L'approche de

Black-Litterman

Motivations

- L'approche de Markowitz suggère d'utiliser les rendements anticipés et la matrice de covariance pour construire la frontière efficiente
- le CAPM nous donne les relation d'équilibre: tout investisseur détient un portefeuille combinaison linéaire du portefeuille de marché et de l'actif sans risque

En absence d'information supplémentaire, il suffit d'acheter l'indice pour être efficient ....

Mais qu'en est il si on possède des anticipations propres (vues) sur les rendements anticipés ? Comment concilier "au mieux" ces anticipations avec celles du marché ?

Exemple: les secteurs européens

Asset	hist. Return	hist. Volatility	Market Cap	beta
AUTO	8.32%	25.09%	1.65%	1.42
BANK	15.14%	21.21%	15.04%	0.96
BRES	7.31%	23.56%	1.22%	1.09
CHEM	12.25%	20.81%	1.80%	0.99
CONS	6.56%	18.92%	1.26%	0.97
CYCL	5.24%	19.94%	2.85%	1.30
CNYL	11.80%	16.66%	2.90%	0.71
ENGY	14.92%	20.72%	10.30%	0.80
FISV	13.01%	20.91%	4.12%	0.60
FBEV	10.47%	15.72%	4.59%	0.78
INDS	13.45%	19.35%	5.19%	1.11
INSU	17.43%	19.68%	6.89%	1.55
MEDA	14.63%	25.17%	3.27%	1.20
PHRM	22.83%	16.20%	10.24%	0.67
RETL	9.49%	16.16%	2.27%	0.74
TECH	25.95%	28.60%	11.03%	1.92
TELE	18.99%	25.69%	10.56%	0.87
UTLY	11.77%	17.25%	4.82%	0.54

gamma 3

Le portefeuille optimal est obtenu en recherchant la solution d'un problème d'optimisation quadratique (voir cours) :

$$\max_w w^T \Pi - \frac{1}{2}\gamma w^T \Sigma w \quad (1)$$

avec w les pondérations, Σ la matrice de covariance, γ l'aversion au risque et Π le vecteur des rendements espérés.

Sans contrainte sur les pondérations (ventes à découvert et emprunts autorisés), cette optimisation conduit à la solution suivante (voir cours) :

$$w^* = \frac{1}{\gamma}\Sigma^{-1}\Pi \quad (2)$$

Notes :

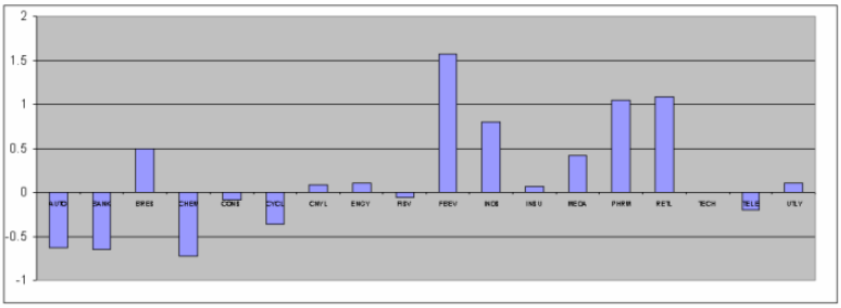
- $w_0 = \sum w_i$ représente la proportion investie dans l'actif sans risque. Il y a emprunt lorsque $w_0 < 0$.
- Π représente le vecteur des rendements en excès du rendement sans risque. Les rendements sont $\Pi' = \Pi + r_{free}$. Le programme d'optisation peut s'écrire de manière équivalente :

$$max_w w^T \Pi' + w_0 r_{free} - \frac{1}{2}\gamma w^T \Sigma w$$

avec $w_0 = 1 - \sum_{i=1,n} w_i$. Dans la suite les rendements sont des rendements en excès $\Pi' - r_{free}$.

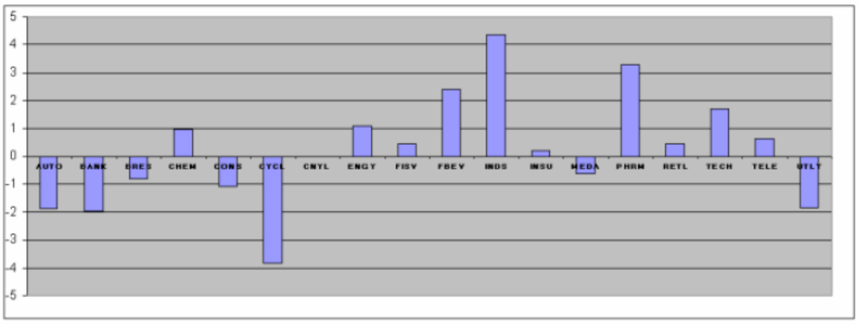
les inconvénients de l'approche de Markowitz

Dans un premier temps, on considère des rendements espérés tous égaux, $\Pi_i = 0.13$ pour tout $i = 1, 18$ On prendra $\gamma = 3$, comme étant l'aversion au risque standard.



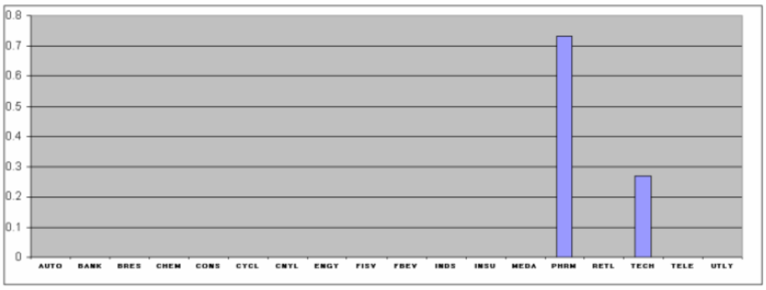
- L'utilisation de rendements espérés arbitraire conduit à un portefeuille peu réaliste ...

Utilisation des rendements historiques



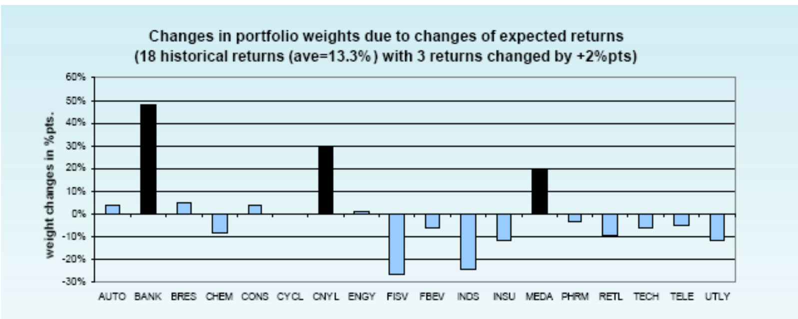
- L'utilisation des rendements historiques conduit à des pondérations extrêmes

- pas de vente à découvert, ni levier



- Il ne reste que quelques actifs ...

- Forte sensibilité aux rendements espérés
  - ☞ de petites modifications des rendements conduisent à des grandes variations des pondérations



Rendements implicites

- **Etant donné un indice que l'on suppose être sur la frontière efficiente (portefeuille de marché), on peut en déduire les rendements attendus implicites**
- **Le portefeuille de marché est proportionnel à**

$$\Sigma w$$

avec

w le vecteur des pondération des capitalisations

Σ la matrice de covariance

$$\Pi = \gamma \Sigma w$$

Π représente les rendements implicites du marché

γ est l'aversion au risque (de l'ordre de 3 à 4)

Estimation de l'aversion au risque

Satchell & Scowcroft and Best & Grauer:

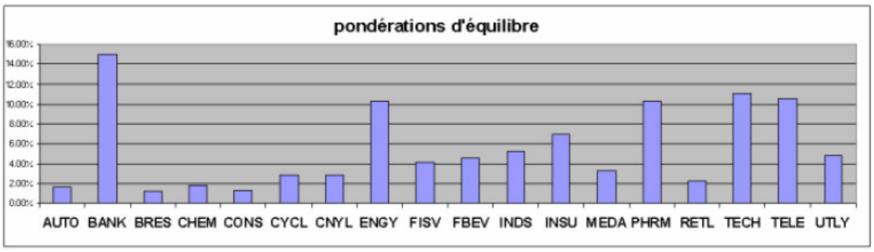
Let $\gamma = (r_M - r_f) / \sigma_M^2$

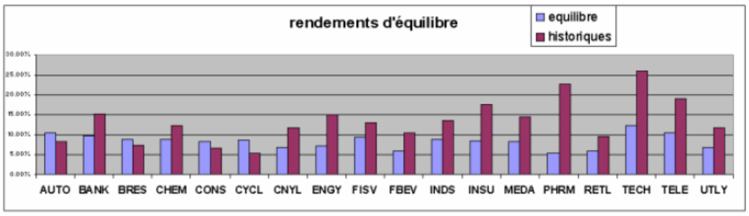
where (a) σ_M = StDev(Market Portfolio) *or* (b) $\sigma_M^2 = w^T \Omega w$, w = market cap.

Zimmermann et al.: (a) σ_M=16.95%p.a. from STOXX-Data (own calculation)
Let γ = 3, which corresponds to a risk premium of 8.6%.
(b) Using $w^T \Omega w$ yields σ_M=16.85%p.a. $\Rightarrow$ risk premium=8.5%.

En pratique, le coefficient d'aversion pour le risque est de l'ordre de 4

$$\Pi = \gamma \Sigma w_{cap}$$





$$\lambda = \frac{E(r) - r_f}{\sigma^2} = \frac{\text{Risk Premium}}{\text{Variance}}$$

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- Assume a Risk Premium of 4%
- The Historical (Long-term) Standard Deviation for the Market Portfolio is 10.84%. The Variance is the Standard Deviatio.n Squared, 1.117%

$$\lambda = \frac{\text{Risk Premium}}{\text{Variance}}$$

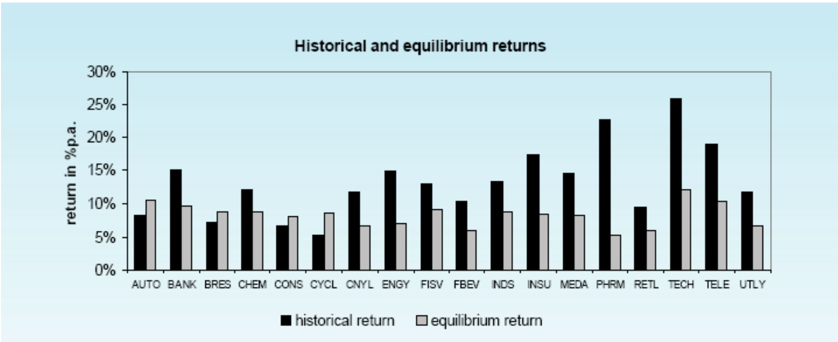
$$\lambda = \frac{4.00\%}{1.117\%}$$

$$\lambda = 3.404$$

Rendements Implicites

Symbol	Historical Return Vector	CAPM Return Vector	Implied Equilibrium Return Vector (Π)
aa	17.30	15.43	13.81
ge	16.91	12.15	13.57
jnj	16.98	9.39	9.75
msft	23.95	14.89	20.41
axp	15.00	15.65	14.94
gm	4.59	13.50	12.83
jpm	5.31	15.89	16.46
pg	7.81	8.04	7.56
ba	-4.18	14.16	11.81
hd	29.38	11.59	12.52
ko	-0.57	10.95	10.92
sbc	10.10	7.76	8.79
c	24.55	16.59	16.97
hon	-0.05	16.89	14.50
mcd	2.74	10.70	10.44
t	-1.24	8.88	10.74
cat	7.97	13.08	10.92
hwp	-4.97	14.92	14.45
mmm	9.03	10.43	8.66
utx	13.39	16.51	15.47
dd	0.44	12.21	10.98
ibm	21.99	13.47	14.66
mo	10.47	7.57	6.86
wmt	30.23	10.94	12.77
dis	-2.59	12.89	12.41
intc	13.59	15.83	18.70
mrk	8.65	8.95	9.22
xom	11.10	8.39	7.88
ek	-17.00	11.08	10.61
ip	1.24	14.80	12.92
Average	9.07	12.45	12.42
Std. Dev.	10.72	2.88	3.22
High	30.23	16.89	20.41
Low	-17.00	7.57	6.86

Rendements implicites vs historiques - Secteurs du STOXX



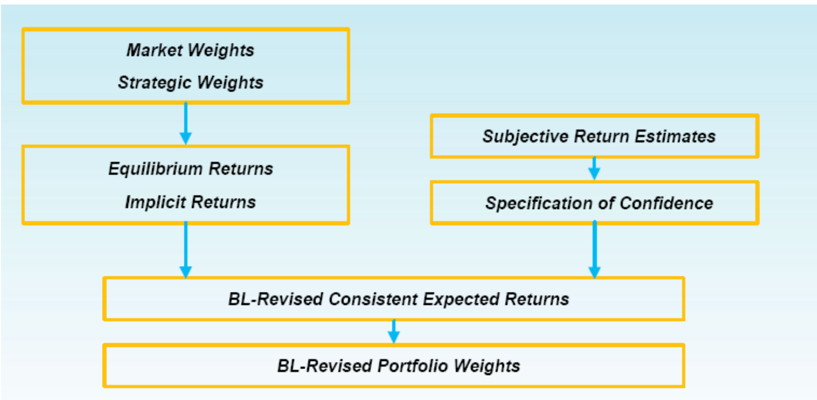
- These implicit returns serve as reference returns for the following investigations („market neutral starting point ")
- Note that equilibrium returns are just calculated; they do not require any estimation procedure.

Rendements implicites vs Historiques - Actions du DJIA

Symbol	Historical Weight	CAPM Weight	Implied Equilibrium Weight	Market Capitalization Weight
aa	223.86%	2.67%	0.88%	0.88%
ge	-65.44%	9.80%	11.62%	11.62%
jnj	-70.08%	6.11%	5.29%	5.29%
msft	3.54%	3.22%	10.41%	10.41%
axp	-15.38%	5.54%	1.39%	1.39%
gm	5.76%	3.44%	0.79%	0.79%
jpm	-213.39%	1.94%	2.09%	2.09%
pg	92.00%	-1.33%	2.99%	2.99%
ba	-111.35%	4.71%	0.90%	0.90%
hd	280.01%	0.11%	3.49%	3.49%
ko	-151.58%	5.70%	3.42%	3.42%
sbc	17.11%	-4.28%	3.84%	3.84%
c	293.90%	5.11%	7.58%	7.58%
hon	15.65%	2.71%	0.80%	0.80%
mcd	-61.68%	1.32%	0.99%	0.99%
t	-86.44%	4.04%	1.87%	1.87%
cat	-70.67%	5.10%	0.52%	0.52%
hwp	-163.02%	6.60%	1.16%	1.16%
mmm	56.84%	4.73%	1.35%	1.35%
utx	-23.80%	4.38%	0.88%	0.88%
dd	-131.99%	1.03%	1.29%	1.29%
ibm	36.92%	5.57%	6.08%	6.08%
mo	136.78%	1.31%	2.90%	2.90%
wmt	21.03%	0.89%	7.49%	7.49%
dis	5.75%	-2.35%	1.23%	1.23%
intc	97.81%	-1.96%	6.16%	6.16%
mrk	144.34%	4.61%	3.90%	3.90%
xom	218.75%	4.10%	7.85%	7.85%
ek	-148.36%	2.04%	0.25%	0.25%
ip	-113.07%	4.76%	0.57%	0.57%
High	293.90%	9.80%	11.62%	11.62%
Low	-213.39%	-4.28%	0.25%	0.25%

Not surprisingly, the Historical Return Vector produces an extreme portfolio. However, despite the similarity between the CAPM Return Vector and the Implied Equilibrium Return Vector (***Π***), the vectors produce two rather distinct portfolios (the correlation coefficient (ρ) is 18%). The CAPM-based portfolio contains four short positions and almost all of the weights are significantly different than the benchmark market capitalization weighted portfolio. As one would expect (since the process of extracting the Implied Equilibrium returns given the market capitalization weights was reversed), the Implied Equilibrium Return Vector (***Π***) leads back to the market capitalization weighted portfolio. In the absence of views that differ from the Implied Equilibrium return, investors should hold the market portfolio. The Implied Equilibrium Return Vector (***Π***) is the market-neutral starting point for the Black-Litterman Model.

L'approche de Black Litterman



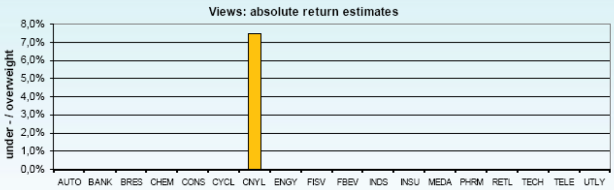
Permet de combiner les relations d'équilibre à long terme avec des anticipations estimées à court terme

Black-Litterman Approach

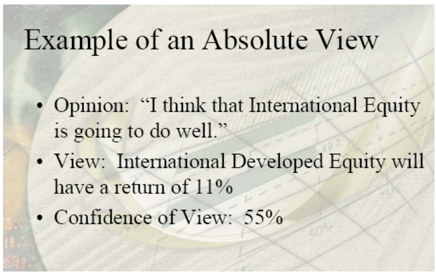
Absolute Views

- An **absolute View** can be formulated as follows: „The sektor of non-cyclical goods will perform better than stated by the equilibrium return of 6.66%. Our target return is 7.5% with 90% of confidence for an interval of ±1.5%":.

$$1 \cdot E(R_{CNYL}) = 7.5\% + (0.91\%)^2$$



abus de notation (0.91%)^2 désignant une erreur normale d'écart type 0.91%



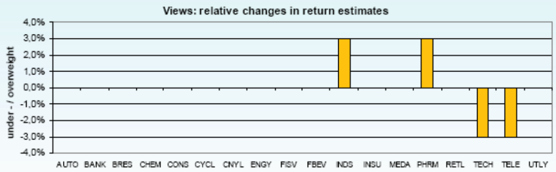
Black-Litterman Approach

Relative Views

- A **relative View** can be formulated as follows: „The sectors Pharmacy and Industry will outperform Telecom and Technology by 3% with a confidence of 90% for ±1%":

$$\left[w_{PHRM}\cdot E(R_{PHRM})+w_{INDU}\cdot E(R_{INDU})\right]$$
$$-\left[w_{TELE}\cdot E(R_{TELE})+w_{TECH}\cdot E(R_{TECH})\right]=3\%+(0.61\%)^2$$

- Thus: A *long*-portfolio with outperformers, a *short*-portfolio with underperformers.



If all views are relative views then only the stocks which are part of those views will have their weights affected. If there are absolute views then all the stocks will have change in returns and weights, since each individual return is linked to the other returns via the covariance matrix of returns.

Example of a Relative View

- Opinion: "I believe that Growth is going to outperform Value."
- View: US Large Growth and US Small Growth will outperform US Large Value and US Small Value by 3%.
- Confidence of View: 80%

Exprimer les vues - formalisme

- Relative and absolute views form a system of linear equations as a constraint to the optimization problem:

$$P \cdot E(R) = V + e$$

where (k = #Views and n = #Assets, with k ≤ n):

E(R)	= n×1 vector of expected asset returns, unknown
P	= k×n matrix, representing the Views
V	= k×1 vector, absolute / relative return expectations (i.e., levels or over-/underperforming)
e	= k×1 vector of squared StDev's

Ω = Diagonal covariance matrix of error terms in expressed views

Combined Views

- Combining the aforementioned Views using $P \cdot E = V + e$, we get:

$$P \cdot \begin{pmatrix} E(R_{AUTO}) \\ \vdots \\ E(R_{UTLY}) \end{pmatrix} = \begin{pmatrix} 3\% \\ 7.5\% \end{pmatrix} + \begin{pmatrix} (0.61\%)^2 \\ (0.91\%)^2 \end{pmatrix}$$

- with

$$P = \begin{pmatrix} View\,1, rel. \\ View\,2, abs. \end{pmatrix} = \begin{pmatrix} 0 \cdots 0 & 0.34 & 0 \cdots 0 & 0.66 & 0 & -0.51 & -0.49 & 0 \\ 0 & \cdots & 0 & 1 & 0 & \cdots & \cdots & \cdots & 0 \end{pmatrix}$$

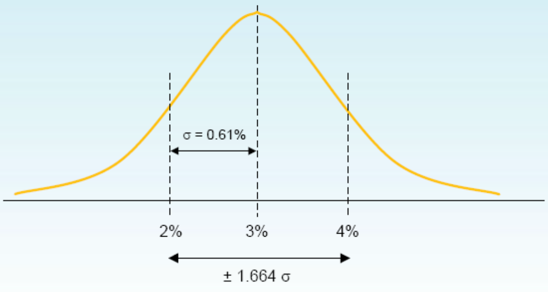
long positions

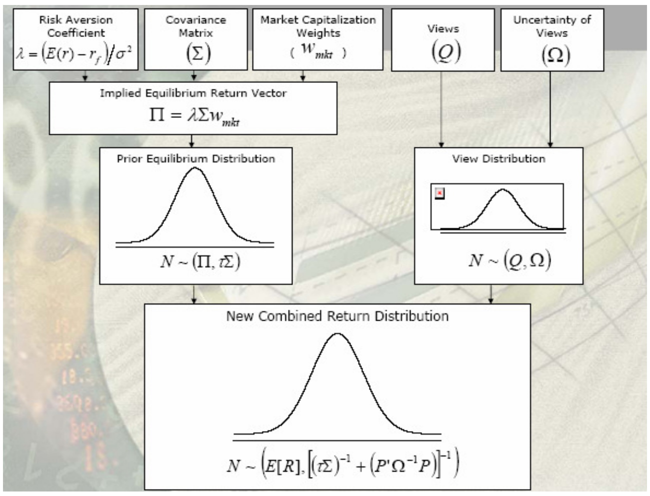
short positions

asset selection for absolute estimate

Remark: Technical note on „confidence"

- Comment on *e* (outlined for the relative return estimate): The fact that the amount of relative outperformance of 3% ±1% is assigned a 90% probability is interpreted within a normal distribution, with ***mean*** **=** μ **= 3%** and ***variance*** **=** ***VAR*** **=** σ^2 **= (0.61%)**$^2 \equiv$ ***e***.





Approche bayesienne - rappel

A=rendements anticipés (les vues)

B=rendements d'équilibre (Π)

Prob(A,B)

=Prob(A|B)*Prob(B)

=Prob(B|A)*Prob(A)

Prob(A|B) = Prob(B|A)*Prob(A) / Prob(B)

Black-Litterman - une approche baysienne

$$\Pr(E(r)|\Pi) = \frac{\Pr(\Pi|E(r))\Pr(E(r))}{\Pr(\Pi)}$$

$$PE(r) = V + e$$
$$PE(r) \sim N(V, \Omega)$$

$$\Pi|E(r) \sim N(E(r), \tau\Sigma)$$

Ω = matrice diagonale k*k
P = matrice des vues

avec Σ la matrice de variance covariance
et τ une pondération que l'on se donne sur les rendements historiques qui dépend de la confiance
de l'investisseur dans les rendements historiques implicites versus ses propres anticipations
τ petit ➔ erreur sur Π petite
➔ confiance plus grande dans les rendements implicites
➔ moins de confiance dans les vues

La meilleure estimation des rendements est obtenue en minimisant la variance autour des rendements d'équilibre

$$\min_{E[R]} \left[E[R]-\Pi\right]^T (\tau\Sigma)^{-1} \left[E[R]-\Pi\right]$$

sous les contraintes

$$P.E[R] = \begin{cases} V & \text{avec des vues certaines} \\ V+e & \text{avec des vues incertaines} \end{cases}$$

La solution est .....

a faire en exercice ...

La formule de Black Litterman

$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1} \left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}V\right]$$

k = number of views
n = number of assets
E[R] =New (posterior) Combined Return Vector (*n x 1* column vector)
τ = Scalar τ = 0.3 "plausible" (used for numerical evaluations throughout).
Σ = Covariance Matrix of Returns (*n x n* matrix)
P = Identifies the assets involved in the views (*k x n* matrix or *1 x n* row vector in the special case of 1 view)
Ω = Diagonal covariance matrix of error terms in expressed views representing the level of confidence in each view (*k x k* matrix)
Π = Implied Equilibrium Return Vector (*n x 1* column vector)
V = View Vector (*k x 1* column vector)

Exemple simple

$$E[R]=\left[(\tau\Sigma)^{-1}+P^{T}\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi+P^{T}\Omega^{-1}V\right]$$

Prenons le cas d'un seul actif

$$\Sigma=(\sigma^{2})$$
$$\Pi=(\pi)$$
$$P=(1)$$
$$V=(v)$$
$$\Omega=(\omega^{2})$$

$$E[R]=\frac{\dfrac{\pi}{\tau\sigma^{2}}+\dfrac{q}{\omega^{2}}}{\dfrac{1}{\tau\sigma^{2}}+\dfrac{1}{\omega^{2}}}$$

$\sigma=20\%$

$\pi=30\%$

$v=35\%$

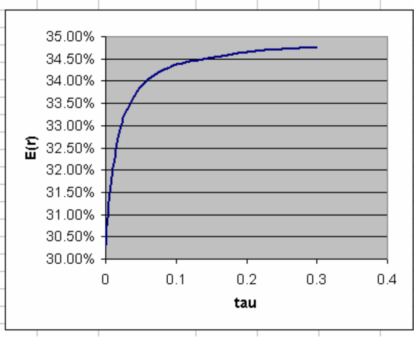
$\omega=3\%$

$\tau=0.3$

$E[R]=34.77\%$

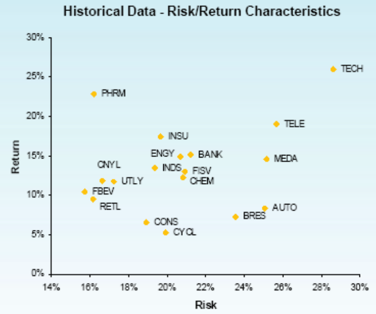
Exemple - simple

A	B	C	D	E	F
rendement implciite					
sigma	σ	25%	sigma2	0.0625	
pi	π	30%			
	rf	5%			
	γ	4	(pi-rf)/sigma^2		
vue					
	v	35%	plus ou moins		5%
	ω	3.0%	avec une confiance de		90%
τ	E(r)				
0.001	30.32%				
0.002	30.61%				
0.005	31.29%				
0.01	32.05%				
0.02	32.90%				
0.03	33.37%				
0.04	33.67%				
0.05	33.88%				
0.06	34.03%				
0.07	34.14%				
0.08	34.24%				
0.09	34.31%				
0.1	34.37%				
0.2	34.66%				
0.3	34.77%				



Rendement mensuels - secteurs STOXX

Asset	hist.Return	hist.Volatility	MarketCap
total:	average:		total:
18	16,22%		100,01%
AUTO	8,32%	25,09%	1,65%
BANK	15,14%	21,21%	15,04%
BRES	7,31%	23,56%	1,22%
CHEM	12,25%	20,81%	1,80%
CONS	6,56%	18,92%	1,26%
CYCL	5,24%	19,94%	2,85%
CNYL	11,80%	16,66%	2,90%
ENGY	14,92%	20,72%	10,30%
FISV	13,01%	20,91%	4,12%
FBEV	10,47%	15,72%	4,59%
INDS	13,45%	19,35%	5,19%
INSU	17,43%	19,68%	6,89%
MEDA	14,63%	25,17%	3,27%
PHRM	22,83%	16,20%	10,24%
RETL	9,49%	16,16%	2,27%
TECH	25,95%	28,60%	11,03%
TELE	18,99%	25,69%	10,56%
UTLY	11,77%	17,25%	4,83%

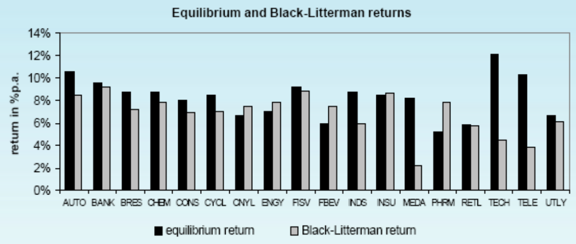


Correlation matrix of Dow Jones STOXX sectors

- Calculation based on monthly returns

	AUTO	BANK	BRES	CHEM	CONS	CYCL	CNYL	ENGY	FISV	FBEV	INDS	INSU	MEDA	PHRM	RETL	TECH	TELE	UTLY
AUTO	100%	74%	73%	83%	78%	75%	73%	55%	73%	71%	79%	72%	46%	43%	68%	69%	65%	64%
BANK	74%	100%	63%	73%	74%	75%	71%	59%	92%	75%	75%	87%	39%	63%	62%	67%	59%	64%
BRES	73%	63%	100%	83%	81%	78%	60%	66%	69%	56%	78%	56%	44%	31%	59%	62%	52%	41%
CHEM	83%	73%	83%	100%	85%	82%	72%	67%	72%	74%	82%	70%	51%	45%	69%	65%	57%	57%
CONS	78%	74%	81%	85%	100%	90%	72%	66%	75%	76%	89%	64%	54%	39%	67%	66%	63%	64%
CYCL	75%	75%	78%	82%	90%	100%	67%	64%	79%	70%	87%	63%	58%	43%	67%	70%	63%	56%
CNYL	73%	71%	60%	72%	72%	67%	100%	55%	69%	75%	69%	74%	41%	58%	73%	53%	59%	71%
ENGY	55%	59%	66%	67%	66%	64%	55%	100%	59%	59%	59%	54%	28%	43%	55%	43%	32%	46%
FISV	73%	92%	69%	72%	75%	79%	69%	59%	100%	75%	73%	85%	39%	61%	58%	64%	57%	58%
FBEV	71%	75%	56%	74%	76%	70%	75%	59%	75%	100%	62%	74%	27%	63%	61%	40%	41%	66%
INDS	79%	75%	78%	82%	89%	87%	69%	59%	73%	62%	100%	65%	72%	38%	68%	82%	77%	67%
INSU	72%	87%	56%	70%	64%	63%	74%	54%	85%	74%	65%	100%	36%	67%	61%	60%	56%	68%
MEDA	46%	39%	44%	51%	54%	58%	41%	28%	39%	27%	72%	36%	100%	21%	42%	75%	77%	57%
PHRM	43%	63%	31%	45%	39%	43%	58%	43%	61%	63%	38%	67%	21%	100%	43%	35%	37%	58%
RETL	68%	62%	59%	69%	67%	67%	73%	55%	58%	61%	68%	61%	42%	43%	100%	52%	53%	57%
TECH	69%	67%	62%	65%	66%	70%	53%	43%	64%	40%	82%	60%	75%	35%	52%	100%	81%	55%
TELE	65%	59%	52%	57%	63%	63%	59%	32%	57%	41%	77%	56%	77%	37%	53%	81%	100%	70%
UTLY	64%	64%	41%	57%	64%	56%	71%	46%	58%	66%	67%	68%	57%	58%	57%	55%	70%	100%

- Covariance matrix via $\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$.



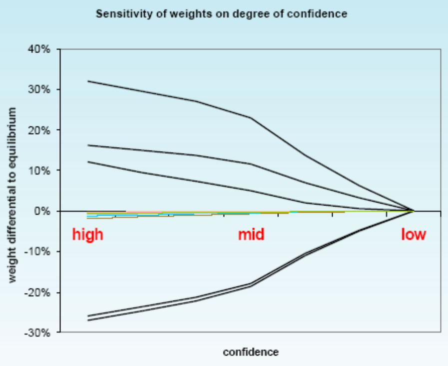
Returns s.t. Views:

- *Short* portfolio Return expectations significantly lowered for TECH und TELE.
- *Long* portfolio Return expectation higher in PHRM but lower in INDS (ok, because the *relative* View „INDS better than TELE und TECH" remains intact !)
- For CNYL, expected return shifts from 6.66% to 7.48% (90% confidence in estimate of 7,5%).
- Via correlations (e.g.) MEDA (75% to TECH, 77% to TELE) has significantly lower return.

Influence of degree of confidence on BL-returns and BL-weights, II

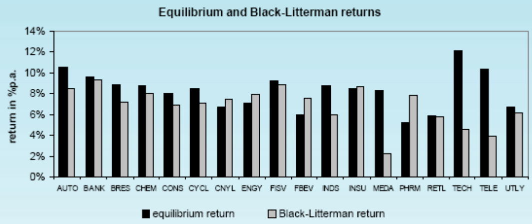
Observations:

- For low degrees of confidence the BL-weights converge to the weights in equilibrium (=market cap's).
- Weights approach equilibrium values from either underweighting (*short*) or overweighting (*long*) path.
- Significant weight changes for the assets with View.

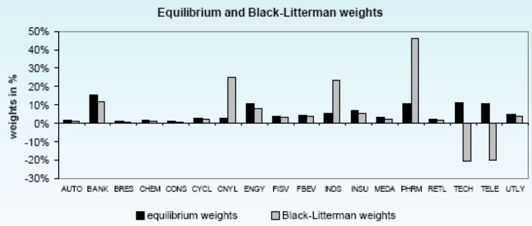


Black-Litterman Approach

Strong confidence

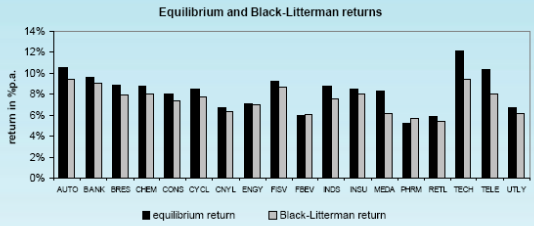


- Large changes in weights due to the "strong views" (change up to 35%pt)

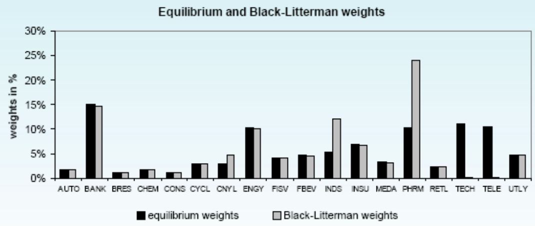


Black-Litterman Approach

Mid confidence

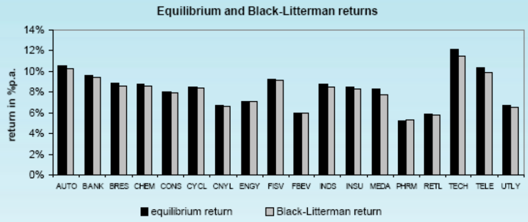


- Moderate changes in weights (change up to 14%pt)

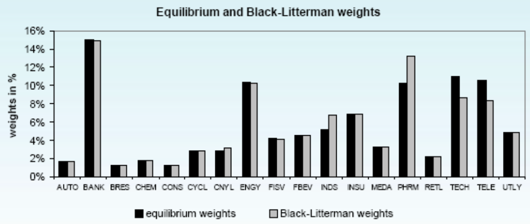


Black-Litterman Approach

Poor confidence



- Weights close to equilibrium weights (change up to 3 %pt)



Black-Litterman Approach - Conclusion I

Traditional vs BL approach

	simple approach	Black-Litterman
Return estimates:	for each asset assumed as certain absolute returns subjective	for selected assets degree of confidence relative or absolute estimates consistent
Reference return	none	equilibrium returns

- Black F. und Litterman R.: *Global Portfolio Optimization*, Fin.Analysts Journal, Sep.1992, p.28-43
- Black F. und Litterman R.: *Asset Allocation: Combining investor views with market equilibrium*, Goldman-Sachs, Fixed Income Research, Sep.1990
- Zimmermann H., Drobetz W. und Oertmann P.: *Global Asset Allocation: New methods and applications, publ. by. Wiley & Sons*, Nov.2002

Annexe - calculs

One minimizes the objective:

$$\min_{\mu}(\mu-\Pi)'(\tau\Sigma)^{-1}(\mu-\Pi)$$

under the constraint $P\mu = Q$, and obtain:

$$\mu' = \Pi + \Sigma P'(P\Sigma P')^{-1}(Q - P\Pi')$$

- We introduce a vector of k Lagrange multipliers λ, and the problem becomes:

$$\min_{\mu}(\mu-\Pi)'\tau\Sigma^{-1}(\mu-\Pi) - 2\lambda'(P\mu - Q)$$

- Differentiating with respect to μ, we get:

$$\mu'\Sigma^{-1} - \Pi'\Sigma^{-1} + \lambda' P = 0$$

and so:

$$\mu = \Pi - \Sigma P'\lambda$$

- Plugging this result into the constraint, we solve for λ:

$$\lambda = (P\Sigma P')^{-1}(P\Pi - Q)$$

and hence:

$$\mu = \Pi + \Sigma P'(P\Sigma P')^{-1}(Q - P\Pi)$$

- When views are associated with some error matrix ε, then the optimization objective is:

$$\min_{\mu}(\mu-\Pi)'(\tau\Sigma)^{-1}(\mu-\Pi) + (P\mu - Q)'\Omega^{-1}(P\mu - Q)$$

- The parameter τ measures the relative confidence in views vs. market equilibrium. A smaller τ indicates more confidence in equilibrium.
- The solution to the above problem is:

$$\mu = ((\tau\Sigma)^{-1} + P'\Omega^{-1}P)^{-1}((\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q)$$

which is known as the **Master Formula**.

- Differentiating the objective, we obtain the equation:

$$((\tau\Sigma)^{-1} + P'\Omega^{-1}P)\mu - ((\tau\Sigma)^{-1}\Pi - P'\Omega^{-1}Q) = 0$$

- and therefore:

$$\mu = ((\tau\Sigma)^{-1} + P'\Omega^{-1}P)^{-1}((\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q)$$

Cap	ω%	σ	Correl	AGL	AMS	BIL	IMP	RCH	SAP	SOL
198,985,914,253	36.66%	31.24%	AGL	100%	56%	78%	44%	49%	55%	42%
23,342,178,717	4.30%	37.80%	AMS	56%	100%	49%	61%	38%	46%	35%
126,862,755,903	23.37%	32.08%	BIL	78%	49%	100%	39%	47%	50%	43%
35,561,190,126	6.55%	38.25%	IMP	44%	61%	39%	100%	38%	37%	32%
81,683,000,000	15.05%	34.96%	RCH	49%	38%	47%	38%	100%	41%	37%
19,197,472,928	3.54%	30.32%	SAP	55%	46%	50%	37%	41%	100%	50%
57,140,413,208	10.53%	30.78%	SOL	42%	35%	43%	32%	37%	50%	100%
542,782,925,135										

K 1.4

Σ	AGL	AMS	BIL	IMP	RCH	SAP	SOL	$\Pi = 1.4\,\Sigma\omega$	r	$\Pi - r$
AGL	0.098	0.066	0.078	0.053	0.053	0.052	0.041	10.44%	7.5%	2.9%
AMS	0.066	0.143	0.059	0.088	0.050	0.052	0.041	8.92%	7.5%	1.4%
BIL	0.078	0.059	0.103	0.048	0.053	0.048	0.042	10.16%	7.5%	2.7%
IMP	0.053	0.088	0.048	0.146	0.050	0.043	0.037	8.00%	7.5%	0.5%
RCH	0.053	0.050	0.053	0.050	0.122	0.043	0.039	8.60%	7.5%	1.1%
SAP	0.052	0.052	0.048	0.043	0.043	0.092	0.046	6.99%	7.5%	-0.5%
SOL	0.041	0.041	0.042	0.037	0.039	0.046	0.095	6.51%	7.5%	-1.0%

Σ^{-1}	AGL	AMS	BIL	IMP	RCH	SAP	SOL	$\Sigma^{-1}(\Pi - r)$	ω%
AGL	31.7	-4.4	-17.9	-0.8	-2.3	-4.2	-0.5	0.391197716	498%
AMS	-4.4	13.7	-0.6	-5.6	-0.3	-2.0	-0.4	0.031805027	41%
BIL	-17.9	-0.6	26.3	-0.2	-2.1	-1.2	-2.2	0.169661903	216%
IMP	-0.8	-5.6	-0.2	11.4	-1.5	-0.6	-0.7	-0.058069774	-74%
RCH	-2.3	-0.3	-2.1	-1.5	11.9	-1.5	-1.5	0.017175307	22%
SAP	-4.2	-2.0	-1.2	-0.6	-1.5	18.5	-5.0	-0.249816103	-318%
SOL	-0.5	-0.4	-2.2	-0.7	-1.5	-5.0	15.3	-0.223473206	-285%

λ 0.07848087

View 1 SOL will have a return of 12% (confidence 60%)
View 2 AGL will outperfrom BIL by 2% (confidence 50%)
View 3 AGL, AMS, BIL, IMP will outperform SAP, SOL by 5% (confidence 70%)

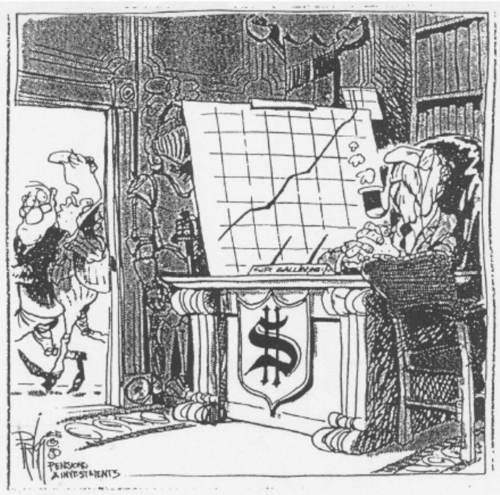
V	Conf	P						
12%	60%	0	0	0	0	0	0	1
2%	50%	1	0	-1	0	0	0	0
5%	70%	0.52	0.06	0.33	0.09	0.00	-0.25	-0.75

c 6% τ 85%

D		
0.11	0	0
0	0.13	0
0	0	0.09

$(\tau\Sigma)^{-1}$	AGL	AMS	BIL	IMP	RCH	SAP	SOL
AGL	37.4	-5.2	-21.1	-0.9	-2.7	-5.0	-0.6
AMS	-5.2	16.1	-0.7	-6.6	-0.4	-2.3	-0.5
BIL	-21.1	-0.7	31.0	-0.3	-2.5	-1.5	-2.5
IMP	-0.9	-6.6	-0.3	13.5	-1.8	-0.7	-0.9
RCH	-2.7	-0.4	-2.5	-1.8	14.0	-1.8	-1.8
SAP	-5.0	-2.3	-1.5	-0.7	-1.8	21.8	-5.9
SOL	-0.6	-0.5	-2.5	-0.9	-1.8	-5.9	18.0

$P'D^{-1}P$	AGL	AMS	BIL	IMP	RCH	SAP	SOL
AGL	10.596	0.339	-5.87	0.52	0	-1.404	-4.18
AMS	0.3387	0.04	0.216	0.06	0	-0.165	-0.49
BIL	-5.869	0.216	8.883	0.33	0	-0.895	-2.66
IMP	0.5159	0.061	0.329	0.09	0	-0.251	-0.75
RCH	0	0	0	0	0	0	0
SAP	-1.404	-0.16	-0.89	-0.25	0	0.6826	2.03
SOL	-4.178	-0.49	-2.66	-0.75	0	2.0317	15.3
	$(\tau\Sigma)^{-1}\Pi$		$P'D^{-1}V$				
AGL	60%		43%				
AMS	7%		3%				
BIL	39%		2%				
IMP	11%		5%				
RCH	25%		0%				
SAP	6%		-14%				
SOL	17%		71%				
μ_v			$\mu_v - r$		ω	$\omega\%$	
AGL	12.36%		4.9%		78%	157%	
AMS	10.51%		3.0%		6%	12%	
BIL	11.70%		4.2%		19%	37%	
IMP	9.42%		1.9%		-6%	-12%	
RCH	9.88%		2.4%		2%	5%	
SAP	8.18%		0.7%		-40%	-81%	
SOL 31	8.49%		1.0%		-9%	-17%	
					50%		



"He uses MPT—Medieval Portfolio Theory."