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Sharing the global outcomes of finite natural resource exploitation: A dynamic coalitional stability perspective

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ABSTRACT

The article explores the implications of natural resource scarcity in terms of global cooperation and trade. We investigate whether there exist stable international long-term agreements that take into account the disparities between countries in terms of geological endowments and productive capacity, while caring about future generations. For that purpose, we build an original cooperative game framework, where countries can form coalitions in order to optimize their discounted consumption stream in the long-run, within the limits of their stock of natural resources. We use the concept of the strong sequential core that satisfies both coalitional stability and time consistency. We show that this set is nonempty, stating that an international long-term agreement along the optimal path will be self-enforcing. The presented model sets out a conceptual framework for exploring the fair sharing of the fruits of global economic growth.

1. Introduction

The nexus between trade and mineral resource extraction is crucial to the understanding of wealth creation dynamics. Early stages of mankind are named after the materials from which tools and weapons were made of: the Bronze Age, which arose fully around 3000 BCE, was preceded by the Chalcolithic or Copper Age. The Near East was the "kernel of the Age of Metals", but was poorly endowed with these materials. Therefore, the "valley urban societies" had to exchange with the "barbarians" (in the technical archaeological sense), before trade expanded from the Near East to Europe (Goody, 2012). Modern resource extraction and trade are of much higher orders of magnitude. Industrialization is characterized by the construction of infrastructures in the sectors of heavy industry, energy, housing, transport and communication, and therefore is inevitably associated with an increase in the consumption of raw materials. The development of trade, necessary to meet an ever-growing demand, is directly linked to the institutional implementation of free trade agreements in free trade areas.

The debate on the "trade-environment divide" (Esty, 2001) is mostly focused on pollution issues (Copeland and Taylor, 1994). The interaction between trade and materially sustainable growth (Dupuy, 2014) was studied by Asheim (1986) who looked at the effects of opening economies on the Hartwick rule. Strategic game theory approaches were also used to explore finite resources issues (Van Long, 2011), and the relationship with trade was studied with two countries (Kagan et al., 2015; Tamasiga and Bondarev, 2014). However, and despite the call of some game theorists to develop cooperative game theory (Samuelson, 2016), the literature on material sustainability neglected to treat the issue through cooperative lenses. Trade can indeed be analyzed through three concepts: "pure competition", "coalitional power" and "fair division". Shapley and Shubik (1969) provide a static market game for the three varieties of solution and show that, under certain assumptions, the outcomes converge. In a distinct stream of literature of cooperative games, the question of renewable and common-pool resources was studied (Funaki and Yamato, 1999), mostly for fishery or river sharing problems (Ambec and Sprumont, 2002; Béal et al., 2013). However, these models do not take into account the dynamic aspects of the issue.

The aim of this article is to show that there exist stable international long-term agreements that take into account the disparities between countries in terms of geological endowments and productive capacity, while caring about future generations. In this context, countries can form coalitions and look for the best agreement to optimize their allocation. Members can break

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an alliance according to their interest, at any point of time. Dynamic cooperative game theory is the most fitted framework to tackle this kind of issues, but the literature on the application of such games is quite limited. Some dynamic cooperative approaches were used to tackle environmental issues, especially fisheries (Munro, 1979) and pollution (Jørgensen and Zaccour, 2001; Hoofe, 2019). The question of non-renewable resource exhaustibility was raised by d'Albis and Ambec (2010), who study cooperative allocations among overlapping generations depleting a natural resource over an infinite future. However, the authors focus on intergenerational allocations and do not include any international aspect in their model.

In order to find such long-term international agreements, the twin issues of coalitional stability *and* time consistency have to be handled at once. First, it is a question of designing a policy that no coalition of the present generation has an incentive to refuse. Traditionally, this issue is tackled by the concept of the core, which is the set of allocations which cannot be improved upon by any coalition of agents. However, this notion does not deal with the dynamic issues relevant to the intergenerational sharing of the benefits generated from resource extraction. It must therefore also ensure that at no point in time the policy initiated then departs from the one originally planned. Recent papers tackle the issue of dynamic core concepts (Predtetchinski et al., 2004; Kranich et al., 2005; Lehrer and Scarsini, 2013), but the first studies date back to the 70s with the introduction of core concepts for production economies (Boehm, 1974; Becker, 1982), as well as the notion of trust in a monetary economy (Gale, 1978). Becker and Chakrabarti (1995) introduced the recursive core solution concept, designed for the specific case of capital accumulation models. They defined it as the set of allocations for which no coalition can improve upon its consumption stream at any time given its accumulation of assets up to that period. This concept is a particular case of the most demanding dynamic solution concept, the strong sequential core, which captures "those situations in which at each stage the grand coalition is formed, its worth is distributed among the players and no coalition has a profitable deviation" knowing that "coalitions are allowed to deviate at any stage of the game" (Kranich et al., 2005). This concept has the strong property to satisfy both coalitional stability and time consistency. It is empty-valued for large classes of economies, as argued by Habis and Herings (2011) who compare various dynamic core concepts. In the present article, we question the emptiness of this solution concept in the case of open and resource-dependent capitalist economies.

The novelty of our approach is to model the geographical heterogeneity of natural endowments that results from geological processes, as well as the geographical heterogeneity of capital and technological endowments that results from historical processes. For that purpose, we settle a cooperative game where countries can form coalitions in order to optimize their discounted consumption stream in the long-run, within the limits of their stock of natural resources. Trade is viewed as a market cooperative TU-game à la Shapley and Shubik (1969) where non-renewable natural resource inputs are exchanged in such a way as to maximize the total value to be shared among the coalition. This game is cast into a traditional Ramsey-type model of intergenerational equity, where the Bellman's dynamic programming framework is used. An international long-term policy is translated into:

- An *action strategy* taken by a coalition S. It consists in a system of quotas for extracted natural resources, a commercial quota system and an investment plan proposed to each country at each time t. An action strategy is *optimal* for S if the commercial quotas optimize the current collective production of S, and if the extraction quotas and the investment plan ensure the maximization of long-term consumption.
- A *distribution policy* of collectively produced consumption goods. Such a policy is said to be *efficient* and *coalitionally rational* if, from some date t onward, a coalition cannot block an international agreement by undertaking a policy increasing its own long-term consumption.

In other words, the coalition decides its extraction and investment plan, and deduces the amount of resources it wishes to trade. It then distributes consumption goods to its members. Note that this definition of a policy is larger than the traditional sense in the field of macroeconomics. Here, it adds to the action the notion of sharing the fruits of economic growth.

This paper addresses the underexplored topic of how to cooperate in a dynamic environment. We show that there exists a unique optimal path of extraction and investment for each country in a coalition S, which could be interpreted as quotas proposed by a social planner. As traded quantities are also unique, we conclude that there exists a unique optimal path for collective production in a coalition S. We then can build an intertemporal cooperative game where the worth of S is the discounted sum of consumption – corresponding to the remaining part of production that is not invested – along this optimal path. We demonstrate that the strong sequential core associated to this game is nonempty. This strong result states that an international long-term agreement along the optimal path will be self-enforcing. The countries, as rational agents, will stick to this agreement, as no other coalition could offer them a better outcome, at any point in time. Furthermore, we show that this strong sequential core contains an infinity of elements, opening an avenue for the research of a fair distribution from both intergenerational and international points of view.

The article is organized as follows. The following section presents our formal apparatus and details the reasoning of the model. In the third section, our main results and their interpretation are provided. The last section is a discussion on the model and the research avenue it opens. All proofs are available in Appendix A.

2. A formal statement of the problem

2.1. Game set-up

Let N be a fixed and finite nonempty set of countries, who are the game players. Countries have the possibility to form a coalition which can be viewed as a free-trade area. Each country $i \in N$ has a natural resource[2] that can be exploited by the infinitely lived country at each period $t \in \mathbb{N}$. Growth is constrained by natural resource availability, its costs of extraction, as well as the accumulation of capital. Countries have three decisions to make at each period: resource extraction, resource trade and investment level through savings that will increase the production capacity of the next period. The remaining created flow of wealth is consumed, and generations seek to maximize their present consumption as well as the discounted consumption of their successors. The present model is therefore a cooperative games adaptation of Ramsey's benchmark model with traded mineral resources. A coalition illustrates contemporary trade agreements between countries with the free movement of natural resources, consumption goods and capital flows (in the form of foreign investments).

At period t and for a nonempty coalition $S \subseteq N$, consider an amount $\check{y}_t(S)$ of a single final good, which can be viewed as Ricardo's metaphorical corn in neoclassical theory. The coalition chooses the amount $\eta_{i,t}$ each country i will invest for tomorrow

[2] Note that this model could be extended to a finite number of resources.

(the seeds to be sowed next year) to increase the country's stock of capital $K_{i,t}$. This leaves $\breve{y}_t(S) - \sum_{i\in S} \eta_{i,t}$ to the present consumption of the coalition. The dynamics of i's capital stock is given by the following law of motion, $\tau \in (0,1)$ being the depreciation rate:

$$K_{i,t+1} = K_{i,t} + \eta_{i,t} - \tau K_{i,t}$$

The coalition also chooses the quantity $e_{i,t}$ of resources each country will extract from its available stock $\Omega_{i,t}$. The dynamics of i's resource stock is given by the following law of motion:

$$\Omega_{i,t+1} = \Omega_{i,t} - e_{i,t}$$

Extracted resources are exchanged in a free-trade area with no transaction cost, and country i ends up with a quantity $z_{i,t}$ of input for its domestic production. It means that at the end of the exchange process, $\sum_{i\in S} z_{i,t} = \sum_{i\in S} e_{i,t}$ holds. The country's productive sector then uses its production capacity $K_{i,t}$ and $z_{i,t}$ to produce $f_i(z_i, K_i)$ through a raw country-specific production function f_i. We assume that, for each $i \in S$, $f_i : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ satisfies:

(i) f_i is continuous, non-decreasing, strictly concave;
(ii) $f_i(z_i, 0) = 0$ for each $z_i \in \mathbb{R}_+$ and $f_i(0, K_i) = 0$ for each $K_i \in \mathbb{R}_+$;
(iii) f_i is differentiable with respect to K_i;
(iv) $\lim_{K_i\to 0} \frac{\partial f_i(z_i,K_i)}{\partial K_i} > \tau$ for each $z_i \in \mathbb{R}_+$;
(v) $\lim_{K_i\to \infty} \frac{\partial f_i(z_i,K_i)}{\partial K_i} < \tau$ for each $z_i \in \mathbb{R}_+$.

Note that these conditions are weaker than the Inada conditions. The cost of extraction $c_i(e_{i,t}, \Omega_{i,t})$ of $i \in N$, expressed in consumption units, is represented through a function depending both on the quantity $\Omega_{i,t}$ available to the country i and the quantity $e_{i,t}$ extracted at date t. We assume that this function, defined on $\mathbb{R}^2_+$, is continuous, convex and non negative when $\Omega_{i,t}$ is larger than $e_{i,t}$.[3]

We consider that each country i has an initial endowment $\breve{y}_{i,0}$ and that

$$\breve{y}_0(S) = \sum_{i\in S} \breve{y}_{i,0}.$$

The production process takes one period to be achieved, since time is necessary for natural resource transportation and capital usage. The choice of the trading strategy is made such that the total net production $\breve{y}_{t+1}(S)$ of each coalition S is maximized through the following static optimization problem:

$$\begin{aligned} \underset{(z_{i,t})_{i\in S}}{\text{maximize}} \quad & \sum_{i\in S}\Big(f_i(z_{i,t}, K_{i,t}) - c_i(e_{i,t}, \Omega_{i,t})\Big) \\ \text{subject to} \quad & \sum_{i\in S} z_{i,t} = \sum_{i\in S} e_{i,t}. \end{aligned} \tag{1}$$

Trade is viewed as a market cooperative TU game à la Shapley and Shubik (1969) where extracted resource inputs are exchanged in such a way as to maximize the total value to be shared among the coalition. Fig. 1 presents a scheme of the model at t and $t+1$.

2.2. Significant sets

In this subsection, a digression is made to define the sets that will be important for the rest of the article.

[3] Of course, this function is economically irrelevant if the stock of resources is smaller than the extracted flow: this feasibility constraint is integrated in the definition of the feasible sets.

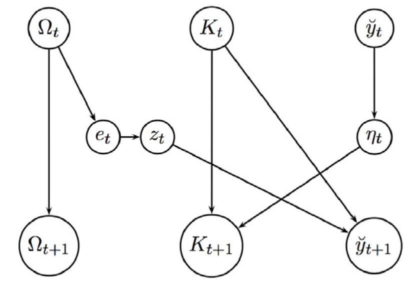

Fig. 1. Scheme of the state and action dynamics.

$\bar{\Omega} \in \mathbb{R}_+$ is the total and finite amount of resources on Earth. Following neoclassical arguments, the assumptions on the function f_i as well as the dynamics of K_i ensure that there exists $\bar{K} \in \mathbb{R}_+$ such that for each $i \in S$ and for each $t \in \mathbb{N}$, $K_{i,t} \leq \bar{K}$. As a consequence, there exists $\bar{\breve{y}} \in \mathbb{R}_+$ such that for each $t \in \mathbb{N}$ and each $S \subseteq N$, $\breve{y}_t(S) \leq \bar{\breve{y}}$.

Let $\Omega(S) = (\Omega_i)_{i\in S}$ (or simply Ω if there is no danger of confusion) be the vector of natural resource stocks and $K(S) = (K_i)_{i\in S}$ (or simply K if there is no danger of confusion) be the vector of capital stocks. The state space $\mathcal{S}(S)$ is defined by:

$$\mathcal{S}(S) := \{(\Omega(S), K(S), \breve{y}(S)) \in \mathbb{R}^S_+ \times \mathbb{R}^S_+ \times \mathbb{R}_+ \mid \breve{y}(S) \leq \bar{\breve{y}} \text{ and } \forall i \in S,\ \Omega_i \leq \bar{\Omega}, K_i \leq \bar{K}\}.$$

Let $e(S) = (e_i)_{i\in S}$ (or simply e if there is no danger of confusion) be the vector of extraction and $\eta(S) = (\eta_i)_{i\in S}$ (or simply η if there is no danger of confusion) be the vector of investment. The action space $\mathcal{A}(S)$ is defined by:

$$\mathcal{A}(S) := \{(e(S), \eta(S)) \in \mathbb{R}^S_+ \times \mathbb{R}^S_+ \mid \forall i \in S,\ e_i \leq \bar{\Omega}, \eta_i \leq \bar{\breve{y}}\}.$$

The set of feasible vectors of extraction $e(S)$ for a coalition S with a stock described by $\Omega(S)$ is

$$E^{\Omega}(S) := \{e(S) \in (\mathbb{R}_+)^S \mid e_i \leq \Omega_i\}.$$

We now define the set of feasible vectors of investment $\eta(S)$ for a coalition S by

$$\mathcal{I}^{\breve{y}(S)}(S) := \{\eta(S) \in (\mathbb{R}_+)^S \mid \sum_{i\in S} \eta_i \leq \breve{y}(S)\}.$$

We can therefore define the set of couples $(e(S), \eta(S))$ of feasible actions by:

$$\Phi^{\Omega,\breve{y}(S)}(S) := \{(e(S), \eta(S)) \mid e(S) \in E^{\Omega}(S), \eta(S) \in \mathcal{I}^{\breve{y}(S)}(S)\}.$$

Now, the set of available resource input after the exchange $z(S) = (z_i)_{i\in S}$ (or simply z if there is no danger of confusion) is:

$$Z^e(S) := \{z(S) \in (\mathbb{R}_+)^S \mid \sum_{i\in S} z_i = \sum_{i\in S} e_i\}.$$

2.3. Coalitional and intergenerational worth

At each period, countries belonging to a coalition S when the production is $\breve{y}(S)$ and the investment is $\eta \in \mathcal{I}^{\breve{y}(S)}(S)$ will earn a worth $v^{\breve{y}(S),\eta}(S)$ that equals their consumption:

$$\forall \breve{y}(S) \in \mathbb{R}_+, \forall \eta \in \mathcal{I}^{\breve{y}(S)}(S) \quad v^{\breve{y}(S),\eta}(S) = \breve{y}(S) - \sum_{i\in S} \eta_i.$$

Let us now zoom up in the time scale, and describe the behavior of countries from an intertemporal viewpoint. Let $\delta \in]0,1[$ be the discount factor. The intertemporal worth $V^{\Omega_0,K_0,\breve{y}_0(S)}(S)$ of the coalition S is defined as the value of the following dynamic programming problem:

$$\underset{(e_t,\eta_t)_{t\in\mathbb{N}}}{\text{maximize}} \quad \sum_{t\in\mathbb{N}} \delta^t v^{\check{y}_t(S),\eta_t}(S)$$

subject to

dynamic constraints

$$\begin{aligned}
\Omega_{t+1} &= \Omega_t - e_t \\
K_{t+1} &= (1-\tau)K_t + \eta_t \\
\check{y}_{t+1}(S) &= \underset{(z_{i,t})_{i\in S}}{\text{maximize}} \sum_{i\in S}\Big(f_i(z_{i,t},K_{i,t}) - c_i(e_{i,t},\Omega_{i,t})\Big) \\
&\qquad \text{subject to} \sum_{i\in S} z_{i,t} = \sum_{i\in S} e_{i,t}
\end{aligned} \tag{2}$$

feasibility constraints

$$\begin{aligned}
&e_{i,t} \le \Omega_{i,t}, \quad i\in S \\
&\sum_{i\in S}\eta_{i,t} \le \check{y}_t(S)
\end{aligned}$$

Observe that the intertemporal utility $\sum_{t\in\mathbb{N}}\delta^t v^{\check{y}_t(S),\eta_t}(S)$ is finite since $\delta \in \,]0,1[$ and that the reward function $(\check{y}(S),\eta) \mapsto v^{\check{y}(S),\eta}(S)$ is bounded on $[0,\bar{\check{y}}]\times[0,\bar{\check{y}}]^S$. Let us now clearly describe the actions that occur on the short and long term.

(i) **On the short run**: Suppose that the coalition S has chosen an extraction and investment plan $\big(e_t(S),\eta_t(S)\big)_{t\in\mathbb{N}}$. The level of extraction and investment is therefore fixed for each one of its members i at each period t. The decision to be made during a time period t only involves the trade of natural resources. The situation is modeled as a market game *à la* Shapley and Shubik (1969). A trading process takes place inside the coalition, and the total extracted amount $\sum_{i\in S} e_{i,t}$ is exactly redistributed among the members. Each country ends up with a level $z_{i,t}$ to contribute to the total net production $\check{y}_t(S)$, which is the production $\sum_{i\in S} f_i(z_{i,t},K_{i,t})$ net from the extraction costs $\sum_{i\in S} c_i(e_{i,t},\Omega_{i,t})$. The market game consists in a choice of distribution of extracted natural resources such that this total net production $\check{y}_t(S)$ is maximized through problem (1). The level of investment $\eta_{i,t}$ being also fixed, this is equivalent to maximizing the total consumption $v^{\check{y}_t(S),\eta_t}(S)$ that is the total net production minus the total investments $\sum_{i\in S}\eta_{i,t}$.

(ii) **On the long run**: the coalition S chooses through problem (2) its extraction and investment plan $\big(e_t(S),\eta_t(S)\big)_{t\in\mathbb{N}}$ for each t to infinity.[4] It does so by maximizing the intertemporal worth $V^{\Omega_0,K_0,\check{y}_0(S)}(S)$. This worth corresponds to the discounted total consumption $\sum_{t\in\mathbb{N}}\delta^t v^{\check{y}_t(S),\eta_t}(S)$ of the coalition S. This choice of the extraction and investment plan on the long term is made while taking into account the dynamics of the state variables $(\Omega_t(S),K_t(S),\check{y}_t(S))$ *and* verifying the feasibility of the chosen plan:

- the extraction flow $e_t(S)$ must be smaller than the resource stock $\Omega_t(S)$;
- the total investment $\sum_{i\in S}\eta_{i,t}$ must be smaller than the total net production $\check{y}_t(S)$. This last point ensures the nonnegativity of the total consumption of S at each period.

[4] By convention, the strategy starts at $t=0$, meaning that the first pair to be chosen is $(e_0(S),\eta_0)$.

3. Existence of distribution policies along the optimal path

3.1. Optimal action strategy

The following proposition establishes that, given a flow of resource extracted by each country, there exists a unique optimal way to trade that can be interpreted as an optimal commercial quota system.

Proposition 1. *The function* $(\Omega,e,K)\mapsto \max\Big\{\sum_{i\in S}\Big(f_i(z_i,K_i) - c_i(e_i,\Omega_i)\Big),\ (z_i)_{i\in S}\in Z^e(S)\Big\}$ *is continuous and strictly concave over* $[0,\bar{\Omega}]^S\times[0,\bar{\Omega}]^S\times[0,\bar{K}]^S$. *Furthermore, for each* $(\Omega,e,K)\in[0,\bar{\Omega}]^S\times[0,\bar{\Omega}]^S\times[0,\bar{K}]^S$, *there exists a unique* $z^*\in Z^e(S)$ *– continuously varying with* (Ω,e,K) *– such that*

$$\max\Big\{\sum_{i\in S}\Big(f_i(z_i,K_i)-c_i(e_i,\Omega_i)\Big),\ (z_i)_{i\in S}\in Z^e(S)\Big\} = \sum_{i\in S}\Big(f_i(z_i^*,K_i)-c_i(e_i,\Omega_i)\Big).$$

The next proposition states that, given the initial endowments of each country, there exists a unique optimal way to extract and to invest that can be interpreted as an optimal extraction quota system and an optimal investment plan.

Proposition 2. *For each nonempty coalition* $S\subseteq N$, *and each* $(\Omega_{i,0})_{i\in S}\in[0,\bar{\Omega}]^S$, $(K_{i,0})_{i\in S}\in[0,\bar{K}]^S$ *and* $\check{y}_0(S)\in[0,\bar{\check{y}}]$, *there exists a unique optimal path of extraction and investment to the dynamic programming problem* (2).

This result is valid for the dynamic structure presented in the set-up description (in particular, the initial conditions and the way the capital is exchanged). The following remarks justify the choices made and show why Propositions 1 and 2 are robust to some changes in the model structure.

Remark 1. It is conventional in macroeconomics to set the stocks (here resource and capital stocks) as state variables to which a transition function is applied. The present model faces an issue of simultaneity for strategic decisions. The extraction vector e and the investment vector η for each country are decided at the same time. But choosing the investment levels requires information on the total net production $\check{y}_0$, which itself depends on the flow of extraction e. The option chosen here to deal with this question is to add the flow variable $\check{y}_0$ to the state variables. Nevertheless, it is possible to find back the standard framework for which only stocks are state variables, in the tradition of optimal growth models. This requires to change the control variables of our set-up: the extraction vector e is decided at each period t. Consequently, the investment must be a function of the extraction "$e\mapsto\eta(e)$". This convention could be adopted at the cost of more complex description of the strategy and readability of the proofs. However, it does not strongly affect our results.[5]

Remark 2. In our set-up, capital flows are mobile internationally, which is not the case for capital stocks (we consider that they

[5] The optimal way to trade remains unique (Proposition 1), as well as the optimal path of extraction (first part of Proposition 2). An optimal path of investment is, with this convention, a time sequence of functions that is not unique anymore. However, when applied to the optimal path of extraction, all the optimal paths of investment functions would have the same image. Therefore, the uniqueness result can be found again (using the same kind of arguments of compactness, continuity and concavity as used in our proofs) but in a much more complex form. This technical point is left to the reader.

only cross borders in the form of a flow[6]). Now, let us assume that the capital stock is also mobile. In that case, it is possible to simply modify the expression of the total net production $\breve{y}_{t+1}(\cdot)$ (as defined in problem (1)) by adding a capital market game besides the natural resource one. Therefore, the capital used at each period by the technology f_i of a country i to produce its contribution to the final good is a capital resulting from an exchange process — which is different from the capital resulting from the accumulation of investments η_i of this country. In this alternative set-up, the existence of an optimal path of extraction and investment still holds by easily adapting the proof of Proposition 1. However, the uniqueness of the investment allocation path $(\eta_{i,t})_{i\in N,t\in\mathbb{N}}$ is not guaranteed. Several investment plans can be optimal, as long as the total capital accumulated equals the total capital exchanged. But losing this uniqueness result for the investment path does not affect the uniqueness of the other stock variables. Indeed, the investment allocation to each country and its accumulation in a capital stock does not impact the total production anymore, since the capital stocks are redistributed to produce. The optimal path for the "after-exchange" capital stock used as an input by each country is therefore still unique. For example, the investment planner could decide to allocate all the investments either to one country i or to another country j. In both cases, the maximum total production would still be the same since the capital stocks would be exchanged in order to maximize this production.

3.2. Efficient and coalitionally rational distribution policy

We showed that there exists a unique optimal path of extraction and investment for each country in a coalition S, which could be interpreted as quotas imposed by a social planner. This leads to a unique path of resource and capital stocks. As traded quantities are also unique, we can conclude that there exists a unique path for collective production in a coalition S. The consumption path of each country, for its part, is not unique and depends upon the sharing rule decided by the coalition. The following result states that an international long-term agreement along this optimal path will be self-enforcing. The countries, as rational agents, will stick to this agreement, as no other coalition could offer them a better outcome, at any point in time. In other words, this agreement within N cannot be dominated by any coalition $S \subseteq N$. For that purpose, we adapt the concept of *strong sequential core* to our framework. This concept satisfies the properties of efficiency, coalitional rationality and also tackles the crucial issue of time consistency. We now formally introduce this concept.

Let $\mathcal{IS}$ be the set $[0,\bar{\Omega}]^N \times [0,\bar{K}]^N \times [0,\bar{\breve{y}}]$ of initial states. For each initial state $(\Omega, K, \breve{y}) \in \mathcal{IS}$ of the grand coalition N, there exists a unique optimal extraction and investment solution $\left(e_t^*(N), \eta_t^*(N)\right)_{t\in\mathbb{N}}$ to the dynamic programming problem (2) according to Proposition 2. Let $\left(\Omega_t^*(N), K_t^*(N), \breve{y}_t^*(N)\right)_{t\in\mathbb{N}}$ be the unique state evolution corresponding to the optimal path when the initial state is $(\Omega, K, \breve{y})$. We now have the material to define the notion of *international distribution policy* which describes the allocation of consumption goods for each country along the international optimal path.

Definition 1. An international distribution policy (or more briefly a policy), π, is a mapping which associates with each initial state $(\Omega, K, \breve{y}) \in \mathcal{IS}$ a path of consumption vector $(\pi_{i,t}(\Omega, K, \breve{y}))_{i\in N,t\in\mathbb{N}}$ such that $\forall t \in \mathbb{N}$:

$$\sum_{i\in N} \pi_{i,t}(\Omega, K, \breve{y}) = \breve{y}_t^*(N) - \sum_{i\in N} \eta_{i,t}^*(N).$$

We denote by Π the set of international distribution policies. A policy is said to be *dominated* when there exists a point in time from which a coalition can achieve by itself a better payoff than the one proposed by the policy.

Definition 2. A policy π is dominated by S at date T if there exists an initial state $(\Omega, K, \breve{y}) \in \mathcal{IS}$ such that:

$$\sum_{t=0}^{+\infty} \delta^t \sum_{i\in S} \pi_{i,t+T}(\Omega, K, \breve{y}) < V^{\Omega_{S,T}^*(N), K_{S,T}^*(N), \breve{y}_{S,T}^*(N)}(S),$$

where for each nonempty coalition S of N and $t \in \mathbb{N}$, $\left(\Omega_{S,t}^*(N), K_{S,t}^*(N), \breve{y}_{S,t}^*(N)\right)$ denotes the projection $\left(\Omega_{i,t}^*(N), K_{i,t}^*(N), \breve{y}_{i,t}^*(N)\right)_{i\in S}$ of $\left(\Omega_t^*(N), K_t^*(N), \breve{y}_t^*(N)\right)$.

The last definition describes the set of policies for which countries stick to the global policy, as no other coalition can offer them a better outcome, at any point in time. The *strong sequential core* in our setting refers to the set of undominated policies.

Definition 3. The strong sequential core $\mathcal{C}$ is the set of policies $\pi \in \Pi$ such that $\forall(\Omega, K, \breve{y}) \in \mathcal{IS}$, $\forall S \subseteq N$, $\forall T \in \mathbb{N}$,

$$\sum_{t=0}^{+\infty} \delta^t \sum_{i\in S} \pi_{i,t+T}(\Omega, K, \breve{y}) \geq V^{\Omega_{S,T}^*(N), K_{S,T}^*(N), \breve{y}_{S,T}^*(N)}(S).$$

The following theorem states that the strong sequential core of this set-up is nonempty whatever the initial state is. The theorem further indicates that this core contains an infinity of elements, corresponding to an infinite way of distributing consumption streams among the countries of the international coalition. Such policies have the property that no coalition can form at any point in time and design a policy that is feasible for the coalition while making all coalition members better off.[7]

Theorem 1. *The strong sequential core $\mathcal{C}$ contains an infinity of elements.*

This theorem states that, if the countries of a coalition follow a long-term cooperation strategy in terms of extraction, investment and induced trade, then the best way to optimize their own consumption while caring about future generations is to form a global coalition. The strong sequential core is constituted of the streams of consumption to be allocated to each country. Since this core is not reduced to a singleton, it is possible to compare the different distribution policies and to select a policy among these allocations. Indeed, the strong sequential core meets a coalitional stability criterion, but several sub-solutions apply to our model that combine stability with other criteria. For instance, a fairness criterion can be investigated in the tradition of the axiomatic methodology in order to explore possible ethic consumption paths. One of the most explored solutions in the literature is the nucleolus solution concept, which always belongs to the core when it is nonempty (Schmeidler, 1969). This solution maximizes recursively the 'welfare' of the worst treated coalitions

[6] This can be justified for both interpretations of the definition of capital as raised by the Cambridge controversy. If a physical capital is at stake, we can take the example of a factory that cannot be moved, but can only be bought or sold. If we are talking about monetary capital, accumulating capital in another country means buying or selling shares of a company, which is the exact definition of a foreign direct investment.

[7] The feasibility constraint on investments in problem (2) ensures that $v^{\breve{y}_t(S),\eta_t}(S)$ is always nonnegative, as well as $V^{\Omega_0,K_0,\breve{y}_0(S)}(S)$. For example, this is true if $S = \{i\}$. So, each country can ensure a nonnegative consumption in autarchy. The nonnegativity of each country's consumption stream follows from the definition of the strong sequential core. Indeed, the level of consumption of each country is given by a policy belonging to this core which guarantees to each country a better consumption than the one secured by autarchy (or any other subcoalition).

and can be understood as an application of the Rawlsian social welfare function (Hamlen et al., 1977). Other interesting solution concepts, that also belong to the core, consider an egalitarian perspective. For instance, the egalitarian core (Arin and Inarra, 2001) is the set of core allocations for which no transfer from a rich country to a poor country is possible without going beyond the core boundaries. The existence of such solutions shows that there are theoretical ways for international cooperation to tackle the North-South divide and related inequality issues in a world where international regulation is delicate without implementing self-enforcing agreements.

4. Discussion and concluding remarks

Dynamic core concepts for cooperative games have drawn some attention in the literature, be it from a technical perspective in general frameworks, but also in models applied to diverse economic situations. An original motivation of this article is to question whether the classical result of the core nonemptiness in a static TU market game remains true in a dynamic framework. The extension is not immediate since, for each coalition and at each period, a static optimization problem – consisting in finding efficient trading quotas – is nested in the dynamic problem which seeks to maximize the level of consumption. It follows that the exogenous basket of goods in the seminal article is endogenized in ours and constructed as part of the coalitions' long-term strategy. This strategy itself is submitted to natural resource and capital stock constraints on the long run. The present article proposes a nonempty core solution to an economic problem modeled through a dynamic cooperative game with transferable utility. Avrachenkov et al. (2013) design a general framework closely related to our setting, by introducing a discrete-time Markov chain of static games, where the utility of the coalitions is transferable and whose transition probabilities depend on the players' actions in each state. They extend the core concept to this framework and find a condition for nonemptiness. Several papers do apply the strong sequential core concept but in a NTU configuration. Among them, only a few show positive results as regards to its nonemptiness. As argued by Habis and Herings (2011), it is empty-valued for large classes of economies. One exception comparable to the present set-up is the paper by Becker and Chakrabarti (1995) designed for the simple case of capital accumulation in a closed economy with no resource dependency. The originality of our methodology is the finding of a demanding result – that is a nonempty core – embedded in complex and endogenous economic dynamics of resource exhaustion, capital accumulation and input trade, by implementing standard convex optimization techniques.

This methodology is also quite neutral with respect to the interpretation of the decision process, due to the self-enforcing feature of the agreements that the core involves. We showed that an international cooperation strategy leads to a unique optimal path of collective production since all the processes involved in this production – namely the extraction of natural resources from a finite stock, their trade and the evolution of the productive capacity – should be unique to be optimal. On the demand side, a given part of this production is optimally allocated to the investment needed to build the productive capacity of each country, in a unique manner. The remaining part of production, corresponding to the consumption of goods, can be shared among countries, this time in different manners. The optimal paths could be interpreted as resulting from quotas chosen by a benevolent planner, but this interpretation that we propose could be objected. Indeed, imposing commercial quotas is not consistent with a common definition of free trade. This objection is fully anchored in the socialist calculation debate (O'Neill, 1996) and is not the subject of this study. Note however that, by definition of optimality, agents should not deviate from these quotas even in a free market interpretation. Another alternative interpretation would be the result of a negotiation procedure between countries. In all cases, no country would have the incentive to refuse a core allocation. This last interpretation is linked to the question of the implementation of cooperative game solutions, also called "Nash program". This could be the subject of a subsequent paper.

This set-up provides a basis on which to progress on questions related to the long-term material development of countries. We support the idea that cooperative game theory gives a fresh and fairly simple framework to tackle traditional issues raised in growth theory, since it better takes into account the heterogeneity of a finite number of countries. An interesting question would be to characterize technological catch-up effects between countries and how it affects the balance of power, by adding a time-dependent production function or an endogenous knowledge capital accumulation. Moreover, recycling or pollution effects could be investigated in our framework and lead to a more complete understanding of the sustainability of the "Spaceship Earth" (Boulding, 1966).

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Appendix A

Proof of Proposition 1

Proof. Let $g : [0, \bar{\Omega}]^S \times [0, \bar{\Omega}]^S \times [0, \bar{K}]^S \times \mathbb{R}_+^S \to \mathbb{R}_+$ be the function defined as

$$g((\Omega, e, K), z) = \sum_{i \in S} \Big(f_i(z_i, K_i) - c_i(e_i, \Omega_i) \Big),$$

and $C : [0, \bar{\Omega}]^S \times [0, \bar{\Omega}]^S \times [0, \bar{K}]^S \rightrightarrows \mathbb{R}_+^S$ be the correspondence defined as

$$C(\Omega, e, K) = Z^e(S).$$

By continuity and strict concavity of $f_i - c_i$ for each $i \in S$, function g is continuous and strictly concave on $[0, \bar{\Omega}]^S \times [0, \bar{\Omega}]^S \times [0, \bar{K}]^S \times \mathbb{R}_+^S$. Clearly, C is a compact-valued continuous correspondence on $[0, \bar{\Omega}]^S \times [0, \bar{\Omega}]^S \times [0, \bar{K}]^S$ and has a convex graph. We have the needed conditions to apply Berge's Maximum Theorem under Convexity (Sundaram, 1996, p.237). We conclude that $\max\{g\big((\Omega, e, K), z\big),\ z \in C(\Omega, e, K)\}$ is a continuous and strictly concave function on $[0, \bar{\Omega}]^S \times [0, \bar{\Omega}]^S \times [0, \bar{K}]^S$ and $C^*(\Omega, e, K) = \operatorname{argmax}\{g\big((\Omega, e, K), z\big),\ z \in C(\Omega, e, K)\}$ is a continuous single-valued function. □

Proof of Proposition 2

Proof. Existence. The existence of an optimal path of extraction and investment is a direct application of a well-established theorem of existence in dynamic optimization (see for instance Theorem 12.19 in Sundaram (1996, p.298)). Indeed,

(i) The reward function $(\check{y}(S), \eta) \mapsto v^{\check{y}(S),\eta}(S)$ is continuous and bounded on $[0, \bar{\check{y}}] \times [0, \bar{\check{y}}]^S$.[8]
(ii) It is clear that $(\Omega, e) \mapsto \Omega - e$ is continuous from $[0, \bar{\Omega}]^S \times [0, \bar{\Omega}]^S$ to $[0, \bar{\Omega}]^S$. In addition, $(K, \eta) \mapsto (1-\tau)K + \eta$ is continuous from $[0, \bar{K}]^S \times [0, \bar{\check{y}}]^S$ to $[0, \bar{K}]^S$. By Proposition 1, $(\Omega, e, K) \mapsto \max\left\{\sum_{i\in S}\Big(f_i(z_i, K_i) - c_i(e_i, \Omega_i)\Big), (z_i)_{i\in S} \in Z^e(S)\right\}$ is continuous on $[0, \bar{\Omega}]^S \times [0, \bar{\Omega}]^S \times [0, \bar{K}]^S$. Hence, the transition function is continuous.
(iii) The feasible action correspondence $(\Omega, \check{y}(S)) \mapsto \Phi^{\Omega,\check{y}(S)}(S)$ is compact-valued and continuous.

Uniqueness. Assume by way of contradiction that there exist two different optimal paths $(e_t^0, \eta_t^0)_{t\in\mathbb{N}}$ and $(e_t^1, \eta_t^1)_{t\in\mathbb{N}}$ when the initial vector is $(\Omega_0, K_0, \check{y}_0(S))$ for a coalition S.

We denote by $(\Omega_t^0)_{t\in\mathbb{N}}$ and $(\Omega_t^1)_{t\in\mathbb{N}}$ the respective sequence of resource stock vectors defined by $\Omega_0^0 = \Omega_0$, $\Omega_0^1 = \Omega_0$ and for each $t > 0$, $\Omega_{t+1}^0 = \Omega_t^0 - e_t^0$ and $\Omega_{t+1}^1 = \Omega_t^1 - e_t^1$. We denote by $(K_t^0)_{t\in\mathbb{N}}$ and $(K_t^1)_{t\in\mathbb{N}}$ the respective sequence of capital stock vectors defined by $K_0^0 = K_0$, $K_0^1 = K_0$ and for each $t > 0$, $K_{t+1}^0 = (1-\tau)K_t^0 + \eta_t^0$ and $K_{t+1}^1 = (1-\tau)K_t^1 + \eta_t^1$. For each $\alpha \in [0, 1]$, we define the path of extraction $(e_t^\alpha)_{t\in\mathbb{N}}$ as follows:

$$\forall t \in \mathbb{N},\ e_t^\alpha = \alpha e_t^1 + (1-\alpha)e_t^0;$$

the path of investment $(\eta_t^\alpha)_{t\in\mathbb{N}}$ is defined as:

$$\forall t \in \mathbb{N},\ \eta_t^\alpha = \alpha\eta_t^1 + (1-\alpha)\eta_t^0.$$

Let $(\Omega_t^\alpha)_{t\in\mathbb{N}}$ be the sequence of resource stock vectors defined by $\Omega_0^\alpha = \Omega_0$ and for each $t \in \mathbb{N}$ by $\Omega_{t+1}^\alpha = \Omega_t^\alpha - e_t^\alpha$. A simple induction leads to $\Omega_t^\alpha = \alpha\Omega_t^1 + (1-\alpha)\Omega_t^0$ for each $t \in \mathbb{N}$.

Let $(K_t^\alpha)_{t\in\mathbb{N}}$ be the sequence of capital stock vectors defined by $K_0^\alpha = K_0$ and for each $t \in \mathbb{N}$ by $K_{t+1}^\alpha = (1-\tau)K_t^\alpha + \eta_t^\alpha$. A simple induction leads to $K_t^\alpha = \alpha K_t^1 + (1-\alpha)K_t^0$ for each $t \in \mathbb{N}$.

We set $\check{y}_0^\alpha(S) = \check{y}_0(S)$ and for each $t > 0$,

$$\check{y}_t^\alpha(S) = \max\left\{\sum_{i\in S}\Big(f_i(z_{i,t-1}, K_{i,t-1}^\alpha) - c_i(e_{i,t-1}^\alpha, \Omega_{i,t-1}^\alpha)\Big),\ (z_{i,t-1})_{i\in S} \in Z^{e_{t-1}^\alpha}(S)\right\}.$$

We define for each $t \in \mathbb{N}$ the following vectors[9]:

$$(z_{i,t}^{*0})_{i\in S} = \operatorname*{argmax}_{(z_{i,t})_{i\in S}\in Z^{e_t^0}(S)} \sum_{i\in S}\Big(f_i(z_{i,t}, K_{i,t}^0) - c_i(e_{i,t}^0, \Omega_{i,t}^0)\Big),$$

and

$$(z_{i,t}^{*1})_{i\in S} = \operatorname*{argmax}_{(z_{i,t})_{i\in S}\in Z^{e_t^1}(S)} \sum_{i\in S}\Big(f_i(z_{i,t}, K_{i,t}^1) - c_i(e_{i,t}^1, \Omega_{i,t}^1)\Big).$$

We also define $(z_t^\alpha)_{t\in\mathbb{N}}$ as follows:

$$\forall t \in \mathbb{N},\ z_t^\alpha = \alpha z_t^{*1} + (1-\alpha)z_t^{*0}.$$

It is straightforward to see that

$$\sum_{i\in S}\eta_{i,0}^\alpha \le \check{y}_0 = \check{y}_0^\alpha, \tag{3}$$

and for each $t \in \mathbb{N}$,

$$\begin{aligned}\sum_{i\in S}\eta_{i,t+1}^\alpha &= \alpha\sum_{i\in S}\eta_{i,t+1}^1 + (1-\alpha)\sum_{i\in S}\eta_{i,t+1}^0\\ &\le \alpha\sum_{i\in S}\Big(f_i(z_{i,t}^{*1}, K_{i,t}^1) - c_i(e_{i,t}^1, \Omega_{i,t}^1)\Big)\\ &\quad + (1-\alpha)\sum_{i\in S}\Big(f_i(z_{i,t}^{*0}, K_{i,t}^0) - c_i(e_{i,t}^0, \Omega_{i,t}^0)\Big).\end{aligned}$$

By concavity of f_i and $-c_i$, we therefore have

$$\sum_{i\in S}\eta_{i,t+1}^\alpha \le \sum_{i\in S}\Big(f_i(z_{i,t}^\alpha, K_{i,t}^\alpha) - c_i(e_{i,t}^\alpha, \Omega_{i,t}^\alpha)\Big). \tag{4}$$

Since for each $t \in \mathbb{N}$, $(z_{i,t}^\alpha)_{i\in S} \in Z^{e_t^\alpha}(S)$, it follows that

$$\begin{aligned}\sum_{i\in S}\Big(f_i(z_{i,t}^\alpha, K_{i,t}^\alpha) - c_i(e_{i,t}^\alpha, \Omega_{i,t}^\alpha)\Big) &\le \max\Bigg\{\sum_{i\in S}\Big(f_i(z_{i,t}, K_{i,t}^\alpha)\\ &\quad - c_i(e_{i,t}^\alpha, \Omega_{i,t}^\alpha)\Big),\ (z_{i,t})_{i\in S} \in Z^{e_t^\alpha}(S)\Bigg\}\\ &= \check{y}_{t+1}^\alpha.\end{aligned} \tag{5}$$

By combining Eqs. (3), (4) and (5), we deduce that for each $t \in \mathbb{N}$,

$$\sum_{i\in S}\eta_{i,t}^\alpha \le \check{y}_t^\alpha. \tag{6}$$

On the other hand, it is clear that

$$e_0^\alpha \le \Omega_0,$$

and for each $t \in \mathbb{N}$,

$$\begin{aligned}e_{t+1}^\alpha &= \alpha e_{t+1}^1 + (1-\alpha)e_{t+1}^0 \le \alpha(\Omega_t^1 - e_t^1)\\ &\quad + (1-\alpha)(\Omega_t^0 - e_t^0) = \Omega_t^\alpha - e_t^\alpha.\end{aligned} \tag{7}$$

Therefore, from Eqs. (6) and (7), we can conclude that $(e_t^\alpha, \eta_t^\alpha)_{t\in\mathbb{N}}$ is a feasible path. It follows that

$$\begin{aligned}V^{\Omega_0,K_0,\check{y}_0(S)}(S) &\ge \sum_{t\in\mathbb{N}}\delta^t v^{\check{y}_t^\alpha(S),\eta_t^\alpha}(S)\\ &= \sum_{t\in\mathbb{N}}\delta^t\Big(\check{y}_t^\alpha(S) - \sum_{i\in S}\eta_{i,t}^\alpha\Big)\\ &\ge \Big(\check{y}_0(S) - \sum_{i\in S}\eta_{i,0}^\alpha\Big)\\ &\quad + \sum_{t>0}\delta^t\Bigg(\sum_{i\in S}\Big(f_i(z_{i,t-1}^\alpha, K_{i,t-1}^\alpha)\\ &\quad - c_i(e_{i,t-1}^\alpha, \Omega_{i,t-1}^\alpha)\Big) - \eta_{i,t}^\alpha\Bigg).\end{aligned} \tag{8}$$

By strict concavity of f_i and $-c_i$, we have for each $t > 0$:

$$\begin{aligned}\sum_{i\in S}\Big(f_i(z_{i,t-1}^\alpha, K_{i,t-1}^\alpha) - c_i(e_{i,t-1}^\alpha, \Omega_{i,t-1}^\alpha)\Big) &> \alpha\sum_{i\in S}\Big(f_i(z_{i,t-1}^{*1}, K_{i,t-1}^1)\\ &\quad - c_i(e_{i,t-1}^1, \Omega_{i,t-1}^1)\Big)\\ &\quad + (1-\alpha)\sum_{i\in S}\\ &\quad \times \Big(f_i(z_{i,t-1}^{*0}, K_{i,t-1}^0)\\ &\quad - c_i(e_{i,t-1}^0, \Omega_{i,t-1}^0)\Big).\end{aligned} \tag{9}$$

[8] Note that the domain of the reward function corresponds to the action space (Sundaram, 1996), not to be confused with the feasible set for which investments cannot exceed production.

[9] It follows from Proposition 1 that both vectors are uniquely defined.

Combining Eqs. (8) and (9) leads to:

$$\begin{aligned}
V^{\Omega_0,K_0,\check{y}_0(S)}(S) > \alpha \Bigg(& \Big(\check{y}_0(S) - \sum_{i\in S} \eta^1_{i,0}\Big) \\
& + \sum_{t>0} \delta^t \Big(\sum_{i\in S} \Big(f_i(z^{*1}_{i,t-1}, K^1_{i,t-1}) \\
& - c_i(e^1_{i,t-1}, \Omega^1_{i,t-1})\Big) - \eta^1_{i,t} \Big)\Bigg) \\
& + (1-\alpha) \Bigg(\Big(\check{y}_0(S) - \sum_{i\in S} \eta^0_{i,0}\Big) \\
& + \sum_{t>0} \delta^t \Big(\sum_{i\in S} \Big(f_i(z^{*0}_{i,t-1}, K^0_{i,t-1}) \\
& - c_i(e^0_{i,t-1}, \Omega^0_{i,t-1})\Big) - \eta^0_{i,t} \Big)\Bigg) \\
& = \alpha V^{\Omega_0,K_0,\check{y}_0(S)}(S) + (1-\alpha) V^{\Omega_0,K_0,\check{y}_0(S)}(S) \\
& = V^{\Omega_0,K_0,\check{y}_0(S)}(S).
\end{aligned}$$

The strict inequality leads to a contradiction. □

Appendix B

First, let us recall some definitions, notations, and useful results of the cooperative game theory in order to demonstrate Theorem 1. Let N denote a fixed finite nonempty set with n members, who will be called agents or players. *Coalitions of players* are nonempty subsets of N.

A *transferable utility* (TU) *game* on N is a pair (N, v) where v is a mapping $v : 2^N \to \mathbb{R}$ satisfying $v(\emptyset) = 0$. We denote by $\mathcal{G}(N)$ the set of all games over N. For any coalition S, $v(S)$ represents the *worth* of S, i.e., what coalition S could earn regardless of other players.

A payoff vector is a vector $x \in \mathbb{R}^N$ that assigns to agent i the payoff x_i. A payoff vector is *efficient* with respect to (N, v) if $\sum_{i\in N} x_i = v(N)$; it is *coalitionally rational* if $\sum_{i\in S} x_i \geq v(S)$ for every possible coalition S.

The *core* of (N, v), denoted by $\mathsf{C}(N, v)$, is the set, possibly empty, of efficient and coalitionally rational payoff vectors:

$$\mathsf{C}(N, v) = \Big\{ x \in \mathbb{R}^N \mid \forall S \subseteq N,\ \sum_{i\in S} x_i \geq v(S) \text{ and } \sum_{i\in N} x_i = v(N) \Big\}.$$

The interpretation of the core is that no group of agents has an incentive to split from the grand coalition N and form a smaller coalition S since they collectively receive at least as much as what they can obtain for themselves as a coalition. The so-called Bondareva–Shapley theorem (Bondareva, 1963; Shapley, 1967) provides a sufficient and necessary condition under which the core of a TU-game is nonempty. First, we introduce the concept of balanced maps. A *balanced map* $\lambda : 2^N \longrightarrow [0, 1]$ is such that:

$$\lambda(\emptyset) = 0, \text{ and } \forall i \in N,\ \sum_{S\ni i} \lambda(S) = 1.$$

Denote by $\mathbb{B}(N)$ the set of balanced maps over N.

Proposition 3 (*Bondareva–Shapley Theorem*). *For each TU-game (N, v), $\mathsf{C}(N, v) \neq \emptyset$ if and only if for each balanced map $\lambda \in \mathbb{B}(N)$, the following inequality holds:*

$$\sum_{S\subseteq N} \lambda(S) v(S) \leq v(N). \tag{10}$$

The following proposition is a direct consequence of Corollary 1 in Gonzalez and Grabisch (2015).

Proposition 4. *If, for each balanced map $\lambda \in \mathbb{B}(N)$ such that $\lambda(N) \neq 1$ the following strict inequality holds:*

$$\sum_{S\subseteq N} \lambda(S) v(S) < v(N),$$

then $\mathsf{C}(N, v)$ contains an infinity of elements.

Proof of Theorem 1

Proof. We proceed in two steps.

Step 1: For each initial state $(\Omega_0(N), K_0(N), \check{y}_0(N)) \in \mathcal{IS}$,

$$\mathsf{C}(N, V^{\Omega_0(N),K_0(N),\check{y}_0(N)}) \neq \emptyset$$

and contains an infinity of elements.

By Proposition 2, for each nonempty coalition $S \subseteq N$, and each initial stock vector

$$\Big(\Omega_0(S), K_0(S), \check{y}_0(S)\Big) \in [0, \bar{\Omega}]^S \times [0, \bar{K}]^S \times [0, \bar{\check{y}}],$$

there exists a unique optimal path $\Big(e^*_t(S), \eta^*_t(S)\Big)_{t\in\mathbb{N}}$ to the dynamic programming problem (2). For each nonempty coalition $S \subseteq N$, denote by $(\Omega^*_t(S))_{t\in\mathbb{N}}$ the sequence of resource stock vectors defined by $\Omega^*_0(S) = \Omega_0(S)$ and for each $t \in \mathbb{N}$, $\Omega^*_{t+1}(S) = \Omega^*_t(S) - e^*_t(S)$. Denote as well by $(K^*_t(S))_{t\in\mathbb{N}}$ the sequence of capital stock vectors defined by $K^*_0(S) = K_0(S)$ and for each $t \in \mathbb{N}$, $K^*_{t+1}(S) = (1-\tau)K^*_t(S) + \eta^*_t(S)$. Finally, denote $(\check{y}^*_t(S))_{t\in\mathbb{N}}$ the sequence of production flow vectors defined by $\check{y}^*_0(S) = \check{y}_0(S)$ and for each $t \in \mathbb{N}$,

$$\begin{aligned}
\check{y}^*_{t+1}(S) = \max \Bigg\{ & \sum_{i\in S} \Big(f_i(z_{i,t}(S), K^*_{i,t}(S)) - c_i(e^*_{i,t}(S), \Omega^*_{i,t}(S)) \Big), \\
& (z_{i,t}(S))_{i\in S} \in Z^{e^*_t(S)}(S) \Bigg\}.
\end{aligned}$$

Let $\lambda \in \mathbb{B}(N)$ be a balanced system of N. If $\lambda(N) = 1$, it is clear that

$$V^{\Omega_0(N),K_0(N),\check{y}_0(N)}(N) = \sum_{S\subseteq N} \lambda(S) V^{\Omega_0(S),K_0(S),\check{y}_0(S)}(S).$$

Assume $\lambda(N) \neq 1$. We define $(e^\lambda_t(N))_{t\in\mathbb{N}} \in (\mathbb{R}^N_+)^{\mathbb{N}}$ and $(\Omega^\lambda_t(N))_{t\in\mathbb{N}} \in (\mathbb{R}^N_+)^{\mathbb{N}}$ as follows:

$$\forall i \in N,\ \forall t \in \mathbb{N}, \quad e^\lambda_{i,t}(N) = \sum_{S\ni i} \lambda(S) e^*_{i,t}(S),$$

$$\forall i \in N,\ \forall t \in \mathbb{N}, \quad \Omega^\lambda_{i,t}(N) = \sum_{S\ni i} \lambda(S) \Omega^*_{i,t}(S).$$

Observe that for each $i \in N$ and each $t \in \mathbb{N}$, we have $\Omega^\lambda_{i,t}(N) \in [0, \bar{\Omega}]$ since $\sum_{S\ni i} \lambda(S) = 1$ and $\Omega^\lambda_{i,t+1}(N) = \Omega^\lambda_{i,t}(N) - e^\lambda_{i,t}(N)$. We also can assert that for each $i \in N$ and each $t \in \mathbb{N}$,

$$e^\lambda_{i,t}(N) \leq \Omega^\lambda_{i,t}(N). \tag{11}$$

We define as well $(\eta^\lambda_t(N))_{t\in\mathbb{N}} \in (\mathbb{R}^N_+)^{\mathbb{N}}$ and $(K^\lambda_t(N))_{t\in\mathbb{N}} \in (\mathbb{R}^N_+)^{\mathbb{N}}$ as follows:

$$\forall i \in N,\ \forall t \in \mathbb{N}, \quad \eta^\lambda_{i,t}(N) = \sum_{S\ni i} \lambda(S) \eta^*_{i,t}(S),$$

$$\forall i \in N,\ \forall t \in \mathbb{N}, \quad K^\lambda_{i,t}(N) = \sum_{S\ni i} \lambda(S) K^*_{i,t}(S).$$

Observe that for each $i \in N$ and each $t \in \mathbb{N}$, we have $K^\lambda_{i,t}(N) \in [0, \bar{K}]$. We set $\check{y}^\lambda_0(N) = \check{y}_0(N)$ and for each $t > 0$,

$$\check{y}^\lambda_t(N) = \max\Bigg\{\sum_{i\in N}\Big(f_i(z_{i,t-1}(N), K^\lambda_{i,t-1}(N)) - c_i(e^\lambda_{i,t-1}(N), \Omega^\lambda_{i,t-1}(N))\Big), (z_{i,t-1}(N))_{i\in N} \in Z^{e^\lambda_{t-1}(N)}(N)\Bigg\}.$$

Observe that for each $i \in N$ and each $t \in \mathbb{N}$, we have $\check{y}^\lambda_{i,t}(N) \in [0, \bar{\check{y}}]$. It is straightforward to see that

$$\sum_{i\in N} \eta^\lambda_{i,0}(N) \le \check{y}_0(N) = \check{y}^\lambda_0(N), \tag{12}$$

and for each $t \in \mathbb{N}$,

$$\begin{aligned}\sum_{i\in N} \eta^\lambda_{i,t+1}(N) &= \sum_{i\in N}\sum_{S\ni i} \lambda(S)\eta^*_{i,t+1}(S)\\ &= \sum_{S\subseteq N}\lambda(S)\sum_{i\in S}\eta^*_{i,t+1}(S) \qquad (13)\\ &\le \sum_{S\subseteq N}\lambda(S)\check{y}^*_{i,t+1}(S)\\ &= \sum_{S\subseteq N}\lambda(S)\sum_{i\in S}\Big(f_i(z^*_{i,t}(S), K^*_{i,t}(S)) - c_i(e^*_{i,t}(S), \Omega^*_{i,t}(S))\Big)\\ &= \sum_{i\in N}\sum_{S\ni i}\lambda(S)\Big(f_i(z^*_{i,t}(S), K^*_{i,t}(S)) - c_i(e^*_{i,t}(S), \Omega^*_{i,t}(S))\Big),\end{aligned}$$

where we define, for each $t \in \mathbb{N}$,

$$z^*_t(S) = \underset{(z_{i,t}(S))_{i\in S}\in Z^{e^*_t}(S)}{\operatorname{argmax}} \sum_{i\in S}\Big(f_i(z_{i,t}, K^*_{i,t}) - c_i(e^*_{i,t}, \Omega^*_{i,t})\Big).$$

By concavity of f_i and $-c_i$, we therefore have

$$\sum_{i\in N}\eta^\lambda_{i,t+1}(N) \le \sum_{i\in N}\Big(f_i(z^\lambda_{i,t}(N), K^\lambda_{i,t}(N)) - c_i(e^\lambda_{i,t}(N), \Omega^\lambda_{i,t}(N))\Big) \tag{14}$$

where $(z^\lambda_t(N))_{t\in\mathbb{N}} \in (\mathbb{R}^N_+)^{\mathbb{N}}$ is defined as

$$\forall i \in N,\ \forall t \in \mathbb{N}, \quad z^\lambda_{i,t}(N) = \sum_{S\ni i}\lambda(S)z^*_{i,t}(S).$$

The following equalities hold:

$$\begin{aligned}\sum_{i\in N} e^\lambda_{i,t}(N) &= \sum_{i\in N}\sum_{S\ni i}\lambda(S)e^*_{i,t}(S)\\ &= \sum_{S\subseteq N}\lambda(S)\sum_{i\in S}e^*_{i,t}(S).\end{aligned}$$

Since $z^*_t(S) \in Z^{e^*_t}(S)$, it follows that

$$\begin{aligned}\sum_{S\subseteq N}\lambda(S)\sum_{i\in S}e^*_{i,t}(S) &= \sum_{S\subseteq N}\lambda(S)\sum_{i\in S}z^*_{i,t}(S)\\ &= \sum_{i\in N}\sum_{S\ni i}\lambda(S)z^*_{i,t}(S)\\ &= \sum_{i\in N}z^\lambda_{i,t}(N),\end{aligned}$$

from which we deduce that $z^\lambda_t(N) \in Z^{e^\lambda_t(N)}(N)$ for each $t \in \mathbb{N}$. It follows that

$$\begin{aligned}\sum_{i\in N}\Big(f_i(z^\lambda_{i,t}(N), K^\lambda_{i,t}(N)) - c_i(e^\lambda_{i,t}(N), \Omega^\lambda_{i,t}(N))\Big) &\le \max\Bigg\{\sum_{i\in N}\Big(f_i(z_{i,t}(N), K^\lambda_{i,t}(N)) - c_i(e^\lambda_{i,t}(N), \Omega^\lambda_{i,t}(N))\Big),\ (z_{i,t}(N))_{i\in N} \in Z^{e^\lambda_t(N)}(N)\Bigg\}\\ &= \check{y}^\lambda_{t+1}(N).\end{aligned} \tag{15}$$

By combining Eqs. (12), (14) and (15), we deduce that for each $t \in \mathbb{N}$,

$$\sum_{i\in N}\eta^\lambda_{i,t}(N) \le \check{y}^\lambda_t(N). \tag{16}$$

We can conclude from Eqs. (11) and (16) that $(e^\lambda_t(N), \eta^\lambda_t(N))_{t\in\mathbb{N}}$ is a feasible path.

By definition of $\check{y}^\lambda_t(N)$ for each $t \in \mathbb{N}$, and since $z^\lambda_t(N) \in Z^{e^\lambda_t(N)}(N)$ for each $t \in \mathbb{N}$, we have

$$\check{y}^\lambda_{t+1}(N) \ge \sum_{i\in N}\Big(f_i(z^\lambda_{i,t}(N), K^\lambda_{i,t}(N)) - c_i(e^\lambda_{i,t}(N), \Omega^\lambda_{i,t}(N))\Big). \tag{17}$$

The hypothesis of strict concavity ensures that

$$\begin{aligned}&\sum_{i\in N}\Big(f_i(z^\lambda_{i,t}(N), K^\lambda_{i,t}(N)) - c_i(e^\lambda_{i,t}(N), \Omega^\lambda_{i,t}(N))\Big)\\ &> \sum_{i\in N}\sum_{S\ni i}\lambda(S)\Big(f_i(z^*_{i,t}(S), K^*_{i,t}(S)) - c_i(e^*_{i,t}(S), \Omega^*_{i,t}(S))\Big)\end{aligned} \tag{18}$$

because for each $i \in N$ and $t \in \mathbb{N}$, $z^\lambda_{i,t}(N)$ (resp. $K^\lambda_{i,t}(N)$, $e^\lambda_{i,t}(N)$, $\Omega^\lambda_{i,t}(N)$) is a convex combination of $(z^*_{i,t}(S))_{S\ni i}$ (resp. $(K^*_{i,t}(S))_{S\ni i}$, $(e^*_{i,t}(S))_{S\ni i}$, $(\Omega^*_{i,t}(S))_{S\ni i}$).

Combining Eqs. (17) and (18), we have:

$$\begin{aligned}\check{y}^\lambda_{t+1}(N) &> \sum_{i\in N}\sum_{S\ni i}\lambda(S)\Big(f_i(z^*_{i,t}(S), K^*_{i,t}(S)) - c_i(e^*_{i,t}(S), \Omega^*_{i,t}(S))\Big)\\ &= \sum_{S\subseteq N}\lambda(S)\sum_{i\in S}\Big(f_i(z^*_{i,t}(S), K^*_{i,t}(S)) - c_i(e^*_{i,t}(S), \Omega^*_{i,t}(S))\Big)\\ &= \sum_{S\subseteq N}\lambda(S)\check{y}^*_{t+1}(S).\end{aligned} \tag{19}$$

Since $(e^\lambda_t(N), \eta^\lambda_t(N))_{t\in\mathbb{N}}$ is a feasible path, and that $\Omega^\lambda_0(N) = \Omega_0(N)$, $K^\lambda_0(N) = K_0(N)$ and $\check{y}^\lambda_0(N) = \check{y}_0(N)$, the definition of $V^{\Omega_0(N),K_0(N),\check{y}_0(N)}(N)$ ensures that

$$V^{\Omega_0(N),K_0(N),\check{y}_0(N)}(N) \ge \sum_{t\in\mathbb{N}}\delta^t\Big(\check{y}^\lambda_t(N) - \sum_{i\in N}\eta^\lambda_{i,t}(N)\Big).$$

Moreover, observe that

$$\check{y}_0(N) = \sum_{S\subseteq N}\lambda(S)\check{y}_0(S). \tag{20}$$

Indeed,

$$\begin{aligned}\sum_{S\subseteq N}\lambda(S)\check{y}_0(S) &= \sum_{S\subseteq N}\lambda(S)\sum_{i\in S}\check{y}_{i,0}\\ &= \sum_{i\in N}\sum_{S\ni i}\lambda(S)\check{y}_{i,0}\\ &= \sum_{i\in N}\check{y}_{i,0}\sum_{S\ni i}\lambda(S)\\ &= \sum_{i\in N}\check{y}_{i,0}.\end{aligned}$$

Using Eqs. (20), (13) and (19), we obtain

$$\begin{aligned} V^{\Omega_0(N),K_0(N),\check{y}_0(N)}(N) &= \Big(\check{y}_0(N) - \sum_{i\in N}\eta_0^{\lambda}(N)\Big) \\ &\quad + \sum_{t>0}\delta^t\Big(\check{y}_t^{\lambda}(N) - \sum_{i\in N}\eta_{i,t}^{\lambda}(N)\Big) \\ &> \Big(\sum_{S\subseteq N}\lambda(S)\check{y}_0(S) - \sum_{S\subseteq N}\lambda(S)\sum_{i\in S}\eta_{i,0}^{*}(S)\Big) \\ &\quad + \sum_{t>0}\delta^t\Big(\sum_{S\subseteq N}\lambda(S)\check{y}_t^{*}(S) - \sum_{S\subseteq N}\lambda(S)\sum_{i\in S}\eta_{i,t}^{*}(S)\Big) \\ &= \sum_{S\subseteq N}\lambda(S)\sum_{t\in\mathbb{N}}\delta^t\Big(\check{y}_t^{*}(S) - \sum_{i\in S}\eta_{i,t}^{*}(S)\Big) \\ &= \sum_{S\subseteq N}\lambda(S)V^{\Omega_0(S),K_0(S),\check{y}_0(S)}(S). \end{aligned}$$

The strict inequality holds by strict concavity and since $\lambda(N) \neq 1$.

By Proposition 3 stating the Bondareva–Shapley theorem, $\mathsf{C}(N, V^{\Omega_0(N),K_0(N),\check{y}_0(N)}) \neq \emptyset$ and by Proposition 4, $\mathsf{C}(N, V^{\Omega_0(N),K_0(N),\check{y}_0(N)})$ contains an infinity of elements.

Step 2: Construction of the policy.
Step 1 states that for each state $(\Omega, K, \check{y})$, the core $\mathsf{C}(N, V^{\Omega,K,\check{y}})$ is nonempty. Hence, for each initial state $(\Omega, K, \check{y})$ we can build a sequence $((X_{i,t}(\Omega, K, \check{y}))_{i\in N})_{t\in\mathbb{N}}$ such that for each $t \in \mathbb{N}$:

(i) $\sum_{i\in N} X_{i,t}(\Omega, K, \check{y}) = V^{\Omega_t^*(N),K_t^*(N),\check{y}_t^*(N)}(N)$,
(ii) $\sum_{i\in S} X_{i,t}(\Omega, K, \check{y}) \geq V^{\Omega_{S,t}^*(N),K_{S,t}^*(N),\check{y}_{S,t}^*(N)}(S)$,

where $(\Omega_t^*(N), K_t^*(N), \check{y}_t^*(N))_{t\in\mathbb{N}}$ is the sequence of state vectors associated with the optimal extraction and investment path of N when the initial state is $(\Omega, K, \check{y})$.

Let π the mapping which associates to each initial state $(\Omega, K, \check{y})$ the allocation path for each country $(\pi_{i,t}(\Omega, K, \check{y}))_{i\in N,t\in\mathbb{N}}$ such that for each $t \in \mathbb{N}$, $\pi_{i,t}(\Omega, K, \check{y}) = X_{i,t}(\Omega, K, \check{y}) - \delta X_{i,t+1}(\Omega, K, \check{y})$.

Let us first show that π is an international distribution policy. For each $t \in \mathbb{N}$,

$$\begin{aligned} \sum_{i\in N}\pi_{i,t}(\Omega, K, \check{y}) &= \sum_{i\in N}X_{i,t}(\Omega, K, \check{y}) - \delta\sum_{i\in N}X_{i,t+1}(\Omega, K, \check{y}) \\ &= V^{\Omega_t^*(N),K_t^*(N),\check{y}_t^*(N)}(N) \\ &\quad - \delta V^{\Omega_{t+1}^*(N),K_{t+1}^*(N),\check{y}_{t+1}^*(N)}(N) \end{aligned}$$

and the Bellman equation ensures that:

$$V^{\Omega_t^*(N),K_t^*(N),\check{y}_t^*(N)}(N) = v^{\check{y}_t^*(N),\eta_t^*(N)}(N) + \delta V^{\Omega_{t+1}^*(N),K_{t+1}^*(N),\check{y}_{t+1}^*(N)}(N).$$

Therefore, we have for each $t \in \mathbb{N}$,

$$\sum_{i\in N}\pi_{i,t}(\Omega, K, \check{y}) = v^{\check{y}_t^*(N),\eta_t^*(N)}(N).$$

Let us now prove that π is an undominated policy. For each $T \in \mathbb{N}$, we have

$$\begin{aligned} \sum_{i\in S}\sum_{t=0}^{\infty}\delta^t\pi_{i,t+T}(\Omega, K, \check{y}) &= \sum_{i\in S}\sum_{t=0}^{\infty}\big(\delta^t X_{i,t+T}(\Omega, K, \check{y}) \\ &\quad - \delta^{t+1}X_{i,t+T+1}(\Omega, K, \check{y})\big) \\ &= \sum_{i\in S}X_{i,T}(\Omega, K, \check{y}) \\ &\geq V^{\Omega_{S,T}^*(N),K_{S,T}^*(N),\check{y}_{S,T}^*(N)}(S). \end{aligned}$$

We conclude that π belongs to the strong sequential core $\mathcal{C}$. Since π was built for each initial state $(\Omega, K, \check{y})$ with an arbitrary element of $\mathsf{C}(N, V^{\Omega,K,\check{y}})$, we conclude from Step 1 that $\mathcal{C}$ also contains an infinity of elements. □

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