

# Estimadores Máximo Verosímiles

---

---

---

---

---



## Estimadores - Método de Máxima Verosimilitud

Si  $X \sim \text{Binomial}(n, p)$ , determinar el estimador de "p" por método de máxima verosimilitud, a través de una M.A.S de tamaño k. Determinar si el estimador obtenido es insesgado y consistente.

$x_1, x_2, \dots, x_k \leftarrow \text{MAS de tamaño } k.$

$X \sim \text{Binomial}(n, p)$

Función de verosimilitud:

$$L(p) = \prod_{i=1}^k P(x_i | p) = \prod_{i=1}^k \left[ \binom{n}{x_i} p^{x_i} (1-p)^{n-x_i} \right]$$

caso continuo:

$$L(\theta) = \prod_{i=1}^k f(x_i, \theta)$$

Función Prob. Binomial

$$P(X = x_i) = \binom{n}{x_i} \cdot p^{x_i} (1-p)^{n-x_i}$$

$$L(p) = \left[ \prod_{i=1}^k \binom{n}{x_i} \right] \cdot \left[ \prod_{i=1}^k p^{x_i} \right] \cdot \left[ \prod_{i=1}^k (1-p)^{n-x_i} \right]$$

$$L(p) = C \cdot p^{\sum_{i=1}^k x_i} \cdot (1-p)^{\sum_{i=1}^k (n-x_i)}$$

$$\underline{a}^x \cdot \underline{a}^y = \underline{a}^{x+y}$$

Logaritmo de  $L(p)$ :

$$\ell(p) = \ln L(p) = \ln C + \ln \left[ p^{\sum_{i=1}^k x_i} \right] + \ln \left[ (1-p)^{\sum_{i=1}^k (n-x_i)} \right]$$

$$\ln(a^b) = b \cdot \ln(a)$$

$$= \ln C + \left[ \sum_{i=1}^k x_i \right] \ln p + \left[ \sum_{i=1}^k (n-x_i) \right] \ln(1-p)$$

Derivar  $l(p)$  respecto a  $p$  e igualar a cero:

$$0 = \frac{\partial l(p)}{\partial p} = 0 + \left( \sum_{i=1}^k x_i \right) \frac{1}{p} + \left( \sum_{i=1}^k (n - x_i) \right) \frac{-1}{1-p}$$

$$(cte \cdot f(x))' = cte \cdot f'(x)$$

$$(cte)' = 0$$

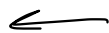
$$(\ln(u))' = \frac{u'}{u}$$

$$\frac{\sum_{i=1}^k (n - x_i)}{1-p} = \frac{\sum_{i=1}^k x_i}{p} \Rightarrow p \left( \sum_{i=1}^k (n - x_i) \right) = (1-p) \sum x_i$$
$$p \cdot k \cdot n - p \sum x_i = \sum x_i - p \sum x_i$$

$$p \cdot k \cdot n = \sum_{i=1}^k x_i$$

$$p = \frac{\sum_{i=1}^k x_i}{k \cdot n} = \frac{\bar{x}}{n}$$

$$\hat{p}_{MV} = \frac{\bar{x}}{n}$$



Estimador M.V.  
de la Binomial  $(1, p)$