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HOMEWORK I — RBC Model with a Government and Endogenous Labor

Rules

- This homework should be answered by a maximum of two people. Collaboration is **Encouraged** as long as all students try at least once every question. Please write with who did you collaborate.
- This homework is part of the foundations we will see later, please take it seriously!
- Any questions, please don't hesitate to ask. We will also be working on this homework!
- The homework is due on April 25 by email to `mgiardau@fen.uchile.cl`.
- The email must include a pdf with your answers and a .zip file with the codes and modules needed.

Model

Households. The economy is populated by infinite identical households that solve the following problem:

$$\max_{\{C_t, L_t, K_t\}_t^\infty} \sum_{s=0}^{\infty} \beta^t u(C_t, L_t)$$
$$s.t.$$
$$K_{t+1} + C_t = (1-\tau_t) W_t L_t + R_t K_t$$

where K_t is capital holdings, C_t is consumption, L_t labor supplied, τ_t is a proportional labor tax, W_t wages, R_t the return on capital, and δ is the depreciation rate. $u_c > 0$, $u_{cc} < 0$ and $u_l < 0$, $u_{ll} < 0$.

Firms. Firms maximize profits and produce according to a Cobb-Douglas production technology. Firms pay the depreciation rate, so the cost of capital is given by $R_t - (1-\delta)$ (in net terms, this is $r_t + \delta$).

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} \tag{0-1}$$

Government. The government follows a balanced budget

$$G_t = \tau_t W_t L_t$$

Exogenous Shocks. We assume there are two exogenous shocks, G_t and A_t that follow AR(1) processes as follows:

$$\log(G_t) =(1-\rho_G)\log(\bar{g}Y)+\rho_G\log(G_{t-1})+\sigma_G\epsilon_t^G$$
$$\log(A_t) =\rho_A\log(A_{t-1})+\sigma_A\epsilon_t^A$$

with $\bar{g}$ government spending as a share of GDP in steady state.

Market Clearing. Goods market clearing holds

$$Y_t = C_t + G_t + I_t$$

Recall that

$$K_{t+1} = (1-\delta)K_t + I_t$$

Warming-up with State Space Methods

1. Solve the model and express the equilibrium conditions given a general $u(.,.)$.
2. (`Python`) Write a function that takes the system of equations and returns an array of the residuals of the different equations. Write it in dynamic form, i.e.; the function should take past, current, and future timing variables, the parameters, and the shocks.
3. (`Python`) Write a `class` with your name in it that decorates the model (with `@decorator`) has some *Operator Overloading* functions which at least analyze the size of the system and takes some default arguments. [*Hint:* try to make the function callable and return some nice print of the name of the model]
4. In your `class`, write a code that finds the Steady State. Be as general as possible. The goal here is to write the function such that we can decorate any system, and it calculates the steady state. Use your own solver to find the steady state. [*Hint:* Use `*args` or `**kwargs`].

From now on, consider three types of utility functions:

$$\text{Inelastic labor: } u(C_t, L=1) = \frac{C_t^{1-\sigma}}{1-\sigma}$$
$$\text{CRRA separable: } u(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \psi\frac{L_t^{1+\varphi}}{1+\varphi}$$
$$\text{GHH nonseparable: } u(C_t, L_t) = \frac{1}{1-\sigma}\left(c - \psi\frac{L_t^{1+\varphi}}{1+\varphi}\right)^{1-\sigma}$$

and the following calibration:

$$\alpha = 0.3,\;\; \beta = 0.96,\;\; \sigma = 2,\;\; \varphi = 1,\;\; \psi = 1,\;\; \delta = 0.01,\;\; \rho_A = 0.75,\;\; \rho_G = 0.75,\;\; \bar{g} = 0.2$$

5. Find the steady state of the economies for the three different utility functions. What's the challenge here? Explain and show the solution method.

6. Plot equilibrium labor (in steady state) for a grid of productivities in $(0.5, 1.5)$ and for $\sigma = [0.8, 1, 2]$. Explain the results for the case with separable and nonseparable utility. What's the role of wealth effects?

7. In your `class`, write a code that linearizes the system of equations and then solves it using *Time Iteration* and *gensys*.

8. Show output, investment, and consumption responses to a government spending shock. Use both time iteration and gensys to solve the model; they should give the same result. Analyze the role of wealth effects through labor supply in the effects of government spending.

9. Study the effects of TFP shocks and the role of wealth effects in the transmission of these shocks. Do negative TFP shocks look like government spending shocks? Why?

Transitional Dynamics

1. Write a function that takes $T \times N_U$ unknowns, where T is the period of truncation (say $T = 300$) of the problem and N_U is the number of unknown paths. The function should return an array (or vector) of residuals of this system for paths of the two shocks A_t and G_t.

2. (`Python`) Write a `class` that decorates the system of equations with `__init__` specs that analyzes the problem.

3. (`Python`) In your `class`, generate a function that calculates the steady state of the problem. You can give an auxiliary function with the system of equations in steady state.

4. In an auxiliary `module`, write your favorite non-linear multivariate solver and find the optimal paths of the unknown variables.

5. Read Boppart et al. (2018) and make a similar exercise. Write a function that solves for an MIT shock of size $eps = 0.000001$ and show that the rescaled responses to these shocks are similar. This is, show that for small shocks, the response to an MIT shock is linear. Show that this holds for both shocks (G_t and A_t). Why is this an important result?

6. (bonus, harder) Read Auclert et al. (2021) sections 1 and 2 (more of this paper later). Explain in a few words what they do. Then, use the same concept of solution in your own problem. This is, express the solution of the problem as a combination of Jacobians of the sequence-space. Is this solution similar to the found in the previous item?

References

Auclert, Adrien, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub, "Using the sequence-space Jacobian to solve and estimate heterogeneous-agent models," *Econometrica*, 2021, *89* (5), 2375–2408.

Boppart, Timo, Per Krusell, and Kurt Mitman, "Exploiting MIT shocks in heterogeneous-agent economies: the impulse response as a numerical derivative," *Journal of Economic Dynamics and Control*, 2018, *89*, 68–92.

Submitted by Mario Giarda on April 11, 2024.