P5

(1) $r = \frac{2}{1+\cos\theta}$ desde $\theta = 0$ a $\theta = \frac{\pi}{2}$

R/=

$$L = \int_0^{\pi/2} \sqrt{\frac{4}{1+2\cos\theta+\cos^2\theta} + \left(\frac{2\operatorname{Sen}\theta}{(1+\cos\theta)^2}\right)^2}\, d\theta$$

$$= \int_0^{\pi/2} \sqrt{\frac{4}{(1+\cos\theta)^2} + \frac{4\operatorname{Sen}^2\theta}{(1+\cos\theta)^4}}\, d\theta$$

$$= \int_0^{\pi/2} \sqrt{\frac{4(1+\cos\theta)^4 + 4\operatorname{Sen}^2\theta(1+\cos\theta)^2}{(1+\cos\theta)^6}}\, d\theta$$

$$= \int_0^{\pi/2} \sqrt{\frac{4(1+\cos\theta)^2 + 4\operatorname{Sen}^2\theta}{(1+\cos\theta)^4}}\, d\theta$$

$$= \int_0^{\pi/2} \sqrt{\frac{8(1+\cos\theta)}{(1+\cos\theta)^4}}\, d\theta$$

$$= \int_0^{\pi/2} \frac{2\sqrt{2}}{\sqrt{(1+\cos\theta)^3}}\, d\theta$$

$$= 2\sqrt{2}\int_0^{\pi/2} \frac{1}{\sqrt{(1+\cos\theta)^3}}\,d\theta = 2\sqrt{2}\int_0^{\pi/2} \frac{1}{\sqrt{(2\cos^2\frac{\theta}{2})^3}}\,d\theta$$

identidad de reducción de potencia

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$= 2\sqrt{2}\int_0^{\pi/2} \frac{1}{\sqrt{8\cos^6\frac{\theta}{2}}}\,d\theta$$

$$= 2\sqrt{2}\int_0^{\pi/2} \frac{1}{\sqrt{8}\cos^3\frac{\theta}{2}}\,d\theta$$

$$= \int_0^{\pi/2} \frac{1}{\cos^3\frac{\theta}{2}}\,d\theta = \int_0^{\pi/2} \sec^3\frac{\theta}{2}\,d\theta$$

sea $w = \frac{\theta}{2}\,d\theta$

$dw = \frac{1}{2}$

$$= 2\int_0^{\pi/2} \sec^3 w\,dw = 2\int_0^{\pi/2} \sec w \cdot \sec^2 w\,dw$$

↳ por partes.

$u = \sec^2 w$ $\quad dv = \sec^2 w\, dw$

$du = \sec w \tan w\, dw$ $\quad v = \tan w$

$$uv - \int v\, du$$

$$= 2\left(\sec w \tan w \Big|_0^{\pi/2} - \int_0^{\pi/2} \tan^2 w \sec w\, dw\right)$$

$$= 2\left(\sec w \tan w \Big|_0^{\pi/2} - \int_0^{\pi/2} (\sec^3 w - \sec w)\, dw\right)$$

$$= 2\left(\sec w \tan w \Big|_0^{\pi/2} - \int_0^{\pi/2} \sec^3 w\, dw + (\ln|\sec w + \tan w|)\Big|_0^{\pi/2}\right)$$

Somar $\int_0^{\pi/2} \sec^3 w\, dw$ a ambos lados:

$$2\int_0^{\pi/2} \sec^3 w\, dw = 2\left(\sec w \tan w \Big|_0^{\pi/2} + (\ln|\sec w + \tan w|)\Big|_0^{\pi/2}\right)$$

$$2\int_0^{\pi/2} \sec^3 w\, dw = 2\left((\sec w \tan w + \ln|\sec w + \tan w|)\Big|_0^{\pi/2}\right)$$

$$= \frac{2}{2}(\sec w \tan w + \ln|\sec w + \tan w|)\Big|_0^{\pi/2}$$

$$= 1\left(\sec\frac{\theta}{2} \tan\frac{\theta}{2} + \ln\left|\sec\frac{\theta}{2} + \tan\frac{\theta}{2}\right|\right)\Big|_0^{\pi/2}$$

$$= \sqrt{2} + \ln(\sqrt{2}+1)$$