ESCUELA POLITÉCNICA NACIONAL
MÉTODOS NUMÉRICOS

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CONJUNTO DE EJERCICIOS

Frontera natural

```
import sympy as sym
from IPython.display import display
def cubic_spline(xs: list[float], ys: list[float]) -> list[sym.Symbol]:
    points = sorted(zip(xs, ys), key=lambda x: x[0]) #ordenar puntos por x

    xs = [x for x, _ in points]
    ys = [y for _, y in points]
    n = len(points) - 1  #Número de splines
    h = [xs[i + 1] - xs[i] for i in range(n)] #distancias entre xs contiguas

    alpha = [0] * (n + 1)
    for i in range(1, n):
        alpha[i] = 3 / h[i]*(ys[i+1] - ys[i])-3 / h[i-1]*(ys[i] - ys[i-1])
    l = [1]
    u = [0]
    z = [0]
    for i in range(1, n):
        l.append(2 * (xs[i + 1] - xs[i - 1]) - h[i - 1] * u[i - 1])
        u.append(h[i] / l[i])
        z.append((alpha[i] - h[i - 1] * z[i - 1]) / l[i])

    l.append(1)
    z.append(0)
    c = [0] * (n + 1)
    b = [0] * n
    d = [0] * n
    a = [0] * n
    x = sym.Symbol("x")
    splines = []
    for j in range(n - 1, -1, -1):
        c[j] = z[j] - u[j] * c[j + 1]
        b[j] = (ys[j + 1] - ys[j]) / h[j] - h[j] * (c[j + 1] + 2 * c[j]) / 3
        d[j] = (c[j + 1] - c[j]) / (3 * h[j])
        a[j] = ys[j]
        S = a[j] + b[j]*(x - xs[j]) + c[j]*(x - xs[j])**2 + d[j]*(x - xs[j])**3
        splines.append(S)
    splines.reverse()
    return splines
```

```python
    points = sorted(zip(xs, ys), key=lambda x: x[0])  #ordenar puntos por x

    xs = [x for x, _ in points]
    ys = [y for _, y in points]
    n = len(points) - 1  #Número de splines
    h = [xs[i + 1] - xs[i] for i in range(n)] #distancias entre xs contiguas

    alpha = [0] * (n + 1)
    for i in range(1, n):
        alpha[i] = 3 / h[i]*(ys[i+1]-ys[i])-3 / h[i-1]*(ys[i]-ys[i-1])
    l = [1]
    u = [0]
    z = [0]
    for i in range(1, n):
        l += [2 * (xs[i + 1] - xs[i - 1]) - h[i - 1] * u[i - 1]]
        u += [h[i] / l[i]]
        z += [(alpha[i] - h[i - 1] * z[i - 1]) / l[i]]
    l.append(1)
    z.append(0)
    c = [0] * (n + 1)
    x = sym.Symbol("x")
    splines = []
    for j in range(n - 1, -1, -1):
        c[j] = z[j] - u[j] * c[j + 1]
        b = (ys[j + 1] - ys[j]) / h[j] - h[j] * (c[j + 1] + 2 * c[j]) / 3
        d = (c[j + 1] - c[j]) / (3 * h[j])
        a = ys[j]
        print(j, a, b, c[j], d)
        S = a + b*(x - xs[j]) + c[j]*(x - xs[j])**2 + d*(x - xs[j])**3
        splines.append(S)
    splines.reverse()
    return splines
```

Frontera condicionada

```python
import sympy as sym

def cubic_spline_clamped(xs: list[float], ys: list[float],
                         d0: float, dn: float)->list[sym.Symbol]:
    points = sorted(zip(xs, ys), key=lambda x: x[0]) #ordenar puntos por x
    xs = [x for x, _ in points]
    ys = [y for _, y in points]
    n = len(points) - 1 #Número de splines
    h = [xs[i + 1] - xs[i] for i in range(n)] #distancias entre xs contiguas

    alpha = [0] * (n + 1)
    alpha[0] = 3 / h[0] * (ys[1] - ys[0]) - 3 * d0
    alpha[-1] = 3 * dn - 3 / h[n - 1] * (ys[n] - ys[n - 1])

    for i in range(1, n):
        alpha[i] = 3 / h[i]*(ys[i+1]-ys[i])-3 / h[i-1]*(ys[i]-ys[i-1])
```

```python
    l = [2 * h[0]]
    u = [0.5]
    z = [alpha[0] / l[0]]

    for i in range(1, n):
        l.append(2 * (xs[i + 1] - xs[i - 1]) - h[i - 1] * u[i - 1])
        u.append(h[i] / l[i])
        z.append((alpha[i] - h[i - 1] * z[i - 1]) / l[i])

    l.append(h[n - 1] * (2 - u[n - 1]))
    z.append((alpha[n] - h[n - 1] * z[n - 1]) / l[n])
    c = [0] * (n + 1)
    b = [0] * n
    d = [0] * n
    a = [0] * n
    x = sym.Symbol("x")
    splines = []
    for j in range(n - 1, -1, -1):
        c[j] = z[j] - u[j] * c[j + 1]
        b[j] = (ys[j + 1] - ys[j]) / h[j] - h[j]*(c[j + 1] + 2 *c[j]) / 3
        d[j] = (c[j + 1] - c[j]) / (3*h[j])
        a[j] = ys[j]
        S = a[j] + b[j]*(x-xs[j]) + c[j]*(x - xs[j])**2 + d[j]*(x - xs[j])**3
        splines.append(S)
    splines.reverse()
    return splines
```

---

1. Dados los puntos $(0,1)$, $(1,5)$, $(2,3)$, determine el spline cúbico.

```python
xs = [0, 1, 2]
ys = [1, 5, 3]

splines = cubic_spline(xs=xs, ys=ys)
_ = [display(s) for s in splines]
print("______")
_ = [display(s.expand()) for s in splines]
```

$$-1.5x^3 + 5.5x + 1$$

$$1.0x + 1.5(x-1)^3 - 4.5(x-1)^2 + 4.0$$

______

$$-1.5x^3 + 5.5x + 1$$

$$1.5x^3 - 9.0x^2 + 14.5x - 2.0$$

2. Dados los puntos $(-1,1)$, $(1,3)$, determine el spline cúbico sabiendo que $f'(x_0) = 1$, $f'(x_n) = 2$.

```python
xs = [-1, 1]
ys = [1, 3]
d0 = 1
dn = 2
```

```
splines = cubic_spline_clamped(xs=xs, ys=ys, d0=d0, dn=dn)
_ = [display(s) for s in splines]
print("______")
_ = [display(s.expand()) for s in splines]
```

$1.0x + 2.0$

______

$1.0x + 2.0$

3. Usando la función anterior, encuentre el spline cúbico para: $xs = [1, 2, 3]$, $ys = [2, 3, 5]$.

```
xs = [1, 2, 3]
ys = [2, 3, 5]

splines = cubic_spline(xs=xs, ys=ys)
_ = [display(s) for s in splines]
print("______")
_ = [display(s.expand()) for s in splines]
```

$0.75x + 0.25(x - 1)^3 + 1.25$

$1.5x - 0.25(x - 2)^3 + 0.75(x - 2)^2$

______

$0.25x^3 - 0.75x^2 + 1.5x + 1.0$

$-0.25x^3 + 2.25x^2 - 4.5x + 5.0$

4. Usando la función anterior, encuentre el spline cúbico para: $xs = [0, 1, 2, 3]$, $ys = [-1, 1, 5, 2]$.

```
xs = [0, 1, 2, 3]
ys = [-1 ,1, 5, 2]

splines = cubic_spline(xs=xs, ys=ys)
_ = [display(s) for s in splines]
print("______")
_ = [display(s.expand()) for s in splines]
```

$1.0x^3 + 1.0x - 1$

$4.0x - 3.0(x - 1)^3 + 3.0(x - 1)^2 - 3.0$

$1.0x + 2.0(x - 2)^3 - 6.0(x - 2)^2 + 3.0$

______

$1.0x^3 + 1.0x - 1$

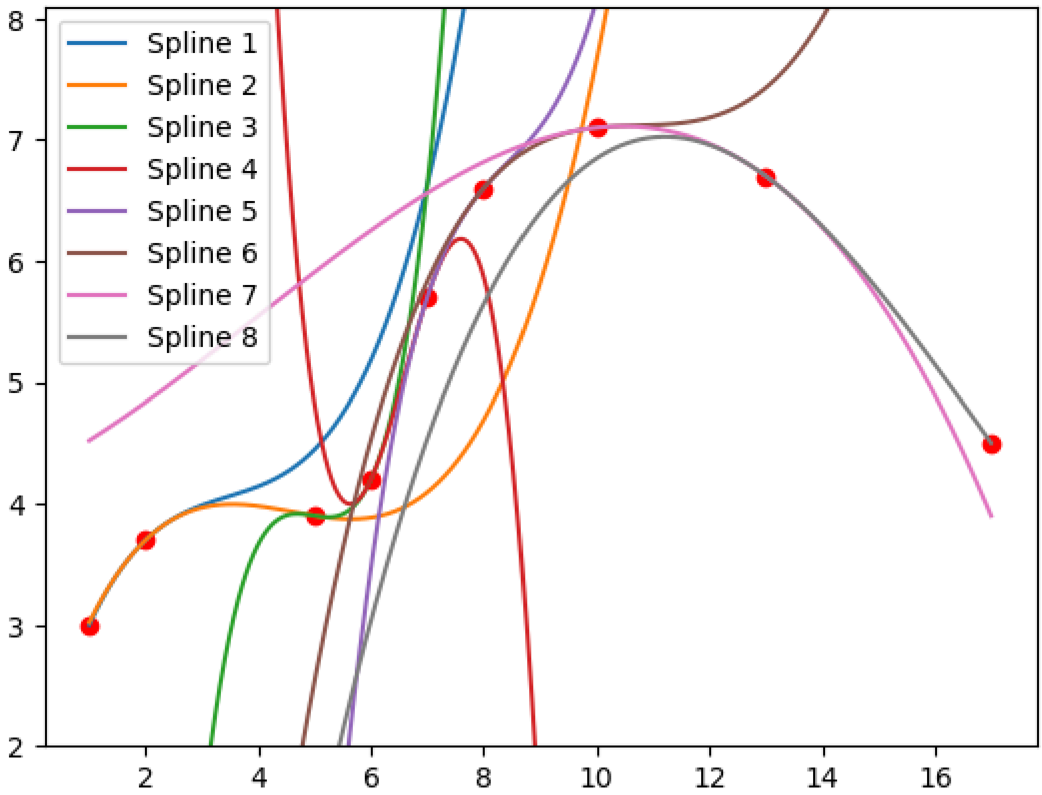
$-3.0x^3 + 12.0x^2 - 11.0x + 3.0$

$$2.0x^3 - 18.0x^2 + 49.0x - 37.0$$

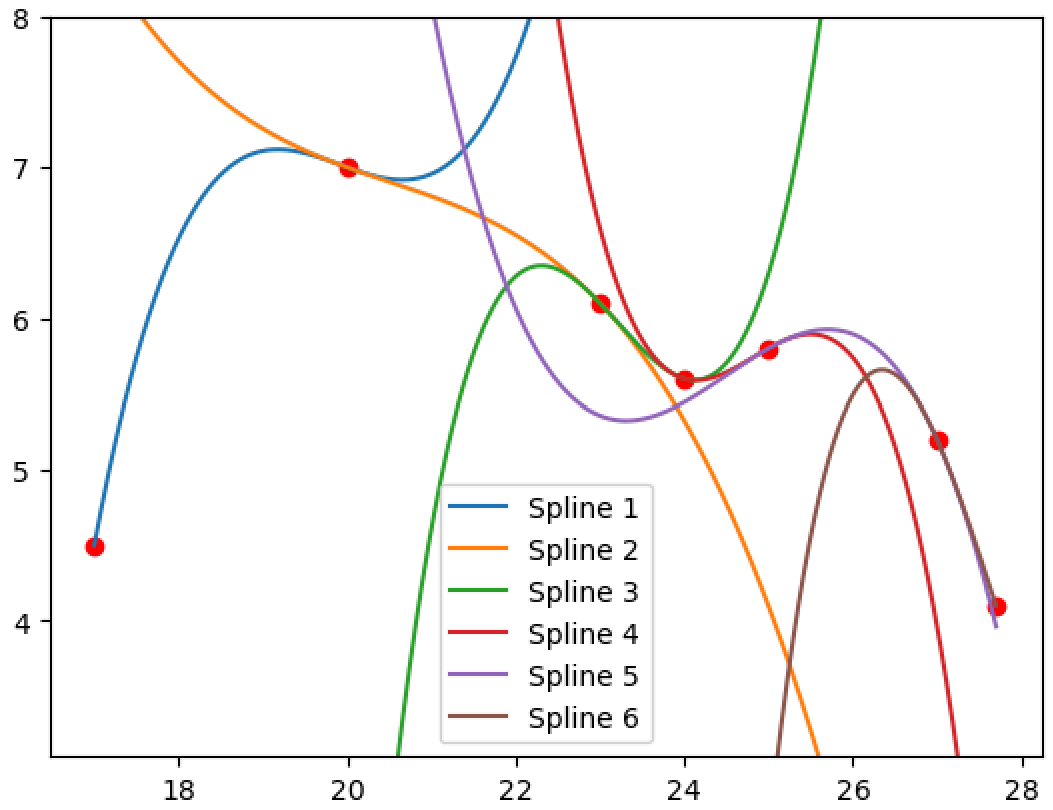
5. Use la función cubic_spline_clamped, provista en el enlace de Github, para graficar los datos de la siguiente tabla.

Curva 1				Curva 2				Curva 3			
i	x_i	$f(x_i)$	$f'(x_i)$	i	x_i	$f(x_i)$	$f'(x_i)$	i	x_i	$f(x_i)$	$f'(x_i)$
0	1	3.0	1.0	0	17	4.5	3.0	0	27.7	4.1	0.33
1	2	3.7		1	20	7.0		1	28	4.3	
2	5	3.9		2	23	6.1		2	29	4.1	
3	6	4.2		3	24	5.6		3	30	3.0	−1.5
4	7	5.7		4	25	5.8					
5	8	6.6		5	27	5.2					
6	10	7.1		6	27.7	4.1	−4.0				
7	13	6.7									
8	17	4.5	−0.67								

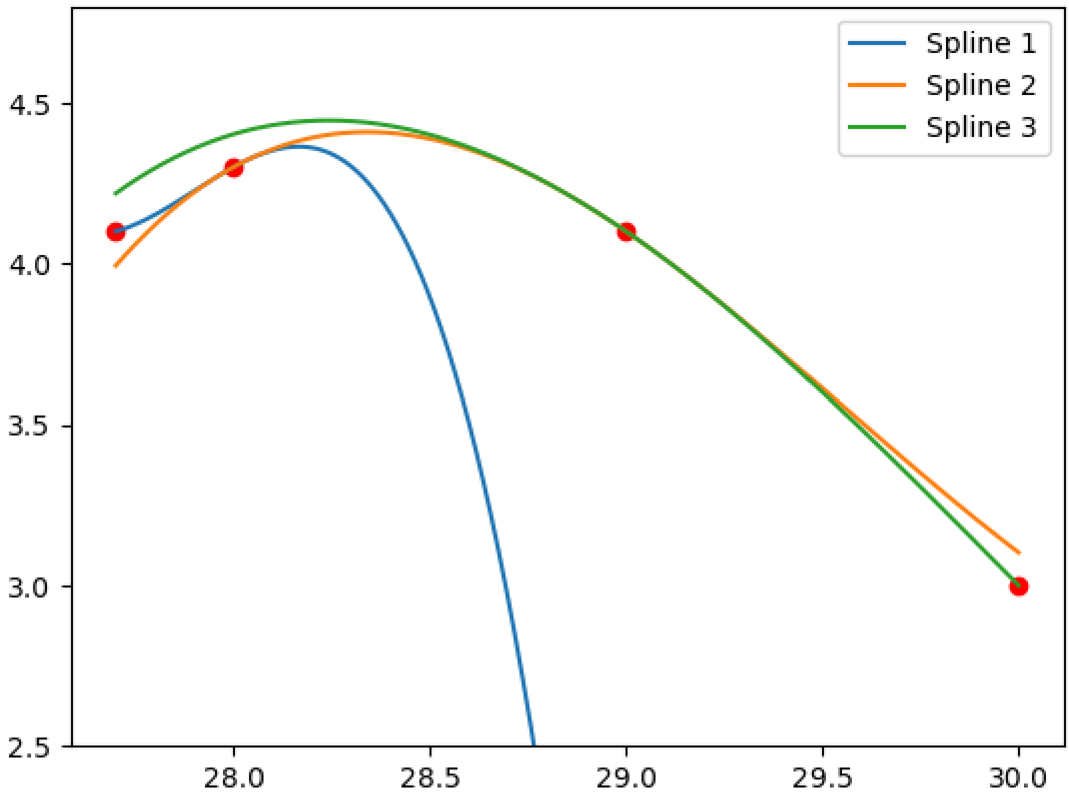
```python
# Curva 1
import matplotlib.pyplot as plt
import numpy as np
import sympy as sym

# Definir los puntos a interpolar y las derivadas en el primer y último punto
xs = [1, 2, 5, 6, 7, 8, 10, 13, 17]
ys = [3, 3.7, 3.9, 4.2, 5.7, 6.6, 7.1, 6.7, 4.5]
d0 = 1
dn = -0.67

# Definir x como una variable simbólica
x = sym.Symbol('x')

# Llamar a la función cubic_spline_clamped
splines = cubic_spline_clamped(xs, ys, d0, dn)

# Crear una gráfica para cada spline
x_vals = np.linspace(min(xs), max(xs), 1000)
for i, spline in enumerate(splines):
    y_vals = [spline.subs(x, val) for val in x_vals]
    plt.plot(x_vals, y_vals, label=f'Spline {i+1}')

# Mostrar la gráfica
plt.scatter(xs, ys, color='red')
plt.ylim(min(ys)-1, max(ys)+1)
plt.legend()
plt.show()
```




```python
# Curva 2
import matplotlib.pyplot as plt
import numpy as np
import sympy as sym

# Definir los puntos a interpolar y las derivadas en el primer y último punto
xs = [17, 20, 23, 24, 25, 27, 27.7]
ys = [4.5, 7, 6.1, 5.6, 5.8, 5.2, 4.1]
d0 = 3
dn = -4

# Definir x como una variable simbólica
x = sym.Symbol('x')

# Llamar a la función cubic_spline_clamped
splines = cubic_spline_clamped(xs, ys, d0, dn)

# Crear una gráfica para cada spline
x_vals = np.linspace(min(xs), max(xs), 1000)
for i, spline in enumerate(splines):
    y_vals = [spline.subs(x, val) for val in x_vals]
    plt.plot(x_vals, y_vals, label=f'Spline {i+1}')

# Mostrar la gráfica
plt.scatter(xs, ys, color='red')
plt.ylim(min(ys)-1, max(ys)+1)
plt.legend()
plt.show()
```




```python
# Curva 3
import matplotlib.pyplot as plt
import numpy as np
import sympy as sym

# Definir los puntos a interpolar y las derivadas en el primer y último punto
xs = [27.7, 28, 29, 30]
ys = [4.1, 4.3, 4.1, 3]
d0 = 0.33
dn = -1.5

# Definir x como una variable simbólica
x = sym.Symbol('x')

# Llamar a la función cubic_spline_clamped
splines = cubic_spline_clamped(xs, ys, d0, dn)

# Crear una gráfica para cada spline
x_vals = np.linspace(min(xs), max(xs), 1000)
for i, spline in enumerate(splines):
    y_vals = [spline.subs(x, val) for val in x_vals]
    plt.plot(x_vals, y_vals, label=f'Spline {i+1}')

# Mostrar la gráfica
plt.scatter(xs, ys, color='red')
plt.ylim(min(ys)-0.5, max(ys)+0.5)
plt.legend()
plt.show()
```



Spline 1
Spline 2
Spline 3
4.5
4.0
3.5
3.0
2.5
28.0
28.5
29.0
29.5
30.0