Support Vector machine

Professor Abdellatif El Afia

Motivation:

Soft margin SVC($C - SVC$) (w, b, ξ)

$$C - SVC \begin{cases} Optimize & f(w,\xi) = (f_1(w), f_2(\xi)) \\ s.t & y_i(w^T x_i + b) \geq 1 - \xi_i, i = 1,..,n \\ & \xi_i \geq 0, w \in \mathbb{R}^d, b \in \mathbb{R} \end{cases}$$

Linear Soft ε-band SVR

$$\text{LS} - \varepsilon\text{–band} - SVR: \begin{cases} Optimize & f(w,\xi) = (f_1(w), f_2(\xi^1), f_3(\xi^2)) \\ & y_i - (w^T x_i - b) \leq \varepsilon + \xi_i^1, \quad i = 1, \dots, n \\ s.\text{à} & w^T x_i + b - y_i \leq \varepsilon + \xi_i^2, \quad i = 1, \dots, n \\ & \xi_i^1 \geq 0, \xi_i^2 \geq 0, (w, b) \in \mathbb{R}^d \times \mathbb{R} \end{cases}$$

Motivation:

Soft margin SVC($C - SVC$): Two Objectives Programming Problem

To optimize $f(w, \xi) = \left(f_1(w), f_2(\xi)\right)$ we have to Minimize $f_1(w)$ and $f_2(\xi)$, where

- $f_1(w) = \frac{1}{Margin}$
- $f_2(\xi)$ is a noise

Linear Soft ε-band SVR: Three Objectives Programming Problem

To optimize $f(w, \xi) = \left(f_1(w), f_2(\xi^1), f_3(\xi^2)\right)$ we have to Minimize $f_1(w)$ $f_1(w), f_2(\xi^1)$, and $f_3(\xi^2)$ where

- $f_1(w) = \frac{1}{Margin}$
- $f_2(\xi^1)$, and $f_3(\xi^2)$ are noises

Motivation

It’s Multi Objective Programming Problem

- How to make it easy to solve?
  - it has several variance
- Under what conditions on its to ansure the solution?
  - Convex Programming Problem
  - Convex Quadratic Programming Problem
- How to fix the Big Data problem?
  - Dual programming problem
- How to solve the NonLinearity problem ?
  - kernel function
- What are methods kind used for their solution?
- How to slove them if their constraints are satified with a probability?
- How to slove them if their Labels are Random Variables ?

Motivation: NonLinearity problem

Soft margin SVC($C-SVC$)

$$C-SVC\begin{cases} Optimize & f(w,\xi)=(f_1(w),f_2(\xi)) \\ s.t & y_i(w^T\varphi(x_i)+b)\geq 1-\xi_i, i=1,..,n \\ & \xi_i\geq 0, w\in\mathbb{R}^d, b\in\mathbb{R} \end{cases}$$

Linear Soft ε-band SVR

$$\mathrm{LS}-\varepsilon\text{–band}-SVR:\begin{cases} \quad Optimize \quad f(w,\xi)=(f_1(w),f_2(\xi^1),f_3(\xi^2)) \\ \quad y_i-(w^T\varphi(x_i)-b)\leq\varepsilon+\xi_i^1, \qquad i=1,\dots,n \\ s.\text{à} \quad w^T\varphi(x_i)+b-y_i \quad \leq\varepsilon+\xi_i^2, \qquad i=1,\dots,n \\ \quad \xi_i^1\geq 0, \xi_i^2\geq 0, (w,b)\in\mathbb{R}^d\times\mathbb{R} \end{cases}$$

Motivation: Probabilistic Approach

Soft margin SVC($C - SVC$)

$$C - SVC \begin{cases} Min & f(w,\xi) = (f_1(w), f_2(\xi)) \\ s.t & P(y_i(w^T x_i + b) \geq 1 - \xi_i,) \geq p_i, \qquad i = 1,..,n \\ & \xi_i \geq 0, w \in \mathbb{R}^d, b \in \mathbb{R} \end{cases}$$

Motivation:

Linear Soft ε-band SVR

$$\text{LS} - \varepsilon\text{–band} - SVR: \begin{cases} min \quad f(w,\xi) = (f_1(w), f_2(\xi^1), f_3(\xi^2)) \\ \quad P(y_i - (w^T x_i - b) \leq \varepsilon + \xi_i^1) \geq p_i^1, \qquad i = 1, \dots, n \\ s.\text{à} \quad P(w^T x_i + b - y_i \quad \leq \varepsilon + \xi_i^2) \geq p_i^2, \qquad i = 1, \dots, n \\ \quad \xi_i^1 \geq 0, \xi_i^2 \geq 0, (w,b) \in \mathbb{R}^d \times \mathbb{R} \end{cases}$$

Motivation: Random Variable $Y_i \rightsquigarrow D_i$

Soft margin SVC($C - SVC$)

$$C - SVC \begin{cases} Min & f(w,\xi) = (f_1(w), f_2(\xi)) \\ s.t & Y_i(w^T x_i + b) \geq 1 - \xi_i, i = 1, .., n \\ & \xi_i \geq 0, w \in \mathbb{R}^d, b \in \mathbb{R} \end{cases}$$

Linear Soft ε-band SVR

$$\mathrm{LS} - \varepsilon\text{–band} - SVR: \begin{cases} min & f(w,\xi) = (f_1(w), f_2(\xi^1), f_3(\xi^2)) \\ & \mathrm{Y}_i - (w^T x_i - b) \leq \varepsilon + \xi_i^1, \quad i = 1, \dots, n \\ s.à & w^T x_i + b - \mathrm{Y}_i \leq \varepsilon + \xi_i^2, \quad i = 1, \dots, n \\ & \xi_i^1 \geq 0, \xi_i^2 \geq 0, (w,b) \in \mathbb{R}^d \times \mathbb{R} \end{cases}$$

Plan

- Part I: Support Vector machine(50%)
  - Duality Theory in the Convex Programming Problem
  - Linear Classification
  - Linear Regression
  - Theory of kernel function
  - NonLinear Classification
  - NonLinear Regression
  - Résolution Method
- Part II: Support Vector machine with Random variables (50%)
  - Robust Support Vector Classification
  - Robust Support Vector Regression
  - Probabilistic Constraints Support Vector Classification
  - Least Squares Probabilistic Support Vector Classification
  - Probabilistic Constraints Support Vector Regression