(σ, δ)-CODES OVER RINGS.

M'Hammed BOULAGOUAZ[1] and André LEROY[2].
[1]Mathematics department, Faculty of Science and technics
Sidi Mohamed ben Abdellah University.
Fès, Maroc.
[2]Mathematics department, Faculty of Science. Artois University.
Lens, France.

Keywords : Sigma-derivation , skew polynomial ring, pseudo-linear transformations, θ-codes, Wedderburn polynomial.

Abstract: In this talk we will introduce the notion of (σ, δ)-codes generalizing the θ-codes as introduced by D. Boucher, F. Ulmer, W. Geiselmann, show how to attach to such code an ideal in some quotient of Ore extension and give the control map of a (σ, δ)-code. We introduce also a definition of (σ, δ) g-polynomial code and we give its generating matrix. A key role will be played by the pseudo-linear transformations.

1 Definitions

1) Let f be a monic invariant polynomial in $R = Ore$.
A (σ, δ)-polynomial code $C(t)$ is a left principal ideal I of R/Rf.
A (σ, δ)-word code C in A^n is the image of a (σ, δ)-polynomial code via the map φ_f described in the above proposition.
2) Let $f \in A[t; \sigma, \delta]$ be a monic polynomial of degree $\mathbf{n}$ and C_f its companion matrix. Then the map

$$T_f : A^n \longrightarrow A^n.$$

$$(a_1, ..., a_n) \longrightarrow T_f(a_1, ..., a_n) := v_{(\sigma(a_1)...\sigma(a_n))C_f} + (\delta(a_1), ..., \delta(a_n)).$$

is a pseudo-linear transformation called the pseudo-linear transformation associated to f.
3) If V_R is a right R-module the action of t on V gives rise to a map

$$T \in End(V, +) \text{ such that: } T(v\alpha) = T(v)\sigma^{-1}(\alpha) + v\delta'(\alpha).$$

For $f = \sum_{i=0}^{n} t^i a_i \in R = Ore$, the right action of t on R/fR is denoted by ${}_fT$.

2 Mains results

Theorem 1 *With the above notations we have:*

- *(a) The code C_g corresponding to $C_g(t) := Rg/Rf$ is of dimension $n-r$ where $\deg(f) = n$ and $\deg(g) = r$.*
- *(b) If $v := (a_0, a_1, \dots, a_{n-1}) \in C_g$ then $T_f(v) \in C_g$.*
- *(c) The rows of the generic matrix of C_g are given by $(T_f)^k(g_0, g_1, \dots, g_r)$ for $1 \le k \le n-r$.*

Theorem 2 *Let $f = gh \in R$ be as above. Then, writing $h = \sum_{i=0}^{l} t^i h_i$. Then we have :*

1) $c = \sum_{i=0}^{l} t^i p_i \in C_g \Leftrightarrow, h({}_fT)(c) = 0$
2) $C_g = \ker h({}_fT) = \ker(\sum_{i=0}^{l}({}_fT^i)h_i)$.

Theorem 3 *Let $f = gh \in R$ be as above. Then, writing $h = \sum_{i=0}^{l} t^i h_i$. Then we have :*

1) $c = \sum_{i=0}^{l} t^i p_i \in C_g \Leftrightarrow, h({}_fT)(c) = 0$
2) $C_g = \ker h({}_fT) = \ker(\sum_{i=0}^{l}({}_fT^i)h_i)$.

Proposition 1 *Let $g \in R = Ore$ be a Wedderburn polynomial and C the corresponding. Then*

- *(a) The generating matrix of a (σ, δ) g-polynomial code C is given by the coefficients of $g(t), tg(t), \dots, t^{n-r-1}g(t)$.*
- *(b) $(c_0, c_1, \dots, c_{n-1}) \in C$ if and only if $(c_0, c_1, \dots, c_{n-1})V_{n\times r}(a_1, \dots, a_r) = (0, \dots, 0)$, where $V_{n\times r}(a_1, \dots, a_r)$ denotes the generalized vandermonde matrix based on $a_1, \dots, a_r$*

References

[1] Boucher, D.Geiselmann, W. and Ulmer, F.*Skew codes*, Applicable Algebra in Engineering, Communication and computing, VOl. 18,pp 379-389,(2007).

[2] A.Leroy , *Pseudo linear transformation and evaluation in Ore extension*, Bull. Belg. Math. Soc.2 (1995), 321-345.

[3] T.Y. Lam and A.Leroy , *Wedderburn polynomial over division rings, I,*, Journal of Pure and Applied Algebra, 186(2004), 43-76.

[4] T.Y. Lam , A.Leroy and A. Osturk: *Wedderburn polynomial over division rings, II,* Proceedeing of conferences held in Chennai ta the Ramanujan Institue (Indes) Contemporary mathematics (456) 2008, pp.73-98.