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Examen - Mecanică teoretică

I 2. $div(\vec{A} \times \vec{r}) = \vec{r}\, rot\, \vec{A}^{\nearrow 0} - \vec{A}\, rot\, \vec{r}^{\nearrow 0} = 0$

3. $\vec{M} = (5r \sin\varphi)\vec{k}$

$$rot\, \vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 0 & 0 & 5r\sin\varphi \end{vmatrix} = \frac{\partial}{\partial \varphi}(5r\sin\varphi)\vec{i} + \frac{\partial}{\partial r}(5r\sin\varphi)\vec{j}$$

$$rot\, \vec{M} = (5r\cos\varphi)\vec{i} + (5\sin\varphi)\vec{j}$$

$$rot\, \vec{M}\,(2, \pi, 0) = (5 \cdot 2\cos\pi)\vec{i} + (5\sin\pi)\vec{j} = -10\vec{i}$$

II 1. $n = 3$

$V = V(\rho)$

a) $L = T - V = \frac{m\vec{v}^2}{2} - V(\rho) = \frac{m}{2}(\dot{\rho}^2 + \rho^2\dot{\varphi}^2 + \dot{z}^2) - V(\rho)$

Având de-a face cu un câmp conservativ:

$$H = T + V = \frac{m}{2}(\dot{\rho}^2 + \rho^2\dot{\varphi}^2 + \dot{z}^2) + V(\rho)$$

b) $p_\rho = \frac{\partial L}{\partial \dot{\rho}} = m\dot{\rho}$

$p_\varphi = \frac{\partial L}{\partial \dot{\varphi}} = m\rho^2\dot{\varphi}$

$p_z = \frac{\partial L}{\partial \dot{z}} = m\dot{z}$

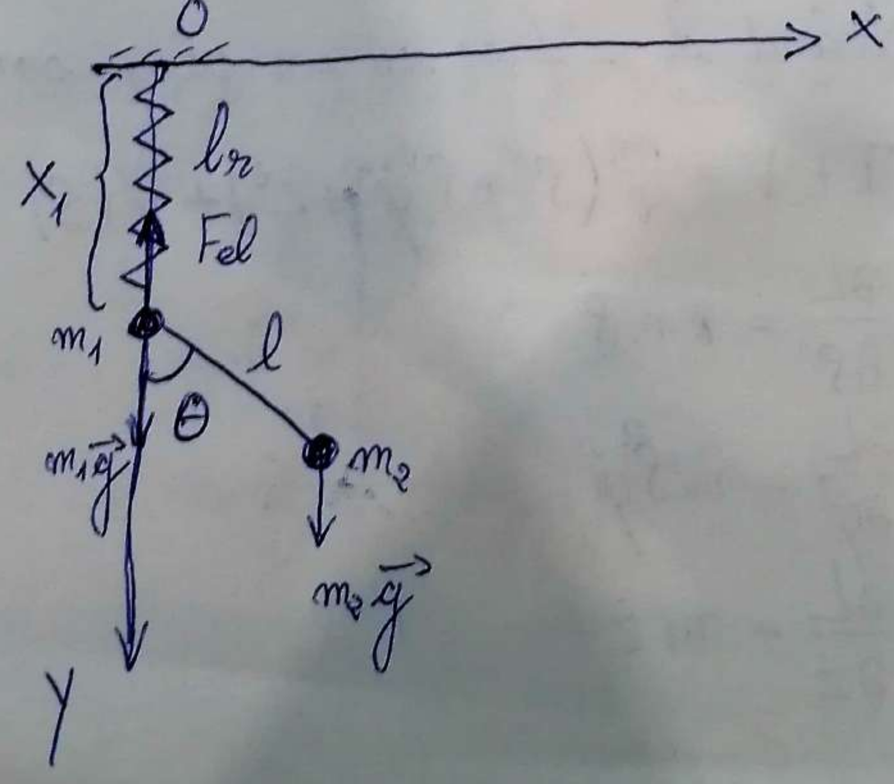
c) $$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\rho}}\right) = \dfrac{\partial L}{\partial \rho} \\ \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\varphi}}\right) = \dfrac{\partial L}{\partial \varphi} \\ \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{z}}\right) = \dfrac{\partial L}{\partial z} \end{cases} \Rightarrow \begin{cases} \dfrac{d}{dt}\left(m\dot{\rho}\right) = -\dfrac{\partial V}{\partial \rho} \\ \dfrac{d}{dt}\left(m\rho^2\dot{\varphi}\right) = 0 \\ \dfrac{d}{dt}\left(m\dot{z}\right) = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} m\ddot{\rho} + \dfrac{\partial V}{\partial \rho} = 0 \Rightarrow \ddot{\rho} = -\dfrac{1}{m}\dfrac{\partial V}{\partial \rho} \\ \rho^2\ddot{\varphi} = 0 \Rightarrow \ddot{\varphi} = 0 \\ \ddot{z} = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \dot{\rho} = \int -\dfrac{1}{m}\dfrac{\partial V}{\partial \rho}\,dt = -\dfrac{1}{m}\dfrac{\partial V}{\partial \rho}t + C_1 \\ \dot{\varphi} = C_2 \\ \dot{z} = C_3 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \rho = -\dfrac{1}{2m}\dfrac{\partial V}{\partial \rho}t^2 + C_1 t + C_4 \\ \varphi = C_2 t + C_5 \\ z = C_3 t + C_6 \end{cases}$$

2.



$$\left.\begin{array}{l} f_1(z_1)=z_1=C_1 \\ f_2(z_2)=z_2=C_2 \\ f_3(x_1)=x_1=C_3 \\ f_4(x_2,y_1,y_2)=(y_2-y_1)^2+\cancel{x_2}^2-l^2=0 \end{array}\right| \Rightarrow$$

$\Rightarrow n=2$ grade de libertate

$$\Downarrow$$

$$\begin{cases} q^1=\cancel{y_1}\equiv y \\ q^2=\theta \end{cases}$$

$$\begin{cases} y_2-y_1=l\cos\theta \\ x_2=l\sin\theta \end{cases} \Rightarrow \begin{cases} \dot{y}_2=\dot{y}-\dot{\theta}l\sin\theta \\ \dot{x}_2=\dot{\theta}l\cos\theta \end{cases}$$

$$L=T-V=\frac{1}{2}m_1\dot{y}^2+\frac{1}{2}m_2(\dot{x}_2^2+\dot{y}_2^2)-\frac{1}{2}k(y-l_r)^2-$$
$$-m_1gy-m_2gy_2$$

Având de-a face cu forțe conservative:

$$H=T+V=\frac{1}{2}m_1\dot{y}^2+\frac{1}{2}m_2\left((\dot{\theta}l\cos\theta)^2+(\dot{y}-\dot{\theta}l\sin\theta)^2\right)+$$
$$+\frac{1}{2}k(y-l_r)^2+m_1gy+m_2g(y+l\cos\theta)$$

$$H=\frac{1}{2}m_1\dot{y}^2+\frac{1}{2}m_2(\dot{y}^2-2\dot{y}\dot{\theta}l\sin\theta+\dot{\theta}^2l^2)+\frac{1}{2}k(y-l_r$$
$$+m_1gy+m_2g(y+l\cos\theta)$$

$$\begin{cases} p_y = \dfrac{\partial L}{\partial \dot y} = m_1\dot y + m_2\dot y - m_2\dot\theta l\sin\theta \\ p_\theta = \dfrac{\partial L}{\partial \dot\theta} = m_2 l^2\dot\theta - m_2\dot y l\sin\theta \end{cases}$$

$$\dot y = \frac{\partial H}{\partial p_y} = \frac{1}{m_1}p_y = \dot y + \frac{m_2}{m_1}\dot y - \frac{m_2}{m_1}\dot\theta l\sin\theta \Rightarrow$$

$$\cancel{H = \frac{1}{2m}(p_y^2 + p_\theta^2) + \frac{1}{2}k(y - l_r)^2 + m_1 g y + m_2 g(y + l\cos\theta)}$$

$$H = \frac{1}{2m_1}p_y^2 + \frac{1}{2m_2}p_\theta^2 + \frac{1}{2}k(y - l_r)^2 + m_1 g y + m_2 g(y + l\cos\theta)$$

$$\Rightarrow \quad \dot y - \dot\theta l\sin\theta = 0$$

$$\dot\theta = \frac{\partial H}{\partial p_\theta} = \frac{1}{m_2}p_\theta = l^2\dot\theta - \dot y l\sin\theta \Rightarrow$$

$$\Rightarrow \begin{cases} (l^2 - 1)\dot\theta - \dot y l\sin\theta = 0 \\ \dot y - \dot\theta l\sin\theta = 0 \;/\cdot l\sin\theta \end{cases} \quad (+)$$

$\cancel{\varepsilon_{ijk}\,\partial_i P_j}$

$$(l^2 - 1)\dot\theta - \dot\theta l^2\sin^2\theta = 0$$

$$\dot\theta\left(l^2 - 1 - l^2\sin^2\theta\right) = 0$$

$k,$

$$\begin{cases} \dot p_y = -\dfrac{\partial H}{\partial y} \\ \dot p_\theta = -\dfrac{\partial H}{\partial \theta} \end{cases} \Rightarrow \begin{cases} \dot p_y = -ky + m_1 g - m_2 g \\ \dot p_\theta = \cancel{\ast} + m_2 g l\sin\theta \end{cases}$$

III 1. $(\vec{K}, K^2) = (K_i \vec{u_i}, K_j K_j) = \vec{u_i}(K_i, K_j K_j) =$

$= \vec{u_j}\left(K_j(K_i, K_j) - (K_i, K_j)K_j\right) =$

$= 2\vec{u_j}\, \cancel{\ }(K_i, K_j) K_j$

$= 2\vec{u_j}\, \varepsilon_{ijk} K_q K_j =$

$= \vec{u_j}\, 2(\vec{K} \times \vec{K})_i = 0$

2. $(\vec{K}, p^n) = (K_i \vec{u_i}, p^n) = \vec{u_i}(\cancel{\ } p_j (K_i, p^{n-1}) - \underbrace{\cancel{(K_i, p_j)}}_{\varepsilon_{ijk} p_k} p^{n-1})$

$(K_i, p_j) = (\varepsilon_{ijk} x_j p_k, p_j) = \varepsilon_{ijk}\left(x_j \cancel{(p_k, p_j)} - (p_j, x_j) p_k\right)$

$\cancel{= \varepsilon_{ijk}\left(x_j \frac{\partial p_k}{\partial x_j} \frac{\partial p_j}{\partial}\right.} = \varepsilon_{ijk} p_k$

$(p_j, x_j) = \delta_{jj} = 1$

$\cancel{(\vec{K}, p^n) = p(K_i, p^{n-1}) = p^2(K_i, p^{n-2}) = \ldots = p^{n-1}(K_i, p_j) = 0}$

$\cancel{div \vec{A} = \frac{1}{2}(\delta_{km}\delta_{in})(\partial_i \partial_m A_m x_i + \partial}$

I.1 $\vec{B} = rot \vec{A}$; $\vec{A} = \frac{1}{2}(\vec{B} \times \vec{r})$

$div \vec{A} = \nabla \cdot \left(\frac{1}{2}(\vec{B} \times \vec{r})\right) = \frac{1}{2} \nabla \cdot \left[(\nabla \times \vec{A}) \times \vec{r}\right]_k =$

$= \frac{1}{2} \partial_i \left[(\nabla \times \vec{A}) \times \vec{r}\right]_i =$

$\varepsilon_{ijk}\varepsilon_{jmn} =$

$= -\varepsilon_{jik}\varepsilon_{jmn} =$

$= -\delta_{im}\delta_{kn} + \delta_{km}\delta_{in}$

$= \frac{1}{2} \partial_i \left(\varepsilon_{ijk} (\nabla \times A)_j x_i\right) =$

$= \frac{1}{2} \varepsilon_{ijk} \partial_i \left(\varepsilon_{jmn} \partial_m A_n x_i\right) =$

$= \frac{1}{2} \varepsilon_{ijk} \varepsilon_{jmn} \left(\partial_i \partial_m A_m x_i + \partial_m A_m \partial_i x_i\right) =$