9!)   c) 8(9!)   d) 9(9!)

17. Suppose that we draw a card from a deck of 52 cards and replace it before the next draw. In how many ways can 10 cards be drawn so that the tenth card is a repetition of a previous draw.
    a) $52\,(51)^{19}$ b) $(52)^{10} - (51)^{9}\,52$ c) $52^{10} - 51^{9}$ d) $52^{10} - 51^{10}$

18. How many ways are there to pick a man and a woman who are not married from 30 married couples ?
    a) 30.(30)   b) 30   c) 30.(29)   d) 29

19. In a certain programming language, an identifier is a sequence of certain number of characters, where the first character must be an upper case letter of the English alphabet and the remaining characters man be either a letter or a digit. How many identifiers are there of length 5 ?
    a) $(36)^5$ b) $36(26)^4$ c) $26(36)^4$ d) $(26)^5$

20. There are 10 telegrams and 2 messenger boys. In how many different ways the telegrams can be distributed to the messenger boys if the telegrams are distinguishable
    a) 11   b) C (10, 2)   c) P (10, 2)   d) 2^{10}

21. In the previous example, if the telegrams are not distinguishable, then how many different ways they can be distributed to the boys ?
    a) 11   b) 2^{10} c) 45   d) 90

22. How many integers between 10^5 and 10^6 have no digits other than 0, 2 5, 8 ?
    a) $3\,(4)^5$ b) $4\,(3)^5$ c) 4^6 d) 3^6

23. A new state flag is to be designed with 6 vertical stripes in yellow, white, blue and red. In how may ways this can be done so that no 2 adjacent stripes have the same color ?
    a) $4\,(3)^5$ b) $3\,(4)^5$ c) 4^6 d) 6^4

24. If 6 men intend to speak at a convention, in how may ways can then do so with B speaking after A ?
    a) 120   b) 360   c) 720   d) P(6,5)

25. The number of ways in which we can post 5 letters in 10 letter boxes is
    a) 5^{10} b) 10^5 c) C (10,5)   d) P (10, 5)

PIGEON HOLE PRINCIPLE :

If A is the average number of pigeons per hole, then some pigeon hole contains atleast $\lceil A \rceil$ pigeons and some pigeon hole contains atmost $\lfloor A \rfloor$ pigeons

Ex: If n + 1 pigeons are distributed among 'n' pigeon holes, then some pigeon hole contains atleast 2 pigeons

Ex : If 2n+1 pigeons are distributed among 'n' pigeon holes, then some pigeon hole contains atleast 3 pigeons

Ex: In general, if k is a positive integer and 'kn+1' pigeons are distributed among 'n' pigeon holes, then some hole contains atleast 'k + 1' pigeons.

Ex : In a group of 61 people atleast 6 people were born in the same month.

Ex: If 401 letters were delivered to 50 apartments, then some apartment received atmost 8 letters.

26. Suppose 50 chairs are arranged in a rectangular array of 5 rows and 10 columns. Suppose that 41 students seated randomly in the chairs. Which o f the following is false.
a) Some row contains at least 9 students b) Some row contains atmost 8 students
c) Some column contains at least 6 students d) Some column contains atmost 4 students

27. The minimum number of cards to be dealt from an arbitrary shuffled deck of 52 cards to guarantee that three cards are from same suit is
a) 3 b) 8 c) 9 d) 13

PRINCIPLE OF MUTUAL INCLUSION & EXCLUSION:

28. Suppose that 200 faculty members can speak French and 50 can speak Russian, while only 20 can speak both French and Russian. How many faculty members can speak either French or Russian ?
a) 130 b) 230 c) 270 d)180

29. From a group of 10 professors how many ways can a committee of 5 members can be formed so that atleast one of professor A and professor B will be included
a) 196 b) 160 c) 148 d)120

30. If there are 200 faculty members that speak French, 50 that speak Russian , 100 that speak Spanish, 20 that speak French and Russian ; 60 that speak French and Spanish, 35 that speak Russian and Spanish, while only 10 speak French, Russian and Spanish. How many speak either French or Russian or Spanish ?
a) 245 b) 160 c) 125 d) 260

EULER'S ∅ - FUNCTION:

If n is a '+ve' integer then ∅(n) = The number of integers x such that $1 \le x \le n$ and such that n and x are relatively prime. $\varnothing(n) = n\,[(1-(1/p_1))\,(1-(1/p_2))\,\ldots\ldots\,(1-(1/p_k))]$ where $p_1, p_2, \ldots, p_k$ are prime divisors of n.

DERANGEMENTS:

Among the permutations of {1, 2, …., n} there are some , called derangements, in which none of the n integers appears in its natural place.

D_n = The number of derangements of n elements

$$D_n = n!\left\{\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \ldots + \frac{(-1)^n}{n!}\right\}$$

In particular,

$$D_5 = 5!\left\{\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right\}$$

$D_5 = 44$, $D_6 = 265$, $D_4 = 9$, $D_3 = 2$

31. Find the number of positive integers less than 91 and relatively prime to 91
a) 72 b) 68 c) 89 d) 54

32. Let 5 different books be distributed to 5 students. Suppose the books are returned and distributed to the students again later on . In how many ways the books be distributed so that no student will get the same book twice ?
a) 5280 b) 3600 c) 2420 d) 6,400

33. Four students take a quiz. Then for the purpose of grading, the teacher ask the students to exchange papers, so that no one is grading his own paper. How many ways this can be done
a) 9 b) 16 c) 256 d) 12

34. The number of five digit telephone numbers having atleast one of their digits repeated is
a) 90000 b) 100000 c) 30240 d) 69760

35. A letter lock consists of three rings marked with 15 different letters. The number of ways in which it is possible to make unsuccessful attempts to open the lock is
a) 3374 b) 2374 c) 4374 d) 1374

36. Four dice are rolled. The number of possible out comes in which atleast one die shows 2 is
a) 1296 b) 625 c) 671 d) 584

37. A is a set containing n elements, A subset P of A is chosen. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen. The number of ways of choosing P and Q so that $P \cap Q = \phi$ is
a) 2^n b) 3^n c) $2^n - 1$ d) $3^n - 1$

38. Eight chairs are numbered 1 to 8. two women and three men wish to occupy one chair each. First the woman choose the chairs amongst the chairs 1 to 4, and then the men select from remaining chairs. The number of possible arrangements are
a) 1440 b) 1290 c) 1160 d) 900

39. The number of rectangles that you can find on a chess board is
a) 1296 b) 64 c) $(56)^2$ d) 1024

40. The number of signals that can be generated by using 6 differently coloured flags, when any number of them may be hoisted at a time is
a) 1956 b) 1957 c) 1958 d) 1959

41. The number of integers greater than 7000 that can be formed with the digits 3, 5, 7, 8 and 9, no digit being repeated is
a) 168 b) 192 c) 240 b) 200

42. Ten persons, amongst whom are A, B and C are to speak at a function, the number of ways in which it can be done if A wants to speak before B and B wants to speak before C is_
a) 8 b) (10 !) / 6 c) 8 ! 3! D) 7 ! 3 !

43. The number of n – bit (digits 0 and 1) strings with exactly k zeros, with no two zeros consecutive, is
a) $C(n-k+1, k)$ b) $C(n+k-1, k)$ c) $C(n+k-1, n)$ d) $C(n+k-1, n-1)$

44. The number of ways in which 5 beads of different colors can form a necklace is
a) 60 b) 24 c) 12 d) 8

45. A Box contains two white, three Black and four Red balls. In how many ways can three balls be drawn so that atleast one black ball is included in the draw ?
a) 16 b) 32 c) 48 d) 64

46. Find the number of ways in which 16 apples can be distributed among four persons so that each of them gets atleast one apple ?
a) 365 b) 455 c) 540 d) 680

47. On Diwali, all the students of a class send greeting cards to one another. If the post man deliver 1640 cards to the students of this class, then the number of students in the class is

a) 41 b) 37 c) 44 d) 36

48. The number of 30- digit ternary sequences (sequences using only the digits 0, 1 ad 2) having exactly ten one's is

a) C(30, 10) b) 2^{20} c) P(30,10) d) C(30, 10) 2^{20}

49. How many 4 digit even numbers have all 4 digits distinct ?

a) 1684 b) 1892 c) 2296 d) 2420

SUMMATION :

1) $C(n, r) . C(r, k) = C(n, k) . C(n-k, r-k)$ for integers $n \geq r \geq k \geq 0$ (Newtons identity)

2) $C(n, r) . r = n. C(n-1, r-1)$

3) $P(n, r) = n. P(n-1, r-1)$

4) Pascals identity
$C(n, r) = C(n-1, r) + C(n-1, r-1)$

5) $C(n, 0) = 1 = C(n, n)$
$C(n, 1) = n = C(n, n-1)$

NOTATION:

$C(n, 0) = C_0,\quad C(n\ 1) = C_1,\quad C(n, 2) = C_2, \ldots\ldots,\quad C(n, r) = C_r, \ldots\ldots,\quad C(n, n) = C_n,$

6) **Row Summation**
$C_0 + C_1 + C_2 + \ldots\ldots + C_n = 2^n$

7) **Row square summation**
$C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2 = C(2n, n)$

8) $C_0 - C_1 + C_2 - C_3 \ldots\ldots + (-1)^n C_n = 0$

9) $(C_0 + C_2 + C_4 + \ldots\ldots) = (C_1 + C_3 + C_5 + \ldots\ldots) = 2^{n-1}$

10) $C_1 + 2C_2 + 3C_3 + \ldots\ldots + n\, C_n = n\, 2^{n-1}$

11) $2^0 C_0 + 2^1 C_1 + 2^2 C_2 + \ldots\ldots + 2^n C_n = 3^n$

12) **Diagonal Summation**
$C(n, 0) + C(n + 1, 1) + C(n + 2, 2) + \ldots + C(n + r, r) = C(n + r + 1, r)$

13) $C(m, 0).C(n, 0) + C(m, 1) . C(n, 1) + \ldots + C(m, n)C(n, n) = C(m + n, n)$
for integers $m \geq n \geq 0$

14) **Column summation**
$C(r, r) + C(r + 1, r) + \ldots + C(n, r) = C(n+1, r+1)$, for any positive integer $n \geq r$

15) $2.1\, C_2 + 3.2\, C_3 + \ldots + n(n-1)C_n = 2^{n-1} n(n-1)$

16) $1^2 C_1 + 2^2 C_2 + \ldots + n^2 C_n = 2^{n-2} n(n+1)$

17) $1^3 C_1 + 2^3 C_2 + \ldots + n^3 C_n = 2^{n-3} n^2 (n+3)$

PROBLEMS

50. $C(2n, 0) + C(2n, 1) + \ldots\ldots + C(2n, 2n) =$
a) 2^n b) 2^{2n} c) 2^{n+2} d) $n\,2^n$

51. $C_0 + 3C_1 + 5C_2 + \ldots\ldots + (2n+1)\,C_n =$
a) $n\,2^n$ b) $(n+1)\,2^n$ c) $n\,2^{n-1}$ d) $(n-1)\,2^n$

52. $C_0^2 - C_1^2 + C_2^2 - C_3^2 \ldots\ldots + (-1)^2\,C_n^2 = 0$ then n is
a) odd b) even c) any positive integer d) prime number

53. If n is even, then $C_0^2 - C_1^2 + C_2^2 - C_3^2 + \ldots + (-1)^n\,Cn^2 =$
a) $(-1)^{n/2}\,C(n, n/2)$ b) 0 c) $(-1)^{n/2}\,C(2n, n)$ d)) $(-1)^{n/2}\,C(2n, n/2)$

54. $C_2 + 2\,C_3 + 3\,C_4 + \ldots\ldots + (n-1)\,C_n =$
a) $(n-2)\,2^n + 1$ b) $(n-2)\,2^{n-1} + 1$ c) $(n-2)\,2^4 - 1$ d) $(n-2)\,2^{n+1} + 1$

55. $C_0 - 2\,C_1 + 3\,C_2 - \ldots\ldots + (-1)^n\,(n+1)\,C_n =$
a) 0 b) 1/n c) 1 / (n + 1) d) 2 / (n+1)

56. $C_1 - 2\,C_2 + 3\,C_3 - 4\,C_4 + \ldots\ldots (-1)^{n-1}\,C_n =$
a) 0 b) 1/n c) 1 / (n-1) d) 1/ (n+1)

57. $3C_0 - 8C_1 + 13C_2 - \ldots\ldots$ (n + 1) terms =
a) 0 b) n c) (n-1) d) (n+1)

58. $\dfrac{C_1}{C_0} + \dfrac{2\,C_2}{C_1} + \dfrac{3\,C_3}{C_2} + \dfrac{n\,C_n}{C_{n-1}} =$
a) n (n + 1) b) n (n + 1) c) n (n – 1) / 2 d) n (n + 1) 2

59. $C_0\,C_1 + C_1\,C_2 + \ldots + C_{n-1}\,C_n =$
a) C (2n, n) b) C (2n, n – 1) c) C (2n, n + 1) d) C (2n, n + 2)

60. $C_0 - 1/2\,C_1 + 1/3\,C_2 + \ldots\ldots + (-1)^n \dfrac{1}{n+1} C_n =$
a) 0 b) 1 / (n – 1) c) 1 / n d) 1 / (n+1)

KEY

1. a	2. a	3. b	4. b	5. a	6. a	7. c	8. a	9. a	10. c	11. c	12. a
13. a	14. b	15. c	16. c	17. b	18. c	19. c	20. d	21. a	22. a	23. a	24. b
25. b	26. c	27. c	28. b	29. a	30. a	31. a	32. a	33. a	34. d	35. a	36. c
37. b	38. a	39. a	40. a	41. b	42. b	43. a	44. c	45. d	46. b	47. a	48. d
49. c	50. b	51. b	52. a	53. a	54. b	55. a	56. a	57. a	58. d	59. b	60. d

PREVIOUS GATE QUESTIONS

01. The exponent of 11 in the prime factorization of 300! is **GATE – 2008**
(a) 27 (b) 28 (c) 29 (d) 30

02. In how many ways can b blue balls and r red balls be distributed in n distinct boxes?

(a) $\frac{(n+b-1)!\,(n+r-1)!}{(n-1)!\,b!\,(n-1)!\,r!}$ (b) $\frac{(n+(b+r)-1)!}{(n-1)!\;(n-1)!\,(b+r)!}$

(c) $\frac{n!}{b!\,r!}$ (d) $\frac{(n+(b+r)-1)!}{n!\,.\,(b+r-1)!}$ **GATE - 2008**

03. In how may ways can we distribute 5 distinct balls $B_1, B_2, \ldots, B_5$ in 5 distinct cells $C_1, C_2, \ldots, C_5$ such that Ball B_i is not in cell C_i, $\forall i = 1, 2, .., 5$ and each cell contains exactly one ball ? **GATE – 2007**
(a) 44 (b) 96 (c) 120 (d) 3125

Linked answer Questions

Suppose that a robot is placed on the Cartesian plane. At each step it is allowed to move either one unit up or one unit right, i.e., if it is at (i, j) Then it can move to either (i + 1, j) or (i, j+1).

04. (i) How many distinct paths are there for the robot to reach the point (10, 10) starting from the initial position (0, 0)?
(a) C(20, 10) (b) 2^{20} (c) 2^{10} (d) P(20, 10)

04. (ii) Suppose the robot is not allowed to traverse the line segment from (4, 4) to (5, 4). With this constraint, how many distinct paths are there for the robot to reach (10, 10) startings from (0, 0)? **GATE – 2007**
(a) 7^9 (b) 2^{19}
(c) C(8, 4) . C(11,5) (d) C(20, 10) – {C(8, 4) . C(11, 5)}

05. Let $n = p^2q$, where p and q are district prime numbers. How many numbers m satisfy $1 \le m \le n$ and GCD (m, n) = 1? **GATE – 2005**
(a) p. (q – 1) (b) p q (c) (q – 1) (d) p(p – 1) . (q – 1)

06. What is the cardinality of the set of integers X defined below?
$X = \{n \mid 1 \le n \le 123$, n is not divisible by either 2, 3 or 5$\}$ **GATE – 2005**
(a) 28 (b) 33 (c) 37 (d) 44

07. The 2^n vertices of a graph G correspond to all subsets of a set of size n, for $n \ge 6$ two vertices of G are adjacent if and only if the corresponding sets interest in exactly two elements. The number of vertices of degree zero in G is **GATE – 2006**
(a) 1 (b) n (c) n + 1 (d) 2^n

08. Let A be the sequence of 8 distinct integers sorted in ascending order. How many distict pairs of sequences, B and C are there such that **GATE – 2003**
(i) each is sorted in ascending order
(ii) B has 5 and C has 3 elements, and
(iii) The result of merging B and C gives A?
(a) 2 (b) 30 (c) 56 (d) 256

09. n couples are invited to a party with the condition that every husband should be accompanied by his wife. However, a wife need not be accompanied by her husband. The number of different gatherings possible at the party is **GATE – 2003**

(a) C (2n, n) . 2^n (b) 3^n (c) $\frac{\lfloor 2n}{2^n}$ (d) C(2n, n)

10. m identical balls are to be placed in n distinct bags. You are given that m ≥ kn, where k is a natural number ≥ 1. In how many ways can the balls be placed in the bags if each hag contains atleast k balls? **GATE – 2003**

(a) C(m – k, n – 1) (b) C (m – kn + n – 1, n – 1)
(c) C (m – 1, n – k) (d) C (m – kn + n + k – 2, n – k)

11. How many 4-digit even numbers have all 4 digits distinct? **GATE - 2001**

(a) 2240 (b) 2296 (c) 2620 (d) 4536

12. Two girls have picked 10 roses, 15 sunflowers and 15 daffodils. What is the number of ways they can divide the flowers among themselves? **GATE – 1999**

(a) 1638 (b) 2100 (c) 2640 (d) 2250

13. How many substrings of different lengths (non zero) can be formed from a character string of length n? **GATE – 1998**

(a) n (b) $\frac{n(n+1)}{2}$ (c) 2^n (d) n (n – 1)

14. In a room containing 28 people, there are 18 people who speak English, 15 people who speak Hindi and 22 people who speak Kannada. 9 persons speak both English and Hindi, 11 persons speak both Hindi and Kannada where as 13 persons speak both Kannada and English. How many people speak all three languages? **GATE – 1998**

(a) 9 (b) 8 (c) 7 (d) 6

KEY

01. a 02. a 03. a 04. (i) a 04. (ii) b 05. d 06. b 07. c 08. c 09. b
10. b 11. b 12. c 13. b 14. d

Set: Set is a collection of well defined objects (called the elements of the set).

Sub Set: Let A and B be any two sets. If every element of A is an element of B, then A is called the subset if B. Symbolically this relation is denoted by $A \subseteq B$.

Note: For any set A, $A \subseteq A$ and $\varnothing \subseteq A$

i.e for any set A, $\varnothing$ and A are called trivial sub sets.

Proper Sub Set: If $A \subseteq B$ and $A \neq B$ then A is called proper sub set of A. This relationship is denoted by $A \subset B$.

Equal Sets: Two sets A and B are equal iff $A \subseteq B$ and $B \subseteq A$. Sets are equal if they have same elements.

Empty Set: A set with no elements is called empty set and it is denoted by $\varnothing$.

Universal Set: The set of all objects under consideration or discussion denoted by U.

Singleton:A set with only one element is called singleton.

Set Difference:

$A - B = \{ x \mid x \in A \text{ and } x \notin B \}$

Set Unoin:

$A \cup B = \{ x \mid x \in A \text{ or } x \in B \text{ or both} \}$

Set Intersection:

$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$

Set Complement:

$A^C = \{ x \mid x \notin A \text{ and } x \in U \}$

Disjoint Sets: Two sets A and B are said to be disjoint sets if $A \cap B = \varnothing$

Symmetric Difference (Boolean Sum):

$A \Delta B = (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

$A \oplus B = \{ x \mid x \in A \text{ or } x \in B \text{ but not both} \}$

Cardinality Of A Set: The number of elements in a set is called the cardinality of that set. It is denoted by $|A|$

Power Set: Let A be any set. Then the set of all subsets of A is called power set of A. It is denoted by P(A)

Note: If a set A contains n elements then its power set P(A) contains 2^n elements.

Idempotent Laws:

$A \cup A = A$

$A \cap A = A$

Commutative Laws:

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Absorption Laws:

$A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

Associative laws:

$(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

Distrbutive Laws:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup \varnothing = A$

$A \cap U = A$

$A \cup U = U$

$A \cap \varnothing = \varnothing$

Demorgan's Laws:

$\sim(A \cup B) = \sim A \cap \sim B$

$\sim(A \cap B) = \sim A \cup \sim B$

$\sim\varnothing = U$

$\sim(\sim A) = A$

$A \cup \sim A = U$

$A \cap \sim A = \varnothing$

1. Which of the following Is false
   a) $A - (A - B) = A \cap B$ b) $(A \cup B) \cap C = A \cup (B \cap C)$
   c) $B - A = B \cap A^C$ d) $(A \cap B) \subset A \subset (A \cup B)$

2. If A is a proper sub set of B, then which of the following statements is not true
   a) $A \cap B = A$ b) $B^C \subseteq A^C$ c) $B \cup A^C = U$ d) $B - A = \varnothing$

3. If $A \subset B$, then which of the following is not true
   a) $A \cup B = B$ b) $A \cap B^C = A$ c) $B^C \subset A^C$ d) $A \cup (B - A) = B$

4. Which of the following is not true
   a) If $A \cap B = \varnothing$ then $A \subset B^C$ b) $A^C - B^C = B - A$
   c) $(B - A) \subset A^C$ d) $A \cup A^C = \varnothing$

5) Which of the following is not true
   a) $B - A^C = B \cap A$ b) If $A \subset B$ then $A - B = \varnothing$
   c) If $A \subset B$ then $A \cap B^C = \varnothing$ d) $A \cap A^C = U$

6) If the power set of A has 256 elements then the number of elements in A =
   a) 8 b) 9 c) 16 d) 3

7) Let A and B be two sets and let A^C and B^C denote the complements of the sets A and B.
   The set $(A - B) \cup (B - A) \cup (A \cap B) =$
   a) $A \cup B$ b) $A^C \cup B^C$ c) $A \cap B$ d) $A^C \cap B^C$

8. Which of the following is not true
   a) If A is a sub set of the null set $\varnothing$, then $A = \varnothing$
   b) $(A \cap B) \cup (A^C \cap B) \cup (A \cap B^C) \cup (A^C \cap B^C) = \varnothing$
   c) $(A \cap B) \cup (A \cap B^C) = A$
   d) $A \cap (A \cup B) = A$

9. Consider the following arguments

   I) H_1 : All my friends are musicians
   H_2 : John is my friend
   H_3: None of my neighbors are musicians
   C : John is not my neighbor.

   II) H_1 : All gold jewelry is expensive
   H_2 : No ear rings are expensive
   C : Ear rings are not made of gold.

   Which of the following is true

   a) Arg .I is valid and Arg. II is not valid b) Arg .I is not valid and Arg. II is valid
   c) Both arguments are valid d) Both Arguments are invalid

10. Consider the following assumptions
S1: No practical car is expensive
S2: Cars with sun roofs are expensive
S3: All wagons are practical
Which of the following conclusions is not valid
a) Some wagons are expensive
b) No practical car has sun roof
c) cars with sun roofs are not practical
d) No wagon has a sun roof

11. Consider the following arguments where S1, S2, S3 are premises and C1, C2 are conclusions

S1: Babies are illogical
S2: Nobody is despised who can manage a crocodile
S3: Illogical people are despised
C1: Babies cannot manage a crocodile
C2: Babies are despised.

Which of the following is true.

a) C_1 is valid and C_2 is not valid
b) C_1 is invalid and C_2 is valid
c) C_1 is valid and C_2 is valid
d) C_1 and C_2 are not valid

12. Let A and B be any two sets. The symmetric difference of A and B is defined by $A \oplus B = (A - B) \cup (B - A)$. Which of the following is false.
a) $A \oplus B = B \oplus A$
b) $A + \varnothing = A$
c) $A \oplus A = \varnothing$
d) $A \oplus B = (A \cap \sim B) \cap (B \cap \sim A)$

13. Consider the statement
S_1: $(A - B) - C = (A - C) - (B - C)$ S_2: $(A \cap B) \cup C = A \cap (B \cup C)$ iff $C \subseteq A$
Which of the following is true
a) S_1 is true and S_2 is false
b) S_1 is false and S_2 is true
c) S_1 and S_2 are false
d) S_1 and S_2 are true

14. Consider the following sets
$S_1 = \{ S \mid S \text{ is a set such that } S \notin S \}$ $S_2 = \{ S \mid S \text{ is a set} \}$

Which of the following is true

a) S_1 exists and S_2 does not exist
b) S_2 exists and S_1 does not exist
c) S_1 and S_2 do not exist
d) S_1 and S_2 both not exist

15. Let $\varnothing$ be an empty set and $P(\varnothing)$ is power set of $\varnothing$. Find the number of elements in $P(P(\varnothing))$
a) 0
b) 1
c) 2
d) 4

16. Which of the following is false.
a) $A \cap (B - A) = B$ b) $A \cup (B - A) = A \cup B$ c) $(A - C) \cap (C - B) = \varnothing$ d) $(A - B) - C \subseteq A - C$

17. Which of the following is false
a) If $A - B = B - A$ then $A = B$
b) If $A - B = A$ then $A \cap B = \varnothing$
c) If $A \cup B = A$ then $A \subseteq B$
d) If $A \cap B = B$ then $B \subseteq A$

18. Which of the following statements is not correct
a) If $A \oplus B = A$ then $B = \varnothing$
b) $(A \oplus B) \oplus B = A$
b) If $A \oplus C = B \oplus C$ then $A \Rightarrow B$
d) $A \oplus B = (A \cup B) \cup (A \cap B)$

19. The bit strings for the sets A{ 1,2,3,4,5 }and B{ 1,3,5,7,9 } are 11 1110 0000 and 101010 1010 respectively. Which of the following is not correct. (Let U = { 1,2,...10 })
   a) The bit string for the union of these sets is 11 1110 1010
   b) The bit strings for the intersection of these sets is 10 1010 0000
   c) The bit string for A – B is 01 0100 0000
   d) The bit string for B^C is 11 01010101

Multi Sets: Multi sets are unordered collections of elements where an element can occur as a member more than once. The notation { $m_1 . a_1, m_2 . a_2, \ldots\ldots m_n . a_n$ } denotes the multi set with element a_1 occurring m_1 times and so on.

m_i = multiplicity if a_i

1) Let P and Q are multi sets. The union of the multi sets P and Q is the multi set where the multiplicity of an element is the maximum of its multiplicities in P and Q.
2) The intersection of P and Q is the multi set where the multiplicity of an element is the minimum of its multiplicities.
3) The difference of P and Q is the multi set where the multiplicity of an element in P less its multiplicity in Q, unless the difference is negative. In which case the multiplicity is zero.
4) The sum of P and Q is denoted by P + Q (Add the multiplicities in P and Q)

20. Let A and B be the multi sets {3.a, 2.b, 1.c } and {2.a, 3.b, 4.d }then which of the following is false
   a) $A \cup B$ = {3.a, 3.b, 1.c, 4.d }
   b) $A \cap B$ = {2.a, 2.b }
   c) A - B = {1.a, 1.c }
   d) B - A = {1.b, 4.d }
   e) A + B = {5.a, 5.b, 2.c, 4.d }

21. The 'successor' of the set A is $A \cup \{A\}$ which of the following is not correct
   a) The successor of $\varnothing$ is $\{\varnothing\}$
   b) The successor of $\{\varnothing\}$ is $\{\varnothing, \{\varnothing\}\}$
   c) The successor of {1,2,3} is {1,2,3, {1,2,3}}
   d) The successor of a set with 'n' elements has n + 1 elements
   e) The successor of $\{\varnothing, \{\varnothing\}\}$ has 4 elements

Cartesian Product:

Cartesian Product of two sets A and B, written as A x B and is defined as

$A \times B = \{ (a,b) \mid a \in A \text{ and } b \in B \}$

$A \times B \times C = \{ (a,b,c) \mid a \in A, b \in B, c \in C \}$

Note:

1) In general, $A \times B \neq B \times A$
2) If $A \times B = B \times A$ then either A = B or (A or B is $\varnothing$)
3) If $|A| = m$, $|B| = n$ and $|C| = p$ then $|A \times B| = m.n$ and $|A \times B \times C| = m.n.p$
4) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
5) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Relaiton: A (binary) relation R from A to B is a sub set of A x B. If A = B , we say R is a relation on A.

Domain and Range: Let R be a relation from A to B. Then

Domain of R = $\{ x \mid x \in A \text{ and } (x,y) \in R \text{ for some } y \in B\}$

Range of R = $\{ y \mid y \in B \text{ and } (x,y) \in R \text{ for some } x \in A\}$

Clearly, dom $R \subseteq A$ and ran $R \subseteq B$. More over, domain of R is the set of first co – ordinates in R and its range is set of all second co- ordinates in R.

We some times write, $(x,y) \in R$ as $x \,^{R} y$ which reads 'x relates y'

Inverse Relation: Let R be a relation from a set A to B. The inverse of R, denoted by R^{-1} is the relation from B to A which consists of those ordered pairs, which when reversed belongs to R i.e , $R^{-1} = \{ (b,a) \mid (a,b) \in R\}$

Equivalence Relation: Let R be relation on A. R is said to be an equivalence relation if the following conditions are satisfied.

1) $x \,^R x \ \forall x \in A$ (R is reflexive)
2) If $x \,^R y$ then $y \,^R x \ \forall x, y \in A$ (R is symmetric)
3) If $x \,^R y$ and $y \,^R z$ then $x \,^R z$ for all $x,y,z \in A$ (R is transitive)

Anti Symmetric Relation: A relation R on a set A is anti symmetric relation if

$(a \,^R b \text{ and } b \,^R a) \Rightarrow a = b$; i.e, whenever $a \,^R b$ and $b \,^R a$ then $a = b$

Note:

1) The properties of being symmetric and being anti symmetric are not negatives of each Other
2) For the sets A and B, A x B is called universal relation from A to B and $\varnothing$ is called the Empty relation from A to B.

Irreflexive relation: A relation R on a set A is irreflexive if $(x , x) \notin R$ for all $x \in A$
Note: Any relation which is not reflexive is not necessarily irreflexive and vice – versa

Assymmetric relation:

$\forall x, y$ if $(x , y) \in R$ then $(y , x) \notin R$

Diagonal Relation: Let A be any set. The diagonal relation consists of all ordered pairs (a,b) such that a = b

i.e, $\Delta = \{(a , a) \mid a \in A\}$

PARTIAL ORDERING RELATION: *A RELATION R ON A SET IS CALLED A PARTIAL ORDER IF R IS REFLEXIVE, ANTISYMMETRIC AND TRANSITIVE.*

Poset : A set S together with a partial order R is called a ***partially ordered set*** or ***poset.***

Complementary Relation:

R is a relation from A to B then

$R^C = \{(a , b) \mid (a , b) \notin R\}$
$= (A \times B) - R$

22. If R_1, R_2, R_3 and R_4 are relations on a set A = { 1,2,3} then which of the following statements is false.
   a) $R_1 = \{(1,3), (3,1), (2,3)\}$ is neither symmetric or antisymmetric
   b) $R_2 = \{(1,1), (2,2)\}$ is both symmetric and antisymmetric
   c) $R_3 = \{(1,1), (2,3), (3,1)\}$ is antisymmetric but not symmetric
   d) $R_4 = \{(1,2), (2,1), (3,3)\}$ is symmetric but not antisymmetric
   e) All of the above statements are false

23. Let A and B are two sets such that $|A| = m$ and $|B| = n$. The number of relations from A to B =
   a) m.n b) 2^{mn} c) 2^{m+n} d) m^2n^2

24. The number of relations from A = { a,b,c } to B = { 1,2,3,4 } is =
   a) 12 b) 144 c) 128 d) 4096

25. Consider the following relations on a set A = { 1,2,3,4}
R_1 = { (1,1), (1,2), (2,3), (1,3), (4,4) }　　R_2 = { (1,1), (1,2), (2,1), (2,2), (3,3), (4,4) }
R_3 = { (1,3), (2,1)}　　R_4 = A x A

Which of the following statements is false

a) R_2 and R_4 are reflexive　　b) R_2 and R_4 are symmetric
c) R_1 , R_2 and R_4 are transitive　　d) R_1 , R_3 and R_4 are antisymmetric
e) R_1 , R_2 and R_4 are equivalences relations

26. Which of the following statements is false
a) The relation ≤ 'less than or equal to' on a set of real numbers is reflexive, transitive and Antisymmetric
b) The relation 'perpendicular to' on the set of lines in a plane is reflexive and symmetric
c) The relation ⊆ (set inclusion) on a collection of sets is a partial ordering relation.
d) The relation 'divides'(a / b) on a set of natural numbers is a partial order.

27. Let R be a non empty relation on a collection of sets, defined by $A \,^R\, B \Leftrightarrow A \cap B = \varnothing$ then which of the following is true.
a) R is reflexive and transitive　　b) R is symmetric and not transitive
c) R is an equivalence relation　　d) R is neither reflexive nor symmetric

28. Let A be a finite set of size n. The number of elements in the power set of A x A is (i,e.. The number of relations on a set A with n elements =)
a) 2^{2^n}　　b) 2^{n^2}　　c) (2^{2n})　　d) 2^n

29. Suppose A is a finite set with n elements. The number of elements in the largest equivalence relation of A is
a) n^2　　b) 2^n　　c) n　　d) 2n

30. The binary relation R = { (1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4)} on the set A= {1,2,3,4} is
a) an equivalence relation　　b) neither reflexive nor irreflexive, but transitive
c) irreflexive, symmetric and transitive　　d) irreflexive and antisymmetric

31. How many reflexive relations are there on a set with n elements?
a) 2^n　　b) 2^{n-1}　　c) $2^{n(n-1)}$　　d) $2^{n(n+1)}$

32. How many irreflexive relations are there on a set with n elements?
a) 2^n　　b) 2^{n-1}　　c) $2^{n(n-1)}$　　d) $2^{n(n+1)}$

33. How many symmetric relations are there on a set with n elements?
a) 2^n　　b) $2^{n(n-1)/2}$　　c) $2^{n(n+1)/2}$　　d) $2^{n(n-1)}$

34. How many antisymmetric relations are there on a set with n elements?
a) $2^n.3^{n(n-1)/2}$　　b) 2^n　　c) n^2　　d) n

35. How many assymmetric relations are there on a set with n elements?
a) $3^{n(n-1)/2}$　　b) 3^n　　c) $3^{n(n+1)/2}$　　d) 2^n

36. How many relations are there on a set with n elements that are reflexive and symmetric?
a) $2^{n(n-1)/2}$　　b) $2^{n(n+1)/2}$　　c) 2^n　　d) n^2

37. Which of the following statements is false
   a) If a relation R on a set A is symmetric and transitive then R is reflexive
   b) A relation R on a set A is symmetric if and only if $R = R^{-1}$ where R^{-1} is a inverse relation
   c) A relation R on a set A is antisymmetric if and only if $R \cap R^{-1}$ is a sub set of the diagonal relation $\Delta = \{ (a,a) \mid a \in A\}$
   d) A relation R on a set A is reflexive $\Leftrightarrow$ The inverse relation R^{-1} is reflexive
   e) A relation on a set A is reflexive $\Leftrightarrow$ The complimentary relation $\overline{R}$ is irreflexive

38) Which of the following statements is false
   a) If R and S are reflexive relation then $R \cap S$ is reflexive
   b) If R and S are symmetric then $R \cap S$ is also symmetric , and $R \cup S$ is symmetric
   c) If R and S are transitive relations on a set A, then $R \cap S$ is also transitive.
   d) If R is reflexive relation on a set A, then $R \cup S$ is reflexive for any relation S on A.
   e) If R and S are transitive relations then $R \cup S$ is also transitive.

39. Which of the following is not true?
   a) If R is an antisymmetric relation on the set A then R^{-1} is antisymmetric.
   b) If R is any relation on A then $R \cup R^{-1}$ is Symmetric.
   c) If R is antisymmetric then $R \cap S$ is antisymmetric for any relation S on A.
   d) If R and S are antisymmetric then $R \cup S$ is antisymmetric.
   e) If R is reflexive then $R \cap R^{-1} = \varnothing$

40. Which of the following is not a partial ordering relation?
   a) The relation $\subseteq$ (Set Inclusion) on any collection of sets.
   b) The relation $\leq$ (less than or equal to) on the set of Real numbers.
   c) The relation 'divides' (a / b) on the set of Natural numbers.
   d) The relation 'divides' on the set of all integers.
   e) The relation R defined over set of all integers as $a \,R\, b$ by $b = a^r$ for some **positive integer 'r'.**

Note: i) A relation R* is the transitive (symmetric, reflexive) closure of R , if R* is the smallest relation containing R which is transitive (symmetric, reflexive).
ii) $R \cup R^{-1}$ is the symmetric closure of R and $R \cup \Delta_A$ is the reflexive closure of R.

41. Which of the following is not a poset
   a) The set of all integers with relation <
   b) The set of all positive integers with relation | (divides)
   c) $[D_6; |]$ d) $[P; \subseteq]$ where P is any collection of sets

42. Which of the following is both a partial ordered set and a totally ordered set
   a) $[D_6; |]$ b) $[N; |]$ c) $[D_8; |]$ d) $[P(U); \subseteq]$
   Where P(U) is the power set of an arbitrary set U = {a, b}

43. Consider the following relations on the set A = { 1,2,3 }
   R = { (1,2), (2,1) }. **Which of the following relations represent the reflexive closure of R**
   a) { (1,2), (2,1), (1,1) } b) { (1,2), (2,1), (1,1), (2,2) }
   c) { (1,2), (2,1), (1,1), (2,2),)3,3) } d) A x A

44. Let R be a relation on A = { 1,2,3 } defined by R = { (1,1), (1,2), (2,3) } which of the following relations represent the symmetric closure of R
   a) { (1,1), (1,2), (2,1), (2,3), (3,2)} b) A x A
   c) a) { (1,1), (1,3) } d) { (1,2), (2,1), (2,3), (3,2) }

45. Let R be a relation on A = { 1,2 ,3 } defined by R = { (1,1), (1,3), (2,2), (3,1), (3,2)} which of the relations represent the transitive closure of R
a) A x A b) A x A – { (2,1), (2,3) } c) $R \cup R^{-1}$ d) $R \cup R^C$

46. Find the transitive closure of the following relation R on A = { a, b, c ,d }
R = { (a, d), (b, a), (b, c), (c, a), (c, d), (d, c) }
a) $R \cup R^{-1}$ b) A x A c) $R \cup \Delta_A$ d) (Ax A) – {(a, b), (b, b), (c, b), (d, c)}

47. Let R be a relation on set A = { a, b, c, d, e }given by
R = { (a, b), (a, c), (b, c), (c, d), (d, b), (d, e), (e, a) }
Then the transitive closure of R is
a) A x A b) $R \cup R^{-1}$ c) $R \cup \Delta_A$ d) $R \cup R^C$

48. Let R be a relation on the set A = {a, b, c, d }given by
R = { (a, b), (b, c), (d, c), (d, a) }
Then the transitive reflexive closure of R is given by
a) { (a, a), (a, b), (a ,c), (b, c), (b, d), (c, d), (d, a) , (d, b), (d, c), (d, d) }
b)A x A c) R ∪ { (a, a), (a, c), (b, d), (c, d), (d, d)} d) $R \cup R^{-1}$

Partition: A partition of S is a sub division of S in to non overlapping, non empty sub sets such that

$A_1 \cup A_2 \cup \ldots\ldots \cup A_n = S$

and $A_i \cap A_j = \varnothing$ if $A_i \neq A_j$

Equivalence Classes: Given any set A and an equivalence relation R on A, the equivalence class [x] of each element x of A is defined by

$[x] = \{ y \in A \mid x\, R\, y \}$

Note: i)We can have [x] = [y] even if x ≠ y, provided $x\, R\, y$

ii) Given any set A and any equivalence relation R on A, S = { [x] |x ∈ A } is a partition of A.

49. How many equivalence classes are there for the following equivalence relation 'Two people are equivalent , if they are born in the same week'.
a) 7 b) 52 c) 2 d) unlimited

50. How many equivalence classes are there for the following equivalence relation 'Two people are equivalent , if they are born in the same year'.
a) 365 b) 2 c) 2003 d) unlimited

51. Which of the following collection of subsets is a partition of { 1,2 ,3,4,5,6 }
a) { 1,2 } ,{2,3,4}, {4,5,6 } b) {1,4,5}, {2,6 }
c){ 1 } ,{2,3,6}, {4},{5 } d) { 1,2 ,3,4}, {5,6,7 }

52. Consider the equivalence relation R = { (x, y)| x – y is an integer}
Which of the following is equivalence class of 1
a) n b) Z c) { 0,1} d) does not exist

53. For the relation given in the previous example, which of the following is the equivalence class of 1/2
a) {(n + 1/2)| n ∈ Z} b) Z c) N d) ∅

54. Determine the number of different equivalence relations on a set with 3 elements.
a) 7 b) 5 c) 3 d) 1

55. Let R be the relation on the set of integers defined by "x - y is divisible by 5". How many equivalence classes are there on R.

a) 5 b) 32 c) 25 d) unlimited

Comparability:

Two elements a and b in a set A are said to be comparable under the relation $\leq$, if either $a \leq b$ or $b \leq a$. Otherwise, they are not comparable.

If every pair of elements of A are comparable, then we say $[A, \leq]$ is a totally ordered set (or) linearly ordered set (or) chain .

Note: If n is a positive integer, then D_n = set of all positive divisors of n.

Ex: $D_{12} = \{1,2,3,4,6,12\}$

Least Upper Bound (lub) (Join or Supremum):

The lub of two elements a and b is denoted by $a \vee b$

If $c = a \vee b$ then c satisfies

i) $c \geq a$ and $c \geq b$

ii) If $d \geq a$ and $d \geq b$ then $d \geq c$

i.e.. c is lub of $\{a, b\}$

Greatest Lower Bound (glb)(meet or infimum)

The g.l.b of two elements a and b is denoted by $a \wedge b$

If $c = a \wedge b$ then c satisfies

i) $c \leq a$ and $c \leq b$

ii) If $d \leq a$ and $d \leq b$ then $d \leq c$

* Both join and meet operations are commutative and associative

Ex: In the poset $[R; \leq]$, $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$

Ex: For the poset $[D; |]$, $a \vee b$ = L.C.M of a and b

$a \wedge b$ = G.C.D of a and b

Ex: If S is any collection of sets, then for the poset $[S; \subseteq]$

$A \cup B$ is the lub and $A \cap B$ is glb of A and B

Poset Diagram (Hasse diagram)

Let $[A; \leq]$ be a poset

On a poset diagram

i) There is a vertex for each element of A

ii) All loops are omitted eliminating the explicit representation of reflexive property.

iii) An edge is not present in a poset diagram if it is implied by transitivity of the relation.

iv) An edge connects a vertex x to a vertex $y \Leftrightarrow y$ covers x i.e...iff there is no element z such that $x < z$ and $z < y$.

Join Semi lattice: A poset $[A; \leq]$ in which each pair of elements a and b of A have a least upper bound is called join semi lattice.

Meet Semi Lattice: A poset $[A; \leq]$ in which each pair of elements a and b of A have a glb (meet) is called 'meet semi lattice'.

Lattice: A lattice is a poset in which each pair of elements has a lub and a glb. In other words, a lattice is both a join semi lattice and a meet semi lattice.

A lattice is often denoted by $[L, \wedge, \vee]$

The following laws hold in L

i) $a \vee b = b \vee a$ and $a \wedge b = b \wedge a$

ii) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$ and $(a \vee b) \vee c = a \vee (b \vee c)$

iii) $a \wedge (a \vee b) = a$ and $a \vee (a \wedge b) = a$ iv) $a \wedge a = a$ and $a \vee a = a$

Note: i) Let L be a lattice, then $a \wedge b = a \Leftrightarrow a \vee b = b$

ii) The Lub and Glb of any pair (a, b), if exists are unique.

Sub Lattice : Suppose M is a non empty subset of a lattice L. We say M is a sub lattice of L if M itself is a lattice.

Bounded lattice: A lattice L is said to have lower bound 0 if $0 \le x \ \forall x \in L$.
Similarly L is said to have an upper bound I, If $x \le I$ for all $x \in L$
We say L is bounded if L has both a lower bound 0 and upper bound I.
$a \vee I = I, \ a \wedge I = a, \ a \vee 0 = a, a \wedge 0 = 0$
Note: Every finite lattice L is bounded.

Distributive Lattice: A lattice $(L, \cup, \cap)$ is said to be distributive if the following dist. Laws hold. $\forall$ a, b, c

$$\left.\begin{aligned} a \cup (b \cap c) &= (a \cup b) \cap (a \cup c) \\ a \cap (b \cup c) &= (a \cap b) \cup (a \cap c) \end{aligned}\right\}$$

Complemented Lattice: Let L be a bounded lattice with lower bound 0 and upper bound I. Let ***a*** be an element of L. An element ***x*** in L is called a complement of ***a*** , if $a \vee x = I$ and $a \wedge I = 0$

Note: In a lattice, compliments need not exist and need not be unique.
Def: A lattice L is said to be a complemented lattice, if L is bounded and every element in L has a complement.

Note: Let L be a bounded distributive lattice. Then complements are unique if they exist.

56. Which of the following pairs of elements are comparable in the poset $(Z^+, |)$
a) 6, 9 b) 3, 5 c) 5, 15 d) 4, 6

57. Which of the following pairs of elements are incomparable in the poset [P{ 0,1,2}, ≤]
a) {0}, {0,1} b) ∅, {0,1,2} c) {1,2}{0,1,2} d) {0}, {1}

KEY

1. b 2. d 3.b 4.d 5. d 6.a 7. a 8. b 9. a 10. a 11. c 12. d

13. d 14.c 15. c 16. a 17. c 18. d 19. d 20. e 21.e 22. e 23. b 24. d

25. e 26. b 27. b 28. b 29. a 30. b 31. c 32. c 33. c 34 .a 35.a 36. a

37. a 38. e 39. e, d 40. d 41. a 42. c 43. c 44. a 45. b 46. d 47. a , d

48. a 49. b 50. d 51. c 52. b 53. a 54 .b 55. a 56. c 57.d

PREVIOUS GATE QUESTIONS

01. What is the possible number of reflexive relations on a set of 5 elements? **(GATE-2010)**
a) 2^{10} b) 2^{15} c) 2^{20} d) 2^{25}

02. Consider the binary relation R = {(x, y), (x, z), (z, x), (z, y)} on the set {x, y, z}. Which one of the following is true **GATE – 2009**
(a) R is symmetric but not antisymmetic (b) R is not symmetric but antisymmetric
(c) R is both symmetric and antisymmetric (d) R is neither symmetric nor antisymmetric

03. If P, Q, R are subsets of the universal set □ then $(P \wedge Q \wedge R) \vee (P^C \wedge Q \wedge R) \vee (Q^C \wedge R^C)$ is **GATE – 2008**
(a) $Q^C \vee R^C$ (b) $P \vee Q^C \vee R^C$ (c) $P^C \vee Q^C \vee R^C$ (d) □

04. Let S be a set with n elements. The number of ordered pairs in the smallest and largest equivalence relations on S are **GATE – 2007**
(a) n and n (b) n and n^2 (c) 0 and n^2 (d) 1 and n

05. Let A and a set with n elements. Let C be a collection of distinct subsets of A such that for any two subsets S_1 and S_2 in C, either $S_1 \subset S_2$ or $S_2 \subset S_1$. What is the maximum ardinality of C **GATE – 2005**
(a) n (b) n + 1 (c) $2^{n-1} + 1$ (d) $\angle n$

06. For the set N of natural numbers and a binary operation $f : N \times N \to N$, an element $Z \in N$ is called an identity for f if f(a, Z) = a = f(Z, a), for all $a \in N$. Which of the following binary operations have an identity?
I) f(x, y) = x + y – 3
II) f(x, y) = max (x, y)
III) $f(x, y) = x^y$ **GATE – 2006**
(a) I and II only (b) II and III only
(c) Neither I nor II (d) Both I and III

07. Let P, Q and R be sets. Let Δ debote the symmetric difference operator defined as $P \Delta Q = (P \vee Q) - (P \wedge Q)$. Which of the following is/are true
I) $P \Delta (Q \wedge R) = (P \Delta Q) \wedge (P \Delta R)$
II) $P \wedge (Q \Delta R) = (P \wedge Q) \Delta (P \wedge R)$
(a) I only (b) II only (c) Neither I nor II (d) Both I and II

08. A relation R is defined on ordered pairs of integers as follows :
(x, y) R (u, v) if x < u and y > v. Then R is **GATE – 2006**
(a) Neither a partial order nor an equivalence relation
(b) A partial order but not a total order
(c) A total order
(d) An equivalence relation

09. Let E, F and G be finite sets.

Let $X = (E \wedge F) - (F \wedge G)$ and $Y = (E - (E \wedge G)) - (E - F)$. Which one of the following is true? **GATE – 2006**

(a) $X \subset Y$ (b) $X \supset Y$
(c) $X = Y$ (d) $X - Y \neq \emptyset$ and $Y - X = \emptyset$

10. Consider the binary relation
$S = \{(x, y) \mid y = x + 1 \text{ and } (x, y) \in \{0, 1, 2, \ldots\}\}$ The reflexive transitive closure of S is **GATE – 2004**

(a) $\{(x, y) \mid y > x \,\&\, x, y \in \{0, 1, 2, \ldots\}\}$ (b) $\{(x, y) \mid y \geq x \,\&\, x, y \in \{0, 1, 2, \ldots\}\}$
(c) $\{(x, y) \mid y < x \,\&\, x, y \in \{0, 1, 2, \ldots\}\}$ (d) $\{(x, y) \mid y \leq x \,\&\, x, y \in \{0, 1, 2, \ldots\}\}$

11. The binary relation $S = \emptyset$ (empty set) on set $A = \{1, 2, 3\}$ is **GATE – 2002**
(a) Neither reflexive nor symmetric (b) Symmetric and reflexive
(c) Transitive and reflexive (d) Transitive and Symmetric

12. Consider the following relations :
R_1 (a, b) iff (a + b) is even over the set of integers
R_2 (a, b) iff (a + b) is odd over the set of integers
R_3 (a, b) iff $ab > 0$ over the set of nonzero rational numbers
R_4 (a, b) iff $|a - b| \leq 2$ over the set of natural numbers
Which of the following statements is correct **GATE – 2001**
(a) R_1 and R_2 are equivalence relations, R_3 and R_4 are not
(b) R_1 and R_3 are equivalence relations, R_2 and R_4 are not
(c) R_1 and R_4 are equivalence relations, R_2 and R_3 are not
(d) R_1, R_2, R_3 and R_4 are equivalence relation

13. Let P(S) denote the power set of a set S. Which of the following is always true? **GATE – 2000**

(a) $P(P(S)) = P(S)$ (b) $P(S) \wedge P(P(S)) = \{\emptyset\}$
(c) $P(S) \wedge S = P(S)$ (d) $S \notin P(S)$

14. A relation R is defined on the set of integers as x R y iff (x - y) is even. Which of the following statements is true? **GATE – 2000**
(a) R is not an equivalence relation
(b) R is an equivalence relation having 1 equivalence class
(c) R is an equivalence relation having 2 equivalence class
(d) R is an equivalence relation having 3 equivalence class

15. How many multisets can be constructed from n distinct elements? **GATE – 2000**
(a) 2^n (b) $\underline{\lfloor n}$ (c) n^n (d) none of these

16. How many multisets of size 4 can be constructed from n distinct elements so that atleast one element occurs exactly twice? **GATE – 2001**

(a) $\frac{n(n-1)}{2}$ (b) $\frac{n.(n+1)}{2}$ (c) $\frac{n(n-1)^2}{2}$ (d) $\frac{n.(n+1)^2}{2}$

17. The number of binary relations on a set with n elements is **GATE - 1999**

(a) n^2 (b) 2^n (c) 2^{n^2} (d) $\underline{\lfloor n}$

KEY : 01. c 02. d 03. d 04. b 05. b 06. a 07. b 08. a 09. c 10. b 11. d
12. b 13. b 14. c 15. d 16. c 17. c

Maximal Element: An element of a poset that is not less than any other element of the poset.
Minimal Element: An element of a poset that is not greater then any other element of the poset.

1) Every finite non empty poset has a minimal element.
2) If a lattice has a universal lower bound (universal upper bound) it is unique.
3) Every totally ordered set is a lattice.
4) Every totally ordered set has a least element and a greatest element.

1. The set [{ 1,2,3,4,6,9 } ; |] is _______
   a) a lattice b) a join semi lattice c) a meet semi lattice d) not a poset

2. The set [{ 2,3,5,30,60,120,180,360 } ; |] is _______
   a) a lattice b) a join semi lattice
   c) a meet semi lattice d) neither a join semi lattice nor a meet semi lattice

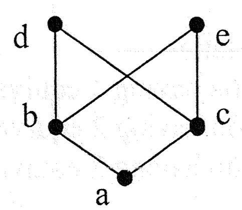
3. The set [{ 2,3,4,9,12,18} ; |] is _______
   a) a lattice b) a join semi lattice
   c) a meet semi lattice d) neither a join semi lattice nor a meet semi lattice

4. The set [D_{12} ; |] is
   a) a lattice b) a join semi lattice c) a meet semi lattice d) not a semi lattice

5. Which of the following is not true
   a) The poset [Z^+, |] is a lattice
   b) The poset [P(S) ; $\subseteq$] is a lattice where S is a set
   c) The poset [D_n ; |] is a lattice where n is a +ve integer
   d) [Z, |] is not a lattice

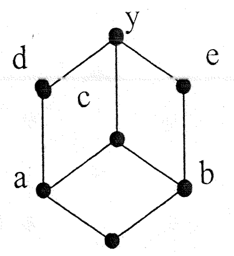
Note: The lub and glb of any pair (a, b) in a poset, if exists, are unique.

6. Consider the poset P = { a, b, c, d, e }shown below



which of the following statements is false
a) P is not a lattice
b) The sub set { a, b, c, d } of P is a lattice
c) The sub set { b, c, d, e } of P is a lattice
d) The sub set { a, b, c, e } of P is a lattice

7. Consider the Lattice L = { x, a, b, c, d, e, y } shown below



which of the following is a sub Lattice of L x

a) { x, a, b, y } b) { x, a, e, y } c) { x, c, d, y } d) { x, a, b, y }

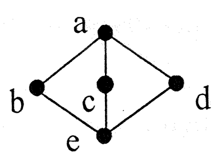
8. Which of the following statements is false, for the Lattice [P(A) ; ⊆]

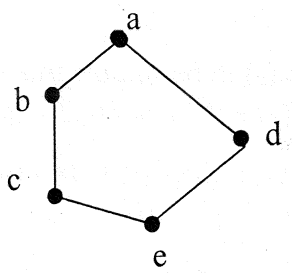
a) The upper bound of [P(A) ; ⊆] is A
b) The lower bound of [P(A) ; ⊆] is ∅
c) The upper bound of [N ; ≤] does not exist
d) The lower bound of [N ; ≤] is 0

9. Which of the following is not true

a) [P(A) ; ⊆] is a distributive lattice
b) In a distributive lattice if $b \wedge \overline{c} = 0$ then $b \leq c$
c) If L is a bounded distributive lattice, the complements are unique, if they exist
d) Every distributive lattice is a complemented lattice

10. Consider the following Lattices L_1* and L_2*





Which of the following is true

a) L_1* is distributive and L_2* is not distributive
b) L_1* is not distributive and L_2* is distributive
c) Both Lattices are distributive
d) Both Lattices are non distributive

11. Which of the following is not a distributive Lattice

a) [D_8 ; |] b) [D_{12} ; |] c) [{1,2,3,5,30}, |] d) [P{a,b,c}, ⊆]

12. Which of the following is not true

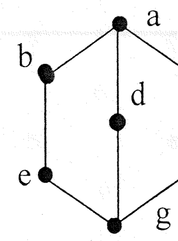
a) The sub lattice of a distributive lattice is distributive
b) Every linearly ordered set is distributive
c) c) Every distributive lattice is bounded
d) A lattice is not distributive iff L has a sub lattice isomorphic to L_1* or L_2* (in Ex: 10)

13. For the Lattice [D_{12} ; |], which of the following is not true

a) complement of 1 is 12
b) complement of 3 is 4
c) complement of 2 is 6
d) complement of 2 and 6 does not exist

14. In the Lattice defined by the Hasse diagram given below, how many complements does the element 'e' have

a) 1 b) 2 c) 3 d) 0



15. For the Lattice given in the previous problem which of the following is true

a) The lattice is a distributive Lattice b) The complement of e is a
c) The complement of f is b d) The complement of c is f

16. For the Lattice $[P(A) ; \subseteq]$, where $A = \{a, b, c\}$ the complement of $\{a\}$ is

a) $\{b, c\}$ b) $\{b\}$ c) $\{c\}$ d) A

17. Which of the following posets is not a Lattice

a) $[\{1,3,6,9,12\} ; |]$ b) $[\{1,5,25,125\} ; |]$ c) $[\{1,2,4,8,16\} ; |]$ d) $[Z ; \geq]$

Boolean Algebras (Boolean Lattice)

A Lattice that contains the element 0 and 1, and which is both distributive and complemented is called a Boolean Algebra.

Ex: Let Π be the set of propositions. Π is a Boolean algebra under the operations $\vee$ and $\wedge$ with negation $\sim$ being the complement, a contradiction F is the zero element and a tautology 1 as the unit element.

Note: A positive integer n is said to be ***square free*** if it does not have any divisors which are perfect squares(other then 1) (i.e. 4, 6, 9, 16, 25, etc......)

Ex: If n is a square free integer, then $[D_n ; |]$ is a Boolean algebra. Where, 1 is zero element, n is unit element and $\overline{}x = (n / x) \quad \forall x \in D_n$

18. Which of the following is not a Boolean algebra

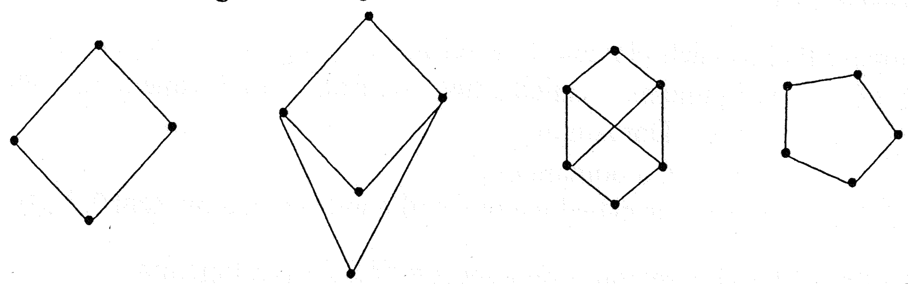
a) $[D_{18} ; |]$ b) $[D_{21} ; |]$ c) $[D_{110} ; |]$ d) $[D_{91} ; |]$

KEY

1. b 2. c 3.d 4. a 5. d 6. c 7. b 8. d 9. d 10. d 11. c 12. c
13. c 14. c 15. c 16. a 17. a 18. a

PREVIOUS GATE QUETSIONS

01. Consider the following Hassi diagrams



Which of the above represent a Lattice ? **GATE – 2008**

(a) 1 and 4 only (b) 2 and 3 only (c) 3 only (d) 1, 2 and 4 only

02. What is the maximum number of different Boolean functions involving n Boolean variables? **GATE – 2006**

(a) n2 (b) 2n (c) 2^{2^n} (d) 22n

03. The inclusion of which of the following sets into
S = {{1, 2}, {1, 2, 3}, {1, 3, 5}, {1, 2, 4}, {1, 2, 3, 4, 5}}
is necessary and sufficient to make S a complete lattice under the partial order defined by set containment? GATE – 2004

(a) { 1 } (b) { 1 }, { 2, 3 }
(c) { 1 }, { 1, 3 } (d) { 1 }, {1, 3}, {1, 2, 3, 4}, {1, 2, 3, 5}

04. Let L be a set with a relation R which is transitive, anti-symmetric and reflexive and for any two elements a, b ∈ L the least upper bound and greatest lower bound exist. Which of the following is/are true **GATE – 1999**

(a) L is a poset (b) L is a Boolean algebra
(c) L is a Lattice (d) None of these

05. Let X = {2, 3, 6, 12, 24}. Let ≤ be a partial order defined by X ≤ Y iff x divides y. Number of edges as in the Hasse diagram of (X, ≤) is **GATE – 1996**

(a) 3 (b) 4 (c) 9 (d) none of these

KEY: 01. a 02. c 03. d 04. a & c 05. b

FUNCTIONS

DEF: A function $f: A \rightarrow B$ is a relation from A to B such that each $a \in A$ belongs to a unique ordered pair (a, b) in f

DEF:. Suppose that to each element of a set A, we assign a unique element of a set B. The collection of such assignments is called a function from A to B. (mapping from A to B)

A = Domain of f

B = Co-domain of f

The set of all image values is called the range of f and denoted by Ran(f), $I_m(f)$ or $f(A)$.

1. Which of the following relations on a set A = {1,2,3} is a function
   a) {(1,3), (2,3), (3,1)}   b) {(1,2), (3,1), (1,3)}
   c) {(1,2), (3,2)}   d) {(1,2), (2,3), (2,3)}

ONE - TO – ONE FUNCTION (INJECTION): A function $f: A \rightarrow B$ is said to be an one – to – one function, if different elements in the domain A have distinct images.

$f: A \rightarrow B$ is one - to – one iff $f(a) = f(b) \Rightarrow a = b$

ON – TO FUNCTION:
A function $f: A \rightarrow B$ is said to be an on – to function, if each element of B is the image of some element of A. i.e Ran $f = B$ or $f(A) = B$

ONE – TO –ONE & ON – TO FUNCTION (BIJECTION):
A function $f: A \rightarrow B$ is said to be a bijection if f is 1-1 and on – to.

INVERSE OF A FUNCTION:
$f: A \rightarrow B$ is invertible if its inverse relation f^{-1} is a function from B to A.

NOTE: A function $f: A \rightarrow B$ invertible if and only if f is a bijection.

CONSTANT FUNCTION:

$f(x) = c \ \forall x$

IDENTITY FUNCTION: A mapping $I : A \rightarrow A$ is called a identity function if
$I = \{(x, x) \mid x \in A\}$

2.The number of subsets of the set {1,2, ….n} with odd cardinality =
   a) 2^{n-1} b) $2^n - 1$ c) 2^{n+1} d) $2^{n(n-1)/2}$

3. The number of functions from an m element set to an n element set is
   a) m + n   b) m . n   c) m^n d) n^m

4. The number of functions on to the set A = {1,2,3,4} are
   a) 16   b)64   c) 256   d) 2^{16}

5. Let A and B be two sets with cardinalities m and n respectively. The number of one-to-one mappings (injections) from A to B (when m<n) is
   a) m^n b) n^m c) P(n,m)   d) C(n,m)

6. If A ={ a, b, c, d, e} then the number of one-to-one function from A to A =
   a) 25   b) 32   c) 120   d) 3125

7. Suppose X and Y are set and $|X|$ and $|Y|$ are their respective cardinalities. It is given that there are exactly 97 functions from X to Y. From this one can conclude that
   a) $|X| = 1, |Y| = 97$
   b) $|X| = 97, |Y| = 1$
   c) $|X| = 97, |Y| = 97$
   d) None of these

8. Which of the following is not true
   a) A constant function is one-to-one $\Leftrightarrow$ The domain consists of exactly one element.
   b) A constant function is on-to $\Leftrightarrow$ The codomain consists of exactly one element.
   c) For any set A, the Identity function on A is a bijection.
   d) $f: A \rightarrow B$ is one-to-one $\Leftrightarrow |A| = |B|$

9. Which of the following is not true
   a) If $f: A \rightarrow B$ is a bijection and A and B are finite sets then $|A| = |B|$
   b) If $A = \{1, 2, \ldots.. n\}$, then any function from A to A which is one-to-one must also be on-to.
   c) Every surjection on a finite set A is also an injection
   d) Every constant function is a surjection.

10. Which of the following is not true
   a) Every surjection on a finite set A is also a bijection.
   b) Every injection on a finite set A is also a bijection
   c) If a set A has n elements, then there are n! bijections from A to A.
   d) Every equivalence relation on A is a function on A.

11. Which of the following is a function from R to R
   a) $f(x) = 1/x$
   b) $f(x) = \sqrt{x}$
   c) $f(x) = \pm\sqrt{x^2+1}$
   d) $f(x) = |x|$

12. Determine whether f is a function from the set of all bit strings to the set of all integers if
   i) f(S) is the position of a 0 bit in S ii) f(S) is the number of 1 bits in S

   Which of the following is true

   a) (i) is a function and (ii) is not a function
   b) (i) is a function and (ii) is a function
   c) Both (i) and (ii) are not functions
   d) (i) is not a function and (ii) is a function

13. Consider the following functions from Z to Z
   S_1: $f(x) = x^3$ is one-to-one but not on-to
   S_2: $f(n) = \lceil n/2 \rceil$ is on-to but not one-to-one
   Which of the following is true
   a) S_1 is true and S_2 is true
   b) S_1 is true and S_2 is false
   c) S_1 is false and S_2 is true
   d) S_1 is false and S_2 is false

14. Let f be a function from the set A to the set B. Let S and T be subsets of A consider the following statements
   S_1: If f is one-to-one then $f(S \wedge T) = f(S) \wedge f(T)$
   S_2: If f is on-to but not 1 –1 then $f(S \wedge T) = f(S) \wedge f(T)$
   Which of the following is true .
   a) S_1 is true and S_2 is false
   b) S_1 is false and S_2 is true
   c) S_1 is true and S_2 is true
   d) S_1 is false and S_2 is false

15. Given $f: A \rightarrow B$ and $g: B \rightarrow C$, which of the following is not true
   a) If f and g are injections then g o f is an injection
   b) If f and g are surjections then g o f is a surjection
   c) If g o f is one-to-one then f is one-to-one
   d) If g o f is on-to then g is on-to

16. Let $A = R - \{3\}$, $B = R - \{1\}$. Let $f : A \rightarrow B$ is defined by $f(x) = (x-2)/(x-3)$. Then which of the following is not true.
a) f is a bijection b) f is one-to-one c) f is on-to d) one-to-one but not on-to

17. Let f and g are functions defined by $f(x) = x / (x+1)$ and $g(x) = x / (1-x)$ then $(f \circ g)^{-1} x$ =
a) x b) 1/x c) 1 d) not defined

18. If $f(x) = 1/(1-x)$ then which of the following is false
a) $f \circ f \circ f$ is an identity function on its domain b) $f \circ f \circ f$ is an bijection on its domain
c) $f \circ f \circ f$ is an one–to-one function d) $f \circ f \circ f$ is a constant function

19. Which of the following is true
a) Every function can be represented graphically
b) The functions $f(x) = x$ and $g(x) = \sqrt{x^2}$ are identical
c) The functions $f(x) = \log x^2$ and $g(x) = 2 \log x$ are identical
d) The domain of $f(x) = 1/(\sqrt{|x|} - x)$ is $(-\infty, 0)$

20. The range of the function
$$f(x) = x^2 / (x^4 + 1) \text{ is}$$
a) (0, 1/2) b)[0, 1/2] c)[0, 2] d)(0, 2)

21. The domain of the function
$$f(x) = \sin \log \{ \sqrt{4 - x^2} / (1-x)\} \text{ is}$$
a) (-2, 1) b) (1, 2) c) (-2, 2) d)(-1, 2)

22. Which of the following functions have inverse defined on their ranges?
a) $f(x) = x^2, x \in R$ b) $f(x) = x^3, x \in R$
c) $f(x) = \sin x, 0<x<2\pi$ d) $f(x) = e^x, x \in R$

23. Let $f : A \rightarrow B$ is a bijection then which of the following is false
a) $f \circ f^{-1} = I_A$ b) $f^{-1} \circ f = I_A$ c) $I_B \circ f = f$ d) $f \circ I_A = f$

NOTE: Let m and n be positive integers with $m > n$. Then, there are
$n^m - C(n,1).(n-1)^m + C(n,2).(n-2)^m \ldots\ldots + (-1)^{n-1}C(n, n-1).1^m$
on-to functions from a set with m elements to a set with n elements

24. How many on-to functions are there from a set with six elements to a set with 3 elements?
a) 240 b) 340 c) 440 d) 540

25. How many on-to functions are there from a set with 5 elements to a set with 4 elements?
a) 240 b) 340 c) 440 d) 540

GROUPS

BINARY OPERATION: The binary operator * is said to be a binary operation on a non empty set A, if
$$a * b \in A \quad \text{for all} \quad a, b \in A$$
ALGEBRAIC SYSTEM: A set 'A' with one or more binary operations defined on it is called an algebraic system.
Ex: (A, *), (A, *, Δ), (R, +, .)
ASSOCIATIVITY: Let * be a binary operation on a set A. The operation * is said to be associative if $(a * b) * c = a * (b * c)$ for all a, b, c in A

IDENTITY: For an algebraic system (A, *) , an element 'e' in A is said to be an identity element of A if

$$a * e = e * a = a \quad \text{for all} \quad a \in A$$

NOTE: For an algebraic system (A, *), the identity element, if exists, is unique.

INVERSE: Let (A, *) be an algebraic system with identity 'e'. Let a be an element in A. An element b is said to be inverse of A if

$$a * b = b * a = e$$

SEMI GROUP: An algebraic system (A, *) is said to be a semi group if

1. * is closed operation on A
2. * is an associative operation, for all a, b, c in A

Monoid: An algebraic system (A, *) is said to be a **monoid** if the following conditions are satisfied.

1) * is a closed operation
2) * is an associative operation
3) There is an identity in A

GROUP: An algebraic system (A, *) is said to be a **group** if the following conditions are satisfied.

1) * is a closed operation
2) * is an associative operation
3) There is an identity in A
4) Every element in A has inverse

ABELIAN GROUP: A group (G, *) is said to be ***abelian*** or ***commutative*** if

$$a * b = b * a \quad \forall a, b \in G$$

1. If A and B are sets of even and odd integers then which of the following is not a semi group
a) (A, .) b) (A, +) c) (B, .) d) (B, +)

2. (N, +) is a
a) Semi group b) monoid c) group d) Abelian group

3. The set of all strings under the operation concatenation of strings is
a) a monoid b) an abelian group c) a group d) not a semi group

4. Let S be a finite set, and let F(S) be the collection of all functions f: S → S under the operation of composition of functions, then F(S) is
a) a monoid b) a group c) an abelian group d) not a semi group

5. Which of the following is true
a) set of all real numbers is a group with respect to multiplication
b) set of all rational numbers is a group with respect to multiplication
c) set of all integers is a group with respect to addition
d) set of all natural numbers in a group with respect to addition

6. Which of the following is false
a) The set of all bijective functions on a finite set forms a group under function composition
b) The set {1, 2 ,...., p – 1} forms a group under multiplication modulo 'p' where p is a prime number
c) The set of all strings over a finite alphabet forms a group under concatenation.
d) In a group, the identity element is unique.

7. If A is set of all non singular matrices of order 'n' and * is matrix multiplication operation then which of the following is false

a) (A, *) is a monoid b) (A, *) is a group
c) (A, *) is a abelian group d) (A, *) is a semi group

8. If (G, *) is a group then which of the following is false

a) $a * b = a * c \Rightarrow b = c$ b) $a * c = b * c \Rightarrow a = b$
c) $a * b = b * a$ d) $(a * b)^{-1} = b^{-1} * a^{-1}$

9. Let (Z, *) be an algebraic structure, where Z is the set of integers and the operation * is defined by $n * m$ = maximum of (n, m). Which of the following is true

a) (Z, *) is a monoid b) (Z, *) is a group
c) (Z, *) is a abelian group d) none of these

10. Which of the following is not true

a) set of all even integers with zero is an abelian group.
b) Set of all non zero complex numbers form an abelian group
c) Set of all rational numbers 'x' such that $0 < x \leq 1$, is a group w.r.t ordinary multiplication
d) Set of all vectors is a group w.r.t vector addition.

11. The set of all positive rational numbers forms an abelian group under the composition * defined by $a * b = (ab)/2$ which of the following is not true

a) The identity element is 2 b) The inverse of a is 2/a.
c) The inverse of 4 is 1 d) The identity element is 1

12. Let R be the set of all real numbers and * is a binary operation defined by $a * b \equiv a + b + ab$. Which of the following is false

a) Identity element is 0 b) the inverse of -1 is 1
c) The inverse of a is $-a/(a+1)$ d) R is not a group

Order of a group is equal to the number of elements in that group.
* In a group, the identity element is its own inverse.
* Every finite group of order less than 6 must be abelian.

13. Which of the following is not true

a) Every group of order 3 is abelain
b) G = {1, -1} is an abelian group under multiplication
c) In a group with 2 elements, each element is its own inverse
d) G = {1, -1, i, -i} is not a group

14. Which of the following is false?

a) If (G, *) is a group and $a \in G$ such that $a * a = a$ then $a = e$
b) If every element of a group is its own inverse, then the group must be abelian
c) In a group of even order there will be atleast one element (other than identity element) which is its own inverse
d) d) In a group of order 3 each element is its own inverse

15. The set G = {0,1,2,3,4,5} is a group with respect to addition modulo 6. Which of the following is false

a) the inverse of 2 is 4 b) the inverse of 3 is 3
c) the inverse of 5 is 2 d) the inverse of 1 is 5

16. The set $G = \{1,2,3,4,5,6\}$ is a group with respect to multiplication modulo 7. Which of the following is false

a) the the inverse of 2 is 5 b) the inverse of 4 is 2
c) the inverse of 3 is 5 d) the inverse of 6 is 6

17. Which of following is a group

a) The set $\{1, 2, 3, 4, 5\}$ is a group under addition modulo 6.
b) The set $\{1, 2, 3, 4, 5\}$ is a group under multiplication modulo 6.
c) The set $\{1, 2, 3, 4, 5, 6\}$ is a group under addition modulo 7.
d) The set $\{1, 2, 3, 4, 5, 6\}$ is a group under multiplication modulo 7

ORDER OF AN ELEMENT OF A GROUP:

Let (G, .) be a group. Let 'a' be an element of G. The smallest integer n such that $a^n = e$ is called order of 'a'. If no such number exists then the order is infinite.

18. $G = \{1, -1, i, -i\}$ is a group w.r.t multiplication . The order $-i$ is

a) 2 b) 3 c) 4 d) 1

19. Which of the following is not true.

a) The order of every element of a finite group is finite and is less than or equal to the order of the group.
b) The order of an element of a group is same as that of its inverse.
c) In the additive group of integers the order of every element except 0 is infinite
d) In the infinite multiplicative group of nonzero rational numbers the order of every element except 1 is infinite.

SUB GROUP

A non empty sub set H of a group (G, *) is a sub group of G, if (H, *) is a group.

NOTE: 1) A non-empty sub set H of a group (G, *) is a sub group of G iff

i) $a * b \in H \quad \forall\, a, b \in H$
ii) $a^{-1} \in H \quad \forall\, a \in H$

2) A necessary and sufficient condition for a non empty subset H of a group (G, *) to be a sub group is that $a \in H, b \in H \Rightarrow a * b^{-1} \in H$

3) A necessary and sufficient condition for a non empty finite subset H of a group (G, *) to be a sub group is that $a * b \in H$ for all $a, b \in H$

4) For any group {G, *}, {e} and G are trivial sub groups.

20. Which of the following is false

a) The union of two sub groups of a group G is a sub group of G
b) The interaction of two sub groups of a group G is a sub group of G
c) Let G be the additive group of integers. Then the set of all multiplies of integers by a fixed integer m is a sub group of G
d) Let 'a' be an element of a group $(G, \cdot)$ the set $H = \{a^n : n \in I\}$ is a sub group of G.

21. Which of the following is false

a) the union of two sub groups is a sub group $\Leftrightarrow$ one sub group is contained in the other
b) an abelian group can have a non abelian sub group
c) a non abelian group can have an abelian sub group
d) a non abelian group can have a non abelian sub group

22. Consider the following statements

S_1: Let G be a multiplicative group of all positive real numbers and R the additive group of all real numbers then G is a sub group of R.

S_2: A group can be expressed as the union of two of its proper sub groups.

Which of the following is true

a) S_1 is true and S_2 is false
b) S_1 is false and S_2 is true
c) S_1 and S_2 are true
d) S_1 and S_2 are false

COSETS

If H is a sub group of G and $a \in G$ then the set

$Ha = \{ha \mid h \in H\}$ is called a right coset of H in G. Similarly

$aH = \{ah \mid h \in H\}$ is called a left coset of H is G.

Note:- 1) Any two left (right) cosets of H in G are either identical or disjoint.

2) Let H be a sub group of G. Then the right cosets of H form a partition of G. i.e., the union of all right cosets of a sub group H is equal to G.

3) Lagrange's theorem: The order of each sub group of a finite group is a divisor of the order of the group.

4) The order of every element of a finite group is a divisor of the order of the group.

5) The converse of the lagrange's theorem need not be true.

CYCLIC GROUPS

A group G is called cyclic, if for same $a \in G$, every element of G is an integral power of a

$$G = \{\ldots, a^{-1}, a^{-2}, a^{-1}, e, a, a^2, a^3, \ldots\ldots.\}$$

The element 'a' is then called generator of G

$G = \{1, \omega, \omega^2\}$ is a cyclic group w.r.t. multiplication. The generators are ω and ω^2.

The set $G = \{1, -1, i, -i\}$ is a cyclic group w.r.t multiplication. The generators are i, -i,

23. Which of the following if false

a) every cyclic group is an abelian group
b) if 'a' is generator of a cyclic group then a^{-1} is also a generator of G.
c) the order of a cyclic group is equal to the order of its generating element.
d) A sub group of a cyclic group need not be cyclic

24. Which of the following is false

a) every group of prime order is cyclic
b) if (G, *) is a group of even order then there in an element a in G such that $a * a = e$
c) every finite group of order less than 6 must be abelian.
d) The group $(\{1, 2, 3, 4, 5, 6\}, X_7)$ is cyclic with 2 as its generator

25. If G is a group of order p, where p is a prime number. Then the number of sub groups of G is

a) 1
b) 2
c) $p - 1$
d) p

26. Which of t he following is false

a) set of all polynomials in x is a group under the operation of polynomial addition
b) {1} and {-1, 1} are the only finite sub groups of set of all non zero real numbers under multiplication.
c) If an abelian group 'G' has subgroups of orders m and n, then G has a subgroup whose order L. C. M of m and n.
d) Let G be a group of order n. If m divides n then there will be a subgroup of order m of G.

27. If a finite group of order n contains an element of order n , then which of the following is false
    a) The group is cyclic b) The group is abelian
    c) The group is cyclic and abelian d) The group is cyclic but not abelian

28. If G is an infinite cyclic group which of the following is not true
    a) G has exactly two generators b) G is isomorphic to (Z, +)
    c) Every proper sub group of G is finite
    d) If a cyclic group is generated by an element 'a' of order n, then a^m is a generator of G $\Rightarrow$ m and n are coprime.

29. How many generators are there of a cyclic group G of order 8 ?
    a) 2 b) 4 c) 6 d) 8

30. How many generators are there for the cyclic group ({1, 2, 3, 4, 5, 6}, X_7)
    a)2 b) 3 c) 4 d) 5

31. If G is a group of prime order p, then G is cyclic and the number of generators =
    a)1 b) 2 c) p – 1 d) p

32. Which of the following is false
    a) A cyclic group with only one generator can have atmost 2 elements
    b) The order of a cyclic group is equal to the order of its generator
    c) The group ({ 1, 2, 3, 4 }, X_5) is cyclic
    d) A group of order 4 is cyclic

NORMAL SUB GROUPS

A sub group H of G is normal if $a^{-1} H a \subseteq H \quad \forall a \in G$

i.e. $a^{-1} h a \in H \quad \forall h \in H$

or $aH = Ha$ for all $a \in G$

Note 1) A group having no proper normal sub groups is called a simple group

2) Let H be a normal sub group of G. then the cosets of H form a group under coset multiplication the group is called Quotient group and denoted by G / H

Homomorphism : Consider the groups (G, *) and (G^1, $\oplus$)

A function $f : G \to G^1$ is called a homomorphism if $f(a * b) = f(a) \oplus f(b)$

If f is a bijection then f is called isomorphism between G and G' we write G ≡G'

33. If G is a normal sub group of prime order ' p ' then the number of normal subgroups in G =
    a) 0 b) 1 c) 2 d) p

34. which of the following is false
    a) Every sub group of a cyclic group is normal
    b) Intersection of two normal sub group is normal
    c) A sub group H of a group is normal if $|H| = ½ |G|$
    d) Every coset of a subgroup H in a group G is also a sub group in G

35. Let G be a reduced residue system modulo 15 say
    G = {1, 2, 4, 7, 8, 11, 13, 14}
    (i,e. .the set of integers between 1 and 15 which are coprime to 15). Then G is a group under multiplication modulo 15. Which of the following is false
    a) inverse of 2 = 8 b) inverse of 7 = 13 c) inverse of 11 = 11 d) inverse of 4 = 9

36. If $f : G \to G^1$ is a group isomorphism, then which of the following is false
a) $f(e) = e^1$ b) $f(a^{-1}) = [f(a)]^{-1}$
c) f is an equivalence relation d) f is not an homomorphism

37. Let G be a group of real numbers under addition and G^1 be a group of real numbers under multiplication. The mapping $f : G \to G^1$ defined by $f(x) = 2^x$ is
a) a homomorphism and a is an isomorphism
b) b) neither isomorphism nor homomorphism
c) a homomorphism but not an isomorphism
d) not a homomorphism but an isomorphism

38. Let A be a non empty set with operation * defined by
$a * b = a$ then
a) (A, *) is a semi group b) * is commutative on A
c) Identity element exists in A d) (A, *) is a group

KEY

FUNCTIONS

1. a 2. a 3. d 4. c 5. c 6. c 7. a 8. d 9. d 10. d 11. d 12. d 13. a
14. a 15. d 16. a 17. a 18. a 19. d 20. b 21. a 22. b, d 23. a 24. d 25. a

GROUPS

1. d 2. a 3. a 4. a 5. c 6. c 7. c 8. c 9. d 10. c 11. d 12. b 13. d
14. d 15. c 16. a 17. d 18. c 19. d 20. a 21. b 22. b 23. d 24. d 25. b 26. d
27. d 28. c 29. b 30. a 31. c 32. d 33. a 34. d 35. d 36. d 37. a 38. a

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Previous GATE questions

01. Consider the set $S = \{1, \omega, \omega^2\}$, where ω and ω^2 are cube roots of unity. If * denotesthe multiplication operation, the structure {S, * } forms (GATE-2010)
a) a group b) a ring c) an integral domain d) a field

02. Which one of the following is not necessarily a property of a group? GATE – 2009
(a) Commutativity (b) Associativity
(c) Existence of inverse for every element (d) Existence of identity

03. For the composition table of a cyclic group shown below GATE – 2009

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$c \to b \to d$

Which one of the following is correct?
(a) a, b are generators (b) b, c are generators
(c) c, d are generators (d) d, a are generators

04. Let f be a function from a set A to a set B, g be a function from B to C, and h be a function from A to C, such that h(a) = g(f(a)) for all a ∈ A. Which of the following statements is always true for all such functions f and g? **GATE – 2005**

(a) g is onto ⇒ h is onto (b) h is onto ⇒ f is onto

(c) h is onto ⇒f and g are onto (d) h is onto ⇒ g is onto

05. The set {1, 2, 3, 4, 5, 6, 7, 8, 9} under multiplication module 10 is not a group. Given below are four plausible reasons. Which one of them is false? **GATE – 2006**

(a) It is not closed (b) 2 does not have an inverse

(c) 3 does not have an inverse (d) 8 does not have an inverse

06. Let X, Y, Z be sets of sizes x, y and z respectively. Let W = X × Y and E be the set of all subsets of W. The number of functions from Z to E is **GATE – 2006**

(a) $Z^{2^{xy}}$ (b) $Z \times 2^{xy}$ (c) $Z^{2^{x+y}}$ (d) 2^{xyz}

07. The following is the incomplete operation table of a 4-element group

*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b				
c				

The last row of the table is **GATE – 2004**

(a) c a e b (b) c b a e

(c) c b e a (d) c e a b

08. Let S = {0, 1, 2, 3, 4, 5, 6, 7} and * denote multiplication modulo 8, that is x * y = (xy) mod 8. Which of the following is not a group w.r.t * **GATE – 2000**

(a) {0, 1} (b) {1, 3} (c) {1, 5} (d) {1, 7}

Key

01.a 02.a 03. c 04.d 05.c 06.d 07.d 08.a

Definition: A recurrence relation is a formula that relates for any integer $n \geq 1$, the n-th term of a sequence a_n to one or more of the terms $a_0, a_1, \ldots a_{n-1}$.

Examples of recurrence relations: If s_n denotes the sum of the first *n* positive integers, then (1) $s_n = n + s_{n-1}$. Similarly if *d* is a real number, then the *n*th term of an arithmetic progression with common difference *d* satisfies the relation (2) $a_n = a_{n-1} + d$. Likewise if p_n denotes the *n*th term of a geometric progression with common ratio *r*, then (3) $p_n = r\, p_{n-1}$. We list other examples as:

(4) $a_n - 3a_{n-1} + 2a_{n-2} = 0$
(5) $a_n - 3a_{n-1} + 2a_{n-2} = n^2 + 1$.
(6) $a_n - (n-1)a_{n-1} - (n-1)a_{n-2} = 0$.
(7) $a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 5n$.
(8) $a_n - 3(a_{n-1})^2 + 2a_{n-2} = n$.
(9) $a_n = a_0 a_{n-1} + a_1 a_{n-2} + \ldots\ldots + a_{n-1} a_0$.
(10) $a_n^2 + (a_{n-1})^2 = -1$.

Definition : suppose *n* and *k* are nonnegative integers. A recurrence relation of the form $c_0(n)\, a_n + c_1(n)\, a_{n-1} + \ldots + c_k(n)\, a_{n-k} = f(n)$ for $n \geq k$, where $c_0(n), c_1(n), \ldots c_k(n)$, and $f(n)$ are functions of *n* is said to be a **linear** recurrence relation. If $c_0(n)$ and $c_k(n)$ are not identically zero, then it is said to be a linear recurrence relation of *degree k*. If $c_0(n), c_1(n), \ldots c_k(n)$ are constants, then the recurrence relation is known as a linear recurrence relation with constant coefficients. If $f(n)$ is identically zero, then the recurrence relation is said to be **homogeneous**; otherwise, it is **inhomogeneous.**

Thus, all the examples above are linear recurrence relations except (8), (9) and (10); the relation (8), for instance, is not linear because of the squared term. The relations in (3), (4), (5) and (7) are linear with constant coefficients. Relations (1), (2) and (3) have degree 1; (4), (5) and (6) have degree 2; (7) has degree 3. Relations (3), (4) and (6) are homogeneous.

Fibonacci relation: The relation $F_n = F_{n-1} + F_{n-2}$ is called the fibonacci relation with the initial condition
$F_0 = 1 = F_1$.

The numbers generated by the Fibonacci relation with the initial condition are called fibonacci numbers.

$\{1, 1, 2, 3, 5, 8, 13, \ldots\ldots\}$

Formation of Recurrence Relation:

1. Find a recurrence relation for the number of ways a person can climb a flight of n steps if the person can skip at most one step at a time ?
   a) $a_n = a_{n-1} + a_{n-2}$ b) $a_n = a_{n-1} + 2a_{n-2}$ c) $a_n = a_{n-1} - a_{n-2}$ d) $a_n + a_{n-1} + a_{n-2} = 0$.

2. Let a_n = the number of ways a person can climb n steps with the condition that the person may take either 1, 2, or 3 steps in each stride. Then the recurrence relation for a_n is
   a) $a_n = a_{n-1} + 2a_{n-2} + 3a_{n-3}$ b) $a_n = a_{n-1} + a_{n-2} + a_{n-3}$
   c) $a_n = 3a_{n-1} + 2a_{n-2} + a_{n-3}$ d) $a_n + a_{n-1} - a_{n-2} - 3a_{n-3} = 0$

3. Suppose that a school principal decides to give a prize away each day. Suppose further that the principal has 3 different kinds of prizes worth \$1 each and 5 different kinds of prizes worth \$4 each. Find a recurrence relation for a_n = the number of different ways to distribute prizes worth *n* dollars.
a) $a_n = 3a_{n-1} + 5a_{n-4}$
b) $a_n = 5a_{n-1} + 3a_{n-1}$
c) $a_n = a_{n-4} + 4 \times 5a_{n-5}$
d) $3a_n = 4a_{n-1} + 5a_{n-4}$

4. Find a recurrence relation for the number of ways to arrange flags on a flagpole *n* feet tall using 4 types of flags: red flags 2 feet high, or white, blue and yellow flags each one foot high.
a) $a_n = a_{n-1} + 3a_{n-2}$ b) $a_n = 3a_{n-1} + a_{n-2}$ c) $a_n = 2a_{n-1} + 3a_{n-2}$ d) $a_n = 3a_{n-1} + 2a_{n-2}$

5. Find a recurrence relation for the number of *n*-digit ternary sequences that have an even number of 0's.
a) $a_n = 2a_{n-1} + 3^{n-1}$ b) $a_n = a_{n-1} + 3^{n-1}$ c) $a_n = 3a_{n-1} + 2^{n-1}$ d) $a_n = 3a_{n-1} + 3^n$

6. Find a recurrence relation for the number of ways to make a pile of *n* chips using garnet, gold, red, white and blue chips such that no two gold chips are together.
a) $a_n = 4a_{n-1} + 4a_{n-2}$
b) $a_n = a_{n-1} + 4a_{n-2}$
c) $a_n = 4a_{n-1} + a_{n-2}$
d) $a_n = a_{n-4} + 4a_{n-1}$

Solution Of Recurrence Relations

Homogeneous linear recurrence relations with constant coefficients:

In general, to solve the equation

$$y_{n+r} + a_1 y_{n+r-1} + a_2 y_{n+r-2} + \ldots\ldots + a_r y_n = 0$$

Where a's are constants:

(i) Write the equation in th symbolic form

$$(E_r + a_1 E^{r-1} + \ldots\ldots + a_r) y_n = 0$$

(ii) Write down the auxillary equation i.e. $E_r + a_1 E^{r-1} + \ldots\ldots + a_r = 0$ and solve it for E.

(iii) Write the solution as follows:

Roots for A.E	Solution i.e. C.F
1. $\lambda_1, \lambda_2, \lambda_3 \ldots.$ (real and distinct roots)	$c_1(\lambda_1)^n + c_2(\lambda_2)^n + c_3(\lambda_3)^n + \ldots.$
2. $\lambda_1, \lambda_1, \lambda_3 \ldots.$ (2 real and equal roots)	$(c_1 + c_2 n)(\lambda_1)^n + c_3(\lambda_3)^n + \ldots.$
3. $\lambda_1, \lambda_1, \lambda_1 \ldots.$ (3 real and equal roots)	$(c_1 + c_2 n + c_3 n^2)(\lambda_1)^n + \ldots.$
4. $\alpha + i\beta, \alpha - i\beta, \ldots.$ (a pair of imaginary roots)	$r^n (c_1 \cos n\theta + c_2 \sin n\theta)$ where $r = \sqrt{(\alpha^2 + \beta^2)}$ and $\theta = \tan^{-1}(\beta/\alpha)$.

7. The solution of recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 0$ where $a_0 = 2$ and $a_1 = 5$ is
a) $a_n = 2^n - 3^n$
b) $a_n = 2^n + 3^n$
c) $a_n = 2 \cdot 3^n + 5 \cdot 2^n$
d) $a_n = 3 \cdot 2^n + 2 \cdot 5^n$

8. The solution of recurrence relation $a_n - 7a_{n-1} + 12a_{n-2} = 0$ for $n \geq 2$. where $a_0 = 2$ and $a_1 = 5$ is
a) $a_n = 3^n + 4^n$ b) $a_n = 3^n - 4^n$ c) $a_n = 3^{n+1} - 4^n$ d) $a_n = 3^{n+1} + 4^n$

9. The solution of the recurrence relation $a_n - 2a_{n-1} + a_{n-2} = 0$ with initial conditions $a_0 = 1$ and $a_1 = 2$ is
a) $a_n = n+2$ b) $a_n = 2^n + 1$ c) $a_n = n-1$ d) $a_n = n+1$

10. The solution of the recurrence relation $a_n - 3a_{n-1} + 3a_{n-2} - a_{n-3} = 0$ where $a_0 = 0$, $a_1 = 1$ and $a_2 = 2$ is
a) $a_n = n^2 + 1$ b) $a_n = n + 2n^2$ c) $a_n = n^2$ d) $a_n = n$

A ***Graph*** G is a pair of sets (V, E) where V = a set of vertices (nodes) and E = a set of edges

V(G) = Set of all vertices in G

E(G) = Set of all edges in G

$|V(G)|$ = Number of vertices in graph G = Order of G

$|E(G)|$ = Number of edges in graph G = Size of the graph

DIRECTED GRAPH: In a ***digraph*** the elements of E are ordered pairs of vertices. In this case an edge (u, v) is said to be from u to v.

NON DIRECTED GRAPH: The elements of E are unordered pairs (sets) of vertices. In this case an Edge {u, v} is said to join u and v or to be between u and v.

NULL GRAPH: A null graph of order n is a graph with n vertices and no edges.

LOOP: An edge drawn from a vertex to itself.

SIMPLE GRAPH: A graph with no loops.

MULTI GRAPH: If one allows more than one edge to join a pair of vertices, the result is then called a multi graph.

DEGREE: Degree of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex. The degree of the vertex 'v' is denoted by deg(v).

IN DEGREE AND OUT DEGREE: In a directed graph an edge (u, v) is said to be incident from u, and to be incident to 'v'. Within a particular digraph, the number of edges incident to a vertex is called the
in-degree of the vertex and the number of vertices incident from a vertex is called its **out-degree**.

- The in degree of a vertex 'v' in a graph G is denoted by $deg^+(v)$. The out degree of a vertex v is denoted by $deg^-(G)$.
- A loop in a digraph is counted once, for both indegree and outdegree of a vertex.

NEIGHBORS: If there is an edge incident from u to v, or incident on u and v, then u and v are said to be ***adjacent*** (or to be neighbors).

$\delta(G)$ = minimum of all the degrees of vertices in a graph G.

$\Delta(G)$ = Maximum of all the degrees of vertices in a graph G.

REGULAR GRAPH: In a graph G, if $\delta(G) = \Delta(G) = k$
i.e., if each vertex of G has degree k, then G is said to be a **regular graph** of degree k (k-regular).

Ex: Polygon is a 2-regular graph .
Ex: A 3-regular graph is a cubic graph.

COMPLETE GRAPH: A simple non directed graph with 'n' mutually adjacent vertices is called a complete graph on n vertices and may be represented by K_n.

NOTE: A complete graph on n vertices has n(n-1)/ 2 edges, and each of its vertices has degree 'n-1'.

CYCLE GRAPH: A cycle graph of order n is a connected graph whose edges form a cycle of length n.

NOTE: A cycle graph 'C_n' of order n has n vertices and n edges.

CYCLIC GRAPH: A graph with atleast one cycle.

Acyclic graph: A simple graph having no cycles is called acyclic graph.

Connected graph: An undirected graph is called ***connected*** if there is a path between every pair of distinct vertices.

Tree: A connected graph with no cycles. A tree with n vertices has (n-1) edges.
A tree with n (n >1)vertices has atleast two vertices of degree 1.

WHEEL GRAPH: A wheel graph of order n is obtained by adding a single new vertex (the hub) to each vertex of a cyclic graph of order n-1.

NOTE: A wheel graph W_n has 'n' vertices and 2(n-1) edges.

BIPARTITE GRAPH: A Bipartite graph is a non directed graph whose set of vertices can be partitioned in to two sets M and N in such a way that each edge joins a vertex in M to a vertex in N.

COMPLETE BIPARTITE GRAPH: A complete Bipartite graph is a Bipartite graph in which every vertex of M is adjacent to every vertex of N.
If $|M| = m$ and $|N| = n$ then the complete Bipartite graph is denoted by $K_{m,n}$. It has 'mn' edges.

DEGREE SEQUENCE: If $v_1, v_2, \ldots\ldots, v_n$ are the vertices of a graph G, then the sequence $(d_1, d_2, \ldots.., d_n)$ where
d_i = degree of v_i is called the degree sequence of G. Usually we order the degree sequence so that the degree sequence is monotonically decreasing.

SUM OF DEGREES THEOREM: If $V = \{v_1, v_2, \ldots.., v_n\}$ is the vertex set of a non directed graph G then

$$\sum_{i=1}^{n} \deg(v_i) = 2.\ |E|$$

NOTE: 1) For directed graph

$$\sum_{i=1}^{n} \deg^+(v_i) = \sum_{i=1}^{n} \deg^-(v_i) = |E|$$

NOTE: 2) An undirected graph has an even number of vertices of odd degree.

3) If $k = \delta(G)$ is the minimum degree of all vertices in a undirected graph G, then

$$k.|V| \leq \sum_{i=1}^{n} \deg(v_i) = 2.|E|$$

4) If G is a k – regular graph, then $k.\ |V| \geq \Sigma \deg(v_i) = 2.\ |E|$

5) $\delta(G) \leq [2|E| / |V|] \leq \Delta(G)$

Complement of a graph: The complement of a graph G is the graph $\overline{G}$ with the same vertices as G. An edge exists in $\overline{G}$ iff it does not exist in G.

Havel-Hakimi Result: Consider the following two sequences and assume the sequence (i) is in descending order

i) $s, t_1, t_2, \ldots\ldots t_s, d_1, d_2, \ldots\ldots d_n$

ii) $t_1 - 1, t_2 - 1, \ldots\ldots, t_{s-1}, d_1, d_2, \ldots\ldots, d_n$

then sequence (i) is graphic iff (ii) is graphic

PROBLEMS

1. Which of the following degree sequences represent a simple non directed graph
   a) {2, 3, 3, 4, 4, 5} b) {2, 3, 4, 4, 5} c) {1, 3, 3, 4, 5, 6, 6} d) {2, 2, 3, 3}

2. Which of the following degree sequences cannot represent a simple non directed graph
   a) {1, 1, 2, 2} b) {3, 3, 3, 3, 2} c) {0, 1, 2, 2, 3} d) {1, 3, 3, 3}

3. A non directed graph contains 16 edges and all vertices are of degree 2. Then the number of vertices in G is ?
   a) 8 b) 6 c) 16 d) 32

4. A simple non directed graph contains 21 edges, 3 vertices of degree 4 and the other vertices are of degree 2. Then the number of vertices in the graph is ?
   a) 8 b) 13 c) 18 d) 21

5. If a simple non directed graph G contains 24 edges and all vertices are of same degree then $|V(G)| =$ ____
   a) 21 b) 22 c) 23 d) 24

6. The largest possible number of vertices in a graph G, with 35 edges and all vertices are of degree atleast 3 is
   a) 24 b) 25 c) 23 d) 26

7. Let G be a simple graph with n vertices. Then the number of edges of G is less than or equal to
   a) 2^n b) n^2 c) $n(n+1)/2$ d) $n(n-1)/2$

8. A cycle graph on n vertices is regular when
   a) $n \leq 2$ b) $n < 3$ c) $n \geq 3$ d) $n \leq 1$

9. A wheel graph on n vertices is regular when
   a) $n = 3$ b) $n = 4$ c) $n = 5$ d) $n > 3$

10. If a complete Bipartite graph $K_{m,n}$ is regular then
    a) $m < n$ b) $m > n$ c) $m = n$ d) None of the above

11. A 4- regular graph have 10 edges. The number of vertices in the graph is
    a) 5 b) 6 c) 7 d) 8

12. Let G be a graph with 5 vertices and 7 edges then
    a) $\delta(G) > 2$ b) $\Delta(G) > 4$ c) $\delta(G) \leq 2$ d) $\Delta(G) \leq 2$

13. If the simple graph G has 5 vertices and 7 edges, how many edges does G have ?
    a) 3 b) 6 c) 8 d) 4

14. If G is a simple graph with 15 edges and $\overline{G}$ has 13 edges, how many vertices does it have
a) 6 b) 8 c) 10 d) 12

14.(a) What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2. 3 vertices of degree 4 and remaining of degree 3?
a) 10 b) 11 c) 18 d) 19 ***(GATE'04[IT])***

14.(b) Which of the following degree sequences represent a simple non directed graph
S_1 : {6, 6, 6, 6, 4, 3, 3, 0}
S_2 : {6, 5, 5, 4, 3, 3, 2, 2, 2}
a) S_1 is graphic but S_2 is not graphical b) S_2 is graphic but S_1 is not graphic
c) S_1 and S_2 are graphic d) neither S_1 nor S_2 is graphic

14.(c) Consider the following statements
S_1 : Every complete graph is regular
S_2 : There is a simple graph with degree sequence {1, 1, 3, 3, 3, 4, 6, 7}
S_3 : Every regular graph is complete
S_4 : A degree sequence with all distinct elements cannot represent a simple graph
Which of the above statements is / are true
a) S_1, S_2 b) S_2, S_3 c) S_3, S_4 d) S_1, S_2 and S_3

14.(d) Let G be a simple graph, all of whose vertices have degree 3 and $|E| = 2|V| - 3$. Which of the following is true
a) G has 6 vertices and 9 edges b) G has 9 vertices and 15 edges
c) G has 5 vertices and 7 edges d) G has 8 vertices and 13 edges

14.(e) The number of edges in a k – regular graph with n vertices is
a) 2nk b) nk/2 c) C(n, k) d) P(n, k)

14.(f) Number of simple graphs possible with 6 vertices and 4 edges is
a) 15 b) 360 c) 1296 d)1315

EULER AND HAMILTONIAN GRAPHS

PATH: In a non directed graph G, a sequence p of zero or more edges of the form $\{v_0, v_1\},\{v_1, v_2\}, \ldots\ldots\{v_{n-1}, v_n\}$or $\{v_0 - v_1 - v_2 - \ldots - v_n\}$is called a path from v_0 to v_n.
v_0 is called initial vertex and v_n is called terminal vertex.
v_0 and v_n are called end points of the path.
We denote p as $v_0 - v_n$ path
If $v_0 \neq v_n$, then p is called open path.
If $v_0 = v_n$, then p is called closed path.

SIMPLE PATH: A path p is simple if all edges and vertices on a path are distinct except possibly the end points.
* An open simple path of length n has n + 1 distinct vertices and n edges.
* A closed simple path of length n has n distinct vertices and n distinct edges.
* A trivial path on $\{v_0\}$ is taken to be simple closed path of length zero.

CIRCUIT: A path of length ≥ 1 with no repeated edges and whose end points are equal are called a circuit.

* A circuit may have repeated vertices other than its end points.
* A ***cycle*** is a circuit with no other repeated vertices except its end points.

* A loop is a cycle of length 1.
* In a graph , a cycle that is not a loop must have length at least 3.
* In a multi graph , there may be a cycle of length 2.
* Two paths in a graph are **edge disjoint** if they don't share a common edge.
* Two paths are **vertex disjoint** if they do not share a common vertex.

*Any circuit in a graph must contain a cycle and that any circuit which is not a cycle contains atleast two cycles.

* An undirected graph is called ***connected*** if there is a path between every pair of distinct vertices.
* A graph that is not connected is the union of two or more connected sub graphs each pair of which has no vertex in common. These disjoint connected sub graphs are called ***connected components*** of G.
* If a graph G is connected and e is an edge such that G – e is not connected, then e is said to be a bridge or cut edge .
* If v is a vertex such that G – v is not connected, then v is a ***cut vertex.***
* A pair of vertices in a digraph are ***weakly connected*** if there is a non directed path between them.
* A pair of vertices in a diagraph are ***unilaterally connected*** if there is a directed path between them.
* A diagraph is ***strongly connected*** if there is a directed path from a to b and from b to a, whenever a and b are vertices in the graph.
* Two vertices u and v of a directed graph G are said to be ***quasi strongly connected,*** if there is a vertex w from which there is a directed path to u and a directed path to v.

NOTE:

1) If there is a directed path from u to v then certainly u and v are strongly connected.
2) The graph G is said to be quasi strongly connected if each pair of vertices in G is quasi strongly connected,
3) G is quasi strongly connected ⇔ There is a vertex r in G such that there is a directed path r to all the remaining vertices of G.
4) If G is quasi strongly connected and 'G – e' is not quasi strongly connected for each edge e of G. hen G is a directed tree.
5) G is a directed tree ⇔ G is quasi strongly connected and contains a vertex r such that the indegree of r is zero and the out degree of all other vertices is 1.
6) G is a directed tree ⇔ G is quasi strongly connected without circuit.
7) A diagraph G has directed spanning tree iff G is quasi strongly connected.
8) Any strongly connected graph is also weakly connected.
9) A connected graph with n vertices has atleast n – 1 edges.
10) In any simple graph there is a path from any vertex of odd degree to some other vertex of odd degree.

TRAVERSABLE MULTI GRAPHS: A multi graph is traversable if it can be drawn with out any breaks in the curve and with out repeating an edge.
i.e... , if there is a path which includes all vertices and uses each edge exactly once.

* A multi graph G with more than two odd vertices is not traversable.

EULER PATH: An Euler path in a multi graph is a path that includes each edge of the

Multi graph exactly once and intersects each vertex of the multi graph atleast once.

* A multi graph is traversable if it has Euler path.
* A non directed multi graph has an Euler path iff it is connected and has zero or exactly two vertices of odd degree.
* Any finite connected graph with exactly two odd vertices is traversable. A traversable trail may begin at either odd vertex and will end at other vertex.
* A connected multi graph has an ***Euler circuit*** if and only if its vertices has even degree.
* A directed multi graph G has an Euler circuit iff G is unilaterally connected and the in degree of every vertex of G is equal to its out degree.

- A directed graph that contains an Euler circuit is strongly connected.

HAMILTONIAN GRAPH: A Hamiltonian Graph is a graph with a closed path that includes every vertex exactly once such a path is a cycle and is called a Hamiltonian cycle.

* An Eulerian circuit uses every edge exactly once but may repeat vertices , while a Hamiltonian cycle uses each vertex exactly once (except for the first and last) but may skip edges.

➢ If a vertex v in a graph G has degree K then a Hamiltonian path must contain atleast one edge incident on v and at most two edges incident on v.

➢ A Hamiltonian cycle contains exactly two edges incident on v. In particular, both edges incident on a vertex of degree 2 will be contained in every Hamiltonian cycle. Finally, there cannot be 3 or more edges incident with one vertex in a Hamiltonian cycle.

➢ No cycle that does not contain all vertices of G can be formed when building a Hamiltonian path on a Hamiltonian cycle.

➢ Once Hamiltonian cycle we are building has passed through a vertex v then all other unused edges incident on v can be deleted because only 2 edges incident on v can be included in a Hamiltonian cycle.

Dirac's Theorem: A simple graph with n vertices ($n \geq 2$) and $\delta(G) \geq n/2$ has a Hamiltonian cycle.

➢ Cor: If G is a complete graph on n vertices ($n \geq 8$) then G has a Hamiltonian cycle (true for digraphs also.)

➢ In a complete graph with n vertices there are $(n - 1)/2$ edge disjoint Hamiltonian cycles, if n is odd number ≥ 3.

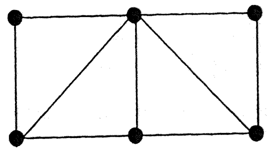
➢ Let G be a simple connected graph with n vertices. G has a Hamiltonian cycle if for any two vertices u and v of G, which are not adjacent, $\deg(u) + \deg(v) \geq n$

➢ G has a Hamiltonian cycle, if $m \geq \frac{1}{2}(n^2 - 3n + 2)$ where m is number of edges and n is number of vertices.

15. Which of the following graphs is not traversable where V(G) = {A, B, C, D} and
 a) E(G) = {(A, B), (A, C), (A, D), (B, D), (C, A), (D, A)}
 b) E(G) = {(A, B), (A, C), (B, C), (C, A), (C, B)}
 c) E(G) = {(A, A), (A, B), (A, C), (B, D), (D, B)}
 d) E(G) = {(A, B), (A, C), (B, A), (B, D), (C, A), (C, D), (D, B) , (D, C)}

16. Which of the following graphs is traversable where $V(G) = \{A, B, C, D\}$ and
 a) $E(G) = [\{A, B\}, \{A, C\}, \{B, C\}, \{C\ D\}, \{D, A\}]$
 b) $E(G) = [\{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{C\ D\}, \{D, A\}]$
 c) $E(G) = [\{A, B\}, \{B, A\}, \{C, D\}, \{D, C\}]$
 d) $E(G) = [\{A, B\}, \{A, C\}, \{A, D\}, \{B, C\}, \{B, D\}, \{C, A\}, \{D, A\}]$

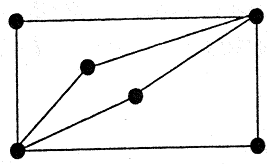
17. Consider the graph shown below



Which of the following is true

a) G is Eulerian and Hamiltonain
b) G is not Eulerian but Hamiltonain
c) G is Eulerian and not Hamiltonain
d) G is neither Eulerian nor Hamiltonain

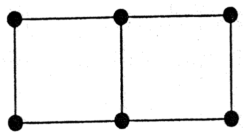
18. Consider the graph G shown below



Which of the following is true

a) G is Eulerian and Hamiltonain
b) G is not Eulerian but Hamiltonain
c) G is Eulerian and not Hamiltonain
d) G is neither Eulerian nor Hamiltonain

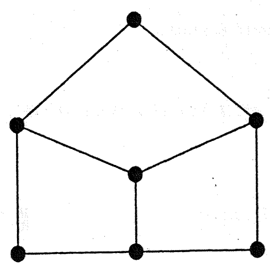
19. Consider the graph G shown in below



Which of the following is true

a) G is Eulerian and Hamiltonain
b) G is not Eulerian but Hamiltonain
c) G is Eulerian and not Hamiltonain
d) G is neither Eulerian nor Hamiltonain

20. Consider the graph G shown in below



Which of the following is true

a) G is Eulerian and Hamiltonain
b) G is not Eulerian but Hamiltonain
c) G is Eulerian and not Hamiltonain
d) G is neither Eulerian nor Hamiltonain

21. For which values of m and n does the complete Bipartite graph $K_{m,\,n}$ has a Hamiltonian cycle.
 a) $m \geq 2, n \geq 2$ b) $m = n \geq 2$ c) $m \leq 1, n \leq 1$ d) $m \neq n$

22. A complete graph on n vertices K_n has a Hamiltonian circuit whenever
 a) $n < 3$ b) $n \geq 3$ c) $n = 2$ d) $n < 2$

PLANARITY

A graph or a multi graph that can be drawn in a plane or on a sphere so that its edges do not cross is called a **planer graph**

Ex : A complete graph on 4 vertices K_4 is a planar graph

Ex : Tree is a planar graph

Map, Connected map : A particular planar representation of a finite planer multigraph is called a **map**. We say that the map is connected if the under lying multigraph is connected

REGION : A given map (planar graph) divide the plane into connected areas called **regions.**

DEGREE OF A REGION : The boundary of each region of a map consists of a sequence of edges forming a closed path. The degree of region 'r' denoted by deg (r) is the length of the closed path bordering r .

* The sum of the degrees of the regions of a map M is equal to twice the number of edges in M. $\Sigma \deg(r) = 2.|E|$

* Let G be a connected graph with 'e' edges and 'v' vertices. Let 'r' be the number of regions in a planar representation of G. Then $r + v = e + 2$

This is known as Euler's formula

* In a planar graph G, if the degree of each region $\geq k$ then $k.|R| \leq 2.|E|$

In particular for a simple connected planar graph

$3.|R| \leq 2.|E|$

* If G is a connected simple planar graph with $|E| > 1$ then,

(a) $|E| \leq 3.|V| - 6$

(b) There exists atleast one vertex v of G such that $\deg(v) \leq 5$

POLYHEDRAL GRAPH: A connected plane graph is said to be polyhedral if $\deg(v) \geq 3$ for all $v \in G$.

* For any polyhedral graph

a) $|V| \geq (2 + \frac{|R|}{2})$

b) $|R| \geq (2 + \frac{|V|}{2})$

c) $(3.|R| - 6) \geq |E|$

Kurtowski Theorem

* A graph G is not planar iff G contains a sub graph homeomorphic to $K_{3,3}$ or K_5

HOMEOMORPHIC GRAPHS: Given any graph G , we can obtain a new graph by dividing an edge of G with additional vertices . Two graphs G and G^1 are said to homeomorphic if they can be obtained from the same graph.

* A complete graph K_n is planar iff $n \leq 4$.

* A complete Bipartite graph $K_{m,n}$ is planar iff $m \leq 2$ or $n \leq 2$.

* Any graph with 4 or fewer vertices is planar.

* If G is a simple connected planar graph with $|V| \geq 3$ and $|R|$ regions then $|R| \leq 2|V| - 4$

* If G is a planar graph with k connected components, each component having atleast 3 vertices then $|E| \leq (3|V| - 6k)$

* If G is a planar graph with k components then $|V| - |E| + |R| = k + 1$ (for k = 1, we get Euler's formula.)

23. A planar graph contains 25 vertices and 60 edges then the number of regions in the graph is

a) 35 b) 36 c) 37 d) 38

24. The maximum number of edges possible in a planar graph with eight vertices is

a) 18 b) 15 c) 12 d) 21

25. The minimum number of vertices necessary for a graph with 11 edges to be planar is
a) 4 b) 5 c) 6 d) 7

26. Consider the following
S_1 : There does not exist a polyhedral graph with exactly seven edges.
S_2 : There is no polyhedral graph with exactly 30 edges and 11 regions.

Which of the following is correct

a) S_1 is true and S_2 is false b) S_1 is false and S_2 is true
c) S_1 is false and S_2 is false d) S_1 is true and S_2 is true

27. Which of the following graphs is not planar
a) K_3 b) $K_{4,2}$ c) K_4 d) $K_{3,3}$

28. Suppose that a connected planar graph has 20 vertices, each of degree 3. In to how many regions does a representation of this planar graph split the plane ?
a) 10 b) 12 c) 16 d) 20

29. The n – cube, denoted by Q_n, is the graph that has vertices representing the 2^n Bit strings of length n. Two bits are adjacent iff the Bit strings that they represent differ in exactly one bit position. Which of the following is false.
a) Q_n has 2^n vertices b) Q_n has $n\,2^{n-1}$ edges
c) Q_3 is planar d) Q_n is not regular

30. A non planar graph with minimum number of vertices has (GATE'92)
a) 9 edges, 6 vertices b) 6 edges, 4 vertices
c) 10 edges, 5 vertices d) 9 edges, 5 vertices

31. A graph G = (V, E) satisfies $|E| \le 3|V| - 6$. The minimum degree of G cannot be
a) 3 b) 4 c) 5 d) 6 (GATE'2003)

32. A graph G is a connected planar graph with 35 regions each of degree 6. Then the number of vertices in G is
a) 70 b) 80 c) 72 d) 62

33. Suppose G is a polyhedral graph with 12 vertices and 30 edges and degree of each region is k then k =
a) 2 b) 3 c) 4 d) 5

COLORING

A **coloring** of a simple graph is the assignment of color to each vertex of the graph so that no two adjacent vertices are assigned the same color.

CHROMATIC NUMBER: The minimum number of colors needed to paint a graph G is called the chromatic number of G, denoted by $\chi(G)$

ADJACENT REGIONS: An assignment of colors to the regions of a map such that adjacent regions have different colors.
* A map 'M' is n – colorable if there exists a coloring of M which uses n colors.
* A planar graph is 5 – colorable

* ***Four color Theorem:*** If the regions of a map M are colored so that adjacent regions have different colors, then no more than 4 colors are required.

* Every planar graph is 4- colorable (vertex coloring)

34. The chromatic number of a complete graph K_n is
a) 4 b) 2 c) n d) n – 1

35. The chromatic number of a complete Bipartite graph $K_{m,n}$ is
a) 4 b) 2 c) m d) n

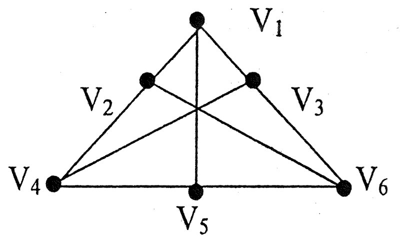
36. If n is even number, then the chromatic number of cyclic graph C_n is
a) 2 b) 4 c) n d) n / 2

37. If every cycle of G has even length then its chromatic number is
a) 2 b) 3 c) 4 d) $\Delta(G)$

38. The chromatic number of a tree on n vertices is
a) 2 b) 4 c) n d) n / 2

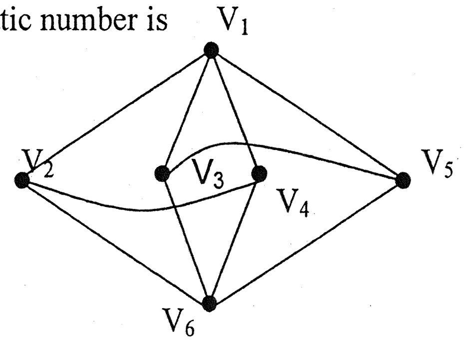
39. Which of the following is false
a) $\chi(G) \le |V(G)|$ for any graph G
b) $\chi(K_3) = 2$
c) $\chi(G) \le \{1 + \Delta(G)\}$
d) $\chi(G) \ge \dfrac{|V(G)|}{|V(G)| - \delta(G)}$

40. Find the chromatic number of the graph shown below



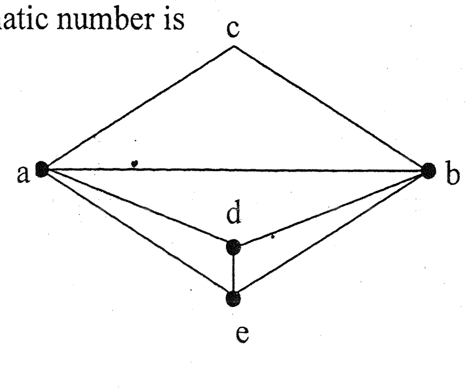
a) 2 b) 3
c) 4 d) 5

41. For the graph shown below, the chromatic number is



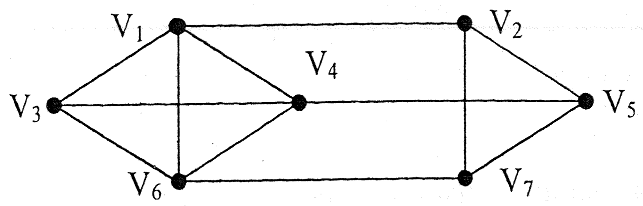
a) 2
b) 3
c) 4
d) 5

42. For the graph shown below, the chromatic number is



a) 2
b) 3
c) 4
d) 5

43. For the graph shown below the chromatic number is



a) 2

b) 3

c) 4

d) 5

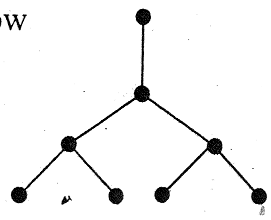
KEY

1. d	2. d	3. c	4. b	5. d	6. c	7. d	8. c	9. a	10. c	11. a	12. c
13. a	14. b	14.(a) d	14.(b) b	14.(c) a	14.(d) a	14.(e) b	14.(f) d	15. b			
16. a	17. b	18. c	19. a	20. d	21. b	22. b	23. c	24. a	25. c	26. d	27. d
28. b	29. d	30. c	31. d	32. c	33. b	34. c	35. b	36. a	37. a	38. a	39. b
40. a	41. b	42. c	43. c								

MATCHING: Given an undirected graph $G = (V, E)$, a matching is a subset of edges $M \subseteq E$ such that for all vertices $v \in V$, at most one edge of M is incident on V. i.e $deg(v) \leq 1 \ \forall v \in V$.

* We say that a vertex $v \in V$ is matched by matching M if some edge in M is incident on v; other wise, v is un matched.
* A maximum matching is a matching of maximum cardinality, that is, a matching M such that for any matching 'M^1', we have $|M| \geq |M^1|$
* A perfect matching is a matching in which every vertex is matched. $deg(v) = 1 \ ; \forall v \in V$.
* A matching in a graph is a sub set of edges in which no two edges are adjacent.
* A single edge in a graph is obviously a matching.
* A maximal matching is a matching to which no edge in the graph can be added.
* A graph may have many different maximal matchings and of different sizes. Among these, the maximal matchings, with the largest number of edges are called the largest maximal matchings. The number of edges in largest maximal matching is called the matching number of the graph.

1. In a complete graph on 3 vertices the number of maximal matchings is
   a) 0 b) 1 c) 2 d) 3
2. The complete graph K_n has a perfect matching if and only if 'n' is____
   a) odd b) even c) prime d) ≥ 4
3. The number of perfect matchings in K_{2n} is
   a) $\frac{\lfloor 2n}{2^n.n!}$ b) $\frac{(2n-1)!}{2^n.(n-1)!}$ c) $\frac{2^n.n!}{2n!}$ d) 0
4. The number of perfect matchings in a complete graph with 6 vertices is
   a) 15 b) 24 c) 30 d)60
5. How many perfect matchings are there in a complete bipartite graph $k_{n,n}$?
   a) 2n b) n^2 c) 2^n d) n!
6. If a graph G has a perfect matching, then the number of vertices in G is
   a) odd b) even c) prime d) a perfect square
7. Consider the following statements
   S_1: Any connected graph with an even number of vertices has a perfect matching
   S_2: The complete bipartite graph $k_{4,6}$ has a perfect matching. Which of the following is true
   a) S_1 is true and S_2 is true b) S_1 is true and S_2 is false
   c) S_1 is false and S_2 is true d) S_1 is false and S_2 is false
8. The number of perfect matchings in a tree with n vertices is
   a) 2 b) ≤ 1 c) $n - 1$ d) $n(n-1)/2$
9. Howe many perfect matchings are there in the tree shown below
   a) 0
   b) 1
   c) 2
   d) 3



10. Which of the following regular graphs has perfect matching?
   a) Q_3 b) W_3 c) $K_{3,3}$ d) K_3

COMPLETE MATCHING

* In a bipartite graph having a vertex partition V_1 and V_2, a complete matching of vertices in set V_1 in to those in V_2 is a matching in which there is one edge incident with every vertex in V_1.
In other words, every vertex in V_1 is matched against some vertex in V_2

* A complete matching (if it exists) is a largest maximal matching, where as the converse is not necessarily true.

* For the existence of a complete matching of set V_1 in to set V_2, first we must have atleast as many vertices in V_2 as there are in V_1. This condition however is not sufficient.

* A complete matching of V_1 in to V_2 in a bipartite graph exists if and only if every subset of r vertices in V_1 is collectively adjacent to r or more vertices in V_2 for all values of r (Hall's Theorem).

* In a bipartite graph a complete matching of V_1 in to V_2 exists if there is a positive integer 'm' for which degree of every vertex in $V_1 \geq m \geq$ degree of every vertex in V_2

NOTE: This condition is a sufficient condition and not necessary for the existence of a complete matching.

* The maximal number of vertices in set V_1 that can be mapped in to V_2 is equal to number of vertices in $V_1 - \delta(G)$

11. Suppose that four applicants a_1, a_2, a_3 and a_4 are available to fill six vacant positions p_1, p_2, p_3, p_4, p_5 and p_6. Applicant a_1 is qualified to fill position p_2 or p_5. Applicant a_2 is qualified to fill position p_2 or p_5. Applicant a_3 is qualified for p_1, p_2, p_3, p_4 or p_6. Applicant a_4 can fill jobs p_2 or p_5. What is the maximum number of positions that can be filled?
a) 4 b) 3 c) 2 d) 1

12. Five senators s_1, s_2, s_3, s_4 and s_5 are members of three committees c_1, c_2 and c_3. c_1 contains $s_1$and s_2, c_2 contains of s_1, s_3 and s_4. c_3 contains of s_3, s_4 and s_5. One member from each committee is to be represented in a super committee. How many representatives can be send to super committee?
a) 1 b) 2 c) 3 d) 4

COVERINGS

In a graph G, a set of edges C is said to **cover** G if every vertex in G is incident on atleast one edge in G.

* A set of edges that covers a graph G is said to be an **edge-covering** (a covering sub graph or simply a **line-covering**) of G

MINIMAL LINE COVERING: A line-covering from which no edge can be removed with out destroying its ability to cover the graph.

* A line-covering exists for a graph iff the graph has no isolated vertex.

* A line-covering of an 'n' vertex graph has atleast $\lceil n/2 \rceil$ edges .

* Every line-covering contains a minimal line-covering.

* No minimal line-covering can contain a circuit.

* A graph, in general, has many line-coverings, and then may be of different sizes (i.e consisting of different number of edges). The number of edges in a minimal line-covering of smallest size is called the **line-covering number** of the graph.

* A line-covering C of a graph G is minimal if G contains no paths of length three or more.

* If a line-covering G contains no paths of length three or more, then all its components must be **star graphs**. (From a star graph no edge can be removed)

13. The number of edges in the minimal line-covering of an n - vertex graph is
 a) $\leq n/2$ b) $\leq (n-1)$ c) 2n d) n

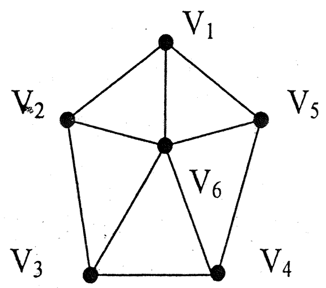
VERTEX COVERING : A covering of a graph $G = (V, E)$ is a subset K of V such that every line of G is incident with a vertex in K. A covering K is called a **minimum vertex-covering** if G has no covering K^1 with $|K^1| < |K|$

* The number of vertices in a minimum vertex-covering of G is called the **covering number** of G, denoted by β.

* A sub set S of V is called an **vertex-independent set** of G if no two vertices of S are adjacent in G.

* The number of vertices in a maximum independent set is called the independence number of G.
* A set S is an independent set of G iff $V - S$ is a vertex-covering of G .

14. For the graph shown below,



Which of the following is false

a) Vertex covering number = 4 b) Vertex independence number = 2
c) $\{V_1, V_3\}$ is a maximum vertex independent set
d) $\{V_1, V_2, V_4\}$ is a vertex covering

KEY

1. d 2. b 3. a 4. a 5. d 6. b 7. d 8. b 9. a 10. d 11. b 12. c
13. b 14. d

CONNECTIVITY

* An undirected graph is called ***connected*** if there is a path between every pair of distinct vertices.
* A graph that is not connected is the union of two or more connected sub graphs each pair of which has no vertex in common. These disjoint connected sub graphs are called ***connected components*** of G.

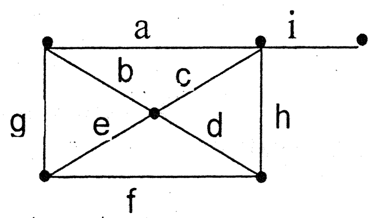
Cut Set: Let G be a connected graph. A cut set in G is a set of edges whose removal from G leaves the graph G disconnected provided no proper subset of these edges disconnects the graph G.

Edge Connectivity: Let G be a connected graph. The edge connectivity of G is the minimum number of edges whose removal results in a disconnected graph. In other words, the number of edges in the smallest cut set of G is defined as the edge connectivity of G denoted by $\lambda(G)$. If G is a connected graph and has a cut edge, then the edge connectivity of G is 1.

Vertex Connectivity: Let G be a connected graph. The minimum number of vertices whose removal results in a disconnected graph is called vertex connectivity of G and is denoted by K(G).

- If G has a cut vertex then K(G) = 1
- If G is a complete graph K_n then $K(G) = n - 1$
- If C_n ($n \geq 4$) is a cycle graph, then $K(C_n) = 2$
- If a graph G has a bridge then K(G) = 1
- The edge connectivity of a connected graph G cannot exceed $\delta(G)$
- For any connected graph G, $K(G) \leq \lambda(G) \leq \delta(G)$
- If G is not connected then G is connected.
- A simple graph with n vertices and k components can have at most $(n - k)(n - k + 1)/2$ edges
- A simple graph with n vertices and k components can have at most $[(n - k)(n - k + 1)]/2$ edges.
- A simple graph G with n vertices is connected if it has more than $[(n - 1)(n - 2)]/2$ edges
-

07. For the graph shown here,



Which of the following is not a cut set.
a) {a, b, g} b) {a, b, e, f} c) {a, c, h, d} d) {a, c, d, f}

08. The edge connectivity and vertex connectivity in the previous example are …… and ….. respectively
a) 1, 1 b) 1, 2 c) 2, 1 d) 2, 2

09. The minimum number of edges in a connected cyclic graph (a graph with atleast one cycle) is *(GATE'95[CS])*
a) $n - 1$ b) n c) $n + 1$ d) $n + 2$

10. Consider a simple graph G with n vertices and n edges ($n > 2$). Then which of the following are true? *(GATE'93[CS])*
a) G has no cycle b) $G - \{e\}$ is not connected where e is any edge
c) G has atleast one cycle
d) The graph obtained by removing 2 edges from G is not connected

11. Which of the following simple graphs is necessarily connected
a) A graph with 6 vertices and 10 edges b) A graph with 7 vertices and 16 edges
c) A graph with 8 vertices and 21 edges d) A graph with 9 vertices and 28 edges

12. A simple graph has 10 vertices and 3 components. The maximum number of edges possible in the graph is
a) 24 b) 26 c) 28 d) 30

ISOMORPHISM

ISOMORPHIC GRAPHS: Two graphs G and G^1 are isomorphic if there is a function
$f : V(G) \rightarrow V(G^1)$ such that
(i) f is a bijection and
(ii) for each pair of vertices u and v of G, $\{u, v\} \in E(G)$ iff $\{f(u), f(v)\} \in E(G^1)$
i.e.. the function preserves adjacency.

- Suppose G and G^1 are two graphs and that f: $V(G) \rightarrow V(G^1)$ is a bijection. Let A be the adjacency matrix for the vertex ordering $V_1, V_2, \ldots\ldots, V_n$ of the vertices of G. Let A^1 be the adjacency matrix for the vertex ordering $f(V_1), f(V_2), \ldots, f(V_n)$. Then f is an isomorphism from V(G) to $V(G^1)$ iff the adjacency matrices A and A^1 are equal.

If $A \neq A^1$, then it may still be the case that graph G and G^1 are isomorphic under some other function.

- Two simple graphs are isomorphic iff their complements are isomorphic.
- If G is isomorphic to G^1 then the following conditions must hold good
- $|V(G)| = |V(G^1)|$
- $|E(G)| = |E(G^1)|$
- The degree sequences of G and G^1 are the same
- If {v, v} is a cycle in G, then {f(v), f(v)} is a loop in G^1, and more generally, if $v_0 - v_1 - v_2 - \ldots, - v_n = v_0$ is a cycle of length k in G, then $f(v_0) - f(v_1) - f(v_2) - \ldots f(v_k)$ is a cycle of length k in G^1.
- If two graphs are isomorphic, then their corresponding subgraphs are isomorphic.
- **Induced Subgraph:** If W is a subset of V(G), then the subgraph induced by W is the subgraph H of G obtained by taking V(H) = W and E(H) to be those edges of G that join pairs of vertices in W.
- If G is isomorphic to $\overline{G}$ then G is said to be self complementary.
- If G is self complementary then G has 4n or 4n + 1 vertices.

16. If a graph G with n vertices is isomorphic to $\overline{G}$ then $|E(G)| =$
a) $n(n-1)/2$ b) $n(n-1)/3$ c) $n(n-1)/4$ d) $n(n+1)/2$

17. If a graph G is self complementary then which of the following is false.
a) $|V(G)| = 5$ b) $|V(G)| = 4$ c) $|V(G)| = 7$ d) $|V(G)| = 9$

18. How many non isomorphic graphs are there of order 4 and size 2?
a) 2 b) 3 c) 4 d) 6

19. How many non isomorphic graphs are there of order 8, size 8 and degree sequence {2, 2, 2, 2, 2, 2, 2, 2}.
a) 2 b) 3 c) 4 d) 5

20. How many non isomorphic graphs are there of order 6, size 6 and degree sequence {2, 2, 2, 2, 2, 2}.
a) 2 b) 3 c) 4 d) 5

21. Let C_n be a cycle graph on n vertices if C_n is isomorphic to C_n then n =
a) 4 b) 7 c) 6 d) 5

22. How many different (pairwise non isomorphic) trees are there of order 4?
a) 2 b) 3 c) 4 d) 5

23. How many different (pairwise non isomorphic) trees are there of order 5?
a) 2 b) 3 c) 4 d) 5

24. How many different (pairwise non isomorphic) trees are there of order 6?
a) 2 b) 8 c) 4 d) 6

Spanning Trees

A sub graph H of a graph G is called a spanning tree of G if
i) H is a tree and
ii) H contains all vertices of G

- In general, if G is a connected graph with n vertices and m edges, a spanning tree of G must have (n – 1) edges. Therefore, the number of edges that must be removed before a spanning tee is obtained must be m – (n – 1). This number is called ***circuit rank*** of G.
- A non directed graph G is connected iff G contains a spanning tree.
- The complete graph K_n has n^{n-2} different spanning trees (Calley's formula)

Kirchoff's Theorem: Let A be the adjacency matrix of a connected graph G and M be the matrix obtained from A by changing all 1's into – 1 and each diagonal element 0 to the degree of the corresponding vertex. Then the number of spanning trees of G is equal to value of any cofactor of M.

* The number of different trees with vertex set $\{v_1, v_2, \ldots, v_n\}$ in which the vertices $v_1, v_2, \ldots, v_n$ have degrees $d_1, d_2, \ldots, d_n$ respectively is

$$\frac{(n-2)!}{(d_1-1)! \quad (d_2-1)! \ldots\ldots (d_n-1)!}$$

Minimal Spanning Tree: Let G be a connected graph where each edge of G is labeled with a non negative cost. A spanning tree T where the total cost C(T) is minimum is called a minimal spanning tree.
Kruskal's Algorithm: (For finding minimal spanning tree of a connected weighted graph).
Input: A connected graph G with non negative values assigned to each edge.
Output: A minimal spanning tree for G.
Method:
1) Select any edge of minimal value that is not a loop. This is the first edge of T(if there is more than one edge of minimal value, arbitrary choose one of these edges)
2) Select any remaining edge of G of having minimal value that does not form a circuit with the edges already included in T.
3) Continue step 2 until T contain (n – 1) edges when n = |V(G)|

Prim's Algorithm: (For finding a minimal spanning tree)

1) Let G be a connected graph with non negative values assigned to each edge. First let T be the tree consisting of any vertex V_1 of G.
2) Among all the edges not in T that are incident on a vertex in T and do not form a circuit when added to T. Select one of minimal cost and add it to T.
3) The process terminates after we have added (n – 1) edges where n = |V(G)|.

Ex(25) The number of spanning trees in a complete graph on 6 vertices is

a) 720 b) 64 c) 36 d) 1296

KEY

1. d	2. b	3. a	4. a	5.	6. b	7. c	8. a	9. b	10. c, d	11. b	
12. c	13.	14.	15.	16. c	17. c	18. a	19. b	20. a	21. d	22. a	23. b
24. d	25. d										

Previous GATE questions

01. Let G = (V , E) be a graph. Define $\xi(G) = \sum i_d \times d$, where id is the number of verticesof degree d in G . If S and T are two different trees with $\xi(S) = \xi(T)$, then

a) |S| = 2 |T| b) |S| = |T| - 1 c) |S| = |T| d) |S| = 2 |T| + 1 (GATE-2010)

02. The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences cannot be the degree sequence of any graph ?

I. 7, 6, 5, 4, 4, 3, 2, 1
II. 6, 6, 6. 6, 3, 3, 2, 2
III. 7,6,6,4,4,3,2,2
IV 8,7,7,6,4,2,1,1

GATE – 2010

(a) I and II (b) III and IV (c) IV only (d) II and IV

Common Data for Questions 03 and 04:

Consider a complete undirected graph with vertex set {0,1,2,3,4}. Entry W_{ij} in the matrix W below is the weight of the edge {i, j}.

$$W = \begin{bmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{bmatrix}$$

03. What is the minimum possible weitht of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T? **GATE – 2010**

(a) 7 (b) 8 (c) 9 (d) 10

04. What is the minimum possible weight of a path P from vertex 1 to vertex 2 in this graph such that P contains at most 3 edges? **GATE – 2010**

(a) 7 (b) 8 (c) 9 (d) 10

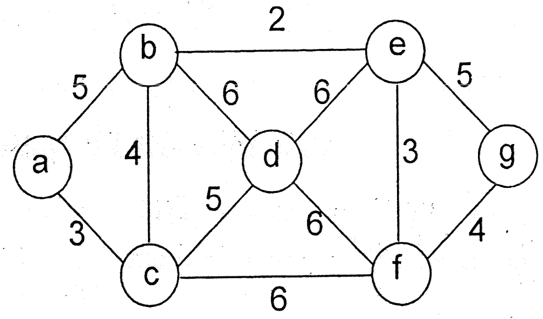
05. What is the chromatic number of an n-vertex simple connected graph which does not contain any odd length cycle? ($n \geq 2$) **GATE – 2009**

(a) 2 (b) 3 (c) $n - 1$ (d) n

06. Which one of the following is true for any simple connected undirected graph with more than two Vertices? **GATE – 2009**

(a) No two vertices have the same degree
(b) Atleast two vertices have the same degree
(c) Atleast three vertices have the same degree
(d) All vertices have same degree

07. Consider the following graph



Which one of the following is not the sequence of edges added to the minimum spanning tree using Krulkal's algorithm? **GATE – 2009**

(a) (b, e), (e, f), (a, c), (b, c), (f, g), (c, d)
(b) (b, e), (e, f), (a, c), (f, g), (b, c), (c, d)
(c) (b, e), (a, c), (e, f), (b, c), (f, g), (c, d)
(d) (b, e), (e, f), (b, c), (a, c), (f, g), (c, d)

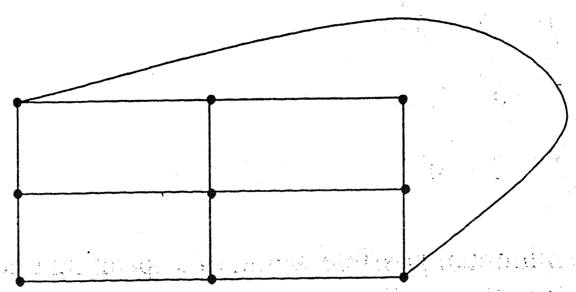
08. Which of the following statements is true for every planar graph on n vertices? **GATE – 2008**

(a) The graph is connected
(b) The graph is Eulerian
(c) The graph has a vertex cover of size at most $\frac{3n}{4}$
(d) The graph has an independent set of size atleast $\frac{n}{3}$

09. What is the size of the smallest MIS (Maximal Independent Set) of a chain of nine nodes **GATE – 2008**

(a) 5 (b) 4 (c) 3 (d) 2

10. What is the chromatic number of the following graph? **GATE – 2008**



(a) 2 (b) 3 (c) 4 (d) 5

11. G is a simple undirected graph. Some vertices of G are of odd degree. Add a node v to G and make it adjacent to each odd degree vertex of G. The resultant graph is sure to be **GATE – 2008**

(a) regular (b) complete (c) Hamiltonian (d) Euler

12. A binary tree with n(n>1) nodes has n_1, n_2 and n_3 nodes of degree one, two and three respectively. The degree of a node is defined as the number of its neighbors.

(i) n_3 can be expressed as **GATE – 2008**

(a) $n_1 + n_2 - 1$ (b) $n_1 - 2$ (c) $\frac{n_1 + n_2}{2}$ (d) $n_2 - 1$

(ii) Starting with the above tree, while there remains a node v of degree 2 in the tree, add an edge between the two neighbors of v and then remove v from the tree. How many edges will remain at the end of the process? **GATE – 2008**

(a) $2n_1 - 3$ (b) $n_2 + 2n_1 - 2$ (c) $n_3 - n_2$ (d) $n_2 + n_1 - 2$

13. What is the largest integer m such that every simple connected graph with n vertices and n edges contain atleast m different spanning trees? **GATE – 2007**

(a) 1 (b) 2 (c) 3 (d) n

14. Let G be the non planar graph with the minimum possible number of edges. They G has **GATE – 2007**

(a) 9 edges and 5 vertices (b) 9 edges and 6 vertices
(c) 10 edges and 5 vertices (d) 10 edges and 6 vertices

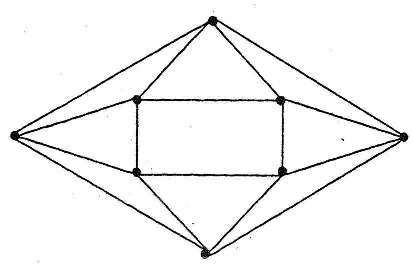
15. Which of the following graphs has an Eulerian circuit? **GATE – 2006**

(a) Any k – regular graph where k is an even number
(b) A complete graph on 90 vertices
(c) The complement of a cycle on 25 vertices
(d) None of the above

16. Consider an undirected graph G defined as follows. The vertices of G are bit strings of length n. We have an edge between vertex u and vertex v if and only if u and v differ in exactly one bit position. The ratio of chromatic number of G to the diameter of G is **GATE–2005**

(a) $\frac{1}{2^{n-1}}$ (b) $\frac{1}{n}$ (c) $\frac{2}{n}$ (d) $\frac{3}{n}$

17. The minimum number of colors required to color the following graph, such that no two adjacent vertices are assigned the same color, **GATE – 2004**



(a) 2 (b) 3 (c) 4 (d) 5

18. How many graphs on n labeled vertices exist which have atleast $(n^2 - 3n)/2$ edges? **GATE – 2004**

(a) $C\left(\frac{n^2-n}{2}, \frac{n^2-3n}{2}\right)$ (b) $C\left(\frac{n^2-n}{2}, n\right)$

(c) $\sum_{k=0}^{(n^2-3n)/2} C\left(\frac{n^2-n}{2}, k\right)$ (d) $\sum_{k=0}^{n} C\left(\frac{n^2-n}{2}, k\right)$

19. Let G be an arbitrary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie between **GATE – 2003**

(a) k and n (b) k – 1 and k + 1
(c) k – 1 and n – 1 (d) k + 1 and n – k

20. How many perfect matching are there in a complete graph of 6 vertices? **GATE – 2003**

(a) 15 (b) 24 (c) 30 (d) 60

21. A graph G = (V, E) satisfies $|E| \le (3|V| - 6)$. The min-degree of G cannot be ____ **GATE – 2003**

(a) 3 (b) 4 (c) 5 (d) 6

22. The minimum number of colors required to color the vertices of a cycle with n nodes in such a way that no two adjacent nodes have the same color is **GATE – 2002**

(a) 2 (b) 3 (c) 4 (d) $n - 2\lfloor n/2 \rfloor + 2$

23. How many undirected graphs (not necessarily connected) can be constructed out of a given set $V = \{v_1, v_2, \ldots, v_n\}$ of n vertices? **GATE – 2001**

(a) $\frac{n(n-1)}{2}$ (b) 2^n (c) $\lfloor n$ (d) $2^{\frac{n(n-1)}{2}}$

Key

01.c	02.d	03.d	04.a	05.a	06.b	07.d	08.c	09.c	10.b	11.d	12. (i)b
12.(ii) b		13.d	14.b	15.a	16.c	17. c	18.c	19.c	20.a	21.d	22. d
23.d											

ROOT FINDING : Consider the equation $f(x) = 0$ .... (1)
If $f(a) < 0$ and $f(b) > 0$ then at least one root of the equation (1) lies between a and b.
$[f(a) . f(b) < 0]$

BISECTION METHOD :

Iterative formula : If x_i and x_{i-1} enclose the root then

$$x_{i+1} = \frac{(x_i + x_{i-1})}{2}$$

* The Bisection method converges slowly. It is however, the simplest iterative method and is guaranteed to converge. Convergence is slow and steady.

REGULA FALSI METHOD : (METHOD OF FALSE POSITION) :
Consider, $f(x) = 0$
If $f(a) . f(b) < 0$ then the iterative formula is

$$x_i = \frac{a . f(b) - b f(a)}{f(b) - b f(a)}$$

* The false position method is guaranteed to converge.
* It is however, slow as it is first order convergent. i.e. order of convergence = 1.
* It is not necessarily a monotonic convergence to the root but most often it will be superior to Bisection method.

NEWTON -RAPHSON METHOD :

The iteration formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)} \quad (n = 0,1,2,..)$$

Where x_0 is approximate root of the equation $f(x) = 0$
* The Newton's method is useful in cases of large valves of $f^1(x)$ i.e, when the graph of $f(x)$ while crossing the x – axis is nearly vertical.
* Sensitive to starting value. Convergence fast if starting point near the root.
  i.e.. The Newton's formulae converges provided the initial approximation x_0 is chosen sufficiently close to the root.
* Newton's method is generally used to improve the result obtained by other methods
* Newton's method has a quadratic convergence. i.e. order of convergence = 2.
  i.e. The subsequent error at each step is proportional to the square of the error at previous step
* The number of functions to be evaluated per iteration is 2

Secant method (Modified version of regula falsi or interpolation method) :

Newton Raphson method is very powerful, but the evaluation of derivative involved may some times be difficult or expensive.

This suggests the idea of replacing $f^1(x_n)$ by the difference quotient.

$$f^1(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

➢ Instead of choosing two values of x such that the function has opposite signs at these values we choose two values nearest the root

➢ This method is applicable only if one is sure that there is a root in the vicinity of x_1 the starting value.

➢ On the average, Secant method is more efficient compared to Newton Raphson method

* In this method, we approximate the graph of the function $y = f(x)$ in the neighborhood of the root by a straight line (secant) passing through the points (x_{i-1}, f_{i-1}) and (x_i, f_i)

The iteration formula is

$$x_{i+1} = \frac{x_{i-1}\ f_i - x_i\ f_{i-1}}{f_i - f_{i-1}}$$

Where x_i and x_{i-1} need not enclose the root.

* The order of convergence is 1.62. converges faster than false position method.
* No guarantee of convergence if not near root. The method fails if $f(x_i) = f(x_{i-1})$
* It may be considered the most economical method giving reasonably rapid convergence at a low cost.
* The amount of computational effort is one function evaluation

1. Using Newton Raphson iteration formulae, find the first approximation to the root of the equation
   $x^4 - x - 10 = 0$ which is nearer to $x = 2$
   a) 1.671 b) 1.871 c) 2.071 d) 2.271

2. Using, Newton's method, find the first approximation to the root of the equation
   $3x = \cos x + 1$ (Take $x_0 = 0$ as initial approximation)
   a) 1/3 b) 1/2 c) 2/3 d) ¾

3. The Newton's iteration formula for finding $\sqrt{N}$ where N is a positive real number is
   a) $x_{n+1} = 1/2\,(x_n + N/x_n)$ b) $x_{n+1} = 1/2\,(x_n - N/x_n)$
   c) $x_{n+1} = x_n\,(2 - N.x_n)$ d) $x_{n+1} = x_n\,(2 + N.x_n)$

4. For $N = 18$ and $x_0 = 4$, The first approximation to $\sqrt{18}$ by Newton's iteration formula is
   a) 4.20 b) 4.25 c) 4.24 d) 4.2426

5. The Newton's iteration formulae for finding 1/N where N is a positive real number is
   a) $x_{n+1} = x_n\ (2 + N/x_n)$ b) $x_{n+1} = x_n\ (2 - N.x_n)$
   c) $x_{n+1} = x_n\ \{2 + (N/x_n)\}$ d) $x_{n+1} = x_n\ \{2 - N/x_n\}$

6. The Newton's – Raphson's iteration formulae for finding $N^{1/3}$ where N is a positive real number is
   a) $x_{n+1} = 1/3\,(2\,x_n + N/xn^2)$ b) $x_{n+1} = 1/3\,(2\,x_n - N/x_n^2)$
   c) $x_{n+1} = 1/3\,(2\,x_n^3 + N/x_n^2)$ b) $x_{n+1} = 1/3\,(2\,x_n^3 - N/x_n^2)$

7. If the initial approximation to a root of the equation $x = e^{-x}$ is $x_0 = 1$, then the first approximation to the root of the equation is
   a) 0.567 b) 0.667 c) 0.767 d) 0.867

8. Using bisection method, find a second approximation to the root of equation
   $x^3 - 4x - 9 = 0$ between 2 and 3
   a) 2.25 b) 2.5 c) 2.75 d) 2.625

9. Find first approximation to a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position between 2 and 3
   a) 2.0588 b) 2.0466 c) 2.0831 d) 2.0614

10. Using secant method, find first approximation to the root of the equation $x\,e^x = \cos x$ between 0 and 1
    a) 0.31467 b) 0.44673 c) 0.51776 d) 0.6451

11. Which of the following methods can not be applied for locating complex roots of an equation
    a) Bisection method b) Regula falsi
    c) Secant method d) Newton's Raphson method

12. After Newton Raphson's method, which of the following has fastest rate of convergence
    a) Bisection method b) Secant method
    b) Method of false position d) Successive approximation

KEY

1. b 2. c 3. a 4. b 5. b 6. a 7. a 8. c 9. a 10. a 11. a 12. b

LU Decomposition for system of Linear equations:

- Consider AX = B a linear system of n equations in n variables, $x_1, x_2, \ldots\ldots x_n$.
- This method is based on the fact that every matrix A can be expressed as the product of a lower trianglar matrix and an upper triangular matrix, provided all the principal minors of A are zero.

Consider AX = B - (1) a system of 3 equations in 3 variables

Let A = L U - (2)

Where

$$L = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \quad \text{and} \quad U = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

then the equation (1) becomes

$$L\,U\,X = B \rightarrow (3)$$

Writing $UX = V \rightarrow (4)$, equation (3) becomes

$$LV = B \rightarrow (5)$$

Where $V = [V_1 \quad V_2 \quad V_3]^T$

Solve equation (5) for V

Substituting in (2) we get X

Both systems (4) & (5) are triangular, so their solving is same as the back substitution in the Gauss elimination. This is called ***Doolittle's method.***

A similar method, ***Crout's method*** is obtained from (2) if U(instead of L) is required to have main diagonal 1,1, ...1. In either case factorization (2) is unique.

We compute the elements of L and U in the following order.

i) First row of U b) First column of L c) Second row of U

iv) Second column of L d) Third row of U

This method is superior to Gauss elimination method and is used for finding the inverse of the matrix.

The crucial idea is that L and U can be computed directly, without solving simultaneous equations (Thus without using Gauss elimination). As a count shows, this needs $(n^3/3)$ operations, about half as many as the Gauss elimination, which needs about $(2n^3/3)$ and once we have (2), we can use it for solving AX = B in two steps, involving about n^2 operations.

Ex(1): Consider the system of linear equations

$$4x_1 + 5x_2 = 7 \quad ; \qquad 12x_1 + 14x_2 = 18$$

If we apply LU decomposition for solving the system where diagonal elements of L are unity, then U =

a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 4 & 5 \\ 0 & -1 \end{pmatrix}$ c) $\begin{pmatrix} 4 & 5 \\ 0 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 4 & -5 \\ 0 & 1 \end{pmatrix}$

Ex(2): Let AX = B be the system given in the previous example. For LU decomposition we can write A = L U so that L U X = B will be the given system. If we write UX = V then the system reduces to LV = B. Now V =

a) $[7 \quad 3]^T$ b) $[7 \quad -3]^T$ c) $[-7 \quad 3]^T$ d) $[-7 \quad -3]^T$

Cholesky's method:

- For symmetric, positive definite matrix A, we can, in equation A = LU, even choose U $= L^T$
- The popular method of solving AX = B is based on this factorization $A = LL^T$ is called cholesky's method.
- If A is symmetric but not positive definite, this method could still be applied but then leads to a complex matrix L, so that it becomes impractical.

Ex(3): Consider the system of linear equations x + 2y = 3, 2x + 5y = 7.

If we apply LU decomposition (where $U = L^T$) for solving the system then L =

a) $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$ d) $\begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$

Ex(4): Let AX = B be the system given in the previous example. For LU decomposition by Choleski method we take $A = LL^T$ so that $LL^TX = B$ be the given system. If we write $L^TX = V$ then the system reduces to LV = B. Now V =

a) $[3 \quad -1]^T$ b) $[3 \quad 1]^T$ c) $[3 \quad 2]^T$ d) $[3 \quad -2]^T$

Ex(5): Consider the system of equations $4x_1 + 2x_2 + 14x_3 = 14$; $2x_1 + 17x_2 - 5x_3 = -101$

$$14x_1 - 5x_2 + 83x_3 = 155$$

If we apply LL^1 decomposition by Choleski's method then L =

a) $\begin{pmatrix} 2 & 0 & 0 \\ 1 & 4 & 0 \\ 7 & -3 & 5 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 0 & 0 \\ -1 & 4 & 0 \\ 7 & 3 & 5 \end{pmatrix}$ c) $\begin{pmatrix} 2 & 0 & 0 \\ 1 & 4 & 0 \\ 7 & 3 & -5 \end{pmatrix}$ d) $\begin{pmatrix} 2 & 0 & 0 \\ 1 & -4 & 0 \\ 7 & 3 & 5 \end{pmatrix}$

NUMERICAL INTEGRATION

It is the process of finding or evaluating a definite integral $I = \int_a^b f(x)\,dx$ from a set of numerical values of the integrand f(x).

- The process of numerical integration is solved by first approximating the integrand by a polynomial with the help of an interpolation formula and then integrating this expression between the desired limits.

  Consider,

$$I = \int_A^b f(x)\,dx$$

Let $y = f(x)$ be given for certain equidistant values of arguments,
say $x_0, x_0 + h, x_0 + 2h,$
Let the range $(b - a)$ be divided into n equal parts, each of which is of width h
i.e., $b - a = nh$
Let $x_0 = a, x_1 = a + h, x_2 = a + 2h, \ldots\ldots\ldots\ldots x_n = a + nh = b$

Compute

x	x_0	x_1	x_2	.....	x_n
$Y = f(x)$	y_0	y_1	y_2	....	y_n

Trapezoidal Rule:

$I = h/2\,[(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})]$

- Here we have taken y, a polynomial of first degree in x or a straight line.
- The error due to trapezoidal rule is given by $E = -(h^3/12)\, f^{11}(0) - (h^4/24)\, f^{111}(0)$

  (In evaluating $\int_0^h f(x)\,dx$)
- As h is numerically smaller than 1, the dominant error term is the h^3- term.
- The rule gives exact value at the integral if f(x) is a polynomial function of degree 0 or 1.
- If the interval at which f(x) is tabulated is halved, the error would be reduced by an eighth.

Simpson's Rule:

- A popular numerical integration technique

  Consider $I = \int_a^b f(x)\,dx$
- Divide [a, b] into n equal parts, where n is even number
- The method is based on approximating the function f(x) by fitting quadratics through sets of 3 points.
- $I = (h/3)[(y_0 + y_n) + 4(y_1 + y_3 + \ldots\ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots\ldots + y_{n-2})]$
- In this rule, we have neglected all differences above second, so y will be a polynomial of second degree only.
- The error term due to simpson's rule is $E= - (h^5/90)\, f^{IV}(0)$
- If f(x) is a polynomial function of degree ≤ 3, then Simpson's rule gives exact value of the integral.
- If the tabulated interval is halved, the error is reduced by a factor of 32
- In general, Simpson's rule is more accurate than Trapezoidal rule. Simpson's rule with n points gives about as much accuracy as Trapezoidal rule with 2n points.
- If a function is tabulated at random unequal intervals then only Trapezoidal rule can be applied.

6) Given that,

x	0.25	0.26	0.27	0.28	0.29
f(x)	0.2474	0.2571	0.2667	0.2764	0.2860

If trapezoidal rule is applied, then

$\int_{0.25}^{0.29} f(x)\,dx =$

a) 0.1057 b) 0.0107 c) 0.0415 d) 0.1156

7) If we integrate the function tabulated below using Simpson's rule between -0.5 and 0.3 then the value of the integral is

a) 2.45 b) 2.68 c) 2.80 d) 3.15

8) If we integrate $f(x) = 5x^3 - 3x^2 + 2x + 1$ from $x = -1$ to $x = 1$ using Simpson's rule (with $h = 1$)then the value of the integral is

a) 0 b) 0.1214 c) 0.0123 d) 0.0012

9) In the previous example, the error due to Simpson's rule is

a) 0 b) -0.1214 c) -0.0123 d) -0.0012

10) A curve is drawn passing through the points given by

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

X	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
f(x)	2	3	8	4	-2	2	3	5	8

By Simpson's rule, the estimated area bounded by the curve, the x–axis, x =1 and x= 4 is

a) 6.846 Sq units b) 7.783 Sq units c) 8.248 Sq. units d) 4.646 Sq. units

11) The value of the integral $\int_0^6 dx / (1 + x^2)$ by Trapezoidal rule (Take $h = 1$) is

a) 1.3662 b) 1.4056 c) 1.4108 d) 1.3891

12) Evaluate $\int_0^4 e^x\, dx$ by Simpson's rule, using the data $e = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$ and $e^4 = 54.60$

a) 53.60 b) 53.873 c) 54.021 d) 53.461

13) A solid of revolution is formed by rotating about the x – axis, the area between the x – axis, the lines $x = 0$, $x = 1$ and a curve through the points with the following co – ordinates

x	0	0.25	0.5	0.75	1
y	1	2	3	1	1

The estimated volume by Simpson's rule is

a) $10(\pi/3)$ b) $13\pi/3$ c) $5\pi/2$ d) 5π

ADDITIONAL PROBLEMS

Ex(14): Using secant method find a root of $f(x) = x^3 + x^2 - 2x - 4 = 0$ between 1 and 2, after two iterations.
a) 1.5 b) 1.6279 c) 1.6623 d) 1.6589

Ex(15): The Newton Raphson iteration formula for finding inverse square root of N is
a) $x_{n+1} = ½ (3x_n - Nx_n^3)$ b) $x_{n+1} = ½ (3x_n + Nx_n^2)$
c) $x_{n+1} = ½ (3x_n + Nx_n^3)$ d) $x_{n+1} = ½ (3x_n - Nx_n^2)$

Ex(16): Newton Raphson method is used to find the root of the equation $x^2 - 2 = 0$. If the iterations are started from -1, the iterations will (GATE-97')
a) converge to –1 b) converge to $\sqrt{2}$ c) converge to $-\sqrt{2}$ d) not converge

Ex(17): Which of the following statements applies to bisection method used for finding roots of function. (GATE-98')
a) converges within few iterations b) guaranteed to work for all continuous functions
c) is faster than Newton Raphson method
d) requires that there will be no error in determining the sign of functions

18) Starting from $x_0 = 1$, one step of Newton Raphson method in solving equation $x^3 + 3x - 7 = 0$ gives the next value x_1 as *(GATE'05[ME])*
a) $x_1 = 0.5$ b) $x_1 = 1.406$ c) $x_1 = 1.5$ d) $x_1 = 2$

19) Consider the following iterative root finding methods and convergence properties

Method	**Property**
Q : False Position	I) Order of convergence is 1.62
R : Newton Raphson	II) Order of convergence is 2
S : Secant	III) Order of convergence is 1 with guarantee of convergence
T : Successive Approximation	IV) Order of convergence = 1 with no guarantee of convergence

The correct matching of methods and properties is *(GATE'04[IT])*
a) Q – II, R – IV, S – III, T – I b) Q – III, R – II, S – I, T – IV
c) Q – II, R – I, S – IV, T – III d) Q – I, R – IV, S – II, T – III

20) The trapezoidal rule for integration gives exact result when the integrand is a polynomial of degree
a) 0 but not 1 b) 1 but not 0 c) 0 or 1 d) 2 *(GATE'02)*

21) The Newton Raphson iteration $x_{n+1} = (x_n/2) + (3/2)x_n$ can be used to solve the equation
a) $x^2 = 3$ b) $x^3 = 3$ c) $x^2 = 2$ d) $x^3 = 2$ *(GATE'02)*

22) The real root of the equation $x \cdot e^x = 2$ is evaluated using Newton – Raphson's method. If the first approximation of the value is 0.8679, the second approximation of the value of x correct to three decimal places is *(GATE'05[PI])*
a) 0.865 b) 0.853 c) 0.849 d) 0.838

23) If the trapezoidal rule is used to evaluate the integral $\int_0^1 x^2\,dx$ then the value obtained
a) is always > 1/3 b) is always < 1/3
c) is always = 1/3 d) may be greater or lesser than 1/3

24) Given $a>0$, we wish to calculate its reciprocal value $1/a$ by using Newton Raphson method for $f(x) = 0$. The Newton Raphson iteration formula for the function will be

a) $x_{k+1} = \frac{1}{2}(x_k + a / x_k)$ b) $x_{k+1} = x_k + a/2\ x_k^2$

c) $x_{k+1} = 2x_k - a\ x_k^2$ d) $x_{k+1} = x_k - a/2\ x_k^2$ [GATE-05-CE]

25) Using the iteration formula in the last example, for $a = 7$ and starting with $x_0 = 0.2$, the first two iterations will be

a) 0.11, 0.1299 b) 0.12, 0.1392 c) 0.12, 0.1416 d) 0.13, 0.1426

KEY

1. b 2. b	3. a	4. b	5. a	6. b	7. c	8. a	9. a	10. b	11. c	12. b
13. a 14. b	15. a	16. b	17. d	18. c	19. b	20. c	21. c	22.	23. a	24. c
25. b										

PREVIOUS GATE QUESTIONS

01. Newton – Raphson formula to find the roots of an equation f(x) = 0 is given by **PI-2005-1M**

(a) $x_{n+1} = x_n - \left[\frac{f(x_n)}{f^{|}(x_n)}\right]$

(b) $x_{n+1} = x_n + \left[\frac{f(x_n)}{f^{|}(x_n)}\right]$

(c) $x_{n+1} = \frac{f(x_n)}{\left[x_n f^{|}(x_n)\right]}$

(d) $x_{n+1} = \frac{x_n f(x_n)}{\left[f^{|}(x_n)\right]}$

02. For solving algebraic and transcendental equation, which one of the following is used? **PI-2005-1M**
(a) Coulomb's theorem
(b) Newton – Raphson Method
(c) Euler's theorem
(d) Stoke's theorem

03. The polynomial $p(x) = x^5 + x + 2$ has **IN-2007-2M**
(a) all real roots
(b) 3 real and 2 complex roots
(c) 1 real and 4 complex roots
(d) all complex roots

04. Identify the Newton – Raphson iteration scheme for the finding the square root of 2 **IN-2007-2M**

(a) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$

(b) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{2}{x_n}\right)$

(c) $x_{n+1} = \frac{2}{x_n}$

(d) $x_{n+1} = \sqrt{2 + x_n}$

05. Given that one root of the equation $x^3 - 10x^2 + 31x - 30 = 0$ is 5 the other tow roots are **CE-2007-2M**
(a) 2 and 3 (b) 2 and 4
(c) 3 and 4 (d) – 2 and – 3

06. The following equation needs to be numerically solved using the Newton – Raphson method $x^3 + 4x - 9 = 0$. The iterative equation for this purpose is (k indicates the iteration level) **CE-2007-2M**

(a) $x_{k+1} = \frac{2x_k^3 + 9}{3x_k^2 + 4}$

(b) $x_{k+1} = \frac{3x_k^3 + 4}{2x_k^2 + 9}$

(c) $x_{k+1} = x_k - 3x_k^2 + 4$

(d) $x_{k+1} = \frac{4x_k^2 + 3}{9x_k^2 + 2}$

07. Matching exercise choose the correct one out of the alternatives A, B, C, D

Group – I

P. 2^{nd} order differential equations
Q. Non – linear algebraic equations
R. Linear algebraic equations
S. Numerical integration

Group – II

1. Range – kulta method
2. Newton – Raphson method
3. Gauss elimination
4. Simpson's rule **PI-2007-2M**

(a) P – 3, Q – 2, R – 4, S – 1
(b) P – 2, Q – 4, R – 3, S – 1
(c) P – 1, Q – 2, R – 3, S – 4
(d) P – 1, Q – 3, R – 2, S – 4

08. A calculator has accuracy upto 8 digits after decimal places. The value of $\int_0^{2\pi} \sin x \, dx$ when evaluated using this calculator by trapezoidal method with 8 equal intervals to 5 significant digits is **ME-2007-2M**

(a) 0.0000 (b) 1.000
(c) 0.00500 (d) 0.00025

09. The equation $x^3 - x^2 + 4x - 4 = 0$ is to be solved using the Newton – Raphson method. If x = 2 is taken as the initial approximation of the solution then the first approximation using this method will be **EC-2008-2M**

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$
(c) 1 (d) $\frac{3}{2}$

10. A differential equation $\frac{dx}{dt} = e^{-2t} u(t)$ has to solved using trapezoidal rule of integration with a step size h = 0.01 s. Function u(t) indicates a unit step function if $x(0^-) = 0$, then value of x at t = 0.01 s will be given by **EE-2008-2M**

(a) 0.00099 (b) 0.00495
(c) 0.0099 (d) 0.0198

11. Equation $e^x - 1 = 0$ is required to be solved using Newton's method with an initial guess $x_0 = -1$. Then after one step of Newton's method, estimate x_1 of the solution will be given by **EE-2008-2M**

(a) 0.71828 (b) 0.36784
(c) 0.20587 (d) 0.00000

12. It is known that two roots of the non – linear equation $x^3 - 6x^2 + 11x - 6 = 0$ are 1 and 3. The third root will be **IN-2008-2M**

(a) j (b) – j
(c) 2 (d) 4

13. The recursion relation to solve $x = e^{-x}$ using Newton – Raphson method is **EC-2008-2M**

(a) $x_{n+1} = e^{-x_n}$

(b) $x_{n+1} = x_n - e^{-x_n}$

(c) $x_{n+1} = (1 + x_n)\frac{e^{-x_n}}{1 + e^{-x_n}}$

(d) $x_{n+1} = \frac{x_n^2 - e^{-x_n}(1 + x_n) - 1}{x_n - e^{-x_n}}$

14. If the interval of integration is divided into two equal intervals of width 1.0, the value of the definite integral $\int_1^3 \log_e x \, dx$, using simpson's one – third rule, will be **PI-2008-2M**

(a) 0.50 (b) 0.80
(c) 1.00 (d) 1.29

15. Let $x^2 - 117 = 0$. The iterative steps for the solution using Newton's – Raphson's method is given by **EE-2009-2M**

(a) $x_{k+1} = \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$

(b) $x_{k+1} = x_k - \frac{117}{x_k}$

(c) $x_{k+1} = x_k - \frac{x_k}{117}$

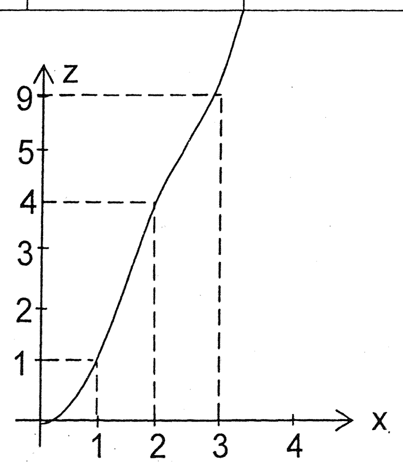
(d) $x_{k+1} = x_k - \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$

16. A cubic polynomial with real coefficients **EE-2009-2M**
   (a) can possibly have no extrema and no zero crossings
   (b) may have up to three extrema and upto 2 zero crossings
   (c) can not have more than two extrema and more than three zero crossings
   (d) will always have an equal number of extrema and zero crossings

17. During the numerical solution of a first under differential equation using the Euler method with step size h the local truncation error is of the order of **PI-2009-1M**
   (a) h^2 (b) h^3
   (c) h^4 (d) h^5

18. The area under the curve shown, b/w $x = 1$ and $x = 5$ is to be evaluated using the trapezoidal rule. The following points on the curve are given

Point	X – coordinate (x)	y – coordinate (x)
1	1	1
2	2	4
3	3	9



The evaluated area (in m^2) will be
   (a) 7 (b) 8.67
   (c) 9 (d) 18

19. Newton – Raphson method is used to compute a root of the equation $x^2 - 13 = 0$ with 3.5 as the initial value. The approximation after one iteration is **CS-2010-2M**
   (a) 3.575 (b) 3.677
   (c) 3.667 (d) 3.607

20. The following algorithm computes the integral $J = \int_a^b f(x)dx$ from the given values $f_j = f(x_j)$ at equidistant points : $x_0 = a, x_1 = x_0 + h, \ldots\ x_{2m} = x_0 + 2mh = b$

Compute $S_0 = f_0 + f_{2m}$

$$S_1 = f_1 + f_3 + \ldots\ldots + f_{2m-1}$$
$$S_2 = f_2 + f_4 + \ldots\ldots + f_{2m-2}$$
$$J = \frac{h}{3}[s_0 + 4s_1 + 2s_2]$$

The rule of numerical integration, which uses the above algorithm is **PI-2010-1M**
   (a) Rectangle rule
   (b) Trapezoidal rule
   (c) Four – point rule
   (d) Simpson's rule

21. The table below gives values of a function f(x) obtained for values of 'x' at intervals of 0.25

x	0	0.25	0.5	0.75	1.0
f(x)	1	0.9412	0.8	0.64	050

The value of the integral of the function between the limits 0 to 1 using Simpson's rule is **CE-2010-2M**
   (a) 0.7854 (b) 2.3562
   (c) 3.1416 (d) 7.5000

KEY:

01. a	02. b	03. c
04. a	05. a	06. a
07. c	08. a	09. b
10.	11. a	12. c
13. c	14. d	15. a
16.	17. a	18. c
19. d	20. d	21. a

* Matrix multiplication is associative, if conformability is assured.

  i.e. $A(BC) = (AB)C$

* Matrix multiplication is distributive w. r. t addition of matrices

  $A.(B + C) = A . B + A . C$

* The matrix multiplication is not always commutative:

  i.e. In general, $AB \neq BA$ (AB need not be equal to BA)

* Whenever $AB = BA$, the matrices A and B are said to commute.

* The equation $AB = O$ does not necessarily imply that atleast one of the matrices A and B is a zero matrix. Eg: $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

  i.e.. The product of two non-zero matrices can be a zero matrix

* If product of two non zero square matrices A & B is a zero matrix then A and B are singular matrices i.e. $|A| = 0$ and $|B| = 0$

* If A is non singular and $A.B = 0$ then B is a zero matrix.

UPPER TRIANGULAR MATRIX: A square matrix is said to be upper triangular if all the elements below its principal diagonal are zeros.

LOWER TRIANGULAR MATRIX: A square matrix is lower triangular if all the elements above its principal diagonal are zeros.

DIAGONAL MATRIX: A square matrix is said to be diagonal if all the elements below and above the principal diagonal are zeros.

SCALAR MATRIX: It is a diagonal matrix with same diagonal elements.

UNIT MATRIX(OR IDENTITY MATRIX): A Scalar matrix whose diagonal elements are all equal to 1.

TRACE: Trace of a matrix is the sum of the elements of the principal diagonal.

* $tr(\lambda A) = \lambda.tr(A)$
* $tr(A + B) = tr(A) + tr(B)$
* $tr(AB) = tr(BA)$

TRANSPOSE: Transpose of a matrix A can be obtained by interchanging the rows and columns of A. It is denoted by A^1 or A^T.

* If A is a matrix of order m x n then A^1 is a matrix of order n x m.
* $(A^1)^1 = A$
* $(A + B)^1 = A^1 + B^1$
* $(kA)^1 = kA^1$
* $(AB)^1 = B^1A^1$

SYMMETRIC AND SKEW SYMMETRIC MATRICES: Let A is square matrix, A is symmetric if $A^1 = A$, A is skew symmetric if $A^1 = -A$.

* If A is a square matrix then
  i) $A + A^1$ is symmetric.
  ii) $A - A^1$ is skew symmetric.
  iii) $A . A^1$ is symmetric.

* Every square matrix can be written as the sum of a symmetric and a skew symmetric matrices.

  i.e. $A = 1/2\,(A + A^1) + 1/2(A - A^1)$

* If A and B are symmetric then $AB + BA$ is symmetric and $AB - BA$ is skew symmetric.
* If A is symmetric then A^n is symmetric($n = 2, 3, 4, \ldots$).
* If A is skew symmetric, then A^n is skew symmetric when n is odd and symmetric when n is even .

INVERSE OF A MATRIX: If A is a square matrix of order n. then a matrix B, if it exists such that $AB = BA = I_n$ is called inverse of A

* A^{-1} exists $\Leftrightarrow |A| \neq 0$
* $A^{-1} = \dfrac{\text{adj } A}{|A|}$
* Every non singular matrix possesses a unique inverse
* $(AB)^{-1} = B^{-1}A^{-1}$

PROPERTIES OF DETERMINANTS :-

01. If 'A' is a square matrix then $|A| = |A^T|$

Note:- In a general manner a row or a column is referred as a line.

02. If two parallel lines of a determinant are interchanged, then the determinant retains its numerical value but changes in sign.

Note:- In general, if any line of a determinant be passed over 'm' parallel lines, the resulting determinant $= (-1)^m \Delta$.(where Δ is the initial determinant value)

03. A determinant vanishes if two parallel lines are identical.

04. If each element of a line be multiplied by the same factor, then the whole determinant is multiplied by that factor.

Ex :- 1. $\begin{vmatrix} ka_1 & kb_1 \\ a_2 & b_2 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

2. $\begin{vmatrix} a_1 & b_1 & lc_1 \\ a_2 & b_2 & lc_2 \\ a_3 & b_3 & lc_3 \end{vmatrix} = l \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Note:- ~~In a determinant~~, if $R_i = k R_j$ (or $C_i = k C_j$) then the value of the determinant is zero.

~~Ex:-~~ $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ ka_1 & kb_1 & kc_1 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix} = k(0) = 0$ (from property-03)

05. $\begin{vmatrix} a_1 & b_1+c_1 & d_1 \\ a_2 & b_2+c_2 & d_2 \\ a_3 & b_3+c_3 & d_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} + \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix}$

06. If to each elements of a line be added equi- multiples of the corresponding elements of one or more parallel lines the determinant remains unaltered.

07. The determinant of an upper / a lower triangular / diagonal / scalar matrix is equal to the product of leading diagonal elements of the matrix.

08. A. B are square matrices of the same order then $|AB| = |A|\ |B| = |BA|$

09. If A is a non singular matrix (i.e. $|A| \neq 0$) then $|A^{-1}| = \frac{1}{|A|}$

10. If A is a square matrix of order-n then (i) $|\text{Adj } A| = |A|^{n-1}$ (ii) $|\text{Adj}(\text{Adj}A)| = |A|^{(n-1)^2}$

11. Determinant of a Skew – symmetric matrix (i.e. $A^T = -A$) of odd order is zero.

12. If A is an orthogonal matrix (i.e. $A^T = A^{-1}$) then $|A| = \pm 1$.

13. If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $\Delta^1 = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$

where A,B,C are co-factors of a,b,c then $\Delta^1 = \Delta^2$, which is called **Reciprocal / Adjucate** determinant of Δ.

14. $|I_n| = 1 \quad \forall \; n \in z^+$

15. If 'A' is a square matrix of order 'n' then $|kA| = k^n \, |A|$

RANK OF A MATRIX: It is the order of its largest non vanishing minor of the matrix.

* ***Rank of the matrix*** is equal to the number of linearly independent rows(cols) in the matrix.
* Rank of matrix A is equal to the number of non zero rows(columns) in the row(column) ***Echelon form*** of A.
* The system of linear equations AX = B has a solution (consistent) if and only if Rank of A = Rank of (A | B)
* The system AX = B has
  i) A unique solution if and only if Rank(A) = Rank(A | B) = number of variables.
  ii) Infinitely many solutions $\Leftrightarrow \rho(A) = \rho(A|B) <$ number of variables.
  iii) No solution if $\rho(A) \neq \rho(A \mid B)$ i. e. $\rho(A) < \rho(A \mid B)$
* The system AX = O has
  i) Unique solution (***zero*** solution or ***trivial*** solution) if $\rho(A)$ = number of variables.
  ii) Infinitely many number of non-zero (or ***non-trivial***) solutions if $\rho(A) < n$.

SOME OTHER OBSERVATIONS :-

01. Product of two diagonal matrices of the same order is a diagonal matrix and follows commutative law (i.e. AB = BA).

02. (i) $(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$, (ii) $(ABCD)^{-1} = D^{-1} C^{-1} B^{-1} A^{-1}$

03. (a) $(A^{-1})^T = (A^T)^{-1}$ (b) If a non-singular matrix A is symmetric $\Rightarrow A^{-1}$ is also symmetric.

04. (a) If A is orthogonal then A^T & A^{-1} are also orthogonal.
(b) If A & B are orthogonal of the same order then AB is also orthogonal.

05. If a matrix A is of rank-r, then A contains 'r' linearly independent *Vectors* (here vector is either row / column of the matrix).

06. In the system of homogenous linear equation $AX = O$
   (i) If A is singular then the system possesses non-trivial solution. (i.e. non-zero solution).

   (ii) If A is non-singular then the system possesses trivial solution (i.e. zero solution).

07. If $\rho(A) = r$, and number of variables $= n$ then, the number of linearly independent solutions of $AX = O$ is $\underline{(n - r)}$.

08. The system of homogeneous linear equations such that the number of unknowns (or variables) exceeds the number of equations necessarily possesses a non-zero solution.

09. If A is a non-singular matrix then all the rows /columns (vectors) of A are linearly independent.

10. If A is a singular matrix then vectors of A are linearly dependent.

11. **(i)** $\rho(A+B) \leq \rho(A) + \rho(B)$ **(ii)** $\rho(A-B) \geq \rho(A) - \rho(B)$
   (iii) $\rho(AB) \leq \min\{\rho(A), \rho(B)\}$

12. Rank of a diagonal matrix is equal to the number of non-zero elements in the diagonal.

EIGEN VALUES AND EIGEN VECTORS: Let A be a square matrix of order n and λ be a scalar. $|A - \lambda I| = 0$ is called the ***characteristic equation*** of A.

The roots of characteristic equation are called ***eigen values*** (***characteristic roots / latent roots***) of A.

Corresponding to each eigen value λ, there exists a *non-zero solution* X such that $(A - \lambda I) X = O$. Then X is called ***eigen vector*** (***characteristic vector or latent vector***) of A.

PROPERTIES OF EIGEN VALUES AND EIGEN VECTORS

01. The sum of the eigen values of a matrix is the sum of the principal diagonal elements (i.e. Trace).

02. The Product of the eigen values of a matrix is equal to the determinant of the matrix.

   Note:- In particular, the eigen values of a Diagonal / Scalar / Triangular (either upper or lower) matrix are just the leading diagonal elements and product of the eigen values is just the product of leading diagonal elements.

03. The eigen values of A^T are same as the eigen values of A.

04. If λ is an eigen value of a non-singular matrix A then
   (a) $1/\lambda$ is an eigen value of A^{-1} ($\because AA^{-1} = I = A^{-1}A$)
   (b) $|A|/\lambda$ is an eigen value of Adj A ($\because A.\text{Adj } A = |A|\, I$)

05. If λ is an eigen value of an Orthogonal matrix A then $1/\lambda$ is also an eigen value of A ($\because A^T = A^{-1}$)

06. If $\lambda_1, \lambda_2, \ldots\ldots\ldots\ldots \lambda_n$ are eigen values of A, then
   (i) The eigen values of kA are $k\lambda_1$, $k\lambda_2 \ldots k\lambda_n$.(where k is a scalar)
   (ii) A^m has eigen values $\lambda_1^m, \lambda_2^m \ldots\ldots \lambda_n^m$ (where $m \in z^+$)
   (iii) A + kI has eigen values $\lambda_1+k, \lambda_2+k\ldots\ldots\ldots \lambda_n+k$
   (iv) $(A - kI)^2$ has eigen values $(\lambda_1 - k)^2$, $(\lambda_2 - k)^2 \ldots\ldots\ldots (\lambda_n - k)^2$

07. The eigen values of an orthogonal matrix have absolute value '1'

08. The eigen values of a symmetric matrix are purely real.

09. The eigen values of skew-symmetric matrix are either purely imaginary or zero.

10. The set of all characteristic roots of a matrix is called ***Spectrum*** of the matrix.

11. Zero is an eigen value of a matrix if and only if the matrix is singular.

12. λ is an eigen value of a non-singular matrix $\Leftrightarrow \lambda \neq 0$.

13. If λ is an eigen value of a matrix A, then the corresponding eigen vector X is not unique. i.e. we have infinite number of eigen vectors corresponding to a single eigen value.

14. If $\lambda_1, \lambda_2 \ldots\ldots\ldots \lambda_n$ be distinct eigen values of an n x n matrix A, then the corresponding eigen vectors X_1, X_2 ......X_n form a linearly independent set.

15. If some eigen values are repeated, it may/may not be possible to get linearly independent eigen vectors corresponding to the equal roots.

PROBLEMS

1. If AB = BA then which of the following need not be true (n is a +ve integer)
   a) $AB^n = B^nA$ b) $(AB)^n = A^nB^n$
   c) $(A + B)(A - B) = A^2 - B^2$ d) A = I or B = I

2. If a diagonal matrix is commutative with every matrix of the same order then it is necessarily a
   a) a zero matrix b) a unit matrix c) a scalar matrix d) a symmetric matrix

3. If A is any m x n matrix such that AB and BA are both defined then B is a matrix of order
   a) n x n b) m x m c) m x n d) n x m

4. If $A_{m \times n}$ and $B_{n \times p}$ are matrices, then the number of multiplications and additions in computing AB are
   a) m p n, mp(n – 1) b) (m – 1) pn, mp(n – 1) c) m p n, (m – 1)(n – 1) d) mn, nm

5. The number of terms in the expansion of the determinant of $A_{n \times n}$ is
   a) n^2 b) 2^n c) n! d) n

6. If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & 1 & -3 \end{pmatrix}$ and $\text{Adj } A = \begin{pmatrix} -3 & 4 & k \\ -3 & -1 & 4 \\ -3 & 1 & 1 \end{pmatrix}$ then k =
   a) –2 b) 4 c) –5 d) 6

7. Rank of unit matrix I_n is
   a) one b) zero c) n d) not defined

8. Rank of a non – singular matrix $A_{n \times n}$ is
   a) one b) zero c) n d) not defined

9. Rank of a singular matrix $A_{n \times n}$ is
   a) n b) zero c) <n d) >n

10. Rank of a diagonal matrix $A_{n \times n}$ is
a) n
b) no. of zeros in the diagonal
c) no. of non zero elements in the diagonal
d) zero

11. If $A_{m \times 1}$ is non zero column matrix and $B_{1 \times n}$ is a non zero row matrix then $\rho(AB) =$
a) m b) n c) 1 d) zero

12. If $A_{n \times n}$ is a non singular matrix and $B_{n \times n}$ is a matrix then $\rho(AB) =$
a) Rank of A b) $\rho(B)$ c) 0 d) 1

13. If $\rho(A)_{n \times n}$ is equal to $(n-2)$ then $\rho(\text{Adj } A) =$
a) $(n-1)$ b) $(n-2)$ c) $(n-3)$ d) zero

14. $\rho(A)_{3 \times 3} = 2$ then $|A| =$
a) 1 b) 0 c) Any non zero number d) 2

15. If $\rho(A) = n$, then which of the following is (are) false
a) $\exists$ at least one non zero minor of order 'n'
b) All the minors of A of order greater than n vanish
c) All the minors of A of order n are not zero.
d) If A is a square matrix then $|A| \neq 0$.

16. If $A = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$ then $\rho(A) =$
a) 2 b) 1 c) 0 d) does not exist

17. If $A = \begin{pmatrix} 3 & -4 \\ -6 & 8 \end{pmatrix}$ then $\rho(A) =$
a) 2 b) 1 c) 0 d) does not exist

18. If $A = \begin{pmatrix} 2 & 3 & -1 \\ 4 & 6 & 1 \end{pmatrix}$ then $\rho(A) =$
a) 0 b) 1 c) 2 d) 3

19. If $A = \begin{pmatrix} 2 & -2 \\ -1 & 1 \\ 3 & -3 \end{pmatrix}$ then $\rho(A) =$
a) 0 b) 1 c) 2 d) 3

20. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 2 & 4 \\ 2 & 3 & -1 \end{pmatrix}$ then $\rho(A) =$
a) 0 b) 1 c) 2 d) 3

21. If $A = \begin{pmatrix} 2 & -3 & -1 \\ -4 & 6 & 2 \\ 6 & -9 & -3 \end{pmatrix}$ then $\rho(A) =$
a) 0 b) 1 c) 2 d) 3

22. If $A = \begin{pmatrix} 2 & -3 & 4 \\ 3 & -2 & 5 \\ 1 & 1 & 1 \end{pmatrix}$ then $\rho(A) =$
a) 0 b) 1 c) 2 d) 3

23. Which of the statements is false.
Rank of the matrix is equal to
a) The no. of its linearly independent rows
b) The no. of its linearly independent columns
c) The no. of non – zero rows
d) The order of its largest non – vanishing minor

24. The system $AX = B$ has no solution if
a) $\rho(A) = \rho(A/B)$ b) $\rho(A) < \rho(A/B)$ c) $\rho(A) > \rho(A/B)$ d) $\rho(A) \geq \rho(A/B)$

25. The system $AX = 0$ in n variables has infinitely many solutions if
a) $\rho(A) = n$ b) $\rho(A) > n$ c) $\rho(A) < n$ d) $\rho(A) \geq n$

26. If A is a square matrix then the system $AX = 0$ has non zero solutions when
a) $|A| \neq 0$ b) $|A| = 0$ c) $\rho(A) =$ no. of variables d) $\rho(A) \geq$ no. of variables

27. If $|A| \neq 0$ then for the system $AX = 0$, which of the following is false
a) The system has unique solution b) The system has a zero solution
c) The system has a trivial solution d) The system has a non - zero solution

28. The system $AX = B$ has a unique solution if
a) $\rho(A) =$ no. of variables in the system $(n) = \rho(A/B)$ b) $\rho(A) < n$
c) $\rho(A) > n$ d) $\rho(A) = \rho(A/B) < n$

29. If $\rho(A) = r$ and number of variables $= n$, then the number of linearly independent solutions of the system $AX = 0$ is
a) n b) r c) $n - r$ d) $n + r$

30. The system
$$2X + 3Y + 4Z = 1$$
$$3Y - Z = 2$$
$$-6Y + 2Z = 3$$
has
a) no solution b) unique solution
c) infinitely many solutions d) two linearly independent solutions

31. If the system
$$X + Y + Z = 0,$$
$$(\lambda + 1)(Y) + (\lambda + 1)(Z) = 0,$$
$$(\lambda^2 - 1) Z = 0$$
has two linearly independent solutions, then $\lambda =$
a) 1 b) –1 c) 0 d) 3

32. The system given in example 31 has only one independent solution when $\lambda =$
a) 1 b) –1 c) 0 d) 3

33. The system given in example 31 has no linearly independent solutions when $\lambda =$
a) 1 b) –1 c) 0 d) ± 1

34. If $\rho(A) = 1$ and number of variables $= 3$, then the system $AX = 0$ has
a) Two linearly independent solutions b) 3 linearly independent solutions
c) 1 linearly independent solutions d) no independent solutions

35. The rank of the matrix, every element of which is unity is =
a) one b) zero c) order of the matrix d) >1

36. If A is a skew symmetric matrix then which of the following is false
a) $\rho(A) = 1$ b) $\rho(A) = 0$ c) $\rho(A) = 2$ d) $\rho(A) = 3$

37. Rank of an elementary matrix =
a) order of the matrix b) one c) zero d) two

38. When elementary transformations are applied on a given matrix then its rank
a) will remain same
b) reduced by one
c) becomes one
d) is equal to the order of matrix

39. $\rho(A)_{m \times n} = r$ then
a) $r \le m, r \le n$
b) $r > m, r \le n$
c) $r > m, r > n$d)
d) $r \le m, r > n$

40. Find the rank of the matrix $\begin{pmatrix} 2 & 3 & -1 & 0 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 5 \end{pmatrix}$ is = 1.5

a) 1
b) 2
c) 3
d) 4

41. If A and B are any two matrices then which of the following is false
a) $\rho(AB) < \rho(A)$
b) $\rho(AB) \le \rho(A)$
c) $\rho(AB) < \rho(B)$
d) $\rho(AB) > \rho(B)$

42. If a square matrix A is orthogonal then
a) $A = A^1$
b) $A^1 = A^{-1}$
c) $A = \overline{A}$
d) $A^1 = \overline{A}$

43. If $(\overline{A^1}) = A$ then A is
a) orthogonal
b) symmetric
c) hermitian
d) unitary

44. If A is skew - hermitian then iA is
a) hermitian
b) symmetric
c) unitary
d) skew – symmetric

45. If a matrix is in row echelon form then its rank is =
a) no. of non – zero rows of the matrix
b) no. of zero rows of the matrix
c) order of the matrix
d) one

46. If $A_{n \times n}$ is a non singular matrix then which of the following is false
a) $\rho(A) = \rho(A^1)$
b) $\rho(A) = \rho(A^{-1})$
c) $\rho(A) = \rho(I_n)$
d) $\rho(A) = 1$

47. $\rho(A) = n$ then $\rho(\text{Adj } A) =$
a) n
b) $n - 1$
c) 1
d) 0

48. $\rho(A) = n - 1$ then $\rho(\text{Adj } A) =$
a) n
b) $n - 1$
c) 1
d) zero

49. The system $3x + 4y - 2z = 4$,
$6x + 8y - 4z = 10$ has
a) no solution
b) infinitely many solutions
c) unique solution
d) one linearly independent solution

50. Rank of a non zero matrix is
a) equal to order of the matrix
b) equal to the number of rows of the matrix
c) < 1
d) ≥ 1

51. Eigen values of a matrix $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$ are
a) (2, 3)
b) (-2, -3)
c) (1, 6)
d) (-1, -6)

52. Eigen values of the matrix $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ are
a) (1, 2, 3)
b) (1, 1, 1)
c) (2, 2, 2)
d) (0, 1, 2)

53. Eigen values of the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ are

a) (3, 2, -1) b) (3, 2, 1) c) (3, 3, -1) d) (4, -1, 0)

54. If '0' and '3' are eigen values of the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ then the third eigen value is

a) 7 b) 8 c) 15 d) 0

55. If $-1 + \sqrt{3}$ is eigen value of matrix $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix}$ then the other two eigen values are

a) $(-1-\sqrt{3}, 2)$ b) $(-1+\sqrt{3}, 1)$ c) $(-1-\sqrt{3}, -2)$ d) $(-1+\sqrt{3}, -2)$

56. If 3 is the eigen value of the singular matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ then the other eigen values of matrix are

a) (0, 15) b) (8, 15) c) (7, 15) d) (0, 3, 2)

57. Which of the following is an eigen vector of the matrix $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

a) (1, 0, 0) b) (1, 0, 1) c) (0, 0, 1) d) (1, 1, 1)

58. Which of the following is an eigen vector of the matrix $\begin{matrix} 5 & 4 \\ 1 & 2 \end{matrix}$ corresponding to eigen value $\lambda = 6$

a) (4, -1) b) (1, -4) c) (-1, -4) d) (4, 1)

59. If 2, 2, 8 are eigen values of a matrix $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ then the matrix is

a) singular b) non – singular c) skew symmetric d) triangular

60. If 2, 2, 8 are eigen values of the matrix $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & k \end{pmatrix}$ then k =

a) 0 b) 1 c) 2 d) 3

For the matrix $A = \begin{pmatrix} 3 & -2 & 5 \\ 0 & 5 & 6 \\ 0 & 0 & -4 \end{pmatrix}$ answer the following

61. The eigen values of A^3 are

a) (3, 5, -4) b) (9, 25, 16) c) (27, 125, -64) d) (9, 15, -12)

62. The eigen values of A^{-1} are

a) (1/3, 1/5, -1/4) b) (1/3, 1/5, 1/4) c) (-3, -5, -4) d) (-1/3, -1/5, 1/4)

63. The eigen values of 9A are

a) (27, 45, -36) b) (12, 14, 5) c) (9, 15, -12) d) $(3^9, 5^9, 4^9)$

64. The eigen values of Adj(A) are
   a) (3, 5, -4)   b) (9, 25, 16)   c) (20, 12, -15)   d) (-20, -12, 15)

65. The eigen values of A^1 are
   a) (3, 5, -4)   b) (-3, -5, 4)   c) (1/3, 1/5, -1/4)   d) does not exist

66. For the matrix $\begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ the eigen values of A^{-1} are
   a) (2, 1, 0)   b) (1/2, 1, $\propto$)   c) (-2, -1, 0)   d) does not exist

67. The number of linearly independent eigen vectors corresponding to any distinct eigen value of the matrix $A_{3 \times 3}$ =
   a) 1   b) 2   c) 3   d) cannot be determined

68. If an eigen value λ is repeated two times for a matrix $A_{3 \times 3}$ then the no. of linearly independent eigen vectors for λ are
   a) 2   b) < 2   c) ≤ 2 d) > 2

69. For the matrix $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ which of the following is an eigen vector
   a) (1, 0, 1)   b) (1, 1, 0)   c) (1, 0, 1)   d) (1, 1, 1)

70. For the matrix $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ which of the following is not a eigen vector
   a) (1, 0, 0)   b) (0, 1, 0)   c) (0, 0, 1)   d) (0, 0, 0)

$a(1-a) - (-a^2+a-1)$
$a - \cancel{a^2} + \cancel{a^2} - a + 1$

71. If $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ then A^8 =
   a) 5I   b) 25I   c) 625I   d) 3125I

$\begin{pmatrix} 1-a & a \\ a^2-a+1 & -1 \end{pmatrix}$

72. If $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 5 & 6 \\ 0 & 0 & i & 7 \\ 0 & 0 & 0 & -i \end{pmatrix}$ then A^4 =
   a) I_4 b) $4I_4$ c) $16I_4$ d) $64I_4$

73. If $\lambda^n + K_1\lambda^{n-1} + K_2\lambda^{n-2} + \ldots\ldots + K_n = 0$ is the characteristic equation of a matrix A then A^{-1} exists if
   a) $K_1 = 0$ b) $K_1 \neq 0$ c) $K_n = 0$ d) $K_n \neq 0$

74. The sum and product of the eigen values of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ are____ and ____
   a) 7, 12   b) 7, 5   c) 12, 5   d) 7, 9

75. The eigen values of a triangular matrix are
   a) diagonal elements of the matrix   b) zero
   c) non – diagonal elements   d) ±1

76. The eigen values of a real skew symmetric matrix are
a) real b) ± 1 c) purely imaginary or zero d) does not exist

77. The eigen values of a orthogonal matrix are
a) real b) ± 1 c) purely imaginary d) of unit modulus

78. For each eigen value of the matrix $A_{3 \times 3}$, the no. of eigen vectors =
a) 1 b) 2 c) 3 d) α

79. If a matrix $A_{3 \times 3}$ has 3 distinct eigen values then the no. of linearly independent eigen vectors for A =
a) 1 b) 2 c) 3 d) α

80. If zero is eigen value of a square matrix A, then A is
a) singular b) non singular c) orthogonal d) symmetric

81. If 2 is eigen value of a scalar matrix $A_{3 \times 3}$, then an eigen value of Adj A is
a) 2 b) 3 c) 4 d) 8

82. Which of the following is not an eigen vector of a 2 x 2 unit matrix
a) (0, 0) b) (1, 1) c) (1, 0) d)(0, 1)

83. What values of x, y, z satisfy the following system of linear equations.
$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ 12 \end{pmatrix}$$
(Gate-2004)

a) x=6, y=3, z=2 b) x=12, y=3, z= -4 c) x=6, y=6, z= -4 d) x=12, y=-3, z=4

84. If matrix $X = \begin{pmatrix} a & 1 \\ -a^2+a-1 & 1-a \end{pmatrix}$ and $X^2 - X + I = 0$. Then the inverse of X is (Gate-2004)

a) $\begin{pmatrix} 1-a & -1 \\ a^2 & a \end{pmatrix}$ b) $\begin{pmatrix} 1-a & -1 \\ a^2-a+1 & a \end{pmatrix}$ c) $\begin{pmatrix} -a & 1 \\ -a^2+a-1 & a-1 \end{pmatrix}$ d) $\begin{pmatrix} a^2-a+1 & a \\ 1 & 1-a \end{pmatrix}$

85. The number of different n×n symmetric matrices with each element being either 0 or 1 is
a) 2^n b) 2^{n^2} c) $2^{\frac{n^2+n}{2}}$ d) $2^{\frac{n^2+n}{2}}$ (Gate-2004)

86. Let A, B, C, D be n×n matrices, each with non-zero determinent. ABCD=I then B^{-1} =
a) $D^{-1}C^{-1}A^{-1}$ b) CDA c) ADC d) does not exist (Gate-2004)

87. How many solutions does the following system of linear equations have
$-x+5y = -1 \quad x-y = 2 \quad x+3y = 3$ (Gate-2004)
a) infinitely many b) two distinct solutions c) unique d) none

88. In an n×n matrix such that non zero entries are covered in a rows and b columns. Then the maximum number of non zero entries, such that no two are on the same row or column is
a) $\leq (a+b)$ b) $\leq \max(a,b)$ c) $\leq \min(M-a, N-b)$ d) $\leq \min(a,b)$ (Gate-2004)

89. Consider the following system of linear equations
$$\begin{pmatrix} 2 & 1 & -4 \\ 4 & 3 & -12 \\ 1 & 2 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha \\ 5 \\ 7 \end{pmatrix}$$
(Gate-2003)

Notice that the second and third columns of the coefficient matrix are linearly dependent. For how many value of α, does the system of equations have infinitely many solutions.
a) 0 b) 1 c) 2 d) infinitely many

90. The rank of the matrix $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ is (Gate-2002)

a) 4 b) 2 c) 1 d) 0

91. Obtain the eigen values of the matrix $A = \begin{pmatrix} 1 & 2 & 34 & 49 \\ 0 & 2 & 43 & 94 \\ 0 & 0 & -2 & 104 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ (Gate-2002)

92. Consider the following statements

S_1 : The sum of two singular matrices may be singular

S_2 : The sum of two non-singular matrices may be non-singular. (Gate-2001)

Which of the following statements is true.

a) S_1 & S_2 are both true
b) S_1 & S_2 are both false
c) S_1 is true and S_2 is false
d) S_1 is false and S_2 is true

93. The determinant of the matrix $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{pmatrix}$ is (Gate-2000)

a) 4 b) 0 c) 15 d) 20

94. An n×n array V is defined as follows

V[i, j] = i-j for all i, j $1 \le i \le n$, $1 \le j \le n$. (Gate-2000)

The sum of elements of the array V is

a)0 b) n-1 c) $n^2 - 3n + 2$ d) n(n+1)

2[1(2) - 7(0)

KEY

1. d	2. c	3. d	4. a	5. c	6. c	7. c	8. c	9. c	10. c	11. c	12. b
13. d	14. b	15. c & d	16. a	17. b	18. c	19. b	20. d	21. b	22. c	23. c	
24. b	25. c	26. b	27. d	28. a	29. c	30. a	31. b	32. a	33. c	34. a	35. a
36. a	37. a	38. a	39. a	40. d	41. d	42. b	43. c	44. d	45. a	46. d	47. a
48. c	49. a	50. d	51. c	52. c	53. b	54. c	55. a	56. a	57. a	58. d	59. b
60. d	61. c	62. a	63. a	64. d	65. a	66. d	67. a	68. c	69. b	70. d	71. c
72. a	73. d	74. b	75. a	76. c	77. d	78. d	79. c	80. a	81. c	82. a	83. c
84. b	85. c	86. b	87. d	88. d	89. c	90. 1,2,-2, -1	91. a	92. a	93. a	94. a	

ADDITIONAL PRPOBLEMS

1. Consider the following system of equations in three real variables x_1, x_2 and x_3 :

$2x_1 - x_2 + 3x_3 = 1$; $3x_1 + 2x_2 + 5x_3 = 2$; $-x_1 + 4x_2 + x_3 = 3$

This system of equations has *(GATE '05)*

a) no solution
b) a unique solution
c) more than one but a finite number of solutions
d) an infinite number of solutions

2. What are the eigen values of the following 2×2 matrix ? *(GATE '05)*

$$\begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$$

a) – 1 and 1 b) 1 and 6 c) 2 and 5 d) 4 and –1

3. Consider the matrices $X_{4\times 3}$ $Y_{4\times 3}$ and $P_{2\times 3}$. The order of $[P\,(X^T Y)^{-1} P^T]^T$ will be

a) 2×2 b) 3×3 c) 4×3 d) 3×4 *(GATE '05)*

4. Consider a non homogeneous system of linear equations representing mathematically an over determined system. Such a system will be *(GATE '05)*

a) Consistent having a unique solution b) Consistent having many solutions
c) inconsistent having a unique solution d) inconsistent having no solution

5. Consider the system of equations *(GATE '05)*

$$A_{n\times n} X_{n\times 1} = \lambda\, X_{n\times 1}$$

Where λ is a scalar. Let (λ_i, X_i) be an eigen value and it corresponding eigen vector for real matrix A. Let $I_{n\times n}$ be unit matrix. Which one of the following statement is not correct.

a) For a homogeneous $n \times n$ system of linear equations, $(A - \lambda I)\,X = 0$, having a non trivial solution, the rank of $(A - \lambda I)$ is less than n
b) For matrix A^m, m being a positive integer, (λ_i^m, X_i^m) will be the Eigen pair for all i
c) If $A^T = A^{-1}$ then $|\lambda_i| = 1$ for all i
d) If $A^T = A$ then λ_i are real for all i

6. The determinant of the matrix given below is *(GATE '05)*

$$\begin{pmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{pmatrix}$$

a) – 1 b) 0 c) 1 d) 2

7. In the matrix equation PX = Q which of the following is a necessary condition for the existence of atleast one solution for the unknown vector X

a) Augmented matrix [P Q] must have the same rank as matrix P
b) Vector Q must have only non zero elements
c) Matrix P must be singular d) Matrix P must be square

8. For the matrix *(GATE '05)*

$$P = \begin{pmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

One of the eigen values is –2. Which of the following is an eigen vector ?

a) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ b) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$ c) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ d) $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

9. If $R = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{pmatrix}$ the top row of R^{-1} is *(GATE '05)*

a) [5 6 4] b) [5 -3 1] c) [2 0 -1] d) [2 -1 0]

10. The eigen values of the matrix M given below are 15, 3 and 0. *(GATE '05)*

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & n & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

The value of the determinant of the matrix is

a) 20 b) 10 c) 0 d) – 10

11. A is a 3×4 matrix and AX = B is an inconsistent system of equations. The highest possible rank of A is *(GATE '05)*

a) 1 b) 2 c) 3 d) 4

12. Which one of the following is an eigen vector of the matrix *(GATE '05)*

$$\begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{pmatrix}$$

a) $[1 \quad -2 \quad 0 \quad 0]^T$ b) $[0 \quad 0 \quad 1 \quad 0]^T$ c) $[1 \quad 0 \quad 0 \quad -2]^T$ d) $[1 \quad -1 \quad 2 \quad 1]^T$

13. Let A be a 3×3 matrix with rank 2. Then AX = O has *(GATE '05)*

a) only the trivial solution X = 0 b) one independent solution

c) two independent solutions d) three independent solutions

14. Identify which one of the following is an eigen vector of the matrix $A = \begin{pmatrix} 1 & 0 \\ -1 & -2 \end{pmatrix}$

a) $[-1 \quad 1]^T$ b) $[3 \quad -1]^T$ c) $[1 \quad -1]^T$ d) $[-2 \quad 1]^T$ *(GATE '05)*

15. Let $A = \begin{pmatrix} 2 & -0.1 \\ 0 & 3 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1/2 & a \\ 0 & b \end{pmatrix}$ then (a + b) = *(GATE '05)*

a) 7 / 20 b) 3 / 20 c) 19/60 d) 11/20

16. Given an orthogonal matrix *(GATE '05)*

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$(A A^T)^{-1}$ is

a) ¼ I_4 b) ½ I_4 c) I d) ¼ I_4

17. If $A = \begin{pmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{pmatrix}$ and adj $A = \begin{pmatrix} -11 & -9 & 1 \\ 4 & -2 & -3 \\ 10 & k & 7 \end{pmatrix}$ then k =

a) – 5 b) 3 c) –3 d) 5 *(GATE '99)*

18. If A and B are real symmetric matrices of order n then which of the following is true

a) $AA^T = I$ b) $A = A^{-1}$ c) AB = BA d) $(AB)^1 = B^1A^1$

(GATE '94)

19. The inverse of the matrix

$$\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$ is *(GATE '94)*

20. The rank of the matrix

$$\begin{pmatrix} 1 & a & a^2 \dots\dots\dots & a^n \\ 1 & a & a^2 \dots\dots\dots & a^n \\ \dots & \dots & \dots & \dots \\ 1 & a & a^2 \dots\dots\dots & a^n \end{pmatrix}$$ of order $(n + 1) \times (n + 1)$ ***(GATE '98)***

Where a is a real number is

a) 1 b) 2 c) n d) depends on a

21. If $\Delta = \begin{pmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{pmatrix}$ then ***(GATE '98)***

Which of the following is a factor of Δ

a) $a + b$ b) $a - b$ c) abc d) $a + b + c$

22. Let $A_{n \times n}$ be a matrix of order n and I_{12} be the matrix obtained by interchanging the first and second rows of I_n. Then $A \cdot I_{12}$ is such that its first ***(GATE '97)***

a) row is the same as its second row b) row is the same as second row of A

c) column is the same as the second column of A

d) row is a zero row

23. Let AX = B be a system of linear equations where A is an $m \times n$ matrix, B is an $m \times 1$ column matrix. Which of the following is false ? ***(GATE '96)***

a) The system has a solution , if $\rho(A) = \rho[A \mid B]$

b) If m = n and B is a non zero vector then the system has a unique solution

c) If m < n and B is a zero vector then the system has infinitely many solutions

d) The system will have a trivial solution when m = n, B us the zero vector and rank of A is n.

KEY

1. b	2. b	3. a	4. d	5. b	6. a	7. a	8. d	9. b	10. c	11. b	12. a
13. b	14. b	15. a	16. c	17. a	18. d	19.	20. a	21. b	22. c	23. b	

PREVIOUS GATE QUESTIONS

01. Lets A be an n×n real matrix such that $A^2 = I$ and y be an n-dimensional vector. Then the linear system of equations AX = Y has **(IN-2007-1M)**
(a) no solution
(b) a unique solution
(c) more than one but infinitely many dependent solutions.
(d) infinitely many dependent solutions

02. Let $A = [a_{ij}]$, $1 \le i, j \le n$ with $n \ge 3$ and $a_{ij} = i \cdot j$. Then the rank of A is **(IN-2007-2M)**
(a) 0 (b) 1
(c) n – 1 (d) n

03. The minimum and maximum eigen values of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ are –2 and 6 respectively. What is other eigen values? **(CE-2007-1M)**
(a) 5 (b) 3
(c) 1 (d) –1

04. For what values of α and β the following simultaneous equations have an infinite number of solutions? **(CE-2007-2M)**
$x+y+z = 5$, $x+3y+3z = 9$, $x+2y+\alpha z=\beta$
(a) 2, 7 (b) 3, 8
(c) 8, 3 (d) 7, 2

05. The inverse of the 2×2 matrix $\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$ is **(CE-2007-2M)**
(a) $\frac{1}{3}\begin{bmatrix} -7 & 2 \\ 5 & -1 \end{bmatrix}$ (b) $\frac{1}{3}\begin{bmatrix} 7 & 2 \\ 5 & 1 \end{bmatrix}$
(c) $\frac{1}{3}\begin{bmatrix} 7 & -2 \\ -5 & 1 \end{bmatrix}$ (d) $\frac{1}{3}\begin{bmatrix} -7 & -2 \\ -5 & -1 \end{bmatrix}$

06. If a square matrix A is real and symmetric then the eigen values **(ME-2007-1M)**
(a) are always real
(b) are always real and positive
(c) are always real and non-negative
(d) occur in complex conjugate pairs.

07. The number of linearly independent eigen vectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is **(ME-2007-2M)**
(a) 0 (b) 1
(c) 2 (d) infinite

08. The determinant $\begin{vmatrix} 1+b & b & 1 \\ b & 1+b & 1 \\ 1 & 2b & 1 \end{vmatrix}$ equals to **(PI-2007-1M)**
(a) 0 (b) 2b(b-1)
(c) 2(1-b)(1+2b) (d) 3b(1+b)

09. If A is square symmetric real valued matrix of dimension 2n, the eigen values of A are **(PI-2007-2M)**
(a) 2n distinct real values
(b) 2n real values not necessarily distinct
(c) n distinct pairs of complex conjugate numbers
(d) n pairs of complex conjugate numbers, not necessarily distinct

10. $q_1, q_2 \ldots q_m$ are n-dimensional vectors with m < n. This set of vectors is linearly dependent. Q is the matrix with $q_1, q_2 \ldots q_m$ as the columns. The rank of Q is **(PI-2007-2M)**
(a) less than m (b) m
(c) between m and n (d) n

11. $X = [x_1, x_2, \ldots x_n]^T$ is an n-tuple non-zero vector. The n×n matrix $V = X \times T$ **(EE-2007-1M)**

(a) has rank zero $\quad$ ~~(b)~~ has rank 1
(c) is orthogonal $\quad$ (d) has rank n

12. If $A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$ then A satisfies the relation **(EE-2007-2M)**

(a) $A+3I+2A^{-1} = 0$ $\quad$ (b) $A^2+2A+2I = 0$
~~(c)~~ $(A+I)(A+2I) = 0$ $\quad$ (d) $e^A = 0$

13. If $A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$ then A^9 equals **(EE-2007-2M)**

(a) $511\,A+510\,I$ $\quad$ (b) $309\,A + 104\,I$
(c) $154\,A + 155\,I$ $\quad$ (d) e^{9A}

15. The characteristic equation of a (3×3) matrix P is defined as $\alpha(\lambda) = |\lambda I - P| = \lambda^3 + \lambda^2 + 2\lambda + 1 = 0$. If I denotes identity matrix then the inverse of matrix P will be **(EE-2008-1M)**

(a) $P^2 + P + 2I$ $\quad$ (b) $P^2 + P + I$
(c) $-(P^2 + P + I)$ $\quad$ (d) $-(P^2 + P + 2I)$

16. A is m × n full rank matrix with m > n and I is an identity matrix Let matrix $A^+ = (A^T A)^{-1} A^T$. Then, which one of the following statement is False? **(EE-2008-2M)**

(a) $A A^+ A = A$ $\quad$ (b) $(A A^+)^2 = A A^+$
(c) $A^+ A = I$ $\quad$ (d) $A A^+ A = A^+$

17. If the rank of a (5×6) matrix Q is 4 then which one of the following statement is correct? **(EE-2008-1M)**

(a) Q will have four linearly independent rows and four linearly independent columns
(b) Q will have four linearly independent rows and five linearly independent columns.
(c) $Q Q^T$ will be invertible
(d) $Q^T Q$ will be invertible

19. All the four entries of the 2×2 matrix $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$ are non-zero and one of the eigen values is zero. Which of the following statements is true? **(EC-2008-1M)**

(a) $p_{11}\,p_{22} - p_{12}\,p_{21} = 1$
(b) $p_{11}\,p_{22} - p_{12}\,p_{21} = -1$
(c) $p_{11}\,p_{22} - p_{12}\,p_{21} = 0$
(d) $p_{11}\,p_{22} + p_{12}\,p_{21} = 0$

20. The system of linear equations $\begin{cases} 4x+2y=7 \\ 2x+y=6 \end{cases}$ has **(EC-2008-1M)**

(a) a unique solution
(b) no solution
(c) an infinite no. of solutions
(d) exactly two distinct solutions

21. Consider the matrix $P = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

The value of e^P is **(EC-2008-2M)**

(a) $\begin{bmatrix} 2e^{-2}-3e^{-1} & e^{-1}-e^{-2} \\ 2e^{-2}-2e^{-1} & 5e^{-2}-e^{-1} \end{bmatrix}$

(b) $\begin{bmatrix} e^{-1}+e^{-2} & 2e^{-2}-e^{-1} \\ 2e^{-1}-4e^{-2} & 3e^{-1}+2e^{-2} \end{bmatrix}$

(c) $\begin{bmatrix} 5e^{-2}-e^{-1} & 3e^{-1}-e^{-2} \\ 2e^{-2}-6e^{-1} & 4e^{-2}+e^{-1} \end{bmatrix}$

(d) $\begin{bmatrix} 2e^{-1}-e^{-2} & e^{-1}-e^{-2} \\ -2e^{-1}+2e^{-2} & -e^{-1}+2e^{-2} \end{bmatrix}$

22. The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ has one eigen value to 3. The sum of the other two eigen values is **(ME-2008-1M)**

(a) p $\quad$ (b) p – 1
(c) p – 2 $\quad$ (d) p – 3

23. The eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ are written in the form of $\begin{bmatrix} 1 \\ a \end{bmatrix}$ & $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is a + b?

(ME-2008-2M)

(a) 0 (b) ½
(c) 1 (d) 2

24. For what values of a, if any, will the following system of equations in x, y and z have a solution?
2x + 3y = 4
x + y + z = 4
x + 2y − z = a **(ME-2008-2M)**

(a) any real number
(b) 0
(c) 1
(d) there is no such value

25. The eigen vector pair of the matrix $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$ is **(PI-2008-2M)**

(a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

26. Inverse of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is

(PI-2008-2M)

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$

27. A square matrix B is symmetric if

(CE-2009-1M)

(a) $B^T = -B$ (b) $B^T = B$
(c) $B^{-1} = B$ (d) $B^{-1} = B^T$

28. In the solution of the following set of linear equations by Gauss-elimination using partial pivoting 5x + y + 2z = 34, 4y − 3z = 12 and 10x − 2y+z = −4. The pivots for elimination of x and y are

(CE-2009-2M)

(a) 10 and 4 (b) 10 and 2
(c) 5 and 4 (d) 5 and −4

29. The eigen values of the following matrix are $\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$

(EC-2009-2M)

(a) 3, 3+5j, 6 − j
(b) − 6+5j, 3+j, 3 − j
(c) 3 + j, 3 − j, 5 + j
(d) 3, −1+3j, −1−3j

30. The eigen values of a (2×2) matrix X are −2 and −3. The eigen values of matrix $(X+I)^{-1}(X + 5I)$ are

(IN-2009-2M)

(a) −3, −4 (b) −1, −2
(c) −1, −3 (d) −2, −4

31. The matrix P = $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ rotates a vector about then axis $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ by angle of

(IN-2009-2M)

(a) 30° (b) 60°
(c) 90° (d) 120°

32. For a matrix [M] = $\begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}$, the transpose of the matrix is equal to the inverse of the matrix, $[M]^T = [M]^{-1}$. The value of x is given by

(ME-2009-1M)

(a) $-\frac{4}{5}$ (b) $-\frac{3}{5}$

(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

33. The trace and determinant of a 2×2 matrix are shown to be –2 and –35 respectively. Its eigen values are **(EE-2009-1M)**
(a) –30 and –5 (b) –37 and –1
(c) –7 and 5 (d) 17.5 and –2

34. The value of the determinant $\begin{vmatrix} 1 & 3 & 2 \\ 4 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix}$ is **(PI-2009-1M)**
(a) –28 (b) –24
(c) 32 (d) 36

35. The value of x_3 obtained by solving the following system of linear equations is
$x_1 + 2x_2 - 2x_3 = 4$
$2x_1 + x_2 + x_3 = -2$
$-x_1 + 2x_2 - x_3 = 2$ **(PI-2009-2M)**
(a) –12 (b) –2
(c) 0 (d) 12

36. An eigen vector of $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ is **(EE-2010-2M)**
(a) $[-1 \quad 1 \quad 1]^T$ **(b)** $[1 \quad 2 \quad 1]^T$
(c) $[1 \quad -1 \quad 2]^T$ (d) $[2 \quad 1 \quad -1]^T$

37. For the set of equations
$$\begin{cases} x_1 + 2x_2 + x_3 + 4x_4 = 2 \\ 3x_1 + 6x_2 + 3x_3 + 12x_4 = 6 \end{cases}$$
The following statement is true **(EE-2010-2M)**
(a) only the trivial solution $x_1 = x_2 = x_3 = x_4 = 0$ exist.
(b) There are no solutions
(c) A unique non-trivial solution exist
(d) Multiple non-trivial solutions exist

38. The eigen values of a skew-symmetric matrix are **(EC-2010-1M)**
(a) always zero
(b) always pure imaginary
(c) either zero or pure imaginary
(d) always real

39. One of the eigen vector of thematrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ is **(ME-2010-2M)**
(a) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$ (b) $\begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$
(c) $\begin{Bmatrix} 4 \\ 1 \end{Bmatrix}$ (d) $\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$

40. A real n×n matrix A = [a_{ij}] is defind as follows: $\begin{cases} a_{ij} = i, & \text{if } i = j \\ \quad = 0, & \text{otherwise} \end{cases}$
The sum of all n eigen values of A is **(IN-2010-1M)**
(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$
(c) $\frac{n(n+1)(2n+1)}{6}$ (d) n^2

41. X and Y are non-zero square matrices of size n×n. If XY = $O_{n\times n}$ then **(IN-2010-2M)**
(a) | X | = 0 and | Y | ≠ 0
(b) | X | ≠ 0 and | Y | = 0
(c) | X | = 0 and | Y | = 0
(d) | X | ≠ 0 and | Y | ≠ 0

42. Consider the following matrix $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$. If the eigen values of A are 4 and 8 then **(CS-2010-2M)**
(a) x = 4, y = 10 (b) x = 5, y = 8
(c) x = -3, y = 9 (d) x = -4, y = 10

43. The inverse of the matrix $\begin{bmatrix} 3+2i & i \\ -i & 3-2i \end{bmatrix}$ is **(CE-2010-2M)**

(a) $\frac{1}{2}\begin{bmatrix} 3+2i & -i \\ i & 3-2i \end{bmatrix}$

(b) $\frac{1}{12}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

(c) $\frac{1}{14}\begin{bmatrix} 3+2i & -i \\ i & 3-2i \end{bmatrix}$

(d) $\frac{1}{14}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

44. The value of q for which the following set of linear algebraic equations $2x+3y=0$, $6x+qy=0$ can have non-trivial solution is **(PI-2010-1M)**

(a) 2 (b) 7
(c) 9 (d) 11

45. If $\{1,0,-1\}^T$ is an eigen vector of the following matrix $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -2 & 1 \end{bmatrix}$ then the corresponding eigen value is **(PI-2010-1M)**

(a) 1 (b) 2
(c) 3 (d) 5

KEY:

01. b	02. b	03. b
04. a	05. a	06. a
07. b	08. a	09. b
10. a	11. b	12. c
13. a	14.	15. d
16. d	17. a	18.
19. c	20. b	21.
22. c	23. b	24. b
25. a	26. b	27. b
28. c	29. d	30. c
31. c	32. a	33. c
34. b	35. b	36. b
37. d	38. c	39. a
40. a	41. c	42. d
43. b	44. c	45. a

Limit: Let f(x) be defined over a deleted neighbourhood of a, and l be any real number. If to each positive real number $\in$ (however small it may be), there exists a positive real number δ such that $|f(x) - 1| < \in$. Whenever $0 < |x - a| < \delta$. Then we say f(x) tends to 1 as x tends to a and l is called limit of f(x) at x = a. It is denoted by $\underset{x \to a}{\text{Lt}} f(x) = l$.

One sided Limits:

Left Limit: If x < a and x → a then we write, $\underset{x \to a^-}{\text{Lt}} f(x)$ and if the limit exists, it is called left limit of f(x) at x = a.

Right Limit: If x > a and x→a then we write, $\underset{x \to a^+}{\text{Lt}} f(x)$ and if the limit exists, it is called limit of f(x) at x = a.

* $\underset{x \to a}{\text{Lt}} f(x)$ exists $\Leftrightarrow \underset{x \to a^-}{\text{Lt}} f(x) = \underset{x \to a^+}{\text{Lt}} f(x)$

- If limit of f(x) at x = a exists; it is then unique.
- If $\underset{x \to a}{\text{Lt}} f(x) = l$ and $\underset{y \to l}{\text{Lt}} g(y) = m$ then $\underset{x \to a}{\text{Lt}} [(g \circ f)(x)] = m$
- If f(x) is a polynomial function, then $\underset{x \to a}{\text{Lt}} f(x) = f(a)$
- $\underset{x \to a}{\text{Lt}} f(x)$ need not be equal to f(a)
- $\underset{x \to a}{\text{Lt}} [(x^n - a^n)/(x - a)] = n \cdot a^{n-1}$
- $\underset{x \to 0}{\text{Lt}} (\sin x / x) = 1 = \underset{x \to 0}{\text{Lt}} (\tan x / x)$
- $\underset{x \to 0}{\text{Lt}} (1 + x)^{1/x} = e = \underset{x \to \infty}{\text{Lt}} (1 + 1/x)^x$
- $\underset{x \to 0}{\text{Lt}} (1 + kx)^{1/x} = e^k = \underset{x \to 0}{\text{Lt}} (1 + k/x)^x$

Indeterminate forms: $0/0$, ∞/∞, $0.\infty$, $\infty - \infty$, 0^0, 1^∞, ∞^0, 0^∞

L. Hospital's Rule: If $\underset{x \to a}{\text{Lt}} [f(x) / g(x)]$ is of the form either 0/0 or ∞/∞ then

$$\underset{x \to a}{\text{Lt}} [f(x) / g(x)] = \underset{x \to a}{\text{Lt}} [f^1(x) / g^1(x)]$$

1) $\underset{x \to 0}{\text{Lt}} 2^{1/x} =$

a) ∞ b) 0 c) indeterminate d) does not exist

2) $\underset{x \to 0}{\text{Lt}} \sin(1/x) =$

a) ∞ b) 0 c) 1 d) does not exist

3) $\lim_{x\to 0} x \cdot \sin(1/x) =$

a) 0 b) 1 c) ∞ d) does not exist

4) $\lim_{x\to\infty} x \cdot \sin(1/x) =$

a) 0 b) 1 c) ∞ d) does not exist

5) $\lim_{x\to 0} |x| / x =$

a) 1 b) -1 c) indeterminate d) does not exist

6) $[x]$ = greatest integer less than or equal to x. If $\lim_{x\to a} [x]$ does not exist, then which of the following is true

a) a is any real number b) a is any rational number
c) a is any integer d) a is a complex number

7) $\lim_{x\to 2} \sqrt{(4 - x^2)} =$

a) 0 b) imaginary c) does not exist d) indeterminate

8) $\lim_{x\to\pi/2} \tan x =$

a) ∞ b) $-\infty$ c) does not exist d) 0

9) $\lim_{x\to 0^+} \log x =$

a) ∞ b) $-\infty$ c) 0 d) does not exist

10) $\lim_{x\to 0} \dfrac{e^{1/x} - 1}{e^{1/x} + 1} =$

a) -1 b) 1 c) ∞ d) does not exist

11) $\lim_{x\to\infty} \left(\dfrac{x - 1}{x + 1}\right)^x =$

a) e b) e^2 c) e^{-2} d) ∞

12) If $\lim_{x\to 0} (\sin 2x + a \sin x) / x^3$ is finite, then a =

a) 0 b) 2 c) -2 d) ½

13) The value of the limit given in the previous example is

a) 0 b) -1 c) 1 d) does not exist

14) If $\lim_{x\to 0} [x(1 + a \cos x) - b \sin x] / x^3 = 1$, then (a, b) =

a) $-5/2, 3/2$ b) $5/2, -3/2$ c) $5/2, 3/2$ d) $-5/2, -3/2$

15) $\lim_{x\to\pi/2} [\sec x / \sec 3x] =$

a) 3 b) -3 c) 1/3 d) $-1/3$

16) $\displaystyle\lim_{x\to 0} [1/x^2 - (1/\sin^2 x)] =$

a) 0 b) 1/3 c) – 1/3 d) cannot be determined

17) $\displaystyle\lim_{x\to 0} (\cos x)^{1/x^2} =$

a) 1 b) e^{-2} c) $e^{-1/2}$ d) e

18) $\displaystyle\lim_{x\to 0} (\log x / \log \operatorname{cosec} x) =$

a) 1 b) – 1 c) 0 d) does not exist

19) $\displaystyle\lim_{x\to \infty} (x^n / e^x) =$

a) 0 b) n! c) 1 d) ∞

20. $\displaystyle\lim_{x\to \infty} (\log x / x^n) =$

a) 0 b) 1/n c) 1 d) –1/n

Continuity: A function f(x) is said to be continuous at x = a if

$$\lim_{x\to a} f(x) = f(a)$$

- A function f(x) is said to be continuous in the interval (a, b) if f(x) is continuous for all values of x in the interval.
- If f(x) and g(x) are two continuous functions then f(x) + g(x), f(x) – g(x), f(x) . g(x) and f(x) / g(x) $(g(x) \neq 0)$ are also continuous.
- Polynomial functions, Exponential functions, Sine and Cosine functions are continuous everywhere.

21) Let $f(x) = (1 + 3x)^{1/x}, x \neq 0$

$= k \quad , x = 0$

If f(x) is continuous at x = 0 then k =

a) e^3 b) 3 c) $e^{1/3}$ d) $e^{-1/3}$

22) Let $f(x) = [\sin^2(ax) / x^2], x \neq 0$

$1 \quad , x = 0$

a) 0, 1 b) -1, 1 c) 0, -1 d) none of these

23) If $f(x) = 1$ when $x < 0$

$= 1 + \sin x$ when $0 \leq x \leq \pi/2$

$= 2 + (x - \pi/2)^2$ when $x > \pi/2$

Then f(x) is

a) continuous at x = 0 but discontinuous at $x = \pi/2$
b) continuous at $x = \pi/2$ but discontinuous at x = 0
c) continuous for all values of x
d) discontinuous at x = 0 and at $x = \pi/2$

24) If $f(x) = -x^2, x \leq 0$
$= 5x - 4, 0 < x \leq 1$
$= 4x^2 - 3x, 1 < x < 2$
$= 3x + 4, x \geq 2$
then f(x) is discontinuous at x =
a) 0 b) 1 c) 2 d) 3

Differentiability: A function f(x) is said to be differentiable at x = a if

$$\operatorname*{Lt}_{h \to 0} \frac{[f(a+h) - f(a)]}{h}$$ exists. The limiting value is denoted by $f^1(a)$

- A function f(x) is said to be differentiable in (a, b), if f(x) is differentiable for all values of x in that interval.
- If f(x) and g(x) are two differential functions then f(x) + g(x), f(x) – g(x), f(x) . g(x) and f(x) / g(x) where g(x) ≠ 0 are also differentiable
- Polynomial functions, Exponential functions, Sine and Cosine functions are differentiable every where.
- Every differentiable function is continuous but a continuous function need not be differentiable.

25) The function f(x) = [x . Sin(1/x)] is
a) continuous at x = 0 but not differentiable at x = 0
b) discontinuous at x = 0 but differentiable at x = 0
c) continuous and differentiable at x = 0
d) neither continuous nor differentiable at x = 0

26) The function f(x) = |x| is
a) continuous at x = 0 but not differentiable at x = 0
b) differentiable at x = 0 but not continuous at x = 0
c) continuous and differentiable at x = 0
d) neither continuous nor differentiable at x = 0

27) If $f(x) = [x^2 . \sin(1/x)]$ then which of the following is true
a) $f^1(0)$ exists but $f^{11}(0)$ does not exist b) both $f^1(0)$ and $f^{11}(0)$ does not exist
c) neither $f^1(0)$ nor $f^{11}(0)$ does not exist d) $f^1(0)$ does not exist but $f^{11}(0)$ exists

28) The function f(x) = {x . [1 + 1/3 Sin log(x)]} is
a) continuous at x = 0 but not differentiable at x = 0
b) differentiable at x = 0 but not continuous at x = 0
c) continuous and differentiable at x = 0
d) neither continuous nor differentiable at x = 0

29) The function f(x) = |x| + |x+1| + |x-2| is differentiable at x =
a) 1 b) – 1 c) 0 d) 2

Note : f(x) = |x – a| is continuous every where
f(x) = |x – a| is differentiable everywhere except at x = a

30) If $f(x) = 2 + x$ when $x \geq 0$
$= 2 - x$ when $x < 0$
then $f(x)$ is
a) continuous but not differentiable at $x = 0$
b) continuous and differentiable at $x = 0$
c) neither continuous nor differentiable at $x = 0$
d) differentiable at $x = 0$ but not continuous

31) If $f(x) = 1$ when $x < 0$
$= 1 + \text{Sin}x$ when $0 \leq x < \pi/2$
$= 2 + (x - \pi/2)^2$ when $x \geq \pi/2$
then at $x = \pi/2$, $f(x)$ is
a) continuous but not differentiable
b) differentiable but not continuous
c) continuous and differentiable
d) neither continuous nor differentiable

MEAN VALUE THEOREMS

Rolle's Theorem: If $f(x)$ is i) continuous in [a, b], (ii) differentiable in (a, b) and (iii) $f(a) = f(b)$ then there exists atleast one value $C \in (a, b)$ such that $f^1(C) = 0$.

32) Find C of the Rolle's theorem for $f(x) = x(x-1)(x-2)$ in [1, 2]
a) 1.5 b) $1 - (1/\sqrt{3})$ c) $1 + (1/\sqrt{3})$ d) 1.25

33) Find C of the Rolle's theorem for $f(x) = e^x \text{Sin}x$ in $[0, \pi]$
a) $\pi/4$ b) $\pi/2$ c) $3\pi/4$ d) does not exist

34) Find C of Rolle's theorem for $f(x) = (x+2)^3 (x-3)^4$ in [-2, 3]
a) 1/7 b) 2/7 c) ½ d) 3/2

35) Find C of Rolle's theorem for $f(x) = e^x (\text{Sin}x - \text{Cos}x)$ in $[\pi/4, 5\pi/4]$
a) $\pi/2$ b) $3\pi/4$ c) π d) does not exist

36) Find C of Rolle's theorem for $f(x) = x(x+3)\, e^{-x/2}$ in [-3, 0]
a) –1 b) –2 c) –0.5 d) 0.5

37) Find C of Rolle's theorem for $f(x) = \log[(x^2 + ab) / (a+b)x]$
a) $(a+b)/2$ b) $\sqrt{ab}$ c) $2ab / (a + b)$ d) $(b - a)/2$

38) Rolle's theorem cannot be applied for the function $f(x) = |x|$ in [-2, 2] because
a) $f(x)$ is not continuous in [-2, 2]
b) $f(x)$ is not differentiable in (-2, 2)
c) $f(-2) \neq f(2)$
d) none of the above

39) Rolle's theorem cannot be applied for the function $f(x) = |x+2|$ in [-2, 0] because
a) $f(x)$ is not continuous in [-2, 0]
b) $f(x)$ is not differentiable in (-2, 0)
c) $f(-2) \neq f(0)$
d) none of these

Lagrange's Mean Value Theorem: If $f(x)$ is continuous in [a, b] and differentiable in (a, b) then there exists atleast one value C in (a, b) such that

$$f^1(C) = [f(b) - f(a)] / (b - a)$$

40) Find C of Lagrange's mean value theorem for $f(x) = (x-1)(x-2)(x-3)$ in [1, 2]
a) $2 - 1/\sqrt{3}$ b) $2 + (1/\sqrt{3})$ c) $1 + (1/\sqrt{3})$ d) $1 - (1/\sqrt{3})$

41) Find C of Lagrange's mean value theorem for $f(x) = \log x$ in [1, e]
a) $e - 2$ b) $e - 1$ c) $(e + 1) / 2$ d) $(e - 1) / 2$'

42) Find C of Lagrange's mean value theorem for $f(x) = lx^2 + mx + n$ in [a, b]
a) $(a + b) / 2$ b) $\sqrt{ab}$ c) $2ab / (a + b)$ d) $(b - a)/2$

43) Find C of Lagrange's theorem mean value theorem for $f(x) = 7x^2 - 13x - 19$ in [-11/7, 13/7]
a) 1/7 b) 2/7 c) 3/7 d) 4/7

44) Find C of Lagrange's mean value theorem for $f(x) = e^x$ in [0, 1]
a) 0.5 b) $\log(e - 1)$ c) $\log(e + 1)$ d) $\log[(e + 1) / (e - 1)]$

45) Lagrange's mean value theorem cannot be applied for the function $f(x) = x^{1/3}$ in [-1, 1] because
a) f(x) is not continuous in [-1, 1] b) f(x) is not differentiable in (-1,1)
c) f(x) is neither continuous not differentiable in [-1, 1]
d) none of the above

Cauchy's Mean Value Theorem:

If f(x) and g(x) are two functions such that
a) f(x) and g(x) are continuous in [a, b]
b) f(x) and g(x) are differentiable in (a, b)
c) $g^1(x) \neq 0$ for all x in (a, b)

then there exists atleast one value C such that

$$[f^1(c) / g^1(c)] = [f(b) - f(a)] / [g(b) - g(a)]$$

46) Find C of Cauchy's mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in [a, b]
a) $(a + b) /2$ b) $\sqrt{ab}$ c) $2ab / (a + b)$ d) $(b - a) / 2$

47) Find C of cauchy's mean value theorem for $f(x) = \sqrt{x}$ and $g(x) = 1/\sqrt{x}$ in [a, b]
a) $(a + b) / 2$ b) $\sqrt{ab}$ c) $2ab / (a + b)$ d) $(b - a) / 2$

48) Find C of Cauchy's mean value theorem for the functions 1/x and $1/x^2$ in [a, b]
a) $(a + b)/2$ b) $\sqrt{ab}$ c) $2ab / (a + b)$ d) $(b - a)/2$

49) Find C of Cauchy's mean value theorem for the functions Sinx and Cosx in $(-\pi/2, 0)$
a) $-\pi/3$ b) $-\pi/4$ c) $-\pi/6$ d) $-\pi/8$

Definite Integrals: The first fundamental theorem of integral calculus:
If f(x) is continuous in [a, b] and F(x) in any antiderivative of f(x) in [a, b] then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

The second fundamental theorem of Integral Calculus: If f(x) is continuous on [a, b] then $F(x) = \int_a^x f(t)\,dt$ is differentiable at every point of x in [a,b] and $dF/dx = d/dx \int_a^b f(t)\,dt = f(x)$

Corollary: If f(x) is continuous on [a, b] then there exists a function F(x) whose derivative on [a, b] is f(x).

Theorem:3: If f(x) is continuous on [a, b] and U(x) and V(x) are differentiable functions of x whose values lie in [a, b], then

$$\frac{d}{dx}\int_{U(x)}^{V(x)} f(t)\,dt = f(V(x))\,(dV/dx) - f(U(x))\,(dU/dx)$$

Properties of Definite Integrals:

1) $\int_a^b f(x)\,dx = \int_a^b f(y)\,dy$

2) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3) If $a < c < b$ then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

4) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

5) $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$ if f(x) is even
$= 0$ if f(x) is odd

6) $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$ if $f(2a-x) = f(x)$
$= 0$ if $f(2a-x) = -f(x)$

7) $\int_0^{na} f(x)\,dx = n\int_0^a f(x)\,dx$ if $f(x+a) = f(x)$
where f(x) is a periodic function with period a

8) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

9) $\int_0^a x\,f(x)\,dx = a/2\int_0^a f(x)\,dx$ if $f(a-x) = f(x)$

10) $\int_0^{\pi/2} \text{Sin}^n x\,dx = \int_0^{\pi/2} \text{Cos}^n x\,dx =$

$[(n-1)/n] \cdot [(n-3)/(n-2)] \cdot [(n-5)/(n-4)] \cdot (2/3)$ if n is odd
$[(n-1)/n] \cdot [(n-3)/(n-2)] \cdot [(n-5)/(n-4)] \ldots\ldots 1/2 \cdot (\pi/2)$ if n is even

11) $\int_0^{\pi/2} \text{Sin}^m x \cdot \text{Cos}^n x\,dx =$

$$\frac{\{(m-1)(m-3)(m-5)\ldots\ldots(2 \text{ or } 1)\} \cdot [(n-1)(n-2)\ldots(2 \text{ or } 1)] \cdot k}{(m+n)(m+n-2)(m+n-4)\ldots\ldots(2 \text{ or } 1)}$$

Where $k = \pi/2$ When both m and n are even, otherwise $k = 1$

50) $\int_0^2 |1 - x|\, dx =$

a) 1 b) – 1 c) 2 d) 3/2

51) $\int_0^1 x(1 - x)^5\, dx =$

a) 1/42 b) 1/48 c) 1/12 d) 1/56

52) $\int_{-1}^{1} \frac{x^2 . \text{Sin}x\, dx}{x^4 + 1} =$

a) 0 b) π c) 2 d) $\pi/2$

53) $\int_{-\pi/2}^{\pi/2} x^{10} . \log |(1 + \text{Sin}x) / (1 - \text{Sin}x)|\, dx =$

a) 0 b) π c) 2 d) $\pi/2$

54) $\int_{-1}^{1} |x|\, dx =$

a) 0 b) 1 c) 2 d) 4

55) $\int_0^{\pi} \text{Sin}^3 x\, dx =$

a) 2/3 **b) 4/3** c) 0 d) $\pi/3$

56) $\int_0^{\pi/2} [(\text{Sin}x - \text{Cos}x) / (1 + \text{Sin}x . \text{Cos}x)]\, dx =$

a) 0 b) π c) $\pi/2$ d) $\pi/4$

57) $\int_0^{\pi/2} dx / (1 + \sqrt{\cot x}) =$

a) 0 b) π c) $\pi/2$ d) $\pi/4$

58) $\int_0^{\pi/2} (\text{Sin}2x . \log(\tan x))\, dx =$

a) 0 b) π c) $\pi/2$ d) $\pi/4$

59) $\int_0^{\pi/2} \log(1 + \tan x)\, dx =$

a) 0 b) $\pi/2 \log 2$ c) $\pi/8 \log 2$ d) $-\pi/4 \log 2$

60) $\int_0^{\pi} [(x . \text{Sin}x) / (1 + \text{Cos}^2 x)]\, dx =$

a) π^2 b) $\pi^2/2$ c) $\pi^2/4$ d) $\pi^2/8$

61) $\int_0^{\pi} dx / (a^2 \cos^2 x + b^2 \sin^2 x) =$

a) 0 b) πab c) π/ab d) $\pi / (a^2 + b^2)$

62) $\int_0^{\pi} x \cdot \sin^6 x \cdot \cos^4 x \, dx =$

a) $3\pi^2 / 512$ b) $5\pi^2/256$ c) $3\pi^2/128$ d) $5\pi^2/128$

63) $\int_0^{\pi} [(x \cdot \tan x) / (\sec x + \tan x)] \, dx =$

a) 0 b) $\pi(\pi - 2)/4$ c) $\pi(\pi - 2)/2$ d) π

64) $\int_0^{\pi^2/4} \sin\sqrt{x} \, dx =$

a) 0 b) 1 c) 2 d) $\pi/2$

65) $\int_2^{3} [\sqrt{x} / (\sqrt{x} + \sqrt{5 - x})] \, dx =$

a) 1 b) 2.5 c) 0.5 d) 1.5

66) $\int_{-\pi}^{\pi} \sin^4 x \, dx =$

a) $\pi/4$ b) $\pi/2$ c) $3\pi/4$ d) 0

67) $\int_0^{\pi} \sin^4 x \cos^5 x \, dx =$

a) 0 b) $3\pi/256$ c) $3\pi/128$ d) $5\pi/128$

68) $\int_0^{2\pi} \sin^4 x \cos^6 x \, dx =$

a) $3\pi/128$ **b) $3\pi/256$** **c) $3\pi/64$** **d) 0**

69) $\int_0^{2\pi} \sin^4 x \cos^5 x \, dx =$

a) 0 b) $3\pi/128$ c) $5\pi/128$ d) $3\pi/256$

IMPROPER INTEGRALS

- $\int_a^b f(x)\,dx$ is said to be an improper integral of first kind if $a = -\infty$ or $b = \infty$ or both.
- $\int_a^b f(x)\,dx$ is said to be an improper integral of second kind if f(x) is infinite for one or more values of x in [a, b].

- $\int_a^b f(x)\,dx$ is said to be convergent if the value of integral is finite.

* If (i) $0 \le f(x) \le g(x)$ for all x and (ii) $\int_a^\infty g(x)\,dx$ converges then $\int_a^\infty f(x)\,dx$ also converges.

* If (i) $f(x) \ge g(x) \ge 0$ for all x and (ii) $\int_a^\infty g(x)\,dx$ diverges then $\int_a^\infty f(x)\,dx$ also converges.

* If f(x) and g(x) are two functions such that $\mathop{Lt}\limits_{n \to \infty} [f(x) / g(x)] = k$ (finite and $\ne 0$) then $\int_a^\infty f(x)\,dx$ and $\int_a^\infty g(x)\,dx$ converge or diverge together.

* $\int_1^\infty (dx / x^p)$ converges when $p > 1$ and diverges when $p \le 1$

* $\int_a^\infty e^{-px}\,dx$ and $\int_{-\infty}^b e^{px}\,dx$ converges for any constant $p > 0$ and diverge for $p \le 0$.

* The integral $\int_a^b dx / (b - x)^p$ is convergent iff $p < 1$

* The integral $\int_a^b dx / (x - a)^p$ converges iff $p < 1$

* Suppose f(x) is continuous in (a, b) and $f(x) \to \infty$ as $x \to b$. If f(x) and g(x) are positive and $\mathop{Lt}\limits_{x \to b} [f(x) / g(x)] = l$ (finite and $\ne 0$) then $\int_a^b f(x)\,dx$ and $\int_a^b g(x)\,dx$ converge or diverge together.

* Suppose f(x) is continuous in (a, b) and $f(x) \to \infty$ as $x \to a$. If f(x) and g(x) are positive and $\mathop{Lt}\limits_{x \to a} [f(x) / g(x)] = l$ (finite and $\ne 0$) then $\int_a^b f(x)\,dx$ and $\int_a^b g(x)\,dx$ converge or diverge together.

70) Which of the following improper integrals is divergent

a) $\int_0^\infty [1 / (1 + x^2)]\,dx$ b) $\int_0^1 [x / \sqrt{(1 - x^2)}]\,dx$ c) $\int_0^1 \log x\,dx$ d) $\int_0^\infty x \,.\, \sin x\,dx$

71) Which of the following improper integrals is divergent

a) $\int_{-\infty}^\infty [1 / (1 + x^2)]\,dx$ b) $\int_0^\infty e^{-x}\,dx$ c) $\int_1^\infty dx / x^5$ d) $\int_0^2 1 / x^3\,dx$

72) Which of the following improper integrals is divergent

a) $\int_0^1 [dx / x^{1/3}]\,dx$ b) $\int_1^\infty [dx / x\sqrt{(x^2 - 1)}]$ c) $\int_1^\infty [1/x^2]\,dx$ d) $\int_1^\infty (1/\sqrt{x})\,dx$

73) Which of the following integrals is divergent

a) $\int_{-\infty}^{-1} 1 / x^4 \, dx$ b) $\int_{0}^{1} 1 / x^2 \, dx$ c) $\int_{0}^{1} [1/\sqrt{(1 - x^2)}]dx$ d) $\int_{0}^{\infty} [1/(x^2 + 2x + 2)]dx$

74) Which of the following improper integrals is divergent.

a) $\int_{0}^{\infty} x^3 . e^{-x} \, dx$ b) $\int_{0}^{\infty} [\log x / x^3] \, dx$ c) $\int_{0}^{1} x . \log x \, dx$ d) $\int_{-1}^{1} dx / (x . x^{1/3})$

75) Consider the integrals

$$I_1 = \int_{1}^{\infty} dx / [x^2 (1 + e^x)] \quad \text{and} \quad I_2 = \int_{1}^{\infty} [(x + 1) / x\sqrt{x}] \, dx$$

Which of the following is true

a) I_1 is convergent and I_2 is divergent
b) I_1 is divergent and I_2 is convergent
c) I_1 and I_2 are convergent
d) I_1 and I_2 are divergent

76) Which of the following integrals is divergent

a) $\int_{0}^{1} [1 / \sqrt{(1 - x)}] \, dx$ b) $\int_{1}^{\infty} [\text{Sin}x / x^3] \, dx$ c) $\int_{-1}^{1} 1/x^2 \, dx$ d) $\int_{0}^{2} [1 / \sqrt{|(1 - x^2)|}] \, dx$

77) Which of the following integrals is divergent

a) $\int_{0}^{1} [1 / \sqrt{(x + 4x^3)}] \, dx$ b) $\int_{e^2}^{\infty} [dx / x(\log x)]^3$ c) $\int_{0}^{\pi/2} \sec x \, dx$ d) $\int_{1}^{e} [dx / x(\log x)^{1/3}]$

78) Which of the following improper integrals is convergent

a) $\int_{1}^{\infty} [(x + \sqrt{(x + 1)} / x^2 + 2(x^4+1)^{1/5}] \, dx$
b) $\int_{2}^{\infty} [(3 + 2x^2)^{1/7} / (x^3 - 1)^{1/5}]dx$

c) $\int_{3}^{\infty} dx / \sqrt{[x(x - 1) (x - 2) \quad (x > 3)]}$
d) $\int_{1}^{\infty} [1 / x^{1/3}(1 + x)^{1/2}]dx$

79) Which of the following improper integrals is divergent

a) $\int_{-1}^{\infty} [x / (1 + x)^3] \, dx$ b) $\int_{1}^{\infty} dx / [(1 + x) \sqrt{x}]$ c) $\int_{0}^{1} dx / [x^{1/3} (1 + x^2)]$ d) $\int_{0}^{1} dx / [x^2(1+x)^2]$

80) Which of the following improper integrals is / are convergent

a) $\int_{0}^{1} [1 / [(x - 1)^6 (x - 2)^{1/5}] \, dx$
b) $\int_{0}^{1} [x^{a-1} / x + 1] \, dx \quad (a > 0)$

c) $\int_{0}^{\pi/2} [\sqrt{x} / \text{Sin}x] \, dx$
d) $\int_{1/2}^{1} dx / [x^{1/3} (1 - x)^{1/3}]$

Partial Derivatives and Total Derivatives

If $u = f(x, y)$ then

$$\partial u / \partial x = \lim_{h\to 0} [f(x + h, y) - f(x, y)] / h \quad \text{and}$$

$$\partial u / \partial y = \lim_{k\to 0} [f(x, y+k) - f(x, y)] / k$$

$(\partial/\partial x) (\partial u / \partial x) = (\partial^2 u / \partial x^2) = f_{xx}$

$(\partial/\partial u) (\partial u / \partial y) = (\partial^2 u / \partial y^2) = f_{yy}$

$(\partial/\partial x) (\partial u / \partial y) = (\partial^2 u / \partial u \partial y) = f_{xy}$

$(\partial/\partial y) (\partial u / \partial x) = (\partial^2 u / \partial y\, \partial x) = f_{yx}$

In general, $f_{xy} = f_{yx}$

Euler's Theorem: If $u = f(x, y)$ is a homogeneous function of degree n then

$$x \, . \, u_x + y \, . \, u_y = nu$$

* If $u = f(x, y, z)$ is a homogeneous function of degree n then

$$x \, . \, u_x + y \, . \, u_y + z \, . \, u_z = nu$$
$$x^2 \, . \, u_{xx} + 2xy \, . \, u_{xy} + y^2 \, . \, u_{yy} = n(n-1)\, u$$

* If $u = f(x, y)$ is not a homogeneous function but $F(u)$ is a homogeneous function of degree n then

i) $x \, . \, u_x + y \, . \, u_y = n\,[F(u) / F^1(u)] = G(u)$

ii) $x^2 \, . \, u_{xx} + 2xy \, . \, u_{xy} + y^2 \, . \, u_{yy} = G(u)\, \{G^1(u) - 1\}$

* If $u = f(x, y) + g(x, y) + h(x, y)$ where f, g, h are homogeneous functions of degrees m, n, p respectively then

i) $x \, . \, u_x + y \, . \, u_y = m \, . \, f + n \, . \, g + p \, . \, h$

ii) $x^2 \, . \, u_{xx} + 2xy\, u_{xy} + y^2 \, . \, u_{yy} = m(m-1)\, f + n(n-1)g + p(p-1)h$

* If $u = f(x, y)$ where x and y are functions of t then the total derivative of u with respect t is given by

$$(du / dt) = (\partial u / \partial x) \, . \, (dx / dt) + (\partial u / \partial y) \, . \, (dy / dt)$$

* Taking $x = t$ in the above equation

$$(du / dx) = (\partial u / \partial x) + (\partial u / \partial y) \, . \, (dy / dx)$$

This formula can be used for finding total derivative of u with respect to x, when x and y are connected by some relation.

* If $f(x, y) = C$ is an implicit function of x and y then

$$(dy / dx) = -\, (f_x / f_y)$$

* If $u = f(x, y)$ where $x = g(r, s)$ and $y = h(r, s)$ then

$$(\partial u / \partial r) = (\partial u/\partial x) \, . \, (\partial x/\partial r) + (\partial u/\partial y) \, . \, (\partial y/\partial r)$$
$$(\partial u / \partial s) = (\partial u/\partial x) \, . \, (\partial x/\partial s) + (\partial u/\partial y) \, . \, (\partial y/\partial s)$$

81) The total derivative of x^2y with respect to x, when x and y are connected by the relation $x^2 + xy + y^2 = 1$ is

a) $2xy - x^2\left(\frac{2x + y}{x + 2y}\right)$ b) $\frac{x^2y + xy^2}{x + 2y}$ c) $\frac{x^2y - xy^2}{y + 2x}$ d) $2xy - y^2\left(\frac{x + 2y}{2x + y}\right)$

82) If $u = x\log(xy)$ where $x^3 + y^3 + 3xy = 1$ then $(du / dx) =$
a) 0
b) $1 + \log(xy) - [x(x^2 + y) / y(y^2 + x)]$
c) $1 + \log(y/x) - [(x^2 + y) / (x + y^2)$
d) $\log(x/y) + [(x^2 + y) / (x + y^2)$

83) If $u = \mathrm{Sin}(x^2 + y^2)$ where $a^2x^2 + b^2y^2 = C^2$ then $du / dx =$
a) $2(1 - a^2/b^2)x\cos(x^2 + y^2)$
b) $2x(a^2 + b^2) / b^2\,\mathrm{Cos}(x^2 + y^2)$
c) $2x(a^2 - b^2)/a^2\,\mathrm{Cos}(x^2 + y^2)$
d) $2x(a^2 - b^2) / b^2\,\mathrm{Cos}(x^2 + y^2)$

84) If $U = \mathrm{Sin}^{-1}(x/y) + \mathrm{Cos}(y/x)$ then $U_x / U_y =$
a) x/y b) y/x c) $-x/y$ d) $-y/x$

85) Let $r^2 = x^2 + y^2 + z^2$ and $V = r^n$ then $V_{xx} + V_{yy} + V_{zz} =$
a) 0 b) $n(n+1)r^{n-2}$ c) $n(n-1)r^{n-2}$ d) $n(n+2)r^{n-2}$

86) If $U = (y/z) + (z/x)$ then $x \,.\, U_x + y \,.\, U_y + z.\, U_z =$
a) 0 b) xy / z^2 c) yz / x^2 d) zx / y^2

87) If $V = (x^2 + y^2 + z^2)^{-2}$ then $V_{xx} + V_{yy} + V_{zz} =$
a) 0 b) $(x^2 + y^2 + z^2)$ c) $(x^2 + y^2 + z^2)^{-2}$ d) $(x^2 + y^2 + z^2)^{-2}$

88) If $U = f(r)$ where $r^2 = x^2 + y^2 + z^2$ then $U_{xx} + U_{yy} + U_{zz} =$
a) $f^{11}(r) + (2/r)f^1(r)$ b) $f^{11}(r) + (1/r^2)f^1(r)$ c) $f^{11}(r) + 3/r\, f^1(r)$ d) $f^{11}(r) - 2/r\, f^1(r)$

89) If $U = f(r)$ where $x = r\cos\theta$ and $y = r\,\mathrm{Sin}\theta$ then $U_{xx} + U_{yy} =$
a) $f^{11}(r) + 1/r\, f^1(r)$ b) $f^{11}(r) + 2/r\, f^1(r)$ c) $f^{11}(r) - 1/r\, f^1(r)$ d) $f^{11}(r) - 2/r\, f^1(r)$

90) If $\mathrm{Sin}U = [(x + 2y + 3z) / (x^8 + y^8 + z^8)]$ then $x\ U_x + y \,.\, U_y + z \,.\, U_z =$
a) $(1/7)\tan U$ b) $-7\tan U$ c) $(1/7)\sec U$ d) $(-1/20)\tan U$

91) If $U = \log[(x^4 + y^4) / (x - y)]$ then $x^2 \,.\, U_{xx} + 2xy \,.\, U_{xy} + y^2 \,.\, U_{yy} =$
a) 0 b) 3 c) -3 d) 1/3

92) If $U = \mathrm{Cosec}^{-1}[(x^{1/4} + y^{1/4}) / (x^{1/5} - y^{1/5})]$ then $x \,.\, U_x + y \,.\, U_y =$
a) $(1/20)\,\mathrm{Cot}\,U$ b) $(-1/20)\,\mathrm{Cot}\,U$ c) $(1/20)\tan U$ d) $(-1/20)\tan U$

93) If $U = [(x^3 + y^3) / (x - y)] + x\,\mathrm{Sin}(x/y)$ then $x^2 \,.\, U_{xx} + 2xy \,.\, U_{xy} + y^2 \,.\, U_{yy} =$
a) 0
b) $2[(x^3 + y^3) / (x - y)]$
c) $[(x^3 + y^3) / (x - y)] - \mathrm{Sin}(x/y)$
d) $[(x^3 + y^3) / (x - y)] - \mathrm{Cos}(x/y)$

94) If $Z = x^n f_1(y/x) + y^{-n}f_2(x/y)$ then $x(\partial z / \partial x) + y(\partial z/\partial y) + x^2 Z_{xx} + 2xy \,.\, Z_{xy} + y^2 \,.\, Z_{yy} =$
a) 0 b) $n(n+1)Z$ c) n^2Z d) $(n^2 - n)Z$

95) If $U = f(r, s)$ where $r = x + y$ and $r = x - y$ then $U_x + U_y =$
a) $2U_r$ b) $2U_s$ c) $-2U_r$ d) $-2U_s$

96) If $U = f(x - y, y - z, z - x)$ then $U_x + U_y + U_z =$
a) 0 b) U c) 2U d) 3 U

97) If $Z = f(x, y)$ where $x = e^u + e^{-v}$ and $y = e^u + e^{-v}$ then $Z_u - Z_v =$
a) $x \,.\, Z_x - y \,.\, Z_y$ b) $x \,.\, Z_x + y \,.\, Z_y$ c) $x \,.\, Z_y + y \,.\, Z_x$ d) $x \,.\, Z_y - y \,.\, Z_x$

98) If $U = f(x + Cy) + g(x - Cy)$ then $U_{xx} / U_{yy} =$
a) C^2 b) C^{-2} c) $-C^2$ d) $-C^{-2}$

99) If $U = \log(x^3 + y^3 + z^3 - 3xyz)$ then $U_x + U_y + U_z =$
a) $-3 / (x + y + z)$ b) $3 / (x + y + z)$ c) $9 / (xy + yz + zx)$ d) $-9 / (xy + yz + zx)$

100) If $U = \log(x^3 + y^3 + z^3 - 3xyz)$ then $(\partial/\partial x + \partial/\partial y + \partial/\partial z)^2 u =$

a) $3 / (x + y + z)^2$ b) $-3/ (x + y + z)^2$ c) $0 / (x + y + z)^2$ d) $-9 / (x + y + z)^2$

101) If $U = e^{ax + by} f(ax - by)$ then $b.u_x + a.u_y$

a) 0 b) 2abU c) $2(a + b) U$ d) $2(a - b) U$

MAXIMA AND MINIMA

Def: A function f(x) has a maximum at x = a if there exists some interval $(a - \delta, a + \delta)$ around 'a' such that $f(a) > f(x)$ for all x in $(a - \delta, a + \delta)$

A function f(x, y) ~~has a~~ minimum at x = a if there exists some interval $(a - \delta, a + \delta)$ around 'a' such that $f(a) < f(x)$ for all values of x in the interval

Extremum: The term used for both for maximum and minimum

A necessary condition for f(a) to be an extreme value of f(x) is $f^1(a) = 0$

The vanishing of $f^1(a) = 0$ is only a necessary but not a sufficient condition for f(a) to be an extreme value of f(x)

Ex: For the function $f(x) = x^3$, f(0) is not an extremum, eventhough $f^1(0) = 0$

Ex: f(0) is a minimum value of $f(x) = |x|$ eventhough $f^1(0)$ does not exist

Stationary Values: A function f(x) is said to be stationary for x = C and f(C) is a stationary value of f(x) if $f^1(C) = 0$

A stationary value may neither be a maximum nor a minimum

Greatest value and least values of a function in the interval [a, b] are f(a) or f(b) or are given by the values of x for which $f^1(x) = 0$

Sufficient conditions for Extrema:
Theorem: f(C) is an extremum of f(x) iff $f^1(x)$ changes sign as x passes through C

Case(i): If $f^1(x)$ changes sign from positive to negative as x passes through C then f(C) is a maximum value of f(x)

Case(ii) : If $f^1(x)$ changes sign from negative to positive as x passes through C then f(C) is a maximum.

Case(iii) : If $f^1(x)$ does not change sign as x passes through C then f(C) is not an extremum.

Theorem: A function f(x) has a maximum at x = a if $f^1(a) = 0$ and $f^{11}(a) < 0$

A function f(x) has a minimum at x = a if $f^1(a) = 0$ and $f^{11}(a) > 0$

102) The function $f(x) = 2x^3 - 3x^2 - 36x + 10$ has a maximum at x =

a) 3 b) 2 c) -3 d) -2

103) The minimum value of $f(x) = 2x^3 - 3x^2 - 36x + 10$ is
a) 0 b) – 13 c) – 17 d) – 71

104) A maximum value of $f(x) = (\log x / x)$ is
a) e b) e^{-1} c) e – 1 d) e + 1

105) The function $f(x) = x^x$ has a minimum at x =
a) e b) e^{-1} c) 0 d) e + 1

106) The minimum value of $f(x) = x \cdot \log x$ is
a) e b) e^{-1} c) – e d) $-e^{-1}$

107) The maximum value of $x \cdot e^{-x}$ is
a) e b) e^{-1} c) 1 d) – e

108) The maximum value of $f(x) = \sin x + \cos 2x$ in the interval $[0, \pi]$ is
a) 2 b) 1.5 c) 5/7 d) 9/8

109) $f(x, y) = x^3 + y^3 - 3xy$ has
a) a maximum at (1, 1) b) a minimum at (1, 1)
c) a saddle point at (1, 1) d) neither maximum nor minimum

110) At (a, a), $f(x, y) = xy + a^3/x + a^3/y$ has
a) a maximum b) a minimum
c) a maximum if a > 0 d) neither maximum nor minimum

111) At $(\sqrt{2}, -\sqrt{2})$, $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ has
a) a minimum b) a maximum
c) a saddle point d) neither maximum nor minimum

112) A rectangular box open at the top is to have a volume 32 C.C. Find the dimensions of the box requiring least material for its construction
a) 4 cm,4 cm,2 cm b) 2 cm,2 cm,8 cm c) 16 cm,1 cm,1cm d) 8 cm,8 cm,½ cm

113) If $f^1(x) = (x + 2)(x - 1)^2(2x - 1)(x - 3)$ then at x = ½; f(x) has
a) a maximum b) a minimum
c) neither maximum nor minimum d) no stationary point

Constrained Maximum or Minimum:
To find maximum or minimum of $U = f(x, y, z)$ where x, y, z are connected by $\phi(x,y,z) = 0$.

Working Rule: a) Write $F(x, y, z) = f(x, y, z) + \lambda\, \phi(x, y, z)$
b) Obtain the equations $F_x = 0$, $F_y = 0$, $F_z = 0$
c) Solve the above equations along with $\phi = 0$ to get stationary point

114) Find the minimum value of $x^2 + y^2 + z^2$ so that $xyz = 8$
a) 8 b) 12 c) 21 d) 27

115) Find the maximum value of $x^2 + y^2 + z^2$ so that $x + y + z = 1$
a) 1 b) ½ c) 1/3 d) ¼

116) Divide 24 into three parts x, y, z so that xy^2z^3 is a maximum
a) 8, 8, 8 b) 4, 8, 12 c) 6, 9, 9 d) 6, 8,10

117) Let $T = 400\, xyz^2$ find maximum value of T so that $x^2 + y^2 + z^2 = 1$

a) 50　b) 100　c) 200　d) 800

118) Find the volume of the greatest rectangular parellelopiped that can be inscribed in the ellipsoid $(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1$

a) $abc / 3\sqrt{3}$　b) $4abc / 3\sqrt{3}$　c) $8abc / 3\sqrt{3}$　d) $2abc / 3\sqrt{3}$

119) What is the value of $\int_0^{2\pi} (x - \pi)^2 . \text{Sin}x\, dx$ ***(GATE '05)***

a) -1　b) 0　c) 1　d) π

KEY

1. d	2. d	3. a	4. b	5. d	6. c	7. c	8. c	9. b	10. d	11. c	12. c
13. b	14. d	15. b	16. c	17. c	18. b	19. a	20. a	21. a	22. b	23. c	24. a
25. a	26. a	27. a	28. a	29. a	30. a	31. c	32. c	33. c	34. a	35. c	36. b
37. b	38. b	39. c	40. a	41. b	42. a	43. a	44. b	45. b	46. a	47. b	48. c
49. b	50. a	51. a	52. a	53. a	54. b	55. b	56. a	57. d	58. a	59. c	60. c
61. c	62. a	63. c	64. c	65. c	66. c	67. a	68. a	69. a	70. d	71. d	72. d
73. b	74. d	75. a	76. c	77. c	78. c	79. d	80. b, c	81. a	82. b	83. a	84. d
85. b	86. a	87. a	88. a	89. a	90. b	91. c	92. d	93. b	94. c	95. a	96. a
97. a	98. b	99. b	100. d	101. b	102. d	103. d	104. b	105. b	106. d	107. b	
108. d	109. b	110. b	111. a	112. a	113. a	114. b	115. c	116. b	117. a	118. c	
119. b.											

PREVIOUS GATE QUESTIONS - "CALCULUS"

1. The integration of $\int \log x\, dx$ has the value ***(GATE'94)***

a) $(x \log x - 1)$　b) $\log x - x$　c) $x(\log x - 1)$　d) None of the above

2. The volume generated by revolving the area bounded by the parabola $y^2 = 8x$ and the line $x = 2$ about y – axis is ***(GATE'94)***

a) $128\pi / 5$　b) $5 / 128\pi$　c) $127 / 5\pi$　d) None above

3. The function, $y = x^2 + (250/x) = 5$ attains ***(GATE'94)***

a) Maximum　b) Minimum　c) Neither　d) 1

4. The value of ξ in the mean value theorem of $f(b) - f(a) = (b - a) f'(\xi)$ for $f(x) = Ax^2 + Bx + C$ in (a,b) is ***(GATE'94)***

a) b + a　b) b-a　c) (b + a)/2　d) (b – a)/2

5. The area bounded by the parabola $2y = x^2$ and the lines $x = y - 4$ is equal to ***(GATE'95)***

a) 6　b) 18　c) ∞　d) none of the above

7. The third term in the Taylor's series expansion of e^x about a would be (GATE'95)
   a) $e^a(x - a)$ b) $e^a/2(x - a)$ c) $e^a/2$ d) $e^a/6(x - a)^3$

8. $\lim_{x\to 0} x \sin(1/x)$ is (GATE'95)
   a) ∞ b) 0 c) 1 d) non – existant

9. The function $f(x) = | x + 1 |$ on the interval $[-2,0]$ is (GATE'95)
   a) continuous and differentiable
   b) continuous on the interval but not differentiable at all points
   c) neither continuous nor differentiable
   d) differentiable but not continuous

10. The function $f(x) = x^3 - 6x^2 + 9x + 25$ has (GATE'95)
   a) a maxima at $x = 1$ and a minima at $x = 3$ b) a maxima at $x = 3$ and a minima at $x = 1$
   c) no maxima, but a minima at $x = 3$ d) a maxima at $x = 1$, but no minima

11. If $f(0) = 2$ and $f(x) = 1 / (5 - x^2)$, the lower and upper bounds of $f(1)$ estimated by the mean value theorem are (GATE'95)
   a) 1.9, 2.2 b) 2.2, 2.25 c) 2.25, 2.5 d) none of the above

12. If a function is continuous at a point its first derivative (GATE'96)
   a) may or may not exist b) exists always c) will not exist d) has a unique value

13. Area bounded by the curve $y = x^2$ and lines $x = 4$ and $y = 0$ is given by, (GATE'97)
   a) 64 b) 64/3 c) 128/3 d) 128/4

14. The curve given by the equation $x^2 + y^2 = 3axy$, is (GATE'97)
   a) symmetrical about x – axis b) symmetrical about y – axis
   c) symmetrical about line $y = x$ d) tangential to $x = y = a/3$

15. e^x is periodic, with a period of (GATE'97)
   a) 2π b) $2i\pi$ c) π d) $i\,\pi$

16. $\lim_{\theta\to 0} \frac{\sin m\theta}{\theta}$, where m is an integer, is one of the following: (GATE'97)
   a) m b) $m\pi$ c) $m\theta$ d) 1

17. If $y=|x|$ for $x<0$ and $y = x$ for $x \geq 0$, then (GATE'97)
   a) dy/dx is discontinuous at $x = 0$ b) y is discontinuous at $x = 0$
   c) y is not defined at $x = 0$ d) Both y and dy/dx are discontinuous at $x = 0$

18. If $\varnothing(x) = \int_0^{x^2} \sqrt{t}\, dt$, then $\frac{d\phi}{dx}$ is (GATE'98)
   a) $2x^2$ b) $\sqrt{x}$ c) 0 d) 1

19. The continuous function f(x, y) is said to have saddle point at (a, b) if (GATE'98)
   a) $f_x(a, b) = f_y(a, b) = 0$; $f^2_{xy} - f_{xx} f_{yy} < 0$ at (a, b)
   b) $f_x(a, b) = 0$; $f_y(a, b) = 0$; $f^2_{xy} - f_{xx} f_{yy} > 0$ at (a, b)
   c) $f_x(a, b) = 0$; $f_y(a, b) = 0$; f_{xx} and $f_{yy} < 0$ at (a, b)
   d) $f_x(a, b) = f_y(a, b) = 0$; $f^2_{xy} - f_{xx} f_{yy} = 0$ at (a, b)

20. The Taylor's series expansion of sinx is *(GATE'98)*
(a) $1 - x^2/2! + x^4/4! + \ldots$ (b) $1 + x^2/4! + x^4/4! + \ldots$
(c) $x + x^3/3! + x^5/5! + \ldots$ (d) $x - x^3/3! + x^5/5! + \ldots$

21. A discontinuous real function can be expressed as *(GATE'98)*
a) Taylor's series and Fourier's series
b) Taylor's series and not by Fourier's series
c) neither Taylor's series nor Fourier's series
d) not by Taylor's series, but by Fourier's series

22. Limit of the function $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+n}}$ is *(GATE'99)*
a) ½ b) 0 c) ∞ d) 1

23. The function $f(x) = e^x$ is *(GATE'99)*
a) Even b) Odd c) Neither even nor odd d) None of the above

24. Value of the function $\lim_{x\to a} (x-a)^{(x-a)}$ is *(GATE'99)*
a) 1 b) 0 c) ∞ d) a

25. Find the maximum and minimum values of the function $f(x) = \sin x + \cos 2x$ over the range $0 < x < 2\pi$. *(GATE'99)*

27. The limit of the function $f(x) = [(1 - a^4) / x^4]$ as $x \to \infty$ is given by *(GATE - 2000)*
a) 1 b) $\exp[-a^4]$ c) ∞ d) zero

28. The maxima and minima of the function $f(x) = 2x^3 - 15x^2 + 36x + 10$ occur, respectively, at *(GATE - 2000)*
a) $x = 3$ & $x = 2$ b) $x = 1$ & $x = 3$ c) $x = 2$ & $x = 3$ d) $x = 3$ & $x = 4$

29. If $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$ *(GATE - 2000)*
$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ is equal to
a) zero b) 1 c) 2 d) $-3(x^2 + y^2 + z^2)^{-5/2}$

30. Consider the following integral $\lim_{a\to\infty} \int_1^a x^{-4}\, dx$ *(GATE - 2000)*
a) diverges b) converges to 1/3 c) converges to $-1/a^3$ d) converges to 0

31. The value of the integral is $I = \int_0^{\pi/4} \cos^2 x\, dx$ *(GATE'01)*
a) $\pi/8 + ¼$ b) $\pi/8 - ¼$ c) $-\pi/8 - ¼$ d) $-\pi/8 + ¼$

32. Which of the following functions is not differentiable in the domain [-1,1] ? *(GATE'02)*
a) $f(x) = x^2$ b) $f(x) = x-1$ c) $f(x) = 2$ d) $f(x) = \text{maximum}(x,-x)$

33. The value of the following definite integral is: *(GATE'02)*

$$\int_{-\pi/2}^{\pi/2} (\mathrm{Sin}2x / 1 + \cos x)\, dx$$

(a) -2 ln2 (b) 2
(c) 0 (d) 1

34. The value of the following improper integral is $\int_0^1 x \ln x$ *(GATE'02)*

a) ¼ b) 0 c) – ¼ d) 1

35. The function $f(x,y) = 2x^2 + 2xy - y^3$ has *(GATE'02)*

a) only one stationary point at (0,0) b) two stationary points at (0,0) and (1/6, -1/3)
c) two stationary points at (0,0) and (1, -1). d) no stationary point

36. $\mathrm{Lt}_{x\to 0} \dfrac{\sin^2 x}{x}$ is equal to *(GATE'03)*

a) 0 b) ∞ c) 1 d) -1

37. The area enclosed between the parabola $y = x^2$ and the straight line $y = x$ is *(GATE'03)*

a) 1/8 b) 1/6 c) 1/3 d) 1/2

38. If $x = a(\theta + \sin\theta)$ and $y = a(1-\cos\theta)$, then (dy/dx) will be equal to *(GATE'04)*

a) sin (θ/2) b) cos (θ/2) c) tan(θ/2) d) cot (θ/2)

40. The value of the function $f(x) = \lim_{x\to 0} \dfrac{x^3 + x^2}{2x^3 - 7x^2}$ is *(GATE'04)*

a) 0 b) – 1 / 7 c) 1 / 7 d) ∞

41. The function $f(x) = 2x^2 - 3x^2 - 36x + 2$ has its maxima at *(GATE'04)*

a) x = -2 only b) x = 0 only c) x = 3 only d) both x = -2 and x = 3

42. $\int_{-a}^{a} (\sin^6 x + \sin^7 x)dx$ is equal to *(GATE'05)*

a) $2\int_0^a \sin^6 x dx$ b) $2\int_0^a \sin^7 x dx$ c) $2\int_0^a (\sin^6 x + \sin^7 x)dx$ d) zero

KEY

1. c 2. d 3. b 4. c 5. b 7. b 8. b 9. b 10. a 11. 12. a

13. b 14. c 15. b 16. a 17. a 18. a 19. a 20. d 21. d 22. d 23. c 24. a

25. 9/8 27. d 28. c 29. a 30. b 31. a 32. d 33. c 34. c 35. b 36. a

37. b 38. c 40. b 41. a 42. a

PREVIOUS GATE QUESTIONS

04. The temperature T(in ^{0}C) at any point (x, y) on a surface is $T = 400\,xy^2$. The highest temperature (in ^{0}C) on the circumference of the circle $x^2 + y^2 = 1$ is

(a) 128 (b) 154 (c) 233 (d) 381

05. A parabolic cable is held betweem two supports at the same level. The horizontal span between the supports is L.
The sag at the mid-span is h. The equation of the parabola is $y = 4h\dfrac{x^2}{L^2}$, where x is the horizontal coordinate and y is the vertical coordinate with the origin at the centre of the cable. The expression for the total length of the cable is **(CE-2010-2M)**

(a) $\int_0^L \sqrt{1+64\dfrac{h^2x^2}{L^4}}\,dx$

(b) $2\int_0^{L/2} \sqrt{1+64\dfrac{h^3x^2}{L^4}}\,dx$

(c) $\int_0^{L/2} \sqrt{1+64\dfrac{h^2x^2}{L^4}}\,dx$

(d) $2\int_0^{L/2} \sqrt{1+64\dfrac{h^2x^2}{L^4}}\,dx$

06. The parabolic arc $y = \sqrt{x}$, $1 \le x \le 2$ is revolved around the x-axis. The volume of the solid of revolution is **(ME-2010-1M)**

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$

(c) $\dfrac{3\pi}{4}$ (d) $\dfrac{3\pi}{2}$

07. $\int_{-a}^{a}\left[\sin^6 x + \sin^7 x\right]dx$ is equal to **(ME-2005-1M)**

(a) $2\int_0^a \sin^6 x\,dx$ (b) $2\int_0^a \sin^7 x\,dx$

(c) $2\int_0^a \left(\sin^6 x + \sin^7 x\right)dx$ (d) zero

08. For the function $f(x) = x^2e^{-x}$, the maximum occurs when x is equal to **(EE-2005-2M)**

(a) 2 (b) 1 (c) 0 (d) −1

09. The value of the integral $\int_{-1}^{1}\dfrac{1}{x^2}dx$ is **(IN-2005-1M)**

(a) 2 (b) does not exists
(c) −2 (d) ∞

10. If $S = \int_1^{\infty} x^{-3}\,dx$ then S has the value **(EE-2005-1M)**

(a) $\dfrac{-1}{3}$ (b) $\dfrac{1}{4}$

(c) $\dfrac{1}{2}$ (d) 1

11. For real x, the maximum value of $\dfrac{e^{\sin x}}{e^{\cos x}}$ is **(IN-2007-2M)**

(a) 1 (b) e (c) $e^{\sqrt{2}}$ (d) ∞

12. Consider the function $f(x) = |x|^3$, where x is real. Then the function f(x) at x = 0 is **(IN-2007-2M)**

(a) continuous but not differentiable
(b) once differentiable but not twice.
(c) twice differentiable but not thrice.
(d) thrice differentiable.

13. The minimum value of function $y = x^2$ in the interval $[1, 5]$ is **(ME-2007-1M)**

(a) 0 (b) 1 (c) 25 (d) undefined

14. $\lim_{x \to 0} \dfrac{e^x - \left(1 + x + \dfrac{x^2}{2}\right)}{x^3} =$ **(ME-2007-2M)**

$e^x - 1 - x$

(a) 0 (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) 1

15. If $y = x + \sqrt{x + \sqrt{x + \sqrt{x + \ldots \infty}}}$ then $y(2) =$ ----- **(ME-2007-2M)**

(a) 4 (or) 1 (b) 4 only
(c) 1 only (d) Undefined

16. What is the value of

$\lim_{x \to \pi/4} \dfrac{\cos x - \sin x}{x - \pi/4} =$ **(PI-2007-1M)**

(a) $\sqrt{2}$ (b) 0
(c) $-\sqrt{2}$ (d) Limit does not exist.

17. For the function $f(x,y) = x^2 - y^2$ defined on R^2; the point (0, 0) is **(PI-2007-2M)**

(a) a local mimimum

(b) neither a local minimum (nor) a local maximum.

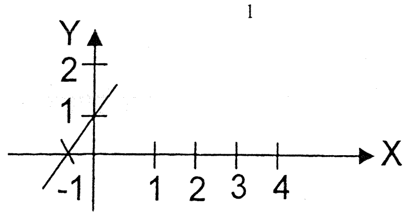
(c) a local maximum

(d) both a local minimum and a local maximum.

18. $\lim_{x \to 0} \dfrac{\sin(\theta/2)}{\theta}$ is **(EC-2007-1M)**

(a) 0.5 (b) 1
(c) 2 (d) not defined

19. The following plot shows a function y which varies linearly with x. The value of the integral $I = \int_1^2 y\,dx$ **(EC-2007-1M)**



(a) 1 (b) 2.5 (c) 4 (d) 5

20. For the function e^{-x}, the linear approximation around $x = 2$ is **(EC-2007-1M)**

(a) $(3 - x)e^{-2}$ (b) $1 - x$

(c) $\left[3 + 2\sqrt{2} - (1 + \sqrt{2})x\right]e^{-2}$

(d) e^{-2}

21. For $|x| << 1$, $\cot h(x)$ can be approximated as **(EC-2007-1M)**

(a) x (b) x^2 (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

22. Consider the function $f(x) = x^2 - x - 2$. The maximum value of f(x) in the closed interval $[-4, 4]$ is **(EC-2007-2M)**

(a) 18 (b) 10
(c) -2.25 (d) indeterminate

23. Consider the function $f(x) = (x^2 - 4)^2$ where x is a real number. Then the function has **(EE-2008-2M)**

(a) Only one minimum
(b) Only two minima
(c) Three minima
(d) Three maxima

24. Given $y = x^2 + 2x + 10$ the value of $\left.\dfrac{dy}{dx}\right|_{x=1}$ is equal to **(IN-2008-1M)**

(a) 0 (b) 4 (c) 12 (d) 13

25. $\lim_{x \to 0} \frac{\sin x}{x}$ is (IN-2008-1M)

(a) indeterminate (b) 0
(c) 1 (d) ∞

26. The expression $e^{-\ln x}$ for $x > 0$ is equal to (IN-2008-2M)

(a) $-x$ (b) x (c) x^{-1} (d) $-x^{-1}$

27. Consider the function $y = x^2 - 6x + 9$. The maximum value of y obtained when x varies over the internal 2 to 5 is (IN-2008-2M)

(a) 1 (b) 3 (c) 4 (d) 9

28. For real values of x, the minimum value of the function $f(x) = e^x + e^{-x}$ is (EC-2008-2M)

(a) 2 (b) 1 (c) 0.5 (d) 0

29. Which of the following function would have only odd powers of x in its Taylor series expansion about the point $x = 0$? (EC-2008-1M)

(a) $\sin(x^3)$ (b) $\sin(x^2)$
(c) $\cos(x^3)$ (d) $\cos(x^2)$

30. In the Taylor series expansion of $e^x + \sin x$ about the point $x = \pi$, the coefficient of $(x - \pi)^2$ is (EC-2008-2M)

(a) e^π (b) $0.5\, e^\pi$
(c) $e^\pi + 1$ (d) $e^\pi - 1$

31. The value of the integral of the function $g(x, y) = 4x^3 + 10y^4$ along the straight line segment from the point (0, 0) to the point (1, 2) in the xy-plane is (EC-2008-2M)

(a) 33 (b) 35 (c) 40 (d) 56

32. In the Taylor series expansion of e^x about $x = 2$, the coefficient of $(x - 2)^4$ is (ME-2008-1M)

(a) $\frac{1}{4!}$ (b) $\frac{2^4}{4!}$
(c) $\frac{e^2}{4!}$ (d) $\frac{e^4}{4!}$

33. The value of $\lim_{x \to 8} \frac{x^{1/3} - 2}{x - 8}$ is (ME-2008-1M)

(a) $\frac{1}{16}$ (b) $\frac{1}{12}$
(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

34. Which of the following integrals is unbounded? (ME-2008-2M)

(a) $\int_0^{\pi/4} \tan x\, dx$ (b) $\int_0^{\infty} \frac{1}{1+x^2} dx$
(c) $\int_0^{\infty} x e^{-x} dx$ (d) $\int_0^1 \frac{1}{1-x} dx$

35. The length of the curve $y = \frac{2}{3}x^{3/2}$ between $x = 0$ & $x = 1$ is (ME-2008-2M)

(a) 0.27 (b) 0.67 (c) 1 (d) 1.22

36. The value of the integral $\int_{-\pi/2}^{\pi/2} (x \cos x)\, dx$ is (PI-2008-1M)

(a) 0 (b) $\pi - 2$ (c) π (d) $\pi + 2$

37. The value of the expression $\lim_{x \to 0} \left[\frac{\sin(x)}{e^x x}\right]$ is (PI-2008-1M)

(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) $\frac{1}{1+e}$

38. The distance between the origin and the point nearest to it on the surface $Z^2 = 1 + xy$ is (ME-2009-2M)

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) 2

39. The area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ is **(ME-2009-2M)**

(a) $\frac{16}{3}$ (b) 8 (c) $\frac{32}{3}$ (d) 16

40. The Taylor series expansion of $\frac{\sin x}{x - \pi}$ at $x = \pi$ is given by **(CE-2009-2M)**

(a) $1 + \frac{(x-\pi)^2}{3!} + \cdots$

(b) $-1 - \frac{(x-\pi)^2}{3!} + \cdots$

(c) $1 - \frac{(x-\pi)^2}{3!} + \cdots$

(d) $-1 + \frac{(x-\pi)^2}{3!} + \cdots$

41. The total derivative of the function 'xy' is **(PI-2009-1M)**

(a) x dy + y dx (b) x dx + y dy
(c) dx + dy (d) dx dy

42. At t = 0, the function $f(t) = \frac{\sin t}{t}$ has **(EE-2010-2M)**

(a) a minimum
(b) a discontinuity
(c) a point of inflection
(d) a maximum

43. The value of the quantity, where $P = \int_0^1 x e^x \, dx$ is **(EE-2010-1M)**

(a) 0 (b) 1 (c) e (d) 1/e

44. If $e^y = x^{1/x}$ then y has a **(EC-2010-2M)**

(a) maximum at x = e
(b) minimum at x = e
(c) masimum at $x = e^{-1}$
(d) minimum at $x = e^{-1}$

$x\frac{dy}{dx} + y$

45. If f(x) = sin |x | then the value of $\frac{df}{dx}$ at $x = -\frac{\pi}{4}$ is **(PI-2010-1M)**

(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $-\frac{1}{\sqrt{2}}$ (d) 1

46. The integral $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-x^2}{2}} dx$ is equal to **(PI-2010-1M)**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) ∞

47. What is the value of $\lim_{n \to \infty} \left(1 - \frac{1}{n}\right)^{2n}$? **(CS-2010-1M)**

(a) 0 (b) e^{-2} (c) $e^{-1/2}$ (d) 1

48. The $\lim_{x \to 0} \frac{\sin\left(\frac{2}{3}x\right)}{x}$ is **(CE-2010-1M)**

(a) $\frac{2}{3}$ (b) 1 (c) $\frac{3}{2}$ (d) ∞

49. Given a function $f(x, y) = 4x^2 + 6y^2 - 8x - 4y + 8$, the optimal values of f(x, y) is **(CE-2010-1M)**

(a) a minimum equal to $\frac{10}{3}$

(b) a maximum equal to $\frac{10}{3}$

(c) a minimum equal to $\frac{8}{3}$

(d) a maximum equal to $\frac{8}{3}$

50. The infinite $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \infty$ Converges to **(ME-2010-1M)**

(a) cos(x) (b) sin(x)
(c) sinh(x) (d) e^x

51. The value of the integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

$\tan^{-1}\infty$

$\frac{\pi}{2} + \frac{\pi}{2}$

(ME-2010-1M)

(a) $-\pi$ (b) $\frac{-\pi}{2}$ (c) $\frac{\pi}{2}$ (d) π

52. The function $y = |2 - 3x|$ **(ME-2010-1M)**

(a) is continuous $\forall x \in R$ and differentiable $\forall x \in R$

(b) is continuous $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = \frac{3}{2}$

(c) is continuous $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = \frac{2}{3}$

(d) is continuous $\forall x \in R$ and except at $x = 3$ and differentiable $\forall x \in R$

53. The integral $\int_{-\infty}^{\infty} \delta\left(t - \frac{\pi}{6}\right) 6\sin(t)\,dt$ evaluates to **(IN-2010-1M)**

(a) 6 (b) 3 (c) 1.5 (d) 0

KEY:

04.	05.d	06. d
07. a	08. a	09. d
10. c	11. c	12. c
13. b	14. b	15. a
16. c	17. b	18. a
19. b	20.	21.
22. c	23. b	24. b
25. c	26. c	27.
28. a	29. a	30. b
31. a	32. d	33. b
34. d	35. d	36. a
37. c	38.	39. a
40. d	41. a	42. d
43. b	44. a	45. c
46. c	47. b	48. a
49. a	50. b	51. d
52. c	53. b	

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