TECHNISCHE UNIVERSITÄT DRESDEN

Institute of Theoretical Computer Science Chair for Automata Theory

CONTEXT REASONING FOR ROLE-BASED MODELS

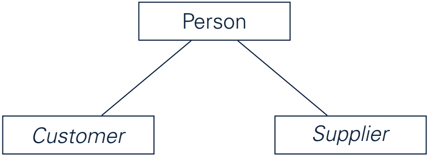
Stephan Böhme

Dresden, 27.10.2017

Motivation

What is a Role?

- Modelling concept from OOP introduced by Bachman in 1973
- Classification of roles with 26 *Features* including identity, behaviour, relationships, players, . . . and about Contexts and Constraints (Kühn, Leuthäuser, et al. 2014; Steimann 2000)



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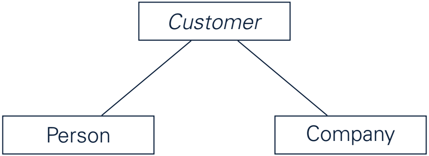

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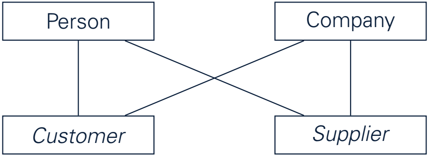

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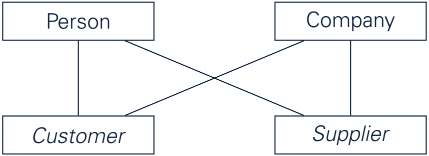

Requirements for modern Software Systems:

- Adaptability
- High expressiveness
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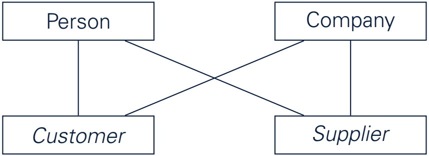

Requirements for modern Software Systems:

- Adaptability → Roles allow for dynamic changes of the system.
- High expressiveness → Roles increase separation of concerns.
- Longevity → Roles enable updating running applications.

Role-Based Systems

- Software systems that use the notion of roles
- Focus on: Compartment Role Object Model (CROM), (Kühn, Leuthäuser, et al. 2014)
  - Well-defined semantical foundation (Kühn, Böhme, et al. 2015)

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Key properties of roles:

- Roles depend on the context.
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↳ Logic-based formalisms

Requirements on logical formalism

- Decidable reasoning tasks
- Express contexts, 'players' and roles as formal objects
- Model contexts and ternary relation of role-playing
- Ability to handle *rigid*, i.e. context-independent, knowledge

Workflow of Automated Analysis of Role-Based Models

Banking application

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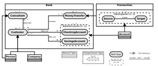
Banking application

Graphical editor



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Graphical notation of a CROM and its formal representation

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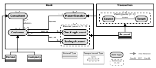

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Contextualized DL ontology

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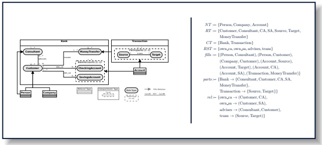

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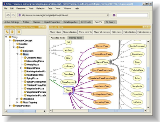

JConHT Reasoner

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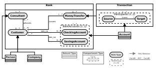

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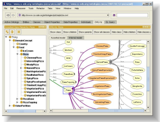

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Inconsistencies?

Can specific role be played?

Partial knowledge satisfiable?

Are even stronger constraints entailed?

Running Example – Banking Application

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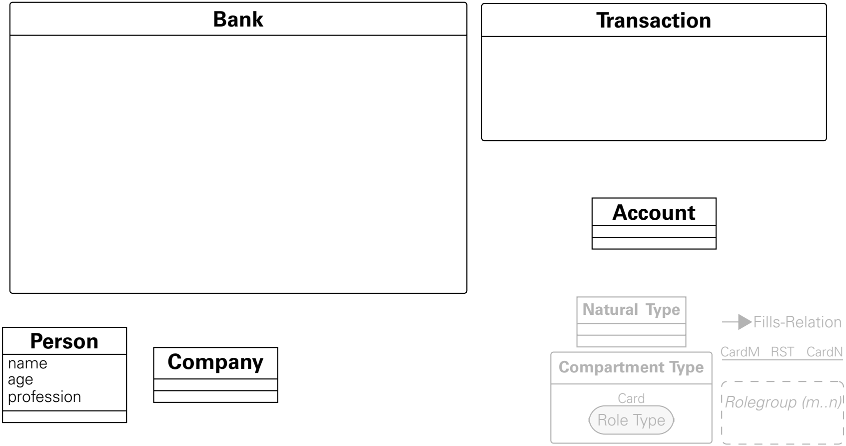

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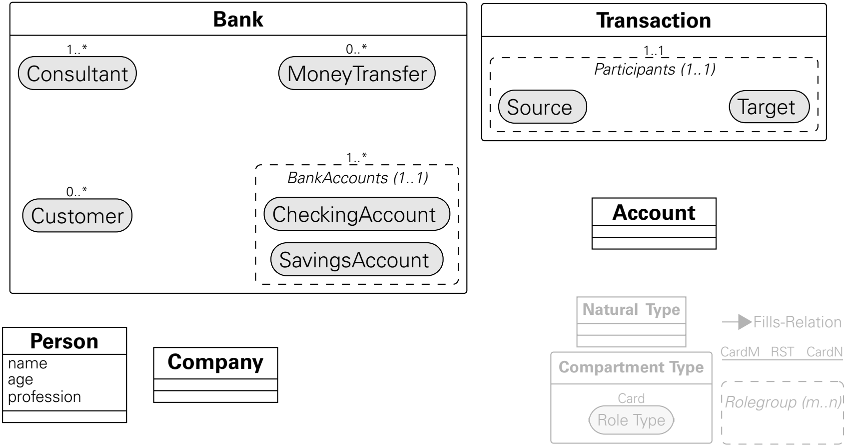

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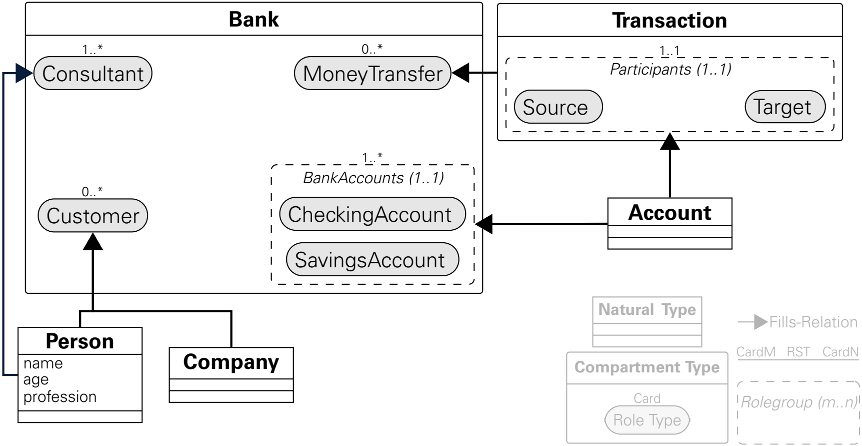

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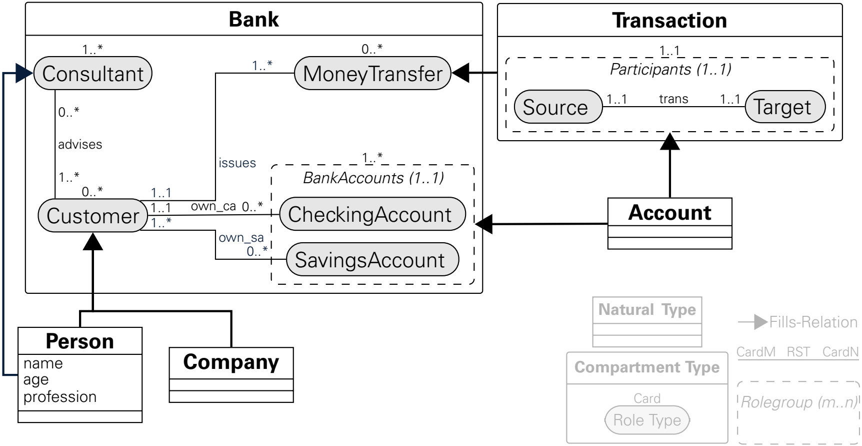

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Syntax and Semantics of the DL $\mathcal{ALC}$

Every consultant advises customers who own an checking account.
$\textsc{Consultant} \sqsubseteq \exists \mathit{advises}.(\textsc{Customer} \sqcap \exists \mathit{own\_ca}.\textsc{CheckingAccount})$

Peter is a consultant. $\textsc{Consultant}(\mathit{Peter})$

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N_I	... individual names	*Peter*

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concept constructors:	$C_1 \sqcap C_2$, $C_1 \sqcup C_2$, $\neg C_1$, $\exists r.C$, $\forall r.C$
set of $\mathcal{ALC}$ concepts:	smallest set that is closed under $\mathsf{N_C}$ and the concept constructors of $\mathcal{ALC}$
General concept inclusion (GCI):	$C \sqsubseteq D$
assertion:	$C(a)$, $r(a, b)$
$\mathcal{ALC}$-axiom:	a GCI or an assertion

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A DL interpretation $\mathcal{I}$ has a domain $\Delta^\mathcal{I}$ and maps

- concept names A to sets $A^\mathcal{I} \subseteq \Delta^\mathcal{I}$,
- DL role names r to binary relations $r^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$, and
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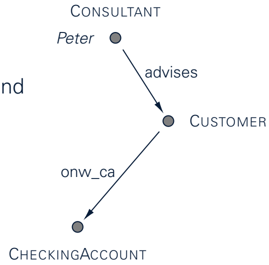

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Intuition of Contextualized DL $\mathcal{L}_M [\![\mathcal{L}_O]\!]$

- Two-dimensional, two-sorted description logic
- $\mathcal{L}_M$ to describe knowledge *about* contexts (meta level)
- $\mathcal{L}_O$ to describe knowledge *within* contexts (object level)
- Contexts $\hat{=}$ possible worlds
- Concepts, axioms of object logic are usual $\mathcal{L}_O$ concepts, axioms
- Object axioms used as meta concepts
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$$\neg[\![(\textsc{Transaction}' \sqcap \exists\, \mathit{plays}.\top) \sqsubseteq \bot]\!] \sqsubseteq \exists\, \mathit{nested}.\textsc{Transaction}$$

Syntax and Semantics of $\mathcal{L}_M [\![\mathcal{L}_O]\!]$

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$$\mathsf{O} = (\mathsf{O}_\mathsf{C}, \mathsf{O}_\mathsf{R}, \mathsf{O}_\mathsf{I}) \qquad \mathsf{M} = (\mathsf{M}_\mathsf{C}, \mathsf{M}_\mathsf{R}, \mathsf{M}_\mathsf{I})$$

$$\mathsf{O}_{\mathsf{Crig}} \subseteq \mathsf{O}_\mathsf{C}$$

$$\mathsf{O}_{\mathsf{Rrig}} \subseteq \mathsf{O}_\mathsf{R}$$

object concept name $A \in \mathsf{O}_\mathsf{C}$

object concept C (using constructors of $\mathcal{L}_O$)

object axioms $C \sqsubseteq D$

$C(a)$

Syntax and Semantics of $\mathcal{L}_M \; [\![\mathcal{L}_O]\!]$

$$\mathsf{O} = (\mathsf{O_C}, \mathsf{O_R}, \mathsf{O_I}) \qquad \mathsf{M} = (\mathsf{M_C}, \mathsf{M_R}, \mathsf{M_I})$$

$$\mathsf{O_{Crig}} \subseteq \mathsf{O_C}$$

$$\mathsf{O_{Rrig}} \subseteq \mathsf{O_R}$$

object concept name $A \in \mathsf{O_C}$

object concept C (using constructors of $\mathcal{L}_O$)

meta concepts $[\![C \sqsubseteq D]\!]$, $[\![C(a)]\!]$

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object concept name	$A \in \mathsf{O}_\mathsf{C}$
object concept	C (using constructors of $\mathcal{L}_O$)
meta concepts	$[\![C \sqsubseteq D]\!]$ $[\![C(a)]\!]$
meta concept name	$B \in \mathsf{M}_\mathsf{C}$
meta concept	E (using constructors of $\mathcal{L}_M$)
meta axioms	$E \sqsubseteq F$ $E(c)$

Syntax and Semantics of $\mathcal{L}_M [\![\mathcal{L}_O]\!]$

$$O = (O_C, O_R, O_I) \qquad\qquad M = (M_C, M_R, M_I)$$

$$O_{Crig} \subseteq O_C$$
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$\mathcal{L}_M [\![\mathcal{L}_O]\!]$ ontology $\mathcal{B}$ ...conjunction of m-axioms

Syntax and Semantics of $\mathcal{L}_M [\![\mathcal{L}_O]\!]$

Nested interpretation $\mathcal{J} = (\mathbb{C}, \cdot^{\mathcal{J}}, \Delta^{\mathcal{J}}, (\cdot^{\mathcal{I}_c})_{c \in \mathbb{C}})$

- $(\mathbb{C}, \cdot^{\mathcal{J}})$ DL interpretation on meta level
- $(\Delta, \cdot^{\mathcal{I}_c})$ DL interpretation on object level for each possible world
- $x^{\mathcal{I}_c} = x^{\mathcal{I}_d}$ for all $c, d \in \mathbb{C}$, $x \in \mathsf{O}_{\mathsf{Crig}} \cup \mathsf{O}_{\mathsf{Rrig}} \cup \mathsf{O}_{\mathsf{I}}$

object concept name	$A^{\mathcal{I}_c} \subseteq \Delta$
object concept	C
meta concepts	$[\![C \sqsubseteq D]\!]$ $[\![C(a)]\!]$
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$\mathcal{L}_M [\![\mathcal{L}_O]\!]$ ontology $\mathcal{B}$

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object concept name	$A^{\mathcal{I}_c} \subseteq \Delta$
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meta concepts	$[\![C \sqsubseteq D]\!]^{\mathcal{J}} := \{c \in \mathbb{C} \mid \mathcal{I}_c \models C \sqsubseteq D\}$ $[\![C(a)]\!]^{\mathcal{J}} := \{c \in \mathbb{C} \mid \mathcal{I}_c \models C(a)\}$
meta concept name	$B^{\mathcal{J}} \subseteq \mathbb{C}$
meta concept	$E^{\mathcal{J}} \subseteq \mathbb{C}$ (acc. to semantics of $\mathcal{L}_M$)
meta axioms	$\mathcal{J} \models E \sqsubseteq F$ $\mathcal{J} \models E(c)$
$\mathcal{L}_M [\![\mathcal{L}_O]\!]$ ontology	$\mathcal{J} \models \mathcal{B}$

$$\left.\begin{array}{l} \mathcal{J} \models E \sqsubseteq F \\ \mathcal{J} \models E(c) \\ \mathcal{J} \models \mathcal{B} \end{array}\right\}\ \text{standard entailment relation}$$

Example of Contextualized DL $\mathcal{L}_M$ $[\![\mathcal{L}_O]\!]$

$$\textsc{Bank} \sqsubseteq [\![\textsc{SavingsAccount} \sqsubseteq \geqslant_1 \mathit{own\_sa}^{-}.\top]\!]$$

$$\textsc{Bank} \sqsubseteq [\![\exists\, \mathit{own\_sa}.\top \sqsubseteq \textsc{Customer}]\!]$$

$$\neg[\![(\textsc{Transaction}' \sqcap \exists\, \mathit{plays}.\top) \sqsubseteq \bot]\!] \sqsubseteq \exists\, \mathit{nested}.\textsc{Transaction}$$

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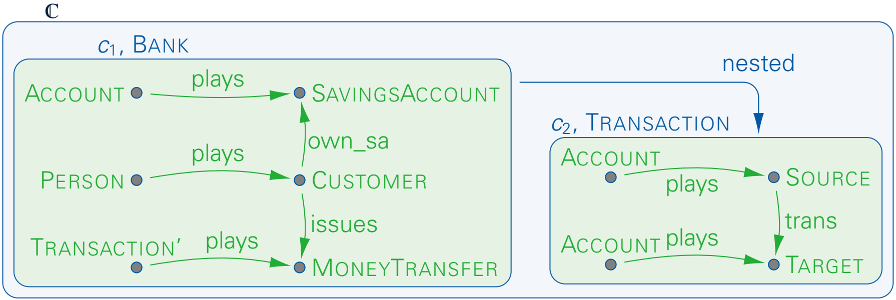

Complexity of Consistency Problem

	$\mathcal{L}_M$ \ $\mathcal{L}_O$	$\mathcal{EL}$	$\mathcal{ALC}-\mathcal{SHOQ}$	$\mathcal{SHOIQ}$
no rigid names	$\mathcal{EL}$	constant	EXP-complete	NEXP-complete
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with rigid roles	$\mathcal{EL}$	constant	2EXP-complete	2EXP-hard and in 2NEXP
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	$\mathcal{SHOIQ}$	NEXP-complete	2EXP-complete	2EXP-hard and in 2NEXP

Upper Bounds for $\mathcal{L}_M \, [\![\mathcal{L}_O]\!]$

Idea: Split consistency problem into two separate decision problems

- *Outer consistency* with $\mathcal{X}$
- *Admissibility* of $\mathcal{X}$

Check whether meta level is consistent.
Check whether the induced o-axioms are consistent.

$$
\begin{aligned}
\mathcal{B}_{ex} = {} & C \sqsubseteq [\![A(a)]\!] \\
& \wedge (C \sqcap [\![A \sqsubseteq \bot]\!])(d) \\
& \wedge (\exists\, r.[\![A \sqsubseteq \bot]\!])(d)
\end{aligned}
$$

$$
\mathcal{X} = \underbrace{\{\varnothing, \{[\![A(a)]\!]\}, \{[\![A \sqsubseteq \bot]\!]\}\}}_{\{[\![A(a)]\!],[\![A \sqsubseteq \bot]\!]\} \notin \mathcal{X}}
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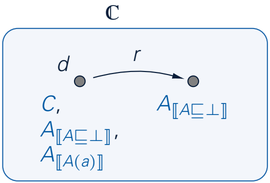

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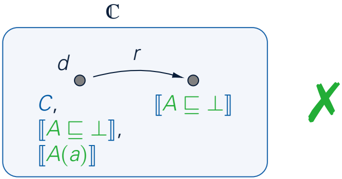

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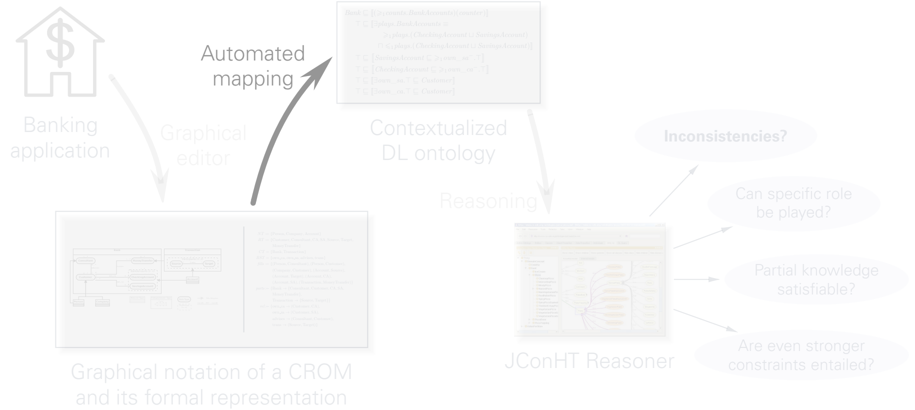
Lemma

The $\mathcal{L}_M [\![\mathcal{L}_O]\!]$ ontology $\mathcal{B}$ is consistent iff there is a set $\mathcal{X}$ such that

1. *$\mathcal{X}$ is admissible, and*
2. *the outer abstraction of $\mathcal{B}$ is outer consistent w.r.t. $\mathcal{X}$.*

Workflow of Automated Analysis of Role-Based Models



Automated Mapping

Objective: Given a CROM $\mathcal{M}$, construct an ontology $\mathcal{O}_{\mathcal{M}}$ s.t.
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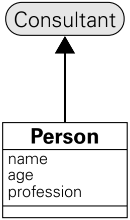

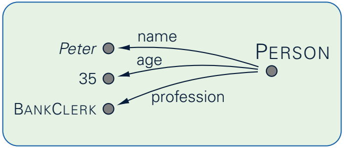

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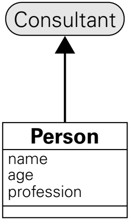

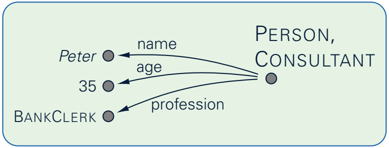

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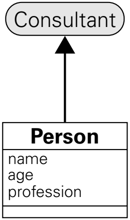

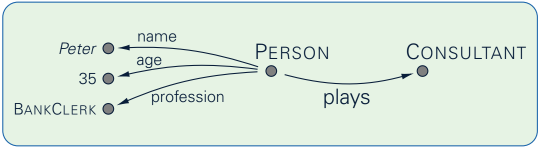

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- Occurrence constraints $\rightsquigarrow$ object individual δ, DL role 'counting'

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Constraints:

- without constraints $\leadsto$ $\mathcal{ALC}$ $[\![\mathcal{ALCIQ}]\!]$
- with occurrence constraints $\leadsto$ $\mathcal{ALC}$ $[\![\mathcal{ALCOIQ}]\!]$
- full $\leadsto$ $\mathcal{ALC}$ $[\![\mathcal{SHOIQ}]\!]$

- current version of CROM $\leadsto$ no rigid DL roles needed
- additional constraints based on fields of natural type $\leadsto$ rigid roles needed!

Complexity of Consistency Problem

<table>
<tr><th colspan="2">$\mathcal{L}_O$ \ $\mathcal{L}_M$</th><th>$\mathcal{EL}$</th><th>$\mathcal{ALC} - \mathcal{SHOQ}$</th><th>$\mathcal{SHOIQ}$</th></tr>
<tr><td rowspan="3">no rigid names</td><td>$\mathcal{EL}$</td><td>constant</td><td rowspan="2">ExP-complete</td><td rowspan="3">CROM ★ NExP-complete</td></tr>
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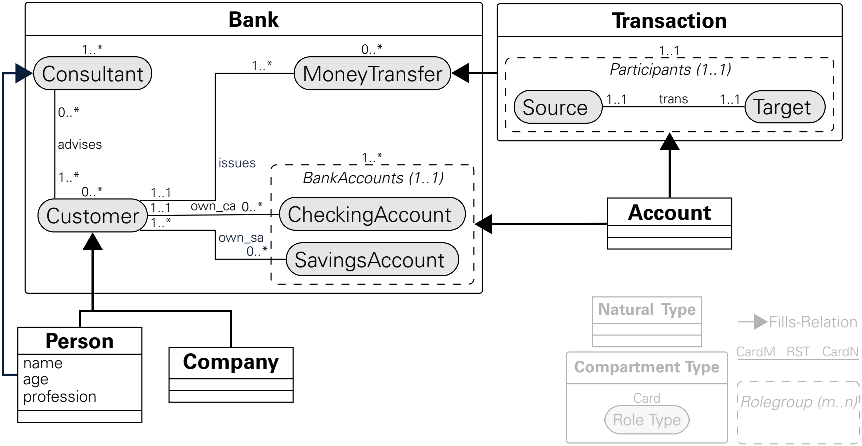
</table>

Example of Mapping



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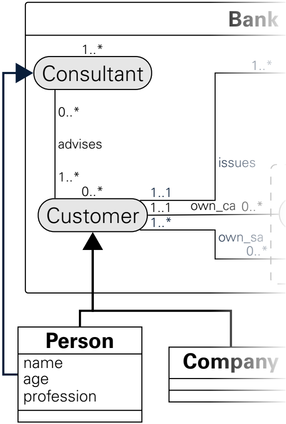

$$\top \sqsubseteq [\![\textsc{Consultant} \sqcup \textsc{Customer} \sqsubseteq =_1 \text{counting}^-.\{\delta\}]\!]$$

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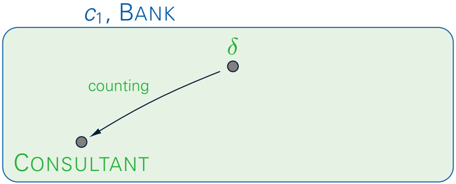

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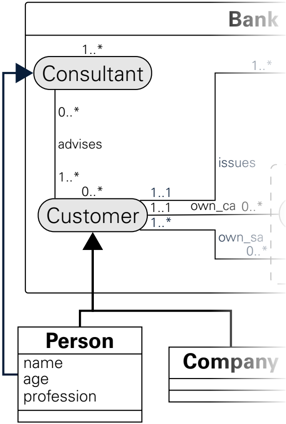

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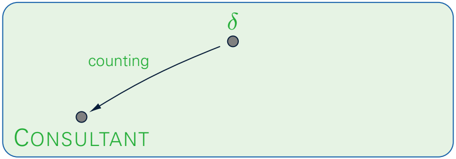
c_1, BANK



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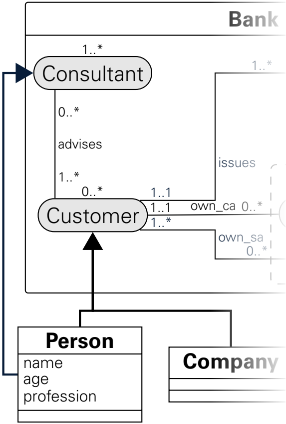

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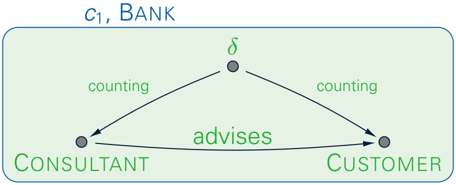

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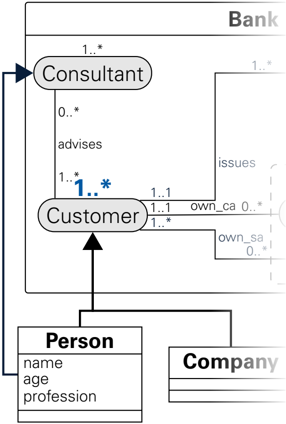

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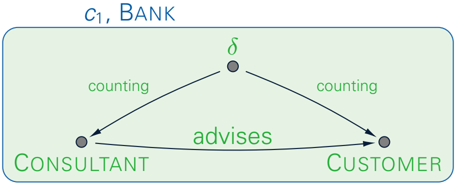
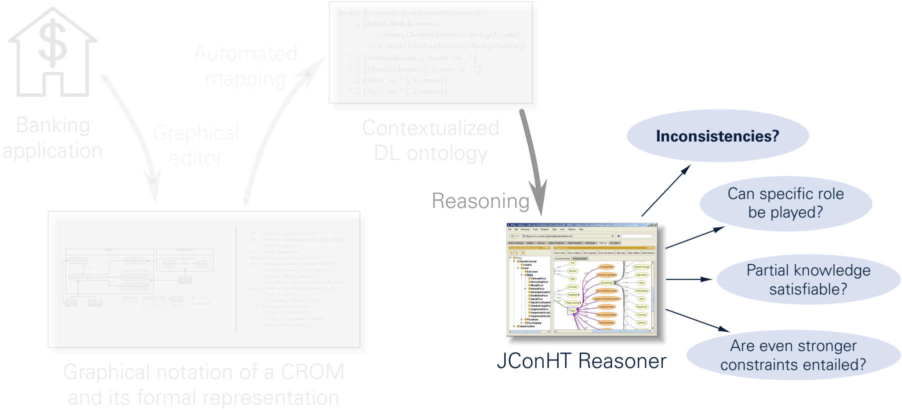


Workflow of Automated Analysis of Role-Based Models



JConHT – A $\mathcal{SHOIQ}$ $[\![\mathcal{SHOIQ}]\!]$ Reasoner

- **J**ava implemented **Con**textualized description logic reasoner based on **H**ermi**T**/**H**yper**T**ableau
- First reasoner capable of processing contextualized DL ontologies
- Utilize Lemma about separation of reasoning tasks
  1. Check consistency of meta level
  2. Check whether induced object axioms consistent
- Reuse existing, highly optimized reasoners
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Algorithm for Checking Consistency

Input : $\mathcal{SHOIQ}$ [[$\mathcal{SHOIQ}$]]-ontology $\mathcal{O}$
Output: true if $\mathcal{O}$ is consistent, false otherwise
Preprocessing (results in $(\mathcal{C}, \mathcal{A})$):

1. Elimination of transitivity axioms, normalization, clausification
2. Repletion of DL-clauses

Let $(\mathcal{T}, \lambda)$ be any derivation for $(\mathcal{C}, \mathcal{A})$.
$\mathfrak{A} := \{\mathcal{A}' \mid$ there exists a leaf node in $(\mathcal{T}, \lambda)$ that is labelled with $\mathcal{A}'\}$

for $\mathcal{A}' \in \mathfrak{A}$ **do**
  if $\mathcal{A}'$ *is clash-free* **then**
    if $\mathcal{O}$ *contains rigid names* **then**
      if $\mathcal{K}_{\mathsf{rig}} := (\mathcal{O}_{\mathcal{A}'}, \mathcal{R}_{\mathsf{O}}{}')$ *is consistent* **then**
        return true
    else
      Let $\{c_1, \ldots, c_k\}$ be the individuals occurring in $\mathcal{A}'$
      if $\mathcal{K}_i := (\mathcal{O}_{c_i}, \mathcal{R}_{\mathsf{O}})$ *is consistent for all* $1 \leq i \leq k$ **then**
        return true

return *false*

Repletion of DL-Clauses

$$\begin{aligned} \mathcal{B}_{\mathrm{ex}} := {} & \neg C(s) \\ & \wedge [\![\neg B(a)]\!] \sqsubseteq C \\ & \wedge \neg C \sqsubseteq [\![B \sqsubseteq \bot]\!] \end{aligned}$$

Repletion of DL-Clauses

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$$\begin{aligned}\mathcal{A}_{\text{ex}} &:= \{\neg C(s)\} \\ \mathcal{C}_{\text{ex}} &:= \{[\![\neg B(a)]\!](x) \to C(x), \\ &\qquad \top \to C(x) \vee [\![B \sqsubseteq \bot]\!](x)\}\end{aligned}$$

Repletion of DL-Clauses

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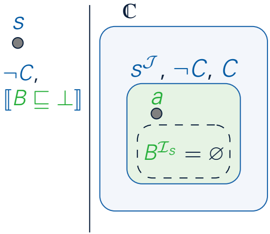

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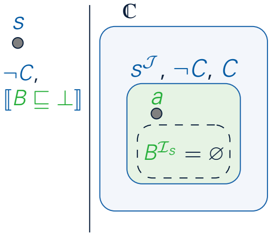

Definition (Repletion)

Let $\mathcal{C}$ be a set of DL-clauses. The **repletion** of $\mathcal{C}$ is obtained from $\mathcal{C}$ by adding the DL-clause $\top \to [\![\alpha]\!](x) \vee [\![\neg\alpha]\!](x)$ for each o-axiom $[\![\alpha]\!]$ occurring in $\mathcal{C}$.

Repletion of DL-Clauses

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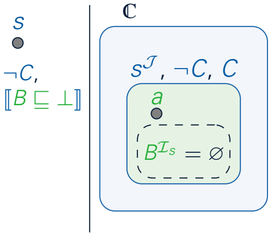

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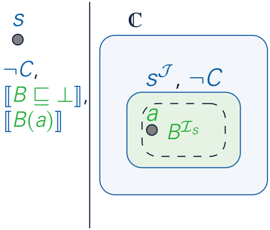

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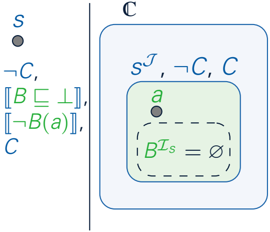

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Thank you for your attention!

Any questions?

Performance Evaluation

- Pseudo-random domain models of increasing complexity:
  - based on parameter *n*
  - *n* natural types, *n* compartment types, for each compartment type *n* role types
- 3 different scenarios
  1. Number of (constrained) relationship types per compartment type
  2. Number of role groups per compartment type
  3. Whether or not compartments can play roles.
- Average execution time of JConHT
  - Translation from CROM models into ontologies neglected
  - For each configuration 100 executions
  - Average time needed to decide consistency

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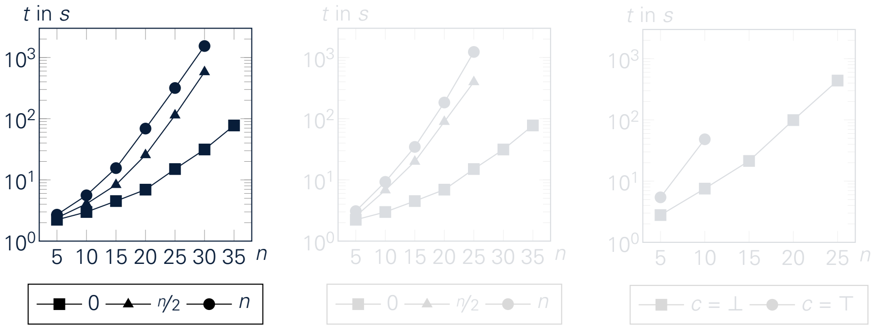
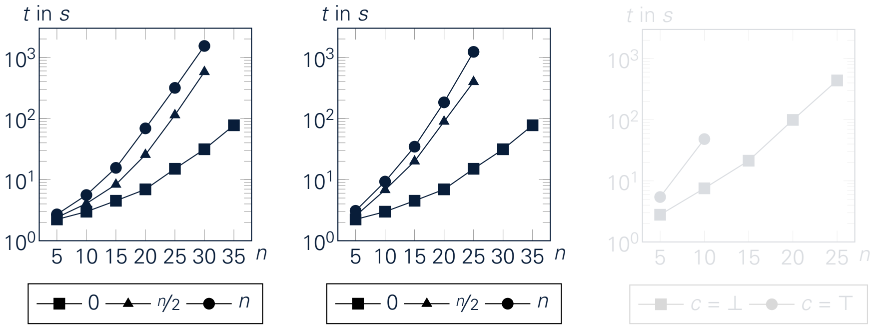


(a) Variation of ♯ RSTs. (b) Variation of ♯ RGs. (c) Flat vs. nested CTs.

Average execution times of JConHT for benchmark ontologies.

Performance Evaluation



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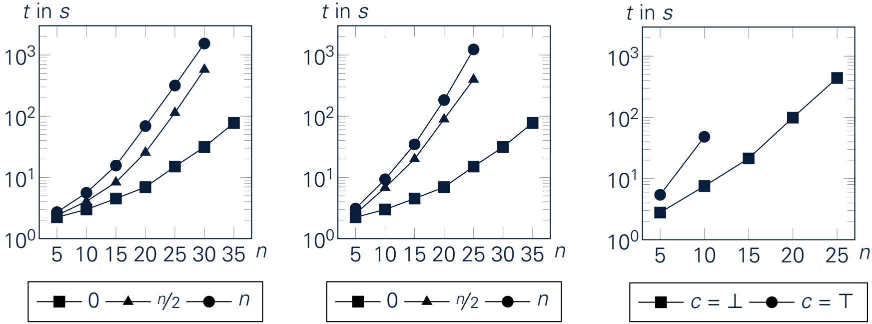

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References I

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