Moraru V.

SI

1) $\nabla \times (r^n \vec{r}) = \vec{u_i} \frac{\partial}{\partial x_i} \times (r^n \vec{r}) = (\vec{u_i} \times \vec{u_j}) \frac{\partial r^{n+1}}{\partial x_i} =$

$= \varepsilon_{ijk} (n+1) r^n \vec{u_k}$

2) $\text{div} (\vec{r} r) = \vec{u_i} \frac{\partial}{\partial x_i} \cdot (\vec{u_j} r^2) = \delta_{ij} \frac{\partial r^2}{\partial x_i} = 2r$

3)

$\text{rot} (\vec{r} \cdot f(r)) = 0 \Leftrightarrow f(r) \cdot \text{rot}(\vec{r}) + \text{grad}(f(r)) \times \vec{r} = 0 \Leftrightarrow$

$\cancel{f(r) \frac{\partial x_j}{\partial x_i} \cdot (\vec{u_i} \times \vec{u_j}) + \vec{u_j} \frac{\partial f(r_j)}{\partial x_i} \times u}$

$f(r) \frac{\partial x_j}{\partial x_i} (\vec{u_i} \times \vec{u_j}) + \frac{\partial f(r_j)}{\partial x_i} (\vec{u_i} \times \vec{u_j}) = 0 \Leftrightarrow$

$f(r) \varepsilon_{ijk} \vec{u_k} \frac{\partial x_j}{\partial x_i} + \frac{\partial (f(r_j))}{\partial x_i} \varepsilon_{ijk} \vec{u_k} = 0$

S II

Moraru V.

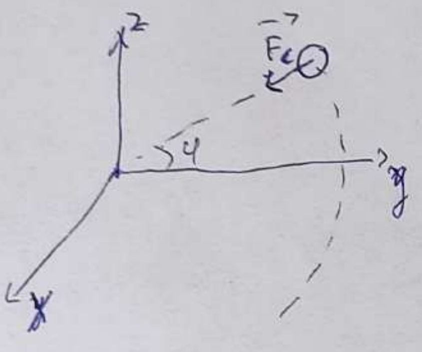
1) N, l ; a) nr. grd. lib : $s = 3N - l$

b) ~~s grade de libertate~~

s coordonate generalizate

c) s ecuații

2)



$V = V(r)$

~~$\rho = x$~~ $x = \rho \sin\varphi$

$y = \rho\cos\varphi$

$z = z$

$$T = \frac{m\dot{x}^2}{2} + \frac{m\dot{y}^2}{2} + \frac{m\dot{z}^2}{2} = \frac{m}{2}\left(\dot{\varphi}^2\rho^2\sin^2\varphi + \dot{\varphi}^2\rho^2\cos^2\varphi + \dot{z}^2\right) \Rightarrow$$

$$\Rightarrow T = \frac{m}{2}\dot{\varphi}^2\rho^2 + \frac{m}{2}\dot{z}^2 \; ; \quad L = T - V = \frac{m}{2}\dot{\varphi}^2\rho^2 + \frac{m}{2}\dot{z}^2 - V(r) \; ;$$

~~$H = p_k\dot{q}_k - L = m(\rho\sin\varphi\cdot\rho$~~ $\quad H = P_k\dot{q}_k - L$

$$\frac{\partial L}{\partial \dot{q}} = P \Rightarrow P_\rho = 0 \; ; \quad P_\varphi = m\rho^2\dot{\varphi} ; \quad P_z = m\dot{z}$$

$$H = m\rho^2\dot{\varphi}\cdot\dot{\varphi} + m\dot{z}\cdot\dot{z} - \frac{m}{2}\dot{\varphi}^2\rho^2 + \frac{m\dot{z}^2}{2} + V(r) \Rightarrow$$

$$\Rightarrow H = \dot{\varphi}^2\rho^2\left(m - \frac{m}{2}\right) + \dot{z}^2\left(m + \frac{m}{2}\right) + V(r) ;$$

$$H = \frac{m}{2}\dot{\varphi}^2\rho^2 + \frac{m}{2}\dot{z}^2 + V(r) ; \quad \text{Var. cicl: } \varphi$$

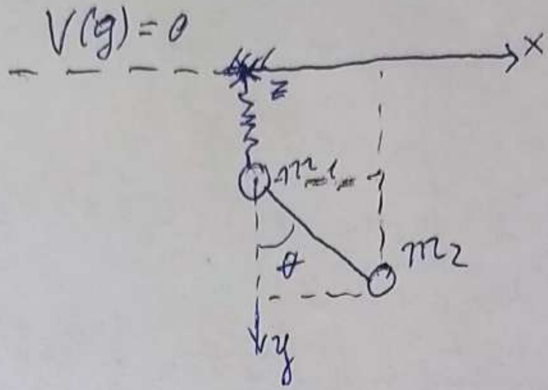
V(r) - integrală primă

Moraru Victorio

S II

3)

m_1, m_2



$m_1 \to f$

$m_2 \to g$

$$\left.\begin{aligned} &f_1(x)=0 \\ &f_2(z)=0 \\ &g_1(x,y)=(y-y_1)^2+x^2-l^2=0 \\ &g_2(z)=0 \end{aligned}\right\} \Rightarrow l=4$$

$3N-l=6-4=2$

$$\left.\begin{aligned} m_1&: y=y_1 \\ m_2&: x=R\sin\theta \\ &\quad y=R\cos\theta+y_1 \end{aligned}\right\} = y=q^1, \ \theta=q^2$$

$$T=\frac{m_1\dot{y}_1^2}{2}+\frac{m_2\dot{x}_2^2}{2}+\frac{m_2\dot{y}_2^2}{2}=\frac{m_1\dot{y}^2}{2}+\frac{m_2}{2}\left(\dot{\theta}^2R^2\cos^2\theta\right)+$$

$$+\frac{m_2}{2}\left(-\dot{\theta}R\sin\theta+\dot{y}\right)^2 \Rightarrow$$

$$T=\frac{m_1\dot{y}^2}{2}+\frac{m_2}{2}\dot{\theta}^2R^2\cos^2\theta+\frac{m_2}{2}\left(\dot{\theta}^2R^2\sin^2\theta-2\dot{\theta}\dot{y}R\sin\theta+\dot{y}^2\right) \Rightarrow$$

$$T=\frac{m_1\dot{y}^2}{2}+\frac{m_2}{2}\dot{\theta}^2R^2+\frac{m_2\dot{y}^2}{2}-\frac{m_2}{\cancel{2}}\cdot\cancel{2}\dot{\theta}\dot{y}R\sin\theta;$$

$$V=-\frac{k\dot{y}^2}{2}+V_{g1}+V_{g2}; \quad V_{g1}=-m_1gy; \ V_{g2}=-m_2g(R\cos\theta+y)$$

$$L = \dot{y}^2\left(\frac{m_1+m_2}{2}\right) + \frac{m_2}{2}\dot{\theta}^2R^2 - m_2\dot{\theta}\dot{y}R\sin\theta - \left[-\frac{ky^2}{2} - m_1gy - m_2g(R\cos\theta + y)\right]$$

$$L = \dot{y}^2\cdot\frac{m_1+m_2}{2} + \dot{\theta}^2\cdot\frac{m_2}{2}R^2 - \dot{\theta}\dot{y}\,m_2R\sin\theta + \frac{ky^2}{2} + m_1gy + m_2gR\cos\theta + m_2gy$$

$$\begin{cases}\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{y}}\right) - \dfrac{\partial L}{\partial y} = 0\\[2ex] \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\theta}}\right) - \dfrac{\partial L}{\partial \theta} = 0\end{cases} \Rightarrow \begin{cases}\dfrac{d}{dt}\left((m_1+m_2)\dot{y} - \dot{\theta}m_2R\sin\theta\right) - ky - m_1g + m_2g = 0\\[2ex] \dfrac{d}{dt}\left(m_2R^2\dot{\theta} - \dot{y}m_2R\sin\theta\right) + \dot{\theta}\dot{y}m_2R\cos\theta + m_2gR\sin\theta = 0\end{cases}$$

$$\Rightarrow \begin{cases}\ddot{y}(m_1+m_2) - \ddot{\theta}m_2R\sin\theta - ky - g(m_1+m_2) = 0\\ \ddot{\theta}m_2R^2 - \ddot{y}m_2R\sin\theta + \dot{\theta}\dot{y}m_2R\cos\theta + m_2gR\sin\theta = 0\end{cases}$$

Moraru V.

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§ III

1) $(\vec{r}, p^n) = \left(\frac{\partial \vec{r}}{\partial x_i} \cdot \frac{\partial p^n}{\partial p_i} - \frac{\partial \vec{r}}{\partial p_i} \cdot \frac{\partial p^n}{\partial x_i}\right) =$

$= (\vec{r}, p \cdot p^{n-1}) = -(p \cdot p^{n-1}, \vec{r}) =$

$= -p(p^{n-1}, \vec{r}) - (p, \vec{r}) p^{n-1} =$

$= -p\left(\frac{\partial p^{n-1}}{\partial x_i} \cdot \frac{\partial \vec{r}}{\partial p_i} - \frac{\partial p^{n-1}}{\partial p_i} \cdot \frac{\partial \vec{r}}{\partial x_i}\right) - \left(\frac{\partial p}{\partial x_i} \cdot \frac{\partial r}{\partial p_i} - \frac{\partial p}{\partial p_i} \cdot \frac{\partial r}{\partial x_i}\right) p^{n-1} =$

$= -p\left(\frac{\partial p^{n-1}}{\partial x_i} \cdot \frac{\partial \vec{r}}{\partial p_i} - (n-1)p^{n-2}\right) - (1-1) p^{n-1} =$

$= -p\left((n-1)p^{n-2} - (n-1)p^{n-2}\right) = 0$

2) $(\vec{p}, r^n) = -(r \cdot r^{n-1}, \vec{p}) = -r(r^{n-1}, \vec{p}) - (r, \vec{p}) r^{n-1} =$

$= -r\left(\frac{\partial r^{n-1}}{\partial x_i} \cdot \frac{\partial \vec{p}}{\partial p_i} - \frac{\partial r^{n-1}}{\partial p_i} \cdot \frac{\partial \vec{p}}{\partial x_i}\right) - \left(\frac{\partial r}{\partial x_i} \cdot \frac{\partial \vec{p}}{\partial p_i} - \frac{\partial r}{\partial p_i} \cdot \frac{\partial \vec{p}}{\partial x_i}\right) r^{n-1} =$

$= -r\left[r^{n-2}(n-1) - r^{n-2}(n-1)\right] - (1-1) r^{n-1} =$

$= -r(0) = 0$