

ESCUELA POLITÉCNICA NACIONAL

MÉTODOS NUMÉRICOS



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GR1CC

Tarea 10 - Descomposición LU

```
%load_ext autoreload
import numpy as np
from src import multiplicar_matrices, descomposicion_LU, resolver_LU
from src import eliminacion_gaussiana_L, eliminacion_gaussiana_U, determinante, inversa
```

Conjunto de Ejercicios

1. Realice las siguientes multiplicaciones matriz-matriz:

a.
$$\begin{bmatrix} 2 & -3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 2 & 0 \end{bmatrix}$$

```
%autoreload 2
A = np.array([[2,-3],[3,-1]])
B = np.array([[1,5],[2,0]])
C = multiplicar_matrices(A,B)
print("El resultado de la multiplicación es: \n",C)
```

El resultado de la multiplicación es:

```
[[ -4 10]
 [ 1 15]]
```

b.
$$\begin{bmatrix} 2 & -3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 5 & -4 \\ -3 & 2 & 0 \end{bmatrix}$$

```
%autoreload 2
A = np.array([[2,-3],[3,-1]])
B = np.array([[1,5,-4],[-3,2,0]])
C = multiplicar_matrices(A,B)
print("El resultado de la multiplicación es: \n",C)
```

El resultado de la multiplicación es:

```
[[ 11  4 -8]
 [ 6 13 -12]]
```

c.

$$\begin{bmatrix} 2 & -3 & 1 \\ 4 & 3 & 0 \\ 5 & 2 & -4 \end{bmatrix} \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & -1 \\ 2 & 3 & -2 \end{bmatrix}$$

```
%autoreload 2
A = np.array([[2,-3,1],[4,3,0],[5,2,-4]])
B = np.array([[0,1,-2],[1,0,-1],[2,3,-2]])
C = multiplicar_matrices(A,B)
print("El resultado de la multiplicación es: \n",C)
```

El resultado de la multiplicación es:

```
[[ -1  5 -3]
 [ 3  4 -11]
 [-6 -7 -4]]
```

d.
$$\begin{bmatrix} 2 & 1 & 2 \\ -2 & 3 & 0 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -4 & 1 \\ 0 & 2 \end{bmatrix}$$

```
%autoreload 2
A = np.array([[2,1,2],[-2,3,0],[2,-1,3]])
B = np.array([[1,-2],[-4,1],[0,2]])
C = multiplicar_matrices(A,B)
print("El resultado de la multiplicación es: \n",C)
```

El resultado de la multiplicación es:

```
[[ -2  1]
 [-14  7]
 [  6  1]]
```

2. Determine cuáles de las siguientes matrices son no singulares y calcule la inversa de esas matrices:

$$\begin{bmatrix} 4 & 2 & 6 \\ 3 & 0 & 7 \\ -2 & -1 & -3 \end{bmatrix}$$

```
%autoreload 2
A = np.array([[4,2,6],[3,0,7],[-2,-1,-3]])
det = determinante(A)
if(det==0):
    print("El determinante es:",det, "\nPor tanto, la matriz es singular y no posee inversa")
else:
    inv = inversa(A)
    print("El determinante es:",det, "\nSu matriz inversa es la siguiente:", inv)
```

El determinante es: 0

Por tanto, la matriz es singular y no posee inversa

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

```
%autoreload 2
A = np.array([[1,2,0],[2,1,-1],[3,1,1]])
det = determinante(A)
if(det==0):
    print("El determinante es:",det, "\nPor tanto, la matriz es singular y no posee inversa")
else:
    inv = inversa(A)
    print("El determinante es:",det, "\nSu matriz inversa es la siguiente:\n", inv)
```

El determinante es: -7.999999999999999

Su matriz inversa es la siguiente:

```
[[ -0.25  0.25  0.25 ]
 [ 0.625 -0.125 -0.125]
 [ 0.125 -0.625  0.375]]
```

c.

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 2 & -4 & -2 \\ 2 & 1 & 1 & 5 \\ -1 & 0 & -2 & -4 \end{bmatrix}$$

```
%autoreload 2
A = np.array([[1,1,-1,1],[1,2,-4,-2],[2,1,1,5],[-1,0,-2,-4]])
det = determinante(A)
if(det==0):
    print("El determinante es:",det, "\nPor tanto, la matriz es singular y no posee inversa")
else:
    inv = inversa(A)
    print("El determinante es:",det, "\nSu matriz inversa es la siguiente:", inv)
```

El determinante es: 0

Por tanto, la matriz es singular y no posee inversa

d.

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 6 & 7 & 0 & 0 \\ 9 & 11 & 1 & 0 \\ 5 & 4 & 1 & 1 \end{bmatrix}$$

```
%autoreload 2
A = np.array([[4,0,0,0],[6,7,0,0],[9,11,1,0],[5,4,1,1]])
det = determinante(A)
if(det==0):
    print("El determinante es:",det, "\nPor tanto, la matriz es singular y no posee inversa")
else:
    inv = inversa(A)
    print("El determinante es:",det, "\nSu matriz inversa es la siguiente:\n", inv)
```

El determinante es: 27.999999999999993

Su matriz inversa es la siguiente:

```

[[ 0.25      0.      0.      0.      ]
 [-0.21428571 0.14285714 0.      0.      ]
 [ 0.10714286 -1.57142857 1.      0.      ]
 [-0.5        1.      -1.      1.      ]]

```

3. Resuelva los sistemas lineales 4 x 4 que tienen la misma matriz de coeficientes:

$$\begin{aligned}
 x_1 - x_2 + 2x_3 - x_4 &= 6, & x_1 - x_2 + 2x_3 - x_4 &= 1, \\
 x_1 - x_3 + x_4 &= 4, & x_1 - x_3 + x_4 &= 1, \\
 2x_1 + x_2 + 3x_3 - 4x_4 &= -2, & 2x_1 + x_2 + 3x_3 - 4x_4 &= 2, \\
 -x_2 + x_3 - x_4 &= 5, & -x_2 + x_3 - x_4 &= -1,
 \end{aligned}$$

Ya que ambos sistemas poseen la misma matriz de coeficientes

$$A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 1 & 0 & -1 & 1 \\ 2 & 1 & 3 & -4 \\ 0 & -1 & 1 & -1 \end{bmatrix}$$

primero realizaré la descomposición LU para usarla posteriormente para resolver cada sistema.

```

%autoreload 2
A = [[1,-1,2,-1],[1,0,-1,1],[2,1,3,-4],[0,-1,1,-1]]
L,U = descomposicion_LU(A)
print("Matriz L:\n",L)
print("Matriz U:\n",U)

```

Matriz L:

```

[[ 1.  0.  0.  0. ]
 [ 1.  1.  0.  0. ]
 [ 2.  3.  1.  0. ]
 [ 0. -1. -0.25 1. ]]

```

Matriz U:

```

[[ 1. -1.  2. -1.]
 [ 0.  1. -3.  2.]
 [ 0.  0.  8. -8.]
 [ 0.  0.  0. -1.]]

```

```

%autoreload 2
b_1 = [6,4,-2,5]
sol_1 = resolver_LU(L,U,b_1)

```

Calculando y

```

y
[ 6. -2. -8.  1.]
Verificación Ly=b:
[ 6.  4. -2.  5.]
Calculando x
x
[ 3. -6. -2. -1.]
Verificación Ux=y:
[ 6. -2. -8.  1.]

```

```

%autoreload 2
b_2 = [1,1,2,-1]
sol_2 = resolver_LU(L,U,b_2)

```

Calculando y

```

y
[ 1.  0.  0. -1.]
Verificación Ly=b:
[ 1.  1.  2. -1.]
Calculando x
x
[1. 1. 1. 1.]
Verificación Ux=y:
[ 1.  0.  0. -1.]

```

Por tanto, la soluciones obtenidas son: $sol_1 = [3, -6, -2, -1]$ y $sol_2 = [1, 1, 1, 1]$

4. Encuentre los valores de A que hacen que la siguiente matriz sea singular

$$A = \begin{bmatrix} 1 & -1 & \alpha \\ 2 & 2 & 1 \\ 0 & \alpha & -\frac{3}{2} \end{bmatrix}$$

Determinante de la matriz A

$$\begin{vmatrix} 1 & -1 & a \\ 2 & 2 & 1 \\ 0 & a & -\frac{3}{2} \end{vmatrix} = 2a^2 - a - 6$$

image.png

Para que cumpla

$$2a^2 - a - 6 = 0$$

Solución

$$a = 2, a = -\frac{3}{2}$$

image.png

5. Resuelva los siguientes sistemas lineales:

a.
$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

```
%autoreload 2
L = np.array([[1,0,0],[2,1,0],[-1,0,1]])
U = np.array([[2,3,-1],[0,-2,1],[0,0,3]])
b = [2,-1,1]
resolver_LU(L,U,b)
```

Calculando y
y
[2. -5. 3.]
Verificación Ly=b:
[2. -1. 1.]
Calculando x
x
[-3. 3. 1.]
Verificación Ux=y:
[2. -5. 3.]

Por tanto, la solución es:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$$

b.
$$\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

```
%autoreload 2
L = np.array([[2,0,0],[-1,1,0],[3,2,-1]])
U = np.array([[1,1,1],[0,1,2],[0,0,1]])
b = [-1,3,0]
resolver_LU(L,U,b)
```

Calculando y
y
[-0.5 2.5 3.5]
Verificación Ly=b:
[-1. 3. 0.]
Calculando x
x
[0.5 -4.5 3.5]
Verificación Ux=y:
[-0.5 2.5 3.5]

Por tanto, la solución es:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -4.5 \\ 3.5 \end{bmatrix}$$

6. Factorice las siguientes matrices en la descomposición LU mediante el algoritmo de factorización LU con $l_{ii} = 1$ para todas las i.

a.

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix}$$

```
%autoreload 2
A = [[2,-1,1],[3,3,9],[3,3,5]]
L,U = descomposicion_LU(A)
print("Matriz L:\n",L)
print("Matriz U:\n",U)
```

Matriz L:

```
[[1.  0.  0. ]
 [1.5 1.  0. ]
 [1.5 1.  1. ]]
Matriz U:
```

$$\begin{bmatrix} 1.012 & -2.132 & 3.104 \\ -2.132 & 4.096 & -7.013 \\ 3.104 & -7.013 & 0.014 \end{bmatrix}$$

b.

```
%autoreload 2
B = [[1.012,-2.132,3.104],[-2.132,4.096,-7.013],[3.104,-7.013,0.014]]
L,U = descomposicion_LU(B)
print("Matriz L:\n",L)
print("Matriz U:\n",U)
```

Matriz L:

```
[[ 1.          0.          0.          ]
 [-2.10671937  1.          0.          ]
 [ 3.06719368  1.19775553  1.          ]]
Matriz U:
```

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 1.5 & 0 & 0 \\ 0 & -3 & 0.5 & 0 \\ 2 & -2 & 1 & 1 \end{bmatrix}$$

c.

```
%autoreload 2
C = [[2,0,0,0],[1,1.5,0,0],[0,-3,0.5,0],[2,-2,1,1]]
L,U = descomposicion_LU(C)
print("Matriz L:\n",L)
print("Matriz U:\n",U)
```

Matriz L:

```
[[ 1.          0.          0.          0.          ]
 [ 0.5         1.          0.          0.          ]
 [ 0.          -2.         1.          0.          ]
 [ 1.          -1.33333333  2.          1.          ]]
Matriz U:
```

$$\begin{bmatrix} 2.1756 & 4.0231 & -2.1732 & 5.1967 \\ -4.0231 & 6.0000 & 0 & 1.1973 \\ -1.0000 & -5.2107 & 1.1111 & 0 \\ 6.0235 & 7.0000 & 0 & -4.1561 \end{bmatrix}$$

d.

```
%autoreload 2
D = [[2.1756,4.0231,-2.1732,5.1967],[-4.0231,6.0000,0,1.1973],
      [-1.0000,-5.2107,1.1111,0],[6.0235,7.0000,0,-4.1561]]
L,U = descomposicion_LU(D)
print("Matriz L:\n",L)
print("Matriz U:\n",U)
```

Matriz L:

```
[[ 1.      0.      0.      0.      ]
 [-1.84919103  1.      0.      0.      ]
 [-0.45964332 -0.25012194  1.      0.      ]
 [ 2.76866152 -0.30794361 -5.35228302  1.      ]]
```

Matriz U:

```
[[ 2.17560000e+00  4.02310000e+00 -2.17320000e+00  5.19670000e+00]
 [ 0.00000000e+00  1.34394804e+01 -4.01866194e+00  1.08069910e+01]
 [ 0.00000000e+00  4.44089210e-16 -8.92952394e-01  5.09169403e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.20361280e+01]]
```

7. Modifique el algoritmo de eliminación gaussiana de tal forma que se pueda utilizar para resolver un sistema lineal usando la descomposición LU y, a continuación, resuelva los siguientes sistemas lineales.

a.

$$\begin{aligned} 2x_1 - x_2 + x_3 &= -1, \\ 3x_1 + 3x_2 + 9x_3 &= 0, \\ 3x_1 + 3x_2 + 5x_3 &= 4 \end{aligned}$$

```
%autoreload 2
A = [[2,-1,1],[3,3,9],[3,3,5]]
b = [1,0,4]
L,U = descomposicion_LU(A)
print("\nMatriz L:\n",L)
print("\nMatriz U:\n",U)
y = eliminacion_gaussiana_L(L,b)
print("\nValor de y:", y,"\n")
x = eliminacion_gaussiana_U(U,y)
print("\nValor de la solución x:", x)
```

Matriz L:

```
[[1.  0.  0. ]
 [1.5 1.  0. ]
 [1.5 1.  1. ]]
```

Matriz U:

```
[[ 2.  -1.  1. ]
 [ 0.  4.5  7.5]
 [ 0.  0. -4. ]]
```

```
[[ 1.  0.  0.  1. ]
 [ 0.  1.  0. -1.5]
 [ 1.5 1.  1.  4. ]]
```

```
[[ 1.  0.  0.  1. ]
 [ 0.  1.  0. -1.5]
 [ 0.  1.  1.  2.5]]
```

```
[[ 1.  0.  0.  1. ]
 [ 0.  1.  0. -1.5]
 [ 0.  0.  1.  4. ]]
```

Valor de y: [1. -1.5 4.]

```
[[ 2.  -1.  1.  1. ]
 [ 0.  4.5  7.5 -1.5]
 [ 0.  0. -4.  4. ]]
```

```
[[ 2.  -1.  1.  1. ]
 [ 0.  4.5  0.  6. ]
 [ 0.  0. -4.  4. ]]
```

```
[[ 2.  -1.  0.  2. ]
 [ 0.  4.5  0.  6. ]
 [ 0.  0. -4.  4. ]]
```

```
[[ 2.      0.      0.      3.33333333]
 [ 0.      4.5      0.      6.      ]
 [ 0.      0.     -4.      4.      ]]
```

Valor de la solución x: [1.6666666 1.3333334 -1.]

b.

$$\begin{aligned}1.012x_1 - 2.132x_2 + 3.104x_3 &= 1.984, \\ -2.132x_1 + 4.096x_2 + -7.013x_3 &= -5.049, \\ 3.104x_1 - 7.013x_2 + 0.014x_3 &= -3.895\end{aligned}$$

```
%autoreload 2
A = [[1.012,-2.132,3.104],[-2.132,4.096,-7.013],[3.104,-7.013,0.014]]
b = [1.984,-5.049,-3.895]
L,U = descomposicion_LU(A)
print("\nMatriz L:\n",L)
print("\nMatriz U:\n",U)
y = eliminacion_gaussiana_L(L,b)
print("\nValor de y:", y,"\n")
x = eliminacion_gaussiana_U(U,y)
print("\nValor de la solución x:", x)
```

Matriz L:

```
[[ 1.      0.      0.      ]
 [-2.10671937  1.      0.      ]
 [ 3.06719368  1.19775553  1.      ]]
```

Matriz U:

```
[[ 1.012    -2.132    3.104    ]
 [ 0.      -0.39552569 -0.47374308]
 [ 0.      0.      -8.93914077]]
```

```
[[ 1.      0.      0.      1.984    ]
 [ 0.      1.      0.      -0.86926877]
 [ 3.06719368  1.19775553  1.      -3.895    ]]
```

```
[[ 1.      0.      0.      1.984    ]
 [ 0.      1.      0.      -0.86926877]
 [ 0.      1.19775553  1.      -9.98031225]]
```

```
[[ 1.      0.      0.      1.984    ]
 [ 0.      1.      0.      -0.86926877]
 [ 0.      0.      1.      -8.93914077]]
```

Valor de y: [1.984 -0.8692688 -8.93914]

```
[[ 1.012    -2.132    3.104    1.98399997]
 [ 0.      -0.39552569 -0.47374308 -0.86926877]
 [ 0.      0.      -8.93914077 -8.93914032]]
```

```
[[ 1.012    -2.132    3.104    1.98399997]
 [ 0.      -0.39552569  0.      -0.39552572]
 [ 0.      0.      -8.93914077 -8.93914032]]
```

```
[[ 1.012    -2.132    0.      -1.11999987]
 [ 0.      -0.39552569  0.      -0.39552572]
 [ 0.      0.      -8.93914077 -8.93914032]]
```

```
[[ 1.012    0.      0.      1.01200026]
 [ 0.      -0.39552569  0.      -0.39552572]
 [ 0.      0.      -8.93914077 -8.93914032]]
```

Valor de la solución x: [1.0000002 1.0000001 0.99999994]

c.

$$\begin{aligned}2x_1 &= 3, \\ x_1 + 1.5x_2 &= 4.5, \\ -3x_2 + 0.5x_3 &= -6.6, \\ 2x_1 - 2x_2 + x_3 + x_4 &= 0.8\end{aligned}$$

```
%autoreload 2
A = [[2,0,0,0],[1,1.5,0,0],[0,-3,0.5,0],[2,-2,1,1]]
b = [3,4.5,-6.6,0.8]
L,U = descomposicion_LU(A)
print("\nMatriz L:\n",L)
print("\nMatriz U:\n",U)
y = eliminacion_gaussiana_L(L,b)
print("\nValor de y:", y,"\n")
x = eliminacion_gaussiana_U(U,y)
print("\nValor de la solución x:", x)
```

Matriz L:

```
[[ 1.      0.      0.      0.      ]
 [ 0.5     1.      0.      0.      ]
 [ 0.      -2.      1.      0.      ]
 [ 1.      -1.33333333 2.      1.      ]]
```

Matriz U:

```
[[2.  0.  0.  0. ]
 [0.  1.5 0.  0. ]
 [0.  0.  0.5 0. ]
 [0.  0.  0.  1. ]]
```

```
[[ 1.      0.      0.      0.      3.      ]
 [ 0.      1.      0.      0.      3.      ]
 [ 0.      -2.      1.      0.      -6.6     ]
 [ 1.      -1.33333333 2.      1.      0.8     ]]
```

```
[[ 1.      0.      0.      0.      3.      ]
 [ 0.      1.      0.      0.      3.      ]
 [ 0.      -2.      1.      0.      -6.6     ]
 [ 1.      -1.33333333 2.      1.      0.8     ]]
```

```
[[ 1.      0.      0.      0.      3.      ]
 [ 0.      1.      0.      0.      3.      ]
 [ 0.      -2.      1.      0.      -6.6     ]
 [ 0.      -1.33333333 2.      1.      -2.2     ]]
```

```
[[ 1.      0.      0.      0.      3.      ]
 [ 0.      1.      0.      0.      3.      ]
 [ 0.      0.      1.      0.      -0.6     ]
 [ 0.      -1.33333333 2.      1.      -2.2     ]]
```

```
[[ 1.  0.  0.  0.  3. ]
 [ 0.  1.  0.  0.  3. ]
 [ 0.  0.  1.  0. -0.6]
 [ 0.  0.  2.  1.  1.8]]]
```

```
[[ 1.  0.  0.  0.  3. ]
 [ 0.  1.  0.  0.  3. ]
 [ 0.  0.  1.  0. -0.6]
 [ 0.  0.  0.  1.  3. ]]
```

Valor de y: [3. 3. -0.6 3.]

```
[[ 2.      0.      0.      0.      3.      ]
 [ 0.      1.5     0.      0.      3.      ]
 [ 0.      0.      0.5     0.      -0.60000002]
 [ 0.      0.      0.      1.      3.      ]]
```

```
[[ 2.      0.      0.      0.      3.      ]
 [ 0.      1.5     0.      0.      3.      ]
 [ 0.      0.      0.5     0.      -0.60000002]
 [ 0.      0.      0.      1.      3.      ]]
```

```
[[ 2.      0.      0.      0.      3.      ]
 [ 0.      1.5     0.      0.      3.      ]
 [ 0.      0.      0.5     0.      -0.60000002]
 [ 0.      0.      0.      1.      3.      ]]
```

```
[[ 2.      0.      0.      0.      3.      ]
 [ 0.      1.5     0.      0.      3.      ]
 [ 0.      0.      0.5     0.      -0.60000002]
 [ 0.      0.      0.      1.      3.      ]]
```

```
[[ 2.      0.      0.      0.      3.      ]
 [ 0.      1.5     0.      0.      3.      ]
 [ 0.      0.      0.5     0.      -0.60000002]
 [ 0.      0.      0.      1.      3.      ]]
```

```
[[ 2.      0.      0.      0.      3.      ]
 [ 0.      1.5     0.      0.      3.      ]
 [ 0.      0.      0.5     0.      -0.60000002]
 [ 0.      0.      0.      1.      3.      ]]
```

```
[[ 2.      0.      0.      0.      3.      ]
 [ 0.      1.5     0.      0.      3.      ]
 [ 0.      0.      0.5     0.      -0.60000002]
```


[0. 0. 0. 1. 3.]]

Valor de la solución x: [1.5 2. -1.2 3.]

d.

$$\begin{aligned} 2.1756x_1 + 4.0231x_2 - 2.1732x_3 + 5.1967x_4 &= 17.102, \\ -4.0231x_1 + 6.0000x_2 + 1.1973x_4 &= -6.1593, \\ -1.0000x_1 - 5.2107x_2 + 1.1111x_3 &= 3.0004, \\ 6.0235x_1 + 7.0000x_2 + -4.1561x_4 &= 0.0000 \end{aligned}$$

```
%autoreload 2
A = [[2.1756,4.0231,-2.1732,5.1967],[-4.0231,6.0000,0,1.1973],[-1,-5.2107,1.1111,0],[6.0235,7.0000,0,-4.
b = [17.102,-6.1593,3.0004,0.0000]
L,U = descomposicion_LU(A)
print("\nMatriz L:\n",L)
print("\nMatriz U:\n",U)
y = eliminacion_gaussiana_L(L,b)
print("\nValor de y:", y,"\n")
x = eliminacion_gaussiana_U(U,y)
print("\nValor de la solución x:", x)
```

Matriz L:

```
[ [ 1.            0.            0.            0.            ]
[ -1.84919103   1.            0.            0.            ]
[ -0.45964332   -0.25012194   1.            0.            ]
[ 2.76866152   -0.30794361   -5.35228302   1.            ]]
```

Matriz U:

```
[ [ 2.17560000e+00   4.02310000e+00   -2.17320000e+00   5.19670000e+00]
[ 0.00000000e+00   1.34394804e+01   -4.01866194e+00   1.08069910e+01]
[ 0.00000000e+00   4.44089210e-16   -8.92952394e-01   5.09169403e+00]
[ 0.00000000e+00   0.00000000e+00   0.00000000e+00   1.20361280e+01]]
```

```
[ [ 1.            0.            0.            0.            17.102        ]
[ 0.            1.            0.            0.            25.46556496]
[ -0.45964332   -0.25012194   1.            0.            3.0004        ]
[ 2.76866152   -0.30794361   -5.35228302   1.            0.            ]]
```

```
[ [ 1.            0.            0.            0.            17.102        ]
[ 0.            1.            0.            0.            25.46556496]
[ 0.            -0.25012194   1.            0.            10.86122       ]
[ 2.76866152   -0.30794361   -5.35228302   1.            0.            ]]
```

```
[ [ 1.            0.            0.            0.            17.102        ]
[ 0.            1.            0.            0.            25.46556496]
[ 0.            -0.25012194   1.            0.            10.86122       ]
[ 0.            -0.30794361   -5.35228302   1.            -47.34964929]]
```

```
[ [ 1.            0.            0.            0.            17.102        ]
[ 0.            1.            0.            0.            25.46556496]
[ 0.            0.            1.            0.            17.23071662]
[ 0.            -0.30794361   -5.35228302   1.            -47.34964929]]
```

```
[ [ 1.            0.            0.            0.            17.102        ]
[ 0.            1.            0.            0.            25.46556496]
[ 0.            0.            1.            0.            17.23071662]
[ 0.            0.            -5.35228302   1.            -39.50769122]]
```

```
[ [ 1.            0.            0.            0.            17.102        ]
[ 0.            1.            0.            0.            25.46556496]
[ 0.            0.            1.            0.            17.23071662]
[ 0.            0.            0.            1.            52.71598078]]
```

Valor de y: [17.102 25.465565 17.230717 52.71598]

```
[ [ 2.17560000e+00   4.02310000e+00   -2.17320000e+00   5.19670000e+00
1.71019993e+01]
[ 0.00000000e+00   1.34394804e+01   -4.01866194e+00   1.08069910e+01
2.54655647e+01]
[ 0.00000000e+00   4.44089210e-16   -8.92952394e-01   5.09169403e+00
1.72307167e+01]
[ 0.00000000e+00   0.00000000e+00   0.00000000e+00   1.20361280e+01
5.27159805e+01]]
```

```
[ [ 2.17560000e+00   4.02310000e+00   -2.17320000e+00   5.19670000e+00
1.71019993e+01]
```

```
[ 0.00000000e+00 1.34394804e+01 -4.01866194e+00 1.08069910e+01
2.54655647e+01]
[ 0.00000000e+00 4.44089210e-16 -8.92952394e-01 0.00000000e+00
-5.06994697e+00]
[ 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.20361280e+01
5.27159805e+01]]

[[ 2.17560000e+00 4.02310000e+00 -2.17320000e+00 5.19670000e+00
1.71019993e+01]
[ 0.00000000e+00 1.34394804e+01 -4.01866194e+00 0.00000000e+00
-2.18670265e+01]
[ 0.00000000e+00 4.44089210e-16 -8.92952394e-01 0.00000000e+00
-5.06994697e+00]
[ 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.20361280e+01
5.27159805e+01]]

[[ 2.17560000e+00 4.02310000e+00 -2.17320000e+00 0.00000000e+00
-5.65857084e+00]
[ 0.00000000e+00 1.34394804e+01 -4.01866194e+00 0.00000000e+00
-2.18670265e+01]
[ 0.00000000e+00 4.44089210e-16 -8.92952394e-01 0.00000000e+00
-5.06994697e+00]
[ 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.20361280e+01
5.27159805e+01]]

[[ 2.17560000e+00 4.02310000e+00 -2.17320000e+00 0.00000000e+00
-5.65857084e+00]
[ 0.00000000e+00 1.34394804e+01 0.00000000e+00 0.00000000e+00
9.49870688e-01]
[ 0.00000000e+00 4.44089210e-16 -8.92952394e-01 0.00000000e+00
-5.06994697e+00]
[ 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.20361280e+01
5.27159805e+01]]

[[ 2.17560000e+00 4.02310000e+00 0.00000000e+00 0.00000000e+00
6.68028266e+00]
[ 0.00000000e+00 1.34394804e+01 0.00000000e+00 0.00000000e+00
9.49870688e-01]
[ 0.00000000e+00 4.44089210e-16 -8.92952394e-01 0.00000000e+00
-5.06994697e+00]
[ 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.20361280e+01
5.27159805e+01]]

[[ 2.17560000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00
6.39593946e+00]
[ 0.00000000e+00 1.34394804e+01 0.00000000e+00 0.00000000e+00
9.49870688e-01]
[ 0.00000000e+00 4.44089210e-16 -8.92952394e-01 0.00000000e+00
-5.06994697e+00]
[ 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.20361280e+01
5.27159805e+01]]
```

Valor de la solución x: [2.9398508 0.07067764 5.677735 4.3798122]

Link del repositorio:

https://github.com/EIAlfa3007/M-todos_Num-ricos.git